Detecting Heterogeneous Treatment Effect with Instrumental Variables

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Introduction

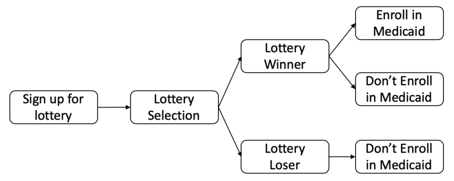
Oregon Health Insurance Experiment

- In 2008, investigators interested in understanding causal effect of **Medicaid** on various financial and health outcomes
- In an ideal world: RCT and randomly force/deny Medicaid on eligible participants
- A (popular) alternative: An encouragement (IV) design that by **random chance** nudges people to enroll in Medicaid.

OHIE website, https://www.nber.org/oregon/1.home.html

Oregon Health Insurance Experiment

IV design for studying effect of Medicaid:



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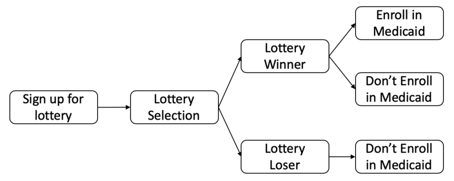
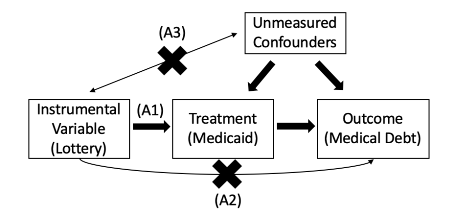


If the core IV assumptions are satisfied, then the complier average treatment effect (CATE) can be estimated as the effect of Medicaid (Angrist et al. 1996)



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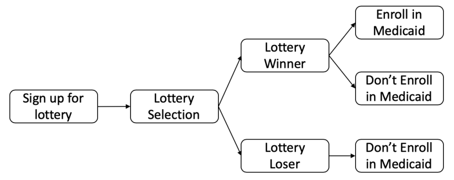

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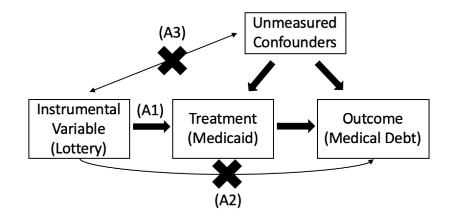

Intent-to-treat measures effect of lottery on the outcome

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- Groups may be known prior to analysis, but what if they're not?
- **Goal**: Estimation & honest inference of discovered heterogeneous CATE

Inferring Heterogeneity from Exploratory Methods with IV

Proposed Two-Staged Solution

Data pre-processing: Pair match lottery winners and losers on pre-lottery covariates

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Stage 1: Find potential effect modifiers of Medicaid through exploratory machine learning methods (CART)

- Compute $|Y|$ to prevent effects of double-dipping data
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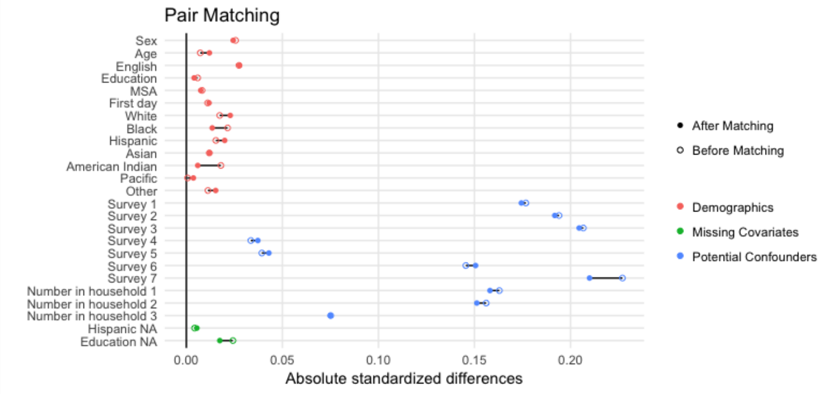
Stage 2: Simultaneous testing of effect modifiers of Medicaid

