Chapter 5 - Introduction to Factorial Designs

Factorial Designs

- So far we discussed experiments involving one factor
  - Compare two conditions; t-tests (Ch 2)
  - Compare many conditions; F-tests, one-way ANOVA (Ch 3)
  - Compare many conditions while blocking for nuisance factors; two-way ANOVA, general linear model (Ch 4)
- Many experiments involve studying two or more factors
  - Factorial designs; multi-factor ANOVA, general linear model (Ch 5);

Factorial Designs

- Factorial design is an experimental design in which in each complete replication of the experiment all possible combinations of the levels of the factors are investigated
  - Factor levels are varied “simultaneously”
  - When factors are arranged in a factorial design the factors are said to be crossed
- Factorial designs allow studying the joint (or interaction) effects of multiple factors

Factorial Designs: Interactions

- Interaction between factors (from Chapter 1):
  - Synergy between factors
  - combined effect of factor levels that is above and beyond the sum of the individual effects of the factors
- Example: Sales of a product in a store
  - Adding a certain amount of shelf space adds 10 units of sales.
  - Adding a certain amount of money to promotion adds 8 units of sales.
  - Add both shelf space and promotion \$s → 18 units of sales?
  - If the gain is > 18 units → Positive interaction

Factorial Designs: A two- factor, two-level experiment

- Two factors, A and B, have potential effects on response y.
  - In the experiment, both factors are tested at two levels
  - # of levels, $a = 2, b = 2$.
  - Denote the two levels as “High” and “Low” or “+” and “-”

Factorial Designs: A two- factor, two-level experiment

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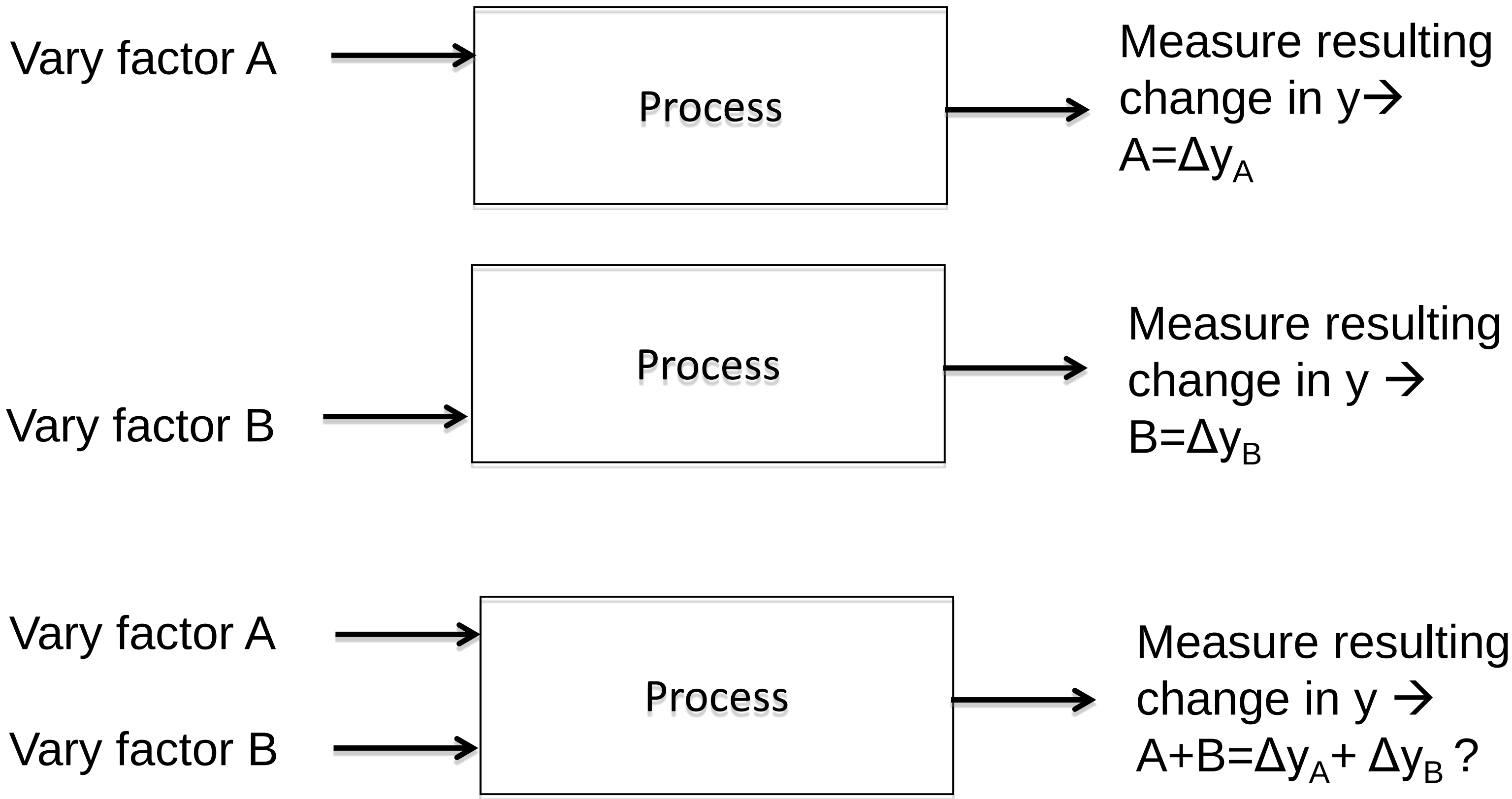
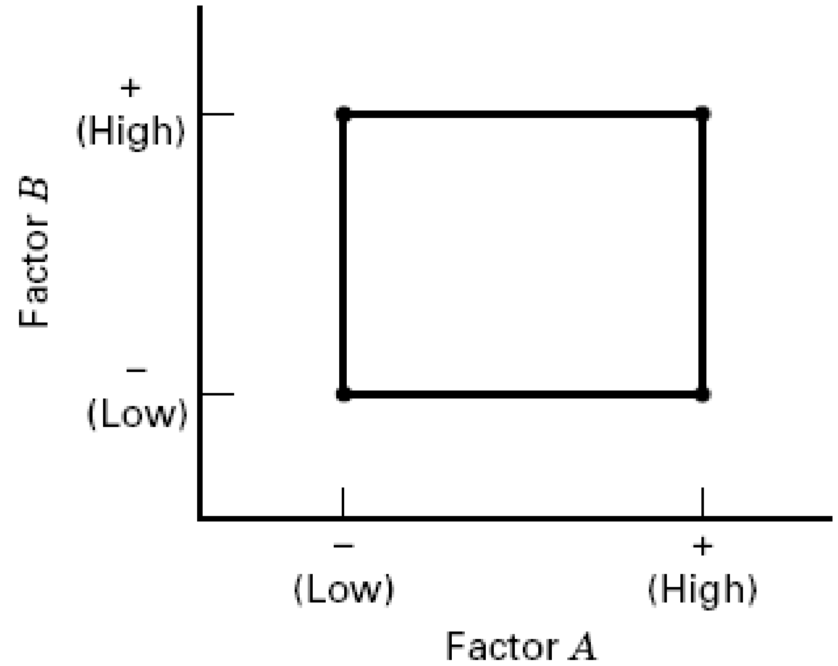
  - In the experiment, both factors are tested at two levels
  - # of levels, $a = 2, b = 2$.
  - Denote the two levels as "High" and "Low" or "+" and "-"
- Effect of a factor A (denoted "A")
  - Change in the response y produced by a change in the level of the factor A
  - <u>Main effect (m.e.)</u> of factor A
- Interaction between A and B (denoted "AB")
  - Change in the response due to change in A not same at all levels of B→ an interaction between A and B
  - <u>Two factor interaction (2 f.i.)</u> between A and B
- Higher order interactions (e.g., 3 f.i., 4 f.i.) may exist

