

Assessment of Multi-Hop Interpersonal Trust in Social Networks by Three-Valued Subjective Logic

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Abstract—Assessing multi-hop interpersonal trust in online social networks (OSNs) is critical for many social network applications such as online marketing but challenging due to the difficulties of handling complex OSNs topology, in existing models such as subjective logic, and the lack of effective validation methods. To address these challenges, we for the first time properly define trust propagation and combination in arbitrary OSN topologies by proposing 3VSL (Three-Valued Subjective Logic). The 3VSL distinguishes the *posteriori* and *priori* uncertainties existing in trust, and the difference between *distorting* and *original* opinions, thus be able to compute multi-hop trusts in arbitrary graphs. We theoretically proved the capability based on the Dirichlet distribution. Furthermore, an online survey system is implemented to collect interpersonal trust data and validate the correctness and accuracy of 3VSL in real world. Both experimental and numerical results show that 3VSL is accurate in computing interpersonal trust in OSNs.

Index Terms—Interpersonal trust, trust establishment, online social networks, three-valued subjective logic, Dirichlet distribution

I. INTRODUCTION

With the emergence of Online Social Networks (OSNs), user-generated content and interactions have dominated web activity and the questions of whom and what to trust have become increasingly important. Trust assessment in OSNs can be applied in various domains such as online marketing, proactive friendships construction [25], and networking security [12], [14], [16], [17], [19], [23], [24]. Various online trust models have been developed but one thing missing from them is that they do not have an interpersonal trust component, which widely exists in real world. Therefore, this paper tries to answer a fundamental question: *how to accurately model and compute the multi-hop interpersonal trust between two users connected within OSNs?*

Trust can be defined in different ways such as rating-based *reputation* in online marketing and local search applications, *preferences* in recommendation systems, and *probability* of being a normal node in network security analysis. We define interpersonal trust as *the probability that a trustee will behave as expected by a trustor*. This general definition of interpersonal trust makes it applicable for a wide range of applications. We also assume trustworthiness is determined by objective evidences, i.e. the cognitive features of trust are not our concern [2], [6]. Subjective logic models trust as a tuple in the form of (*belief, distrust, uncertainty*) and is widely used in assessing multi-hop interpersonal trust [9]. However,

it can only handle series-parallel social network topology, i.e. complex trust topology need to be simplified by removing or selecting edges. Such simplification will result information loss, which is confirmed in our numerical analysis.

To cope with arbitrary topology in OSNs, we propose the 3VSL (Three-Valued Subjective Logic), developed from subjective logic, as a solution for assessing multi-hop interpersonal trust in OSNs. Unlike subjective logic, we define interpersonal trust as trinary event (belief, distrust, neutral) instead of binary event (belief, distrust), hence extend Beta distribution to Dirichlet distribution. Neutral state expresses the *posteriori* uncertainty in trust generated by trust propagation, which is ignored in subjective logic. *The introduction of neural state makes operations in 3VSL different from subjective logic. More importantly, it completes the underlying theory of 3VSL.* Leveraging on this new definition, operations (discounting and combining) on trust are redesigned in 3VSL. Furthermore, a solution for applying 3VSL to assess trust in arbitrary social network graph is given and proved to be correct. To the best of our knowledge, 3VSL is the first model to compute multi-hop interpersonal trust in arbitrary network topology.

Also, we implement an online survey system to collect interpersonal trust data. 100 participants are invited to evaluate the trustworthiness of their 1st and 2nd hop friends by filling out a questionnaire designed for measuring interpersonal trust [8]. In addition, we conduct a numerical analysis to show the features of 3VSL. Experimental and numerical results indicate that 3VSL is accurate in assessing interpersonal trust.

Major contributions of this paper are as follows:

1. The shortcomings of subjective logic-based approaches are identified and addressed in the 3VSL model. 3VSL distinguishes the *priori* and *posteriori* uncertainties, the *distorting* and *original* opinions, and redefine the discounting and combining operations on opinions. 3VSL is proved to be able to assess multi-hop interpersonal trust in arbitrary network topology.

2. The 3VSL model is validated by the interpersonal trust data collected from 100 participants. To the best of our knowledge, this is the first established dataset for assessing interpersonal trust in real-life multi-hop social networks.

The rest of this paper is organized as follows: In sections II and III, we introduce related work and the problem statement. In section IV, we present details of 3VSL. In sections V and VI, we validate 3VSL through experimental and numerical

analysis. A conclusion will be given in section VII.

II. RELATED WORK

Many previous efforts have been devoted to the study of using probability distribution to express and model trust in a computational way. In probability distribution-based methods, trust is defined as a probability distribution of a certain event. For example, Beta distribution [9] is used to model binary discrete random variables, i.e. whether a person is trustworthy or not. In [1], [18], Gaussian distribution is proposed to handle the situation when possible outcomes are continuous random variables. Beta distribution is then expanded to the Dirichlet distribution to handle multiple discrete random variables [3]. Probability distributions are often associated with Bayesian analysis [1], [15], [18] so that they can model trust by integrating information from various sources such as reputations, preference and group behaviors.

Based on Beta distribution, Josang, etc. proposed subjective logic [9] to assess interpersonal trust in multi-hop trust graph through trust propagation and fusion among people who do not have direct social connections. In [10], [11], how to apply subjective logic in trust graph is studied. Subsequently, it is redefined in [13], [20]–[22] and the accuracy of trust estimation is improved. In [5], the authors introduce the selection operation which selects the strongest path to compute multi-hop trust when several trust paths exists in social networks. In [26], trust information computed by subjective logic is used in the routing domain.

Nevertheless, all previous work are not able to correctly define how interpersonal trust changes during trust propagation. Due to this issue, subjective logic cannot handle complex topology in social networks. Unlike subjective logic, 3VSL considers certain evidences are distorted and transferred into the neural state when trust propagates from one person to another. As a complementary of one-hop trust estimation, we believe 3VSL is an improvement to the framework of social trust computation.

III. SYSTEM MODEL AND PROBLEM STATEMENT

A *trust social network* is modeled as a directed graph $G(V, E)$ where a vertex $u \in V$ represents a person, and an edge $e(u, v) \in E$ denotes how much u trusts v (from his/her direct interaction experience). Each edge is expressed as an *opinion*, which means v 's trustworthiness to u .

In a social network, two edges are in *series* if they are incident to a vertex of degree 2 and are *parallel* if they join the same pair of distinct vertices. As shown in Fig. 1(a) and Fig. 1(b), two users can be simply connected in a *serial* topology or *parallel* topology. They can also be connected in a *bridge* topology (in Fig. 1(c)), where the connection from A to C cannot be decomposed into series and parallel topologies.

