Chapter 19
Hammered Strings

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In the next three chapters we consider the science of hammered string instruments. In this chapter, we present a brief discussion of vibrating strings excited by a hard or soft hammer. Chapter 20 discusses the most important hammered string instrument, the piano – probably the most versatile and popular of all musical instruments. Chapter 21 discusses hammered dulcimers, especially the American folk dulcimer.

19.1 Dynamics of the Hammer–String Interaction

The dynamics of the hammer–string interaction has been the subject of considerable research, beginning with von Helmholtz (1877). The problem drew the attention of C.V. Raman and Banerji (1920), R.N. Ghosh (1926), and other Indian researchers in the 1920s and 1930s. More recently, the subject has been reviewed by Askenfelt and Jansson (1985, 1993), Chaigne and Askenfelt (1994), Conklin (1996), and Hall (1986, 1987a–b, 1992), Hall and Askenfelt (1988).

When a string is plucked, we assume an initial displacement but zero initial velocity (see Chap. 2). The opposite set of initial conditions, zero initial velocity with a specified initial velocity, could be imparted by hard hammer striking the string at $t = 0$. Of course, a blow by a real hammer does not instantly impart a given velocity to the string. The initial velocity depends in a rather complicated way on a number of factors, such as the width, mass, and compliance of the hammer (Hall 1986, 1987a–b).

Suppose that the string is struck by a hard, narrow hammer with velocity V. After a short time t, a portion of the string with length $2\,ct$ is set into motion. Because this mass increases and becomes comparable to the hammer mass M, the hammer is slowed down and will eventually be stopped. With a string of finite length,

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however, reflected impulses return while the hammer has appreciable velocity, and these reflected impulses interact with the hammer in a rather complicated way, causing it to be thrown back from the string.

At the point of contact, the string and hammer together satisfy the equation

$$M\frac{\partial^2 y}{\partial t^2} = T\Delta\left(\frac{\partial y}{\partial x}\right), \tag{19.1}$$

where elsewhere the string continues to satisfy the equation for transverse waves on a vibrating string.

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2} = c^2\frac{\partial^2 y}{\partial x^2} \tag{19.2}$$

The discontinuity in the string slope $\Delta(\partial y/\partial x)$ according to (19.1) is responsible for the force that slows down the hammer. Equation (19.1) is satisfied at the contact point by a velocity

$$V(t) = V\mathrm{e}^{-t/\tau}, \tag{19.3}$$

where $\tau = Mc/2T$ may be termed the deceleration time (Hall 1986). The corresponding displacement is

$$y(t) = V\tau(1 - \mathrm{e}^{-t/\tau}). \tag{19.4}$$

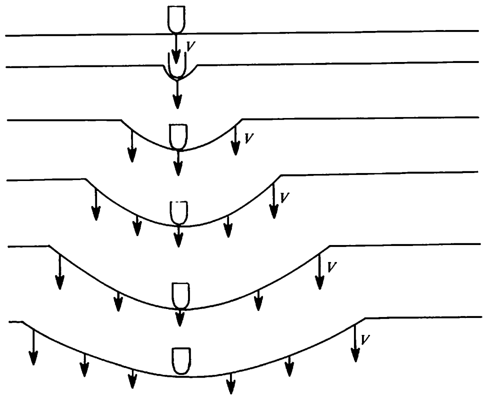
The displacement at the contact point approaches $VMc/2T$, and the velocity approaches zero. If the string were very long, the displacement and velocity elsewhere on the string could be found by substituting $t - [(x - x_0)/c]$ for t in (19.2) and (19.3), as shown in Fig. 19.1.

Only when the string is very long or the hammer is very light does the hammer stop, as in Fig. 19.1. In a string of finite length, reflected pulses return from both ends of the strings and interact with the moving hammer in a fairly complicated way. Eventually the hammer is thrown clear of the string, and the string vibrates freely in its normal modes.

In general, the harmonic amplitudes in the vibration spectrum of a struck string fall off less rapidly with frequency than those of plucked strings (see Chap. 2). For a very light hammer with mass M much less than the mass of the string M_s, the spectrum dips to zero for harmonic numbers that are multiples of $1/\beta$ (where the string is struck at a fraction β of its length), but otherwise does not fall off with frequency, as shown in Fig. 19.2a. (The spectrum of sound radiated by a piano may be quite different from the vibration spectrum of the string.)