Factorial Designs: Two- factor, two-level experiment



Example: A two- factor, two-level experiment

- Factors A and B tested at two-levels
  - 4 combinations. Response is measured at each combination once



- Main effect of A:

(avg response at High level of A) –

(avg response at Low level of A)

$$A = \bar{y}_{A^+} - \bar{y}_{A^-}$$

- Similarly, main effect of B:

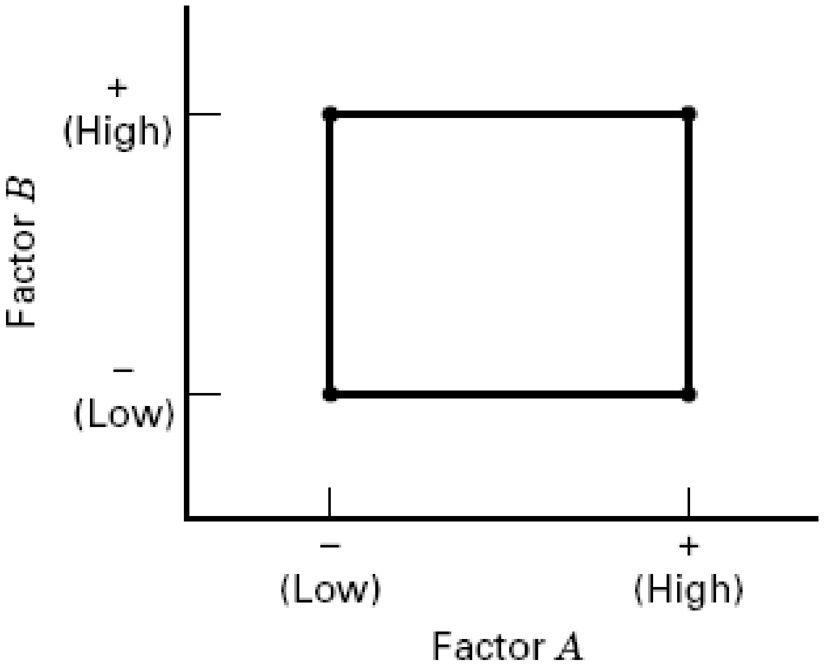
$$B = \bar{y}_{B^+} - \bar{y}_{B^-}$$

Example: A two- factor, two-level experiment

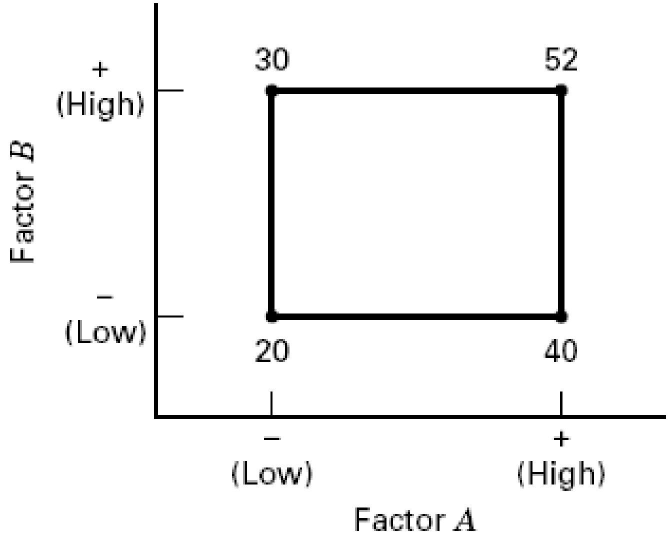
- Factors A and B tested at two-levels
  - 4 combinations. Response is measured at each combination once
- Interaction between A and B:

[(Main effect of A when B is high)-(Main effect of A when B is low)]/ 2

$$AB = \frac{A\ at\ B^{+} - A\ at\ B^{-}}{2}$$



Example 1 (Fig 5.1)



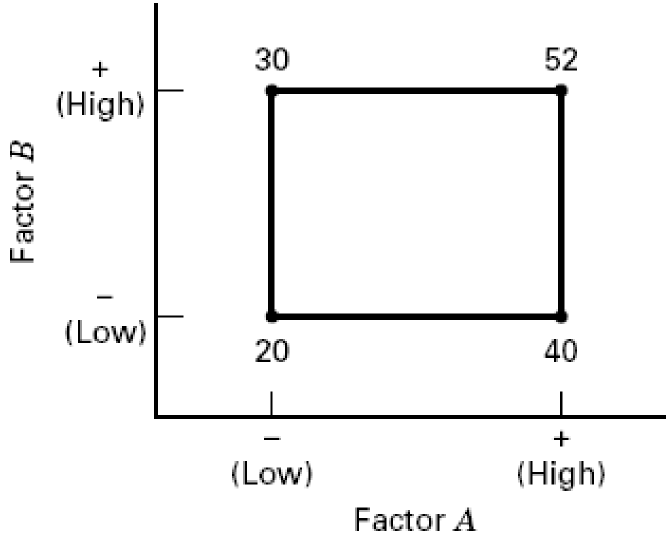
Main Effect of A--- $A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{40+52}{2} - \frac{20+30}{2}$

$$= 46 - 25 = 21$$

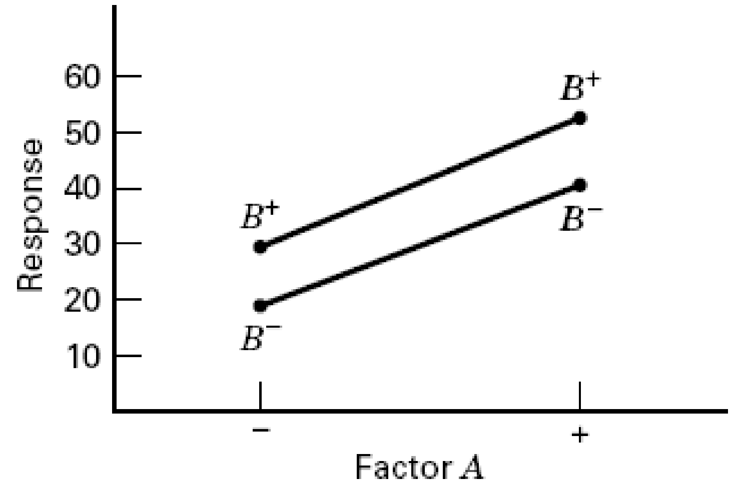
Main Effect of B---$B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{30+52}{2} - \frac{20+40}{2}$

$$= 41 - 30 = 11$$

Example 1 (Fig 5.1)



