Single Qubit Quantum Gates

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Problem 1

Answer: $|1\rangle$

Problem 2

- ▷ It is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$ in the computational basis $|0\rangle$, $|1\rangle$.
- ▷ It is a unitary operator.
- ▷ It is the same as a phase gate with phase $\frac{\pi}{2}$.

Problem 3

- ▷ Measuring both states yields either outcome 0, 1 with equal probabilities.
- ▷ If we applied a Hadamard gate then $H|\varphi\rangle = |0\rangle$ and $H|\psi\rangle = \frac{1}{2}((1+i)|0\rangle + (1-i)|1\rangle)$.
- ▷ The two states can be distinguished upon the application of the Hadamard gate and measurement. One of the states always yields $|0\rangle$ while the other yields $|0\rangle$ and $|1\rangle$ with equal probabilities.

Problem 4

- ▷ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- ▷ $\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$
- ▷ $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

The option $\frac{1}{\sqrt{2}}\begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix}$ is also correct. However, it's considered wrong.