Problem 1

Answer: Stock A

β is a measure of price fluctuation for a given stock. Then a small β indicates less fluctuations. This is precisely what a risk-averse investor wants. Therefore, I would recommend **stock A**.

Problem 2

Answer: 1.0

By definition, β is

$$\beta = \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}(R_m)}, \tag{1}$$

where R_i denotes the rate of return for the stock of interest, and R_m denotes the rate of return for some stock market index.
This question is about what happens when $R_i = R_m$. By using the fact that $\mathrm{Cov}(X, X) = \mathrm{Var}(X)$, we can write β as follows:

$$\begin{aligned}\beta &= \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}(R_m)} \\ &= \frac{\mathrm{Cov}(R_m, R_m)}{\mathrm{Var}(R_m)} \\ &= \frac{\mathrm{Var}(R_m)}{\mathrm{Var}(R_m)} \\ &= 1.\end{aligned} \tag{2}$$

Therefore, the value of β for the market is 1.

Problem 3

Answer: Firm A

Problem 4

Answer: 21.25%

We begin by summarizing the information we were given:
▷ The company's beta is $\beta = 1.5$;
▷ the risk-free rate is 7%, i.e., $r_{\mathrm{RF}} = 0.07$;
▷ the equity premium is 9.5%, i.e., $\mathrm{EP} = 0.095$.
As explained in the lectures, the cost of equity R_e can be computed as follows:

$$R_e = r_{\mathrm{RF}} + \beta \cdot \mathrm{EP}. \tag{3}$$

By substituting the known values into the RHS of the above equation, we get

$$\begin{aligned}R_e &= r_{\mathrm{RF}} + \beta \cdot \mathrm{EP} \\ &= 0.07 + 1.5 \cdot 0.095 \\ &= 0.2125.\end{aligned} \tag{4}$$

Therefore, the expected return is 21.25%.

Problem 5

Answer: 0.89

To answer this question, we're going to use the equation for the cost of equity. By solving this equation for β, we get

$$\begin{aligned} R_e &= r_{\mathrm{RF}} + \beta \cdot \mathrm{EP} \\ R_e - r_{\mathrm{RF}} &= \beta \cdot \mathrm{EP} \\ \beta &= \frac{R_e - r_{\mathrm{RF}}}{\mathrm{EP}} \end{aligned} \tag{5}$$

We were given all the values on the RHS of the last equation:
▷ the expected return is 10.2%, i.e., $R_e = 0.102$;
▷ the risk-free rate is 4%, i.e., $r_{\mathrm{RF}} = 0.04$;
▷ the market risk premium is 7%, i.e., $\mathrm{EP} = 0.07$.
By substituting these values into our formula for β, we obtain

$$\begin{aligned} \beta &= \frac{R_e - r_{\mathrm{RF}}}{\mathrm{EP}} \\ &= \frac{0.102 - 0.04}{0.07} \\ &= 0.89. \end{aligned} \tag{6}$$

Therefore, the value of beta for this stock is $\beta = 0.89$.

Problem 6

Answer: Yes

We begin by summarizing the information we were given. If we denote the value of the assets by A, then we have the following:
▷ equity is $E = 0.5A$;
▷ debt is $D = 0.5A$;
▷ cost of equity is $R_e = 0.1$;
▷ cost of debt is $R_d = 0.05$;
▷ tax rate is $t = 0$.
We can use the values above to compute the WACC. As explained in the lectures, this quantity is given by

$$\mathrm{WACC} = \frac{E}{E+D} R_e + \frac{D}{E+D}(1-t) R_d. \tag{7}$$

By substituting the known values into the RHS of this equation, we get

$$\begin{aligned} \mathrm{WACC} &= \frac{E}{E+D} R_e + \frac{D}{E+D}(1-t) R_d \\ &= \frac{0.5A}{0.5A + 0.5A} R_e + \frac{0.5A}{0.5A + 0.5A}(1-0) R_d \\ &= 0.5R_e + 0.5R_d \\ &= 0.5(R_e + R_d) \\ &= 0.5(0.1 + 0.05) \\ &= 0.5 \cdot 0.15 \\ &= 0.075. \end{aligned} \tag{8}$$

Then the cost of capital for this company is 7.5%.
The next step is to use the WACC to compute the present value of \$600 million in 2 years. With the WACC as the discount rate, we get the following for this PV:

$$\mathrm{PV} = \frac{600}{(1+0.075)^2} = 519.2. \tag{9}$$

The present value of the income is \$519.2 million. Since this amount is greater than the required investment, the company should take on the project.

