Qubits

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Problem 1

- ▷ Measuring it yields 0 and 1 with equal probabilities $\left(\frac{1}{2}\right)$.
- ▷ Upon measurement the super position collapses to one of the pure states $|0\rangle$ or $|1\rangle$.

Problem 2

- ▷ $\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$
- ▷ 0
- ▷ 1

Problem 3

The general form of $|-\rangle$ is

$$|-\rangle = c_0|0\rangle + c_1|1\rangle\,, \tag{1}$$

where $|c_0|^2 + |c_1|^2 = 1$. Since we want $|+\rangle$ and $|-\rangle$ to be orthogonal, the following must hold:

$$\langle +|-\rangle = 0 \qquad \Rightarrow \qquad \frac{1}{\sqrt{2}}c_0 + \frac{1}{\sqrt{2}}c_1 = 0 \qquad \Rightarrow \qquad c_0 + c_1 = 0. \tag{2}$$

Then we have $c_1 = -c_0$. Assuming these constants are real numbers, we can write the other condition as follows:

$$2c_0^2 = 1 \qquad \Rightarrow \qquad c_0 = \frac{1}{\sqrt{2}}. \tag{3}$$

Hence:

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right). \tag{4}$$