Detecting Heterogeneous Treatment Effect with Instrumental Variables

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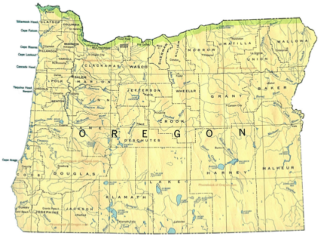
Introduction

Oregon Health Insurance Experiment

- In 2008, investigators interested in understanding causal effect of health coverage for low-asset people on various financial and health outcomes
- Lottery selection in Oregon for opportunity to enroll in Medicaid
- Randomized controlled design to analyze effect of expanding health care access



OHIE website, https://www.nber.org/oregon/1.home.html

Oregon Health Insurance Experiment

- Comparison of outcomes between treated and control
    - Treated: adults selected by lottery and so have opportunity to apply for Medicaid
    - Control: adults not selected by lottery and so cannot apply for Medicaid
- Estimate causal effect of insurance coverage
    - Use lottery selection as instrument for insurance coverage
        - Randomized instrument satisfies core IV assumptions except perhaps exclusion restriction
        - Access to Medicaid only if a lottery winner ensures near one-sided compliance

Heterogeneous Treatment Effect

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- Groups may be known prior to analysis, but what if they're not?

Inferring Heterogeneity from Exploratory Methods

General Procedure

Given observed outcome R, binary instrument Z, exposure D, and covariates:

1. Pair match on observed covariates
2. Calculate absolute value of adjusted pairwise differences,

$$\left|Y_i\right| = \left|\left(Z_{i1} - Z_{i2}\right)\left(R_{i1} - \lambda D_{i1} - (R_{i2} - \lambda D_{i2})\right)\right|$$

3. Use exploratory methods on $|Y|$ versus X to detect patterns in data and form groups
    - Formulate the hypotheses for inference
4. Use closed testing to test hypotheses $H_{0_s} : \lambda_s = \lambda_0$ and strongly control family wise error rate

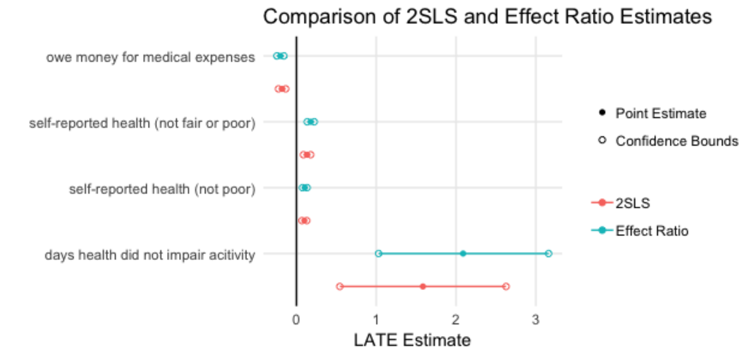
Covariates used to partition groups that was found to have a significant treatment effect are considered effect modifiers

Step 1: Matching

- **Potential exposure**: d_{zij} ; **Potential outcome**: $r_{zij}^{(d_{zij})}$ for $z \in \{0, 1\}$
- The estimand of interest is the effect ratio

$$\lambda = \frac{\sum_{i=1}^{I} \sum_{j=1}^{n_i} r_{1ij}^{(d_{1ij})} - r_{0ij}^{(d_{0ij})}}{\sum_{i=1}^{I} \sum_{j=1}^{n_i} d_{1ij} - d_{0ij}}$$

$$\overset{\text{IV}}{=} E[R_1 - R_0 \mid C = complier]$$



Step 2: Calculate Absolute Difference $|Y|$

- **Potential exposure**: d_{zij} ; **Potential outcome**: $r_{zij}^{(d_{zij})}$ for $z \in \{0, 1\}$
- Similar to Fisher's sharp null,
  assume $r_{1ij}^{(d_{1ij})} - r_{0ij}^{(d_{0ij})} = \lambda(d_{1ij} - d_{0ij})$ for all i, j
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Pair i	Individual 1	Individual 2
Possibility 1	$Z_{i1} = 1$	$Z_{i2} = 0$
	$r_{1i1}^{(d_{1i1})} - \lambda d_{1i1}$	$r_{0i2}^{(d_{0i2})} - \lambda d_{0i2}$
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- $|Y_i| = \left| \pm \left(r_{0i1}^{(d_{0i1})} - \lambda d_{0i1} - (r_{0i2}^{(d_{0i2})} - \lambda d_{0i2})\right)\right|$

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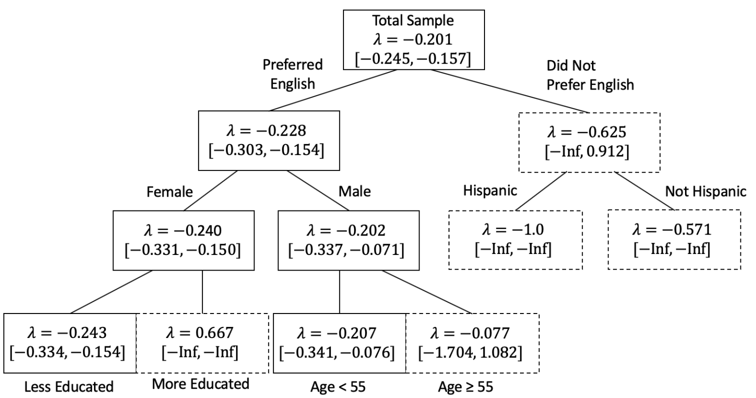
- $|Y_i| = \left|\left(r_{0i1}^{(d_{0i1})} - \lambda d_{0i1} - (r_{0i2}^{(d_{0i2})} - \lambda d_{0i2})\right)\right|$

Step 3: CART with $|Y|$ and covariates X

- Whether or not an individual currently owes money for medical expenses
- $H_{0_s} : \lambda = \lambda_s$



Step 4: Closed Testing for Strong Familywise Error Control

Test procedure for converting a method testing a global hypothesis into multiple inferences for subhypotheses.