To apply subjective logic, the graph between two users must be a *directed series-parallel graph*, i.e. subjective logic only works on *directed two-terminal series-parallel graphs* (DTTSPG) [7]. Since the bridge topology is not a DTTSPG, it is unsolvable in subjective logic.

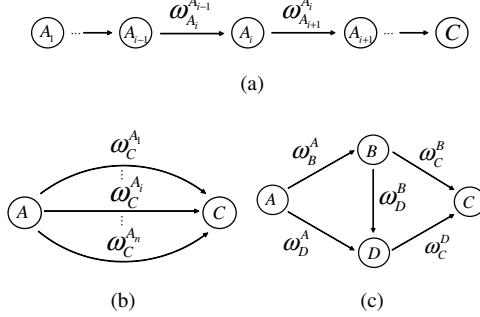


Fig. 1: Examples of serial, parallel and bridge topologies

In fact, the *bridge* (or even *arbitrary*) topology is solvable in 3VSL by distinguishing the distorting opinion(AB) and original opinion (DC), which will be discussed in the following sections. Since *bridge* topology is common in social networks, it is important to design a model to compute the trustworthiness between users connected by a *bridge* or *arbitrary* topology without losing information by removing edges. Formally, we define the problem of computing interpersonal trust in social networks as follows:

Problem Statement

Given an arbitrary trust social network $G(V, E)$, $\forall u$ and v s.t. $e(u, v) \notin E$ and \exists at least one path from u to v , how to compute the trustworthiness of v to u , i.e. how u should trust a stranger v based on her existing social connections.

IV. THREE-VALUED SUBJECTIVE LOGIC

To compute interpersonal trust in any arbitrary topology, we propose the Three-Valued Subjective Logic (3VSL) by distinguishing 1) the *priori* and *posteriori* uncertainties, and 2) distorting and original opinions. In 3VSL, the trustworthiness/opinion of a person is considered as an trinary event (true, false, neutral). Neutral state, also called *posteriori uncertainty*, keeps the the evidences distorted from certain spaces while trust propagates from one person to another. Leveraging on this new definition of trust, we redesign the discounting and combining operations on opinions, and theoretically prove that 3VSL is capable to handle arbitrary network topology.

A. Subjective Logic

To better understand 3VSL, we first briefly introduce the subjective logic [9]. Considering two users A and X , the trustworthiness of X to A can be described by an *opinion vector* ω_X^A :

$$\omega_X^A = (b_X^A, d_X^A, u_X^A, a_X^A) \\ b_X^A + d_X^A + u_X^A = 1$$

where b_X^A , d_X^A , u_X^A and a_X^A refer to the belief, distrust, uncertainty, and base rate. a_X^A is a constant formed from an existing impression without solid evidences, e.g. prejudice, preference and general opinion obtained from hearsay. For example, if A always distrusts/trusts the persons from a certain group where X belongs to, then a_X^A will be smaller/greater than 0.5. Based

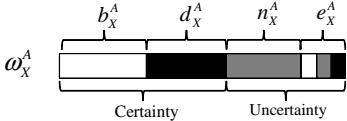


Fig. 2: Components of an opinion vector in 3VSL

on the Beta distribution, two opinions $\omega_1 = (b_1, d_1, u_1, a_1)$ and $\omega_2 = (b_2, d_2, u_2, a_2)$ can be combined as follows:

$$\left\{ \begin{array}{l} b_{12} = \frac{b_1 u_2 + b_2 u_1}{u_1 + u_2 - u_1 u_2} \\ d_{12} = \frac{d_1 u_2 + d_2 u_1}{u_1 + u_2 - u_1 u_2} \\ u_{12} = \frac{u_1 u_2}{u_1 + u_2 - u_1 u_2} \\ a_{12} = a_2 \end{array} \right.$$

Let A and B be two persons where $\omega_B^A = (b_1, d_1, u_1, a_1)$ is A 's opinion about B 's trustworthiness, and let C be another person where $\omega_C^B = (b_2, d_2, u_2, a_2)$ is B 's opinion about C . Then, subjective logic applies the discounting operation to compute ω_C^A as follows.

$$\left\{ \begin{array}{l} b_{12} = b_1 b_2 \\ d_{12} = b_1 d_2 \\ u_{12} = 1 - b_{12} - d_{12} - u_2 \\ a_{12} = a_2 \end{array} \right.$$

Finally, the expected belief of an opinion ω_X^A is computed by $E(\omega_X^A) = b_X^A + u_X^A a_X^A$.

B. Opinion Vector in 3VSL

In 3VSL, an opinion vector ω_X^A is defined as:

$$\omega_X^A = (b_X^A, d_X^A, n_X^A, e_X^A) | a_X^A \\ b_X^A + d_X^A + n_X^A + e_X^A = 1$$

where b_X^A , d_X^A , n_X^A , e_X^A and a_X^A refer to belief, distrust, posterior uncertainty, and priori uncertainty, and base rate. b_X^A , d_X^A , n_X^A represent the probabilities that X is trustworthy, not trustworthy, and neutral, respectively. e_X^A represents priori uncertainty (without evidences) of whether X is trustworthy, not trustworthy, or neutral. Priori uncertainty exists due to the lack of evidences, while posteriori uncertainty exists because of evidence distortions. The definition of a_X^A is the same as that in subjective logic and it keeps unchanged in 3VSL, for simplicity we will not mention it unless necessary.

As shown in Fig 2, the certainty of an opinion comes from b_X^A and d_X^A , while uncertainty from n_X^A and e_X^A . For example, if A has no interactions with X , then her opinion on the trustworthiness of X is $(0, 0, 0, 1)$. Later on, if A receives 1, 2 and 4 evidences to support X is trustworthy, not trustworthy and neutral, then ω_X^A becomes $(0.1, 0.2, 0.4, 0.3)$. The details of how to compute e_X^A will be shown in the following sections. Compared to subjective logic, 3VSL further distinguishes the uncertainty as priori and posteriori.

C. Dirichlet Distribution in 3VSL

To support mathematical operations on opinion vectors in 3VSL, we first define a mapping between the probability density function representation and the opinion representation by introducing a evidence space. As shown in Fig. 3, the

operations on opinions can be considered as the operations on the Dirichlet distributions.

