

Practicalities of Simulation-Based Inference in Cosmology and Astrophysics

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Simulation-Based Inference (SBI): Contents

1. SBI: Motivation & Background
2. Forward Models
3. Types of Simulation-Based Inference (ABC, NDE & SNDE)
4. Data Compression
5. Model Testing & Misspecification with SBI
6. Diagnosing & Testing SBI
7. Conclusion & Outlooks

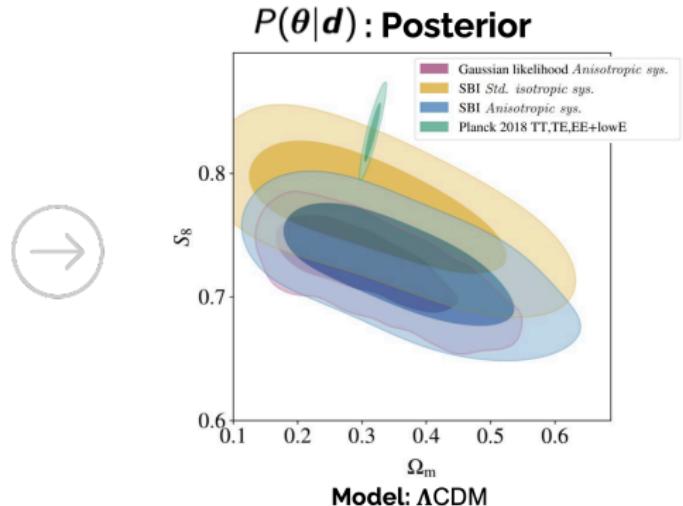
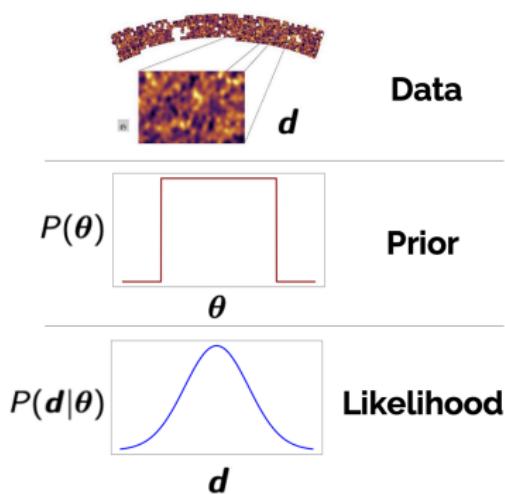
SBI: Motivation & Background

Bayesian Inference

Probability of a statement being true given an update to a prior belief and a specific model.

⇒ Bayes' Theorem:

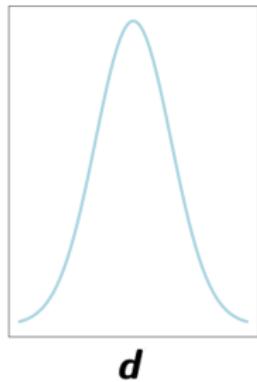
$$P(\theta | d) = \frac{P(d | \theta) P(\theta)}{P(d)} \quad (1)$$



Modelling Likelihood Distributions

Given a
model!

$P(d|\theta)$



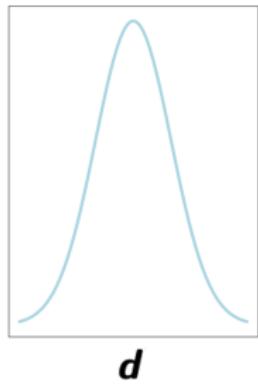
Analytic

e.g. $P(d) \propto \exp\left(-\frac{(d-\mu)^2}{2\sigma^2}\right)$

Modelling Likelihood Distributions

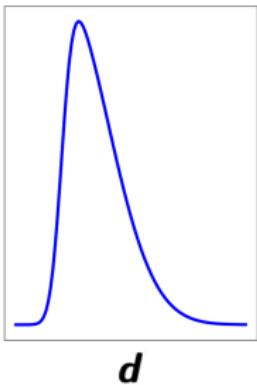
Given a model!

$$P(d|\theta)$$



Analytic

$$\text{e.g. } P(d) \propto \exp\left(-\frac{(d-\mu)^2}{2\sigma^2}\right)$$



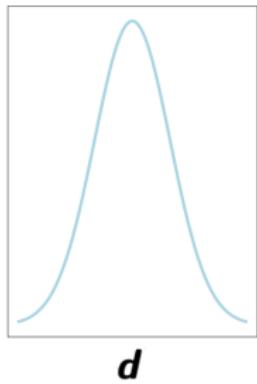
Biased

e.g. Systematics

Modelling Likelihood Distributions

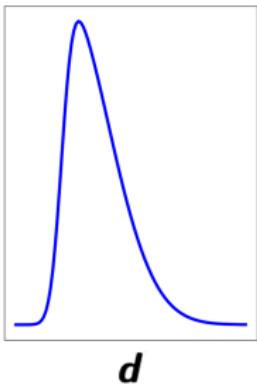
Given a model!

$$P(d|\theta)$$



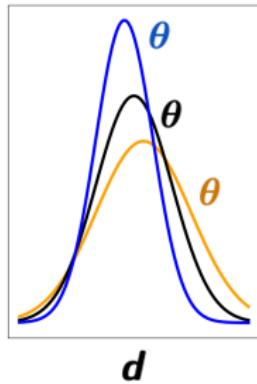
Analytic

$$\text{e.g. } P(d) \propto \exp\left(-\frac{(d-\mu)^2}{2\sigma^2}\right)$$



Biased

e.g. Systematics



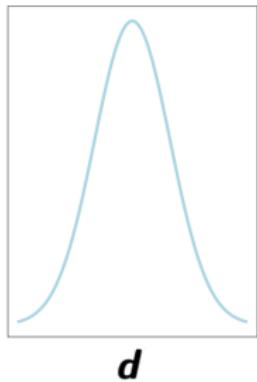
Signal-dependent uncertainty

e.g. Non-random noise

Modelling Likelihood Distributions

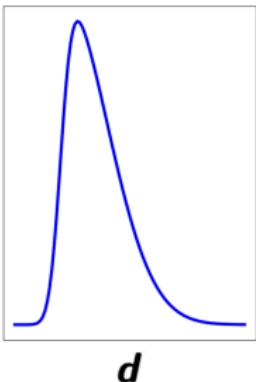
Given a model!

$$P(d|\theta)$$



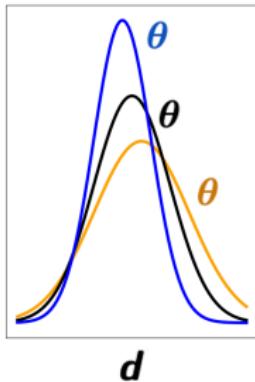
Analytic

$$\text{e.g. } P(d) \propto \exp\left(-\frac{(d-\mu)^2}{2\sigma^2}\right)$$



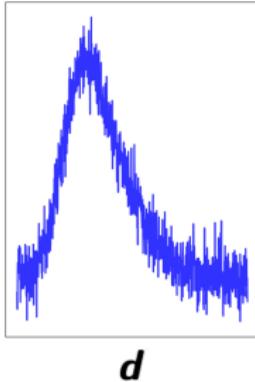
Biased

e.g. Systematics



Signal-dependent uncertainty

e.g. Non-random noise



Intractable

e.g. Many sources of uncertainty

Zooming Out: the Joint Probability

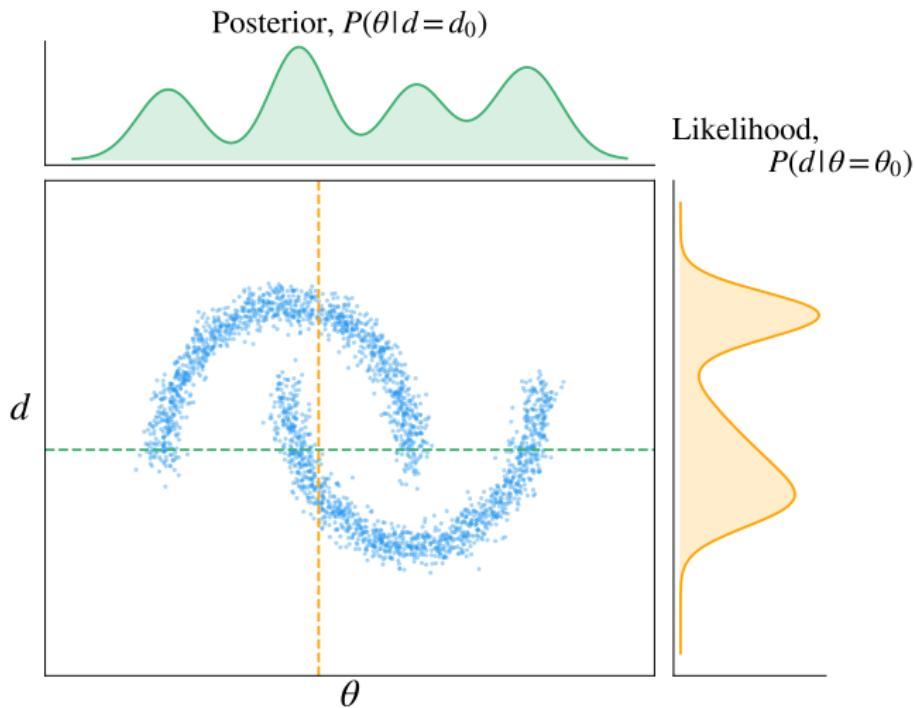
$$P(\boldsymbol{\theta} \mid \mathbf{d}) = \frac{P(\mathbf{d} \mid \boldsymbol{\theta}) P(\boldsymbol{\theta})}{P(\mathbf{d})} \propto P(\boldsymbol{\theta}, \mathbf{d}) P(\boldsymbol{\theta}) \quad (2)$$

