Pecuniary externalities in economies with downward wage rigidity: Online Appendix.

A Normative analysis

This Appendix complements the normative analysis from Section 3. In Appendix A.1, we first repeat the planning problem from Definition 2. We then derive the constrained-efficient labor demand curve from Proposition 1. In Appendix A.2, we provide details on the implied macroprudential regulation, including the proof of Proposition 2.

A.1 The constrained-efficient equilibrium

From Definition 2, the constrained-efficient equilibrium solves

$$\max E_0 \sum_{t\geq 0} \beta^t U(C_t - G(H_t))$$

subject to the set of constraints

$$\begin{array}{llr}
i) & P_t C_t + B_{t+1}/R = P_t a_t F(H_t) + B_t & \text{(multiplier: } \iota_t) \\
ii) & (U'(t)/P_t) = \beta R E_t (U'(t+1)/P_{t+1}) & \text{(multiplier: } \nu_t) \\
iii) & W_t \geq P_t G'(H_t) & \text{(multiplier: } \zeta_t) \\
iv) & W_t \geq \psi W_{t-1} & \text{(multiplier: } \lambda_t) \\
v) & P_t a_t F'(H_t) \geq W_t & \text{(multiplier: } \gamma_t)
\end{array}$$

where $U'(t) \equiv U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{a_t, \bar{P}_t\}_{t\geq 0}$.

We show first that constraint ii) is slack, implying that we can omit this constraint from the maximization. We proceed as in Bianchi (2016): we consider the maximization without constraint ii) and show that constraint ii) is implied as an optimality condition.

Assume constraint ii) is never binding ($\nu_t = 0$). Taking first order conditions with respect to C_t and B_{t+1} gives

$$U'(t) - \iota_t P_t = 0$$
$$-\iota_t / R + \beta E_t \iota_{t+1} = 0.$$

Combining both yields constraint ii). As a result, constraint ii) is never binding in equilibrium, as claimed.

Now we take first order conditions with respect to W_t and H_t

$$-\gamma_t + \zeta_t + \lambda_t - \beta\psi E_t \lambda_{t+1} = 0 \tag{A.1}$$

$$-U'(t)G'(H_t) + \gamma_t \frac{1}{\varepsilon_t^F}\frac{W_t}{H_t} - \zeta_t \frac{1}{\varepsilon_t^G}\frac{W_t}{H_t} + U'(t)a_t F'(H_t) = 0 \tag{A.2}$$

where we define $\varepsilon_t^F < 0$ and $\varepsilon_t^G > 0$ as the wage elasticities of the labor demand and supply curve, respectively. The demand elasticity is negative under our assumptions imposed on F

(labor demand slopes downward in wages), whereas the supply elasticity is positive under our assumptions imposed on G (labor supply slopes upwards).[39]

We proceed by distinguishing the cases where downward wage rigidity is slack and binding, respectively.

Case 1: Downward nominal wage rigidity is slack

If downward nominal wage rigidity is slack ($W_t > \psi W_{t-1}$) then $\lambda_t = 0$ by complementary slackness. We show that in this case, labor supply iii) must hold with equality. Assume not. In this case, $\zeta_t = 0$. Using $\zeta_t = 0$ in equation (A.2) gives

$$\gamma_t = \varepsilon_t^F \frac{H_t}{W_t} U'(t) \left(G'(H_t) - a_t F'(H_t) \right).$$

Because of labor demand v) and because labor supply iii) holds with strict inequality by assumption, the expression in brackets is strictly negative. Because also $\varepsilon_t^F < 0$, it follows that $\gamma_t > 0$. Imposing $\zeta_t = 0$ in (A.1) yields $-\gamma_t = \beta\psi E_t \lambda_{t+1} \geq 0$. This is a contradiction. Labor supply iii) thus holds with equality.

We next show that $\gamma_t = 0$. Assume not, $\gamma_t > 0$. In this case, labor demand v) holds with equality. Because constraints v) and iii) both hold with equality, equation (A.2) yields

$$\gamma_t \frac{1}{\varepsilon_t^F} \frac{W_t}{H_t} = \zeta_t \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t}.$$

Because $\gamma_t > 0$ and $\varepsilon_t^F < 0$, the left hand side is strictly negative. But the right hand side is weakly positive ($\zeta_t \geq 0$ and $\varepsilon_t^G > 0$). This is a contradiction. The multiplier for labor demand v) therefore equals zero: $\gamma_t = 0$.

We summarize our results. When downward nominal wage rigidity is slack, $\lambda_t = \gamma_t = 0$ and constraint ii) holds with equality. Using this to combine (A.2) and (A.1), by replacing ζ_t, then yields

$$a_t F'(H_t) = \frac{W_t}{P_t} + \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta\psi E_t \lambda_{t+1}. \quad (9)$$

This is constrained-efficient labor demand when downward nominal wage rigidity is slack, equation (9) in the main text.

Case 2: Downward nominal wage rigidity binds

Assume now that downward nominal wage rigidity iv) binds. We need to make a case distinction. We first study a region where labor supply iii) holds with equality (as when downward nominal wage rigidity is slack). We call this the region where downward nominal wage rigidity "binds lightly". We move on to the case where labor supply iii) is rationed. We call this the region where downward nominal wage rigidity "binds strongly".

[39] Labor demand is

$$F'^{-1}\left(\frac{W_t}{P_t}\frac{1}{a_t}\right) = H_t = H_t(W_t).$$

The demand elasticity is defined as $\varepsilon_t^F \equiv H_t'(W_t)(W_t/H_t) < 0$. Labor supply is

$$G'^{-1}\left(\frac{W_t}{P_t}\right) = H_t = H_t(W_t).$$

The supply elasticity is defined as $\varepsilon_t^G \equiv H_t'(W_t)(W_t/H_t) > 0$.

Assume first that labor supply iii) holds with equality. Intuitively, this implies that W_t is determined by downward nominal wage rigidity iv) whereas hours H_t are determined by labor supply iii). We have already shown that, when labor supply iii) holds with equality, it must be that $\gamma_t = 0$. We can thus solve for ζ_t, from (A.2)

$$\zeta_t = \varepsilon_t^G \frac{H_t}{W_t} U'(t) \left(a_t F'(H_t) - \frac{W_t}{P_t} \right),$$

where we have used that labor supply iii) holds with equality (we have replaced $G'(H_t) = W_t/P_t$). Recall that, when downward nominal wage rigidity is slack, $a_t F'(H_t) > W_t/P_t$, in which case $\zeta_t > 0$ is strictly positive (recall that $\varepsilon_t^G > 0$). The previous expression shows that ζ_t can remain positive when downward nominal wage rigidity starts to bind. However, W_t/P_t rises and $a_t F'(H_t)$ falls when $W_t = \psi W_{t-1}$ gradually rises, because hours H_t are determined by labor supply which slopes upward in wages. At some point, ζ_t will therefore become equal to zero.

At this point, $\zeta_t = 0$ and labor supply iii) holds with strict inequality: labor supply now becomes rationed. We can show that in this case, labor demand v) holds with equality and $\gamma_t > 0$. Assume not, $\gamma_t = 0$. Then from (A.2), $a_t F'(H_t) = G'(H_t)$. Because labor supply is rationed, $G'(H_t) < W_t/P_t$, this implies $a_t F'(H_t) < W_t/P_t$, which is a contradiction because of labor demand v).

Thus $\gamma_t > 0$, implying that labor demand v) holds with equality, $a_t F'(H_t) = W_t/P_t$. We can then infer γ_t from (A.2)

$$\gamma_t = \varepsilon_t^F \frac{H_t}{W_t} U'(t) \left(G'(H_t) - \frac{W_t}{P_t} \right)$$

where we use that labor demand v) holds with equality to replace $a_t F'(H_t) = W_t/P_t$. Thus, $\gamma_t > 0$ because labor supply iii) is rationed: $G'(H_t) < W_t/P_t$ (recall that $\varepsilon_t^F < 0$).

The multiplier λ_t can be inferred from combining (A.1) and (A.2)

$$\lambda_t = -U'(t) \left(\varepsilon_t^F \frac{H_t}{W_t} \left(\frac{W_t}{P_t} - G'(H_t) \right) + \varepsilon_t^G \frac{H_t}{W_t} \left(a_t F'(H_t) - \frac{W_t}{P_t} \right) \right) + \beta \psi E_t \lambda_{t+1} \tag{10}$$

which is equation (10) in the main text. When downward nominal wage rigidity binds lightly, $W_t/P_t = G'(H_t)$, hence the multiplier can be simplified to

$$\lambda_t = -U'(t) \varepsilon_t^G \frac{H_t}{W_t} \left(a_t F'(H_t) - \frac{W_t}{P_t} \right) + \beta \psi E_t \lambda_{t+1}.$$

When downward nominal wage rigidity binds strongly, $a_t F'(H_t) = W_t/P_t$, hence the multiplier becomes

$$\lambda_t = -U'(t) \varepsilon_t^F \frac{H_t}{W_t} \left(\frac{W_t}{P_t} - G'(H_t) \right) + \beta \psi E_t \lambda_{t+1}.$$

A.2 Implications for regulation

In this section we provide the proof of Proposition 2. We also show that decentralization can be achieved through a tax on labor supply. Finally, we derive the closed-form expression for the optimal tax in equation (15).

A.2.1 Proof of Proposition 2

To prove Proposition 2, we proceed in two steps. We first show that the policy maker facing the regulated equilibrium can choose τ_t^w to replicate the constrained-efficient equilibrium. That is, this allocation is *feasible* for the policy maker. Second, we show that the constraints faced by the policy maker are at least as stringent as the constraints faced by the constrained-efficient planner. Because the policy maker and the constrained-efficient planner both have the same objective (they maximize household utility), it follows that the policy maker chooses to implement the constrained-efficient allocation.

We state the equilibrium conditions of the regulated equilibrium. An equilibrium is a path $\{C_t, H_t, B_{t+1}, W_t\}_{t\geq 0}$ such that

$$1 = \beta R E_t \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}}$$
$$P_t C_t + \frac{B_{t+1}}{R} = P_t a_t F(H_t) + B_t$$

as well as the labor market conditions

$$a_t F'(H_t) = \frac{W_t(1+\tau_t^w)}{P_t}$$
$$G'(H_t) = \frac{W_t}{P_t}$$

if $W_t \geq \psi W_{t-1}$ (*slack*), or else

$$a_t F'(H_t) = \frac{W_t(1+\tau_t^w)}{P_t}$$
$$W_t = \psi W_{t-1}$$

(*binds*), where $U'(t) \equiv U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t\geq 0}$ and $\{\tau_t^w \geq 0\}_{t\geq 0}$, are all satisfied. Note that the resource constraint is the same as in the equilibrium under laissez-faire. This reflects that the payroll tax is rebated lump-sum to firms in equilibrium.

We state the equilibrium conditions of the constrained-efficient planner. An equilibrium is a path $\{C_t, H_t, B_{t+1}, W_t, \lambda_t^{sp}\}_{t\geq 0}$ such that

$$1 = \beta R E_t \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}}$$
$$P_t C_t + \frac{B_{t+1}}{R} = P_t a_t F(H_t) + B_t$$

as well as the labor market conditions

$$a_t F'(H_t) = \frac{W_t}{P_t} + \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}^{sp}$$
$$G'(H_t) = \frac{W_t}{P_t}$$

if $W_t \geq \psi W_{t-1}$ (*slack*), or else

$$W_t = \psi W_{t-1}$$
$$G'(H_t) = \frac{W_t}{P_t}$$

if $a_t F'(H_t) > W_t/P_t$ (*binds lightly*), or else

$$W_t = \psi W_{t-1}$$
$$a_t F'(H_t) = \frac{W_t}{P_t},$$

(*binds strongly*), where the multiplier λ_t^{sp} is given by

$$\lambda_t^{sp} = -U'(t)\left(\varepsilon_t^F \frac{H_t}{W_t}\left(\frac{W_t}{P_t} - G'(H_t)\right) + \varepsilon_t^G \frac{H_t}{W_t}\left(a_t F'(H_t) - \frac{W_t}{P_t}\right)\right) + \beta\psi E_t \lambda_{t+1}^{sp}$$

where $U'(t) = U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t\geq 0}$, are all satisfied.

We first discuss feasibility. Comparing the two allocations reveals that the policy maker can replicate the constrained-efficient allocation by i) setting

$$\tau_t^w = \left(\frac{W_t}{P_t}\right)^{-1} \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta\psi E_t \lambda_{t+1}^{sp} \tag{A.3}$$

whenever downward nominal wage rigidity is slack, by ii) setting τ_t^w such that

$$a_t F'(H_t) = \frac{(1+\tau_t^w)W_t}{P_t}$$

if downward nominal wage binds, as long as this implies $\tau_t^w \geq 0$ (this corresponds to the region where downward nominal wage rigidity binds lightly) and by iii), setting

$$\tau_t^w = 0$$

otherwise. The tax in (A.3) is non-negative, because $\varepsilon_t^G > 0$ and $\lambda_t \geq 0$.

We second discuss optimality. The constrained-efficient planner faces as constraints (compare Definition 2): i) the resource constraint ii) the Euler equation iii) labor supply holding with weak inequality iv) downward nominal wage rigidity v) a no-subsidy requirement on labor demand and vi) a fixed price level.

The policy maker also faces constraints i)-ii) and vi). Concerning constraint iii), labor supply as faced by the policy maker always satisfies $G'(H_t) \leq W_t/P_t$, which is the constraint faced by the planner. The same applies to downward nominal wage rigidity, constraint iv). Concerning constraint v), the requirement $a_t F'(H_t) \geq W_t/P_t$ is equivalent to $a_t F'(H_t) = (1+\tau_t^w)(W_t/P_t)$ plus $\tau_t^w \geq 0$. In sum, the constraints faced by the policy maker are at least as stringent as the constraints faced by the constrained-efficient planner.

A.2.2 Decentralization through taxes on households

Assume that a tax $\tilde{\tau}_t^w$ applies to households' wage income, rebated lump-sum to households in equilibrium. We show that, in case downward nominal wage rigidity applies to the *net* wage $(1-\tilde{\tau}_t^w)W_t$, this instrument can be used to decentralize the constrained-efficient allocation.

Households' budget constraint (2) becomes

$$P_t C_t + \frac{B_{t+1}}{R} = (1-\tilde{\tau}_t^w)W_t H_t + \Pi_t + B_t + \mathcal{T}_t,$$

where $\mathcal{T}_t$ denotes the lump-sum transfer.

For downward nominal wage rigidity we assume that

$$(1-\tilde{\tau}_t^w)W_t \geq \psi(1-\tilde{\tau}_{t-1}^w)W_{t-1},$$

which replaces equation (3).

The labor supply curve (4) becomes

$$G'(H_t) \leq \frac{(1-\tilde{\tau}_t^w)W_t}{P_t}.$$

The firms' problem is unchanged. As a result, labor demand is still given by (6).

The tax is rebated lump-sum to households in equilibrium $\mathcal{T}_t = \tilde{\tau}_t^w W_t H_t$. As a result, the resource constraint (8) is also unchanged.

An equilibrium is a path $\{C_t, H_t, B_{t+1}, W_t\}_{t\geq 0}$ such that

$$1 = \beta R E_t \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}}$$

$$P_t C_t + \frac{B_{t+1}}{R} = P_t a_t F(H_t) + B_t$$

as well as the labor market conditions

$$a_t F'(H_t) = \frac{W_t}{P_t}$$

$$G'(H_t) = \frac{(1-\tilde{\tau}_t^w)W_t}{P_t}$$

if $(1-\tilde{\tau}_t^w)W_t \geq \psi(1-\tilde{\tau}_{t-1}^w)W_{t-1}$ (*slack*), or else

$$a_t F'(H_t) = \frac{W_t}{P_t}$$

$$(1-\tilde{\tau}_t^w)W_t = \psi(1-\tilde{\tau}_{t-1}^w)W_{t-1}$$

(*binds*), where $U'(t) = U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for given initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t\geq 0}$ and $\{\tilde{\tau}_t^w \geq 0\}_{t\geq 0}$, are all satisfied.