The Dirichlet distribution is a family of continuous multivariate probability distributions parametrized by a vector $\alpha = (\alpha_1, \dots, \alpha_K)$. Its probability density function returns the belief that the probabilities of K rival events are x_i given that each event has been observed $\alpha_i - 1$ times. Since evidences in an opinion vector are in three possible states (belief, distrust, neutral), we use a trinomial Dirichlet distribution:

$$f(P_b, P_d | \alpha, \beta, \gamma) = \frac{\Gamma(\alpha + \beta + \gamma)}{\Gamma(\alpha) \cdot \Gamma(\beta) \cdot \Gamma(\gamma)} \cdot P_b^{\alpha-1} \cdot P_d^{\beta-1} \cdot P_n^{\gamma-1}$$

where (α, β, γ) is the controlling vector, P_b , P_d and P_n represents the probability distribution of belief, distrust, and neutral events. Let r , s and o be the number of evidences observed to support a person is trustworthy, not trustworthy, and neutral, respectively. According to the definition of the Dirichlet distribution, we have $\alpha = r + 1$, $\beta = s + 1$, and $\gamma = o + 1$.

Here we assume that a person already had one evidence of each event (belief, distrust and neutral). The assumption is reasonable because the Dirichlet distribution still works even when no event is observed, i.e. $(\alpha = 1, \beta = 1, \gamma = 1)$ and the probability of each event will be $1/3$. These three priori evidences are considered as a priori uncertainty.

According to the definition of ω_X^A , its four components can be expressed as:

$$\begin{aligned} b_X^A &= \frac{r}{r + s + o + 3} \\ d_X^A &= \frac{s}{r + s + o + 3} \\ n_X^A &= \frac{o}{r + s + o + 3} \\ e_X^A &= \frac{3}{r + s + o + 3} \end{aligned} \quad (1)$$

where the amount of priori evidences is set as 3, and its ratio to the number of observed evidences is e_X^A .

Since the expected probability of each event in Eq. 1 is,

$$\begin{aligned} E(P_b) &= \frac{\alpha}{\alpha + \beta + \gamma} = \frac{r + 1}{r + s + o + 3} \\ E(P_d) &= \frac{\beta}{\alpha + \beta + \gamma} = \frac{s + 1}{r + s + o + 3} \\ E(1 - P_b - P_d) &= \frac{\gamma}{\alpha + \beta + \gamma} = \frac{o + 1}{r + s + o + 3} \end{aligned} \quad (2)$$

we have the following equations:

$$\begin{aligned} E(P_b) &= \frac{r + 1}{r + s + o + 3} = b_X^A + \frac{1}{3} e_X^A \\ E(P_d) &= \frac{s + 1}{r + s + o + 3} = d_X^A + \frac{1}{3} e_X^A \\ E(1 - P_b - P_d) &= \frac{o + 1}{r + s + o + 3} = n_X^A + \frac{1}{3} e_X^A \end{aligned} \quad (3)$$

where the probability of each event is determined by certain posterior evidences and priori evidences. So far we have built the mapping between opinion model and the Dirichlet distribution by introducing an evidence space (r, s, o) .

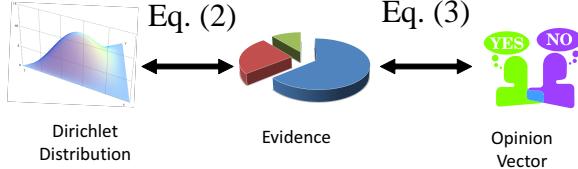


Fig. 3: Mapping between the opinion model and the Dirichlet distribution

Although prior uncertainty in 3VSL is similar to u in subjective logic, posterior uncertainty is considered a parallel state to distrust and belief. Posterior uncertainty, belief and distrust form trinary events, so Beta distribution in subjective logic is extended to Dirichlet distribution in 3VSL.

D. Opinion Combining in 3VSL

In a parallel topology as shown in Fig. 1(b), opinions from parallel paths should be combined or fused in a fair and equal way so that the resulting opinion reflects all opinions. We first introduce the theorem of combining two opinions and then generalized it to support multiple opinions.

Theorem 1 Let $\omega_1 = (b_1, d_1, n_1, e_1)$ and $\omega_2 = (b_2, d_2, n_2, e_2)$ be the opinions on two parallel paths between two users, then the combining operation $\Theta(\omega_1, \omega_2)$ is carried out as follows:

$$\Theta(\omega_1, \omega_2) = \begin{cases} b_{12} = \frac{e_2 b_1 + e_1 b_2}{e_1 + e_2 - e_1 e_2} \\ d_{12} = \frac{e_2 d_1 + e_1 d_2}{e_1 + e_2 - e_1 e_2} \\ n_{12} = \frac{e_2 n_1 + e_1 n_2}{e_1 + e_2 - e_1 e_2} \\ e_{12} = \frac{e_2 e_1}{e_1 + e_2 - e_1 e_2} \end{cases}$$

PROOF According to the mapping relationship between opinion model and Dirichlet distribution, ω_1 and ω_2 can be represented as $f(P_b, P_d | \alpha_1, \beta_1, \gamma_1)$ and $f(P_b, P_d | \alpha_2, \beta_2, \gamma_2)$, respectively. If posterior evidences are independent, these two Dirichlet distributions can be aggregated into one distribution:

$$f(P_b, P_d | \alpha_1 + \alpha_2 - 1, \beta_1 + \beta_2 - 1, \gamma_1 + \gamma_2 - 1)$$

It was shown that posterior evidences from the same category are accumulated except for the prior evidences. 3VSL considers the prior uncertainty from different individuals as identical. Then, the expected probability of belief event in the aggregated distribution is:

$$\begin{aligned} E(P_b) &= \frac{\alpha_1 + \alpha_2 - 1}{\alpha_1 + \alpha_2 + \beta_1 + \beta_2 + \gamma_1 + \gamma_2 - 3} \\ &= \frac{r_1 + r_2 + 1}{r_1 + r_2 + s_1 + s_2 + o_1 + o_2 + 3} \end{aligned} \quad (4)$$

Similarly, the expected probabilities of other events can be computed. We can see the number of posterior evidences of each events are increased to $r_1 + r_2$, $s_1 + s_2$, and $o_1 + o_2$. However, the amount of prior evidences is still 3 and does not change. 3VSL considered posterior evidences of different single hop opinions are independent while prior evidences are dependent.

