Assignment 5

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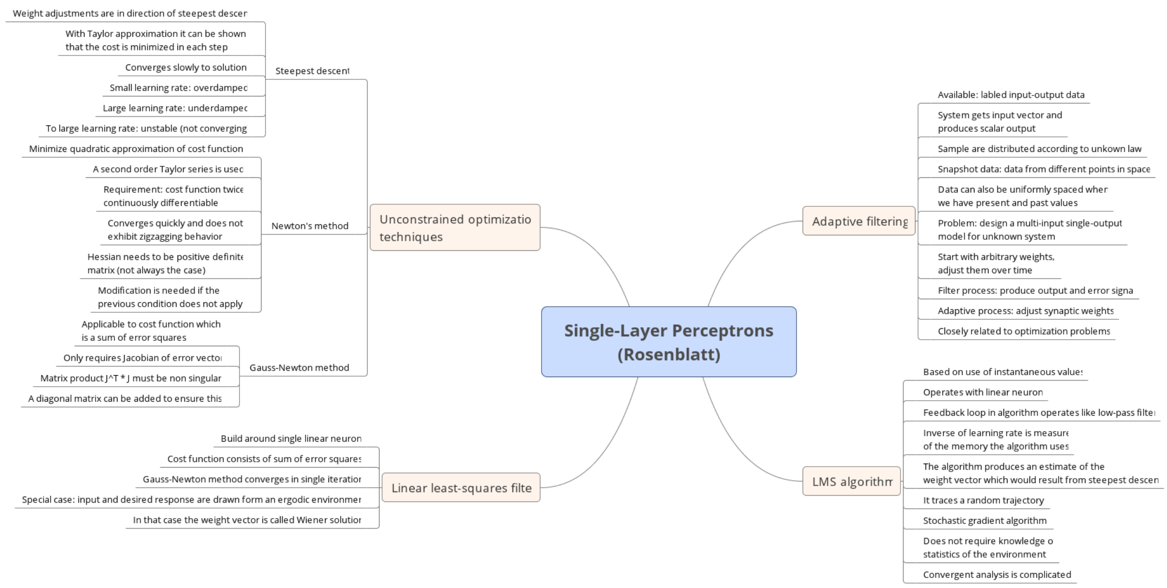
Task 1

In [1]:

```
from IPython.display import Image
Image("Single-Layer_Perceptrons.png")
```

Out[1]:



Task 2 (Haykin 3.1)

In [2]:

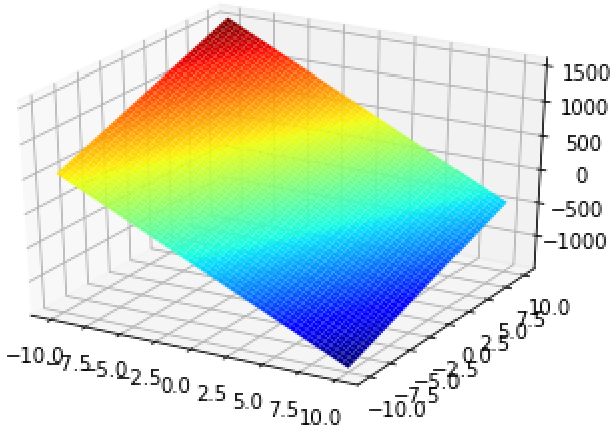
```
import numpy as np
import sympy as sp
sp.init_printing()
```

In [3]:

```
w_1,w_2 = sp.symbols("w_1,w_2 ")

r_xd=100;
r_x=100;
o=100;

def function(x,y):
    return (1/2)*o-r_xd*x+(1/2)*r_x*y
```