- Use closed testing on CART tree

Data Pre-Processing: Matching



Stage 1: Absolute Value of Pairwise Differences $|\mathbf{Y}|$

- **Potential treatment**: d_{zij}; **Potential outcome**: r_{zij}, for $z \in \{0, 1\}$
- Similar to the usual Fisher's sharp null,
  assume $r_{1ij} - r_{0ij} = \lambda(d_{1ij} - d_{0ij})$ for all i, j
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Matched Pair i	Individual 1	Individual 2
Possibility 1	Lottery Winner	Lottery Loser
Observe	$r_1 - \lambda d_1$	$r_0 - \lambda d_0$
Under null	$r_0 - \lambda d_0$	$r_0 - \lambda d_0$
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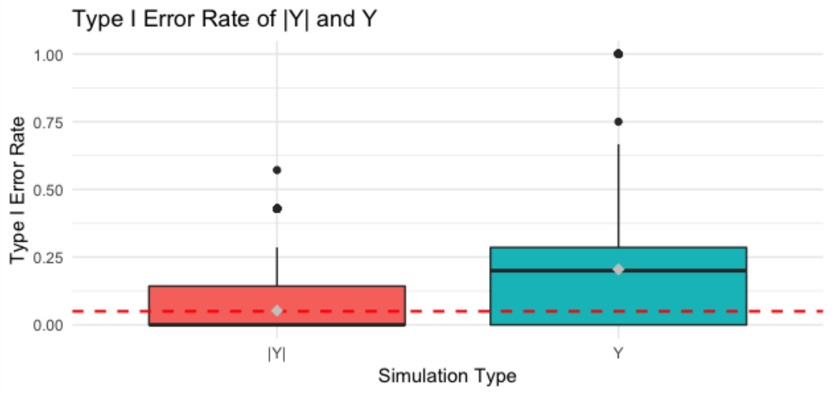
- $\left|Y_i\right| = \left|\left(r_{0i1} - \lambda d_{0i1} - (r_{0i2} - \lambda d_{0i2})\right)\right|$

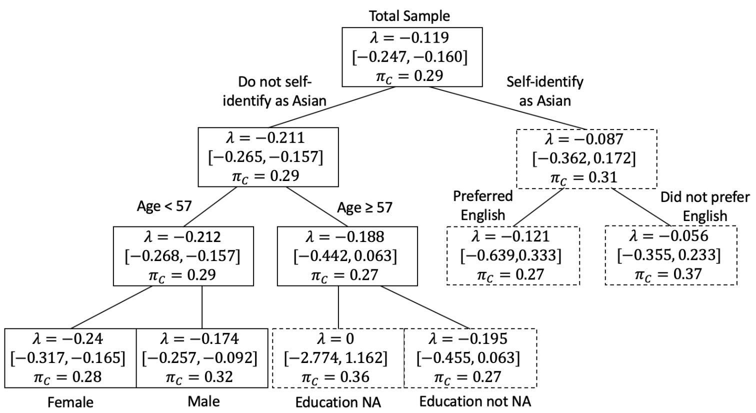
Stage 1: Why $|Y|$, not Y?



- **tl;dr** Without $|Y|$, the data is double-dipped and the type I error is inflated

Stage 1: Using |Y| and CART to Discover Potential Effect Modifiers

- Y: Whether or not an individual currently owes money for medical expenses
- $H_{0_s} : \lambda = \lambda_s = 0$



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- Solution: Closed testing (Marcus et al. 1976) for nested or tree-structured hypotheses

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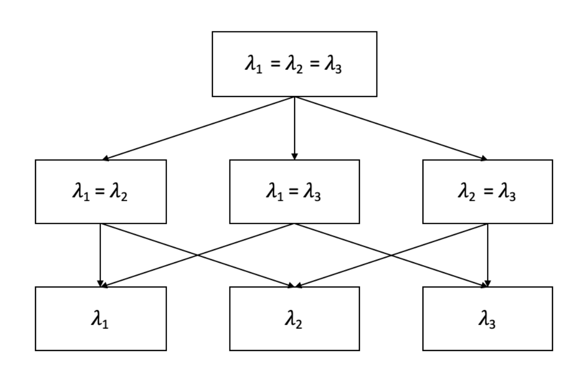

- A family of hypotheses is closed under intersection
- Each hypothesis is level α
    - Kang et al. (2016) test for matching and IV setting

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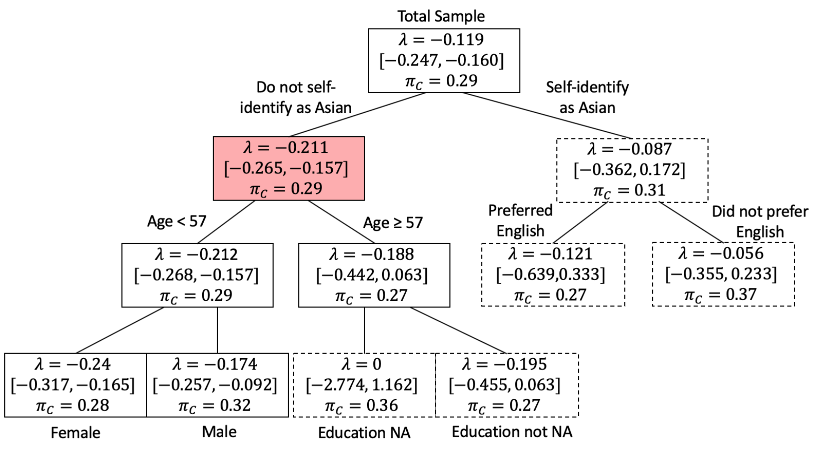

