Fiscal Stagnation

Luca Fornaro and Martin Wolf*

April 2025

Abstract

We study public debt sustainability in an economy with endogenous productivity growth. Our model has two key features: i) financing large primary surpluses entails fiscal distortions that depress investment and growth, ii) low growth increases the primary surpluses needed to stabilize the public debt-to-GDP ratio. Negative shocks to fundamentals or pessimistic animal spirits may drive the economy into a state of fiscal stagnation, characterized by high public debt, large fiscal distortions and low productivity growth. We discuss policy options to avoid/escape fiscal stagnation.

JEL Codes: E22, E62, H63, O40.

Keywords: Fiscal Policy, Public Debt, Endogenous Productivity Growth, Stagnation, Investment.

---

*Fornaro: CREI, Universitat Pompeu Fabra, Barcelona School of Economics and CEPR; LFornaro@crei.cat. Wolf: University of St Gallen and CEPR; martin.wolf@unisg.ch. We thank Olivier Blanchard, Max Croce, Christoph Trebesch, Jaume Ventura and participants at several seminars and conferences for very helpful comments. We are grateful to Iginio Marchesini and Niklas Henrik Schüler for excellent research assistance. Luca Fornaro acknowledges financial support from the European Research Council through the Consolidator Grant MACROGROWTH-101169982, from the Spanish Ministry of Economy and Competitiveness through the Severo Ochoa Programme for Centres of Excellence in R&D (CEX2019-000915-S), and from the Generalitat de Catalunya, through CERCA and SGR Programme (2021-SGR-01599). Martin Wolf acknowledges financial support from the Swiss National Science Foundation (SNSF) under the career grants program, SNSF Starting Grant (TMSGI1_226520-DEBTANDGROWTH).

1 Introduction

Public debt-to-GDP ratios have climbed to record highs in many advanced countries, triggering a heated debate about their sustainability and macroeconomic implications. This debate is often led on the premise that productivity growth is exogenous to fiscal policy (Blanchard, 2023). Recent empirical evidence, however, suggests that firms' investment and productivity growth do react to fiscal interventions. For instance, increases in corporate taxes and cuts in spending on public R&D seem to depress business investment and productivity growth (Croce et al., 2019; Antolin-Diaz and Surico, 2022; Cloyne et al., 2022; Fieldhouse and Mertens, 2023).

In some historical episodes, moreover, the correlation between fiscal policy and productivity growth is visible to the naked eye. Take the case of Italy, illustrated in Figure 1. The Italian public debt-to-GDP ratio rose substantially during the 1980s, due to high public expenditure. Since then, Italian governments had to come up with large primary surpluses - financed with a combination of high taxes and low public investment - to sustain the public debt. Interestingly, high primary surpluses have been accompanied by a sharp slowdown in productivity growth.[1] The fact that in many advanced economies public debt-to-GDP ratios are reaching Italian levels makes one wonder whether similar dynamics will soon happen elsewhere.

In this paper, we propose a framework to study the connections between productivity growth, fiscal policy and public debt. Our model has two key features. First, as is typical in modern endogenous growth frameworks, productivity growth is the outcome of investment by profit-maximizing firms. Second, financing large primary surpluses requires the use of distortionary taxes. These two elements generate a feedback loop between fiscal policy and growth. Intuitively, large primary surpluses are associated with high fiscal distortions, which depress firms' investment and productivity growth. In turn, low growth calls for high primary surpluses to ensure the sustainability of public debt. Our key insight is that this amplification mechanism may push the economy into a state of fiscal stagnation, that is a protracted period of low growth and high fiscal distortions.

More precisely, we study a standard endogenous growth model augmented with public debt and distortionary taxes. As in Aghion and Howitt (1992), firms invest in innovation to increase their future profits. The government sets the primary surplus to ensure the sustainability of the public debt. There are two sources of public revenue: non-distortionary taxes on households, and profit taxes on firms. Profit taxes are distortionary, because they reduce firms' incentives to invest. To make the model interesting, as well as realistic, we assume that there is an upper bound on the revenue that the government can extract from the private sector through non-distortionary taxes. Hence, a low primary surplus does not generate fiscal distortions. Once the primary surplus exceeds a threshold, however, fiscal distortions kick in.[2]

[1] Bastasin and Toniolo (2023) provide a broad overview of the macroeconomic history of Italy. They also discuss several potential causes, including fiscal distortions, behind the Italian productivity growth slowdown.

[2] Profit taxes are a particularly tractable source of fiscal distortions, which explains why we focus on them. But there are other channels through which a high primary surplus could depress investment and growth. For instance, following Barro (1990), we could assume that private investment is complementary to public services. A high primary surplus would then depress investment, by reducing the amount of public services provided by the government to private firms.

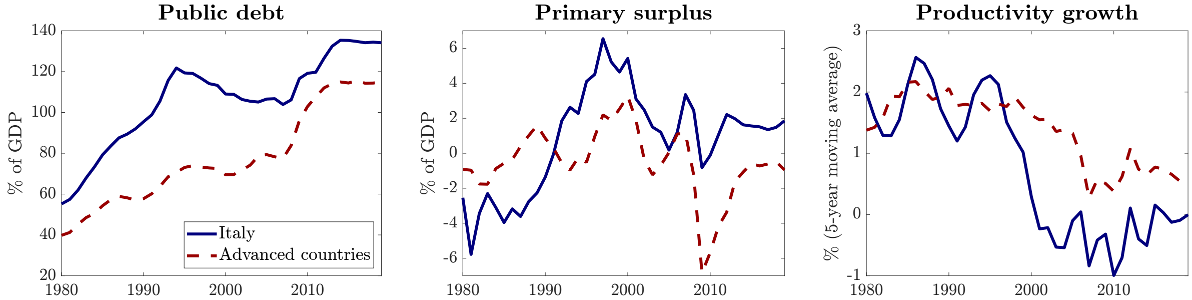

Figure 1: Motivating facts. This figure contrasts the evolution of public debt, primary surpluses and productivity growth in Italy, against a sample of advanced economies, during 1980-2019. See Appendix C for details on the data sources and the procedures used to construct these figures.