In [4]:

```
sp.plotting.plot3d(function(w_1,w_2))
```

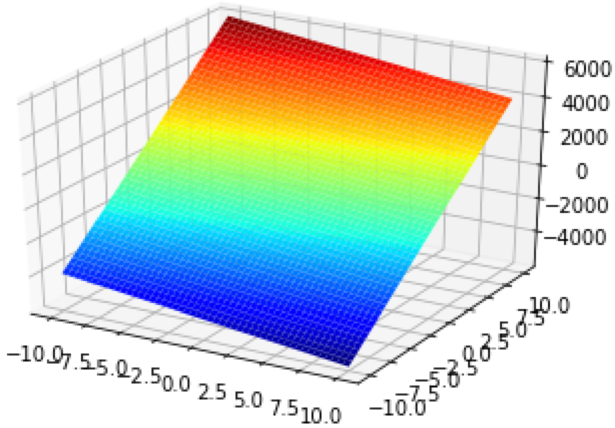


Out[4]:

```
<sympy.plotting.plot.Plot at 0x7f3aabc3feb8>
```

In [5]:

```
r_xd=100;
r_x=1000;
o=100;

sp.plotting.plot3d(function(w_1,w_2))
```

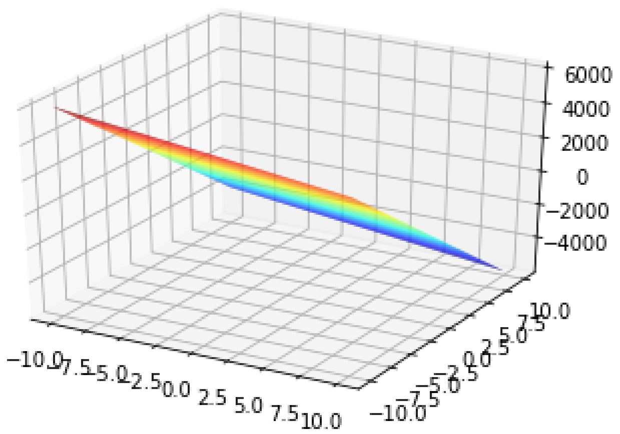


Out[5]:

```
<sympy.plotting.plot.Plot at 0x7f3aa6762e10>
```

In [6]:

```
r_xd=100;
r_x=-1000;
o=100;

sp.plotting.plot3d(function(w_1,w_2))
```



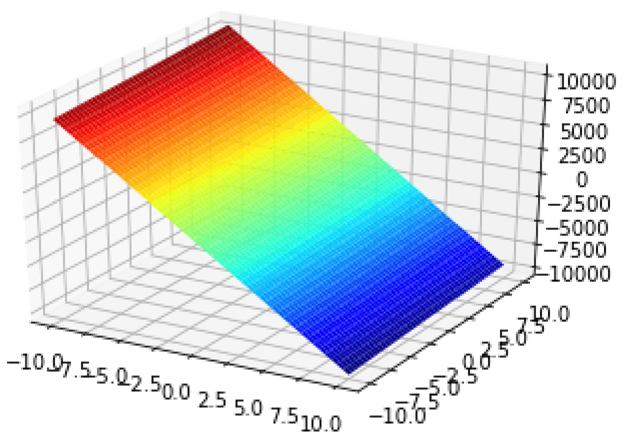
Out[6]:

```
<sympy.plotting.plot.Plot at 0x7f3aa662f0b8>
```

r_x rotates the hyperplane around x axis

In [7]:

```
r_xd=1000;
r_x=100;
o=100;

sp.plotting.plot3d(function(w_1,w_2))
```

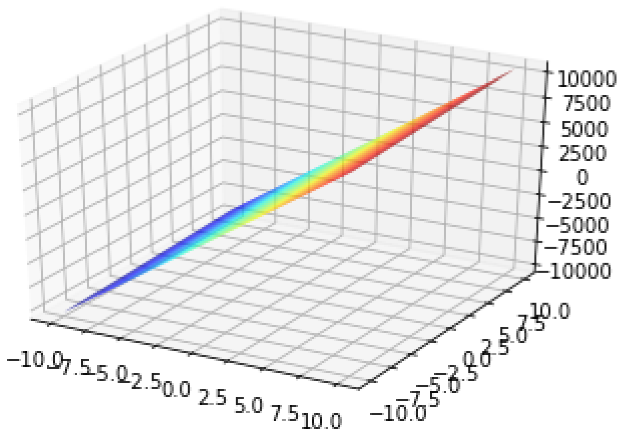


Out[7]:

```
<sympy.plotting.plot.Plot at 0x7f3aa66270b8>
```

In [8]:

```
r_xd=-1000;
r_x=100;
o=100;

sp.plotting.plot3d(function(w_1,w_2))
```



