Problem 1

Answer: 1 in 700.

The smallest 300-digit number is $n = 10^{299}$. Then the probability is approximately

$$P \approx \frac{1}{\ln(n)} = \frac{1}{\ln\left(10^{299}\right)} = \frac{1}{299\,\ln(10)}. \tag{1}$$

Since $\ln(10) \approx 2.3$, we can write

$$P \approx \frac{1}{299 \cdot 2.3} = \frac{1}{687.7}. \tag{2}$$

Therefore, the probability is close to 1 in 700.

Problem 2

Answer: All prime numbers that are no greater than the square root of N.

Problem 3

Answer: the General Number Field Sieve.

Problem 4

Answer: Carmichael numbers.

Problem 5

Answer: a Fermat witness to the compositeness of n.

Problem 6

Answer: a Fermat liar.

Problem 7

Answer: that there are exactly two square roots of 1, namely 1 and $N-1$, $(\text{mod } N)$.

Problem 8

Answer: 174

Problem 9

Answer: decreases as the size of the prime number sought increases.

Problem 10

Answer: at least $\frac{3}{4}$.