Fully Polynomial Time Approximation Scheme

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Problem 1

Answer: $100\,n^2$

We begin by computing the value of ε:

$$\begin{aligned}(1-\varepsilon)\,\mathrm{OPT} &= 0.9\,\mathrm{OPT}\\ \mathrm{OPT}-\varepsilon\,\mathrm{OPT} &= 0.9\,\mathrm{OPT}\\ \mathrm{OPT}-0.9\,\mathrm{OPT} &= \varepsilon\,\mathrm{OPT}\\ 0.1\,\mathrm{OPT} &= \varepsilon\,\mathrm{OPT}\\ 0.1 &= \varepsilon\\ \varepsilon &= 0.1\\ \varepsilon &= \frac{1}{10}\end{aligned} \tag{1}$$

Next, we use this result to evaluate the time cost:

$$n^2\left(\frac{1}{\varepsilon}\right)^2 = n^2\cdot 10^2 = 100\,n^2. \tag{2}$$

Problem 2

Answer: $\max\{v | 0 \leq v \leq 5 \,\text{and}\, W(4,v) \leq 6\}$

Problem 3

Answer:
▷ $W(0,0) = 0$
▷ $W(1,1) = 2$
▷ $W(1,1) = \min(W(0,1), W(1,0)+2)$

Two of the options are easy to check. We were given the equation $W(0,0) = 0$. Then the corresponding option is **correct**. The expression for $W(0,1)$ is also simple to evaluate. After all, we were given the equation for $W(0,v)$. Since this value is infinity for $v > 0$, the option involving $W(0,1)$ is **wrong**.
To make a decision on the remaining options, we need to compute the value of $W(1,1)$. This can be done as follows:

$$\begin{aligned}W(1,1) &= \min(W(1-1,1), W(1-1,1-v_1)+w_1)\\ &= \min(W(0,1), W(0,1-1)+2)\\ &= \min(W(0,1), W(0,0)+2)\\ &= \min(W(0,1), 0+2)\\ &= \min(\infty, 2)\\ &= 2\end{aligned} \tag{3}$$

Therefore, the option containing a value is **correct**. The other option is also **right**, since $W(0,0) = W(1,0) = 0$. This allows us to write

$$\min(W(0,1), W(1,0)+2) = \min(W(0,1), W(0,0)+2) = W(1,1). \tag{4}$$