Held-Karp Algorithm

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All problems are related to the graph shown below. Black edges have weight 1, and red edges have weight 2.

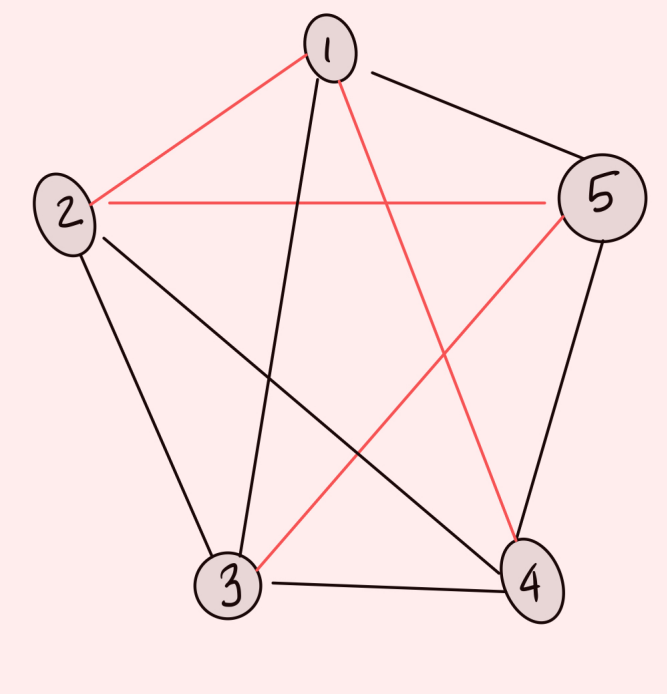

Figure 1: TSP instance

Problem 1

$$\text{minCostPath}\left(\{2,3\},4\right) = \min\left(\text{minCostPath}\left(\{2\},3\right) + C_{3,4}, \text{minCostPath}\left(\{3\},2\right) + C_{2,4}\right) \tag{1}$$

Problem 2

We begin by computing the arguments of the min function on the RHS of the above equation:

$$\begin{aligned}\text{minCostPath}\left(\{2\},3\right) + C_{3,4} &= C_{1,2} + C_{2,3} + C_{3,4} = 2+1+1 = 4,\\ \text{minCostPath}\left(\{3\},2\right) + C_{2,4} &= C_{1,3} + C_{3,2} + C_{2,4} = 1+1+1 = 3.\end{aligned}$$

Hence:

$$\text{minCostPath}\left(\{2,3\},4\right) = \min(4,3) = 3. \tag{2}$$

Problem 3

I believe the correct answer is vertex 2. After all, from the above calculations, visiting vertex 2 before vertex 4 yields the optimal solution. However, the answer that is accepted as correct is **vertex 3**. Perhaps I'm wrong. But it wouldn't be the first instructor mistake I catch.

Problem 4

The correct formula for the recurrence is

$$\text{minCostTSPTour}(C) = \min \begin{cases} \text{minCostTour}\left(\{2,3,5\},4\right) + C_{4,1} \\ \text{minCostTour}\left(\{2,3,4\},5\right) + C_{5,1} \\ \text{minCostTour}\left(\{3,4,5\},2\right) + C_{2,1} \\ \text{minCostTour}\left(\{2,4,5\},3\right) + C_{3,1} \end{cases}. \tag{3}$$

Therefore, the missing portions are given by

$$??_1 = C_{4,1} = 2, \tag{4}$$
$$??_2 = C_{5,1} = 1, \tag{5}$$
$$??_3 = 2, \tag{6}$$
$$??_4 = C_{2,1} = 2, \tag{7}$$
$$??_5 = \{2,4,5\}, \tag{8}$$
$$??_6 = C_{3,1} = 1. \tag{9}$$