Joint probability: $P(\boldsymbol{\theta}, \mathbf{d} \mid \text{Model})$

Simulator: $\mathbf{d}_i \sim P(\mathbf{d} \mid \boldsymbol{\theta}, \text{Model})$

Zooming Out: the Joint Probability

Joint probability: $P(\theta, d \mid \text{Model})$



Generalised Bayesian Inference (GBI)

Standard Bayesian inference: Derived from Boolean logic when introducing uncertainty

→ Joint probability, $P(\theta, d)$ defines everything for a given model.

Generalised Bayesian Inference: Considers the case of imperfect models → Based on Decision theory Berger (2013)

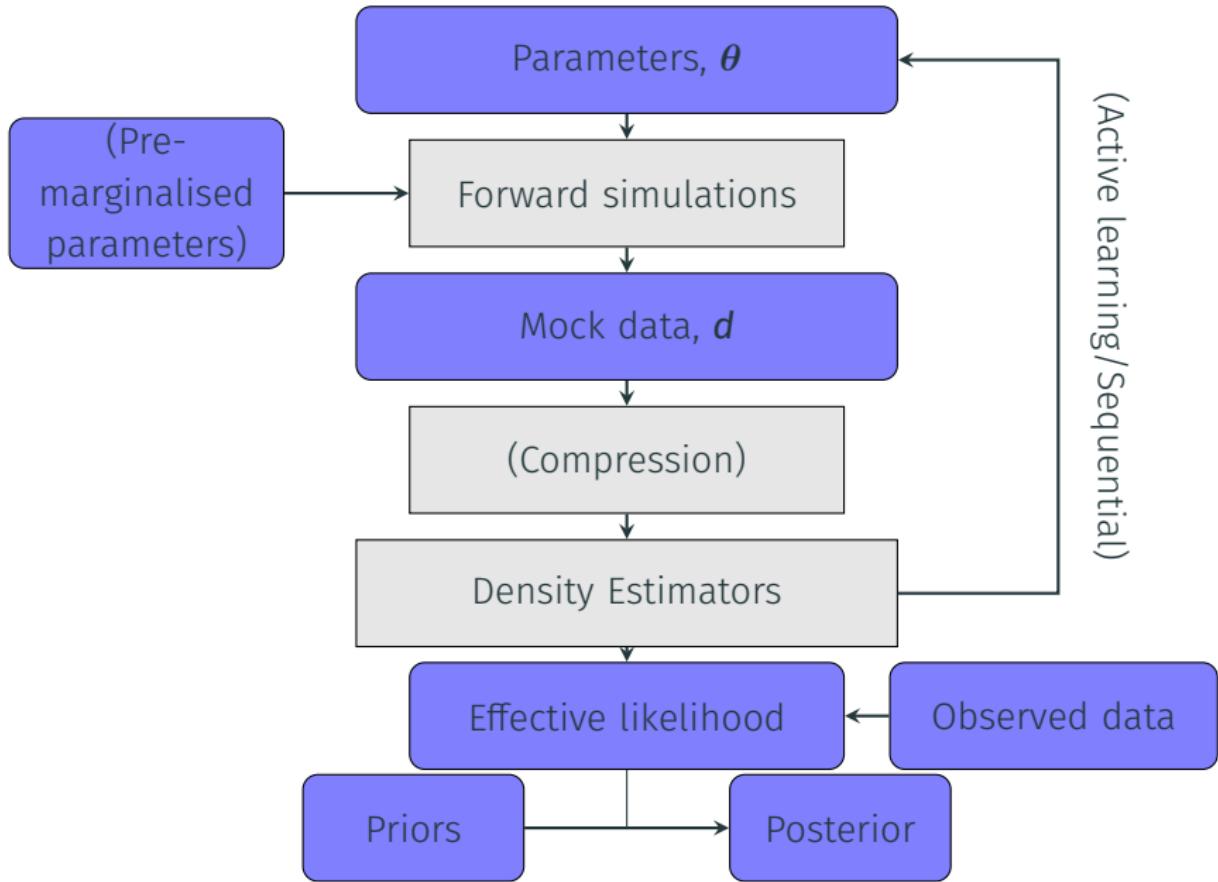
→ Finds optimal belief given an **imperfect model** and a specific goal

→ Model is characterised through a **Loss function**, $L(\theta, d)$, Bissiri et al. (2013):

$$\underbrace{P_G(\theta | d)}_{\text{Generalised Posterior}} \propto \underbrace{\exp(-\eta L(\theta, d))}_{\text{Generalised Likelihood}} \times \underbrace{P(\theta)}_{\text{Prior}}, \quad (3)$$

where η quantifies the degree of match/mismatch between the data and the model ($\eta = 1$ corresponds to standard Bayes' theorem).
 $L(\theta, d) = -\ln P(d | \theta)$ returns Bayes' theorem.

Simulation-Based Inference



Simulation-Based Inference (SBI) - FAQs

- How do I pick a forward model?
- What type of SBI suits my problem?
- My posterior is robust and reliable, but is my model accurate?
- How do I get the most out of the results of my SBI?

Forward Models

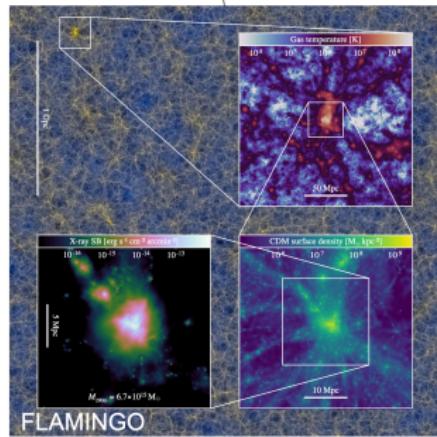
What can we get away with?

1. In what are we interested? - Parameters, model comparison...
2. What is our measurement? - Summary statistics, images...
3. To which systematics will the measurement be sensitive?
4. What type of SBI suits our problem? - NPE, NLE, NRE, active learning...

Forward Models: Capturing the Signal

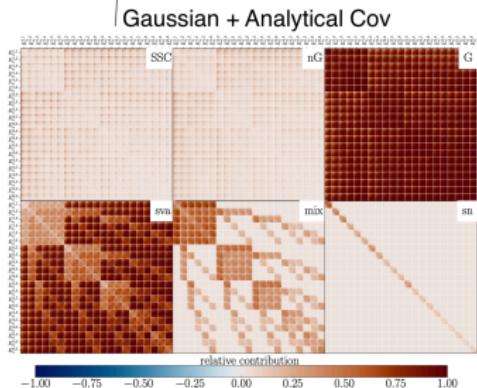
Complexity

$$\begin{aligned} C_{\epsilon\epsilon}^{(pq)}(\ell) &= C_{\text{gg}}^{(pq)}(\ell) + C_{\text{gl}}^{(pq)}(\ell) + C_{\text{lg}}^{(pq)}(\ell) + C_{\text{ll}}^{(pq)}(\ell) \\ C_{ab}^{(pq)}(\ell) &= \int_0^\infty \frac{d\chi}{f_k^2(\chi)} W_a^{(p)}(\chi) W_b^{(q)}(\chi) P_\delta\left(\frac{\ell + 1/2}{f_k(\chi)}, \chi\right) \\ C_{\epsilon\epsilon,\mu}^{(pq)}(\ell, \Theta) &= \sum_{\ell'=0}^{\ell''_{\max}} \sum_{\ell''=0}^{\ell''_{\max}} \sum_{v=1}^3 \sum_{v'=1}^3 M_{\mu\nu', \ell\ell''} M_{v'v, \ell''\ell'}^{(pq)} C_{\epsilon\epsilon,v}^{(pq)}(\ell'; \Theta) \end{aligned}$$