Our main result is that two stable steady states may coexist. In the first one, which we dub the fiscally sound steady state, the primary surplus is low, meaning that fiscal distortions are low and productivity growth is high. Because growth is high, running a low primary surplus is enough to stabilize the debt-to-GDP ratio. In the second one, which we dub the fiscal stagnation steady state, the primary surplus and fiscal distortions are high, depressing investment and productivity growth. Low growth, in turn, forces the government to run a large primary surplus to prevent the debt-to-GDP ratio from exploding. Since these two steady states may coexist, putting in place fiscal policies which ensure the existence of the fiscally sound steady state is not enough to rule out the risk of fiscal stagnation.[3]

What makes the economy fall into fiscal stagnation? If the feedback loop between fiscal policy and growth is of moderate strength, the fall into fiscal stagnation happens through hysteresis effects. That is, in the aftermath of a shock causing a modest rise in the public debt-to-GDP ratio, the economy converges back to the fiscally sound steady state. In contrast, once public debt relative to GDP exceeds a threshold, a vicious cycle of declining growth and increases in fiscal distortions leads the economy to the fiscal stagnation steady state. Temporary shocks or small differences in initial conditions can therefore have large long-run effects.

If the feedback loop between fiscal policy and growth is sufficiently strong, moreover, fiscal stagnation can also be the result of pessimistic animal spirits. To see why, imagine that private agents anticipate high future taxes on profits. As a result, they will cut back on investment, and productivity growth will decline. Low productivity growth, in turn, will push up the public debt-to-GDP ratio. If this effect is sufficiently strong, the government will then have to tax profits to stabilize debt, validating the initial expectations of the private sector. A wave of pessimism can thus shift the economy from fiscal soundness to stagnation.

Irrespective of whether it is driven by hysteresis effects or pessimistic animal spirits, a shift from the fiscally sound to the stagnation steady state can be interpreted as the result of self-defeating

[3]The coexistence of two steady states is due to the fact that our economy features a dynamic Laffer curve. Increasing the primary surplus in the present, in fact, decreases the future public debt-to-GDP ratio. But the expectations of a high future surplus, insofar as it is associated with high expected profit taxes, lead to lower investment and growth. Lower growth, in turn, pushes up the future debt-to-GDP ratio. The Laffer curve arises because, under certain circumstances, the second effect dominates the first one.

austerity (Fatás and Summers, 2018; Angeletos et al., 2024). During the transition toward fiscal stagnation, in fact, the increase in the primary surplus depresses growth so much that the debt-to-GDP ratio ends up not declining, or may even rise.

In the last part of the paper, we discuss how to optimally exit fiscal stagnation. Escaping fiscal stagnation requires a reduction in the public debt-to-GDP ratio to reach the fiscally sound steady state. In our model, the government can employ two strategies to attain this objective. First, public debt can be reduced through austerity, that is by increasing the primary surplus in the present. Second, public debt can be reduced through pro-growth policies. A credible promise to lower future taxes, in fact, stimulates investment and growth, reducing the debt-to-GDP ratio by boosting output.

We first show that a government lacking the ability to commit relies too heavily on austerity to exit fiscal stagnation, due to a time-consistency problem. Before firms have made investment plans, the government has an incentive to promise low future taxes to stimulate growth. But once investment is sunk, the government will tax profits at a higher rate than promised to reduce the future public debt-to-GDP ratio. Anticipating this outcome, firms' investment will be low to begin with, making the pro-growth approach to exiting fiscal stagnation infeasible. Lack of credibility thus depresses investment and growth during the transition to the final steady state. Moreover, it leaves the economy vulnerable to pessimistic animal spirits. Indeed, private agents' expectations determine by how much the debt-to-GDP ratio has to drop to reach the fiscally sound steady state.

Under commitment, the government complements fiscal austerity with pro-growth policies to exit stagnation. Growth is thus higher on average during the transition to the fiscally sound steady state, leading to higher welfare. Moreover, the ability to make credible promises allows the government to rule out self-fulfilling pessimism, by coordinating expectations on the best possible equilibrium. Our model thus assigns an important role to expectations and government credibility to successfully exit fiscal stagnation.

This paper is related to several strands of the literature. First, it is motivated by the revival of the literature on public debt sustainability. Recent examples are Blanchard (2023), Mehrotra and Sergeyev (2021), Mian et al. (2022), Angeletos et al. (2024) and Jiang et al. (2024), which all treat productivity growth as exogenous. A few works have connected public debt and fiscal policy to productivity growth (Croce et al., 2012; Fatás and Summers, 2018; Croce et al., 2019, 2021; Elfsbacka-Schmöller and McClung, 2024).[4] Relative to this literature, we show that the interplay between growth and fiscal policy can generate steady state multiplicity, and discuss the options to exit fiscal stagnation.

Our paper is also related to the literature arguing that a high stock of public debt exposes the economy to the risk of equilibrium multiplicity. Some prominent contributions to this literature are Calvo (1988), Cole and Kehoe (2000), Lorenzoni and Werning (2019) and Galli (2021). In these works, multiple equilibria arise because of the risk that the government will default on its

[4] In turn, this literature builds on earlier work on fiscal policy in endogenous growth models. Barro (1990) and Saint-Paul (1992) are two seminal contributions.

obligations. We explore a different source of multiple equilibria, which arises also when public debt is totally risk free. Benigno and Fornaro (2018) show that pessimistic expectations can push the economy into a stagnation trap, characterized by a self-reinforcing cycle of depressed demand, low investment and weak growth. Instead, in this paper multiple equilibria arise from the interaction between fiscal policy and endogenous growth, and in particular from the fact that legacy debt may give rise to fiscal distortions.

Our focus on the negative macroeconomic effects of legacy debt connects this paper to the literature on debt overhang. An important strand of this literature considers the relationship between debt overhang and default risk in open economies (Krugman, 1988). In Lamont (1995), corporate debt overhang gives rise to multiple equilibria due to profit externalities among monopolistically competitive firms. We abstract from this channel, and centre our analysis on public debt and fiscal externalities.

