CSCE 554

Chapter 4 - Randomized Blocks, Latin Squares and related designs

Paired t-test

- What is a paired test?
- Why is it important?

- Let's talk about Nuisance variables…
- A **nuisance** factor is a factor that probably has some effect on the response, but it's of no interest to the experimenter…however, the variability it transmits to the response needs to be minimized

Types of Nuisance Factors

- nuisance factor is unknown (and uncontrolled)
  - "lurking" variable
  - hope that randomization balances out its impact across the experiment (Chapter 3)
- nuisance variable known and controllable
  - use blocking to eliminate effects of nuisance variable on statistical comparisons (Chapter 4)
- nuisance factor is known and uncontrollable,
  - use the analysis of covariance (see Chapter 15) to remove the effect of the nuisance factor from the analysis

- Sometimes several sources of variability are combined in a block, so the block becomes an aggregate variable

The Blocking Principle

- Blocking is controlling the factors that are not of primary interest ("nuisance factors") but that if uncontrolled will add to the variability in the data and obscure the true effects of the factors that are of interest
  - Nuisance factors: factors that may influence the experimental response; but in which we are not interested
- Typical nuisance factors: batches of raw material, operators, pieces of test equipment, time of experiment (shifts, days, etc.)

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- Blocking in design: Randomize and compare treatments within the same block
  - Randomized Complete Block Design (RCBD)
  - Block is a group of homogeneous experimental units (E.g.: samples from same batch)→ small variability within block
  - by making the comparisons within the same block, we can improve the precision of treatment comparisons

The Hardness Testing Example (p. 140)

- Determine whether 4 different tips produce different (mean) hardness reading on a Rockwell hardness tester
  - Press the tip on a metal coupon and measure resulting depth
  - metal coupon = experimental unit
  - Tips 1,…,4 = treatments
  - Take 4 observations for each tip

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  - Press the tip on a metal coupon and measure resulting depth
  - metal coupon = experimental unit
  - Tips 1,…,4 = treatments
  - Take 4 observations for each tip
- Completely randomized design (CRD): 16 different metal coupons
  - Randomly assign treatments to experimental units

Tip 3	Tip 1	Tip 2	Tip 3
Tip 4	Tip 2	Tip 2	Tip 1
Tip 2	Tip 3	Tip 4	Tip 3
Tip 1	Tip 1	Tip 4	Tip 4

Coupon

The Hardness Testing Example

- CRD: If the test coupons differ in their properties this will contribute to the experimental error
  - The difference between test coupons is a source of nuisance variability
  - Experimental error will include both random error and variability between coupons
- Blocking: To make experimental error as small as possible:
  - Test all 4 tips once on each of four coupons and perform comparisons within coupons→ Coupons are called "blocks" (<u>assuming</u>: coupons are large enough)
  - Blocking allows to remove the block-to-block variation from the experimental error →Improves the accuracy of comparisons among the treatments

The Hardness Testing Example

- Randomized Complete Block Design (RCBD)
  - Within a block, the tips are tested in random order.
  - A block represents a restriction on randomization
- A complete replicate of experiment in each block
  - Each block contains all treatments; "complete" block
  - Test each tip (treatment) once on each of four coupons (blocks)

Tip 3	Tip 3	Tip 2	Tip 1
Tip 1	Tip 4	Tip 1	Tip 4
Tip 4	Tip 2	Tip 3	Tip 2
Tip 2	Tip 1	Tip 4	Tip 3

Coupon (block)

ANOVA for blocked design

■ TABLE 4.4

Analysis of Variance for the Vascular Graft Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Treatments (extrusion pressure)	178.17	3	59.39	8.11	0.0019
Blocks (batches)	192.25	5	38.45		
Error	109.89	15	7.33		
Total	480.31	23			

- Critical value $F_{0.05,a-1,(a-1)(b-1)} = F_{0.05,3,15} = 3.29$
- Observed F statistic, 8.11, is within critical region→ Reject H0, extrusion pressure affects the defect occurrence