Two factor interaction plot



Main Effect of A--- $A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{40+52}{2} - \frac{20+30}{2}$

$$= 46 - 25 = 21$$

Main Effect of B---$B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{30+52}{2} - \frac{20+40}{2}$

$$= 41 - 30 = 11$$

Interaction between A and B

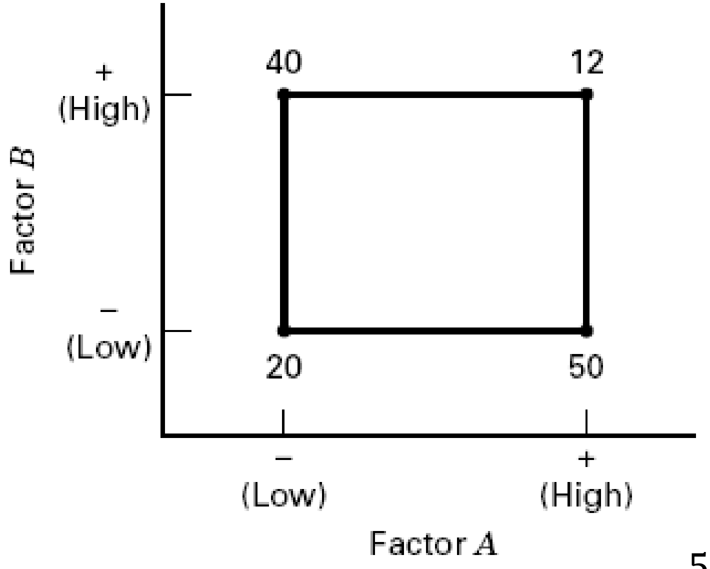
$$A \; at \; B^+ = 52 - 30 = 22$$

$$A \; at B^- = 40 - 20 = 20$$

$$AB = \frac{A \; at \; B^+ - A \; at \; B^-}{2} = \frac{22 - 20}{2} = 1$$

Interaction is small

Example 2 (Fig 5.2)



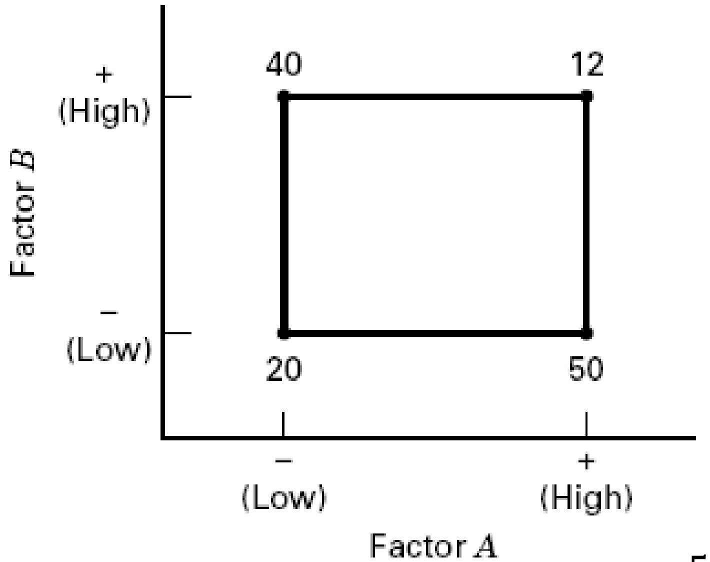
Main Effect of A----$A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{50+12}{2} - \frac{20+40}{2}$

$$= 31 - 30 = 1$$

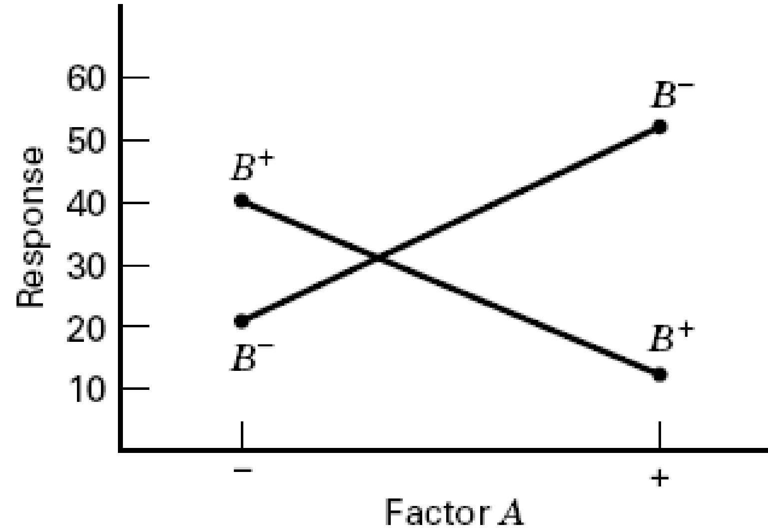
Main Effect of B--- $B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{40+12}{2} - \frac{20+50}{2}$

$$= 26 - 35 = -11$$

Example 2 (Fig 5.2)



Two factor interaction plot



Main Effect of A----$A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{50+12}{2} - \frac{20+40}{2}$

$$= 31 - 30 = 1$$

Main Effect of B--- $B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{40+12}{2} - \frac{20+50}{2}$

$$= 26 - 35 = -11$$

Interaction between A and B

$$A\ at\ B^+ = \ 12 - 40 \ = \ -28$$

$$A\ at\ B^- \ = \ 50 \ - 20 \ = 30$$

$$AB = \frac{A\ at\ B^+ - A\ at\ B^-}{2} = \frac{-28 - 30}{2} = -29$$

Interaction is large

Interpretation of Interaction Effect

- When interaction is significant main effects are not meaningful
  - In the case of Fig 5.2: if we consider only main effects, we may conclude that effect of A is not important
  - However, factor A has an effect, but it depends on the level of factor B
  - Significant interactions may mask the importance of main effects

