

NN_LindaKoine_JensWeidmann_161017

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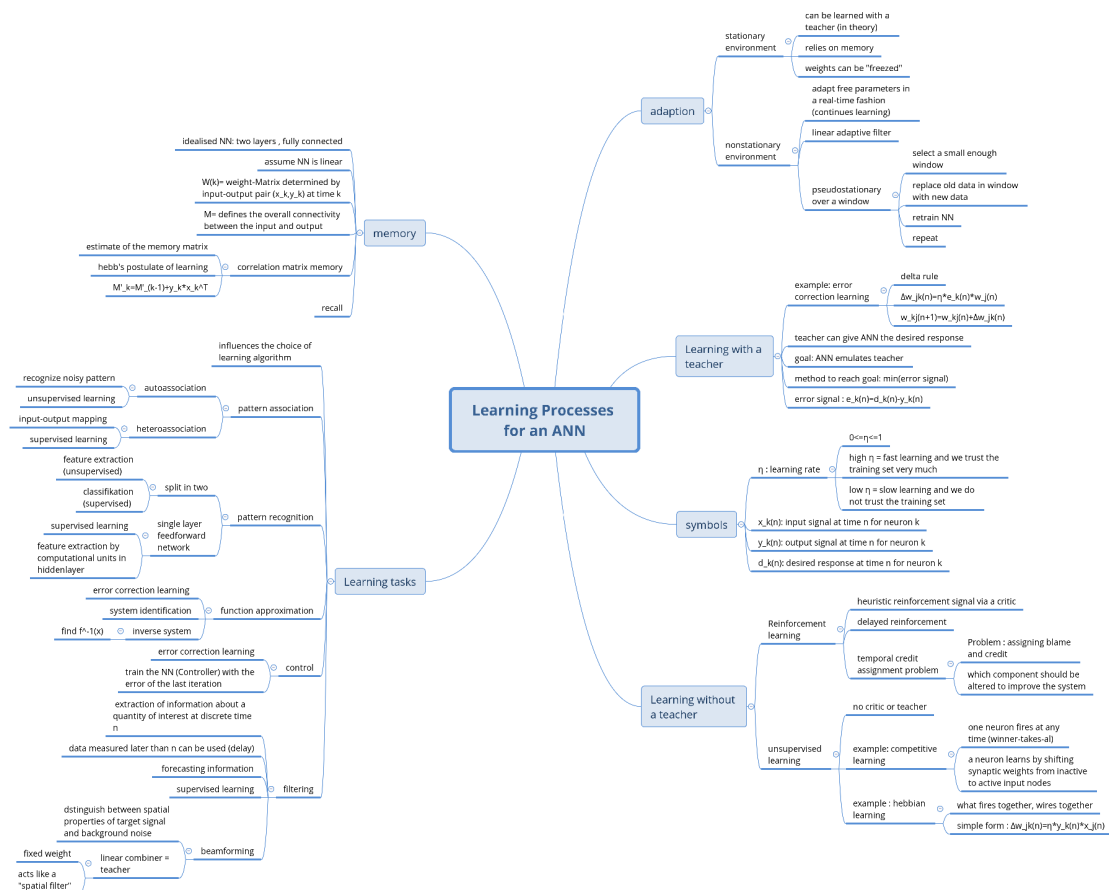
Assignment2 Neural Networks

```
In [32]: import numpy as np
import sympy as sp
sp.init_printing()
```

1 mindmap:

```
In [33]: from IPython.display import Image
Image("ANN.png")
```

Out[33]:



2 Aufgabe 1.13

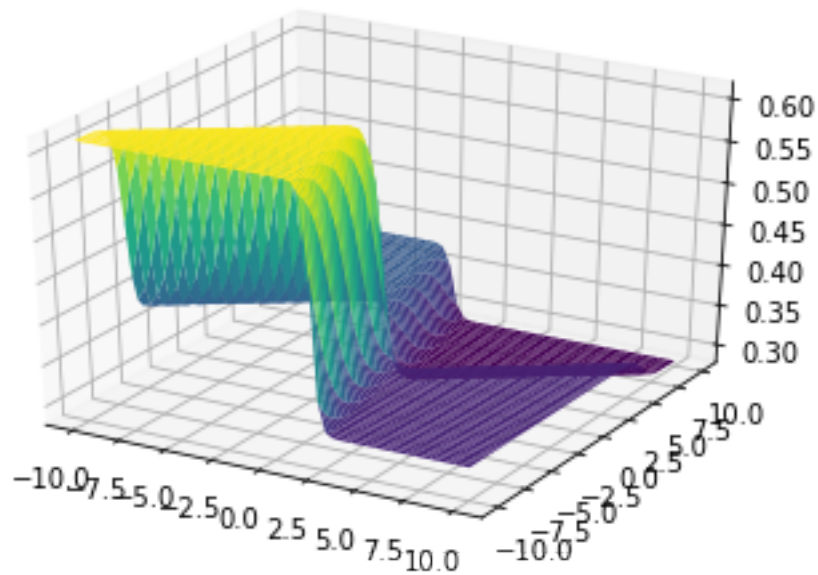
(a)

```
In [34]: a, b = sp.symbols("a, b")
```

```
def phi(x):  
    return 1 / (1 + sp.exp(-x))
```

```
In [35]: a1 = a * 5 + b  
b1 = -3 * b + 2 * a  
a2 = phi(a1)  
b2 = phi(b1)  
a3 = 3 * a2 - b2  
b3 = 6 * b2 + 4 * a2  
a4 = phi(a3)  
b4 = phi(b3)
```

```
In [36]: sp.plotting.plot3d(phi(-2 * a4 + b4))
```



```
Out [36]: <sympy.plotting.plot.Plot at 0x7fddf8a9a240>
```

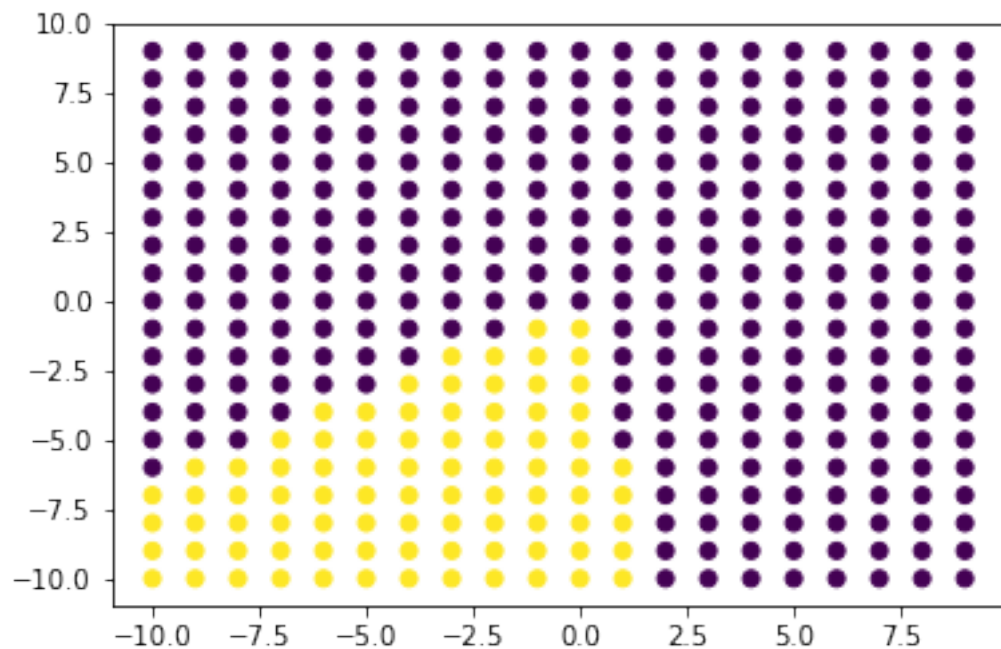
```
In [37]: def net(_a, _b):  
    return phi(-2 * a4 + b4).subs(a, _a).subs(b, _b).evalf()
```