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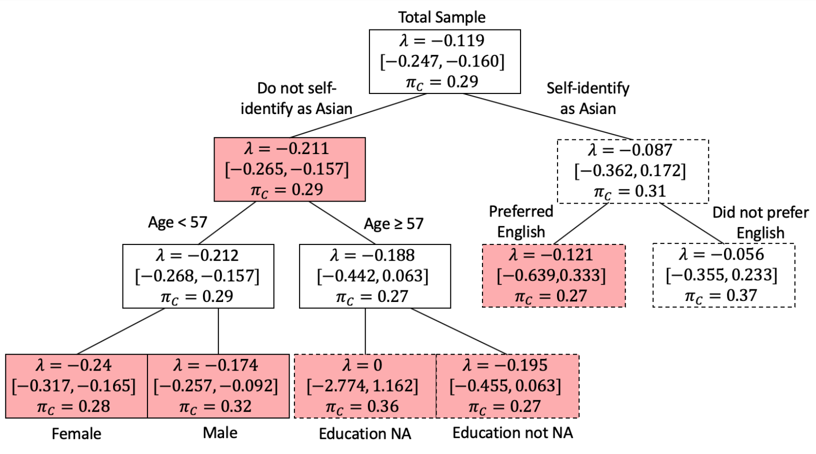

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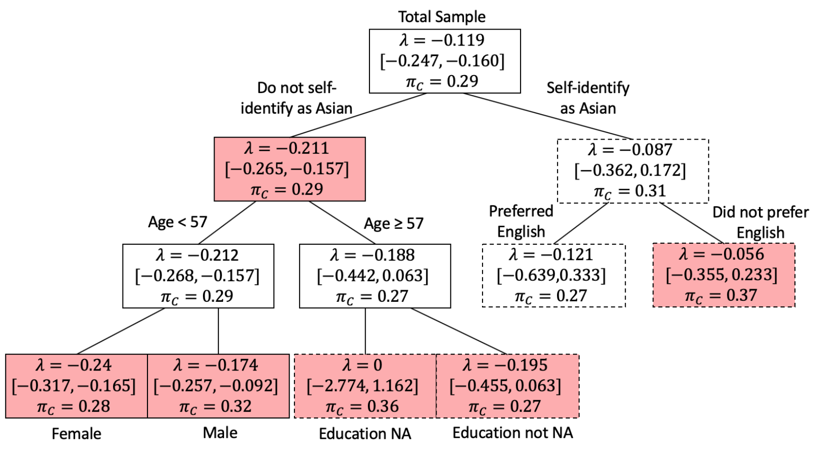

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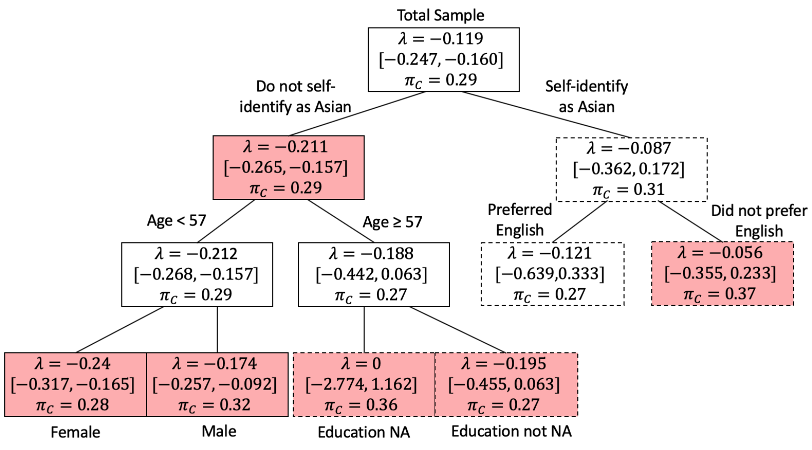

Theorem

Above procedure **strongly** *controls FWER.*

Conclusion

Summary and Future Work

Summary

- Valid simultaneous discovery and inference of heterogeneous treatment effect using instrumental variables
- Use $|Y|$ with CART to discover potential effect modifiers
- Use closed testing on CART to test for true potential effect modifiers and strongly control for family wise error rate
- Medicaid had a larger impact on complying young men and women who do not self-identify as Asian in reducing medical debt

Future Work

- Determine how CART splits based on treatment effect and compliance rate
- Investigate method with other machine learning exploratory methods
  - LASSO, Random Forests

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Matching & IV

- **Potential treatment**: d_{zij} ; **Potential outcome**: $r_{zij}^{(d_{zij})}$ for $z \in \{0, 1\}$
- The estimand of interest is the effect ratio

$$\lambda = \frac{\sum_{i=1}^{I} \sum_{j=1}^{n_i} r_{1ij}^{(d_{1ij})} - r_{0ij}^{(d_{0ij})}}{\sum_{i=1}^{I} \sum_{j=1}^{n_i} d_{1ij} - d_{0ij}}$$

$$\stackrel{\text{IV}}{=} E[R_1 - R_0 \mid C = complier]$$