If the hammer mass is small but not negligible compared to the mass of the string, the spectrum envelope falls off as $1/n$ (6 dB/octave) above a mode number given by $n_m = 0.73\ M_s/M$, as shown in Fig. 19.2b. Note that for high harmonic (mode) numbers, there are missing modes between those in Fig. 19.2a.

Fig. 19.1 Displacement and velocity of a long string at successive times after being struck by a hard narrow hammer having a velocity V



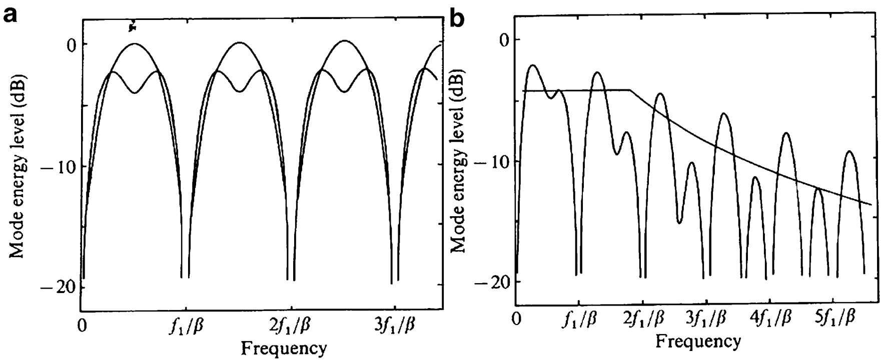


Fig. 19.2 Spectrum envelopes for a string struck at a fraction β of its length. (**a**) Hammer mass $M \ll$ string mass M_s; (**b**) $M = 0.4/\beta M_s$ (from Hall 1986)

19.2 Piano Hammers

Hammer masses in a modern grand piano vary over a factor of 2–3, from about 10 g (bass) to 3.8 g (treble). The ratio of hammer mass to string mass varies even more widely (about 8–0.08). The hammers have hardwood cores of graduated sizes, which are covered by one or two layers of felt that increase in thickness from treble to bass. The outer layer for each set of hammers is formed from a strip of felt that increases in thickness lengthwise from treble to bass, and this strip is attached to the hammer in a press that is long enough to accommodate a complete set of 88 hammers. After pressing, the outer felt generally has a density in the range of 0.6–0.7 g/cm^3 (Conklin 1996).

Hammer hardness is an important factor in piano sound. Hard hammers better excite high-frequency modes in a string than soft hammers, and thus treble hammers are considerably harder than bass hammers. The static hardness of hammers can be tested with a durometer or hardness tester. Felt does not obey Hooke's law but rather behaves as a hardening spring. Force–compression measurements on piano hammers can be characterized by a power law

$$F - K\xi^p, \tag{19.5}$$

where F is force, ξ is the compression, K is a generalized stiffness, and the exponent p describes how much the stiffness changes with force, ranging from 2.2 to 3.5 for hammers taken from pianos and 1.5–2.8 for unused hammers (Hall and Askenfelt 1988).

Dynamic measurements of the force and compression show hysteresis: K and p take on different values for compression and relaxation. Hard hammers tend to have a larger value of the exponent p than soft hammers. Using a nonlinear hammer model along with a flexible string, Hall (1992) found that increasing exponent p leads to a smoother and more gradual rise of force as the hammer compression begins and a correspondingly steeper falloff in string mode amplitudes toward higher frequencies.

One way to test the dynamic hardness of a piano hammer is to observe the force–time pulse shape of the hammer as it strikes a rigid surface. Hall and Askenfelt (1988) determined that the contact duration τ is related to the maximum force $F_{\max}$ by $\tau\alpha(F_{\max})^{(1-p)/2p}$, where p is the same as in (19.5). The residual shock spectrum provides related information about what string modal frequencies would be most effectively excited by a given hammer (Russell and Rossing 1998).