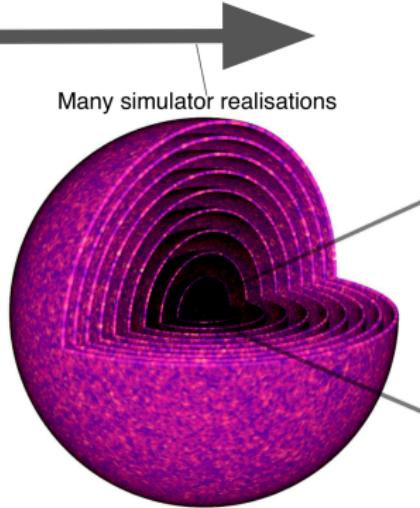


Forward Models: Modelling the Uncertainty

Complexity



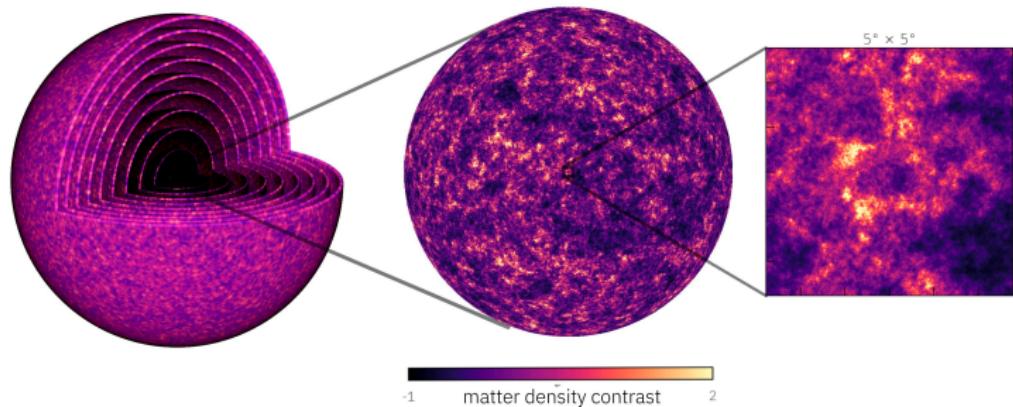
Many simulator realisations



Example: Large-Scale Structure and Weak Lensing

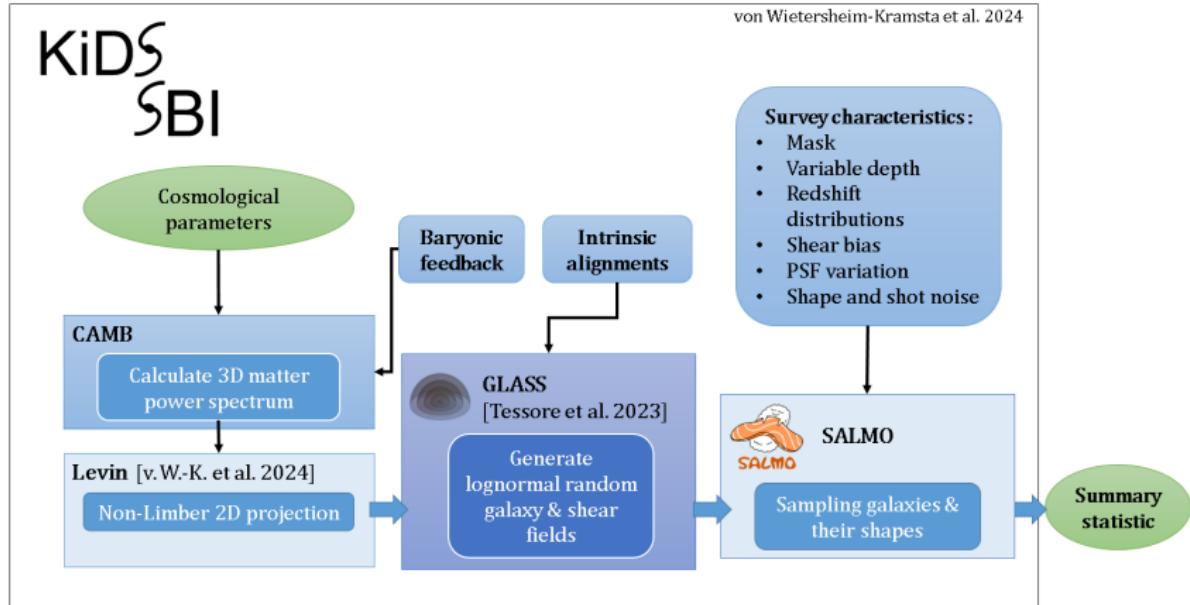
1. Interested in parameter inference of cosmology
2. Measure 2-point statistics → Capture 2-point signal and its uncertainty (up to n -point)
3. A range of systematics:
 - Physical: intrinsic alignments, baryonic feedback, (galaxy bias)
 - Observational: photometric redshift uncertainty, masking, galaxy shape calibration uncertainties, anisotropies in the selection function
4. Suited for Neural Likelihood Estimation (only one Universe to observe + prior dependence)

Example: Large-Scale Structure and Weak Lensing



Statistical simulations, e.g. lognormals (Tessore et al., 2023).

Example: Large-Scale Structure and Weak Lensing



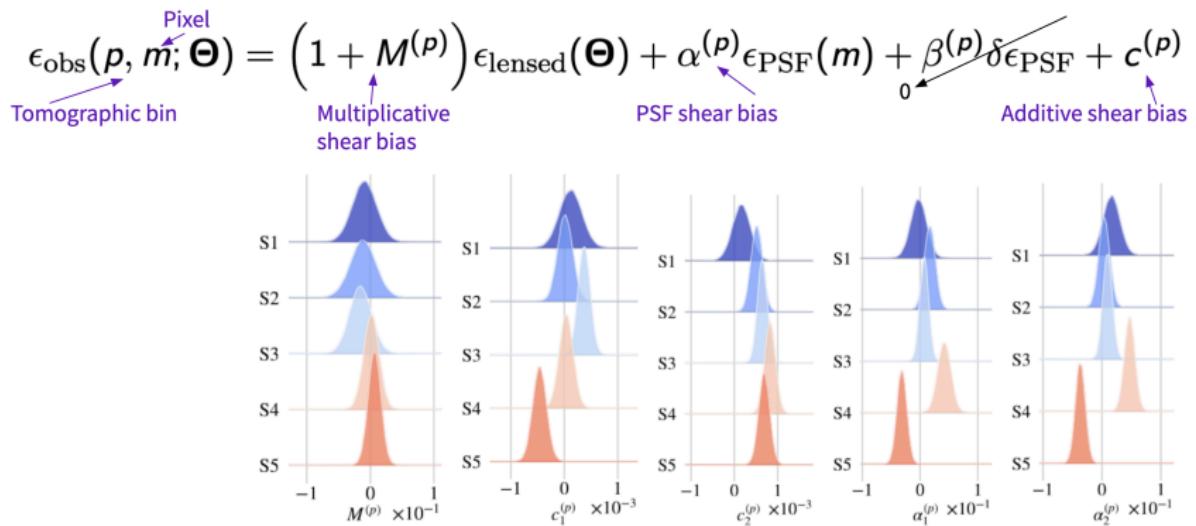
von Wietersheim-Kramsta et al. (2024)

Parameters, Priors, and Pre-Marginalisation

Parameter	Symbol	Prior type	Prior range	Fiducial
Density fluctuation amp.	S_8	Flat	[0.1, 1.3]	0.76
Hubble constant	h_0	Flat	[0.64, 0.82]	0.767
Cold dark matter density	ω_c	Flat	[0.051, 0.255]	0.118
Baryonic matter density	ω_b	Flat	[0.019, 0.026]	0.026
Scalar spectral index	n_s	Flat	[0.84, 1.1]	0.901
Intrinsic alignment amp.	A_{IA}	Flat	[-6, 6]	0.264
Baryon feedback amp.	A_{bary}	Flat	[2, 3.13]	3.1
Redshift displacement	δ_z	Gaussian	$\mathcal{N}(\mathbf{0}, \mathbf{C}_z)$	$\mathbf{0}$
Multiplicative shear bias	$M^{(p)}$	Gaussian	$\mathcal{N}(\bar{M}^{(p)}, \sigma_M^{(p)})$	$\bar{M}^{(p)}$
Additive shear bias	$c_{1,2}^{(p)}$	Gaussian	$\mathcal{N}(\bar{c}_{1,2}^{(p)}, \sigma_{c_{1,2}}^{(p)})$	$\bar{c}_{1,2}^{(p)}$
PSF variation shear bias	$\alpha_{1,2}^{(p)}$	Gaussian	$\mathcal{N}(\bar{\alpha}_{1,2}^{(p)}, \sigma_{\alpha_{1,2}}^{(p)})$	$\bar{\alpha}_{1,2}^{(p)}$

von Wietersheim-Kramsta et al. (2024)