To see the equivalence to the payroll tax on firms, define $\tilde{W}_t \equiv (1-\tilde{\tau}_t^w)W_t$ and rewrite all optimality conditions accordingly. This yields for the labor market market conditions

$$a_t F'(H_t) = \frac{\tilde{W}_t/(1-\tilde{\tau}_t^w)}{P_t}$$

$$G'(H_t) = \frac{\tilde{W}_t}{P_t}$$

if $\tilde{W}_t \geq \psi \tilde{W}_{t-1}$ (*slack*), or else

$$a_t F'(H_t) = \frac{\tilde{W}_t/(1-\tilde{\tau}_t^w)}{P_t}$$

$$\tilde{W}_t = \psi \tilde{W}_{t-1}$$

(*binds*). These are identical once we replace W_t by $\tilde{W}_t$ and set $1+\tau_t^w = 1/(1-\tilde{\tau}_t^w)$ (see Appendix A.2.1). Therefore, the allocation induced by using payroll taxes on firms, or by using income taxes on households, are identical.

A.2.3 A closed-form expression for τ_t^w

Here we derive formula (15) from the main text. We start with equation (A.3), repeated here for convenience

$$\tau_t^w = \left(\frac{W_t}{P_t}\right)^{-1} \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta\psi E_t \lambda_{t+1}^{sp}.$$

We assume that downward nominal wage rigidity is binding strongly in $t+1$, and again slack thereafter (in periods $t+2,\ t+3$ etc). This implies for λ_{t+1}^{sp}, from equation (10)

$$\lambda_{t+1}^{sp} = -U'(t+1)\varepsilon_{t+1}^F \frac{H_{t+1}}{W_{t+1}} \left(\frac{W_{t+1}}{P_{t+1}} - G'(H_{t+1})\right),$$

where we use that downward nominal wage rigidity binds strongly to replace $a_{t+1}F'(H_{t+1}) - W_{t+1}/P_{t+1} = 0$, and that is it slack thereafter to replace $\lambda_{t+2}^{sp} = 0$.

Using our functional forms $G(H_t) = H_t^{1+\varphi}/(1+\varphi)$ and $F(H_t) = H_t^\alpha$, we can replace $\varepsilon_{t+1}^F = -1/(1-\alpha)$ and $G'(H_t) = H_t^\varphi$. Using this, and using equations (13) and (14) we rewrite the multiplier λ_{t+1}^{sp}

$$\lambda_{t+1}^{sp} = U'(t+1)\frac{1}{1-\alpha}\frac{H_{t+1}^p}{W_{t+1}}(1-u_{t+1})(H_{t+1}^p)^\varphi(1-(1-u_{t+1})^\varphi).$$

We again replace $(H_{t+1}^p)^\varphi = W_t/P_t$ by using (13) and cancel the W_{t+1} to arrive at

$$\lambda_{t+1}^{sp} = U'(t+1)\frac{1}{1-\alpha}\frac{H_{t+1}^p}{P_{t+1}}(1-u_{t+1})(1-(1-u_{t+1})^\varphi).$$

We insert this into the equation for τ_t^w

$$\tau_t^w = \frac{\varphi}{1-\alpha}\psi E_t \beta \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}} \frac{H_{t+1}^p}{H_t^p}(1-u_{t+1})(1-(1-u_{t+1})^\varphi)$$

where we use $\varepsilon_t^G = 1/\varphi$ and replace $H_t = H_t^p$ because by assumption, labor supply in the current period is not rationed.

Finally, we use again (13) to replace H_t^p and H_{t+1}^p, and we use that downward nominal wage rigidity binds in the next period to replace $W_{t+1} = \psi W_t$

$$\tau_t^w = \frac{\varphi}{1-\alpha}\psi^{1+\frac{1}{\varphi}} E_t \beta \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}} \left(\frac{P_t}{P_{t+1}}\right)^{\frac{1}{\varphi}} (1-u_{t+1})(1-(1-u_{t+1})^\varphi), \tag{15}$$

which is equation (15) in the main text.

B A monopsony model

This Appendix complements the analysis of the monopsony model from Section 4. In Section B.1 we derive households' labor supply and firms' labor demand in the monopsony model, and we provide a summary of equilibrium conditions. We study the problem of a single monopsonist in Section B.2. We derive the analogue of Proposition 2, and more generally we show that all results from Section 3.2 go through (largely) unchanged for the monopsony model, in Section B.3. In Section B.4 we report and explain the analogue of Figure 3 from the main text for the case $\eta = 5$.

B.1 Additional derivations

B.1.1 Deriving firm-specific labor supply

We first derive the firm-specific labor supply curves from Section 4.1.

Set up the Lagrangian

$$\mathcal{L} = \int_0^1 W_t(i)H_t(i)di + W_t\left(H_t - \left(\int_0^1 H_t(i)^{1+\frac{1}{\eta}}di\right)^{1/\left(1+\frac{1}{\eta}\right)}\right),$$

where we denote W_t the Lagrange multiplier (which, by the Envelope theorem, measures the increase in $\int_0^1 W_t(i)H_t(i)di$ following a marginal rise in H_t).

The first order conditions for hours $H_t(i)$ are

$$W_t(i) - W_t H_t^{-\frac{1}{\eta}} H_t(i)^{\frac{1}{\eta}} = 0, \quad i \in [0,1].$$

To determine the multiplier W_t, use that

$$W_t(i)^{1+\eta} = W_t^{1+\eta}\left(\frac{H_t(i)}{H_t}\right)^{\frac{1+\eta}{\eta}}$$

$$\Rightarrow \int_0^1 W_t(i)^{1+\eta}di = W_t^{1+\eta}\int_0^1 \left(\frac{H_t(i)}{H_t}\right)^{\frac{1+\eta}{\eta}} di = W_t^{1+\eta}$$

where the integral in the second equation is one by the definition of the aggregator H_t.

Provided that $\eta > 0$ the second order condition is negative,

$$-W_t H_t^{-\frac{1}{\eta}}\frac{1}{\eta}H_t(i)^{\frac{1}{\eta}-1} < 0.$$

This establishes that we study a local maximum.

B.1.2 Deriving labor demand

We repeat the dynamic program of the firms from Definition 4

$$\Gamma_t(W_{t-1}(i)) = \max_{(H_t(i),W_t(i))} \left\{U'(t)\left(a_t F(H_t(i)) - \frac{W_t(i)}{P_t}H_t(i)\right) + \beta E_t \Gamma_{t+1}(W_t(i))\right\}$$

subject to the set of constraints

$$\begin{array}{lrl} i) & (W_t(i)/W_t)^\eta H_t \geq H_t(i), & \text{(multiplier: } \xi_t(i)) \\ ii) & W_t(i) \geq \psi W_{t-1}(i), & \text{(multiplier: } \lambda_t(i)) \end{array}$$

for given aggregate states $\{a_t, P_t, H_t, W_t, U'(t)\}$. The first order conditions are

$$U'(t)\left(a_t F'(H_t(i)) - \frac{W_t(i)}{P_t}\right) - \xi_t(i) = 0 \tag{B.1}$$

for hours $H_t(i)$ as well as

$$-U'(t)\frac{1}{P_t}H_t(i) - \beta\psi E_t\lambda_{t+1}(i) + \lambda_t(i) + \xi_t(i)\eta(W_t(i)^{\eta-1}/W_t^\eta)H_t = 0 \tag{B.2}$$

for wages $W_t(i)$, where we have already used the Envelope condition

$$\frac{\partial}{\partial W_{t-1}(i)}\Gamma_t(W_{t-1}(i)) = -\lambda_t(i)\psi.$$

As in the constrained-efficient equilibrium (Appendix A.1), we proceed by distinguishing the cases where downward nominal wage rigidity is slack and binds, respectively. Then we study the symmetric equilibrium.

Case 1: Downward nominal wage rigidity is slack

Assume that constraint i) is slack. In this case, it must be that downward wage rigidity, constraint ii), is binding. Namely, if not, it were always possible to choose the same $H_t(i)$ but a strictly lower $W_t(i)$, which is feasible because constraint i) is slack, and which raises $\Gamma_t(W_{t-1}(i))$ because current profits increase ($a_tF(H_t(i))$ is unchanged but the wage bill $W_t(i)H_t(i)/P_t$ is reduced) and because there is a non-negative effect on the continuation value, because $\Gamma_{t+1}(W_t(i))$ is weakly decreasing in the individual state $W_t(i)$.

The contraposition is that, once constraint ii) is slack, it must be that constraint i) is binding. By using that $\lambda_t(i) = 0$ once constraint ii) is slack, we solve for multiplier $\xi_t(i)$ from equation (B.2)

$$\xi_t(i) = U'(t)\frac{1}{\eta}\frac{W_t(i)}{P_t} + \frac{1}{\eta}\frac{W_t(i)}{H_t(i)}\beta\psi E_t\lambda_{t+1}(i),$$

where we have used that constraint i) is binding to replace H_t in (B.2). This expression shows that the multiplier $\xi_t(i) > 0$ is strictly positive, which verifies formally that constraint i) must be binding. Combining the last equation with (B.1) then yields

$$a_tF'(H_t(i)) = \frac{\eta+1}{\eta}\frac{W_t(i)}{P_t} + \frac{1}{U'(t)}\frac{1}{\eta}\frac{W_t(i)}{H_t(i)}\beta\psi E_t\lambda_{t+1}(i) \tag{B.3}$$

which is firm i's labor demand when downward nominal wage rigidity is slack.

Case 2: Downward nominal wage rigidity binds

Assume now that constraint ii) binds. Assume also that constraint i) binds. In this case, $H_t(i)$ is determined by constraint i) which holds with equality. Since $W_t(i)$ is determined by constraint ii), $W_t(i)$ and $H_t(i)$ are both determined by $\psi W_{t-1}(i)$. The multiplier $\xi_t(i)$ is determined in (B.1):

$$\xi_t(i) = U'(t)\left(a_tF'(H_t(i)) - \frac{W_t(i)}{P_t}\right). \tag{B.4}$$

Recall that, when constraint ii) is slack, $\xi_t(i) > 0$ is strictly positive because of $a_tF'(H_t(i)) > W_t(i)/P_t$. When constraint ii) starts to bind, both $W_t(i)$ and $H_t(i)$ increase in $\psi W_{t-1}(i)$, implying that $a_tF'(H_t(i))$ falls and that $W_t(i)/P_t$ increases. This implies that $\xi_t(i)$ falls as $\psi W_{t-1}(i)$ increases (see equation (B.4)). However, $\xi_t(i)$ may remain positive as long as constraint ii) binds only "lightly". Instead, $\xi_t(i)$ will hit zero when constraint ii) binds strong enough.

Therefore, equilibrium in the monopsony model features a similar "intermediate region" as the constrained-efficient equilibrium, recall Section 3.1 and Appendix A.1. Intuitively, when downward nominal wage rigidity binds, firms find it optimal to absorb the additional labor supply associated with higher wages as long as the marginal product is still above the real wage.

This is, in fact, a classical finding in monopsonies: as Manning (2003) points out in the context of minimum wages, "a minimum wage that just binds must raise employment". An equivalent way of saying this is that firms, when downward nominal wage rigidity binds, find it optimal to endogenously reduce their monopsonistic mark-ups to zero.

From (B.4), when constraint ii) binds strong enough employment is determined purely by labor demand:

$$a_t F'(H_t(i)) = \frac{W_t(i)}{P_t} \tag{B.5}$$

and the multiplier $\xi_t(i) = 0$.

The multiplier $\lambda_t(i)$ can be inferred from (B.2):

$$\lambda_t(i) = U'(t)\frac{1}{P_t}H_t(i) + \beta\psi E_t\lambda_{t+1}(i) - \xi_t(i)\eta(W_t(i)^{\eta-1}/W_t^{\eta})H_t. \tag{B.6}$$

Notice that, when constraint ii) binds strongly, implying that $\xi_t(i) = 0$, $\lambda_t(i)$ simplifies to the following:

$$\lambda_t(i) = U'(t)\frac{1}{P_t}H_t(i) + \beta\psi E_t\lambda_{t+1}(i). \tag{B.7}$$

Symmetric equilibrium

We now study the symmetric equilibrium. All firms are identical, hence we set $W_t(i) = W_t$, $H_t(i) = H_t$, $\xi_t(i) = \xi_t$ and $\lambda_t(i) = \lambda_t$. Note that this implies that constraint i) always holds with equality.

When constraint ii) is slack, employment is determined from (B.3)

$$a_t F'(H_t) = \frac{\eta+1}{\eta}\frac{W_t}{P_t} + \frac{1}{U'(t)}\frac{1}{\eta}\frac{W_t}{H_t}\beta\psi E_t\lambda_{t+1}, \tag{19}$$

which is equation (19) in the main text.

When downward nominal wage rigidity binds, wages are determined by constraint ii), in equilibrium: $W_t = \psi W_{t-1}$. When downward nominal wage rigidity binds lightly, equilibrium employment is determined by aggregate labor supply (4). When it binds strongly, equilibrium employment is determined by labor demand

$$a_t F'(H_t) = \frac{W_t}{P_t}.$$

In the symmetric equilibrium, the multiplier λ_t can be written as

$$\lambda_t = -U'(t)\eta\frac{H_t}{W_t}\left(a_t F'(H_t) - \frac{\eta+1}{\eta}\frac{W_t}{P_t}\right) + \beta\psi E_t\lambda_{t+1}, \tag{20}$$

where we have combined (B.4) and (B.6) in the symmetric equilibrium. This is equation (20) from the main text. When downward nominal wage rigidity binds strongly such that $a_t F'(H_t) = W_t/P_t$, this reduces to

$$\lambda_t = U'(t)\frac{1}{P_t}H_t + \beta\psi E_t\lambda_{t+1} \tag{B.8}$$

which is (B.7) after imposing the symmetric equilibrium.

B.1.3 Summary of equilibrium conditions

Here we summarize the equilibrium conditions in the monopsony model. An equilibrium is a path $\{C_t, H_t, B_{t+1}, W_t, \lambda_t\}_{t\geq 0}$ such that

$$
\begin{aligned}
1 &= \beta R E_t \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}} \\
P_t C_t + \frac{B_{t+1}}{R} &= P_t a_t F(H_t) + B_t
\end{aligned}
$$

as well as the labor market conditions

$$
\begin{aligned}
a_t F'(H_t) &= \frac{\eta+1}{\eta} \frac{W_t}{P_t} + \frac{1}{U'(t)} \frac{1}{\eta} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1} \\
G'(H_t) &= \frac{W_t}{P_t}
\end{aligned}
$$

if $W_t \geq \psi W_{t-1}$ (*slack*), or else

$$
\begin{aligned}
W_t &= \psi W_{t-1} \\
G'(H_t) &= \frac{W_t}{P_t}
\end{aligned}
$$

if $a_t F'(H_t) \geq W_t / P_t$ (*binds lightly*), or else

$$
\begin{aligned}
W_t &= \psi W_{t-1} \\
a_t F'(H_t) &= \frac{W_t}{P_t},
\end{aligned}
$$

(*binds strongly*), where the multiplier λ_t is given by

$$
\lambda_t = -U'(t) \eta \frac{H_t}{W_t} \left(a_t F'(H_t) - \frac{\eta+1}{\eta} \frac{W_t}{P_t} \right) + \beta \psi E_t \lambda_{t+1},
$$

where $U'(t) \equiv U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t\geq 0}$, are all satisfied.