Not considering blocks

- Suppose we ran this experiment as a CRD and obtained the same data.
- P-value<0.05, but MSE more than doubled
  - P-value=0.0019 for RCBD
  - P-value =0.0239 for CRD
  - At $\alpha = 0.01$ we would fail to reject H_0 and conclude no difference between extrusion pressures
  - With CRD the effects of factors is not as easy to detect
  - RCBD is an effective noise reducing technique

TABLE 4.5

Incorrect Analysis of the Vascular Graft Experiment as a Completely Randomized Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	*P*-Value
Extrusion pressure	178.17	3	59.39	3.95	0.0235
Error	302.14	20	15.11		
Total	480.31	23			

4.1.1 Statistical Analysis of the RCBD

- Suppose a treatments to be compared in b blocks (total number of observations $N = ab$)
- A statistical model for the RCBD is

$$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij} \quad \begin{cases} i = 1,2,\ldots,a \\ j = 1,2,\ldots,b \end{cases}$$

- μ overall mean
- τ_i effect of i-th treatment
- β_j effect of j-th block
- $\epsilon_{ij} \sim NID(0, \sigma^2)$ experimental error

Basic principles

- Sum of Squares is additive
- Degrees of freedom is additive

- Check for assumptions:
  - ❑ Normality of residuals
  - ❑ Same variance across batches
  - ❑ Same variance across treatments

ANOVA Display for the RCBD

- ANOVA partitioning of sum of squares and resulting Mean Squares summarized in a table

■ **TABLE 4.2**

Analysis of Variance for a Randomized Complete Block Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}}$	$a - 1$	$\dfrac{SS_{\text{Treatments}}}{a-1}$	$\dfrac{MS_{\text{Treatments}}}{MS_E}$
Blocks	SS_{Blocks}	$b - 1$	$\dfrac{SS_{\text{Blocks}}}{b-1}$	
Error	SS_E	$(a-1)(b-1)$	$\dfrac{SS_E}{(a-1)(b-1)}$	
Total	SS_T	$N - 1$		

Estimates of the treatment and block effects

- Overall mean

$$\hat{\mu} = \bar{y}_{..}$$

- Effect of i-th treatment

$$\hat{\tau}_i = \bar{y}_{i.} - \bar{y}_{..}$$

- Effect of j-th block

$$\hat{\beta}_i = \bar{y}_{.j} - \bar{y}_{..}$$

- The prediction of y_{ij}

$$\begin{aligned} \hat{y}_{ij} &= \hat{\mu} + \hat{\tau}_i + \hat{\beta}_j \\ &= \bar{y}_{i.} + \bar{y}_{.j} - \bar{y}_{..} \end{aligned}$$

- Residuals

$$\begin{aligned} e_{ij} &= y_{ij} - \hat{y}_{ij} \\ &= y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..} \end{aligned}$$

Latin squares

- Extending blocking to 2 variables

- Basic principles
  - Sum of Squares is additive
  - Degrees of freedom is additive

  - Check for assumptions:
    - Normality of residuals
    - Same variance across blocked variable 1
    - Same variance across blocked variable 2
    - Same variance across treatments

ANOVA table for a Latin Square Design

Analysis of Variance for the Latin Square Design

Source of Variation	Sum of Squares	Degrees of Freedom
Treatments	$SS_{\text{Treatments}} = \frac{1}{p}\sum_{j=1}^{p} y_{.j.}^2 - \frac{y_{...}^2}{N}$	$p - 1$
Rows	$SS_{\text{Rows}} = \frac{1}{p}\sum_{i=1}^{p} y_{i..}^2 - \frac{y_{...}^2}{N}$	$p - 1$
Columns	$SS_{\text{Columns}} = \frac{1}{p}\sum_{k=1}^{p} y_{..k}^2 - \frac{y_{...}^2}{N}$	$p - 1$
Error	SS_E (by subtraction)	$(p - 2)(p - 1)$
Total	$SS_T = \sum_i \sum_j \sum_k y_{ijk}^2 - \frac{y_{...}^2}{N}$	$p^2 - 1$