In [3]:

```
import numpy as np
import sympy as sp
sp.init_printing()
```

In [4]:

```
a,v = sp.symbols("a, v")
```

1.2

The follwoing cell shows the sigmoid function:

In [5]:

```
f1 = (1 - sp.exp(-a*v)) / (1 + sp.exp(-a*v))
f1
```

Out[5]:

$$\frac{1 - e^{-av}}{1 + e^{-av}}$$

The drivative is given in the next cell:

In [6]:

```
sp.diff(f1, v)
```

Out[6]:

$$\frac{a\left(1 - e^{-av}\right)e^{-av}}{\left(1 + e^{-av}\right)^2} + \frac{ae^{-av}}{1 + e^{-av}}$$

When we divide $[1 - \varphi^2(v)]$ from the derivative we get the following result:

In [7]:

```
sp.simplify(sp.diff(f1, v) / (1 - f1**2))
```

Out[7]:

$$\frac{a}{2}$$

Since $\frac{a}{2}$ is the result of dividing $[1 - \varphi^2(v)]$ from the derivative, the derivative can be written as $\frac{a}{2}[1 - \varphi^2(v)]$.

The value of the derivative at the origin is computed in the next cell:

In [8]:

```
sp.diff(f1, v).subs(v, 0)
```

Out[8]:

$$\frac{a}{2}$$

The value of the derivative at the origin is $\frac{a}{2}$.

If a is infinitely large then e^{-av} will converge to 0 so the whole function $\varphi(v)$ will be a straight line parallel to the x axis at the y value 1.

1.3

The following cell shows the sigmoid function:

In [12]:

```
f3 = v / (sp.sqrt(1 + v**2))
f3
```

Out[12]:

$$\frac{v}{\sqrt{v^2 + 1}}$$

If we differentiate this function and compare it to the given $\frac{\varphi^3(v)}{v^3}$ by subtracting both, we see that the result is 0:

In [34]:

```
sp.simplify(sp.diff(f3, v) - (f3**3 / v**3))
```

Out[34]:

$$0$$

That proves, that the derivative of $\varphi(v)$ is given by $\frac{\varphi^3(v)}{v^3}$.

In the following cell the value of the derivative at $v = 0$ is computed:

In [11]:

```
sp.diff(f3, v).subs(v, 0)
```

Out[11]:

$$1$$

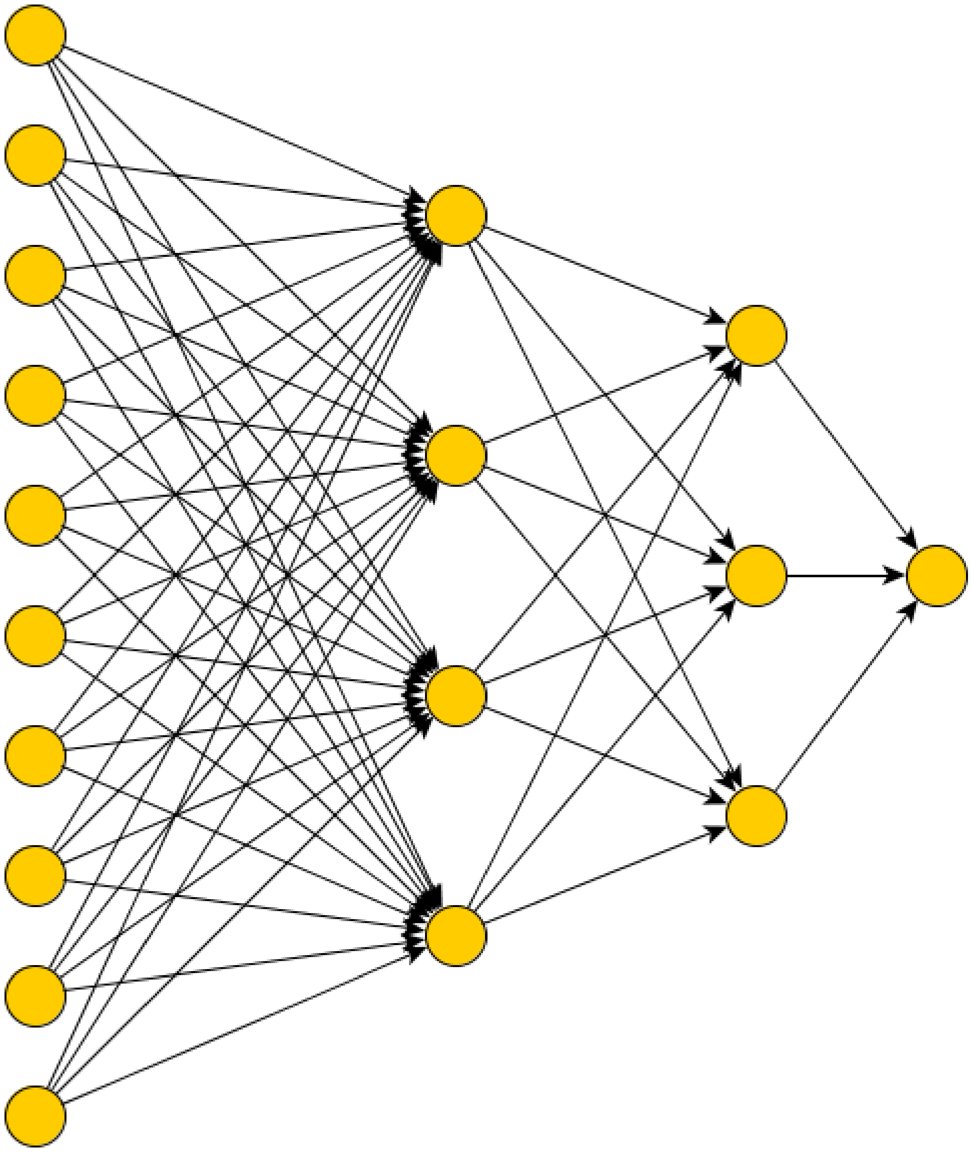
The result is 1, but we should keep in mind that the derivate is undefined for $v = 0$ since v is in the denominator and dividing by 0 is not allowed. However, the function converges to 1 for $v \to 0$.

1.12

In [32]:

```
from IPython.display import Image
Image("1_12_graph.png")
```

Out[32]:



1.13

a)

In [30]:

```
a, b = sp.symbols("a, b")
phi = sp.Function("varphi")
```

In [37]:

```
a1 = a * 5 + b
b1 = -3 * b + 2 * a
a2 = phi(a1)
b2 = phi(b1)
a3 = 3 * a2 - b2
b3 = 6 * b2 + 4 * a2
a4 = phi(a3)
b4 = phi(b3)

phi(-2 * a4 + b4)
```

Out[37]:

$$\varphi(-2\varphi(-\varphi(2a - 3b) + 3\varphi(5a + b)) + \varphi(6\varphi(2a - 3b) + 4\varphi(5a + b)))$$

For two given inputs a and b the input-output mapping is given in the previous cell.

b)

If the last neuron is linear instead of the function φ then the input-output mapping is calculated in the next call, where w is the weight of the last neuron.

In [43]:

```
w = sp.Symbol("w")
w * (-2 * a4 + b4)
```

Out[43]:

$$w\left(-2\varphi(-\varphi(2a - 3b) + 3\varphi(5a + b)) + \varphi(6\varphi(2a - 3b) + 4\varphi(5a + b))\right)$$