

Problem 1

Answer: Player 2 does not play R.

Player 2 is rational, and Player 1 knows it. So Player 1 knows that, no matter which strategy he chooses, Player 2 will choose the strategy that maximizes his payoff. Then consider the best reply by Player 2 for each strategy of Player 1:

- ▷ Player 1 chooses U: Player 2 chooses L;
- ▷ Player 1 chooses B: Player 2 chooses C.

Therefore, Player 2 will never respond with strategy R, and Player 1 knows it.

Problem 2

Answer: Only A.

First, consider the possible replies by Player 1 for each strategy of Player 2:

- ▷ Player 2 chooses X:
 - C is the best reply;
 - A is worse;
 - B is the worst reply.
- ▷ Player 2 chooses Y:
 - B is the best reply;
 - C is worse;
 - A is the worst reply.
- ▷ Player 2 chooses Z:
 - C is the best reply;
 - A is worse;
 - B is the worst reply.

This makes it clear that C is always a better strategy than A. In other words, A is dominated by C.

Next, consider the possible replies by Player 2 for each strategy of Player 1:

- ▷ Player 1 chooses A:
 - X is the best reply;
 - Y is worse;
 - Z is the worst reply.
- ▷ Player 1 chooses B:
 - Z is the best reply;
 - Y is worse;
 - X is the worst reply.
- ▷ Player 1 chooses C:
 - Z is the best reply;
 - X is worse;
 - Y is the worst reply.

In this case, there is no dominated strategy. Therefore, the only dominated strategy in this game is A.

Problem 3

Answer: Player 2 knows that Player 1 never chooses A. Given that, Z is always best for Player 2. Therefore, Player 2 chooses Z.

Player 1 is rational, and Player 2 knows it. So Player 2 knows that, no matter which strategy he chooses, Player 1 will choose the strategy that maximizes his payoff. Then, based on our previous analysis, Player 1 will never select strategy A. Player 2 is aware of this fact. In other words, Player 2 knows that his opponent will choose either B or C. In both of these cases, since Player 2 is rational, he will choose the best reply. As we've seen, the

best reply to B is Z. This is also the best strategy against C. Therefore, Player 1 will never select A, and Player 2 will select Z.

Problem 4

Answer: Both players are rational. Both players know that they are both rational. Both players know that they know that they are both rational.

In the previous problem, we've examined the situation in which

- ▷ both players are rational, and
- ▷ Player 2 knows that Player 1 is rational.

In this situation (Case 1), Player 1 chooses either B or C, and Player 2 chooses Z.

Next, consider a similar situation (Case 2):

- ▷ both players are rational, and
- ▷ Player 1 knows that Player 2 is rational.

Since Player 2 is rational, he'll never play strategy Y. Player 1 is aware of this fact. This player is rational. So he'll select C, which is the best reply to both X and Z. Therefore, in this situation, Player 1 chooses C, and Player 2 chooses either X or Z.

Let us combine the above conclusions into a single general statement. When both players are rational and one of them knows about the rationality of his opponent, the player with more knowledge has only one option, and his opponent has two options. Then both players knowing that they're both rational isn't enough for the Nash equilibrium to be played.

Finally, consider the following case:

- ▷ both players are rational,
- ▷ both players know that they're both rational, and
- ▷ both players know that they know that they're both rational.

First, we start from the perspective of Player 1. This player is rational, and he knows that Player 2 is also rational. As a result, Player 1 will reach the same conclusion we did for Case 2. In other words, this player will select strategy C. But now Player 2 knows this will be his opponent's choice. Since Player 2 is rational, he'll select his best option, strategy Z. In this case, the Nash equilibrium is played.

To conclude, we now start from Player 2's perspective. This player is rational, and he knows that his opponent is also rational. As a result, Player 2 will reach the same conclusion we did for Case 1. In other words, this player will select strategy Z. But now Player 1 knows this will be his opponent's choice. Since Player 1 is rational, he'll select his best option, strategy C. Once again, the Nash equilibrium is played. Notice that, for this to occur, Player 1 has to know that both players know that they're both rational.

Problem 5

Answer: Strategy B does not provide a strictly higher payoff than Strategy A, but Strategy A sometimes provides a strictly higher payoff.

Problem 6

Answer: When there is uncertainty, payoffs represent utility from each outcome, which describes players' attitude to risks.

Problem 7

Answer: They do not choose 0 because choosing $1/4$ is always better (irrespective of the opponent's location). Similarly, they do not choose 1 either.