We also build on the empirical evidence on the impact of fiscal interventions on investment and productivity. Croce et al. (2012) find that high public debt depresses the market value of innovative firms, and argue that this is due to the negative impact of high future taxes on investment in innovation. Cloyne et al. (2022) show that increases in corporate taxes depress business investment and productivity growth. Antolin-Diaz and Surico (2022) and Fieldhouse and Mertens (2023) document empirically that drops in public R&D spending cause declines in private R&D and productivity growth. Our model builds on the idea that the relationships between primary surpluses, taxes and growth is likely to be non-linear. Jaimovich and Rebelo (2017) provide empirical evidence in support of this notion, and propose a model to explore some of its macroeconomic implications. Finally, our model suggests that the relationship between public debt and growth is a complex one, and depends on the structural characteristics of the economy, as well as on animal spirits. These implications accord well with the existing empirical evidence (Eberhardt and Presbitero, 2015).

The rest of the paper is composed of four sections. Section 2 introduces our model. Section 3 shows how the economy can fall into fiscal stagnation. Section 4 discusses several strategies to escape fiscal stagnation. Section 5 concludes. The Appendix contains all the mathematical proofs, and some additional derivations.

2 Model

Consider an infinite-horizon closed economy. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The economy is inhabited by households, firms, and by a government that sets fiscal policy. For simplicity, we will assume perfect foresight.

2.1 Households

There is a continuum of measure one of identical households deriving utility from consumption of a homogeneous "final" good. The lifetime utility of the representative household is

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma}, \tag{1}$$

where C_t denotes consumption, $0 < \beta < 1$ is the subjective discount factor, and $\gamma \geq 0$ the inverse of the elasticity of intertemporal substitution. Each household supplies inelastically one unit of labor on the market.

Taking the consumption good as the numeraire, the households' budget constraint is

$$C_t + \frac{B_{t+1}}{R_t} = W_t + D_t + B_t - T_t, \tag{2}$$

where B_t are one-period bonds paying the gross interest rate R_t, W_t the wage, D_t the dividends distributed to the households by the firms, and T_t lump-sum taxes levied by the government.

At each time t, households allocate their total income between consumption expenditure and bonds purchases. Optimal saving behavior implies

$$R_t = \frac{1}{\beta} \left(\frac{C_{t+1}}{C_t} \right)^{\gamma}. \tag{3}$$

The optimal consumption path also satisfies a standard transversality condition. The parameter γ is going to play an important role in our model. In fact, it determines the sensitivity of the equilibrium interest rate to changes in the growth rate of the economy.[5] For reasons that will become clear below, we find plausible to focus on scenarios in which the interest rate responds less than one-for-one to changes in productivity growth. Throughout the paper, we will therefore assume that $0 < \gamma < 1$.[6]

2.2 Final good production

The final good is produced by competitive firms using labor L_t and a continuum of measure one of intermediate inputs $x_{j,t}$, indexed by $j \in [0,1]$. Denoting by Y_t the output of the final good, the production function is

$$Y_t = (ZL_t)^{1-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^{\alpha} dj, \tag{4}$$

[5] Strictly speaking, equation (3) connects the interest rate to the growth rate of households' consumption. However, in our simple model, which is designed to capture medium to long-run dynamics, productivity growth will be the key determinant of consumption growth.

[6] The assumption of an intertemporal elasticity of substitution above 1 is in line with the literature on the macroeconomic consequences of long-run growth risks (Bansal and Yaron, 2004; Croce, 2014). Moreover, as shown by Carroll et al. (2000), in models with consumption habits the intertemporal elasiticity of substitution is increasing in the horizon considered. Since our focus is on medium to long-run dynamics, this observation lends support to our focus on an intertemporal elasticity of substitution above 1. Let us also note that a high aggregate elasticity of intertemporal substitution could be perfectly reconciled with a low elasticity at the individual level, for instance, by assuming an overlapping generations framework.

where $Z \equiv \left(\alpha^{2\alpha/(1-\alpha)}(1-\alpha^2)\right)^{-1}$ is a normalizing constant, $0 < \alpha < 1$ determines the share of intermediate inputs in gross output, and $A_{j,t}$ is the productivity, or quality, of input j.[7]

Profit maximization implies the demand functions

$$(1-\alpha)Z^{1-\alpha}L_t^{-\alpha}\int_0^1 A_{j,t}^{1-\alpha}x_{j,t}^{\alpha}dj = W_t \tag{5}$$

$$\alpha\,(ZL_t)^{1-\alpha}\,A_{j,t}^{1-\alpha}x_{j,t}^{\alpha-1} = p_{j,t}, \tag{6}$$

where $p_{j,t}$ is the price of intermediate input j. Due to perfect competition, firms in the final good sector do not make any profit in equilibrium.

2.3 Intermediate goods production and profits

Every intermediate good is produced by a single monopolist. One unit of final output is needed to manufacture one unit of the intermediate good, regardless of quality. In order to maximize profits, each monopolist sets the price of its good according to

$$p_{j,t} = \frac{1}{\alpha}. \tag{7}$$

In words, each monopolist charges a constant markup $1/\alpha > 1$ over its marginal cost. Equations (6) and (7) imply that the quantity produced of a generic intermediate good j is

$$x_{j,t} = \alpha^{\frac{2}{1-\alpha}}ZA_{j,t}L_t. \tag{8}$$

Combining equations (4) and (8) gives

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}}ZA_tL_t, \tag{9}$$

where $A_t \equiv \int_0^1 A_{j,t}dj$ is an index of average productivity of the intermediate inputs. Hence, production of the final good is increasing in the average productivity of intermediate goods, in the exogenous component of labor productivity, and in the amount of labor employed in production. From now on, to streamline notation, we will impose the labor market clearing condition $L_t = 1$.

The profits earned by the monopolist in sector j are given by

$$(p_{j,t}-1)x_{j,t} = \varpi A_{j,t},$$

where $\varpi \equiv (\alpha-\alpha^2)/(1-\alpha^2)$. According to this expression, the producer of an intermediate input of higher quality earns higher profits. This is the reason why firms will want to invest to increase their productivity.

[7] More precisely, for every good j, $A_{j,t}$ represents the highest quality available. In principle, firms could produce using a lower quality of good j. However, the structure of the economy is such that in equilibrium only the highest quality version of each good is used in production.

We introduce fiscal distortions by assuming that the government taxes profits at rate τ_t^p. As we will see, this tax will influence firms' investment decisions, and so the path of productivity growth.