Pre-Marginalisation



von Wietersheim-Kramsta et al. (2024)

Example: Galaxy-Scale Strong Lensing

1. Interested in parameter inference of cosmology and model comparison between dark matter models
2. Observe full images of lenses
3. A range of systematics:
 - Physical: external shear, multipoles, lens light, source complexity, etc.
 - Observational: point-spread function, pixel noise, selection effects, resolution, etc.
4. Suited for Neural Posterior Estimation (many lenses which can be amortised)

Source:

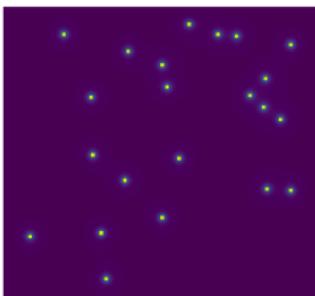
- Elliptical Core-Sersic
- $z = 1$
- **Axis ratio $\in [0.3, 0.85]$**
- **Axial tilt $\in [30, 70]^\circ$**

Lens:

- Power law mass
- $z = 0.5$
- No external shear
- $R_E \in [1.0, 1.5]$ "

Perturbers:

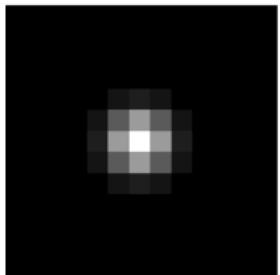
- Warm Dark Matter
- Truncated NFW mass
- $M_{\text{hf}} = 10^7$
- $n_{\text{subhalos}} \in [0, 30]$

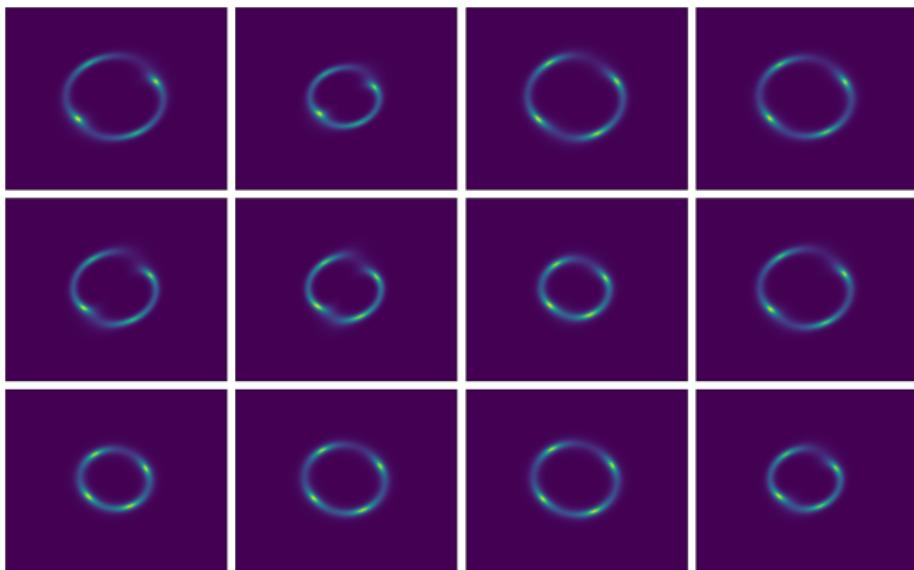


He et al. (2022), von Wietersheim-Kramsta et al. (in prep.)

**Observational Effects
(HST-like)**

- Exposure = 8000s
- Sky background = 0.1
- Pixel scale = 0.05"
- $\sigma_{\text{PSF}} = 0.05"$
- + Poisson noise





von Wietersheim-Kramsta et al. (in prep.)

Types of SBI

Types of Simulation-Based Inference

a.k.a. Likelihood-free or implicit likelihood inference

- Approximate Bayesian Computation (ABC)
- Neural Density Estimation (NDE)
 - Neural Posterior Estimation (NPE)
 - Neural Likelihood Estimation (NLE)
 - Neural Ratio Estimation (NRE)
 - Neural Posterior Score Estimation (NPSE)
- Sequential Methods

Simplest Case: Approximate Bayesian Computation

1. Draw simulations from simulator within prior space:

$$d^* \sim P(d | \theta^*); \quad \theta^* \sim P(\theta). \quad (4)$$

2. Define a distance metric between simulated data, d^* , and observed data, d_0 (e.g. Euclidean):

$$D = D(d_0, d^*). \quad (5)$$

3. Accept or reject according to arbitrary threshold, ϵ and repeat Rubin (1984):

$$\text{if } D < \epsilon, \text{ keep } \theta^*. \quad (6)$$

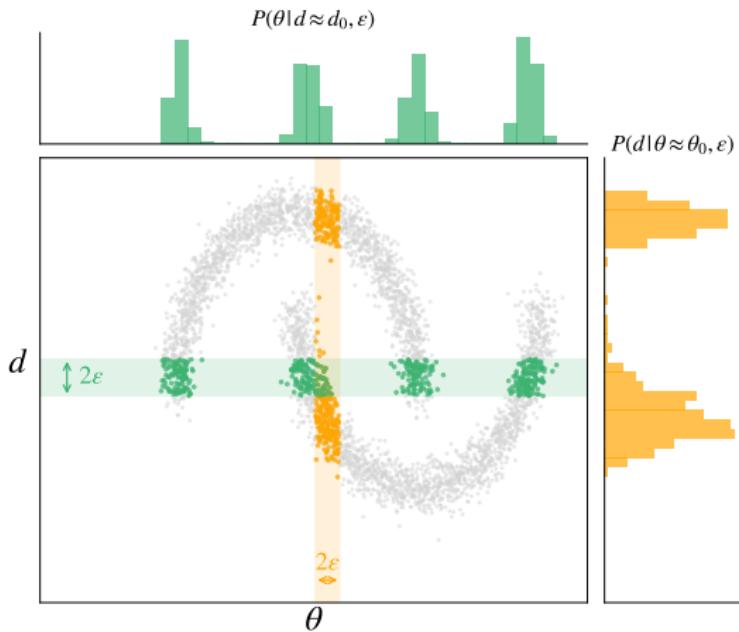
In the context of GBI, the Loss function is:

$$L(\theta, d) = \begin{cases} 0, & \text{if } D < \epsilon, \\ \infty, & \text{otherwise.} \end{cases} \quad (7)$$

Simplest Case: Approximate Bayesian Computation

Converges given:

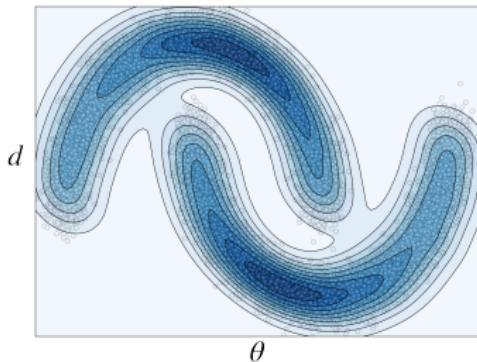
$$\lim_{\epsilon \rightarrow 0} P_{\text{ABC}}(\theta \mid d_0) = P(\theta \mid d_0). \quad (8)$$



Simplest Case: Approximate Bayesian Computation

- **Curse of Dimensionality:** Performance degrades significantly with the dimensionality the data vector and model.
- Requires significant **compression**.
- The results sensitive to the choice of distance metric and ϵ .
- **Inefficiency:** Large number of simulations may be rejected, especially for small ϵ .
- **Not amortised:** Reevaluation required for each new d_0 .
- **Recent applications:** e.g. galaxy morphology (Cameron and Pettitt, 2012; Tortorelli et al., 2021), diffuse X-ray background (Baxter et al., 2022), galaxy-scale strong lenses (He et al., 2022)

Neural Posterior Estimation (NPE)



$$D_{KL}(P \parallel Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)}$$

See Papamakarios and Murray (2016); Lueckmann et al. (2017); Greenberg et al. (2019); Cranmer et al. (2020)

1. Draw simulations:

$$d^* \sim P(d \mid \theta^*); \quad \theta^* \sim P(\theta). \quad (9)$$

2. Find an estimator of the posterior, $\hat{P}_w(\theta \mid d)$, with its weights, w , such that:

$$w^* = \arg \min_w \mathbb{E}_{P(d)} [D_{KL}(P(\theta \mid d) \parallel \hat{P}_w(\theta \mid d))], \quad (10)$$

$$w^* = \arg \max_w \mathbb{E}_{P(\theta, d)} [\ln(\hat{P}_w(\theta \mid d))]. \quad (11)$$