Assume the network is a binary-classifier:

```
In [38]: values = np.zeros((400,3))
        idx = 0
        for x in range(-10, 10, 1):
            for y in range(-10, 10, 1):
                v=net(x,y)
                if(v>=0.5):
                    c=1
                else:
                    c=0
                values[idx] = np.array([x, y, c])
                idx = idx + 1
```

```
In [39]: import matplotlib.pyplot as plt
        %matplotlib inline
        plt.scatter(values[:,0], values[:,1], c=values[:,2])
```

Out[39]: <matplotlib.collections.PathCollection at 0x7fdddf8527a20>



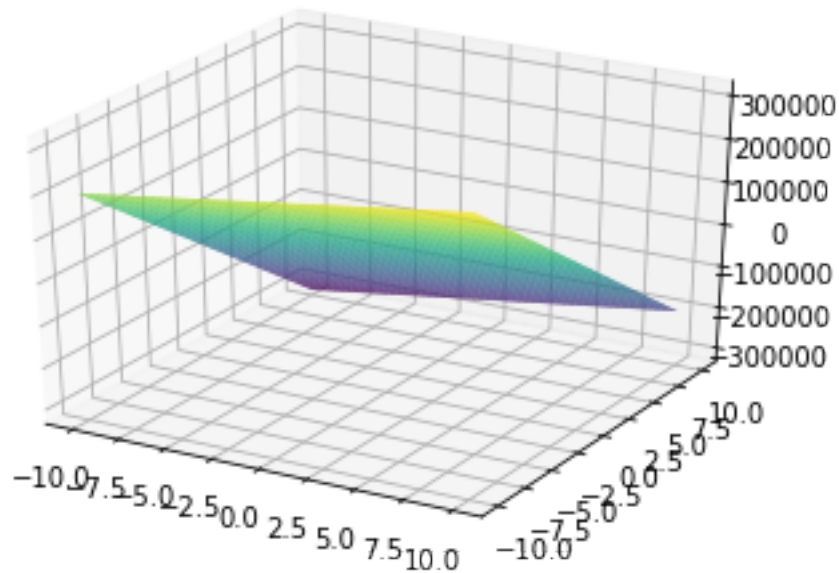
b)

Assume ϕ is linear:

```
In [40]: def phi_lin(x):
        return x*10
```

```
In [41]: a1 = a * 5 + b
        b1 = -3 * b + 2 * a
        a2 = phi_lin(a1)
        b2 = phi_lin(b1)
        a3 = 3 * a2 - b2
        b3 = 6 * b2 + 4 * a2
        a4 = phi_lin(a3)
        b4 = phi_lin(b3)

In [42]: sp.plotting.plot3d(phi_lin(-2 * a4 + b4))
```



```
Out[42]: <sympy.plotting.plot.Plot at 0x7fddf8a4aac8>
```

```
In [43]: phi_lin(-2 * a4 + b4)
```

```
Out[43]:
```