2.4 Investment and productivity growth

Firms operating in the intermediate sector can invest in innovation in order to improve the quality of their products. In particular, a firm that invests $I_{j,t}$ units of the final good sees its productivity evolve according to

$$A_{j,t+1} = A_{j,t} + \chi I_{j,t}, \tag{10}$$

where $\chi > 0$ determines the productivity of investment.

Innovation-based endogenous growth models typically assume that knowledge is only partly excludable. For instance, this happens if inventors cannot prevent others from drawing on their ideas to innovate. For this reason, in most endogenous growth frameworks, the social return from investing in innovation is higher than the private one.[8] A simple way to introduce this effect in the model is to assume that every period, after production takes place, there is a constant probability $1 - \eta$ that the incumbent firm dies, and is replaced by another firm that inherits its technology. This assumption encapsulates all the factors that might lead to the termination of the rents from innovation, including patent expiration and imitation by competitors.

To simplify the exposition, from now on we will assume that the rents from innovation last for only one period, so that $\eta = 1$. Firms producing intermediate goods then choose investment in innovation to maximize after-tax profits net of investment costs

$$\beta \left(\frac{C_t}{C_{t+1}} \right)^{\gamma} (1 - \tau_{t+1}^p) \varpi A_{j,t+1} - I_{j,t}, \tag{11}$$

subject to (10) and a non-negativity constraint on investment ($I_{j,t} \geq 0$). Since firms are owned by the households, they discount profits using the households' discount factor $\beta(C_t/C_{t+1})^{\gamma}$.

Optimal investment in innovation requires

$$\frac{1}{\chi} \geq \beta \left(\frac{C_t}{C_{t+1}} \right)^{\gamma} (1 - \tau_{t+1}^p) \varpi, \tag{12}$$

holding with equality if investment is positive. Intuitively, firms equalize the marginal cost from performing research $1/\chi$, to its marginal benefit discounted using the households' discount factor. The marginal benefit is given by the marginal increase in next period profits ($(1 - \tau_{t+1}^p)\varpi$). If the marginal cost of investment exceeds the marginal benefit, then the firms set investment equal to zero.

Notice that firms' incentives to invest are decreasing in the profit tax τ_{t+1}^p. As it is natural, the fact that the government appropriates part of the returns from investment discourages innovation activities by the private sector. Moreover, since firms are forward looking, it is the expectation of high future taxes that depresses investment in the present. This is the channel through which

[8] See for instance Romer (1990) and Aghion and Howitt (1992).

fiscal policy will distort economic activity and productivity growth in our framework.

2.5 Aggregation and market clearing

Using equations (8) and (9) we can write GDP as

$$Y_t - \int_0^1 x_{j,t} dj = A_t, \tag{13}$$

where $A_t \equiv \int_0^1 A_{j,t} dj$. Market clearing for the final good then implies

$$A_t = C_t + I_t, \tag{14}$$

where $I_t \equiv \int_0^1 I_{j,t} dj$. This expression captures the fact that all the GDP has to be consumed or invested.

Long run growth in this economy takes place through increases in the quality of the intermediate goods, captured by increases in the productivity index A_t. To simplify notation, we will focus on a symmetric equilibrium in which all the firms are identical. Defining $g_t \equiv A_t/A_{t-1}$, we can then write equation (10) as

$$g_{t+1} = 1 + \chi \frac{I_t}{A_t}. \tag{15}$$

This expression implies that higher investment in research in period t is associated with faster productivity growth between periods t and $t+1$. More precisely, the rate of productivity growth is determined by the ratio of investment in innovation I_t over the existing stock of knowledge A_t. In turn, the stock of knowledge depends on all past investment in innovation, that is on the R&D stock. Hence, there is a positive link between R&D intensity, captured by the ratio I_t/A_t, and future productivity growth.

2.6 Fiscal policy

The government issues default-free one-period bonds.[9] The government budget constraint is given by

$$S_t + \frac{D_{t+1}}{R_t} = D_t, \tag{16}$$

where S_t denotes the primary surplus, and D_t is the stock of public debt maturing at time t. In equilibrium, all the public debt has to be held by households so that $D_t = B_t$. The primary surplus is

$$S_t = T_t + \tau_t^p \varpi A_t. \tag{17}$$

[9] It is well known that the possibility of default can give rise to equilibrium multiplicity (Calvo, 1988; Cole and Kehoe, 2000; Lorenzoni and Werning, 2019). We thus abstract from default to focus on the novel source of multiplicity highlighted by our model. To do so, we assume that default is infinitely costly, that fundamentals are such that at least one equilibrium without default exists, and that households' expectations never coordinate on the self-fullfilling default equilibria described by Calvo (1988), Cole and Kehoe (2000) and Lorenzoni and Werning (2019).

The term T_t encapsulates the part of the primary surplus financed with non-distortionary taxes, while $\tau_t^p \varpi A_t$ denotes the fiscal revenue raised through taxing profits. To make things interesting, as well as realistic, we will assume that the government has a limited ability to finance itself through non-distortionary taxes. We capture this notion by imposing the upper bound

$$T_t \leq \bar{s} A_t, \tag{18}$$

so that the government can extract from the private sector at most a fraction $\bar{s}$ of GDP using non-distortionary taxes. We will also assume that the government minimizes the use of distortionary taxes on firms' profits, so that

$$\tau_t^p = \max\left(0, \frac{S_t - \bar{s} A_t}{\varpi A_t}\right). \tag{19}$$

In words, the government uses distortionary taxes only if the primary surplus-to-GDP ratio exceeds the threshold $\bar{s}$.[10]

Before moving on, two remarks are in order. First, in our model the relationship between the primary surplus and fiscal distortions (and growth) is non-linear. More precisely, we want to capture a scenario in which governments can sustain normal levels of primary surpluses without generating significant fiscal distortions. It is only when the primary surpluses needed to balance the budget get exceptionally high that the government is forced to resort to distortionary taxes.[11] The parameter $\bar{s}$, which determines the extent to which the government can sustain a primary surplus without generating fiscal distortions, could be interpreted as a measure of institutional quality or fiscal capacity.

Second, to maintain the model tractable we introduce fiscal distortions through taxes on profits. But other sources of fiscal distortions would play a similar role. For instance, one could assume that the return to private investment depends on productive public expenditure (Barro, 1990). A high primary surplus could then lower the profitability of investment, by forcing the government to cut productive public expenditure.

