Problem 1

Answer: 5

There are five decision variables, one for each type of loan.

Problem 2

Answer: $0.026x_1$

Let's denote the expected net return by R_1. This value can be expressed as

$$R_1 = r_1 x_1 (1 - p_1) - x_1 p_1, \tag{1}$$

where r_1 represents the interest rate, and p_1 is the probability of bad debt. By substituting the values of these quantities into the last equation, we get

$$\begin{aligned} R_1 &= r_1 x_1 (1 - p_1) - x_1 p_1 \\ &= 0.14(1 - 0.1)x_1 - 0.1x_1 \\ &= 0.126x_1 - 0.1x_1 \\ &= 0.026x_1. \end{aligned} \tag{2}$$

Problem 3

Answer: $0.0509x_2$

Let's denote the expected net return by R_2. This value can be expressed as

$$R_2 = r_2 x_2 (1 - p_2) - x_2 p_2, \tag{3}$$

where r_2 represents the interest rate, and p_2 is the probability of bad debt. By substituting the values of these quantities into the last equation, we get

$$\begin{aligned} R_2 &= r_2 x_2 (1 - p_2) - x_2 p_2 \\ &= 0.13(1 - 0.07)x_2 - 0.07x_2 \\ &= 0.1209x_2 - 0.07x_2 \\ &= 0.0509x_2. \end{aligned} \tag{4}$$

Problem 4

Answer:

$$\max \quad Z = 0.026x_1 + 0.0509x_2 + 0.0864x_3 + 0.06875x_4 + 0.078x_5$$

To derive the formula for the objective function, we need to perform calculations similar to the ones in Problems 2 and 3. Specifically, we have to compute the expected net return for the three remaining types of loan. We won't present the details here. However, it's straightforward to show that the expected returns associated with Home, Farm and Commercial loans can be expressed as follows:

▷ Home: $0.0864x_3$
▷ Farm: $0.06875x_4$
▷ Commercial: $0.078x_5$

Then the total expected return is

$$Z = 0.026x_1 + 0.0509x_2 + 0.0864x_3 + 0.06875x_4 + 0.078x_5. \tag{5}$$

It should be clear that the bank wants to maximize its return. Therefore, we need to find $\max Z$.

Problem 5

Answer: $x_1 + x_2 + x_3 + x_4 + x_5 \leq 10$

The total amount the bank will spend in loans is $x_1 + x_2 + x_3 + x_4 + x_5$. At most, the total spent will be \$10 million. Since the unit of all x_i is \$1 million, we can write this constraint as follows:

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 10. \tag{6}$$

Problem 6

Answer: $0.3x_1 + 0.3x_2 + 0.3x_3 - 0.7x_4 - 0.7x_5 \leq 0$

The amount allocated to Farm and Commercial loans is $x_4 + x_5$. This amount should be at least

$$0.3\left(x_1 + x_2 + x_3 + x_4 + x_5\right).$$

Therefore, this constraint can be written as

$$\begin{aligned} x_4 + x_5 &\geq 0.3\left(x_1 + x_2 + x_3 + x_4 + x_5\right) \\ x_4 + x_5 &\geq 0.3x_1 + 0.3x_2 + 0.3x_3 + 0.3x_4 + 0.3x_5 \\ 0.3x_1 + 0.3x_2 + 0.3x_3 + 0.3x_4 + 0.3x_5 &\leq x_4 + x_5 \\ 0.3x_1 + 0.3x_2 + 0.3x_3 - 0.7x_4 - 0.7x_5 &\leq 0 \end{aligned} \tag{7}$$

Problem 7

Answer: $0.4x_1 + 0.4x_2 - 0.6x_3 \leq 0$

Recall that the amount spent in Home loans is x_3. The amount allocated to Personal, Car and Home loans is given by $x_1 + x_2 + x_3$. Since Home loans must equal at least 40% of this sum, the following must hold:

$$\begin{aligned} x_3 &\geq 0.4\left(x_1 + x_2 + x_3\right) \\ x_3 &\geq 0.4x_1 + 0.4x_2 + 0.4x_3 \\ 0.4x_1 + 0.4x_2 + 0.4x_3 &\leq x_3 \\ 0.4x_1 + 0.4x_2 - 0.6x_3 &\leq 0 \end{aligned} \tag{8}$$

Problem 8

Answer: $0.06x_1 + 0.03x_2 - 0.01x_3 + 0.01x_4 - 0.02x_5 \leq 0$

For the amount x_i, we expect the bank to lose $x_i p_i$ due to bad debt. Then the total loss can be expressed as $x_1p_1 + x_2p_2 + x_3p_3 + x_4p_4 + x_5p_5$. By using the probability values, we can rewrite this loss: $0.1x_1 + 0.07x_2 + 0.03x_3 + 0.05x_4 + 0.02x_5$. The overall ratio of bad debts corresponds to the total loss divided by the total spent. Since this ratio is not to exceed 0.04, we have

$$\begin{aligned} \frac{0.1x_1 + 0.07x_2 + 0.03x_3 + 0.05x_4 + 0.02x_5}{x_1 + x_2 + x_3 + x_4 + x_5} &\leq 0.04 \\ 0.1x_1 + 0.07x_2 + 0.03x_3 + 0.05x_4 + 0.02x_5 &\leq 0.04x_1 + 0.04x_2 + 0.04x_3 + 0.04x_4 + 0.04x_5 \\ 0.06x_1 + 0.03x_2 - 0.01x_3 + 0.01x_4 - 0.02x_5 &\leq 0 \end{aligned} \tag{9}$$

Problem 9

Answer: \$0.8388 million

Problem 10

Answer: 2

Problem 11

Answer: $7 million in home loans and $3 million in commercial loans