3. Train a neural network from this loss function:

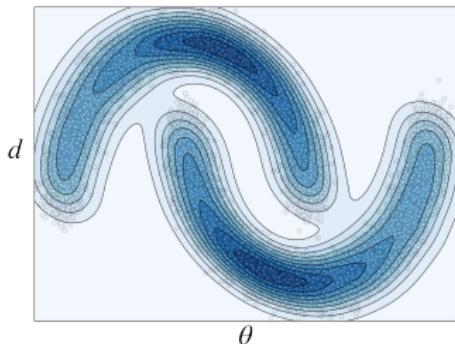
$$L(w) = -\mathbb{E}_{P(\theta, d)} [\ln(\hat{P}_w(\theta \mid d))] \quad (12)$$

4. Use network to directly sample $\hat{P}_w(\theta \mid d)$.

Neural Posterior Estimation (NPE)

- **Amortised***: Direct mapping from d to $P(\theta | d)$.
- ***Amortisation gap**: If observed data, d_0 , changes, $\hat{P}_w(\theta | d)$ may not be well characterised at new $d_0 \rightarrow$ Additional training required.
- ***Prior-dependent**: For new parameters priors, retraining is necessary.
- **Recent applications**: e.g. galaxy clustering (Lemos et al., 2023b), exoplanets (Vasist et al., 2023), gravitational waves (Leyde et al., 2024), X-ray spectra (Barret and Dupourqué, 2024), lensed quasars (Erickson et al., 2024)
- **Implemented in**: *sbi* (Tejero-Cantero et al., 2020), *lampe* (Rozet et al., 2021) and *ltu-ili* (Ho et al., 2024).

Neural Likelihood Estimation (NLE)



$$D_{KL}(P||Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)}$$

See Papamakarios and Murray (2016); Papamakarios et al. (2017); Alsing et al. (2019); Cranmer et al. (2020); Boelts et al. (2022); Glöckler et al. (2022)

1. Draw simulations:

$$\mathbf{d}^* \sim P(\mathbf{d} | \boldsymbol{\theta}^*); \quad \boldsymbol{\theta}^* \sim P(\boldsymbol{\theta}). \quad (13)$$

2. Find an estimator of the likelihood, $\hat{P}_w(\mathbf{d} | \boldsymbol{\theta})$, with its weights, w , such that:

$$\mathbf{w}^* = \arg \min_w \mathbb{E}_{P(\mathbf{d})} [D_{KL}(P(\mathbf{d} | \boldsymbol{\theta}) || \hat{P}_w(\mathbf{d} | \boldsymbol{\theta}))], \quad (14)$$

$$\mathbf{w}^* = \arg \max_w \mathbb{E}_{P(\boldsymbol{\theta}, \mathbf{d})} [\ln(\hat{P}_w(\mathbf{d} | \boldsymbol{\theta}))]. \quad (15)$$

3. Train a neural network from this loss function:

$$L(\mathbf{w}) = -\mathbb{E}_{P(\boldsymbol{\theta}, \mathbf{d})} [\ln(\hat{P}_w(\mathbf{d} | \boldsymbol{\theta}))]. \quad (16)$$

4. Define priors, $P(\boldsymbol{\theta})$, and sample posterior with an MCMC.

Neural Likelihood Estimation (NLE)

- **Prior-independent:** Learnt likelihood can be reused for sampling when varying the priors.
- **Useful for model testing:** Likelihood evaluation allows for model comparison.
- **Sampling required:** Requires sampling with MCMC or other sampler to obtain posteriors.
- **Compression:** Can struggle with high-dimensional data → Requiring compression.
- **Recent applications:** e.g. weak lensing (Jeffrey et al., 2024; von Wietersheim-Kramsta et al., 2024), seismology (Saoulis et al., 2025)
- **Implemented in:** *sbi* (Tejero-Cantero et al., 2020), *delfi* (Alsing et al., 2019) and *ltu-ili* (Ho et al., 2024).

NDE: Normalising Flows

- Used for **NPE** and **NLE**, as they naturally encode the normalisation of the probability densities (Papamakarios et al., 2021).
- Learns **invertible and differentiable** transformations between any distribution and a Gaussian.
- Usually train ensembles in parallel for robustness.
- Examples: Masked Autoregressive Flows (MAF), Neural Spline Flows (NSF), etc.

Neural Ratio Estimation (NRE)

1. Typically, learn the likelihood-to-evidence:

$$r(d | \theta) = P(d | \theta) / P(d) = P(\theta | d) / P(\theta). \quad (17)$$

2. Train a neural classifier, $Q_w(d, \theta)$, to distinguish draws from:
 - The “true” joint distribution $P(d, \theta) = P(d | \theta) P(\theta)$.
 - The product of the marginal distributions $P(d) P(\theta)$.
3. When optimised, we find:

$$Q^*(d, \theta) = P(d, \theta) / [P(d, \theta) + P(d)P(\theta)]. \quad (18)$$

$$\hat{r}(d | \theta) = \frac{Q_w(d, \theta)}{1 - Q_w(d, \theta)} \approx \frac{P(d | \theta)}{P(d)}. \quad (19)$$

4. Sample posterior from $P(\theta | d_0) \propto \hat{r}(d_0 | \theta) P(\theta)$.

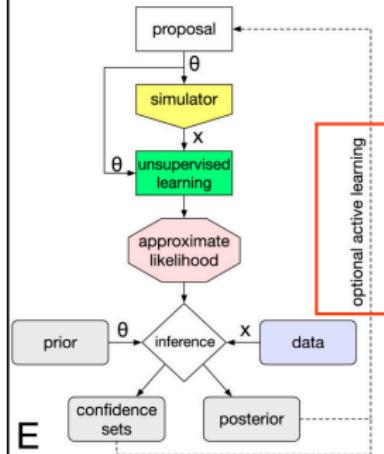
See Hermans et al. (2019); Cranmer et al. (2020); Durkan et al. (2020); Delaunoy et al. (2022); Miller et al. (2022)

Neural Ratio Estimation (NRE)

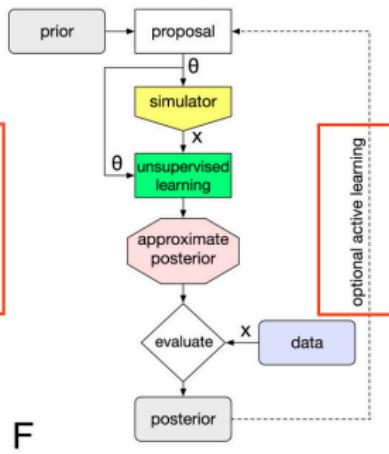
- **Robust ratio estimates:** Useful to directly compute likelihood ratios for model testing.
- **Scalability:** Can scale well in high-dimensional parameter spaces, e.g. Truncated Marginal Neural Ratio Estimation (TMNRE).
- **Density-Chasm problem:** joint distribution and marginal distributions may have little overlap in high dimensions.
- **Recent applications:** e.g. CMB (Cole et al., 2022), strong lensing (Anau Montel et al., 2023; Filipp et al., 2024), pulsars (Berteaud et al., 2024), supernova cosmology (Karchev and Trotta, 2024)
- **Implemented in:** *sbi* (Tejero-Cantero et al., 2020), *swyft* (Miller et al., 2022) and *ltu-ili* (Ho et al., 2024).

Sequential SBI and Active Learning

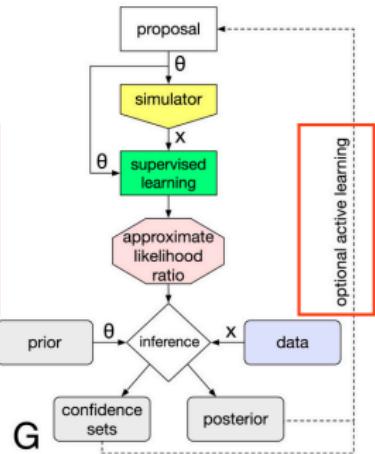
Amortized likelihood



Amortized posterior



Amortized likelihood ratio



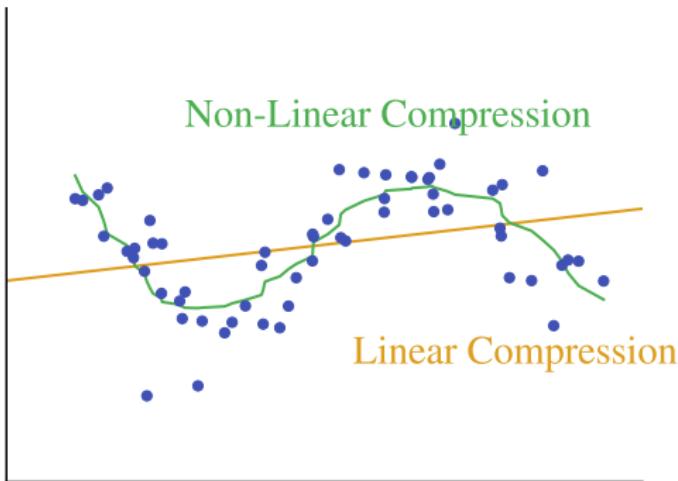
Cranmer et al. (2020)

Sequential SBI and Active Learning

1. Draw initial simulations, (θ_i, d_i) .
2. Train initial density estimate $\hat{P}_{W^{(1)}}(\theta | x)$ or $\hat{P}_{W^{(1)}}(d | \theta)$.
3. For subsequent rounds $r > 1$, construct a proposal distribution, $\tilde{P}^{(r)}(\theta)$; e.g., $\tilde{P}^{(r)}(\theta) \propto \hat{P}_{W^{(r-1)}}(\theta | d_0)$.
4. Draw new simulations with $\theta_j \sim \tilde{P}^{(r)}(\theta)$ and $d_j \sim P(d | \theta_j)$.
5. Retrain the density estimator using all accumulated simulations.