According to Eq. 1 and the definition of opinion vector, we have:

$$\begin{aligned} b_1 &= \frac{r_1}{r_1 + s_1 + o_1 + 3} \\ &= \frac{b_1}{(1 - e_1) + e_1} = \frac{\frac{e_2}{e_1} b_1}{(1 - e_1) \frac{e_2}{e_1} + e_2} \\ b_2 &= \frac{r_2}{r_2 + s_2 + o_2 + 3} = \frac{b_2}{(1 - e_2) + e_2} \end{aligned}$$

e_1 and e_2 are different if $r_1 + s_1 + o_1 \neq r_2 + s_2 + o_2$, so we normalized them in the above equations. Combining these two opinions yields:

$$b_{12} = \frac{\frac{e_2}{e_1} b_1 + b_2}{(1 - e_1) \frac{e_2}{e_1} + (1 - e_2) + e_2} = \frac{e_2 b_1 + e_1 b_2}{e_1 + e_2 - e_1 e_2}$$

Similarly, the other components of resulting opinion can be calculated as follows:

$$\begin{aligned} d_{12} &= \frac{e_2 d_1 + e_1 d_2}{e_1 + e_2 - e_1 e_2} \\ n_{12} &= \frac{e_2 n_1 + e_1 n_2}{e_1 + e_2 - e_1 e_2} \\ e_{12} &= \frac{e_2 e_1}{e_1 + e_2 - e_1 e_2} \end{aligned}$$

■

According to Theorem 1, we could expect the combining operation is both commutative and associative, i.e. $\Theta(\omega_1, \omega_2) \equiv \Theta(\omega_2, \omega_1)$ and $\Theta(\omega_1, \Theta(\omega_2, \omega_3)) \equiv \Theta(\Theta(\omega_1, \omega_2), \omega_3)$.

If there exist multiple parallel opinions $\omega_1, \omega_2 \dots \omega_n$ between two users, the overall opinion can be calculated as $\Theta(\Theta(\Theta(\omega_1, \omega_2), \dots), \omega_n)$. Since combining operation is commutative and associative, the above equation can be simplified as $\Theta(\omega_1, \omega_2, \dots, \omega_n)$. Compared to subjective logic, new operation rules on priori and posterior uncertainties are introduced in 3VSL.

E. Opinion Discounting in 3VSL

The purpose of opinion discounting operation is to implement trust propagation along trust connections within a social network. We first introduce how to discount one opinion from another, and then generalize it to support multiple opinions. Considering a simple case where A trusts B who trusts C , the discounting operation allows A discounts B 's opinion over C to obtain her own opinion on C .

Definition 1 (Discounting Operation) Let A and B be two persons where $\omega_B^A = (b_1, d_1, n_1, e_1)$ is A 's opinion about B 's trustworthiness, and let C be another person where $\omega_C^B = (b_2, d_2, n_2, e_2)$ is B 's opinion about C . Then, the discounting operation $\Delta(\omega_B^A, \omega_C^B)$ is carried out as follows:

$$\Delta(\omega_B^A, \omega_C^B) = \begin{cases} b_{12} = b_1 b_2 \\ d_{12} = b_1 d_2 \\ n_{12} = 1 - b_{12} - d_{12} - e_2 \\ e_{12} = e_2 \end{cases}$$

where $\Delta(\omega_B^A, \omega_C^B)$ is called the discounting of ω_C^B by ω_B^A , expressing A 's opinion about C as a result of B 's advice to A .

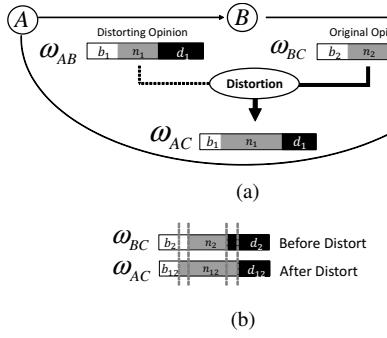


Fig. 4: Illustration of the discounting operation in 3VSL

Since A discounts B 's opinion (of C) to obtain her own opinion upon C , some certain evidences from ω_C^B will be distorted since A does not absolutely trust B and his opinion. The distorted evidences from r_C^B and s_C^B will be saved into the posteriori uncertainty space of the resulting opinion ω_C^A . However, the priori evidences will be kept unchanged. As shown in Fig. 4, distorted evidences from belief and distrust spaces are saved into the posteriori uncertainty space, resulting the same amount of evidences.

Discounting operation is analogical to electromagnetic wave propagation where original signal is distorted into a weak one at the receiving side. Trust propagating from A to C is considered as the opinion ω_C^B being distorted by ω_B^A (in the inverse direction). Since certain evidences from ω_B^A are distorted and saved into the posteriori uncertainty space of ω_C^A , ω_C^A and ω_C^B have the same amount of evidences.

Definition 2 (Distorting and Original Opinions) Given a discounting operation $\Delta(\omega_1, \omega_2)$ on two opinions ω_1 and ω_2 , we treat ω_1 as the distorting opinion, and ω_2 the original opinion.

Since certain evidences from ω_2 are distorted by ω_1 and transferred into the posteriori uncertainty space of ω_2 , the evidence space of opinion $\Delta(\omega_1, \omega_2)$ is the same as ω_2 's. Therefore, we can conclude *the resulting opinion of a discounting operation shares exactly the same evidence space with the original opinion*.

It is easy to prove that discounting operation is associative but not commutative, i.e. $\Delta(\omega_1, \omega_2) \neq \Delta(\omega_2, \omega_1)$ but $\Delta(\Delta(\omega_1, \omega_2), \omega_3) \equiv \Delta(\omega_1, \Delta(\omega_2, \omega_3))$. Given a serial topology where opinions are ordered as $\omega_1, \omega_2, \dots, \omega_n$, the final opinion can be calculated as $\Delta(\Delta(\Delta(\omega_1, \omega_2), \dots), \omega_n)$. Since discounting operation is associative, this equation is simplified as $\Delta(\omega_1, \omega_2, \dots, \omega_n)$.

Compared to subjective logic, posteriori uncertainty is introduced in 3VSL to store neutral evidences eliminated from certainty spaces as trust propagates while priori uncertainty is kept unchanged.

F. Difference between Distorting and Original Opinions in 3VSL

To further understand the difference between distorting and original opinions, we investigate two special cases shown in Fig. 5. By analyzing these two topologies, we discover the shortcomings of subjective logic, and point out original

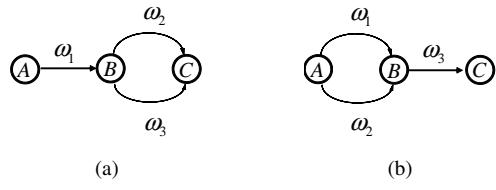


Fig. 5: Difference between distorting and original opinions

opinions can be used only once in trust computation while distorting opinions are not.