2.7 Equilibrium

To describe the equilibrium, it is useful to express some variables in terms of their ratio with respect to GDP. So, for a generic variable X_t, we will define $x_t \equiv X_t/A_t$.

The equilibrium of the economy can be described by three expressions. The first one is the *growth* equation, which captures how fiscal policy affects firms' investment and productivity growth.

[10] In principle, we could allow the government to subsidize investment, that is to set $\tau^p < 0$. Allowing for investment subsidies, however, would complicate the exposition, without affecting the key insights of the paper.

[11] The model proposed by Jaimovich and Rebelo (2017) shares a similar feature. They argue that the effects of taxation on growth are highly nonlinear: low tax rates have a very small impact on growth, but as tax rates rise, their negative impact on growth rises dramatically. A similar non-linearity arises between the primary surplus and growth in our model.

It is obtained by combining expressions (12) and (19) to get

$$g_{t+1} = \begin{cases} \frac{c_t}{c_{t+1}} \left(\beta\chi\varpi\right)^{1/\gamma} \equiv \bar{g}_{t+1} & \text{if } s_{t+1} \leq \bar{s} \\ \bar{g}_{t+1} \left(1 - \frac{s_{t+1}-\bar{s}}{\varpi}\right)^{1/\gamma} & \text{if } \bar{s} < s_{t+1} \leq \bar{s} + \varpi \left(1 - \bar{g}_{t+1}^{-\gamma}\right) \\ 1 & \text{if } \bar{s} + \varpi \left(1 - \bar{g}_{t+1}^{-\gamma}\right) < s_{t+1}. \end{cases} \tag{20}$$

While it may look complicated, the intuition behind this expression is simple. If the primary surplus is smaller than $\bar{s}$, the government does not need to resort to distortionary taxes and growth is fully determined by non-fiscal variables. In this case, growth is equal to $\bar{g}_{t+1}$, which we assume to be positive. Once the primary surplus exceeds $\bar{s}$, distortionary taxes kick in, which explains why productivity growth becomes decreasing in the primary surplus. Finally, if the primary surplus is higher than $\bar{s} + \varpi \left(1 - \bar{g}_{t+1}^{-\gamma}\right)$, distortionary taxes are so high that firms stop investing and productivity growth drops to zero.

The second one is the *fiscal* equation. It is obtained by combining (3) and (16)

$$s_t = d_t - d_{t+1}\beta \left(\frac{c_t}{c_{t+1}}\right)^{\gamma} g_{t+1}^{1-\gamma}. \tag{21}$$

This equation shows how productivity growth affects the primary surplus needed to balance the public budget. There are two effects at play. On the one hand, a higher rate of productivity growth reduces the primary surplus needed to achieve a given future debt-to-GDP ratio (d_{t+1}). Intuitively, higher productivity growth helps stabilize the debt-to-GDP ratio by increasing future output. On the other hand, faster productivity growth pushes up the equilibrium interest rate, increasing the primary surplus needed to achieve a given d_{t+1}.[12] Under our assumption $\gamma < 1$, this second effect is always weaker than the first one, explaining why equation (21) implies a negative link between productivity growth and the primary surplus.

The third equation combines the goods market clearing condition (14) and the fact that $I_t/A_t = (g_{t+1} - 1)/\chi$ to obtain

$$1 = c_t + \frac{g_{t+1} - 1}{\chi}. \tag{22}$$

This equation implies a negative relationship between the consumption-to-GDP ratio and productivity growth, because to generate faster growth the economy has to devote a larger fraction of output to investment, reducing the resources available for consumption.

We are now ready to define an equilibrium as a path for $\{g_{t+1}, c_t, d_{t+1}\}_{t=0}^{\infty}$ satisfying equations (20), (21) and (22) for given fiscal policy $\{s_t\}_{t=0}^{\infty}$ and an initial stock of public debt d_0.

[12] Intuitively, forward-looking consumers want to save less in a fast-growing economy because they anticipate that they will be richer in the future compared to the present.

3 Falling into fiscal stagnation

We now derive the key result of the paper, by showing how our economy can fall into fiscal stagnation, that is a persistent state characterized by low growth and high fiscal distortions. To do so, in this section we will frame fiscal policy in terms of some simple rules. We will derive the optimal fiscal policy later on, in Section 4.

3.1 A constant debt policy

Consider a simple fiscal policy that maintains a constant debt-to-GDP ratio equal to d. It is useful to start the analysis by studying non-stochastic steady states, that is equilibria characterized by constant values for productivity growth g, normalized consumption c, primary surpluses s, and public debt d satisfying

$$g = \begin{cases} \bar{g} \equiv (\beta\chi\varpi)^{1/\gamma} & \text{if } s \leq \bar{s} \\ \bar{g}\left(1 - \frac{s-\bar{s}}{\varpi}\right)^{1/\gamma} & \text{if } \bar{s} < s \leq \bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\right) \\ 1 & \text{if } \bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\right) < s \end{cases} \tag{GG}$$

$$s = d\left(1 - \beta g^{1-\gamma}\right) \tag{FF}$$

$$1 = c + \frac{g-1}{\chi}. \tag{MK}$$

We now show that two steady state equilibria can coexist: a fiscally sound steady state, and one characterized by fiscal stagnation.

The fiscally sound steady state. We define a steady state as fiscally sound if $s \leq \bar{s}$, so that primary surpluses are fully financed with non-distortionary taxes. In this case, productivity growth is equal to $\bar{g}$. We will assume that $1 < \bar{g} < \beta^{\frac{1}{\gamma-1}}$. In words, we assume that growth in a fiscally sound steady state is positive, but low enough so that households' utility is finite. This condition also implies that the interest rate is higher than the growth rate of the economy.

The fiscal equation (FF) then implies that a fiscally sound steady state exists if[13]

$$d\left(1 - \beta\bar{g}^{1-\gamma}\right) \leq \bar{s}. \tag{23}$$

As it is intuitive, a fiscally sound steady state exists if public debt is not too high, compared to the government's ability to fund itself through non-distortionary taxes. Moreover, strong growth fundamentals, i.e. a high $\bar{g}$, make it more likely that a fiscally sound steady state exists. The reason is that robust productivity growth reduces the primary surplus needed to balance the public budget.