Data Compression

Data Compression: Linear vs. Non-Linear Compression



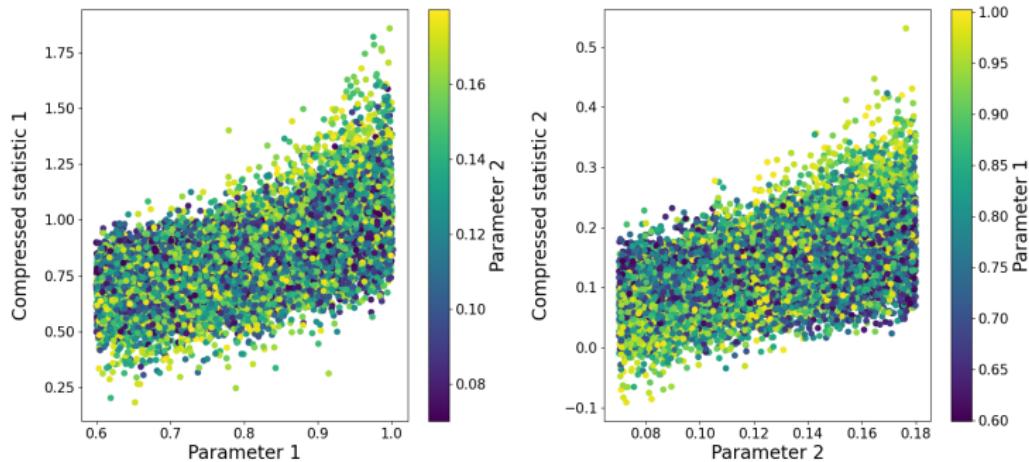
- **Linear methods** may lose non-Gaussian and non-linear information. Sometimes also require an analytic likelihood.
- **Non-linear methods** can address this, but can come at the expense of efficiency and interpretability.

Data Compression: Linear Compression

- Principal Component Analysis (PCA): Projects data onto principal components capturing maximal variance (e.g. Barret and Dupourqué 2024).
- Canonical Correlation Analysis (CCA): Finds linear combinations of two sets of model parameters, θ , and data, d , that are maximally correlated (e.g. Park et al. 2025).
- Score Compression: Utilises the score function (gradient of the log-likelihood with respect to parameters, $\nabla_{\theta} \ln P(d | \theta)$), for compression. Can be lossless in Fisher information (Alsing and Wandelt, 2018).
- MOPED and e-MOPED: Lossless compression in Fisher information under Gaussian likelihood assumption (Heavens et al., 2000).

Linear Compression: Score Compression in Weak Lensing

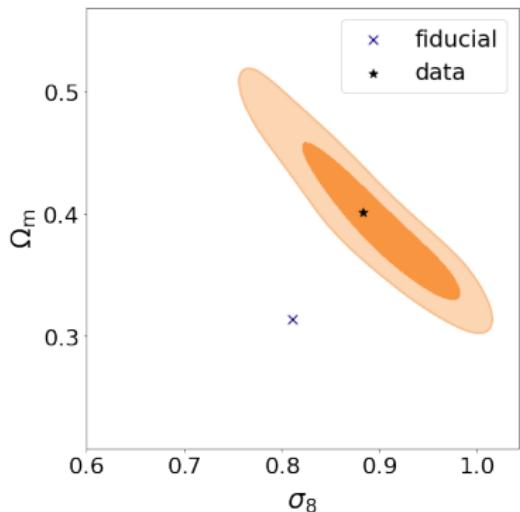
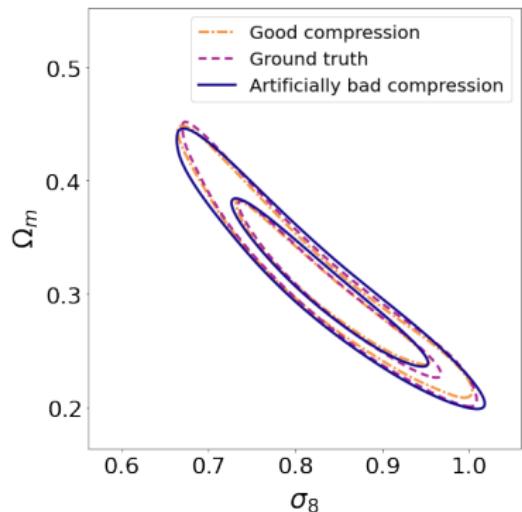
Linearly score compressed data vs. parameters



Alsing et al. (2018); Lin et al. (2023);
von Wietersheim-Kramsta et al.
(2024)

$$t = \nabla d$$

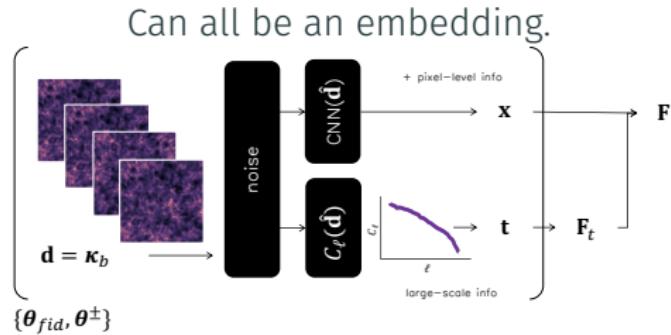
Linear Compression: Score Compression in Weak Lensing



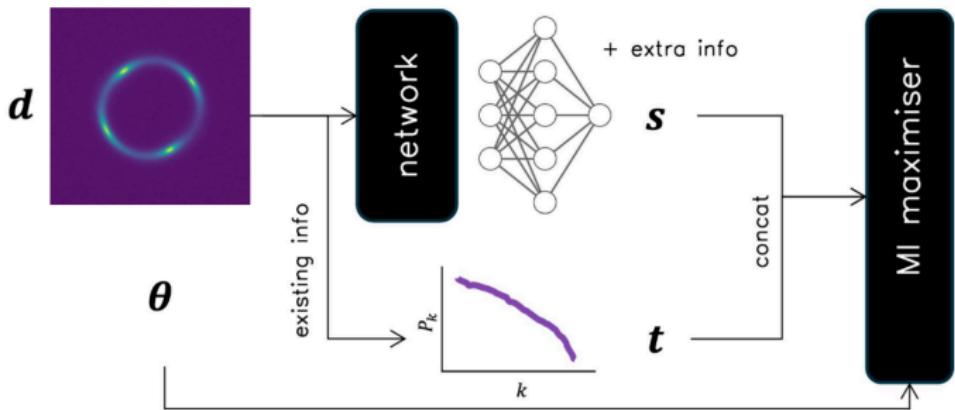
Lin et al. (2023); von Wietersheim-Kramsta et al. (2024)

Data Compression: Non-Linear Compression

- **Deep Auto-Encoders:** Unsupervised neural networks consisting of an encoder that maps data to a lower dimensional space.
- **Convolutional Neural Networks (CNN):** Type of encoder that involves filter/kernel optimization (e.g. Lemos et al. 2023b; Jeffrey et al. 2024).
- **Information-Maximising Neural Networks:** Compress with neural networks which find non-linear summary statistics that explicitly maximising the Fisher information (e.g. Charnock et al. 2018; Makinen et al. 2025).

