Problem 1

Answer: 2

There are two decision variables, x_1 and x_2. x_1 denotes the number of toy soldiers produced, and x_2 denotes the number of toy trains produced.

Problem 2

Answer: \$3

The cost of producing a toy soldier is \$24 (materials + labor). Since a soldier sells for \$27, the profit margin is \$3.

Problem 3

Answer: \$2

The cost of producing a toy train is \$19 (materials + labor). Since a train sells for \$21, the profit margin is \$2.

Problem 4

Answer:

$$\max \quad Z = 3x_1 + 2x_2$$

Producing x_1 soldiers generates a profit of $3x_1$. Similarly, producing x_2 trains generates a profit of $2x_2$. Then the total profit is $Z = 3x_1 + 2x_2$. The goal is to maximize profit. Therefore, Giapetto needs to find $\max Z$.

Problem 5

Answer: 3

There are three constraints:

▷ one that specifies the upper bound for the number of finishing hours;
▷ one that specifies the upper bound for the number of carpentry hours;
▷ one that specifies the upper bound for the number of toy soldiers.

Problem 6

Answer: $2x_1 + x_2 \leq 100$

To produce x_1 soldiers, $2x_1$ hours of finishing labor are required. Similarly, to produce x_2 trains, x_2 hours of finishing labor are required. Then the total number of finishing hours is $2x_1 + x_2$. We know this number is at most 100 hours. Hence:

$$2x_1 + x_2 \leq 100.$$

Problem 7

Answer: $x_1 + x_2 \leq 80$

To produce x_1 soldiers, x_1 hours of carpentry labor are required. Similarly, to produce x_2 trains, x_2 hours of carpentry labor are required. Then the total number of carpentry hours is $x_1 + x_2$. We know this number is at most 80 hours. Hence:

$$x_1 + x_2 \leq 80.$$

Problem 8

Answer: No

Actually, it's not necessary to plot the feasible region. We can answer this question by using the constraints directly. For $x_1 = 40$ and $x_2 = 30$, we have

$$\begin{aligned} 2x_1 + x_2 &= 2 \cdot 40 + 30 \\ &= 80 + 30 \\ &= 110. \end{aligned}$$

This shows that the constraint $2x_1 + x_2 \leq 100$ is not satisfied. Therefore, the point under consideration is **not** in the feasible region.

Problem 9

Answer: $(x_1 = 20, x_2 = 60)$

The plot clearly shows that the solution corresponds to the point G. This point lies in the intersection between the lines defined by the following equations:

$$\begin{aligned} 2x_1 + x_2 &= 100, \\ x_1 + x_2 &= 80. \end{aligned}$$

Using the second equation, we can express x_2 as

$$x_2 = 80 - x_1.$$

Next, we substitute this result into the first equation, and solve for x_1:

$$\begin{aligned} 2x_1 + x_2 &= 100 \\ 2x_1 + 80 - x_1 &= 100 \\ x_1 + 80 &= 100 \\ x_1 &= 100 - 80 \\ x_1 &= 20 \end{aligned}$$

Hence:

$$x_2 = 80 - 20 = 60.$$

Therefore, the optimal solution is given by $(x_1, x_2) = (20, 60)$. Notice that this result is consistent with what is shown in the graph.

Problem 10

Answer: \$180

According to the graph, this value is $Z = 180$. Just to be sure, let's use the optimal solution to compute the maximum profit:

$$\begin{aligned} Z &= 3x_1 + 2x_2 \\ &= 3 \cdot 20 + 2 \cdot 60 \\ &= 60 + 120 \\ &= 180. \end{aligned}$$