Problem 1

Answer: 12

For every month, we need two decision variables: the number of pilots who are hired, and the number of pilots who are fired. Since we're considering 6 months, there are 12 decision variables.

Problem 2

Answer: $120 + x_{H_1} - x_{F_1}$

At the beginning of the first month, the company has 120 pilots. x_{H_1} pilots are hired, and x_{F_1} pilots are fired. Therefore, the number of available pilots is

$$120 + x_{H_1} - x_{F_1}.$$

Problem 3

Answer: $120 + x_{H_1} - x_{F_1} \geq 100$

In month 1, the company needs at least 100 pilots. We've just derived an expression for the number of available pilots. By using this result, we can write

$$120 + x_{H_1} - x_{F_1} \geq 100. \tag{1}$$

Problem 4

Answer: `D5:I6`

Problem 5

Answer: $4(120 + x_{H_1} - x_{F_1}) + 5x_{H_1} + 3x_{F_1}$

To write down a formula for this cost, we'll express all the amounts that represent money in units of $1,000. Every pilot who is still a staff member has to be paid his salary of $4k. The number of such pilots has been computed in Problem 2. By using that result, we can conclude that the cost related to salaries is $4(120 + x_{H_1} - x_{F_1})$. Furthermore, each of the x_{H_1} pilots who were hired represents a cost of $5k. Then the corresponding cost is $5x_{H_1}$. Finally, there's the cost of $3k for every pilot who were fired. In total, this cost is $3x_{F_1}$. By adding it all up, we get

$$4(120 + x_{H_1} - x_{F_1}) + 5x_{H_1} + 3x_{F_1}.$$

Problem 6

Answer: `C11+D5-D6`

Problem 7

Answer: `D11*$C$13+D5*$C$8+D6*$C$9`

Problem 8

Answer: $3740

To answer this question and the ones that follow, first we need to complete the mathematical formulation of this problem. We already know what happens during the first month. So consider month 2 next. At the beginning of this month, there are $N_1 = 120 + x_{H_1} - x_{F_1}$ pilots working for the company. x_{F_2} of them will be fired, and x_{H_2} new pilots will be hired. Then, in month 2, there will be $N_2 = N_1 + x_{H_2} - x_{F_2}$ available pilots.
As we have done for month 1, we can derive the constraint for N_2 and compute the corresponding contribution to the cost. To obtain the constraint, we can do something very similar to what we've done in Problem 3. What we get is

$$N_1 + x_{H_2} - x_{F_2} \geq 110. \tag{2}$$

The cost for the second month can be calculated as we've done in Problem 5. Very similar reasoning allows us to express this cost as follows:

$$4(N_1 + x_{H_2} - x_{F_2}) + 5x_{H_2} + 3x_{F_2}. \tag{3}$$

We could repeat this process four more times to get the desired results for the remaining months. We won't present the details, and simply write down these results. First consider the number N_i of pilots available in month i:

$$\begin{aligned} N_1 &= 120 + x_{H_1} - x_{F_1}, \\ N_2 &= N_1 + x_{H_2} - x_{F_2}, \\ N_3 &= N_2 + x_{H_3} - x_{F_3}, \\ N_4 &= N_3 + x_{H_4} - x_{F_4}, \\ N_5 &= N_4 + x_{H_5} - x_{F_5}, \\ N_6 &= N_5 + x_{H_6} - x_{F_6}. \end{aligned} \tag{4}$$

By introducing $N_0 = 120$, we can summarize this set of equations as follows:

$$N_i = N_{i-1} + x_{H_i} - x_{F_i}, \qquad i = 1, \ldots, 6. \tag{5}$$

There's a constraint for each of these numbers. These constraints guarantee that, during months 1-6, the company has enough pilots. These inequalities are the following:

$$\begin{aligned} N_1 &\geq 100, \\ N_2 &\geq 110, \\ N_3 &\geq 115, \\ N_4 &\geq 140, \\ N_5 &\geq 110, \\ N_6 &\geq 200. \end{aligned} \tag{6}$$

Finally, consider the cost C_i for the i-th month. This amount can be written as

$$C_i = 4N_i + 5x_{H_i} + 3x_{F_i}, \qquad i = 1, \ldots, 6. \tag{7}$$

The objective function for this problem corresponds to the total cost. The total is obtained from adding up all the C_i:

$$Z = \sum_{i=1}^{6} C_i = \sum_{i=1}^{6} (4N_i + 5x_{H_i} + 3x_{F_i}). \tag{8}$$

We need to minimize this function subject to the 6 constraints listed above, and to non-negativity constraints for all the decision variables x_{H_i} and x_{F_i}.

Problem 9

Answer: 3

Problem 10

Answer: 25