B.2 A single monopolist

A single monopsonist faces the dynamic problem

$$
\Gamma_t(W_{t-1}) = \max_{\{H_t, W_t\}} \left\{ U'(t) \left(a_t F(H_t) - \frac{W_t}{P_t} H_t \right) + \beta E_t \Gamma_{t+1}(W_t) \right\}
$$

subject to the set of constraints

$$
\begin{aligned}
&i) \qquad & W_t &\geq P_t G'(H_t), \qquad & \text{(multiplier: } \xi_t) \\
&ii) \qquad & W_t &\geq \psi W_{t-1}, \qquad & \text{(multiplier: } \lambda_t)
\end{aligned}
$$

by taking as given the aggregate variables $\{a_t, P_t, U'(t)\}$.[40]

[40] We assume the single monopsonist takes $U'(t)$ as given even though H_t enters through G: $U'(t) \equiv U'(C_t - G(H_t))$. Nothing material changes if we assume the monopsonist internalizes the effect on $U'(t)$.

We take first order conditions with respect to W_t and H_t

$$-U'(t)\frac{1}{P_t}H_t + \xi_t + \lambda_t - \beta\psi E_t\lambda_{t+1} = 0$$
$$U'(t)\left(a_tF'(H_t) - \frac{W_t}{P_t}\right) - \xi_t\frac{1}{\varepsilon_t^G}\frac{W_t}{H_t} = 0,$$

where we have already used the Envelope condition

$$\frac{\partial}{\partial W_{t-1}}\Gamma_t(W_{t-1}) = -\lambda_t\psi$$

and where the wage elasticity of labor supply $\varepsilon_t^G > 0$ is defined as before.

We derive the labor demand curve when downward nominal wage rigidity is slack, as well as the multiplier λ_t when it is strongly binding.

In the former case, $\lambda_t = 0$ and $\xi_t > 0$. Combining the two first order conditions yields

$$a_tF'(H_t) = \frac{\varepsilon_t^G + 1}{\varepsilon_t^G}\frac{W_t}{P_t} + \frac{1}{U'(t)}\frac{1}{\varepsilon_t^G}\frac{W_t}{H_t}\beta\psi E_t\lambda_{t+1}.$$

When comparing this with equation (19) in the main text we spot two differences. First, the monopsonist discounts the future utility cost of downward nominal wage rigidity by using the wage elasticity of *aggregate* labor supply ε_t^G. This shows that the monopsonist is not affected by the pecuniary externality, unlike the monopsonistic competitors. Second, the mark-up charged by the monopsonist is $(\varepsilon_t^G + 1)/\varepsilon_t^G$. This reflects that the monopsonist exploits the aggregate labor supply curve when maximizing profits.

When downward nominal wage rigidity binds strongly, $\xi_t = 0$ and $\lambda_t > 0$. Using this in the first order condition with respect to W_t yields

$$\lambda_t = U'(t)\frac{1}{P_t}H_t + \beta\psi E_t\lambda_{t+1},$$

which is the same as in the case of monopsonistic competitors (see equation (20) in the main text after imposing $a_tF'(H_t) = W_t/P_t$, reflecting that downward nominal wage rigidity binds strongly; or see equation (B.8) earlier above).

This shows that $\lambda_t \neq \lambda_t^{sp}$ reflects a distortion due to firms' market power rather than a distortion due to the pecuniary externality.

B.3 Implications for regulation

B.3.1 Proposition 2

We derive the analogue of Proposition 2 in the monopsony model.

Assume a payroll tax on firms is in place. The households' problem is unchanged relative to the economy without intervention: in particular, the first order conditions (18) and (4) are unchanged. Downward nominal wage rigidity is still given by $W_t(i) \geq \psi W_{t-1}(i)$.

The firms' problem changes as follows

$$\Gamma_t(W_{t-1}(i)) = \max_{(H_t(i),W_t(i))}\left\{U'(t)\left(a_tF(H_t(i)) - \frac{(1+\tau_t^w)W_t(i)}{P_t}H_t(i) + \frac{\mathcal{T}_t}{P_t}\right) + \beta E_t\Gamma_{t+1}(W_t(i))\right\}$$

subject to the set of constraints

$$
\begin{array}{ll}
i) & (W_t(i)/W_t)^{\eta} H_t \geq H_t(i), \\
ii) & W_t(i) \geq \psi W_{t-1}(i),
\end{array}
$$

by taking as given the aggregate variables $\{a_t, P_t, H_t, W_t, U'(t), \tau_t^w, \mathcal{T}_t\}$.

The first order conditions are given by

$$U'(t)\left(a_t F'(H_t(i)) - \frac{(1+\tau_t^w)W_t(i)}{P_t}\right) - \xi_t(i) = 0$$

for hours $H_t(i)$ as well as

$$-U'(t)\frac{(1+\tau_t^w)}{P_t}H_t(i) - \beta\psi E_t\lambda_{t+1}(i) + \lambda_t(i) + \xi_t(i)\eta(W_t(i)^{\eta-1}/W_t^{\eta})H_t = 0$$

for wages $W_t(i)$. We proceed along the lines of Appendix B.1, then impose the symmetry condition $W_t(i) = W_t$ and $H_t(i) = H_t$. This yields labor demand curves

$$a_t F'(H_t) = \frac{\eta+1}{\eta}\frac{(1+\tau_t^w)W_t}{P_t} + \frac{1}{U'(t)}\frac{1}{\eta}\frac{W_t}{H_t}\beta\psi E_t\lambda_{t+1}$$

when downward nominal wage rigidity is slack, and

$$a_t F'(H_t) = \frac{(1+\tau_t^w)W_t}{P_t}$$

when downward nominal wage rigidity binds (strongly), respectively. The multiplier λ_t is given by

$$\lambda_t = -U'(t)\eta\frac{H_t}{W_t}\left(a_t F'(H_t) - \frac{\eta+1}{\eta}\frac{(1+\tau_t^w)W_t}{P_t}\right) + \beta\psi E_t\lambda_{t+1}.$$

The tax is rebated lump-sum to firms in equilibrium: $\mathcal{T}_t = \tau_t^w W_t H_t$. Profits are given by $\Pi_t = P_t a_t F(H_t) - (1+\tau_t^w)H_t W_t + \mathcal{T}_t$. By inserting the equilibrium $\mathcal{T}_t$, then inserting profits in households' budget constraint (2), this implies that the resource constraint is still given by equation (8).

We summarize the equilibrium conditions in the monopsony model with intervention. A regulated equilibrium is a path $\{C_t, H_t, B_{t+1}, W_t, \lambda_t\}_{t\geq 0}$ such that

$$
\begin{aligned}
1 &= \beta R E_t \frac{U'(t+1)}{U'(t)}\frac{P_t}{P_{t+1}} \\
P_t C_t + \frac{B_{t+1}}{R} &= P_t a_t F(H_t) + B_t
\end{aligned}
$$

as well as the labor market conditions

$$
\begin{aligned}
a_t F'(H_t) &= \frac{\eta+1}{\eta}\frac{W_t(1+\tau_t^w)}{P_t} + \frac{1}{U'(t)}\frac{1}{\eta}\frac{W_t}{H_t}\beta\psi E_t\lambda_{t+1} \\
G'(H_t) &= \frac{W_t}{P_t}
\end{aligned}
$$

if $W_t \geq \psi W_{t-1}$ (*slack*), or else

$$W_t = \psi W_{t-1}$$
$$G'(H_t) = \frac{W_t}{P_t}$$

if $a_t F'(H_t) \geq (1+\tau_t^w)(W_t/P_t)$ (*binds lightly*), or else

$$W_t = \psi W_{t-1}$$
$$a_t F'(H_t) = \frac{(1+\tau_t^w)W_t}{P_t},$$

(*binds strongly*), where the multiplier λ_t is given by

$$\lambda_t = -U'(t)\eta \frac{H_t}{W_t}\left(a_t F'(H_t) - \frac{\eta+1}{\eta}\frac{(1+\tau_t^w)W_t}{P_t}\right) + \beta\psi E_t \lambda_{t+1},$$

where $U'(t) \equiv U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t\geq 0}$ and $\{\tau_t^w \geq 0\}_{t\geq 0}$, are all satisfied.

We consider the Ramsey problem of maximizing (1) over regulated equilibria, restricted to non-negative taxes $\tau_t^w \geq 0$. As in Proposition 2, the outcome of the Ramsey problem and the constrained-efficient equilibrium coincide. To show this, we proceed as in Appendix A.2. We first show that the constrained-efficient equilibrium is feasible for the policy maker. Then we show that the constraints faced by the policy maker are at least as stringent as the constraints faced by the planner. Because both the policy maker and the planner maximize the same objective, it follows that the policy maker chooses to implement the constrained-efficient allocation.

Comparing the regulated equilibrium with the constrained-efficient equilibrium in Appendix A.2 reveals that the policy maker can replicate the constrained-efficient allocation by i) setting

$$\tau_t^w = \frac{\eta}{\eta+1}\left(\left(\frac{W_t}{P_t}\right)^{-1}\frac{1}{U'(t)}\frac{W_t}{H_t}\beta\psi\left(\frac{1}{\varepsilon_t^G}E_t\lambda_{t+1}^{sp} - \frac{1}{\eta}E_t\lambda_{t+1}\right) - \frac{1}{\eta}\right) \tag{B.9}$$

when downward nominal wage rigidity is slack, by ii) setting τ_t^w such that

$$a_t F'(H_t) = \frac{(1+\tau_t^w)W_t}{P_t}$$

if downward nominal wage rigidity is binding as long as this implies $\tau_t^w \geq 0$ and by iii), setting

$$\tau_t^w = 0$$

otherwise.

One complication relative to the baseline model becomes apparent. The tax in equation (B.9) is non-negative as long as η is sufficiently large. Instead, for low values of η, it can be the case that τ_t^w becomes negative. This arises from the fact that the monopsonistic mark-up $(\eta+1)/\eta$, which reduces labor demand and which declines in η, itself calls for *labor* *subsidies*.[41] In this case, Proposition 2 continues to be valid once we additionally assume that labor subsidies are available in expansions, but not to outright support labor demand when downward nominal wage rigidity binds in recessions.

Having discussed feasibility, we now discuss optimality. We note that the constraints faced by the policy maker all satisfy *i)* $W_t/P_t \leq a_t F'(H_t)$ (because of $\tau_t^w \geq 0$), *ii)* $G'(H_t) \leq W_t/P_t$ and *iii)* $W_t \geq \psi W_{t-1}$, which are constraints iii)-v) faced by the constrained-efficient planner (see Definition 2). Because all other constraints are identical, we conclude that the constraints faced by the policy maker are at least as stringent as those faced by the constrained-efficient planner.

All other results from Section 3.2 go through unchanged for the monopsony model. The Ramsey outcome is time consistent, because the constrained-efficient equilibrium is time consistent. Second, decentralization is possible through sales taxes. Third, decentralization is possible through payroll taxes on households, but only of downward nominal wage rigidity applies to after-tax wages.[42] The last two points can be shown in an analogous way as in the baseline model, see Appendix A.2.

B.4 Policy functions in the monopsony model

Here we report the analogue of Figure 3 in the main text in the case of $\eta = 5$. All other parameters are set as in the baseline calibration. The result is in Figure B.1.

Policy functions under $\eta = 5$ are the green dotted lines. The figure also reproduces the case of perfect competition ($\eta = \infty$) in blue solid, and the constrained-efficient equilibrium in red-dashed, as shown in the main text.

Under $\eta = 5$, the result is that firms compress hiring and wage increases in expansions more strongly than does the constrained-efficient planner, which turns results from Figure 3 on its head. We argued in Section 4 that firms underestimate the social cost of downward nominal wage rigidity even when firms' market power is *substantial*. How is this compatible with Figure B.1?

When firms have market power, differences to the constrained-efficient equilibrium arise both due to the pecuniary externality and due to firms' charging monopsonistic mark-ups. Under $\eta = 5$, firms' mark-ups are $(\eta + 1)/\eta = 120\%$. In Figure B.1, this leads firms to reduce hiring more strongly in expansions than does the constrained-efficient planner. This implies that to decentralize the constrained-efficient allocation, policy makers need to give subsidies in expansions, as explained in Appendix B.3. However, note that we continue to assume that the planner has no power to relax downward nominal wage rigidity in recessions: when this rigidity binds, the payroll tax is $\tau_t^w = 0$.

To see that the externality is still at play, inspect the lower left panel, which shows the wedge in the labor demand curve that arises from the expected utility loss of downward nominal wage rigidity. In the relevant (slack) region, the wedge by firms is less than half as large as in the constrained-efficient allocation, which establishes that firms substantially underestimate the welfare cost of downward nominal wage rigidity even when $\eta = 5$. In other words, the fact that firms reduce labor demand strongly reflects their monopsonistic mark-ups, rather than them internalizing the pecuniary externality.

[41] Moreover, for $\eta < \varepsilon_t^G$, the tax may be negative due to firms' compressing wage increases in expansions more strongly than does the constrained-efficient planner. However, as we argued in Section 4.2, this requires an unreasonably strong degree of market power.

[42] That is, if we assume $(1 - \tilde{\tau}_t^w)W_t \geq \psi(1 - \tilde{\tau}_{t-1}^w)W_{t-1}$.

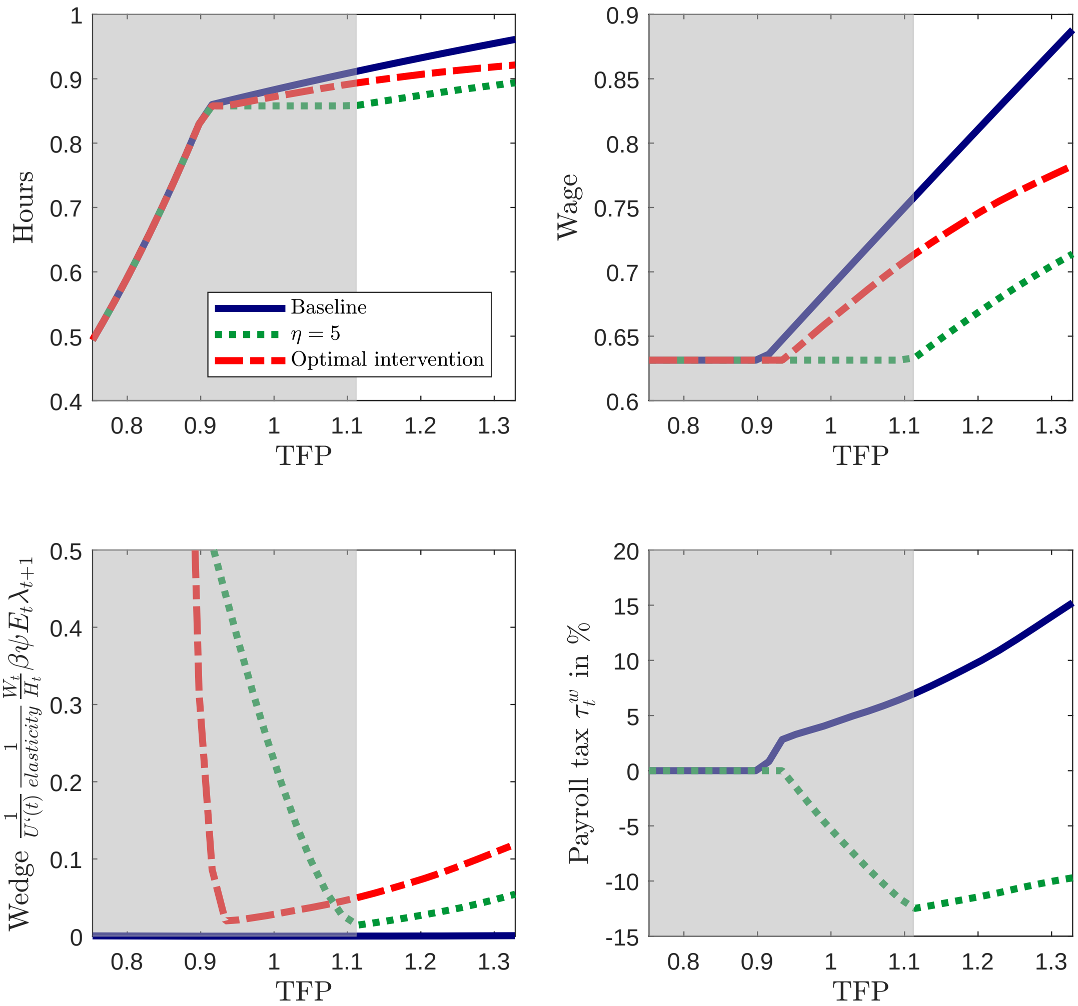

Figure B.1: Policy functions under $\eta = \infty$, $\eta = 5$ and in the constrained-efficient equilibrium. The gray area indicates the region where downward nominal wage rigidity binds under laissez-faire in the model with $\eta = 5$.

