Problem 1

Answer: Project B

We begin by summarizing the relevant information in the problem statement. For project A, we have the following:
▷ the value of the initial investment is $1,000;
▷ the value of the annual income is $500.
The data related to project B is the following:
▷ the value of the initial investment is $2,000;
▷ the value of the annual income is $900.
In both cases, the discount rate is $r = 0.05$, and the number of time periods is $t = 3$ years.
First, we compute the NPV for project A. In this case, the NPV can be expressed as

$$\mathrm{NPV}_A = -1000 + \sum_{t=1}^{3} \frac{500}{(1+0.05)^t} = -1000 + 500 \sum_{t=1}^{3} \frac{1}{1.05^t} \approx 361.62. \tag{1}$$

Therefore, the net present value for project A is $361.62. Next, consider the NPV for project B. This amount can be computed as follows:

$$\mathrm{NPV}_B = -2000 + \sum_{t=1}^{3} \frac{900}{(1+0.05)^t} = -2000 + 900 \sum_{t=1}^{3} \frac{1}{1.05^t} \approx 450.92. \tag{2}$$

Since this amount is greater than NPV_A, it's better to select project B.

Problem 2

Answer: Project A

First, consider project A. In this case, it's clear that we get the initial investment back in two years. On the other hand, for project B, in the same period we would get back only $1,800, which is less than the initial investment. Then the payback period of B is greater than 2 years. Therefore, based on payback period, we should select project A.

Problem 3

Answer: NPV decreases

If the initial investment increases, then we subtract a larger number from the present value of the cash flows. Assuming this PV doesn't change, we get a smaller difference as a result. Therefore, the NPV decreases.

Problem 4

Answer: NPV decreases

Problem 5

Answer: IRR decreases

To answer this question, we're going to compute the IRR in both cases. In the first case, the NPV can be written as follows:

$$\mathrm{NPV}_1 = -1000 + \frac{50}{(1+r_1)^1} + \frac{50}{(1+r_1)^2} + \frac{1000}{(1+r_1)^2}, \tag{3}$$

where r_1 denotes the discount rate in case 1. By definition, this quantity becomes the IRR when $\text{NPV}_1 = 0$. By imposing this condition, we get

$$\frac{50}{1+\text{IRR}_1} + \frac{1050}{(1+\text{IRR}_1)^2} = 1000. \tag{4}$$

To actually compute IRR_1, first we need to re-write this equation:

$$\begin{aligned}
\frac{50}{1+\text{IRR}_1} + \frac{1050}{(1+\text{IRR}_1)^2} &= 1000 \\
\frac{1}{1+\text{IRR}_1} + \frac{21}{(1+\text{IRR}_1)^2} &= 20 \\
(1+\text{IRR}_1)^2\left[\frac{1}{1+\text{IRR}_1} + \frac{21}{(1+\text{IRR}_1)^2}\right] &= 20(1+\text{IRR}_1)^2 \\
1+\text{IRR}_1+21 &= 20(1+\text{IRR}_1)^2 \\
\text{IRR}_1+22 &= 20(1+2\text{IRR}_1+\text{IRR}_1^2) \\
\text{IRR}_1+22 &= 20+40\text{IRR}_1+20\text{IRR}_1^2 \\
20\text{IRR}_1^2+39\text{IRR}_1-2 &= 0
\end{aligned} \tag{5}$$

The above equation can be solved easily with the aid of the quadratic formula. One can show that this equation has two solutions: $\text{IRR}_1 = -2$ and $\text{IRR}_1 = \frac{1}{20}$. A discount rate is a strictly positive number. Then the negative root isn't a valid solution. This allows us to conclude that

$$\text{IRR}_1 = \frac{1}{20} = 0.05. \tag{6}$$

In other words, in the first case, the internal rate of return is 5%.
To obtain the IRR in the second case, we can follow a similar procedure. For this reason, we won't present a detailed explanation. Instead, we'll simply write down the important results. It's straightforward to show that IRR_2 satisfies the following quadratic equation:

$$25\text{IRR}_2^2+49\text{IRR}_2-2 = 0. \tag{7}$$

With the aid of the quadratic formula, it's simple to prove that the solutions to the above equation are -2 and 0.04. Once again, we discard the negative root. Then we have $\text{IRR}_2 = 0.04$. This clearly shows that the IRR decreases.

Problem 6

Answer: IRR decreases

The initial situation is the same as in the previous problem. If we increase the investment to $1,050, the IRR is the positive solution to the following equation:

$$21\text{IRR}_3^2+41\text{IRR}_3-1 = 0. \tag{8}$$

Once again, it's possible to show that this equation has two real roots, one negative and the other positive. This positive root is given approximately by $\text{IRR}_3 \approx 0.024$. This value is close to half the original one. Therefore, as in the previous problem, the IRR decreases.

Problem 7

Answer: ROI doubles

It's very clear that the return doubles when the annual incomes double. As a consequence, the ROI also doubles.