Lemma 1 Let $\omega_1, \omega_2, \omega_3$ be three opinions, then $\Delta(\omega_1, \Theta(\omega_2, \omega_3)) \equiv \Theta(\Delta(\omega_1, \omega_2), \Delta(\omega_1, \omega_3))$. However, $\Delta(\Theta(\omega_1, \omega_2), \omega_3) \neq \Theta(\Delta(\omega_1, \omega_2), \Delta(\omega_2, \omega_3))$.

PROOF We first prove $\Delta(\omega_1, \Theta(\omega_2, \omega_3)) \equiv \Theta(\Delta(\omega_1, \omega_2), \Delta(\omega_1, \omega_3))$.

Let $\omega_i = (b_i, d_i, n_i, e_i)$ where $i = 1, 2, 3$, then both $\Delta(\omega_1, \Theta(\omega_2, \omega_3))$ and $\Theta(\Delta(\omega_1, \omega_2), \Delta(\omega_1, \omega_3))$ yield the same result:

$$\left\{ \begin{array}{l} b = \frac{b_1 b_2 e_3 + b_1 b_3 e_2}{e_2 + e_3 - e_3 e_2} \\ d = \frac{b_1 d_2 e_3 + b_1 d_3 e_2}{e_2 + e_3 - e_3 e_2} \\ n = \frac{e_3 n_2 + e_2 n_3}{e_2 + e_3 - e_3 e_2} \\ e = \frac{e_3 e_2}{e_2 + e_3 - e_3 e_2} \end{array} \right.$$

where

$$\left\{ \begin{array}{l} n_2 = 1 - b_1 b_2 - b_1 d_2 - e_2 \\ n_3 = 1 - b_1 b_3 - b_1 d_3 - e_3 \end{array} \right.$$

Now we prove $\Delta(\Theta(\omega_1, \omega_2), \omega_3) \neq \Theta(\Delta(\omega_1, \omega_3), \Delta(\omega_2, \omega_3))$. According to the aggregation constraint, two Dirichlet distributions can be combined if and only if their evidence spaces are independent. Since opinions $\Delta(\omega_1, \omega_3)$ and $\Delta(\omega_2, \omega_3)$ share the same evidence space from the opinion ω_3 , they cannot be combined. Therefore, $\Delta(\Theta(\omega_1, \omega_2), \omega_3)$ is the only correct solution, and is not equal to $\Theta(\Delta(\omega_1, \omega_3), \Delta(\omega_2, \omega_3))$. ■

In subjective logic, $\Delta(\omega_1, \Theta(\omega_2, \omega_3))$ and $\Theta(\Delta(\omega_1, \omega_2), \Delta(\omega_1, \omega_3))$ will give different results, which is contradictory to the common sense. From Lemma 1, we also note that reusing ω_1 is allowed, but not ω_3 . The difference between ω_1 and ω_3 is that ω_1 is a distorting opinion while ω_3 is an original opinion. Therefore, we conclude that *in trust computation, original opinions can be combined only once, while distorting opinions can be combined any number of times because they do not impact the overall evidence space of the final opinion*.

G. Expected Belief in 3VSL

Given a computed opinion ω_X^A , we need to calculate the expected probability that A is affirmative that X will perform the desired actions. Opinion ω_X^A contains four component $(b_X^A, d_X^A, n_X^A, e_X^A)$ which corresponds to vector in the evidence space $(r_X^A, s_X^A, o_X^A, 3)$. In this vector, the expected number of positive evidences are $(r_X^A + a_X^A o_X^A + 1)$ where a_X^A is the base rate which is set as 0.5 in this paper. According to Eq. 1, the expected probability of positive events occurring is:

$$E(\omega_X^A) = b_X^A + a_X^A n_X^A + \frac{1}{3} e_X^A \quad (5)$$

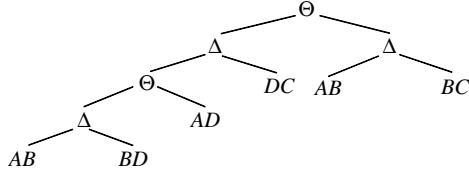


Fig. 6: Binary decomposition tree of bridge topology

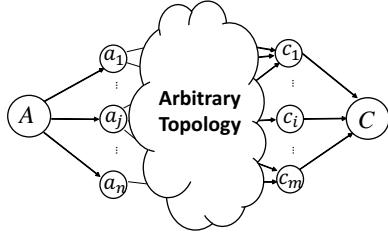


Fig. 7: Illustration of an arbitrary topology

H. Bridge and Arbitrary Topologies

Starting from node C backwards to A , the bridge topology can be represented in a binary decomposition tree as shown in Fig. 6. Note that only edge \widehat{AB} is used twice (both as distorting opinions), which is allowed by Lemma 1. In fact, 3VSL can handle any arbitrary topology.

Theorem 2 *Given an arbitrary two-terminal directed graph (TTDG) $G(V, E)$ where A, C are the first and second terminals. Node u represents a person, edge $e(u, v)$ denotes an opinion ω_v^u . By applying discounting and combining operations, the overall opinion ω_C^A of A upon C is solvable and unique.*

PROOF We prove the theorem in a recursive manner, i.e. reducing the original problem into sub-problem(s) and keep reducing sub-problems until the base case is solvable and the solution is unique.

As shown in Fig. 7, we assume there are m nodes (c_1, c_2, \dots, c_m) connecting to C , i.e. $e(c_i, C) \in E$ where $i \in [1, m]$. There are n nodes (a_1, a_2, \dots, a_n) being connected from A , i.e. $e(A, a_j) \in E$ where $j \in [1, n]$.

Reduction

Case 1: If there is only one node connecting to C , i.e. $m = 1$, then $\omega_C^A = \Delta(\omega_{c_1}^A, \omega_C^{c_1})$.

Case 2: If there are more than one node connecting to C , i.e. $m > 1$, according to Lemma 1, then $\omega_C^A = \Theta(\Delta(\omega_{c_1}^A, \omega_C^{c_1}), \Delta(\omega_{c_2}^A, \omega_C^{c_2}), \dots, \Delta(\omega_{c_m}^A, \omega_C^{c_m}))$. It is shown that ω_C^A is solvable and unique if and only if the $\omega_{c_i}^A$ is solvable and unique where $\omega_{c_i}^A$ is the result of the sub-problem with sub-graph $G' = G - \Sigma e(c_i, C) - C$.