We summarize our results about the fiscally sound steady state in a proposition.

[13]For completeness, a fiscally sound steady state exists only if it is associated with positive consumption, that is if $\bar{g} < 1 + \chi$, which we assume to always hold.

Proposition 1 *Suppose that* $1 < \bar{g} < \beta^{\frac{1}{\gamma-1}}$ *and that condition (23) holds. Then, there exists a unique fiscally sound steady state with* $s < \bar{s}$*. The fiscally sound steady state is characterized by positive growth* $g = \bar{g} > 1$*.*

The fiscally sound steady state can be thought as the normal state of affairs of the economy, in which the distortions related to fiscal policy do not interfere significantly with the growth process. In what follows, we will assume that conditions are such that a fiscally sound steady state exists.

The fiscal stagnation steady state. Even if a fiscally sound steady state exists, the economy may not necessarily end up there. It may rather fall into a state of stagnation, characterized by low growth and high fiscal distortions.

To see this point, suppose that distortionary taxes are so high that firms have no incentives to invest, so that productivity growth drops all the way to zero ($g = 1$). The growth equation (GG) implies that this happens if $s > \bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\right)$. The fiscal equation (FF) then indicates that a fiscal stagnation steady state exists if[14]

$$d(1-\beta) > \bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\right), \tag{24}$$

and so if public debt is sufficiently high. Naturally, having a high debt-to-GDP ratio makes it more likely that the government will have to resort to distortionary taxes to balance the budget. The following proposition summarizes our results about the fiscal stagnation steady state.

Proposition 2 *Suppose that condition (24) holds. Then a fiscal stagnation steady state exists, and it is characterized by zero growth* $g = 1$ *and high fiscal distortions* $s > \bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\right)$*.*

When the economy is stuck in the fiscal stagnation steady state high fiscal distortions and low growth self-perpetuate themselves. Notice that for values of debt satisfying

$$\frac{\bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\right)}{1-\beta} < d < \frac{\bar{s}}{1 - \beta\bar{g}^{1-\gamma}}, \tag{25}$$

the fiscally sound and fiscal stagnation steady states coexist. In this case, the same public debt-to-GDP ratio can be sustained through a combination of high growth and low distortionary taxes (in the fiscally sound steady state), or low growth and high distortionary taxes (in the fiscal stagnation steady state).[15]

At the heart of this result there is an amplification effect between fiscal policy and productivity growth. When productivity growth slows down, the government needs to increase its primary surplus to prevent the debt-to-GDP ratio from increasing. In turn, a higher primary surplus is associated with larger fiscal distortions, which end up depressing investment and growth. When

[14] To be precise, another requirement for this steady state to exist is that the profits tax does not exceed 100%. This is the case if $d(1-\beta) < \bar{s} + \varpi$, which we assume to hold.

[15] To simplify the analysis, we are focusing on fiscal stagnation steady states with zero growth. However, depending on parameter values, our model may generate fiscal stagnation steady states in which growth is positive, albeit below $\bar{g}$.

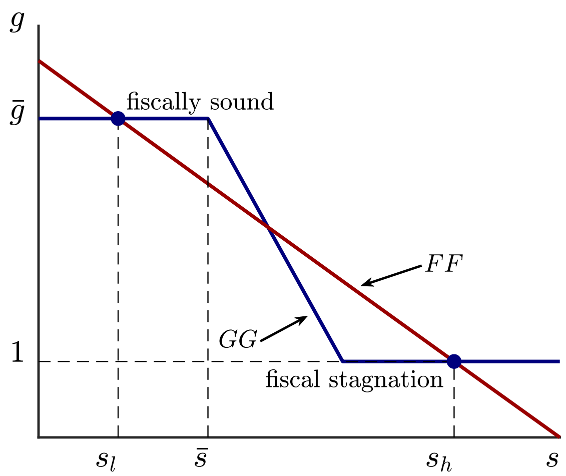

Figure 2: Steady states with constant public debt.

this amplification effect is strong enough, the two steady states coexist. The debt-to-GDP ratio is an important determinant of the strength of this amplification effect. In fact, only when public debt is high enough changes in productivity growth have a substantial impact on the government budget, which explains why the fiscal stagnation steady state arises only if public debt is sufficiently high relative to GDP.[16]

Figure 2 illustrates graphically these results. The GG schedule shows the negative relationship between productivity growth and the primary surplus implied by the growth equation (GG). In particular, it shows how investment and growth are particularly sensitive to fiscal policy for intermediate values of the primary surplus ($\bar{s} < s < \bar{s} + \varpi (1 - \bar{g}^{-\gamma})$). The FF schedule, which follows from the fiscal equation (FF), captures the fact that lower growth requires a higher primary surplus to maintain the debt-to-GDP ratio constant. When conditions (23) and (24) are satisfied, which is the case depicted in Figure 2, two stable steady states are possible.[17] The fiscally sound steady state, with its combination of high growth ($g = \bar{g}$) and low primary surplus ($s = s_l < \bar{s}$), and the fiscal stagnation one, characterized by low growth ($g = 1$) and high primary surpluses ($s = s_h > \bar{s} + \varpi (1 - \bar{g}^{-\gamma})$).

We are left with determining what makes the economy settle in one of the two steady states. Under a constant debt policy, this role is fulfilled by expectations. Suppose that agents expect that the economy will settle on the fiscally sound steady state. Then, the prospect of low fiscal distortions fosters private investment and growth. In turn, high growth helps the government

[16] Condition (25) is more likely to hold when ϖ (the profit share of GDP) is low, when $\bar{g}$ (potential growth) is large, and when γ (the sensitivity of the interest rate with respect to changes in growth) is low. Intuitively, a low ϖ means a lower tax base, so distortionary taxes are more sensitive to changes in the primary surplus. This implies that distortions rise more quickly when the government stabilizes debt-to-GDP in face of low growth. A high $\bar{g}$ implies that debt-to-GDP destabilizes by more once growth falls to zero. Finally, a low γ that a decline in growth has a larger negative impact on the government's budget, because interest rates fall by less in response to the growth decline.

[17] The middle intersection corresponds to an unstable steady state. This steady state is unstable because it is characterized by a high sensitivity of investment and growth with respect to fiscal policy. Around this point, in fact, a drop in the primary surplus generates an increase in growth large enough so that the debt-to-GDP ratio falls.