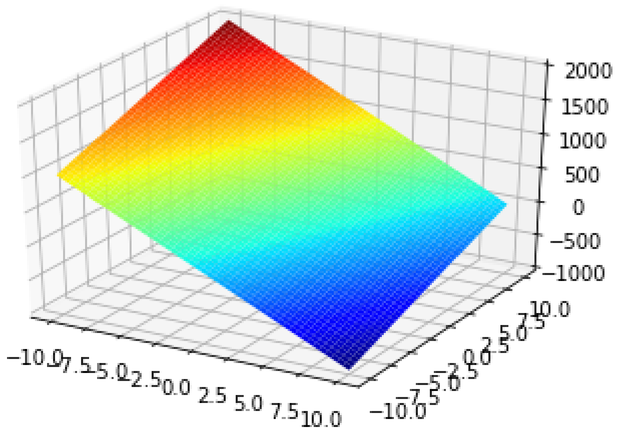
Out[8]:

```
<sympy.plotting.plot.Plot at 0x7f3aa6342e80>
```

r_{xd} rotates the hyperplane around y axis

In [9]:

```
r_xd=100;
r_x=100;
o=1000;

sp.plotting.plot3d(function(w_1,w_2))
```

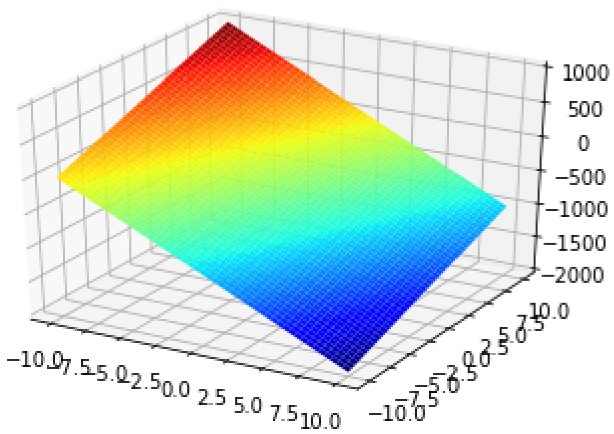


Out[9]:

```
<sympy.plotting.plot.Plot at 0x7f3aa649d588>
```

In [10]:

```
r_xd=100;
r_x=100;
o=-1000;

sp.plotting.plot3d(function(w_1,w_2))
```



Out[10]:

<sympy.plotting.plot.Plot at 0x7f3aa6047b00>

o pushes the hyperplane up and down (bias)

Task 3 (Haykin 3.2)

a

In [11]:

```
import sympy as sp
from scipy.optimize import minimize
from sympy.utilities.lambdify import lambdify
sp.init_printing()
```

In [12]:

```
w1 = sp.Symbol("w_1")
w2 = sp.Symbol("w_2")
w = sp.Matrix([[w1], [w2]])
sigma = sp.Symbol("sigma")

r = sp.Matrix([[0.8182], [0.354]])
R = sp.Matrix([[1, 0.8182], [0.8182, 1]])

a = r.transpose() * w
b = w.transpose() * R * w

#cost_func = 0.5 * sigma - a[0] + 0.5 * b[0]
cost_func = - a[0] + 0.5 * b[0]
lambdified = lambdify((w1, w2), cost_func)

def minimizable_function(params):
    w1, w2 = params
    return lambdified(w1, w2)

cost_func
```

Out[12]:

$$0.5w_1\left(w_1 + 0.8182w_2\right) - 0.8182w_1 + 0.5w_2\left(0.8182w_1 + w_2\right) - 0.354w_2$$