For Case 2, the network topology from $\{a_j\}$ to $\{c_i\}$ is unknown and maybe complex, it is possible that all c_i are connected at a certain node b . If $b = A$, then \widehat{Ab}_iC are parallel and can be combined. If $b \neq A$, ω_b^A can be computed in as 1) the combination of \widehat{Ab}_iC , or 2) combining \widehat{bc}_iC first and discounting it by ω_b^A . According to Lemma 1, both methods yields the same result, i.e. although we use the first method, the final result is unique.

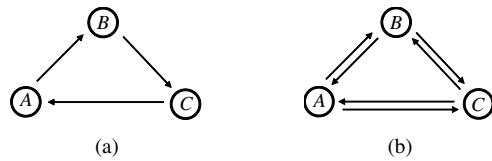


Fig. 8: Illustration of cycles in directed trust graph

After each reduction, G is reduced into G' in the way that $|E| = |E| - m$ and $|V| = |V| - 1$. Applying reductions on sub-problems recursively, finally the base case will arise, i.e. $|E| = 1$ and $|V| = 2$.

Base Case

The sub-graph of base case contains only one edge from A to a_j where $j \in [1, n]$.

Since $\omega_{a_j}^A$ is known from the original graph G , the base case is solvable and the result is unique. Applying the equations in Case 1 and 2 repeatedly, we can obtain an unique ω_C^A of the original problem. ■

Based on Theorem 2, we design the ASSESS-TRUST algorithm to apply 3VSL in arbitrary topology, where G denotes the corresponding graph, A and C denote the trustee and trustor, respectively. ASSESS-TRUST algorithm can also handle cyclic graphs, which is elaborated by two examples as shown in Fig. 8. For any two node in Fig. 8(a), e.g. A and C , ω_C^A can be computed from $\Delta(\omega_B^A, \omega_C^B)$ which ignores edge \widehat{AC} because only incoming edges are considered in ASSESS-TRUST. Similarly, in Fig. 8(b), the overall opinion of A on C can be computed as $\Theta(\Delta(\omega_B^A, \omega_C^B), \omega_C^A)$. To the best of our knowledge, this is the first algorithm which takes cyclic graph into consideration.

Algorithm 1 ASSESS-TRUST(G, A, C)

Input: A directed graph G with source node A and destination node C .

Output: A 's opinion upon C , ω_C^A .

```

1:  $n \leftarrow 0$ 
2: for each incoming edges  $e(c_i, C) \in G$  do
3:   if  $c_i = A$  then
4:      $\omega_i \leftarrow \omega_{c_i}^A$ 
5:   else
6:      $G' \leftarrow G - e(c_i, C)$ 
7:      $\omega_{c_i}^A \leftarrow \text{ASSESS-TRUST}(G', A, c_i)$ 
8:      $\omega_i \leftarrow \Delta(\omega_{c_i}^A, \omega_C^{c_i})$ 
9:   end if
10:   $n \leftarrow n + 1$ 
11: end for
12: if  $n > 1$  then
13:    $\omega_C^A = \Theta(\omega_1 \dots \omega_n)$ 
14: else
15:    $\omega_C^A = \omega_n$ 
16: end if

```

V. EXPERIMENTAL EVALUATION

Experimental validations of the 3VSL trust computational model are hard due to the difficulties in measuring generic in-

terpersonal trust and the lack of established datasets, especially those that could support more than a scalar representation of trust. To address these challenges, we design an online system aiming to collect generic trust data in the opinion format and then validate the discounting and combining operations in 3VSL.

A. Evaluation Setup and Trust Opinion Construction

100 participants are invited to evaluate the trustworthiness of their 1st and 2nd hop friends (target persons) by filling out the questionnaire based on prior work on the psychology of trust [8]. *The participants are randomly selected from people around us, with various character and background.* The questionnaire is composed of 12 example questions, and the answers of these questions are used to construct a scale for measuring interpersonal trust. The judgment score (X) for each question is scaled in 9 levels, where 8 represents “strongly trust” and 0 as “strongly distrust”. In addition, we add another question to let participants indicate how certain they are for their answers to the 12 questions. The uncertainty score (Y) is scaled in 5 levels, where 0 represents “not sure at all” and 4 as “very confident”.

After logging into the online system, a participant A will first be asked to identify and evaluate the trustworthiness of her two direct friends B and D . Then, A is told that her friend B trusts C with an opinion ω_C^B , and she is asked to evaluate her opinion on C . Finally, A is told D also knows C (with an opinion ω_C^D), and she is asked to evaluate C again, by integrating opinions from B and C . More details about the online system could be found in [4].

From collected data, we construct trust opinions as follows. The average score of X , denoted as T , reflects the portion of evidences (in a participant’s mind) that supports the target person is trustworthy. The uncertainty Y score is transferred to priori uncertainty e , i.e. the more certain evidences exist, the less uncertain a participant will be in judging the target person. It’s difficult to obtain the accurate value of e , since participants may not recall the exact number of evidences they used to make their judgments. However, it is noticed that people tends to form their opinions according to most recent experiences. Hence, assuming 20 recent evidences are good enough for a person to make fair judgments, the value of e is set as 1 when $Y = 0$ and $3/(5Y)$, otherwise. Each pair of T and e is transferred into a opinion vector according to the following equation:

$$(b, d, n, e) = (T \cdot (1 - e), (1 - T) \cdot (1 - e), 0, e) \quad (6)$$

Notice that we assume the posteriori uncertainty in each non-computed opinion as 0 because when human make decisions, neutral evidences are usually ignored and only those positive and negative ones take affect. In other words, we consider posterior uncertainty only occurs due to distortion when trust propagates.

