∘ Critical Limit Theorem. (중심극한정리)

기초지식 : 모집단이 정규분포이면. 표본의 평균 역시. 정규분포를 따른다.

; If the population distribution is $N(\mu, \sigma^2)$

; $X_1, X_2 \dots X_n$ are iid $N(\mu, \sigma^2)$

; $\bar{X} \sim N(\mu, \sigma^2/n)$

√ The critical limit theorem provides an important extension to these results.

√ Regardless of the actual distribution of the individual random variables X_i, the distribution of their average $\bar{X}$ is closely approximately by a $N(\mu, \sigma^2/n)$ for large sample size.

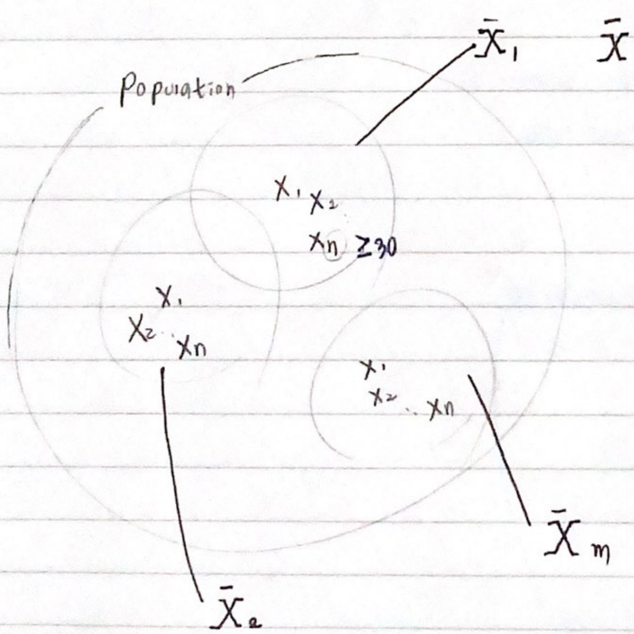
√

심플하게 정리

If $X_1, X_2 \dots X_n$ is a sequence (배열)
of indipendent (독립적이고) identically distributted random variables. with a mean μ
동일한 분산 ,, 을 따름 이.
and a variance σ^2,
then the distribution of their average $\bar{x}$ can be approximated by
$N(\mu, \sigma^2/n)$ for large n (in general $n \geq 30$)

n, m

$\bar{X}_1, \dots \bar{X}_m \sim N(\mu, \frac{\sigma^2}{n})$



° Two Independent populations.

(any distributions) ; 모집단이 2개인 경우.

1. $\bar{X}_1 - \bar{X}_2$ 목표, $\mu_1 - \mu_2$

- pop. 1 $(\mu_1, \sigma_1^2) \rightarrow$

; Sample 1 with n_1 obsns $\rightarrow \bar{X}_1$

- pop 2. $(\mu_2, \sigma_2^2) \rightarrow$

; Sample 2 with n_2 obsns $\rightarrow \bar{X}_2$

° Expected Value of $\bar{X}_1 - \bar{X}_2$

$$E[\bar{X}_1 - \bar{X}_2] = E[\bar{X}_1] - E[\bar{X}_2]$$

(독립이기에 가능)

$= \mu_1 - \mu_2$

° Variance of $\bar{X}_1 - \bar{X}_2$

- Note: $\bar{X}_1$ and $\bar{X}_2$ are "independent random variables."

$$V(\bar{X}_1 - \bar{X}_2) = V(\bar{X}_1) + V(\bar{X}_2)$$

$$= \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

If, pop. 1 is $N(\mu_1, \sigma_1^2)$, and,

pop. 2 is $N(\mu_2, \sigma_2^2)$,

- $\bar{X}_1 - \bar{X}_2$ is distributed

$$N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

예제) $\bar{X}_1 - \bar{X}_2 \sim N(-8, 0.69)$ 라면,

$P[\bar{X}_1 - \bar{X}_2 \le -6]$?

표준정규분포화

$$= P\left[\frac{\bar{X}_1 - \bar{X}_2 - (-8)}{\sqrt{0.69}} \le \frac{-6 - (-8)}{\sqrt{0.69}}\right]$$

$$= P[Z \le 2.12]$$

$$= \Phi(2.12) = 0.9830$$

2. s^2의 확률?

° Sample Variance 가산 정리

$$s^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (X_i - \bar{X})^2$$

Note $\sum_{i=1}^{n} (X_i - \bar{X})^2 = s^2 \underbrace{(n-1)}_{}$ "

자유도라는 의미이다.

Assume $X_1, X_2, \ldots X_n$ are iid $N(\mu, \sigma^2)$

Then. $(n-1) s^2 / \sigma^2 \sim \chi^2(n-1)$

Continent (상수)

주의

$s^2 \sim \chi^2(n-1)$ " 카이스퀘어 chi-square를 따른다고 할 수있음.