C Constrained efficiency of labor supply

In the main text, we argued firms' hiring decisions are constrained inefficient. We now show that also households' labor supply choices are constrained inefficient.

In Appendix C.1 we study the baseline model, but we consider the problem of a different planner: a planner choosing labor supply on behalf of households, rather than labor demand on behalf of firms. This allows us to show that labor supply decisions by households are constrained inefficient.

In Appendix C.2 we introduce unions which choose labor supply on behalf of households in monopolistic competition (Benigno and Ricci, 2011). Similarly as in the monopsony model from the main text, we show that the externality is not resolved by considering purposeful wage-setters.

C.1 Planning problem

The planning problem is

$$\max E_0 \sum_{t\geq 0} \beta^t U(C_t - G(H_t))$$

subject to the set of constraints

$$
\begin{array}{llr}
i) & P_t C_t + B_{t+1}/R = P_t a_t F(H_t) + B_t & \text{(multiplier: } \iota_t) \\
ii) & (U'(t)/P_t) = \beta R E_t (U'(t+1)/P_{t+1}) & \text{(multiplier: } \nu_t) \\
iii) & W_t = P_t a_t F'(H_t) & \text{(multiplier: } \gamma_t) \\
iv) & W_t \geq \psi W_{t-1} & \text{(multiplier: } \lambda_t)
\end{array}
$$

where $U'(t) \equiv U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{a_t, \bar{P}_t\}_{t\geq 0}$.

This planning problem replaces the problem in Definition 2 from the main text. Note the key difference: labor demand iii) as chosen by private agents is taken as a constraint for the planner. However, labor supply is not taken as a constraint.

We proceed as in Appendix A.1. First, the multiplier $\nu_t = 0$, because the Euler equation is not a constraint in equilibrium. We can thus omit it from the maximization. Moreover, $\iota_t = U'(t)/P_t$.

First order conditions with respect to W_t and H_t are given by

$$
\begin{aligned}
\gamma_t + \lambda_t - \beta\psi E_t \lambda_{t+1} &= 0 \\
-U'(t)G'(H_t) - \gamma_t \frac{1}{\varepsilon_t^F} \frac{W_t}{H_t} + U'(t) a_t F'(H_t) &= 0,
\end{aligned}
$$

where $\varepsilon_t^F < 0$ continues to denote the wage elasticity of labor demand. Combining both by replacing γ_t yields

$$U'(t)G'(H_t) = \frac{1}{\varepsilon_t^F} \frac{W_t}{H_t} (\lambda_t - \beta\psi E_t \lambda_{t+1}) + U'(t) a_t F'(H_t).$$

In a period when downward nominal wage rigidity is slack $\lambda_t = 0$, this yields

$$G'(H_t) = \frac{W_t}{P_t} - \frac{1}{U'(t)} \frac{1}{\varepsilon_t^F} \frac{W_t}{H_t} \beta\psi E_t \lambda_{t+1}, \tag{C.1}$$

where we used that $a_t F'(H_t) = W_t/P_t$.

In turn, λ_t becomes positive in periods when downward nominal wage rigidity binds

$$\lambda_t = U'(t)\varepsilon_t^F \frac{H_t}{W_t} \left(G'(H_t) - \frac{W_t}{P_t} \right) + \beta\psi E_t \lambda_{t+1}.$$

We summarize the constrained-efficient equilibrium as follows. An equilibrium is a path $\{C_t, H_t, B_{t+1}, W_t, \lambda_t^{sp}\}_{t\geq 0}$ such that

$$
\begin{aligned}
1 &= \beta R E_t \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}} \\
P_t C_t + \frac{B_{t+1}}{R} &= P_t a_t F(H_t) + B_t
\end{aligned}
$$

labor demand

$$a_t F'(H_t) = \frac{W_t}{P_t},$$

as well as labor supply

$$G'(H_t) = \frac{W_t}{P_t} - \frac{1}{U'(t)} \frac{1}{\varepsilon_t^F} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}^{sp},$$

if $W_t \geq \psi W_{t-1}$ (*slack*), or else

$$W_t = \psi W_{t-1}$$

(*binds*), where the multiplier λ_t^{sp} is given by

$$\lambda_t^{sp} = U'(t) \varepsilon_t^F \frac{H_t}{W_t} \left(G'(H_t) - \frac{W_t}{P_t} \right) + \beta \psi E_t \lambda_{t+1}^{sp}.$$

where $U'(t) = U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t \geq 0}$, are all satisfied.

Clearly, the equilibrium under laissez-faire is constrained inefficient, because the equilibrium conditions from Definition 1 and the previous equilibrium conditions do not coincide. That is, households' labor supply decisions are constrained inefficient.

The externality can be understood in similar terms as the externality affecting labor demand. When households supply less labor, they fail to internalize that less competition between households for firms raises the aggregate wage, as the economy moves along a downward sloping labor demand curve. In the context of downward nominal wage rigidity, this makes the laissez-faire outcome constrained inefficient.

Note that the planner addresses the externality by *raising* labor supply in periods when downward nominal wage rigidity is slack, as this reduces wages. In equation (C.1), this can be seen because $\lambda_t^{sp} \geq 0$ and $\varepsilon_t^F < 0$, such that $G'(H_t)$ becomes larger. Therefore, the decentralization would involve *subsidizing* labor supply.

C.2 A model with household-unions

We now turn to the case of monopolistic competition. As in the baseline model, the economy is populated by a large number of households, but we now make the household index specific, $j \in [0,1]$. Each household j maximizes

$$E_0 \sum_{t \geq 0} \beta^t U(C_t(j) - G(H_t(j))) \tag{C.2}$$

subject to the budget constraint

$$P_t C_t(j) + \frac{B_{t+1}(j)}{P_t} = W_t(j) H_t(j) + \Pi_t + B_t(j).$$

We assume that households receive symmetric firm profits Π_t. All variables and the functions U and G are defined as in the main text.

Following Benigno and Ricci (2011), we assume consumption risk sharing across households through a set of state-contingent claims to monetary units. This yields the Euler equation

$$1 = \beta R \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}},$$

where $U'(t) = U'(C_t(j) - G(H_t(j)))$ for all $j \in [0,1]$.[43]

Households supply differentiated labor types $H_t(j)$ which for firms are imperfectly substitutable. This gives market power to households. Note the difference to the monopsony model: there workers within households could not easily substitute their employer, which gave market power to firms.

Formally, a representative firm operates the technology $a_t F(H_t)$, where F is defined as in the main text, and where H_t is a composite

$$H_t = \left(\int_0^1 H_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1.$$

Imperfect substitutability of workers types for firms is reflected by $\theta < \infty$, whereas $\theta = \infty$ corresponds to the baseline case of perfect competition.

Here $H_t(j)$ is the demand for labor supplied by household j. Every household sells labor to the representative firm. The demand for household-specific labor on the part of wage-taking firms is the solution of

$$\min_{\{H_t(j)\}_{j\in[0,1]}} \int_0^1 W_t(j) H_t(j) dj \quad \text{s.t.} \quad H_t \leq \left(\int_0^1 H_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1.$$

This yields a set of labor demand curves

$$H_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\theta} H_t, \tag{C.3}$$

where W_t is the Lagrange multiplier given by $W_t \equiv (\int_0^1 W_t(j)^{1-\theta} dj)^{1/(1-\theta)}$.

Moreover, aggregate labor demand is

$$a_t F'(H_t) = \frac{W_t}{P_t}$$

and profits are given by $\Pi_t = P_t a_t F(H_t) - W_t H_t$.

We now turn to labor supply. Each household j maximizes utility (C.2), subject to household-specific labor demand (C.3), subject to the budget constraint, and subject to downward nominal wage rigidity $W_t(j) \geq \psi W_{t-1}(j)$.

As shown in Benigno and Ricci (2011), this is equivalent to solving the following problem

$$\Gamma_t(W_{t-1}(j)) = \max_{(H_t(j), W_t(j))} \left\{ U'(t) \left(\frac{W_t(j) H_t(j)}{P_t} - G(H_t(j)) \right) + \beta E_t \Gamma_{t+1}(W_t(j)) \right\}$$

subject to the set of constraints

$$\begin{array}{ll} i) & H_t(j) = (W_t(j)/W_t)^{-\theta} H_t, \\ ii) & W_t(j) \geq \psi W_{t-1}(j), \end{array}$$

[43] Strictly speaking, we do not require the consumption-risk-sharing assumption, because households are identical and downward nominal wage rigidity is symmetric (and we assume symmetric initial conditions, i.e., $B_0(j) = B_0$ and $W_{-1}(j) = W_{-1}$). In Benigno and Ricci (2011), this assumption is strictly required because households are subject to idiosyncratic preference shocks. In the literature studying Calvo wages, this assumption is also commonly made because households have ex-post different wage incomes even though they are ex-ante identical.

by taking as given the aggregate variables $\{H_t, W_t, U'(t), P_t\}$.

Inserting constraint $i)$ to replace $H_t(j)$ and denoting the Lagrange multiplier with respect to constraint $ii)$ $\lambda_t(j)$, we obtain the first order condition

$$U'(t)\left((1-\theta)\frac{H_t(j)}{P_t} - G'(H_t(j))(-\theta)\frac{H_t(j)}{W_t(j)}\right) + \lambda_t(j) - \beta\psi E_t\lambda_{t+1}(j) = 0,$$

where we have already used the Envelope condition

$$\frac{\partial}{\partial W_{t-1}(j)}\Gamma_t(W_{t-1}(j)) = -\psi\lambda_t(j).$$

When downward nominal wage rigidity is slack $\lambda_t(j) = 0$, this can be rearranged to

$$G'(H_t(j)) = \frac{\theta-1}{\theta}\frac{W_t(j)}{P_t} + \frac{1}{U'(t)}\frac{1}{\theta}\frac{W_t(j)}{H_t(j)}\beta\psi E_t\lambda_{t+1}(j).$$

Notice that under perfect labor market competition ($\theta \to \infty$), labor supply reduces to the familiar expression

$$G'(H_t(j)) = \frac{W_t(j)}{P_t}.$$

Conversely, when labor market competition is imperfect, labor supply is increased as this reduces wages, because households internalize that downward nominal wage rigidity may bind in future periods.

When downward nominal wage rigidity binds, the multiplier $\lambda_t(j)$ turns positive

$$\lambda_t(j) = -U'(t)\left((1-\theta)\frac{H_t(j)}{P_t} - G'(H_t(j))(-\theta)\frac{H_t(j)}{W_t(j)}\right) + \beta\psi E_t\lambda_{t+1}(j).$$

We study the symmetric equilibrium where $W_t(j) = W_t$ and $H_t(j) = H_t$ for all $j \in [0,1]$. We also assume symmetric initial conditions ($B_0(j) = B_0$ and $W_{-1}(j) = W_{-1}$).

An equilibrium is a set of processes $\{C_t, H_t, B_{t+1}, W_t, \lambda_t\}_{t\geq 0}$ such that

$$1 = \beta E_t\frac{U'(t+1)}{U'(t)}\frac{P_t}{P_{t+1}}$$

$$P_tC_t + \frac{B_{t+1}}{R} = P_t a_t F(H_t) + B_t$$

labor demand

$$a_tF'(H_t) = \frac{W_t}{P_t},$$

as well as labor supply

$$G'(H_t) = \frac{\theta-1}{\theta}\frac{W_t}{P_t} + \frac{1}{U'(t)}\frac{1}{\theta}\frac{W_t}{H_t}\beta\psi E_t\lambda_{t+1}, \tag{C.4}$$

if $W_t \geq \psi W_{t-1}$ (*slack*), or else

$$W_t = \psi W_{t-1}$$

(*binds*), where the multiplier λ_t is given by

$$\lambda_t = -U'(t)\left((1-\theta)\frac{H_t}{P_t} - G'(H_t)(-\theta)\frac{H_t}{W_t}\right) + \beta\psi E_t\lambda_{t+1},$$

where $U'(t) = U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t\geq 0}$, are all satisfied.

By comparing this equilibrium with the constrained-efficient equilibrium from Appendix C.1, we note that all equilibrium conditions coincide, except labor supply when downward nominal wage rigidity is slack—compare equations (C.1) and (C.4). There are three differences. First, the monopolistic mark-up $(\theta - 1)/\theta$ is absent in the constrained-efficient equilibrium. Second, we have $\lambda_t \neq \lambda_t^{sp}$. Third, there is the pecuniary externality: households discount the expected utility loss due to downward nominal wage rigidity by using their *individual* labor demand elasticity $\theta > 1$, whereas the planner uses the *aggregate* labor demand elasticity $-\varepsilon_t^F > 0$.

We have shown that households' labor supply decisions are constrained inefficient. The general picture that emerges is thus that the externality affects labor market outcomes—regardless whether the wage setting power is on firms or on households (unions).

D Demand externalities and links with Schmitt-Grohé and Uribe (2016)

We now augment the baseline model by a non-tradable sector. This extension is interesting for two reasons. First, it allows us to link our results to the analysis in Schmitt-Grohé and Uribe (2016). Second, more generally, the demand externality discussed in the literature reappears, allowing us to demonstrate that the demand and pecuniary externality are distinct, but that they may arise simultaneously.

In Section D.1 we introduce the economic environment. In order to enhance comparability with the results in Schmitt-Grohé and Uribe (2016), we follow their set-up as much as possible. First, we assume that production takes place only in the non-tradable sector. Second, the cycle is driven by shocks to the tradable endowment and to the real borrowing rate facing domestic households. Third, we assume endogenous labor supply and depart from GHH preferences. This corresponds to a variant of the model of Schmitt-Grohé and Uribe in their Online Appendix. Instead, Schmitt-Grohé and Uribe assume inelastic labor supply in their baseline model. Here we study endogenous labor supply, in order to make addressing the pecuniary externality an interesting problem: as explained in the main text, exogenous labor supply leads to the degenerate policy implication to tax labor as much as possible, because the trade-off associated with taxing labor disappears.

In Section D.2 we define the constrained-efficient equilibrium. Section D.3 provides a discussion of the theoretical results. In Section D.4 we solve the model numerically, which allows us to study the interaction between the two externalities quantitatively.

