Homework 2.3-2.3

Ex. 2.3.1. No air resistance Suppose that a crossbow bolt is shot straight upward from the ground ($y_0 = 0$) with initial velocity $v_0 = 49$ (m/s). Then with $g = 9.8$ gives $\frac{dv}{dt} = -9.8$, so$v(t) = -(9.8)t + v_0 = -(9.8)t + 49$. Hence the bolt's height function y(t) is given by $y(t) = \int [-(9.8)t + 49]\, dt = -(4.9)t^2 + 49t + y_0 = -(4.9)t^2 + 49t$. The bolt reaches its maximum height when $v = -(9.8)t + 49 = 0$, hence when $t = 5$ (s). Thus its maximum height is $y_{\max} = y(5) = -(4.9)\left(5^2\right) + (49)(5) = 122.5\,(\mathrm{m})$. The bolt returns to the ground when $y = -(4.9)t(t - 10) = 0$, and thus after 10 seconds aloft.

Ex. 2.3.2. Velocity-proportional resistance We again consider a bolt shot straight upward with initial velocity $v_0 = 49$m/s from a crossbow at ground level. But now we take air resistance into account, with $\rho = 0.04$ in . We ask how the resulting maximum height and time aloft compare with the values found in .

Problems 9 through 12 illustrate resistance proportional to the velocity.

Pr. 2.3.7. Suppose that a car starts from rest, its engine providing an acceleration of $10\mathrm{ft/s}^2$, while air resistance provides $0.1\mathrm{ft/s}^2$ of deceleration for each foot per second of the car's velocity.

(a) Find the car's maximum possible (limiting) velocity.

(b) Find how long it takes the car to attain 90% of its limiting velocity, and how far it travels while doing so.

Problems 17 and 18 apply – to the motion of a crossbow bolt.

Pr. 2.3.9. A motorboat weighs 32,000 lb and its motor provides a thrust of 5000 lb. Assume that the water resistance is 100 pounds for each foot per second of the speed v of the boat. Then $1000\frac{dv}{dt} = 5000 - 100v$. If the boat starts from rest, what is the maximum velocity that it can attain?

Answer Key

Ex. 2.3.2. We substitute $y_0 = 0, v_0 = 49$, and $v_\tau = -g/\rho = -245$ in and , and obtain $\begin{aligned} v(t) &= 294e^{-t/25}-245, \\ y(t) &= 7350-245t-7350e^{-t/25}. \end{aligned}$ To find the time required for the bolt to reach its maximum height (when $v = 0$), we solve the equation $v(t) = 294e^{-t/25} - 245 = 0$ for $t_m = 25\ln(294/245) \approx 4.558$ (s). Its maximum height is then $y_{\max} = v\left(t_m\right) \approx 108.280$ meters (as opposed to 122.5 meters without air resistance). To find when the bolt strikes the ground, we must solve the equation $y(t) = 7350-245t-7350e^{-t/25} = 0$. Using Newton's method, we can begin with the initial guess $t_0 = 10$ and carry out the iteration $t_{n+1} = t_n - y\left(t_n\right)/y'\left(t_n\right)$ to generate successive approximations to the root. Or we can simply use the `Solve` command on a calculator or computer. We find that the bolt is in the air for $t_{\text{f}} \approx 9.411$ seconds (as opposed to 10 seconds without air resistance). It hits the ground with a reduced speed of $|v\left(t_{\text{f}}\right)| \approx 43.227$m/s (as opposed to its initial velocity of 49 m/s). Thus the effect of air resistance is to decrease the bolt's maximum height, the total time spent aloft, and its final impact speed. Note also that the bolt now spends more time in descent ($t_{\text{f}} - t_m \approx 4.853$ s) than in ascent ($t_m \approx 4.558$ s).

Pr. 2.3.7. (a) 100 ft/sec ; (b) about 23 sec and 1403 ft to reach 90 ft/sec

Pr. 2.3.9. 50 ft/s