5.3. Two-Factor Factorial Design

- General two-factor factorial design
  - Factor A has a levels. Factor B has b levels.
  - ab treatment combinations. Each combination is replicated n times
  - all abn observations taken in random order (Completely randomized design)
- Two-way layout for observations: y_{ijk}

Factor B (Column treatments)

Factor A (Row treatments)	1	2	$\ldots$	b
1	$y_{111}, y_{112}, \ldots, y_{11n}$	$y_{121}, y_{122}, \ldots, y_{12n}$		$y_{1b1}, y_{1b2}, \ldots, y_{1bn}$
2	$y_{211}, y_{212}, \ldots, y_{21n}$	$y_{221}, y_{222}, \ldots, y_{22n}$		$y_{2b1}, y_{2b2}, \ldots, y_{2bn}$
$\vdots$				
a	$y_{a11}, y_{a12}, \ldots, y_{a1n}$	$y_{a21}, y_{a22}, \ldots, y_{a2n}$		$y_{ab1}, y_{ab2}, \ldots, y_{abn}$

Example: battery life experiment (p. 187)

- Design of a battery with maximum life that will be subject to extreme temperatures. Test 3 plate materials at 3 temperature levels. Take $n = 4$ replications
  - Factors : Material type (A) and Temperature (B)
  - Response: Battery life (hours)
  - 3^2 factorial design.
- What effects do material type & temperature have on life? Main effects and Joint effects? Choice of material that would give long life regardless of temperature (a robust product)?

TABLE 5.1
Life (in hours) Data for the Battery Design Example

Material Type	Temperature (°F) 15		70		125	
1	130	155	34	40	20	70
	74	180	80	75	82	58
2	150	188	136	122	25	70
	159	126	106	115	58	45
3	138	110	174	120	96	104
	168	160	150	139	82	60

General Statistical Model for a Two-Factor Factorial Experiment

- A statistical model for the two-factor experiment

$$y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \epsilon_{ijk} \begin{cases} i = 1,2,\ldots,a \\ j = 1,2,\ldots,b \\ k = 1,2,\ldots,n \end{cases}$$

Effects of factors A and B defined as deviations from μ

- μ :overall mean
- τ_i :effect of i-th level of factor A
- β_j :effect of j-th level of factor B
- $\tau\beta_{ij}$:effect of interaction between i-th level of factor A and j-th level of factor B
- $\epsilon_{ijk} \sim NID(0, \sigma^2)$ experimental error

General Statistical Model for a Two-Factor Factorial Experiment

- We are interested in testing 3 hypotheses:
- Factor A does not have effect on response (Response has equal means under different levels of factor A)

$$H_0 : \tau_1 = \tau_2 = \cdots = \tau_a = 0$$
$$H_1 : \text{at least one } \tau_i \neq 0$$

- Factor B does not have effect on response (Response has equal means under different levels of factor B)

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_b = 0$$
$$H_1 : \text{at least one } \beta_j \neq 0$$

- Interaction of the levels of factor A and factor B does not have effect on the response

$$H_0 : \tau\beta_{ij} = 0 \text{ for all i and j}$$
$$H_1 : \text{at least one } \tau\beta_{ij} \neq 0$$

5.3.2. Statistical Analysis of the Factorial Design

- To estimate the model parameters, define row, column, cell and grand averages

$$\bar{y}_{i..} = \frac{1}{bn}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk} \quad , i = 1,2, \dots , a$$

$$\bar{y}_{.j.} = \frac{1}{an}\sum_{i=1}^{a}\sum_{k=1}^{n} y_{ijk} \quad , j = 1,2, \dots , b$$

$$\bar{y}_{ij.} = \frac{1}{n}\sum_{k=1}^{n} y_{ijk} , \qquad \forall\, i, j$$

$$\bar{y}_{...} = \frac{1}{abn}\sum_{j=1}^{b}\sum_{i=1}^{a}\sum_{k=1}^{n} y_{ijk}$$

- Row, column and cell averages

		Factor B			
		1	2	. . .	b
Factor A	1	$y_{111}, y_{112}, \ldots, y_{11n}$	$y_{121}, y_{122}, \ldots, y_{12n}$		$y_{1b1}, y_{1b2}, \ldots, y_{1bn}$
	2	$y_{211}, y_{212}, \ldots, y_{21n}$	$y_{221}, y_{222}, \ldots, y_{22n}$		$y_{2b1}, y_{2b2}, \ldots, y_{2bn}$
	$\vdots$				
	a	$y_{a11}, y_{a12}, \ldots, y_{a1n}$	$y_{a21}, y_{a22}, \ldots, y_{a2n}$		$y_{ab1}, y_{ab2}, \ldots, y_{abn}$