D.1 Economic environment

We assume that welfare is of the form

$$E_0 \sum_{t\geq 0} \beta^t (U(C_t) - G(H_t)), \quad \beta \in (0, 1).$$

Consumption $C_t = A(C_t^T, C_t^N)$ is composed of tradable and non-tradable consumption. For U we assume the CRRA type as in the main text, with $\sigma > 0$ denoting households' risk aversion coefficient. Moreover, as in Schmitt-Grohé and Uribe (2016), for A we assume an Armington aggregator with elasticity of substitution between tradables and non-tradables equal to $1/\sigma$. As a result, period utility becomes[44]

$$U(A(C_t^T, C_t^N)) = \frac{\omega(C_t^T)^{1-\sigma} + (1-\omega)(C_t^N)^{1-\sigma}}{1-\sigma},$$

where $\omega \in (0,1)$ determines the share of tradable in total consumption.

We assume that tradable output Y_t^T is exogenous and time-varying, and that labor income $W_t H_t$ and profits $\mathit{\Pi}_t$ derive from firms which operate in the non-tradable sector exclusively. Households' budget constraint is thus given by

$$P_t^T C_t^T + P_t^N C_t^N + \frac{B_{t+1}}{R_t} = P_t^T Y_t^T + W_t H_t + \mathit{\Pi}_t + B_t.$$

We assume that the interest rate R_t is exogenous and time-varying. We continue to assume that the price of tradables $P_t^T = \bar{P}_t^T$ is exogenously fixed. However, the price index of non-tradables P_t^N is endogenous.

The Euler equation of households is given by

$$1 = \beta R_t E_t \frac{(C_{t+1}^T)^{-\sigma}}{(C_t^T)^{-\sigma}} \frac{P_t^T}{P_{t+1}^T}. \tag{D.1}$$

A novel equilibrium condition is the optimal allocation of expenditure across tradable and non-tradable consumption, given by

$$\frac{P_t^N}{P_t^T} = \frac{1-\omega}{\omega} \left(\frac{C_t^N}{C_t^T}\right)^{-\sigma}. \tag{D.2}$$

The relative price of non-tradables P_t^N/P_t^T is an increasing function in tradable consumption C_t^T. Intuitively, when tradables and non-tradables are imperfect substitutes, households increase their demand for non-tradables as they consume more tradables, which raises the price of non-tradables.

Labor supply is given by

$$\frac{G'(H_t)}{A(C_t^T, C_t^N)^{-\sigma}} \leq \frac{W_t}{P_t^A}, \tag{D.3}$$

where the price level P_t^A is an expenditure-based measure of the consumer price index, linked to the price of tradables and non-tradables as[45]

$$P_t^A = \frac{P_t^T}{A(C_t^T, C_t^N)^{\sigma}\omega(C_t^T)^{-\sigma}} = \frac{P_t^N}{A(C_t^T, C_t^N)^{\sigma}(1-\omega)(C_t^N)^{-\sigma}}. \tag{D.4}$$

Underlying the weak inequality in (D.3) is the assumption of downward nominal wage rigidity, as before:

$$W_t \geq \psi W_{t-1}.$$

[44] U is given by $C^{1-\sigma}/(1-\sigma)$. The aggregator A is given by $(\omega(C_t^T)^{1-\sigma} + (1-\omega)(C_t^N)^{1-\sigma})^{1/(1-\sigma)}$.

[45] To see this, consider the problem of optimally allocating expenditure

$$\min \{P_t^T C_t^T + P_t^N C_t^N\} \ \text{ s.t. } \ A(C_t^T, C_t^N) \equiv \left[\omega(C_t^T)^{1-\sigma} + (1-\omega)(C_t^N)^{1-\sigma}\right]^{1/(1-\sigma)}.$$

The Lagrange multiplier of this problem, which by the Envelope theorem represents an expenditure-based measure of the consumer price index, is denoted P_t^A.

Firms are owned by the households. They operate in the non-tradable sector, maximizing profits $\Pi_t = P_t^N F(H_t) - W_t H_t$, where $F(H_t) = H_t^\alpha$, $\alpha \in (0, 1)$, by taking prices and wages as given. This yields labor demand

$$F'(H_t) = \frac{W_t}{P_t^N}. \tag{D.5}$$

With respect to the baseline model, the important difference is that the sales price of firms' output P_t^N is now endogenous.

The market clears when all non-tradables are consumed in each period domestically: $F(H_t) = C_t^N$. Using market clearing and inserting $W_t H_t + \Pi_t = P_t^N F(H_t)$ in the budget constraint yields the resource constraint

$$P_t^T C_t^T + \frac{B_{t+1}}{R} = P_t^T Y_t^T + B_t. \tag{D.6}$$

We state the following definition of equilibrium. Equilibrium under laissez-faire is a set of processes $\{P_t^N, C_t^T, H_t, B_{t+1}, W_t\}_{t\geq 0}$ such that equations (D.1)-(D.2) and (D.5)-(D.6) as well as either

$$\begin{aligned} &i)\ [slack]\ \text{(D.3) with equality if } W_t \geq \psi W_{t-1},\ \text{ or else} \\ &ii)\ [binds]\ W_t = \psi W_{t-1}, \end{aligned}$$

where $C_t^N = F(H_t)$, where $P_t^A = P_t^T / (A(C_t^T, C_t^N)^\sigma \omega (C_t^T)^{-\sigma})$, where $P_t^T = \bar{P}_t^T$, for given initial conditions $W_{-1} > 0$ and B_0, and for given exogenous $\{Y_t^T, R_t, \bar{P}_t^T\}_{t\geq 0}$, are all satisfied.

D.2 Normative analysis

This model features both the pecuniary externality and an aggregate demand externality. The nature of the demand externality has been described in Schmitt-Grohé and Uribe (2016) as follows (pp. 1468-69): "The nature of the externality is that expansions in aggregate demand drive up wages, putting the economy in a vulnerable situation.... Agents understand this mechanism but are too small to internalize the fact that their individual expenditure decisions collectively cause inefficiently large increases in wages during expansions."

To take account of this aspect, we consider a planner that chooses aggregate demand (demand for tradables C_t^T) on behalf of households. Furthermore, we consider two different constrained-efficient equilibria as follows.

First, we study a planner that respects labor demand as chosen by firms. This corresponds to the planner considered by Schmitt-Grohé and Uribe (2016). Second, we study a planner that, additionally to choosing aggregate demand, chooses labor demand on behalf of firms, as in the main text. We introduce both planners, because we will compare both constrained-efficient allocations in our numerical analysis below.

D.2.1 Intervention in financial markets only (Schmitt-Grohé and Uribe, 2016)

The planning problem is given by

$$\max\ E_0 \sum_{t\geq 0} \beta^t (U(C_t) - G(H_t))$$

subject to the set of constraints

$$\begin{aligned}
&i) && P_t^T C_t^T + B_{t+1}/R_t = P_t^T Y_t^T + B_t && \text{(multiplier: } \iota_t\text{)} \\
&ii) && W_t \geq P_t^T \frac{G'(H_t)}{\omega (C_t^T)^{-\sigma}} && \text{(multiplier: } \zeta_t\text{)} \\
&iii) && W_t \geq \psi W_{t-1} && \text{(multiplier: } \lambda_t\text{)} \\
&iv) && P_t^T F'(H_t) \frac{1-\omega}{\omega} \left(\frac{F(H_t)}{C_t^T} \right)^{-\sigma} = W_t && \text{(multiplier: } \gamma_t\text{)}
\end{aligned}$$

where $C_t = A(C_t^T, F(H_t))$, and where $P_t^T = \bar{P}_t^T$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{Y_t^T, R_t, \bar{P}_t^T\}_{t \geq 0}$.

Importantly, constraint iv) holds always with equality: the planner respects labor demand as chosen by firms. This corresponds to the planner studied in Schmitt-Grohé and Uribe (2016).

The planning problem reveals the benefit of regulating demand C_t^T. Because C_t^T enters labor demand (and supply), demand matters for wages, and therefore interacts with downward nominal wage rigidity.[46]

Formally, taking first order conditions with respect to C_t^T and B_{t+1}

$$\begin{aligned}
\omega (C_t^T)^{-\sigma} - P_t^T \iota_t + \sigma \frac{W_t}{C_t^T} (\gamma_t - \zeta_t) = 0 \\
-\iota_t / R_t + \beta E_t \iota_{t+1} = 0.
\end{aligned}$$

Combining both yields the Euler equation

$$1 = \beta R_t E_t \frac{\omega (C_{t+1}^T)^{-\sigma} + \sigma (W_{t+1}/C_{t+1}^T)(\gamma_{t+1} - \zeta_{t+1})}{\omega (C_t^T)^{-\sigma} + \sigma (W_t / C_t^T)(\gamma_t - \zeta_t)} \frac{P_t^T}{P_{t+1}^T}.$$

Taking first order conditions with respect to W_t and H_t yields restrictions for the multipliers λ_t, γ_t and ζ_t

$$\begin{aligned}
-\gamma_t + \zeta_t + \lambda_t - \psi \beta E_t \lambda_{t+1} = 0 \\
(1-\omega)(F(H_t))^{-\sigma} F'(H_t) - G'(H_t) + \gamma_t \frac{1}{\varepsilon_t^F} \frac{W_t}{H_t} - \zeta_t \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} = 0,
\end{aligned}$$

where, as in the main text, $\varepsilon_t^F < 0$ and $\varepsilon_t^G > 0$ denote the wage elasticities of labor demand and supply, respectively. In the current model the two elasticities are given by

$$\varepsilon_t^F = -\frac{1}{1 - \alpha + \alpha\sigma}$$

for labor demand, and for labor supply

$$\varepsilon_t^G = \frac{1}{\varphi}.$$

[46] From equation (D.3), C_t^T enters labor supply both through the consumer price index P_t^A, and through $U'(C_t)$ via a wealth effect. The wealth effect channel is distinct from the demand externality channel and is discussed separately in the context of the baseline model in Appendix E.1.

Relative to the baseline model, the wage elasticity of labor demand thus becomes smaller (in absolute value). Intuitively, a change in the nominal wage affects firms' hiring by less, because the sales price P_t^N is endogenous: firms partially pass on the higher nominal wage to higher domestic prices.

We distinguish the regions where downward nominal wage rigidity is slack and binds, respectively.

Case 1: Downward nominal wage rigidity is slack

In this case, $W_t > \psi W_{t-1}$ such that $\lambda_t = 0$. In this region, constraint ii) cannot hold with strict inequality, for the implied $\zeta_t = 0$ yields a contradiction: first, $-\gamma_t = \psi\beta E_t\lambda_{t+1} \geq 0$ from the first order condition with respect to W_t, but second, $\gamma_t > 0$ from the first order condition with respect to H_t. Hence, constraint ii) must hold with equality. This implies for equilibrium employment

$$(1-\omega)(F(H_t))^{-\sigma}F'(H_t) = G'(H_t). \tag{D.7}$$

From the first order condition with respect to H_t, this implies $\gamma_t = (\varepsilon_t^F/\varepsilon_t^G)\zeta_t$. Inserting this into the first order condition with respect to W_t then yields

$$\gamma_t = (\varepsilon_t^F/\varepsilon_t^G)\zeta_t = (\varepsilon_t^F/\varepsilon_t^G)\frac{1}{1-(\varepsilon_t^F/\varepsilon_t^G)}\beta\psi E_t\lambda_{t+1},$$

implying that $\gamma_t < 0$ is strictly negative and $\zeta_t > 0$ is strictly positive whenever $E_t\lambda_{t+1} > 0$ (recall that $\varepsilon_t^F < 0$ and $\varepsilon_t^G > 0$).

Case 2: Downward nominal wage rigidity binds

We need to make a case distinction. Assume that $W_t = \psi W_{t-1}$. Assume constraint ii) holds with equality. Then taking the same steps as before, we obtain

$$\gamma_t = (\varepsilon_t^F/\varepsilon_t^G)\zeta_t = (\varepsilon_t^F/\varepsilon_t^G)\frac{1}{1-(\varepsilon_t^F/\varepsilon_t^G)}(\beta\psi E_t\lambda_{t+1} - \lambda_t),$$

which is compatible with $\gamma_t < 0$ and $\zeta_t > 0$ as long as $\lambda_t < \beta\psi E_t\lambda_{t+1}$. What determines the multiplier λ_t? The answer is the consumption Euler equation. Because constraint ii) still holds with equality, H_t is determined from equation (D.7). Because W_t is determined from downward nominal wage rigidity, C_t^T is determined from constraint ii). The multiplier λ_t is then determined from the Euler equation. We say that in this region, downward nominal wage rigidity binds "lightly".

When $\lambda_t > \beta\psi E_t\lambda_{t+1}$, the multiplier ζ_t hits zero and labor supply constraint ii) becomes rationed ("binds strongly").

This implies for λ_t

$$\lambda_t = -\varepsilon_t^F\frac{H_t}{W_t}((1-\omega)(F(H_t))^{-\sigma}F'(H_t) - G'(H_t)) + \beta\psi E_t\lambda_{t+1},$$

where the first summand (which equals γ_t) becomes strictly positive when employment in rationed.

In this region, W_t is determined from downward nominal wage rigidity. The variables λ_t, H_t and C_t^T are determined jointly from the previous equation, the Euler equation and from labor demand (constraint iv).

We summarize the constrained-efficient equilibrium with intervention in financial markets as follows. Equilibrium is a set of processes $\{P_t^N, C_t^T, H_t, B_{t+1}, W_t, \lambda_t^{sp}\}$ such that

$$1 = \beta R_t E_t \frac{\omega(C_{t+1}^T)^{-\sigma} + \sigma(W_{t+1}/C_{t+1}^T)(\lambda_{t+1}^{sp} - \beta\psi E_{t+1}\lambda_{t+2}^{sp})}{\omega(C_t^T)^{-\sigma} + \sigma(W_t/C_t^T)(\lambda_t^{sp} - \beta\psi E_t\lambda_{t+1}^{sp})} \frac{P_t^T}{P_{t+1}^T}$$

$$P_t^T C_t^T + \frac{B_{t+1}}{R_t} = P_t^T Y_t^T + B_t$$

$$\frac{P_t^N}{P_t^T} = \frac{1-\omega}{\omega}\left(\frac{F(H_t)}{C_t^T}\right)^{-\sigma}$$

$$P_t^N F'(H_t) = W_t$$

as well as the conditions

$$\lambda_t^{sp} = 0$$

$$\frac{G'(H_t)}{\omega(C_t^T)^{-\sigma}} = \frac{W_t}{P_t^T}$$

if $W_t \geq \psi W_{t-1}$ (*slack*), or else

$$W_t = \psi W_{t-1}$$

$$\frac{G'(H_t)}{\omega(C_t^T)^{-\sigma}} = \frac{W_t}{P_t^T}$$

if $\lambda_t^{sp} \leq \beta\psi E_t\lambda_{t+1}^{sp}$ (*binds lightly*), or else

$$W_t = \psi W_{t-1}$$

$$\lambda_t^{sp} = -\varepsilon_t^F \frac{H_t}{W_t}((1-\omega)(F(H_t))^{-\sigma}F'(H_t) - G'(H_t)) + \beta\psi E_t\lambda_{t+1}^{sp},$$

(*binds strongly*), where $P_t^T = \bar{P}_t^T$, for given initial conditions $W_{-1} > 0$ and B_0, for given exogenous processes $\{Y_t^T, R_t, \bar{P}_t^T\}_{t\geq 0}$, are all satisfied.