B. Errors in Discounting and Combining Operations

To validate the discounting and combining operations, we calculate the errors between expected beliefs obtained from

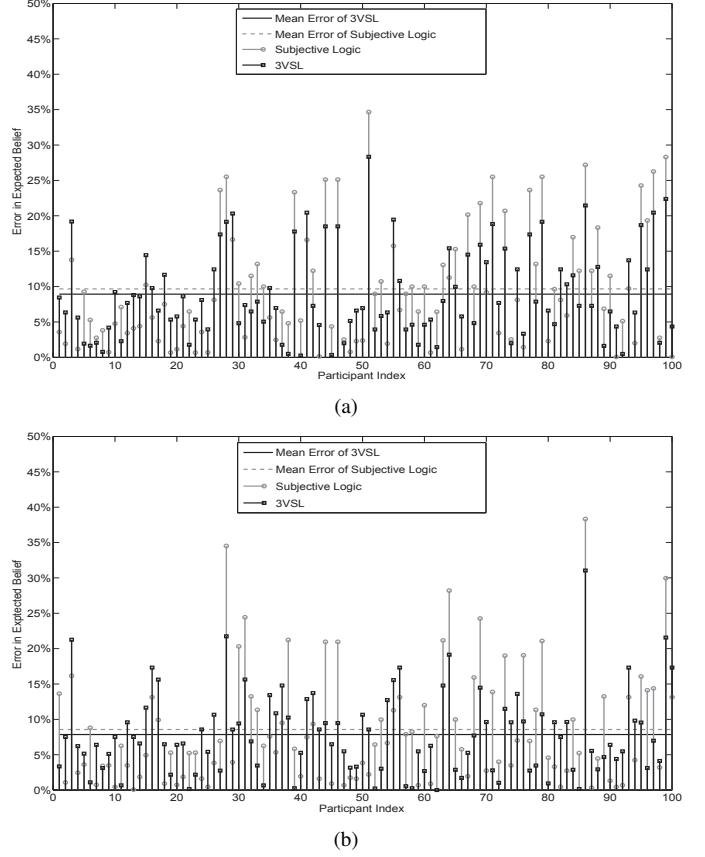


Fig. 9: a) Errors in expected belief of discounting operation b) Errors in expected belief of combining operation

the questionnaire results and the computed ones as follows:

$$\Delta_{Err} = \left| \frac{E(\Delta(\omega_B^A, \omega_C^B)) - E(\omega_C^A)}{E(\omega_C^{A \rightarrow B})} \right|$$

$$\Theta_{Err} = \left| \frac{E(\Theta(\Delta(\omega_B^A, \omega_C^B), \Delta(\omega_D^A, \omega_C^D))) - E(\omega_C^{A \rightarrow B, D})}{E(\omega_C^{A \rightarrow B, D})} \right|$$

where $\omega_C^{A \rightarrow B}$ denotes the opinion A hold upon C through B ’s opinion, $\omega_C^{A \rightarrow B, D}$ denotes the opinion A hold upon C through both B and D ’s opinions.

The errors of combining and discounting operations can be seen in Fig 9. As a comparison, we also plot the results computed by subjective logic. The average error of 3VSL is less than subjective logic while both of them are less than 10%, i.e. 3VSL is an accurate model in computing interpersonal trust. We further plot the CDF of the error data in Fig. 10. For the combining operation in 3VSL, we can see around 95% results have errors less than 20%. For the discounting operation in 3VSL, 90% results have errors less than 20%. However, the number of trust value with error $< 20\%$ computed by subjective logic is smaller. This indicates 3VSL provides higher accuracy in multi-hop interpersonal trust assessments.

The errors between measured and computed results are caused mainly by two reasons. First, the scale used in the questionnaire is discrete, while interpersonal trusts are actu-

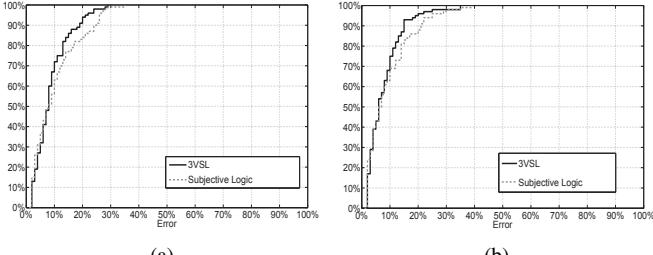


Fig. 10: a) CDF of errors in expected belief of discounting operation
b) CDF of errors in expected belief of combining operation

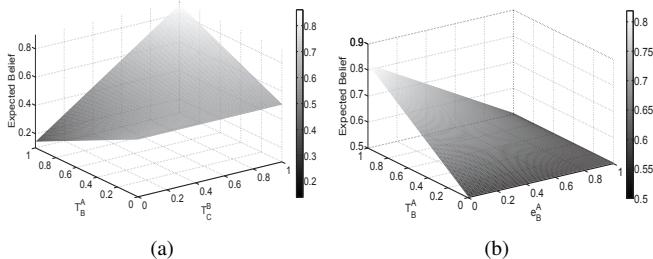


Fig. 11: a) Influence of belief on discounting operation b) Influence of belief and priori uncertainty on discounting operation

ally continuous, meaning selected option may not accurately convey participants' opinion. Second, a participant's feeling on 2nd hop friends is assumed to be vague, which indicates the evaluation of multi-hop trust could not be 100% accurate.

VI. NUMERICAL EVALUATION

To further understand the features of the 3VSL model, we conduct a set of numerical analysis on the two basic operations: discounting and combining. In the numerical evaluation, we use the same parallel topology as in the previous section, and adjusts parameters T and e to see whether the model works as designed.

A. Discounting Operation

In the serial topology shown in Fig. 1(a), we first set two opinions ω_B^A and ω_C^B with fixed priori uncertainty (i.e. $e_B^A = e_C^B = 0.2$), and vary their belief parts by changing T_B^A and T_C^B . According to Eq. 6, ω_B^A and ω_C^B can be written as $(0.8T_B^A, 0.8(1-T_B^A), 0, 0.2)$ and $(0.8T_C^B, 0.8(1-T_C^B), 0, 0.2)$, respectively. As shown in Fig. 11(a), the expected belief $E(\Delta(\omega_B^A, \omega_C^B))$ tends to approach T_C^B when T_B^A is high. When T_B^A is low, $E(\Delta(\omega_B^A, \omega_C^B))$ approaches to 0.5. That means A tends to believe B 's opinion on C when A highly trusts B ; however, A 's opinion on C tends to neutral when she does not trust B .

Secondly, we evaluate the impact of both belief and priori uncertainty on discounting operations. We vary T_B^A and e_B^A from 0 to 1, which results an opinion $(T_B^A \cdot (1 - e_B^A), (1 - T_B^A) \cdot (1 - e_B^A), 0, e_B^A)$. At the same time, we keep $\omega_C^B = (0.7, 0.1, 0, 0.2)$. As can be seen in Fig. 11(b), $E(\Delta(\omega_B^A, \omega_C^B))$ are close to T_B^A when e_B^A is low, and it becomes neutral when e_B^A is high. This phenomena indicates when A is more certain about her discretion on B (i.e. smaller e_B^A), she will rely more on B to form her opinion on C . Otherwise, A 's opinion on C tends to be neutral as she cannot judge due to the lack of evidences.