$\bar{y}_{i..}$

$\bar{y}_{.j.}$

$\bar{y}_{ij.}$

5.3.4 Estimates of model parameters

- Overall mean

$$\hat{\mu} = \bar{y}_{...}$$

- Effect of i-th level of factor A

$$\hat{\tau}_i = \bar{y}_{i..} - \bar{y}_{...}$$

- Effect of j-th level of factor B

$$\hat{\beta}_i = \bar{y}_{.j.} - \bar{y}_{...}$$

- Interaction between i-th level of A and j-th level of B

$$\widehat{\tau\beta}_{ij} = \bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}$$

- The fitted values of y_{ij}

$$\begin{aligned} \hat{y}_{ijk} &= \hat{\mu} + \hat{\tau}_i + \hat{\beta}_j + \widehat{\tau\beta}_{ij} \\ &= \bar{y}_{ij.} \end{aligned}$$

- Residuals

$$e_{ijk} = y_{ijk} - \hat{y}_{ijk}$$

5.3.2. Statistical analysis - Decomposition of the total sum of squares

- ANOVA model can be written in terms of the estimated parameters

$$y_{ij} = \hat{\mu} + \hat{\tau}_i + \hat{\beta}_j + \widehat{\tau\beta}_{ij} + e_{ijk}$$

- Substitute estimates, bring $\bar{y}_{...}$ to the left side, take square of both sides, sum over i, j, k

$$y_{ijk} - \bar{y}_{...} = (\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}) + (y_{ijk} - y_{ij.})$$

5.3.2. Statistical analysis - Decomposition of the total sum of squares

- Take square of both sides, sum over i, j, k

$$y_{ijk} - \bar{y}_{...} = (\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}) + (y_{ijk} - y_{ij.})$$

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \bar{y}_{...})^2 = bn\sum_{i=1}^{a}(\bar{y}_{i..} - \bar{y}_{...})^2 + an\sum_{j=1}^{b}(\bar{y}_{.j.} - \bar{y}_{...})^2$$

$$+ n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \bar{y}_{ij.})^2$$

$$\boxed{SS_T = SS_A + SS_B + SS_{AB} + SS_E}$$

- Degree of freedom breakdown

$$abn - 1 = (a-1) + (b-1) + (a-1)(b-1) + ab(n-1)$$

5.3.2. Statistical analysis - Decomposition of the total sum of squares

- Effect of A: If large differences between row treatment means, MS_A will be larger than MS_E

$$F_{0,A} = MS_A/MS_E$$

  - If the F statistic is large enough, then the effect of A is significant

- Effects of B and AB use $F_{0,B} = MS_B/MS_E$ and $F_{0,AB} = MS_{AB}/MS_E$

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{...})^2 = bn\sum_{i=1}^{a}(\bar{y}_{i..}-\bar{y}_{...})^2 + an\sum_{j=1}^{b}(\bar{y}_{.j.}-\bar{y}_{...})^2$$

$$+n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...})^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{ij.})^2$$

$$\boxed{SS_T = SS_A + SS_B + SS_{AB} + SS_E}$$

Variability between row means

Variability between column means

Variability between cell means(interaction)

Variability within cells

ANOVA Table for Two-Factor Factorial Fixed Effects Model

■ **TABLE 5.3**

The Analysis of Variance Table for the Two-Factor Factorial, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A treatments	SS_A	$a-1$	$MS_A = \frac{SS_A}{a-1}$	$F_0 = \frac{MS_A}{MS_E}$
B treatments	SS_B	$b-1$	$MS_B = \frac{SS_B}{b-1}$	$F_0 = \frac{MS_B}{MS_E}$
Interaction	SS_{AB}	$(a-1)(b-1)$	$MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)}$	$F_0 = \frac{MS_{AB}}{MS_E}$
Error	SS_E	$ab(n-1)$	$MS_E = \frac{SS_E}{ab(n-1)}$	
Total	SS_T	$abn-1$		

NOTE: We must have at least two replications (n>=2) if all possible interactions are included, to determine SSE

Example 5.1: battery life experiment (two-factor experiment)

- Design of a battery with maximum life that will be subject to extreme temperatures. Test 3 plate materials at 3 temperature levels
  - ❑Factors : A = Material type; B = Temperature
  - ❑Response: battery life (hours)
  - ❑3^2 factorial design. With replications $n = 4$

- What effects do material type & temperature have on life? Main effects and Joint effects? Choice of material that would give long life regardless of temperature (a robust product)?