19.3 String Excitation by a Piano Hammer

In Sect. 19.1 we discussed what happens when a string is struck by a hammer having considerably less mass than the string. In the most extreme case (which may apply at the bass end of the piano), the hammer is thrown clear of the string by the first reflected pulse. The theoretical spectrum envelope (shown in Fig. 19.2a is missing harmonics numbered n/β (where β is the fraction of the string length at which the hammer strikes), but it does not fall off systematically at high frequency. When the hammer mass is a slightly greater portion of the string mass, as in Fig. 19.2b, the spectrum envelope falls off as $1/n$ (6 dB/octave) above a certain mode number.

A heavier hammer is less easily stopped and thrown back by the string. It may remain in contact with the string during the arrival of several reflected pulses, or it may make multiple contacts with the string. Analytical models of hammer behavior are difficult to construct, but simulations can be of some value. Hall (1987a–b), for example, showed that in the case of the hard, narrow hammer a mode energy spectrum envelope with a slope of −6 dB/octave at high frequency emerges, although the individual mode amplitudes are not easily predicted.

Askenfelt and Jansson (1985) investigated the effect of hammer mass by exciting the C4 string with three hammers: the original one, a heavy bass hammer, and a light treble hammer. They obtained a longer than normal contact time with the bass hammer and a shorter than normal contact time with the treble hammer, as expected. Although the shapes of the string vibrations did not look very different for the different hammers, the sounds they produced were markedly different. The lighter, harder treble hammer, for example, produced a sound somewhat like that of a harpsichord.

19.4 Hammer Position on the String

Figure 19.3 compares the hammer striking position along the strings *d*/*L* for a 1720 Cristofori piano with a modern grand piano. The striking position *d*/*L* varies considerably in Cristofori pianos for no apparent purpose. Later in the eighteenth century, a consensus developed that for the best tone the hammers should strike the strings between 1/7 and 1/9 of the speaking length. Figure 19.3 suggest that nowadays that criterion is followed in the lower half of the piano, but that the striking position *d*/*L* is smaller in the treble. The optimum striking position at key 88 varies with hammer weight and shape and also with string size and contact conditions but normally occurs for *d*/*L* between 1/12 and 1/17 (Conklin 1996).

In theory, an ideal stretched string struck at a certain point will not vibrate at frequencies for which vibrational nodes exist at the striking point. For actual piano strings, a large (~40 dB) but finite attenuation of the *L*/*d*-th partial does occur, but this is not a major factor in selecting the striking position. In the mid-treble, the best value is often a compromise between fundamental strength and best tone. Reducing *d*/*L* will generally make the tone sound “thinner” because of less fundamental, but too large a ratio will reduce clarity due to excessive hammer dwell time (Conklin 1996).

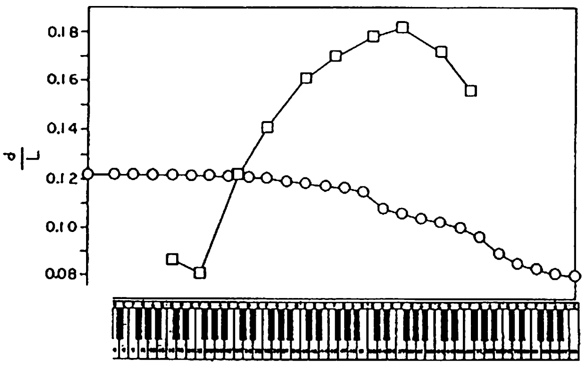


Fig. 19.3 Hammer striking ratio *d*/*L* for a Cristofori piano (*open square*) and a modern grand piano (*open circle*) (Conklin 1996)

19.5 String Excitation in a Hammered Dulcimer

Hammered dulcimers are typically played with wooden hammers having a mass of about 8 g. Because that mass is distributed along the length of the hammer, however, they act as light hard hammers. Unlike a piano hammer, which is essentially free of the action at the moment it strikes the string, the dulcimer hammer is still in contact with the player's fingers. It is not uncommon for the wooden hammer to lose and regain string contact one or more times (Peterson 1995).

The strike position in the hammered dulcimer is not only variable, but it is also near the bridge, rather than near the terminal end as in a piano. The strike ratio α, the strike point to bridge distance divided by the string length, is of particular interest. Increasing the strike ratio α increases the hammer contact time, decreases the hammer force, and generates a more complex hammer force (see Chap. 21).

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