$$6000a - 26000b$$

Assume the network is a binary-classifier:

```
In [44]: def net_lin(_a, _b):
        return phi_lin(-2 * a4 + b4).subs(a, _a).subs(b, _b).evalf()

In [45]: values_lin = np.zeros((400,3))
        idx = 0
        for x in range(-10, 10, 1):
            for y in range(-10, 10, 1):
```

```

v=net_lin(x,y)
if(v>=0):
    c=1
else:
    c=0
values_lin[idx] = np.array([x, y, c])
idx = idx + 1

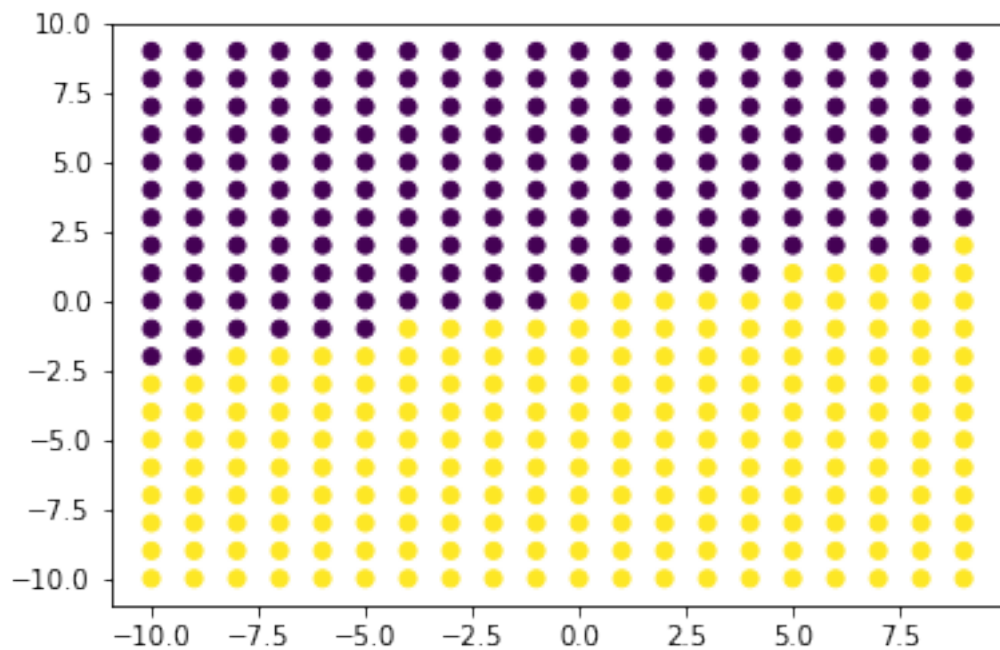
```

```

In [46]: import matplotlib.pyplot as plt
         %matplotlib inline
         plt.scatter(values_lin[:,0], values_lin[:,1], c=values_lin[:,2])

```

Out[46]: <matplotlib.collections.PathCollection at 0x7fddf82dedd8>



3 Bias

Adjust the data at the "New Classification Example (now *with* bias)" slide, such that a bias becomes necessary (not 0).

$C1 = \{(1,4), (1, 2), (0, 2)\}$

$C2 = \{(-1,2), (-1,4), (0,4)\}$

Trick for bias:

$C1' = \{(1,1,4), (1,1, 2), (1,0, 2)\}$

$C2' = \{(1,-1,2), (1,-1,4), (1,0,4)\}$

Replace tuples from $C2'$:

$C2' = \{(-1,1,-2), (-1,1,-4), (-1,-0,-4)\}$

$$w(1) = [1, 0, 0]^T = 1$$

iteration 1:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(1, 0, 0)	1	No	(1, 0, 0)
(1, 1, 2)	(1, 0, 0)	1	No	(1, 0, 0)
(1, 0, 2)	(1, 0, 0)	1	No	(1, 0, 0)
(-1, 1, -2)	(1, 0, 0)	-1	Yes	(0, 1, -2)
(-1, 1, -4)	(0, 1, -2)	9	No	(0, 1, -2)
(-1, 0, -4)	(0, 1, -2)	8	No	(0, 1, -2)

iteration 2:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(0, 1, -2)	-7	Yes	(1, 2, 2)
(1, 1, 2)	(1, 2, 2)	7	No	(1, 2, 2)
(1, 0, 2)	(1, 2, 2)	5	No	(1, 2, 2)
(-1, 1, -2)	(1, 2, 2)	-3	Yes	(0, 3, 0)
(-1, 1, -4)	(0, 3, 0)	3	No	(0, 3, 0)
(-1, 0, -4)	(0, 3, 0)	0	Yes	(-1, 3, -4)

iteration 3:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(-1, 3, -4)	-14	Yes	(0, 4, 0)
(1, 1, 2)	(0, 4, 0)	4	No	(0, 4, 0)
(1, 0, 2)	(0, 4, 0)	0	Yes	(1, 4, 2)
(-1, 1, -2)	(1, 4, 2)	-1	Yes	(0, 5, 0)
(-1, 1, -4)	(0, 5, 0)	5	No	(0, 5, 0)
(-1, 0, -4)	(0, 5, 0)	0	Yes	(-1, 5, -4)