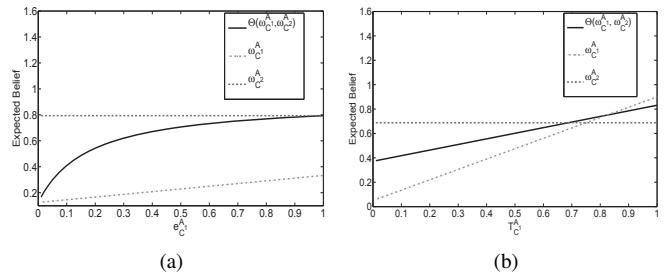


Fig. 12: a) Influence of priori uncertainty on combining operation b) Influence of belief on combining operation

B. Combining Operation

Considering the topology shown in Fig. 1(b), we set $\omega_C^{A_2}$ as a fixed opinion $(0.7, 0.1, 0, 0.2)$ (high belief) while $\omega_C^{A_1}$ as an variable opinion $(0.125 \times (1 - e_C^{A_1}), 0.875 \times (1 - e_C^{A_1}), 0, e_C^{A_1})$ (high distrust) with various priori uncertainty. As shown in Fig. 12(a), when $e_C^{A_1}$ is less than $e_C^{A_2}$, the expected belief of combined opinion $E(\Theta(\omega_C^{A_1}, \omega_C^{A_2}))$ tends to approach $\omega_C^{A_1}$. When $e_C^{A_1}$ is greater than $e_C^{A_2}$, $E(\Theta(\omega_C^{A_1}, \omega_C^{A_2}))$ gets close to $\omega_C^{A_2}$. That means combining two opinions always yields a result which is closer to the opinion with less priori uncertainty or more evidences.

We then evaluate the impact of belief on the combining operation, by setting $\omega_C^{A_2} = (0.7, 0.1, 0, 0.2)$ (high belief) while $\omega_C^{A_1} = (0.8T_C^{A_2}, 0.8 \cdot (1 - T_C^{A_2}), 0, 0.2)$ with various beliefs. In Fig. 12(b), when $T_C^{A_2}$ and $T_C^{A_1}$ are close, the expected belief $E(\Theta(\omega_C^{A_1}, \omega_C^{A_2}))$ is close to but a little higher than both $E(\omega_C^{A_1})$ and $E(\omega_C^{A_2})$. When $T_C^{A_2}$ and $T_C^{A_1}$ become different, $E(\Theta(\omega_C^{A_1}, \omega_C^{A_2}))$ gets close to the mean of $E(\omega_C^{A_1})$ and $E(\omega_C^{A_2})$. We conclude that combining two opinions with the same priori uncertainty and belief will enhance the original opinions, due to the increased evidence space. On the other hand, combining opinions with same priori uncertainty but different beliefs will neutralize these two opinions.

C. Impact of Bridge Opinion

3VSL is able to handle the bridge topology as shown in Fig. 1(c), while subjective logic could not because it has to remove a certain edge (e.g. ω_D^B). We believe the being removed edge (bridge opinion) is also important in assessing trust in the bridge topology.

By eliminating the bridge opinion (as subjective logic does), the bridge topology becomes a parallel topology where A connects to C by two paths \overrightarrow{ABC} and \overrightarrow{ADC} . We set opinion vectors ω_B^A , ω_D^A , ω_C^B and ω_D^C as $(0.7, 0.1, 0, 0.2)$, $(0.5, 0.3, 0, 0.2)$, $(0.5, 0.3, 0, 0.2)$, and $(0.5, 0.3, 0, 0.2)$, respectively. Then, we vary the bridge opinion ω_D^B as $((1 - e_D^B)T_D^B, (1 - e_D^B)(1 - T_D^B), 0, e_D^B)$.

We pick a high priori uncertainty ($e_D^B = 0.6$) and a low priori uncertainty ($e_D^B = 0.15$), and vary the belief by changing T_D^B from 0 to 1. As shown in Fig. 13(a), when the $e_D^B = 0.15$, the expected belief of the bridge topology are closer to T_D^B than the parallel topology. When $e_D^B = 0.6$, it approaches T_D^B as well yet not as much as $e_D^B = 0.15$. It is shown that when bridge opinion is a certain one, its impact on the final result could not be omitted.

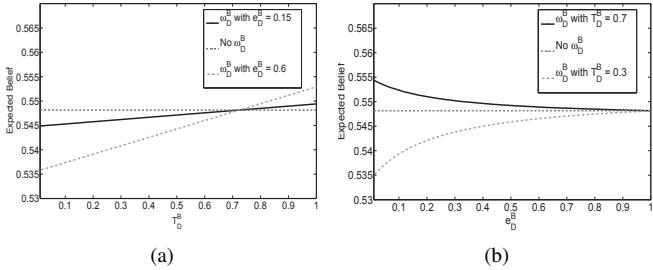


Fig. 13: a) Influence of bridge opinion's belief on the resulting expected belief b) Influence of bridge opinion's prior uncertainty on the resulting expected belief

Then, we vary e_D^B from 0 to 1 and pick a high belief ($T_D^B = 0.7$) and a low belief ($T_D^B = 0.3$) of the opinion ω_D^B . As can be seen in Fig. 13(b), when e_D^B is low, the impact of the bridge opinion should not be ignored. When e_D^B is high, the expected belief gets close to the parallel topology. That means when e_D^B is high (no enough evidences), the bridge opinion can be ignored as what the subjective logic does. However, when the e_D^B is low, it has significant impact on the final result and must be taken into account. Moreover, we note that a higher belief of the bridge opinion leads to a larger expected belief than the parallel topology and vice versa.

VII. CONCLUSION AND FUTURE WORK

Three-valued subjective logic is proposed to compute the interpersonal trust between any two persons who have not had interactions before. 3VSL introduces posterior uncertainty space to store the evidences distorted from certain spaces as trust propagates, and prior uncertainty space to control the evidence size as trusts combine. We also discover the differences between distorting and original opinions, i.e. original opinions are so unique that they can be reused in trust computation while distorting opinions are not. We validate 3VSL both in theory and real world evaluation. The results indicate that 3VSL is sound and can be applied in computing trust with high accuracy. For the future work, we will improve 3VSL by employing stochastic process to model a trust opinion. Also, we will evaluate 3VSL in real word scenarios with more complicated topology. In addition, Bayesian analysis will be integrated to make 3VSL be able to handle evidences from multiple sources.

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