D.2.2 Intervention in the labor market and in financial markets

We next study the case where the planner intervenes both in financial markets and in the labor market. In this case, the constrained-efficient allocation solves

$$\max \; E_0 \sum_{t\geq 0} \beta^t (U(C_t) - G(H_t))$$

subject to the set of constraints

i)	$P_t^T C_t^T + B_{t+1}/R_t = P_t^T Y_t^T + B_t$	(multiplier: ι_t)
ii)	$W_t \geq P_t^T \frac{G'(H_t)}{\omega(C_t^T)^{-\sigma}}$	(multiplier: ζ_t)
iii)	$W_t \geq \psi W_{t-1}$	(multiplier: λ_t)
iv)	$P_t^T F'(H_t) \frac{1-\omega}{\omega}\left(\frac{F(H_t)}{C_t^T}\right)^{-\sigma} \geq W_t$	(multiplier: γ_t)

where $C_t = A(C_t^T, F(H_t))$, and where $P_t^T = \bar{P}_t^T$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{Y_t^T, R_t, \bar{P}_t^T\}_{t\geq 0}$.

The key difference relative to the case where the planner intervenes in financial markets only is constraint iv), which now needs to hold only with weak inequality. This also implies that the multiplier γ_t associated with this constraint is now restricted to be non-negative.

The first order conditions are the same as in the case of financial markets intervention only. As a result, the Euler equation is still given by:

$$1 = \beta R_t E_t \frac{\omega(C_{t+1}^T)^{-\sigma} + \sigma(W_{t+1}/C_{t+1}^T)(\lambda_{t+1} - \psi\beta E_{t+1}\lambda_{t+2})}{\omega(C_t^T)^{-\sigma} + \sigma(W_t/C_t^T)(\lambda_t - \psi\beta E_t\lambda_{t+1})} \frac{P_t^T}{P_{t+1}^T}. \tag{D.8}$$

Moreover, the first order conditions with respect to W_t and H_t are still given by:

$$-\gamma_t + \zeta_t + \lambda_t - \psi\beta E_t\lambda_{t+1} = 0$$
$$(1-\omega)(F(H_t))^{-\sigma}F'(H_t) - G'(H_t) + \gamma_t \frac{1}{\varepsilon_t^F}\frac{W_t}{H_t} - \zeta_t \frac{1}{\varepsilon_t^G}\frac{W_t}{H_t} = 0,$$

We again distinguish the regions where downward nominal wage rigidity is slack and binds, respectively.

Case 1: Downward nominal wage rigidity is slack

Taking identical steps as in Appendix A.1, we find that when downward nominal wage rigidity is slack, $\lambda_t = 0$, labor supply ii) holds with equality, and the multiplier $\gamma_t = 0$. This yields the labor demand curve

$$\frac{P_t^N F'(H_t)}{P_t^A} = \frac{W_t}{P_t^A} + \frac{1}{A(C_t^T, C_t^N)^{-\sigma}} \frac{1}{\varepsilon_t^G}\frac{W_t}{H_t}\beta\psi E_t\lambda_{t+1}. \tag{D.9}$$

Here we have used equation (D.4) and we have used that labor supply ii) holds with equality. This is the analogue of equation (9) in the main text.

Case 2: Downward nominal wage rigidity binds

Assume that $\lambda_t > 0$, such that downward nominal wage rigidity binds.

As in Appendix A.1, we make a case distinction. Focus first on the case where downward nominal wage rigidity binds "lightly". In this region, labor demand continues to hold with strict inequality, such that $\gamma_t = 0$.

We show that firms hire the full labor supply as long as the marginal product is above the real wage. Using that $\gamma_t = 0$, we solve for the multiplier ζ_t from the first order condition with respect to H_t

$$\begin{aligned}\zeta_t &= \varepsilon_t^G \frac{H_t}{W_t}((1-\omega)(F(H_t))^{-\sigma}F'(H_t) - G'(H_t)) \\ &= A(C_t^T, C_t^N)^{-\sigma}\varepsilon_t^G \frac{H_t}{W_t}\left(\frac{P_t^N F'(H_t)}{P_t^A} - \frac{W_t}{P_t^A}\right) \geq 0,\end{aligned}$$

where we have used (D.4) and that labor supply ii) holds with equality. When downward nominal wage rigidity binds, $F'(H_t)$ falls and W_t rises, which shrinks ζ_t. However, because $\zeta_t > 0$ when downward nominal wage rigidity is slack, ζ_t can still remain strictly positive. Hence, the extra labor supply will be absorbed by firms in this case.

Instead, when downward nominal wage rigidity binds strong enough, $\zeta_t = 0$ and firms ration labor supply according to

$$\frac{P_t^N F'(H_t)}{P_t^A} = \frac{W_t}{P_t^A}.$$

In this region, multiplier γ_t becomes strictly positive. It is given by

$$\gamma_t = -\varepsilon_t^F \frac{H_t}{W_t}\left(A(C_t^T, C_t^N)^{-\sigma}\frac{W_t}{P_t^A} - G'(H_t)\right),$$

where we have used equation (D.4) and that labor demand iv) holds with equality. By using the first order condition with respect to W_t, the multiplier λ_t is thus given by

$$\lambda_t = -\omega(C_t^T)^{-\sigma}\left(\varepsilon_t^F \frac{H_t}{W_t}\left(\frac{W_t}{P_t^T} - \frac{G'(H_t)}{\omega(C_t^T)^{-\sigma}}\right) + \varepsilon_t^G \frac{H_t}{W_t}\left(\frac{P_t^N F'(H_t)}{P_t^T} - \frac{W_t}{P_t^T}\right)\right) + \beta\psi E_t \lambda_{t+1},$$

where we have used equation (D.4) to replace P_t^A.

We summarize the constrained-efficient equilibrium with two interventions as follows. Equilibrium is a set of processes $\{P_t^N, C_t^T, H_t, B_{t+1}, W_t, \lambda_t^{sp}\}$ such that

$$1 = \beta R_t E_t \frac{\omega(C_{t+1}^T)^{-\sigma} + \sigma(W_{t+1}/C_{t+1}^T)(\lambda_{t+1}^{sp} - \beta\psi E_{t+1}\lambda_{t+2}^{sp})}{\omega(C_t^T)^{-\sigma} + \sigma(W_t/C_t^T)(\lambda_t^{sp} - \beta\psi E_t \lambda_{t+1}^{sp})}\frac{P_t^T}{P_{t+1}^T}$$

$$P_t^T C_t^T + \frac{B_{t+1}}{R_t} = P_t^T Y_t^T + B_t$$

$$\frac{P_t^N}{P_t^T} = \frac{1-\omega}{\omega}\left(\frac{F(H_t)}{C_t^T}\right)^{-\sigma}$$

as well as the labor market conditions

$$\frac{P_t^N F'(H_t)}{P_t^T} = \frac{W_t}{P_t^T} + \frac{1}{\omega(C_t^T)^{-\sigma}}\frac{1}{\varepsilon_t^G}\frac{W_t}{H_t}\beta\psi E_t \lambda_{t+1}^{sp}$$

$$\frac{G'(H_t)}{\omega(C_t^T)^{-\sigma}} = \frac{W_t}{P_t^T}$$

if $W_t \geq \psi W_{t-1}$ (*slack*), or else

$$W_t = \psi W_{t-1}$$

$$\frac{G'(H_t)}{\omega(C_t^T)^{-\sigma}} = \frac{W_t}{P_t^T}$$

if $P_t^N F'(H_t) \geq W_t$ (*binds lightly*), or else

$$W_t = \psi W_{t-1}$$

$$P_t^N F'(H_t) = W_t$$

(*binds strongly*), where the multiplier λ_t solves the recursive expression

$$\lambda_t^{sp} = -\omega(C_t^T)^{-\sigma}\left(\varepsilon_t^F \frac{H_t}{W_t}\left(\frac{W_t}{P_t^T} - \frac{G'(H_t)}{\omega(C_t^T)^{-\sigma}}\right) + \varepsilon_t^G \frac{H_t}{W_t}\left(\frac{P_t^N F'(H_t)}{P_t^T} - \frac{W_t}{P_t^T}\right)\right) + \beta\psi E_t \lambda_{t+1}^{sp},$$

where $P_t^T = \bar{P}_t^T$, for given initial conditions $W_{-1} > 0$ and B_0, for given exogenous processes $\{Y_t^T, R_t, \bar{P}_t^T\}_{t\geq 0}$, are all satisfied.

D.3 Discussion of the theoretical results

In this model two externalities interact. First, there is the demand externality, which is addressed by the planner by regulating aggregate demand (Euler equation (D.8)). Second, there is the externality in the labor market, which the planner addresses by regulating labor demand. Indeed, the constrained-efficient labor demand curve (D.9) resembles closely its counterpart (9) in the model from the main text.

The constrained-efficient equilibrium also makes transparent that the two externalities are interlinked. This is because the same multiplier λ_t^{sp} enters both the constrained-efficient Euler equation and labor demand curve. For example, a severe recession is indicated by a large multiplier $E_t\lambda_{t+1}^{sp}$. Ex-ante, this leads the planner to reduce labor demand (as in the baseline model), and from the Euler equation it leads the planner to reduce borrowing, as current period's marginal utility of tradable consumption is regulated upwards.

This also implies that the two interventions operate as partial substitutes: to the extent that intervention in the labor market reduces the (expected) severity of the recession, there is less incentive for the planner to intervene in financial markets. We come back to this point in the numerical analysis below.

At this stage we reemphasize that, while the externality in the labor market is present in their model, it has gone unnoticed by Schmitt-Grohé and Uribe (2016). This is because they impose that labor demand is always determined competitively: the planner that they consider coincides with the planner that we had studied in Appendix D.2.1.[47]

In contrast to the baseline model, one difference in the current model is that the planner also intervenes ex-post in recessions: when downward nominal wage rigidity binds $\lambda_t^{sp} > 0$, this leads the planner to *increase* borrowing, from the Euler equation (D.8). This is because a higher demand for tradables reduces unemployment in the non-tradable sector, a feature highlighted in Schmitt-Grohé and Uribe (2016). However, as in the main text, we impose that the planner cannot demand labor when the marginal product falls below the real wage (no labor subsidies in the decentralized allocation).

Lastly, in terms of decentralization, the constrained-efficient allocation from Appendix D.2.1 can be achieved by policy makers via charging capital taxes, while the constrained-efficient allocation from Appendix D.2.2 can be achieved via charging appropriate labor and capital taxes *jointly*. We explore this more in the numerical analysis below.

D.4 Quantitative analysis

We now solve the extended model numerically, in order to assess quantitatively the interaction between the demand and pecuniary externality. In doing so we contrast three equilibria: the equilibrium under laissez-faire, the equilibrium where the planner intervenes optimally in financial markets (the case studied in Schmitt-Grohé and Uribe (2016)), and the equilibrium where the planner intervenes optimally both in the labor market *and* in financial markets.

D.4.1 Calibration

We follow largely the calibration in Schmitt-Grohé and Uribe (2016) to Argentina, from 1983Q1-2001Q4, at quarterly frequency. Price inflation of tradables is zero $\bar{P}_t = \bar{P}_{t-1}$. The

[47] In Schmitt-Grohé and Uribe (2016), this can be seen by noting that the constrained planner (their equation (32)) takes as a constraint the labor demand curve holding with equality (their equation (16)).

degree of downward nominal wage rigidity is $\psi = 0.99$. For the elasticity of substitution between tradables and non-tradables we assume a value of 0.44. This implies for the risk aversion coefficient $\sigma = 1/0.44 = 2.27$, which is close to the value $\sigma = 2$ in the calibration of the baseline model. For the labor share we use $\alpha = 2/3$. For the share of tradables in total consumption we set $\omega = 0.26$. For the inverse Frisch elasticity of labor supply, we use the same value as in the baseline model which is $\varphi = 3$.

The stochastic structure is also taken from Schmitt-Grohé and Uribe (2016). We consider a bivariate process in logs for the (real) borrowing rate and for the tradable endowment, as follows

$$\begin{bmatrix} \log(Y_t^T) \\ \log(R_t/R) \end{bmatrix} = \begin{bmatrix} 0.7901 & -1.3570 \\ -0.0104 & 0.8638 \end{bmatrix} \begin{bmatrix} \log(Y_{t-1}^T) \\ \log(R_{t-1}/R) \end{bmatrix} + v_t,$$

where $R = 1.0316$, and where $v_t \sim^{iid} \mathcal{N}([0,0]', \Sigma_v)$, with the variance-covariance matrix Σ_v given by

$$\Sigma_v = \begin{bmatrix} 0.0012346 & -0.0000776 \\ -0.0000776 & 0.0000401 \end{bmatrix}.$$

We calibrate the time discount factor β such that net foreign assets to annual GDP, in the mean of the ergodic distribution, equal -26%. As borrowing constraint we impose 150% debt to annual tradable steady-state GDP (as in the baseline model). This calibration strategy yields $\beta = 0.961$.

D.4.2 Results of the quantitative analysis

We first study policy functions and stationary distributions, then we move on to implications for welfare.

Figure D.1 plots policy functions against the lagged wage W_{t-1}. The variables shown are hours H_t and the nominal wage W_t (upper row), foreign assets to GDP $B_t/(4P_t^A Y_t)$ and unemployment u_t, both in percent (middle row), and the implied labor tax τ_t^w and capital tax τ_t^c for the constrained-efficient allocations, in percent (lower row).[48]

When W_{t-1} is high enough, downward nominal wage rigidity binds. In this region, hours sharply decline and unemployment turns positive. Moreover, foreign debt to GDP is reduced in this region.

Turn now to optimal financial markets intervention (green dotted lines). In boom times (W_{t-1} is low), the policy maker finds it optimal to charge capital taxes (i.e., the planner reduces borrowing, which is implemented in the decentralized economy by using capital taxation). This reduces demand for tradables (which can be seen by noting that foreign assets to GDP increase), which reduces wages in the non-tradable sector. In recessions, the opposite occurs: it becomes optimal to subsidize capital, as the resulting capital inflows sustain employment in the non-tradable sector (i.e., there is less unemployment under the intervention).

When both interventions are in place (red dashed), the policy maker still uses capital taxes in a comparable fashion, however, quantitatively, the use of capital controls is more limited. Instead, the policy maker taxes labor in boom times in order to reduce hours and therefore wages in the non-tradable sector. The fact that less capital taxes are necessary when

[48] Nominal GDP is given by $P_t^A Y_t = P_t^T Y_t^T + P_t^N Y_t^N$. Unemployment is defined as $u_t \equiv 1 - H_t/H_t^p$, where H_t^p is potential employment defined in $W_t = P_t^T G'(H_t^p)/(\omega(C_t^T)^{-\sigma})$. The taxes are given by $1 = \beta R_t(1 + \tau_t^c)E_t(C_{t+1}^T/C_t^T)^{-\sigma}(P_t^T/P_{t+1}^T)$, from equation (D.8), as well as $P_t^N F'(H_t) = (1 + \tau_t^w)W_t$, from equation (D.9).