iteration 4:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(-1, 5, -4)	-12	Yes	(0, 6, 0)
(1, 1, 2)	(0, 6, 0)	6	No	(0, 6, 0)
(1, 0, 2)	(0, 6, 0)	0	Yes	(1, 6, 2)
(-1, 1, -2)	(1, 6, 2)	1	No	(1, 6, 2)
(-1, 1, -4)	(1, 6, 2)	-3	Yes	(0, 7, -2)
(-1, 0, -4)	(0, 7, -2)	-8	Yes	(-1, 7, -6)

iteration 5:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(-1, 7, -6)	-18	Yes	(0, 8, -2)

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 2)	(0, 8, -2)	4	No	(0, 8, -2)
(1, 0, 2)	(0, 8, -2)	-4	Yes	(1, 8, 0)
(-1, 1, -2)	(1, 8, 0)	7	No	(1, 8, 0)
(-1, 1, -4)	(1, 8, 0)	7	No	(1, 8, 0)
(-1, 0, -4)	(1, 8, 0)	-1	Yes	(0, 8, -4)

iteration 6:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(0, 8, -4)	-8	Yes	(1, 9, 0)
(1, 1, 2)	(1, 9, 0)	10	No	(1, 9, 0)
(1, 0, 2)	(1, 9, 0)	1	No	(1, 9, 0)
(-1, 1, -2)	(1, 9, 0)	8	No	(1, 9, 0)
(-1, 1, -4)	(1, 9, 0)	8	No	(1, 9, 0)
(-1, 0, -4)	(1, 9, 0)	-1	Yes	(0, 9, -4)

iteration 7:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(0, 9, -4)	-7	Yes	(1, 10, 0)
(1, 1, 2)	(1, 10, 0)	11	No	(1, 10, 0)
(1, 0, 2)	(1, 10, 0)	1	No	(1, 10, 0)
(-1, 1, -2)	(1, 10, 0)	9	No	(1, 10, 0)
(-1, 1, -4)	(1, 10, 0)	9	No	(1, 10, 0)
(-1, 0, -4)	(1, 10, 0)	-1	Yes	(0, 10, -4)

iteration 8:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(0, 10, -4)	-6	Yes	(1, 11, 0)
(1, 1, 2)	(1, 11, 0)	12	No	(1, 11, 0)
(1, 0, 2)	(1, 11, 0)	1	No	(1, 11, 0)
(-1, 1, -2)	(1, 11, 0)	10	No	(1, 11, 0)
(-1, 1, -4)	(1, 11, 0)	10	No	(1, 11, 0)
(-1, 0, -4)	(1, 11, 0)	-1	Yes	(0, 11, -4)

iteration 9:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(0, 11, -4)	-5	Yes	(1, 12, 0)
(1, 1, 2)	(1, 12, 0)	13	No	(1, 12, 0)
(1, 0, 2)	(1, 12, 0)	1	No	(1, 12, 0)
(-1, 1, -2)	(1, 12, 0)	11	No	(1, 12, 0)
(-1, 1, -4)	(1, 12, 0)	11	No	(1, 12, 0)

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(-1, 0, -4)	(1, 12, 0)	-1	Yes	(0, 12, -4)

iteration 10:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(0, 12,-4)	-4	Yes	(1,13,0)
(1, 1, 2)	(1, 13, 0)	14	No	(1, 13, 0)
(1, 0, 2)	(1, 13, 0)	1	No	(1, 13, 0)
(-1, 1, -2)	(1, 13, 0)	12	No	(1, 13, 0)
(-1, 1, -4)	(1, 13, 0)	12	No	(1, 13, 0)
(-1, 0, -4)	(1, 13, 0)	-1	Yes	(0, 13, -4)