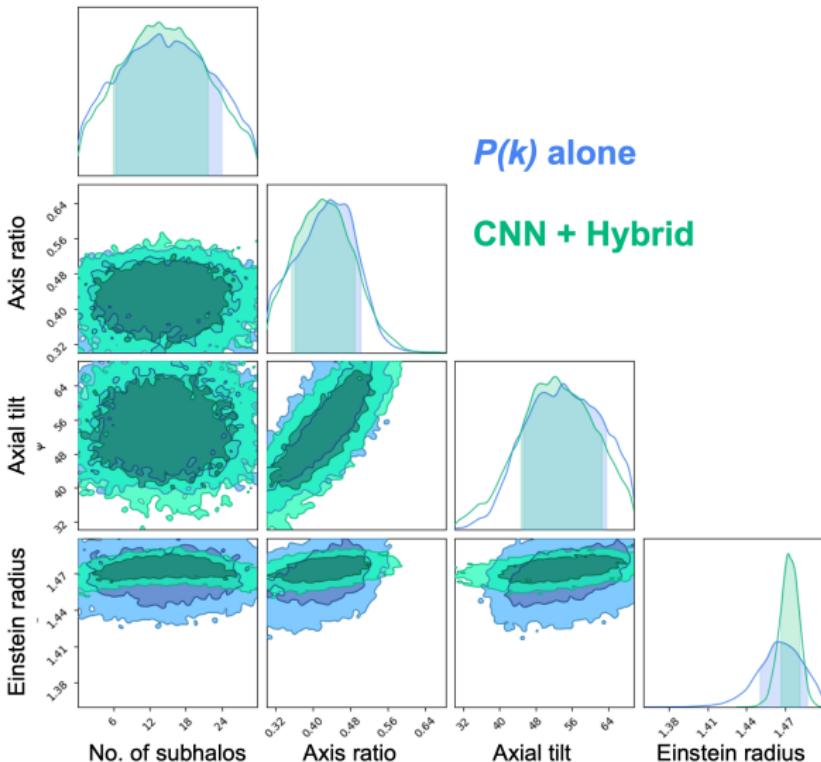


Example: IMNN Hybrid Summaries in Strong Lensing



Makinen et al. (2025)

Example: IMNN Hybrid Summaries in Strong Lensing



von Wietersheim-Kramsta et al. (in prep.)

Model Testing & Misspecification with SBI

Bayesian Model Testing

For a given model, \mathcal{M} , and observed data, d , the Bayesian evidence, $P(d | \mathcal{M})$, is defined as :

$$P(d | \mathcal{M}) = \int P(d | \theta, \mathcal{M}) P(\theta | \mathcal{M}) d\theta. \quad (20)$$

When comparing two models, \mathcal{M}_1 and \mathcal{M}_2 , one may define the Bayes factor:

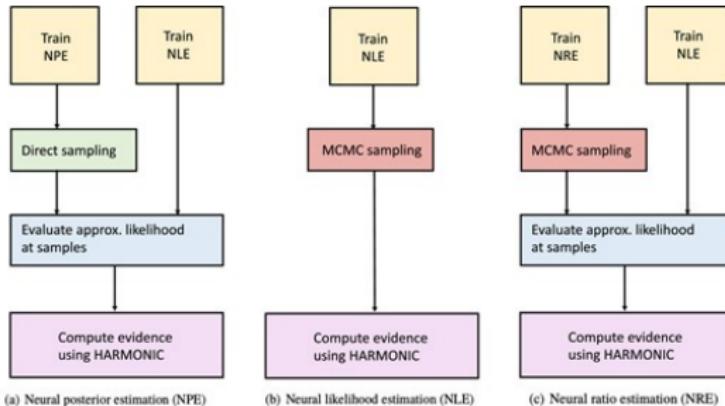
$$B_{12} = \frac{P(d | \mathcal{M}_1)}{P(d | \mathcal{M}_2)}. \quad (21)$$

→ Ratio of posterior odds to prior odds of the two models and provides a measure of the evidence in favor of \mathcal{M}_1 over \mathcal{M}_2 .

Naturally incorporates Occam's razor: overly complex models are disfavoured.

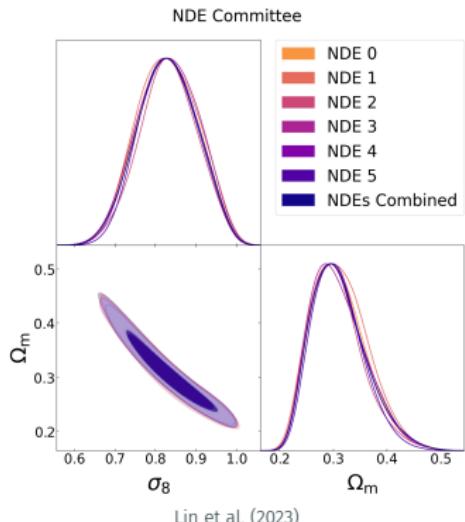
Bayesian Model Testing with SBI

- **NLE-based:** Integrate the learnt likelihood.
- **NLE/NPE or NRE:** If all are available, can compute $\hat{P}(d|\mathcal{M}) \approx \frac{\hat{P}_{w'}(d|\theta_0, \mathcal{M}) P(\theta_0|\mathcal{M})}{\hat{P}_w(\theta_0|d, \mathcal{M})}$ (Spurio Mancini et al., 2023).
- **floZ:** Training a normalising flow directly on the evidence (Srinivasan et al., 2024).
- **Classifier-based:** Train classifier to distinguish $d \sim \mathcal{M}_1$ from $d \sim \mathcal{M}_2$ (e.g. Jeffrey and Wandelt 2024).



Model Misspecification in SBI

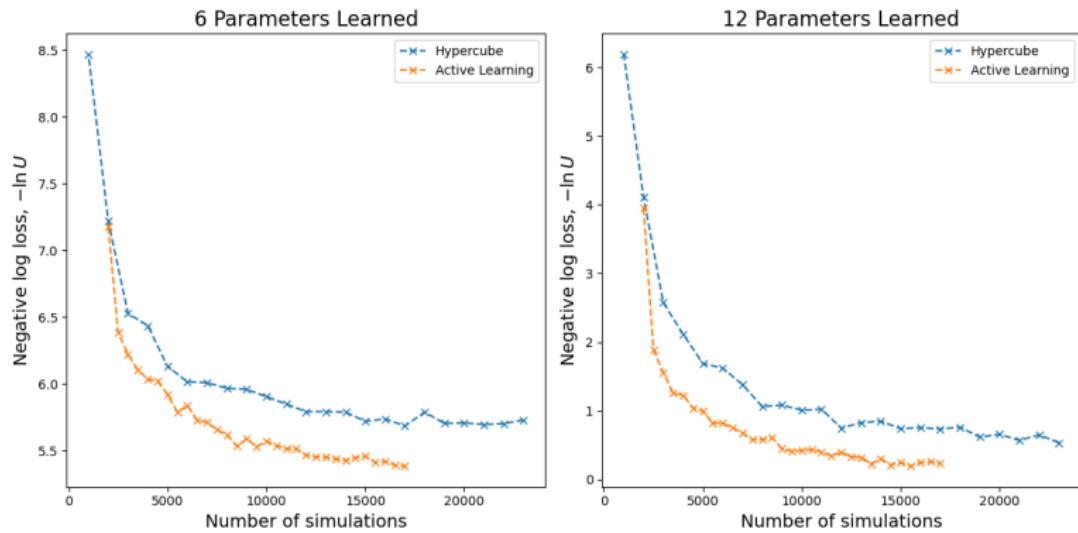
- Model misspecification can cause **biased posterior estimates** despite a self-consistent SBI.
- **Excessive extrapolation:** If the observed data is out-of-distribution (OOD), the trained NNs are no longer describing the correct probability distribution.
- **Mitigation:**
 - GBI framing
 - Error modelling within NDE (Huang et al., 2023; Kelly et al., 2025)
 - Ensembling



Lin et al. (2023)

Diagnosing & Testing SBI

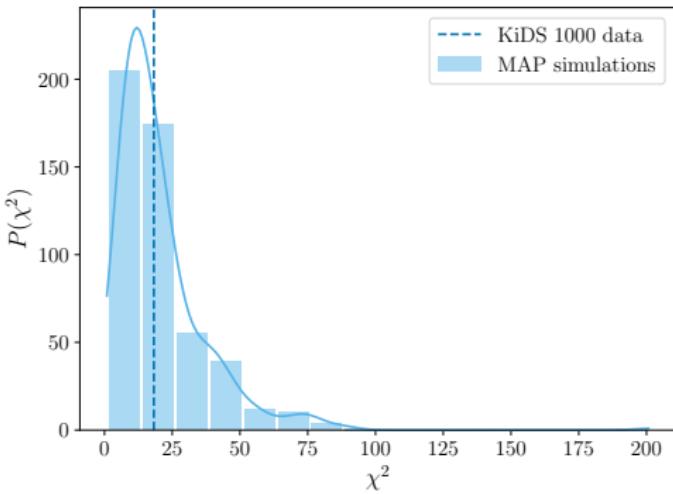
Convergence of Training



Lin et al. (2023); von Wietersheim-Kramsta et al. (2024)

Testing for OOD: Goodness-of-Fit Tests

$$\chi^2(\mathbf{d}|\Theta) = (\mathbf{d}_i - \mathbb{E}[\mathbf{d}_*|\Theta_*])^T (\text{Cov}(\mathbf{d}_*|\Theta_*))^{-1} (\mathbf{d}_i - \mathbb{E}[\mathbf{d}_*|\Theta_*]) \quad (22)$$