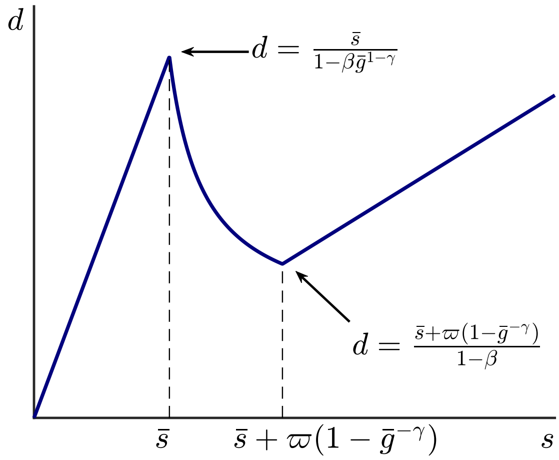

Figure 3: The Laffer curve in steady state.

to service its debt without recurring to distortionary taxation. Conversely, suppose that agents expect that the economy will get stuck into fiscal stagnation. In this case, the prospect of high fiscal distortions depresses investment and growth. Facing low growth, the government will have no choice but to impose distortionary taxes in order to service its public debt. Hence, expectations can be self-fulfilling and animal spirits can determine real outcomes.

This does not mean, however, that fundamentals are not important. As we discussed above, a fiscal stagnation steady state is possible only if the fundamentals of the economy are weak. For instance, a low productivity of investment (low χ), a weak fiscal capacity (low $\bar{s}$), or a high equilibrium interest rate (low β) make it more likely that a fiscal stagnation steady state exists. Under a constant debt policy, therefore, the economy falls into fiscal stagnation due to a combination of weak fundamentals and pessimistic animal spirits.

A Laffer curve interpretation. To provide further intuition, Figure 3 shows an example of a Laffer curve - tracing the steady state debt-to-GDP ratio d as a function of the primary surplus-to-GDP ratio s - generated by our model. The Laffer curve may be non-monotonic because of the growth effect of fiscal policy, i.e. the fact that taxing profits at a higher rate lowers investment, productivity growth, and so the future tax base. This effect operates for values of $\bar{s} < s < \bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\right)$, explaining why in this region increasing the primary surplus-to-GDP ratio may lead to a reduction in the debt-to-GDP ratio that the government can sustain.[18] In this case, which is the one shown in Figure 3, there is a range of values for d for which multiple equilibria are possible.

We conclude this section with a remark. A recurrent theme in the fiscal policy debate is that austerity may be self-defeating, meaning that a rise in the primary surplus can depress growth so

[18]The growth effect does not operate when $s < \bar{s}$, because in this case the government fully finances itself with non-distortionary taxes. It does not operate either when $s > \bar{s} + \varpi(1 - \bar{g}^{-\gamma})$, because there taxes on profits are already so high that firms stopped investing. This explains why in these two regions the relationship between s and d is positive.

much that the debt-to-GDP ratio ends up not declining, or may even rise (Fatás and Summers, 2018; Angeletos et al., 2024). This is exactly what happens when the economy moves from the fiscally sound to the fiscal stagnation steady state. The primary surplus rises relative to GDP, but the debt-to-GDP ratio ends up being the same. Conversely, one can think of a shift from the fiscal stagnation to the fiscally sound steady state as a case in which deficits are self-financing, since the growth boost coming from lower taxes allows the government to maintain the same debt-to-GDP, in spite of a lower primary surplus as a share of GDP. We will go back to these points in Section 4, where we will discuss the strategies to exit fiscal stagnation.

3.2 A gradual fiscal adjustment policy

We now turn to a more realistic configuration of fiscal policy, in which the government adjusts gradually its fiscal stance to stabilize the public debt-to-GDP ratio (Bohn, 1998). We will see that under this policy the economy may exhibit hysteresis effects, that is initial conditions may determine long-run outcomes. Moreover, the fall into a state of fiscal stagnation is now a gradual phenomenon, which takes time to unfold.

To keep things simple, consider the linear fiscal rule

$$s_t = -\delta + \phi d_t, \tag{26}$$

where $\delta > 0$ and $\phi > 0$. Under this policy, the primary surplus increases with the inherited stock of public debt relative to GDP. A higher ϕ corresponds to stronger efforts to contain the growth of public debt by adjusting the primary surplus.

Steady states. Also in this case, a fiscally sound and a fiscal stagnation steady state may coexist.[19] But now, due to the positive link between the primary surplus and debt implied by the rule (26), the fiscal stagnation steady state features a higher debt-to-GDP ratio. More precisely, in a fiscally sound steady state the debt-to-GDP ratio is equal to

$$\frac{\delta}{\bar{g}^{1-\gamma}\beta + \phi - 1}, \tag{27}$$

while in a fiscal stagnation steady state with no growth the debt-to-GDP ratio is given by

$$\frac{\delta}{\beta + \phi - 1}. \tag{28}$$

Following the same steps described in the previous section, one can show that both steady states coexist if

$$(1 - \bar{g}^{1-\gamma}\beta)\left(1 + \frac{\delta}{\bar{s}}\right) \leq \phi \leq (1 - \beta)\left(1 + \frac{\delta}{\bar{s} + \varpi(1 - \bar{g}^{-\gamma})}\right). \tag{29}$$

To understand the logic behind this condition, consider that a higher ϕ is associated with a lower debt-to-GDP ratio in steady state. So if ϕ is too small the fiscally sound steady state does not

[19] See Appendix A.3 for the detailed mathematical derivations backing the results presented in this Section.

exist, because steady state debt is too high to be financed exclusively with non-distortionary taxes. Instead, if ϕ is too large the fiscal stagnation steady state does not exist, because steady state debt is so small that distortionary taxes are not needed to balance the public budget. This is the same intuition that we derived under the constant debt policy of the previous section. To keep things interesting, from now on we will assume that condition (29) holds. In addition, we will also assume that $\phi > 1 - \beta$, to ensure that the two steady states are locally stable.

Transitional dynamics. Compared to a constant debt policy, the fiscal rule (26) gives rise to more complex transitional dynamics. To derive analytic results, we will study debt dynamics under the assumption that the ratio of investment in innovation-to-GDP is close to zero, so that $c_t \approx 1$. This approximation is useful, but the insights that we will derive are by no means specific to it.