■ **TABLE 5.1**

Life (in hours) Data for the Battery Design Example

Material Type	Temperature (°F) 15		70		125	
1	130	155	34	40	20	70
	74	180	80	75	82	58
2	150	188	136	122	25	70
	159	126	106	115	58	45
3	138	110	174	120	96	104
	168	160	150	139	82	60

Example 5.1: Battery Life Experiment – Cont' d

■ **TABLE 5.4**

Life Data (in hours) for the Battery Design Experiment

Material Type	Temperature (°F) 15		$y_{ij.}$	$\bar{y}_{ij.}$	70				125				$y_{i..}$
	130	155	539	134.75	34	40	229	57.25	20	70	230	57.5	
1	74	180			80	75			82	58			998
	150	188	623	155.75	136	122	479	119.75	25	70	198	49.5	
2	159	126			106	115			58	45			1300
	138	110	576	144	174	120	583	145.75	96	104	342	85.5	
3	168	160			150	139			82	60			1501
$y_{.j.}$	1738				1291				770				$3799 = y_{...}$

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{...})^2 = bn\sum_{i=1}^{a}(\bar{y}_{i..}-\bar{y}_{...})^2 + an\sum_{j=1}^{b}(\bar{y}_{.j.}-\bar{y}_{...})^2$$

$$+n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...})^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{ij.})^2$$

$$
\begin{aligned}
SS_T &= \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{\ldots}^2}{abn} \\
&= (130)^2 + (155)^2 + (74)^2 + \cdots \\
&\quad + (60)^2 - \frac{(3799)^2}{36} = 77{,}646.97
\end{aligned}
$$

$$
\begin{aligned}
SS_{\text{Material}} &= \frac{1}{bn}\sum_{i=1}^{a} y_{i..}^2 - \frac{y_{\ldots}^2}{abn} \\
&= \frac{1}{(3)(4)}[(998)^2 + (1300)^2 + (1501)^2] \\
&\quad - \frac{(3799)^2}{36} = 10{,}683.72
\end{aligned}
$$

$$
\begin{aligned}
SS_{\text{Temperature}} &= \frac{1}{an}\sum_{j=1}^{b} y_{.j.}^2 - \frac{y_{\ldots}^2}{abn} \\
&= \frac{1}{(3)(4)}[(1738)^2 + (1291)^2 + (770)^2] \\
&\quad - \frac{(3799)^2}{36} = 39{,}118.72
\end{aligned}
$$

$$
\begin{aligned}
SS_{\text{Interaction}} &= \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{\ldots}^2}{abn} - SS_{\text{Material}} \\
&\quad - SS_{\text{Temperature}} \\
&= \frac{1}{4}[(539)^2 + (229)^2 + \cdots + (342)^2] \\
&\quad - \frac{(3799)^2}{36} - 10{,}683.72 \\
&\quad - 39{,}118.72 = 9613.78
\end{aligned}
$$

and

$$
\begin{aligned}
SS_E &= SS_T - SS_{\text{Material}} - SS_{\text{Temperature}} - SS_{\text{Interaction}} \\
&= 77{,}646.97 - 10{,}683.72 - 39{,}118.72 \\
&\quad - 9613.78 = 18{,}230.75
\end{aligned}
$$

Example 5.1: Battery Life Experiment – Cont'd

TABLE 5.5
Analysis of Variance for Battery Life Data

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Material types	10,683.72	2			
Temperature	39,118.72	2			
Interaction	9,613.78	4			
Error	18,230.75	27			
Total	77,646.97	35			

- Threshold value to test effect of Material Type and Temperature

$$F_{0.05,2,27} = 3.35$$

- Threshold value to test effect of interaction

$$F_{0.05,4,27} = 2.73$$

Example 5.1: Battery Life Experiment – Cont'd

■ TABLE 5.5

Analysis of Variance for Battery Life Data

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Material types	10,683.72	2	5,341.86		
Temperature	39,118.72	2	19,559.36		
Interaction	9,613.78	4	2,403.44		
Error	18,230.75	27	675.21		
Total	77,646.97	35			

- Threshold value to test effect of Material Type and Temperature

$$F_{0.05,2,27} = 3.35$$

- Threshold value to test effect of interaction

$$F_{0.05,4,27} = 2.73$$

Example 5.1: Battery Life Experiment – Cont'd

■ TABLE 5.5

Analysis of Variance for Battery Life Data

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Material types	10,683.72	2	5,341.86	7.91	0.0020
Temperature	39,118.72	2	19,559.36	28.97	0.0001
Interaction	9,613.78	4	2,403.44	3.56	0.0186
Error	18,230.75	27	675.21		
Total	77,646.97	35			

