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Integer and binary integer linear programming – Laboratory 27/04/20

A. Integer programming problem: Task 2

1. Solution to the LR problem

$$\min \leftarrow x_1 + x_2 + 2x_3$$

under constraints

$$2x_1 + 3x_2 \geq 5$$

$$x_2 + 2x_3 \geq 4$$

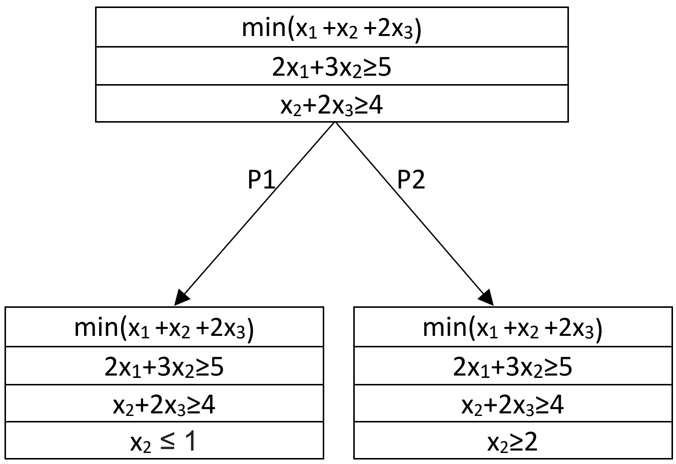
$$x_i \geq 0,\ x_i \in Z,\ i=1,2,3$$

Solution: $x_1=0$, $x_2=1.6667$, $x_3=1.1667$ with the objective value f(x) = 4

x_1 is an integer, therefore x_2 and x_3 are natural choices for the first branch steps. The obtained solution trees are presented below.

2. Solution tree

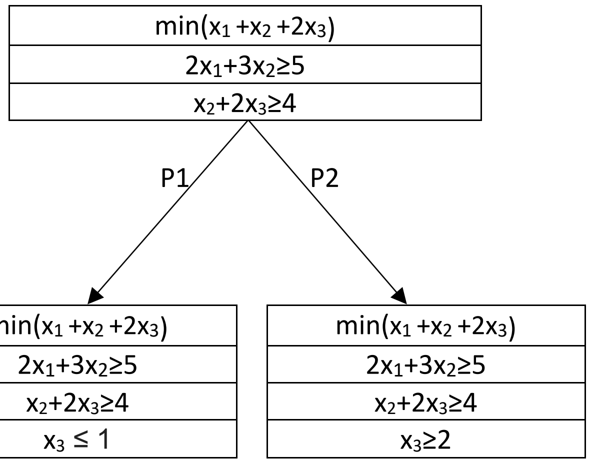
Taking x_2 into account as a first branch step.



Solutions for branches:

- For first subproblem(P1). Its LR solution is **$x_1 = 1$, $x_2 = 1$, $x_3 = 1.5$** and the objective value **f(x) = 5**. Therefore, the objective value of the solution of the IP for this problem is at least 5, which is worse than 4.
- For second subproblem(P2). Its LR solution is **$x_1 = 0$, $x_2=2$, $x_3=1$** and the objective value **f(x) = 4**. Since all variable values are integers, it is also the solution to the IP problem.

Taking x_3 into account as a first branch step.



Solutions for branches:

- For first subproblem(P1). Its LR solution is **x_1 = 0, x_2 = 2, x_3 = 1** and the objective value **f(x) = 4**, which is the same solution as solution with x_2 as a first branching step.
- For second subproblem(P2). Its LR solution is **x_1 = 0, x_2=1.6667, x_3=2** and the objective value **f(x) = 5.6667**. Therefore, the objective value of the solution of the IP for this problem is at least 6, which is worse than 4.

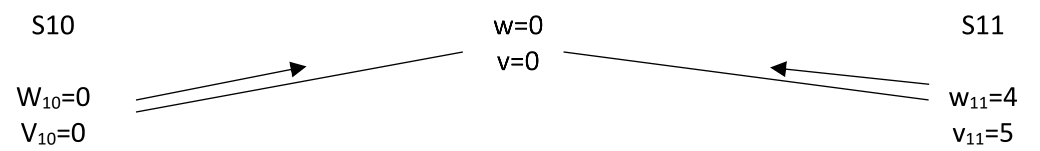
B. Knapsack problem: Task 3

$w_{max} = 6$

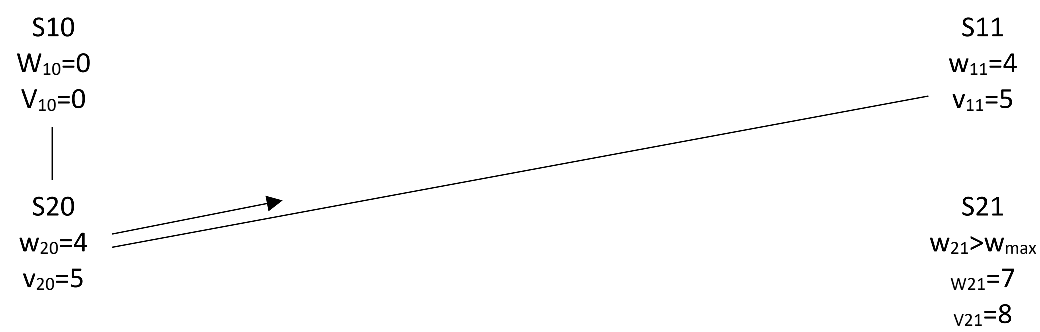
i	v_i	w_i
1	5	4
2	3	3
3	4	2
4	3	2

Decision tree for given problem was shortened, due to its size.

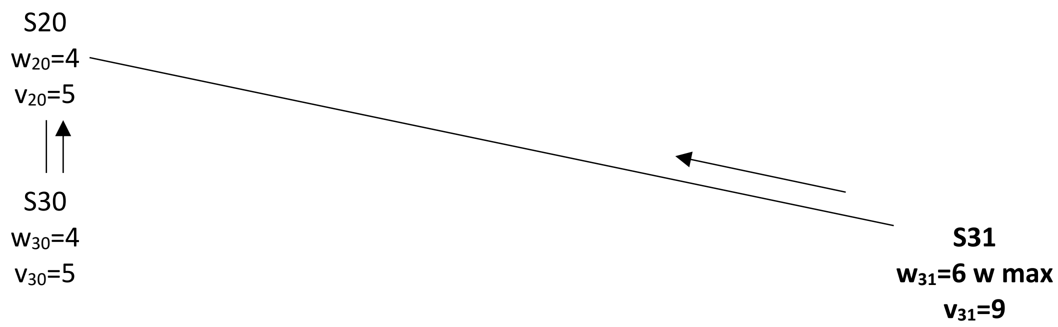
At first stage x_1:



At second stage x_2: $w^*_{21} = w_{11}+3 = 7$, $w_{21}>w_{max}$ and $w_{20} = 4$, $v_{20} = 5$.



At third stage x_3:



Solution: $x_1=1$, $x_2=0$, $x_3=1$, $x_4=0$
w=6
v=9