Problem 7

Answer: No

In this case, the present value corresponding to the income is

$$\mathrm{PV} = \frac{600}{(1+0.075)^3} = 482.98. \tag{10}$$

The PV of the income is \$482.98 million. Since this amount is less than the required investment, the company should NOT take on the project.

Problem 8

Answer: 33.3%

To solve this problem, we're going to use the accounting equation:

$$A = D + E, \tag{11}$$

where A denotes the amount in assets, D denotes the debt amount, and E denotes the equity amount. We know that $D = 0.25A$. Hence:

$$\begin{aligned} A &= D + E \\ A &= 0.25A + E \\ A - 0.25A &= E \\ E &= 0.75A \end{aligned} \tag{12}$$

The company's debt-equity ratio can now be computed as follows:

$$\begin{aligned} \frac{D}{E} &= \frac{0.25A}{0.75A} \\ &= \frac{0.25}{0.75} \\ &= \frac{\frac{1}{4}}{\frac{3}{4}} \\ &= \frac{1}{3} \end{aligned} \tag{13}$$

As a percentage, this ratio is approximately 33.3%.

Problem 9

Answer: 66.6%

First, we use the debt-equity ratio to obtain an expression for D:

$$\frac{D}{E} = 0.5 \qquad \Rightarrow \qquad D = 0.5E. \tag{14}$$

Next, we substitute this expression into the accounting equation:

$$\begin{aligned}
A &= D + E \\
A &= 0.5E + E \\
A &= 1.5E \\
A &= \frac{3}{2}E \\
E &= \frac{2}{3}A
\end{aligned} \tag{15}$$

Therefore, two thirds of the company's assets are financed by equity. As a percentage, this fraction is approximately 66.6%.

Problem 10

Answer: 97%

We begin by re-writing the definition of WACC. The goal is to make the debt-equity ratio appear in that equation, and then solve for this ratio. This can be done as follows:

$$\begin{aligned}
\text{WACC} &= \frac{E}{E+D}R_e + \frac{D}{E+D}(1-t)R_d \\
(E+D)\text{WACC} &= ER_e + D(1-t)R_d \\
\frac{(E+D)\text{WACC}}{E} &= \frac{ER_e + D(1-t)R_d}{E} \\
\left(1 + \frac{D}{E}\right)\text{WACC} &= R_e + \frac{D}{E}(1-t)R_d \\
\text{WACC} + \frac{D}{E}\text{WACC} &= R_e + \frac{D}{E}(1-t)R_d \\
\frac{D}{E}\left[\text{WACC} - (1-t)R_d\right] &= R_e - \text{WACC} \\
\frac{D}{E} &= \frac{R_e - \text{WACC}}{\text{WACC} - (1-t)R_d} \\
\frac{D}{E} &= \frac{\text{WACC} - R_e}{(1-t)R_d - \text{WACC}}
\end{aligned} \tag{16}$$

Now it's just a matter of using the values given in the problem statement:
▷ $\text{WACC} = 0.098$;
▷ $R_e = 0.13$;
▷ $R_d = 0.065$.
The value of the tax rate wasn't specified. So we assume that $t = 0$. By substituting the above values into our first equation for the debt-equity ratio, we get

$$\begin{aligned}
\frac{D}{E} &= \frac{R_e - \text{WACC}}{\text{WACC} - (1-t)R_d} \\
&= \frac{R_e - \text{WACC}}{\text{WACC} - R_d} \\
&= \frac{0.13 - 0.098}{0.098 - 0.065} \\
&= 0.97.
\end{aligned} \tag{17}$$

Therefore, the debt-equity ratio for this company is 97%.