iteration 11:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(0, 13,-4)	-3	Yes	(1,14,0)
(1, 1, 2)	(1, 14, 0)	15	No	(1, 14, 0)
(1, 0, 2)	(1, 14, 0)	1	No	(1, 14, 0)
(-1, 1, -2)	(1, 14, 0)	13	No	(1, 14, 0)
(-1, 1, -4)	(1, 14, 0)	13	No	(1, 14, 0)
(-1, 0, -4)	(1, 14, 0)	-1	Yes	(0, 14, -4)

iteration 12:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(0, 14,-4)	-2	Yes	(1,15,0)
(1, 1, 2)	(1, 15, 0)	16	No	(1, 15, 0)
(1, 0, 2)	(1, 15, 0)	1	No	(1, 15, 0)
(-1, 1, -2)	(1, 15, 0)	14	No	(1, 15, 0)
(-1, 1, -4)	(1, 15, 0)	14	No	(1, 15, 0)
(-1, 0, -4)	(1, 15, 0)	-1	Yes	(0, 15, -4)

iteration 13:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(0, 15,-4)	-1	Yes	(1,16,0)
(1, 1, 2)	(1, 16, 0)	17	No	(1, 16, 0)
(1, 0, 2)	(1, 16, 0)	1	No	(1, 16, 0)
(-1, 1, -2)	(1, 16, 0)	15	No	(1, 16, 0)
(-1, 1, -4)	(1, 16, 0)	15	No	(1, 16, 0)
(-1, 0, -4)	(1, 16, 0)	-1	Yes	(0, 16, -4)

iteration 14:

Adjusted pattern	Weight applied	w(n)x(n)	Update?	New weight
(1, 1, 4)	(0, 16,-4)	0	Yes	(1,17,0)
(1, 1, 2)	(1, 17, 0)	18	No	(1, 17, 0)
(1, 0, 2)	(1, 17, 0)	1	No	(1, 17, 0)
(-1, 1, -2)	(1, 17, 0)	16	No	(1, 17, 0)
(-1, 1, -4)	(1, 17, 0)	16	No	(1, 17, 0)
(-1, 0, -4)	(1, 17, 0)	-1	Yes	(0, 17, -4)

iteration 15:

Adjusted pattern	Weight applied	w(n)x(n)	Update?	New weight
(1, 1, 4)	(0, 17,-4)	1	No	(0,17,-4)
(1, 1, 2)	(0, 17, -4)	9	No	(0, 17, -4)
(1, 0, 2)	(0, 17, -4)	-8	Yes	(1, 17, -2)
(-1, 1, -2)	(1, 17, -2)	20	No	(1, 17, -2)
(-1, 1, -4)	(1, 17, -2)	24	No	(1, 17, -2)
(-1, 0, -4)	(1, 17, -2)	7	No	(1, 17, -2)

iteration 16:

Adjusted pattern	Weight applied	w(n)x(n)	Update?	New weight
(1, 1, 4)	(1, 17, -2)	10	No	(1,17,-2)
(1, 1, 2)	(1, 17, -2)	14	No	(1, 17, -2)
(1, 0, 2)	(1, 17, -2)	-3	Yes	(2, 17, 0)
(-1, 1, -2)	(2, 17, 0)	15	No	(2, 17, 0)
(-1, 1, -4)	(2, 17, 0)	15	No	(2, 17, 0)
(-1, 0, -4)	(2, 17, 0)	-2	Yes	(1, 17, -4)

iteration 17:

Adjusted pattern	Weight applied	w(n)x(n)	Update?	New weight
(1, 1, 4)	(1, 17, -4)	2	No	(1,17,-4)
(1, 1, 2)	(1, 17, -4)	10	No	(1, 17, -4)
(1, 0, 2)	(1, 17, -4)	-7	Yes	(2, 17, -2)
(-1, 1, -2)	(2, 17, -2)	11	No	(2, 17, -2)
(-1, 1, -4)	(2, 17, -2)	7	No	(2, 17, -2)
(-1, 0, -4)	(2, 17, -2)	6	No	(2, 17, -2)

iteration 18:

Adjusted pattern	Weight applied	w(n)x(n)	Update?	New weight
(1, 1, 4)	(2, 17, -2)	11	No	(2,17,-2)
(1, 1, 2)	(2, 17, -2)	15	No	(2, 17, -2)
(1, 0, 2)	(2, 17, -2)	-2	Yes	(3, 17, 0)
(-1, 1, -2)	(3, 17, 0)	14	No	(3, 17, 0)

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(-1, 1, -4)	(3, 17, 0)	14	No	(3, 17, 0)
(-1, 0, -4)	(3, 17, 0)	-3	Yes	(2, 17, -4)

iteration 19:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(2, 17, -4)	3	No	(2,17,-4)
(1, 1, 2)	(2, 17, -4)	11	No	(2, 17, -4)
(1, 0, 2)	(2, 17, -4)	-6	Yes	(3, 17, -2)
(-1, 1, -2)	(3, 17, -2)	18	No	(3, 17, -2)
(-1, 1, -4)	(3, 17, -2)	14	No	(3, 17, -2)
(-1, 0, -4)	(3, 17, -2)	5	No	(3, 17, -2)

iteration 20:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(3, 17, -2)	13	No	(3, 17,-2)
(1, 1, 2)	(3, 17, -2)	16	No	(3, 17, -2)
(1, 0, 2)	(3, 17, -2)	-1	Yes	(4, 17, 0)
(-1, 1, -2)	(4, 17, 0)	13	No	(4, 17, 0)
(-1, 1, -4)	(4, 17, 0)	13	No	(4, 17, 0)
(-1, 0, -4)	(4, 17, 0)	-4	Yes	(3, 17, -4)

iteration 21:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(3, 17, -4)	4	No	(3, 17,-4)
(1, 1, 2)	(3, 17, -4)	12	No	(3, 17, -4)
(1, 0, 2)	(3, 17, -4)	-5	Yes	(4, 17, -2)
(-1, 1, -2)	(4, 17, -2)	9	No	(4, 17, -2)
(-1, 1, -4)	(4, 17, -2)	5	No	(4, 17, -2)
(-1, 0, -4)	(4, 17, -2)	4	No	(4, 17, -2)

iteration 22:

Adjusted pattern	Weight applied	$w(n)x(n)$	Update?	New weight
(1, 1, 4)	(4, 17, -2)	14	No	(4, 17, -2)
(1, 1, 2)	(4, 17, -2)	17	No	(4, 17, -2)
(1, 0, 2)	(4, 17, -2)	0	Yes	(5, 17, 0)
(-1, 1, -2)	(5, 17, 0)	12	No	(5, 17, 0)
(-1, 1, -4)	(5, 17, 0)	12	No	(5, 17, 0)
(-1, 0, -4)	(5, 17, 0)	-5	Yes	(4, 17, -4)

iteration 23:

Adjusted pattern	Weight applied	w(n)x(n)	Update?	New weight
(1, 1, 4)	(4, 17, -4)	5	No	(4, 17, -4)
(1, 1, 2)	(4, 17, -4)	13	No	(4, 17, -4)
(1, 0, 2)	(4, 17, -4)	-4	Yes	(5, 17, -2)
(-1, 1, -2)	(5, 17, -2)	16	No	(5, 17, -2)
(-1, 1, -4)	(5, 17, -2)	20	No	(5, 17, -2)
(-1, 0, -4)	(5, 17, -2)	3	No	(5, 17, -2)

iteration 24:

Adjusted pattern	Weight applied	w(n)x(n)	Update?	New weight
(1, 1, 4)	(5, 17, -2)	15	No	(5, 17, -2)
(1, 1, 2)	(5, 17, -2)	18	No	(5, 17, -2)
(1, 0, 2)	(5, 17, -2)	1	No	(5, 17, -2)
(-1, 1, -2)	(5, 17, -2)	16	No	(5, 17, -2)
(-1, 1, -4)	(5, 17, -2)	20	No	(5, 17, -2)
(-1, 0, -4)	(5, 17, -2)	3	No	(5, 17, -2)

function : $17x_1 - 2x_2 + 5$