TSP Basics

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Problems 1 and 2 are related to the graph below. Black edges have weight 1, and red edges have weight 2.

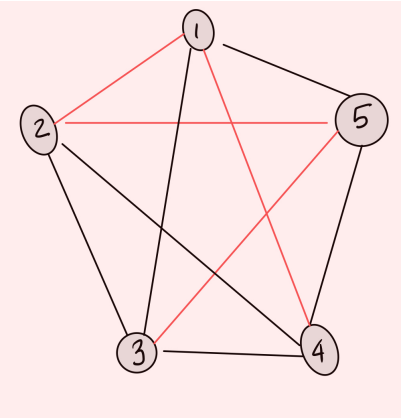

Figure 1: TSP instance

Problem 1

▷ There is a TSP tour whose cost is 5, i.e., it involves only the black edges in the graph.
▷ The sequence of vertices 1, 2, 3, 4, 5, 1 (1 is the start/end point) is a valid TSP tour of cost 6.

Problem 2

Answer: True

Problem 3

This problem is related to the following graph:

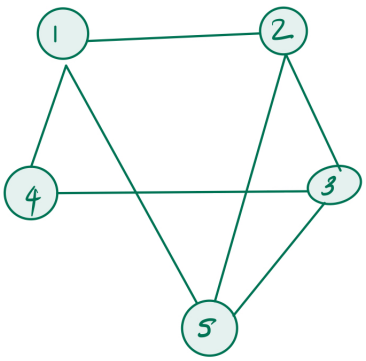

Figure 2: Graph for question on Hamiltonian cycles

- ▷ The reduction will need to assign a weight of 1 to each edge in the original graph and add the missing edges $(4,5)$, $(1,3)$ and $(2,4)$ with some weight $W > 1$.
- ▷ Once we reduce to the TSP, we conclude the presence of a Hamiltonian cycle if the optimal TSP tour has weight 6.
- ▷ Let $W > 1$ be the weight given to the missing edges that we will need to add back for the reduction to the TSP. The optimal TSP tour cost will be at least $5 + W$ if there is no Hamiltonian cycle.