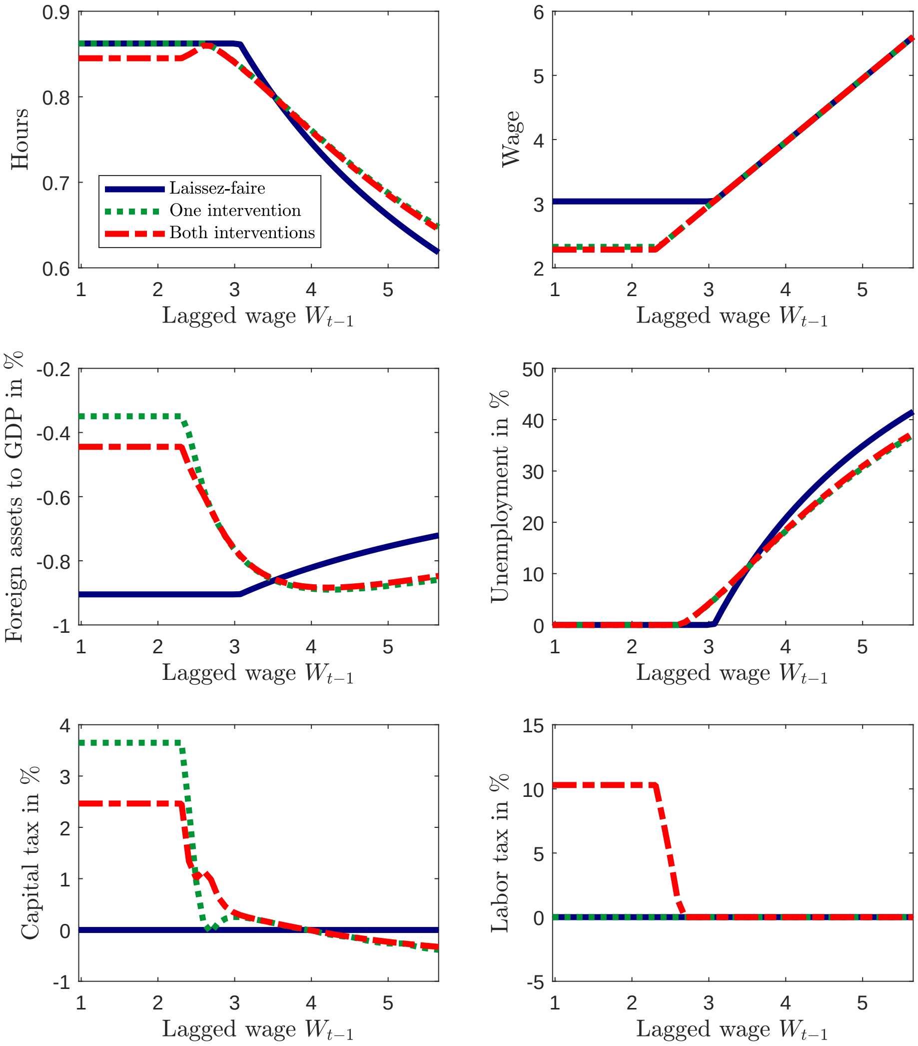

Figure D.1: Policy functions in the model with non-tradable sector, plotted against the lagged wage W_{t-1}. Shown are outcomes under laissez-faire, outcomes with the optimal intervention in financial markets in place, and outcomes under both interventions.

the optimal intervention in the labor market is in place reflects that the two instruments are partial substitutes, as explained earlier above. This fact will also become visible in the stationary distributions for both taxes. When both interventions are in place, wage inflation in boom times thus becomes even more limited even though quantitatively, the additional impact of labor taxation is small.

In a similar fashion, Figure D.2 plots policy functions against the interest rate R_t. Downward nominal wage rigidity binds when R_t is sufficiently large. The pattern of the intervention is similar as in Figure D.1. When both interventions are in place, the policy maker charges both capital taxes and payroll taxes on firms, which reduces wage inflation by more compared to the case where only capital taxes are in place.

In Figure D.3 we show stationary distributions for foreign assets to GDP $B_t/(4P_t^AY_t)$ and unemployment u_t as well as for the two taxes τ_t^w and τ_t^c, all in percent. Both interventions shift the distribution of foreign assets to the right, reflecting that the private economy overborrows.

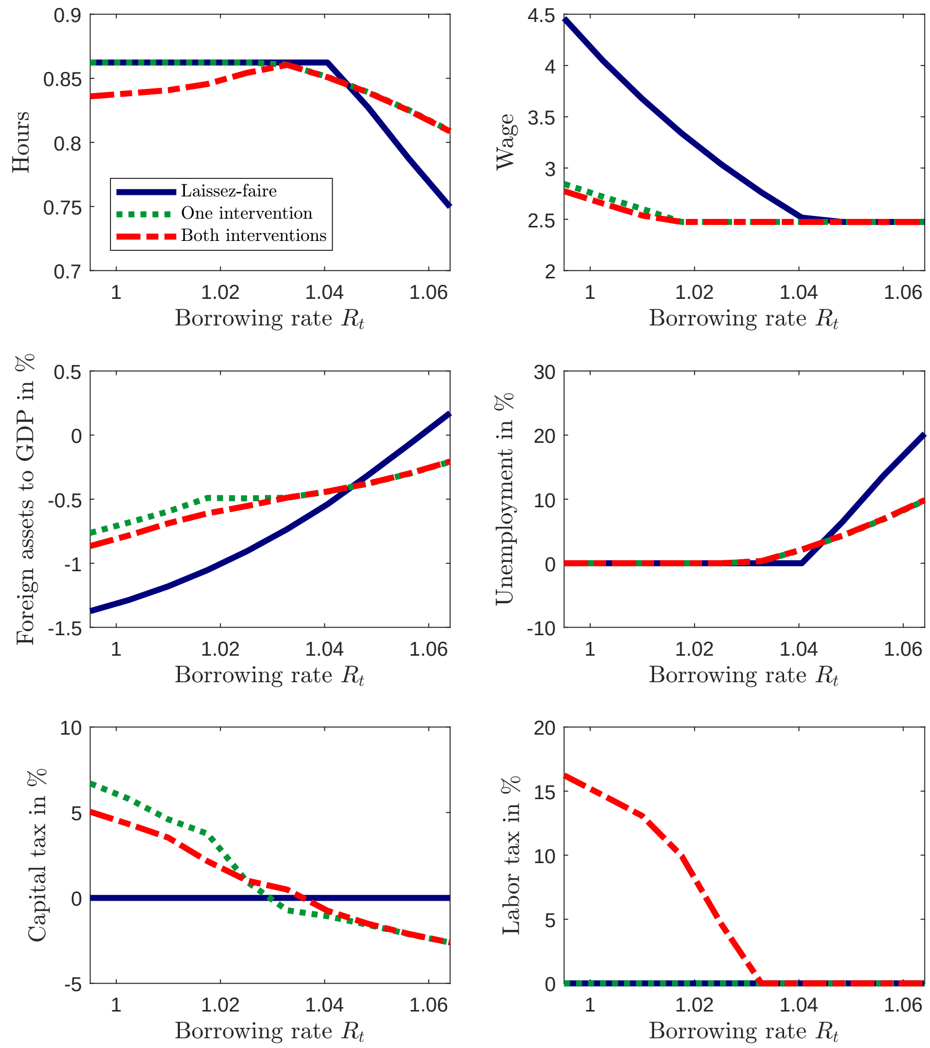

Figure D.2: Policy functions in the model with non-tradable sector, plotted against the borrowing rate R_t. Shown are outcomes under laissez-faire, outcomes with the optimal intervention in financial markets in place, and outcomes under both interventions.

However, the distribution is shifted by more to the right when only capital taxes are in place, whereas some more borrowing is allowed for when the policy maker also uses labor taxes. The stationary distribution for unemployment is squeezed to the vertical axis under both interventions, and slightly more so when both interventions are in place (the mass point at zero is larger). In terms of the underlying taxes, the mean capital tax is about 0.09% under both types of intervention. However, the distribution for capital taxes is narrower when both interventions are in place, which reflects again that the two taxes are partial substitutes. In turn, the mean labor tax is around 4.8%.

We turn to welfare implications. We study the mean consumption equivalent relative to first best in percent. We thus follow the same procedure as in the main text: we compare welfare losses to a benchmark where downward nominal wage rigidity is absent in the first place. To evaluate these losses, we use the analogue of equation (22) for the current model

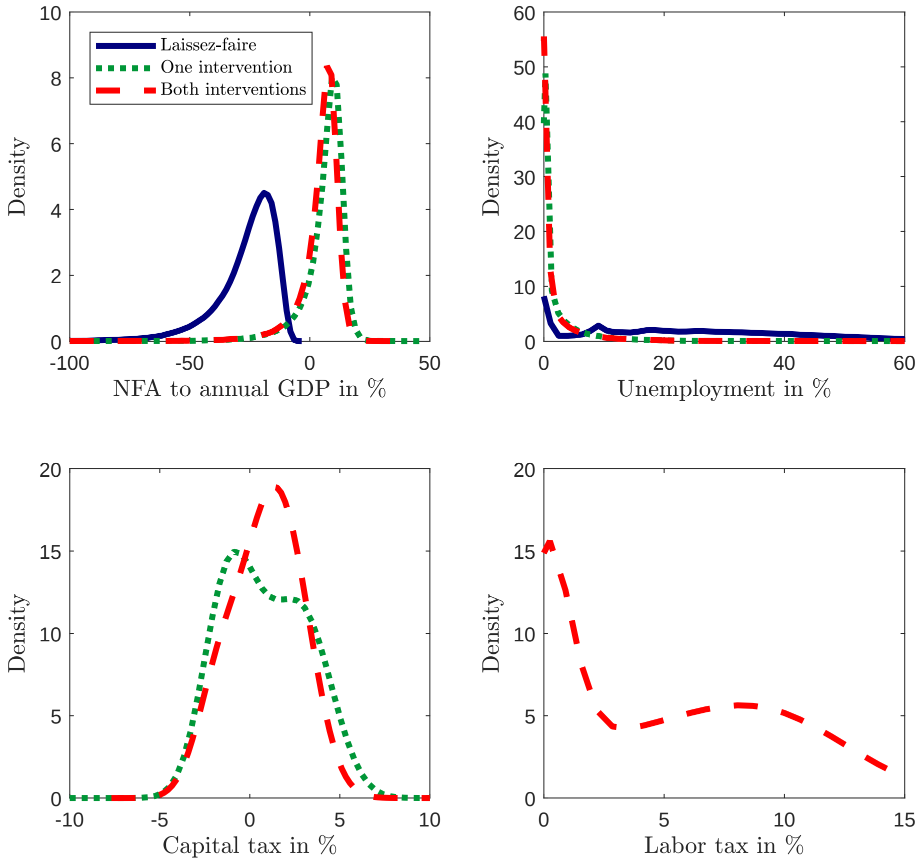

Figure D.3: Stationary distributions in the model with non-tradable sector.

	$mean(u_t)$	$mean(B_{t+1}/(4P_t^A Y_t))$	$mean(\iota_0)$	$mean(\tau_t^c)$	$mean(\tau_t^w)$
Laissez-faire	23.6%	-26.5%	4.4%	0%	0%
One intervention	2%	6%	0.3%	0.9%	0%
Both interventions	1.78%	4.2%	0.27%	0.8%	4.8%

Table D.1: Summary statistics in the model with non-tradable sector.

environment:

$$E_0 \sum_{t\geq 0} \beta^t (U(C_t(1+\iota_0)) - G(H_t)) \equiv E_0 \sum_{t\geq 0} \beta^t (U(C_t^{fb}) - G(H_t^{fb})),$$

where ι_0 measures the consumption equivalent.

The results are in Table D.1. The third column contains the mean consumption equivalent. In the other columns, we also report mean unemployment, mean foreign assets to GDP, and the mean required capital and labor taxes (recall Figure D.3).

Consumption losses under laissez-faire relative to first best are 4.4% on average, in line with the numbers reported in Schmitt-Grohé and Uribe (2016). This loss declines to 0.3% when the optimal capital tax intervention is in place. In turn, when both interventions are in place the loss declines further to 0.27%. Thus the additional welfare effect of charging labor taxes is small.

Finally, the mean unemployment rate declines from 23.6% under laissez-faire to 2% and 1.78%, respectively, under the two types of interventions. Mean assets to GDP climb from -26.5% to 6% and 4.2% respectively.

D.4.3 Discussion of the numerical results

These results suggest that if macroprudential capital taxes are available to policy makers in addition to labor taxes, the latter contribute only little to reducing unemployment. However, this conclusion is not necessarily correct.

First, recall that in the model without non-tradable sector (the main text), capital taxes cannot be used to decentralize the constrained-efficient allocation, because private agents' borrowing and consumption decisions are efficient. On the contrary, in this model capital taxes would be *detrimental* to welfare, while the instrument useful for decentralizing the constrained-efficient allocation is labor taxation.

Second, even in a model where capital taxes are useful, the finding that this instrument outperforms labor taxes is likely not general. This is for two reasons: i) in the model studied above, all shocks arise in the tradable sector and ii) the elasticity of substitution between tradables and non-tradables is small (it is 0.44). Both imply that by controlling tradable consumption, the policy maker has a strong lever on developments in the non-tradable sector. This fact reduces the usefulness of having an additional instrument in the non-tradable sector.

Third, the small additional benefit of labor taxes also reflects that both instruments are partial substitutes, as explained earlier. Indeed, studying the constrained optimum where labor taxes are available, but not capital taxes, would likely also yield a strong decline of equilibrium unemployment.[49]

More research is thus needed to understand better the relationship between capital and labor taxes in an environment where demand and pecuniary externalities interact.

E Other model extensions

This Appendix studies three further model extensions. Appendix E.1 considers a utility function which allows for wealth effects on labor supply. Appendix E.2 considers a different wage rigidity. Appendix E.3 considers a different utility function such that unemployment arises at the extensive margin, following Galí et al. (2012).

E.1 Wealth effects on labor supply

Here we study how our results change once we allow for wealth effects on labor supply.

Households' welfare (1) is now given by

$$E_0 \sum_{t \geq 0} \beta^t (U(C_t) - G(H_t)), \quad \beta \in (0, 1)$$

[49] However, solving the case where only ex-ante labor taxes are in place, but no macroprudential capital taxes, is numerically much more challenging. This is because the resulting constrained-efficient optimum is not time consistent. This analysis is therefore left for future research.

where U and G are specified as before. Under laissez-faire, the only equilibrium condition that is affected by this change is labor supply

$$\frac{G'(H_t)}{U'(C_t)} \leq \frac{W_t}{P_t},$$

which replaces (4) from the main text.

We study a constrained social planner as in Definition 2, however, in addition to assuming that the planner chooses labor allocations on behalf of firms, also assuming that the planner chooses consumption demand on behalf of households.

The problem of the planner becomes

$$\max \ E_0 \sum_{t\geq 0} \beta^t \{U(C_t) - G(H_t)\}$$

subject to the set of constraints

$$\begin{array}{llr} i) & P_t C_t + B_{t+1}/R = P_t a_t F(H_t) + B_t & \text{(multiplier: } \iota_t) \\ ii) & W_t \geq P_t G'(H_t)/U'(C_t) & \text{(multiplier: } \zeta_t) \\ iii) & W_t \geq \psi W_{t-1} & \text{(multiplier: } \lambda_t) \\ iv) & P_t a_t F'(H_t) \geq W_t & \text{(multiplier: } \gamma_t) \end{array}$$

where $P_t = \bar{P}_t$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{a_t, \bar{P}_t\}_{t\geq 0}$.

The first order conditions with respect to C_t and B_{t+1} are

$$U'(C_t) - P_t \iota_t - \zeta_t \nu_t^G \frac{W_t}{C_t} = 0$$

$$-\iota_t/R + \beta E_t \iota_{t+1} = 0,$$

where $\nu_t^G > 0$ denotes the elasticity of wages with respect to a change in C_t. It is positive, because a higher C_t implies a positive wealth effect which shifts labor supply upwards.