- Threshold value to test effect of Material Type and Temperature

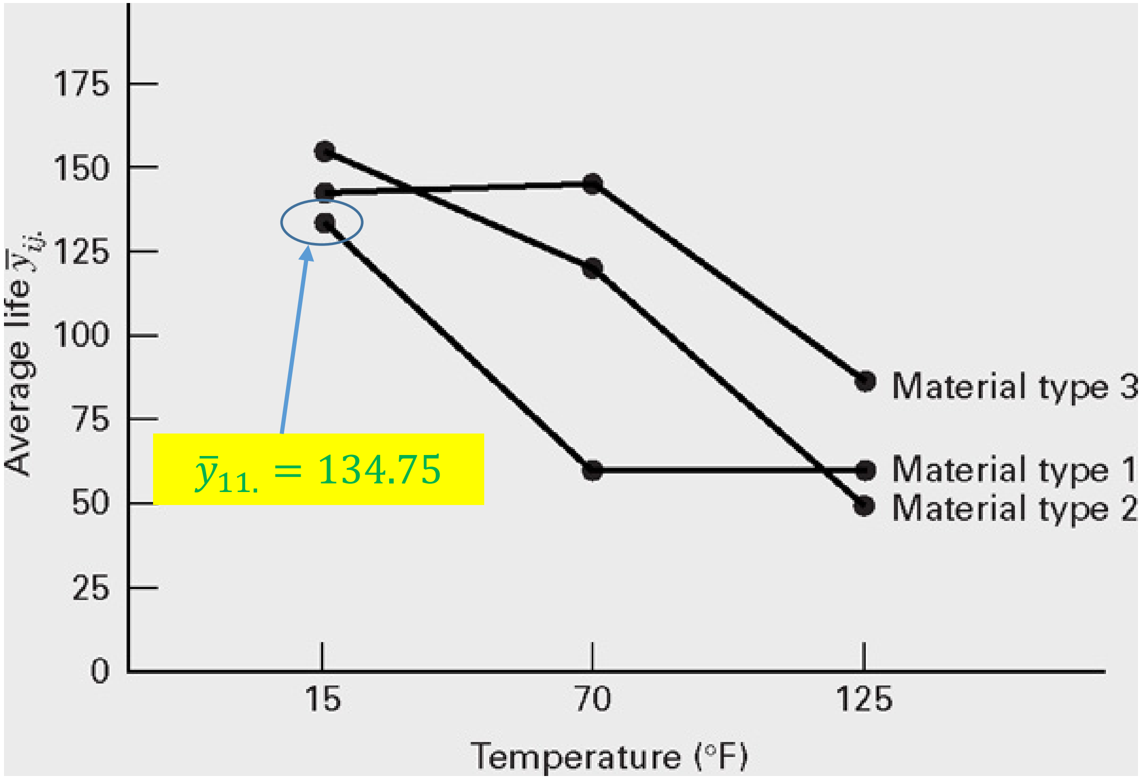
$$F_{0.05,2,27} = 3.35$$

- Threshold value to test effect of interaction

$$F_{0.05,4,27} = 2.73$$

Example 5.1: Battery Life Experiment – Cont'd

- Two factor interaction plot
  - Average response at each treatment combination
  - Long life attained at low temperatures regardless of material
  - Material 3 gives smallest loss of life with temperature
  - Lines are not parallel: interaction effect is significant
  - formal test of significance: use p-values in ANOVA table



Classwork

- Problem 5.8

5.4. Factorials with More Than Two Factors

- General case:
  - Factor A has a levels,
  - Factor B has b levels,
  - Factor C has c levels, …,
  - Factor K has k levels
  - Run n replications of each combinations
  - All $abc \dots kn$ treatment combinations are run in random order
- ANOVA identity is

$$
\begin{gathered}
SS_T = SS_A + SS_B + \ldots + SS_K + \\
SS_{AB} + SS_{AC} + \ldots + \\
SS_{ABC} + SS_{ABD} + \ldots + \\
SS_{AB\ldots K} + \\
SS_E
\end{gathered}
$$

Three factor factorial experiment

- A statistical model for the three-factor experiment

$$y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + \tau\beta_{ij} + \tau\gamma_{ik} + \gamma\beta_{jk} + \tau\beta\gamma_{ijk}$$

$$+\epsilon_{ijkl} \quad \begin{cases} i = 1,2,\ldots,a \\ j = 1,2,\ldots,b \\ k = 1,2,\ldots,c \\ l = 1,2,\ldots,n \end{cases}$$

■ **TABLE 5.12**

The Analysis of Variance Table for the Three-Factor Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A	SS_A	$a-1$	MS_A	$F_0 = \frac{MS_A}{MS_E}$
B	SS_B	$b-1$	MS_B	$F_0 = \frac{MS_B}{MS_E}$
C	SS_C	$c-1$	MS_C	$F_0 = \frac{MS_C}{MS_E}$
AB	SS_{AB}	$(a-1)(b-1)$	MS_{AB}	$F_0 = \frac{MS_{AB}}{MS_E}$
AC	SS_{AC}	$(a-1)(c-1)$	MS_{AC}	$F_0 = \frac{MS_{AC}}{MS_E}$
BC	SS_{BC}	$(b-1)(c-1)$	MS_{BC}	$F_0 = \frac{MS_{BC}}{MS_E}$
ABC	SS_{ABC}	$(a-1)(b-1)(c-1)$	MS_{ABC}	$F_0 = \frac{MS_{ABC}}{MS_E}$
Error	SS_E	$abc(n-1)$	MS_E	
Total	SS_T	$abcn-1$		

NOTE: We must have at least two replications (n>=2) to determine SSE if all possible interactions are included in the model