GoF measure under Gaussian likelihood assumption (von Wietersheim-Kramsta et al., 2024)

Testing for OOD: Goodness-of-Fit Tests

Localized deviation tests



$$t_i(\mathbf{x}) = -2 \ln \frac{p_{\text{sim}}(\mathbf{x})}{p_{\text{dist}}(\mathbf{x}|i)}$$

- ↓
1) p-values for
anomaly detection
↓

- 2) Residual analysis

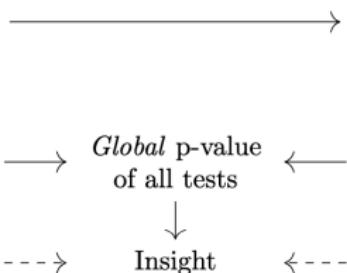
Aggregated deviation tests



$$t_{\text{sum}}(\mathbf{x}) = \sum_i t_i(\mathbf{x})$$

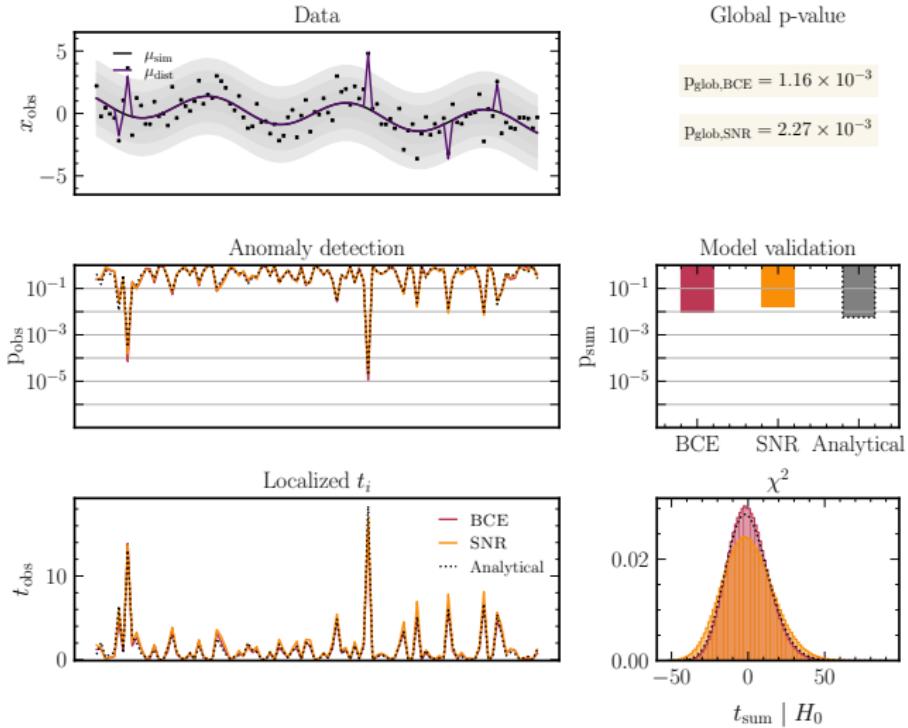
- ↓
1) p-value for
model validation
↓

- 2) Residual variance analysis



Posterior predictive tests (Anau Montel et al., 2025)

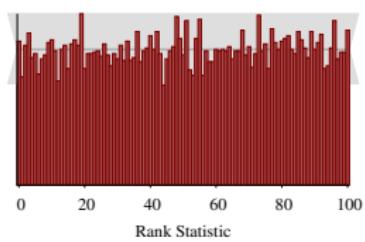
Testing for OOD: Goodness-of-Fit Tests



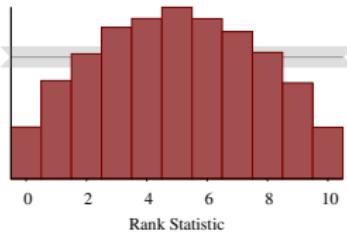
Posterior predictive tests (Anau Montel et al., 2025)

Consistency between SBI & Simulator: Simulation-Based Calibration

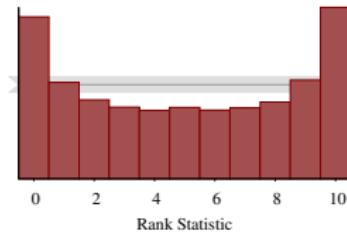
1. For a fixed θ^* , draw N posterior samples, θ_i .
2. Evaluate the probability that the true parameter θ^* is less than some drawn value, according to the recovered posterior (i.e. the rank).
3. Averaging over many simulations, we evaluate the posterior cumulative density distribution.



Accurate posterior



Learnt posterior too wide

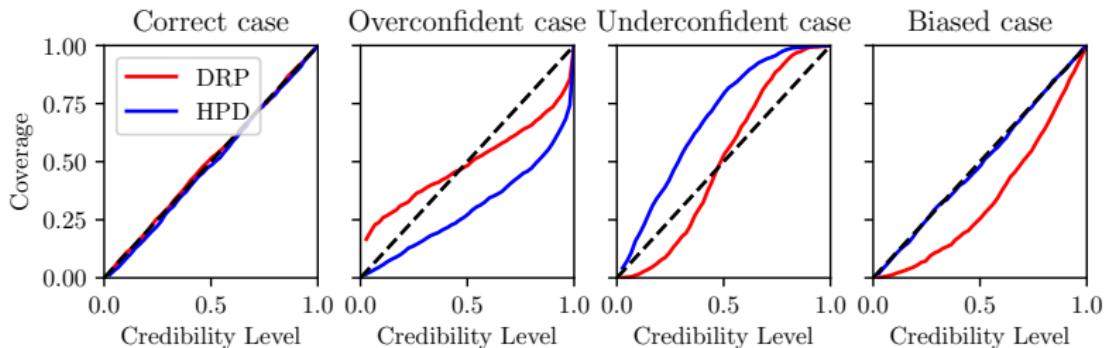


Learnt posterior too narrow

Simulation-Based Calibration (Talts et al., 2018). Available in [lzu-ili](#).

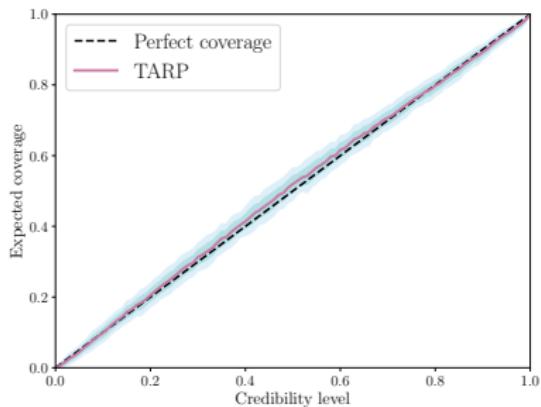
Consistency between SBI & Simulator: Coverage/TARP

1. Draw parameter samples from the learnt posterior.
2. Run forward simulations at the parameters.
3. Evaluate fraction (f) of parameter samples per sim (i) where:
 $P(\theta_i|d_i) < P(\theta_i^*|d_i)$.
4. Evaluate fraction of sims where f is within the expected fraction.

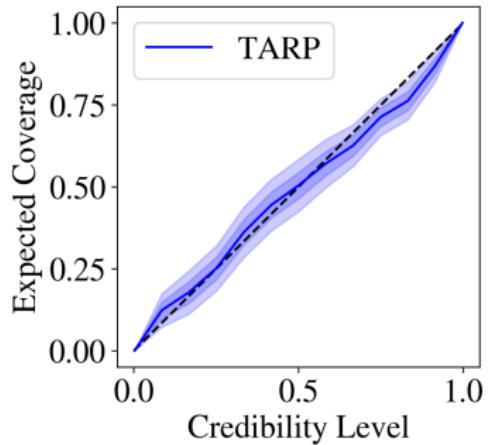


Coverage tests/TARP (Deistler et al., 2022; Lemos et al., 2023a). Available in
ltu-ili.

Examples: TARP in Weak and Strong Lensing SBI



von Wietersheim-Kramsta
et al. (2024)



von Wietersheim-Kramsta, et
al. (in prep.)

Conclusion & Outlooks

Outlooks

- SBI is a powerful method allowing for increasingly complex modelling.
 - Some **caveats** → Extensive testing and validation required.
 - Still limited by computational resources, particularly, with higher dimensional data.
-
- **Future avenues:**
 - Neural Posterior Score Estimation (NPSE) and Flow Matching Posterior Estimation (FMPE)
 - Transfer Learning to combine constraints
 - Training with multifidelity simulators (Krouglova et al., 2025)
 - **Exenstive applications:** large datasets from surveys, simulations and high-resolution imaging.

Questions?

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