In [13]:

```
solution = minimize(minimizable_function, [0, 0])
solution
```

Out[13]:

```
      fun: -0.48524726287868303
 hess_inv: array([[ 2.93450335, -2.42312918],
       [-2.42312918,  2.99531171]])
      jac: array([  3.01748514e-06,   1.37835741e-06])
  message: 'Optimization terminated successfully.'
     nfev: 36
      nit: 8
     njev: 9
   status: 0
  success: True
        x: array([ 1.5990351 , -0.95432915])
```

In [14]:

```
w = solution.x
print("w* = ")
w
```

```
w* =
```

Out[14]:

```
array([ 1.5990351 , -0.95432915])
```

In the previos code we use scipy to minimize the function. The constant σ^2 does not matter for the minimization, since it could be chose arbitrarily negative.

b

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

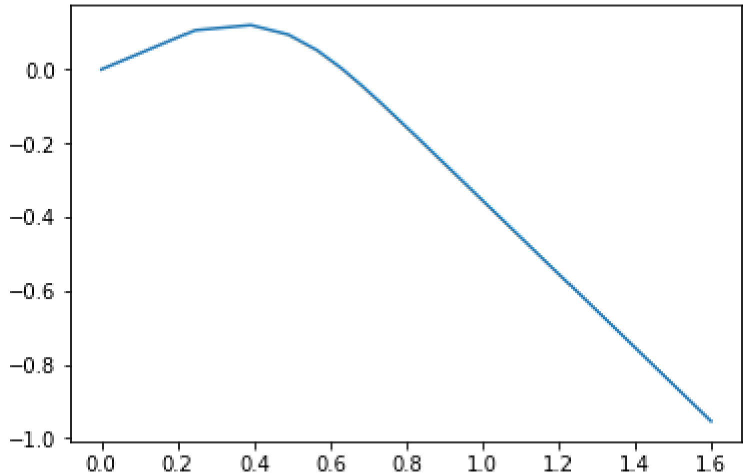
In [16]:

```
def steepest_descent(epsilon, max_iteration, learning_rate, init_guess, function):
    guess = init_guess
    all_guesses = np.array(guess.T)

    counter = 0
    while counter < max_iteration:
        counter = counter + 1
        old_value = function.subs({w1: guess[0], w2: guess[1]})

        steepest_descent = sp.Matrix([[sp.diff(function, w1)],
                                      [sp.diff(function, w2)]]).subs({w1: guess[0],

        guess = guess - learning_rate * steepest_descent
        all_guesses = np.row_stack((all_guesses, guess.T))
        new_value = function.subs({w1: guess[0], w2: guess[1]})

        if abs(new_value - old_value) < epsilon:
            break

    if counter == max_iteration:
        print("Algorithm did not stop, w =", guess)
    else:
        print("Epsilon reached after", counter, "iterations, w =", guess)

    plt.plot(all_guesses[:,0], all_guesses[:,1])
    plt.show()
    return guess
```

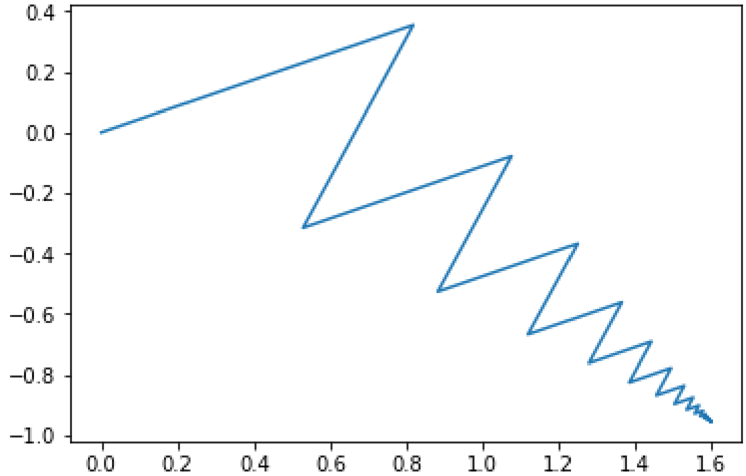
In [24]:

```
epsilon = 1e-9
max_iteration = 10000


guess = np.array([[0],[0]])

steepest_descent(epsilon, max_iteration, 0.3, guess, cost_func)
steepest_descent(epsilon, max_iteration, 1, guess, cost_func)
```

```
Epsilon reached after 155 iterations, w = Matrix([[1.59881526814624],
[-0.954111715181767]])
```




```
Epsilon reached after 49 iterations, w = Matrix([[1.59897818042613],
[-0.954239995718932]])
```