Closed testing requires:

- Hypotheses to imply subsequent subhypotheses
- A family of hypotheses to be closed under intersection
- Each hypothesis is level α

Closed testing provides a way to **strongly control family wise error rate**, since rejecting a true hypothesis at level α is only possible if the intersection of all true hypotheses is also tested and rejected.

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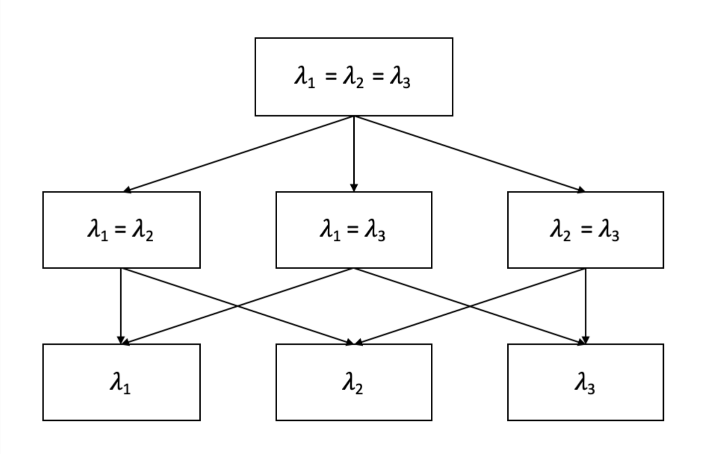

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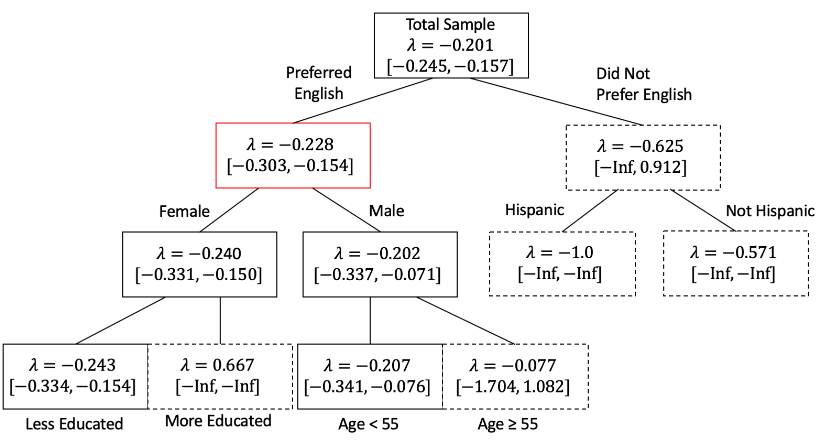

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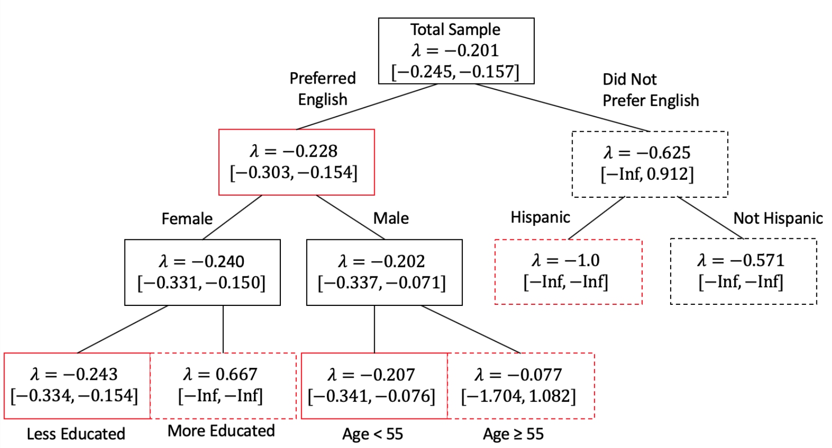

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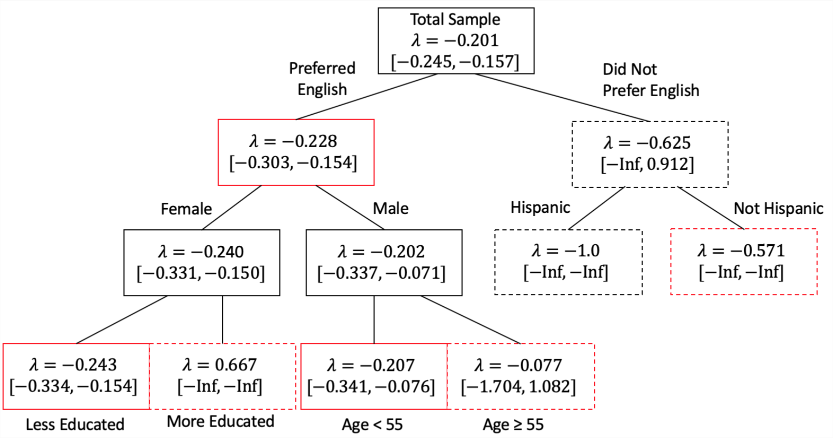

Conclusion

Summary and Future Work

Summary

- Simultaneous inference on effect modification using instrumental variables and discovered with exploratory methods can be valid
- Use $|Y|$ to obscure instrument assignment
- Closed testing to control for family wise error rate

Future Work

- Investigate method's performance under weak instruments
- Pursue Bayesian methods for determining interesting groups

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