Combining both gives the constrained-efficient Euler equation

$$1 = \beta R E_t \frac{U'(C_{t+1}) - \zeta_{t+1}\nu_{t+1}^G (W_{t+1}/C_{t+1})}{U'(C_t) - \zeta_t \nu_t^G (W_t/C_t)} \frac{P_t}{P_{t+1}}, \tag{E.1}$$

which needs to be compared with

$$1 = \beta R E_t \frac{U'(C_{t+1})}{U'(C_t)} \frac{P_t}{P_{t+1}},$$

under laissez-faire.

This is the main difference of the model with wealth effects on labor supply: the planner has an incentive to regulate aggregate demand, for aggregate demand matters for wealth effects which impact labor supply. In turn, regulating labor supply matters due to the pecuniary externality in the labor market.

The first order conditions for W_t and H_t are (largely) unchanged from the baseline model (compare equations (A.1) and (A.2) in Appendix A.1)

$$-\gamma_t + \zeta_t + \lambda_t - \beta\psi E_t \lambda_{t+1} = 0$$

$$-G'(H_t) + \gamma_t \frac{1}{\varepsilon_t^F} \frac{W_t}{H_t} - \zeta_t \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} + \iota_t P_t a_t F'(H_t) = 0.$$

These can be combined to yield

$$a_t F'(H_t) = \frac{G'(H_t)}{\iota_t}\frac{1}{P_t} + \frac{1}{\varepsilon_t^G}\frac{1}{P_t}\frac{1}{\iota_t}\frac{W_t}{H_t}\beta\psi E_t \lambda_{t+1}$$

when downward nominal wage rigidity is slack, which compares with equation (9) in the baseline model.

In sum, in the model with wealth effects, in addition to regulating labor demand, the planner also regulates aggregate demand.

Are capital controls prudential? Taking identical steps as in Appendix A.1, $\zeta_t > 0$ when downward nominal wage rigidity is slack, but $\zeta_t = 0$ when downward nominal wage rigidity is (strongly) binding. Assume now that downward nominal wage rigidity is binding with certainty next period (i.e., $\zeta_{t+1} = 0$). This implies for the Euler equation (E.1)

$$U'(C_t) = \beta R E_t U'(C_{t+1})\frac{P_t}{P_{t+1}} + \zeta_t \nu_t^G \frac{W_t}{C_t},$$

where $\zeta_t > 0$ in case downward nominal wage rigidity is slack in the current period. Recalling that $\nu_t^G > 0$, we see that the planner regulates today's marginal utility of consumption upwards, such that capital controls are indeed prudential (in the sense that the planner decreases current borrowing in anticipation of the binding constraint).

The intuition for the intervention is as follows. Reducing current consumption reduces the wealth effect on labor supply, shifting labor supply downwards. This reduces current wage inflation, which in turn helps to alleviate the pecuniary externality.

It is important to note that, even though capital controls are required to decentralize the constrained-efficient equilibrium, the motive for doing so is different than in the literature studying demand externalities, because here the inefficiency of consumption demand operates through the labor market. We explore the demand externality in detail in Appendix D.

E.2 A different wage rigidity

We show that, under the alternative wage rigidity $W_t \geq \bar{W}$, $\bar{W} > 0$, the need for macroprudential intervention disappears.

The equilibrium under laissez-faire under the alternative wage rigidity can be summarized as follows. An equilibrium is a path $\{C_t, H_t, B_{t+1}, W_t\}_{t\geq 0}$ such that

$$1 = \beta R E_t \frac{U'(t+1)}{U'(t)}\frac{P_t}{P_{t+1}}$$

$$P_t C_t + \frac{B_{t+1}}{R} = P_t a_t F(H_t) + B_t$$

as well as the labor market conditions

$$a_t F'(H_t) = \frac{W_t}{P_t}$$

$$G'(H_t) = \frac{W_t}{P_t}$$

if $W_t \geq \bar{W}$ (*slack*), or else

$$a_t F'(H_t) = \frac{W_t}{P_t}$$

$$W_t = \bar{W}$$

(*binds*), where $U'(t) \equiv U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t\geq 0}$ and $\{\tau_t^w \geq 0\}_{t\geq 0}$, are all satisfied.

We show that the equilibrium under laissez-faire is constrained efficient. Along the lines of Definition 2, the constrained-efficient equilibrium solves

$$\max E_0 \sum_{t\geq 0} \beta^t U(C_t - G(H_t))$$

subject to the set of constraints

$$\begin{array}{llr} i) & P_t C_t + B_{t+1}/R = P_t a_t F(H_t) + B_t & (\text{multiplier: } \iota_t) \\ ii) & (U'(t)/P_t) = \beta R E_t (U'(t+1)/P_{t+1}) & (\text{multiplier: } \nu_t) \\ iii) & W_t \geq P_t G'(H_t) & (\text{multiplier: } \zeta_t) \\ iv) & W_t \geq \bar{W} & (\text{multiplier: } \lambda_t) \\ v) & P_t a_t F'(H_t) \geq W_t & (\text{multiplier: } \gamma_t) \end{array}$$

where $U'(t) \equiv U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{a_t, \bar{P}_t\}_{t\geq 0}$.

Taking identical steps as in Appendix A.1, we find that the Euler equation ii) is slack in equilibrium ($\nu_t = 0$), and that $\iota = U'(t)/P_t$. Taking first order conditions with respect to W_t and H_t then yields

$$-\gamma_t + \zeta_t + \lambda_t = 0 \tag{E.2}$$

$$-U'(t)G'(H_t) + \gamma_t \frac{1}{\varepsilon_t^F}\frac{W_t}{H_t} - \zeta_t \frac{1}{\varepsilon_t^G}\frac{W_t}{H_t} + U'(t) a_t F'(H_t) = 0, \tag{E.3}$$

where elasticities $\varepsilon_t^F < 0$ and $\varepsilon_t^G > 0$ are defined as before. We proceed by distinguishing the cases where downward nominal wage rigidity is slack and binds, respectively.

Case 1: Downward nominal wage rigidity is slack

When downward nominal wage rigidity is slack ($W_t > \bar{W}$), then $\lambda_t = 0$ implying $\gamma_t = \zeta_t$ from (E.2). Using this in (E.3) yields

$$\gamma_t = \zeta_t = \left(\frac{1}{\varepsilon_t^G} - \frac{1}{\varepsilon_t^F}\right)^{-1} U'(t)\frac{H_t}{W_t}(a_t F'(H_t) - G'(H_t)).$$

Assume that both $\gamma_t = \zeta_t > 0$. Then both constraints iii) and v) must bind, implying that $a_t F'(H_t) = G'(H_t)$. But then $\gamma_t = \zeta_t = 0$ from the previous equation, a contradiction. Hence, $\gamma_t = \zeta_t = 0$, and $a_t F'(H_t) = G'(H_t)$, such that constraints iii) and v) both hold with equality.

We have thus verified that the constrained-efficient equilibrium coincides with the equilibrium under laissez-faire, whenever downward nominal wage rigidity is slack. Let us now turn to the case where the rigidity binds.

Case 2: Downward nominal wage rigidity binds

Assume downward nominal wage rigidity binds: $\lambda_t > 0$, implying that $W_t = \bar{W}$. We first show that in this case, $\zeta_t = 0$. Assume not, $\zeta_t > 0$. In this case, from (E.2), $\gamma_t > 0$ as well, such that both constraints iii) and v) hold with equality. From (E.3), this implies that

$$\gamma_t \frac{1}{\varepsilon_t^F}\frac{W_t}{H_t} = \zeta_t \frac{1}{\varepsilon_t^G}\frac{W_t}{H_t}.$$

The left hand side is negative, because $\varepsilon_t^F < 0$, whereas the right hand side is positive. This is a contradiction.

Hence $\zeta_t = 0$. From (E.2), this implies that $\gamma_t = \lambda_t > 0$. Moreover, labor demand v) must hold with equality: $a_t F'(H_t) = W_t/P_t$. Equation (E.3) then yields

$$\gamma_t = \varepsilon_t^F \frac{H_t}{W_t}\left(G'(H_t) - \frac{W_t}{P_t}\right),$$

which is positive whenever $G'(H_t) < W_t/P_t$, because $\varepsilon_t^F < 0$.

We have thus verified that, when downward nominal wage rigidity binds, $W_t = \bar{W}$, labor demand holds with equality, $a_t F'(H_t) = W_t/P_t$ and labor supply is rationed, $G'(H_t) < W_t/P_t$. These are the same conditions as in the equilibrium under laissez-faire.

E.3 Unemployment at the extensive margin

We consider household preferences as in Galí et al. (2012) (see also Galí (2015) and Galí (2011)). The economy is inhabited by a large number of households. Each household consists of a large number of household *members*, denoted $j \in [0, 1]$.

Household member j contributes disutility of work to the household as

$$\mathbf{1}_t j^{\varphi}, \quad \varphi > 0,$$

where φ is a parameter. The variable $\mathbf{1}_t$ is an indicator variable, taking the value of 1 in case member j is employed in period t, and 0 else.

As in Galí et al. (2012), we also assume perfect consumption risk sharing among household members $C_t(j) = C_t$, for all j.

The household's period utility is given by the integral of its members' utilities and can thus be written as follows

$$E_0 \sum_{t\geq 0} \beta^t U\left(C_t - \int_0^{H_t} \mathbf{1}_t j^{\varphi} dj\right), \quad \beta \in (0, 1),$$

where we have maintained the assumption of GHH preferences at the household level, as in the main text. Here, H_t is the fraction of household members that are employed at time t. In case firms demand H_t workers, only the first $[0, H_t]$ household members choose to work as they contribute the lowest work disutility to the household. By evaluating the indicator variable accordingly, the integral can be computed as

$$\int_0^{H_t} \mathbf{1}_t j^{\varphi} dj = \frac{H_t^{1+\varphi}}{1+\varphi}.$$

We next derive aggregate participation. Following Galí et al. (2012), household member j—using household welfare as a criterion and taking as given current labor market conditions (i.e., the real wage)—will find it optimal to participate in the labor market at time t if and only if

$$U'(t)\frac{W_t}{P_t} \geq U'(t) j^{\varphi},$$

for this is the marginal contribution to household utility of member j. This implies that the *marginal supplier* of labor is given by

$$\frac{W_t}{P_t} = (H_t^p)^{\varphi},$$

the variable H_t^p thus defining aggregate participation or the labor force (i.e., participation is given by the interval $[0, H_t^p]$).

Finally, as in Galí et al. (2012), this allows us to define unemployment as aggregate participation minus aggregate employment, in relative terms

$$u_t = \frac{H_t^p - H_t}{H_t^p} = 1 - \frac{H_t}{H_t^p},$$

which corresponds to the definition of unemployment (14) from the main text.

In line with the discussion in Galí (2015), introducing household preferences that allow for unemployment at the *extensive* margin, therefore leads to the same reduced form expressions as in the case where "unemployment" arises at the *intensive* margin (i.e., an involuntary reduction of hours of all workers). The framework presented here thus involves a *reinterpretation* of the framework from the main text, rather than a modification or extension of that framework (Galí, 2015).

F Model equilibrium conditions in stationary terms

We define the real wage $w_t \equiv W_t/P_t$ and real foreign assets $b_{t+1} \equiv B_{t+1}/P_{t+1}$. We also define the shadow value of relaxing downward nominal wage rigidity in real terms $\tilde{\lambda}_t \equiv \lambda_t P_t$ (and analogously $\tilde{\lambda}_t^{sp} \equiv \lambda_t^{sp} P_t$). The real interest rate can be defined as $R^r \equiv R/\bar{\pi}$.

F.1 Baseline model

Equilibrium is a path $\{C_t, H_t, b_{t+1}, w_t\}_{t\geq 0}$ such that

$$1 = \beta R^r E_t \frac{U'(t+1)}{U'(t)}$$
$$C_t + \frac{b_{t+1}}{R^r} = a_t F(H_t) + b_t$$

as well as the labor market conditions

$$a_t F'(H_t) = w_t$$
$$G'(H_t) = w_t$$

if $w_t \geq (\psi/\bar{\pi})w_{t-1}$ (*slack*), or else

$$G'(H_t) = w_t$$
$$w_t = (\psi/\bar{\pi})w_{t-1}$$

(*binds*), where $U'(t) \equiv U'(C_t - G(H_t))$, for given initial conditions $w_{-1} > 0$ and b_0, and for given exogenous $\{a_t\}_{t\geq 0}$, are all satisfied.

F.2 Constrained-efficient equilibrium

Equilibrium is a path $\{C_t, H_t, b_{t+1}, w_t, \tilde{\lambda}_t^{sp}\}_{t\geq 0}$ such that

$$1 = \beta R^r E_t \frac{U'(t+1)}{U'(t)}$$
$$C_t + \frac{b_{t+1}}{R^r} = a_t F(H_t) + b_t$$

as well as the labor market conditions

$$a_t F'(H_t) = w_t + \frac{1}{\bar{\pi}} \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{w_t}{H_t} \beta \psi E_t \tilde{\lambda}_{t+1}^{sp}$$
$$G'(H_t) = w_t$$

if $w_t \geq (\psi/\bar{\pi})w_{t-1}$ (*slack*), or else

$$w_t = (\psi/\bar{\pi})w_{t-1}$$
$$G'(H_t) = w_t$$

if $a_t F'(H_t) \geq w_t$ (*binds lightly*), or else

$$w_t = (\psi/\bar{\pi})w_{t-1}$$
$$a_t F'(H_t) = w_t,$$

(*binds strongly*), where the multiplier $\tilde{\lambda}_t^{sp}$ is given by

$$\tilde{\lambda}_t^{sp} = -U'(t) \left(\varepsilon_t^F \frac{H_t}{w_t}(w_t - G'(H_t)) + \varepsilon_t^G \frac{H_t}{w_t}(a_t F'(H_t) - w_t) \right) + \frac{1}{\bar{\pi}} \beta \psi E_t \tilde{\lambda}_{t+1}^{sp}$$

where $U'(t) \equiv U'(C_t - G(H_t))$, for given initial conditions $w_{-1} > 0$ and b_0, and for given exogenous $\{a_t\}_{t \geq 0}$, are all satisfied.

F.3 Monopsony model

Equilibrium is a path $\{C_t, H_t, b_{t+1}, w_t, \tilde{\lambda}_t\}_{t \geq 0}$ such that

$$1 = \beta R^r E_t \frac{U'(t+1)}{U'(t)}$$
$$C_t + \frac{b_{t+1}}{R^r} = a_t F(H_t) + b_t$$

as well as the labor market conditions

$$a_t F'(H_t) = \frac{\eta + 1}{\eta} w_t + \frac{1}{\bar{\pi}} \frac{1}{U'(t)} \frac{1}{\eta} \frac{w_t}{H_t} \beta \psi E_t \tilde{\lambda}_{t+1}$$
$$G'(H_t) = w_t$$

if $w_t \geq (\psi/\bar{\pi})w_{t-1}$ (*slack*), or else

$$w_t = (\psi/\bar{\pi})w_{t-1}$$
$$G'(H_t) = w_t$$

if $a_t F'(H_t) \geq w_t$ (*binds lightly*), or else

$$w_t = (\psi/\bar{\pi})w_{t-1}$$
$$a_t F'(H_t) = w_t,$$

(*binds strongly*), where the multiplier $\tilde{\lambda}_t$ is given by

$$\tilde{\lambda}_t = -U'(t)\eta \frac{H_t}{w_t}(a_t F'(H_t) - \frac{\eta + 1}{\eta} w_t) + \frac{1}{\bar{\pi}} \beta \psi E_t \tilde{\lambda}_{t+1},$$

where $U'(t) \equiv U'(C_t - G(H_t))$, for given initial conditions $w_{-1} > 0$ and b_0, and for given exogenous $\{a_t\}_{t \geq 0}$, are all satisfied.