Out[24]:

$$\begin{bmatrix} 1.59897818042613 \\ -0.954239995718932 \end{bmatrix}$$

3.4

Correlation matrix:

$$Rx = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

Condition for the learning rate to be convergent:

$0 < \eta < \frac{1}{lambda_{\max}}$
or
$0 < \eta < \frac{1}{trace_{\mathrm{Rx}}}$

Calculate eigenvalues and trace:

In [27]:

```
m = [[1, 0.5], [0.5, 1]]
eigenv = np.linalg.eig(m)
lambda_max = max(eigenv[0])

trace = np.trace(m)
```

In [28]:

```
print("eigenvalues = ", eigenv[0],"\n",
      "trace = ", trace,"\n")
```

```
eigenvalues =  [ 1.5  0.5]
 trace =  2.0
```

Result:

Upperbound (eigenvalues): $\frac{1}{lambda_{\max}} = \frac{1}{1.5} = \frac{2}{3}$
Upperbound (trace): $\frac{1}{lambda_{\max}} = \frac{1}{2}$
LMS is converging for (using eigenvalues) : $0 < \eta < \frac{2}{3}$
LMS is converging for (using trace) : $0 < \eta < \frac{1}{2}$

3.8

a

$J(w) = \frac{1}{2}\sigma_d^2 - r_{xd}^T w + \frac{1}{2}w^T R_x w$

$J(w) = \frac{1}{2}E[d^2(n)] - E[x(n)d(n)]^T w + \frac{1}{2}w^T E[x(n)x^T(n)]w$

$J(w) = \frac{1}{2}E[d^2(n)] - E[x^T(n)d(n)]w + \frac{1}{2}w^T E[x(n)x^T(n)]w$

$J(w) = \frac{1}{2}E[d^2(n)] - E[x^T(n)d(n)]w + E[\frac{1}{2}w^T x(n)x^T(n)w]$

$J(w) = E[\frac{1}{2}d^2(n) - x^T(n)wd(n) + \frac{1}{2}w^T x(n)x^T(n)w]$

$J(w) = \frac{1}{2} * E[d^2(n) - 2 * x^T(n)wd(n) + w^T x(n)x^T(n)w]$

$J(w) = \frac{1}{2} * E[d^2(n) - 2 * x^T(n)wd(n) + (x^T(n)w)^2]$

$J(w) = \frac{1}{2} * E\left[\left(d(n) - x^T(n)w\right)^2\right]$

b

$$J(w) = \frac{1}{2} * E[d^2(n) - 2 * x^T(n)wd(n) + (x^T(n)w)^2]$$

$$J'(w) = \frac{1}{2} * E[-2 * x^T(n)d(n) + x^T(n) * 2 * x^T(n)w]^T$$

$$J'(w) = E[-x^T(n)d(n) + x^T(n) * x^T(n)w]^T$$

$$J'(w) = E[-x(n)d(n)] + E[x(n) * x^T(n)]w$$

$$J'(w) = -r_{xd} + R_x w$$

$$J'(w) = E[-x^T(n)d(n) + x^T(n) * x^T(n)w]^T$$

$$J''(w) = E[x^T(n) * x^T(n)]^T$$

$$J''(w) = E[x(n) * x^T(n)]$$

$$J''(w) = R_x$$