The law of motion for debt is then given by

$$d_{t+1} g_{t+1}^{1-\gamma} \beta = \delta + (1 - \phi) d_t \tag{30}$$

$$g_{t+1} = \begin{cases} \bar{g} & \text{if } d_{t+1} \leq \frac{\delta + \bar{s}}{\phi} \\ \bar{g} \left(1 - \frac{\phi d_{t+1} - \delta - \bar{s}}{\varpi}\right)^{1/\gamma} & \text{if } \frac{\delta + \bar{s}}{\phi} < d_{t+1} \leq \frac{\delta + \bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\right)}{\phi} \\ 1 & \text{if } \frac{\delta + \bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\right)}{\phi} < d_{t+1}. \end{cases} \tag{31}$$

Equation (30) describes how debt evolves for a given productivity growth rate, and it is obtained by combining (21) with (26). Expression (31) pins down productivity growth as a function of debt, and results from combining (20) and (26). Figure 4, to which we will come back shortly, shows graphically the law of motion for debt when condition (29) holds, so that two stable steady states are possible.

It turns out that debt dynamics are shaped by two effects, as can be seen by differentiating equation (30) to obtain

$$\frac{\partial d_{t+1}}{\partial d_t} = \underbrace{\frac{1 - \phi}{g_{t+1}^{1-\gamma} \beta}}_{\text{direct effect}} \left(1 + \underbrace{(1 - \gamma) \frac{d_{t+1}}{g_{t+1}} \frac{\partial g_{t+1}}{\partial d_{t+1}}}_{\text{growth effect}}\right)^{-1}. \tag{32}$$

Let us start by describing the direct effect, which has a straightforward interpretation. Holding growth constant, a marginal increase in d_t leads to a rise in d_{t+1} equal to $(1 - \phi)/(g_{t+1}^{1-\gamma} \beta)$. The reason is that a higher stock of inherited debt increases the fiscal revenue needed to balance the public budget. Under the fiscal rule (26), a fraction ϕ of this fiscal revenue is covered through a rise in the primary surplus, while the rest is financed by issuing more debt. This logic explains why a higher ϕ reduces the strength of this effect.

Importantly, the direct effect is the only force shaping debt dynamics around the fiscally sound and the fiscal stagnation steady states, because around these steady states marginal changes in debt do not affect productivity growth ($\partial g_{t+1}/\partial d_{t+1} = 0$). This observation clarifies why the assumption

$\phi > 1 - \beta$ guarantees the stability of these two steady states by implying that $\partial d_{t+1}/\partial d_t < 1$ in their neighborhood. Graphically, this condition ensures that the law of motion for debt is flatter than the 45 degrees line for $d_{t+1} \leq (\delta + \bar{s})/\phi$ and $d_{t+1} \geq \left(\delta + \bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\right)\right)/\phi$, that is when growth is not affected by marginal changes in debt.

For intermediate values of d_{t+1}, however, marginal increases in debt lower productivity growth ($\partial g_{t+1}/\partial d_{t+1} < 0$), because they are associated with higher distortionary taxes and lower investment. This is where the growth effect kicks in, acting as an amplification mechanism that generates macroeconomic instability. Intuitively, lower growth increases the future debt-to-GDP ratio, forcing the government to increase future distortionary taxes, which further lowers investment and growth. The strength of this amplification mechanism is decreasing in γ, because the parameter γ determines the sensitivity of the public budget to changes in growth. Graphically, the growth effect implies that law of motion for debt is steeper than the 45 degrees line for intermediate values of d_{t+1}, and more so the lower γ is.

To make progress, let us now consider a case in which the growth effect is not too strong. More precisely, suppose that[20]

$$\gamma > \frac{\delta + \bar{s} + \varpi(1 - \bar{g}^{-\gamma})}{\delta + \bar{s} + \varpi}. \tag{33}$$

When this condition holds, d_{t+1} increases monotonically with d_t.[21] This case is represented graphically by the left panel of Figure 4. Condition (29) ensures that the law of motion for debt intersects three times with the 45 degrees line, meaning that there are three possible steady states. The two extreme ones are the fiscally sound and fiscal stagnation steady states. The middle one is an unstable steady state, because it lies in the region in which the growth effect operates. Let us denote by d^* the level of debt in this unstable steady state.[22]

The interesting result here is that long run outcomes are determined by the initial level of debt, so the interplay between fiscal policy and productivity growth may amplify substantially small differences in initial conditions (or small shocks). The reason, of course, is the amplification

[20]Besides γ, the other parameters determine this condition in an intuitive way. For instance, a higher ϖ means a higher tax base, and so a lower sensitivity of the tax rate on profits with respect to the primary surplus. This explains why a higher ϖ reduces the amplification mechanism linked to the growth effect of fiscal policy. Instead, a higher δ or a higher $\bar{s}$ weaken the strength of the growth effect, because both are associated with a lower need to raise fiscal resources through distortionary taxes.

[21]As we said before, this is always true for $d_{t+1} \leq \frac{\delta+\bar{s}}{\phi}$ and $d_{t+1} \geq \frac{\delta+\bar{s}+\varpi\left(1-\bar{g}^{-\gamma}\right)}{\phi}$. For intermediate values of d_{t+1}, we have that

$$\frac{\partial d_{t+1}}{\partial d_t} = \left(\frac{\varpi + \bar{s} + \delta - \phi d_{t+1}}{\varpi + \bar{s} + \delta - \phi\frac{d_{t+1}}{\gamma}}\right)\frac{(1-\phi)}{g_{t+1}^{1-\gamma}\beta}.$$

The right hand side of this expression is positive if

$$\phi d_{t+1} < \gamma\left(\varpi + \bar{s} + \delta\right).$$

Inserting the highest possible d_{t+1}, given by $d_{t+1} = \frac{\delta+\bar{s}+\varpi\left(1-\bar{g}^{-\gamma}\right)}{\phi}$, and rearranging, yields condition (33).

[22]To be precise, d^* is the solution to

$$d^*\left(\bar{g}^{1-\gamma}\left(1 - \frac{\phi d^* - \delta - \bar{s}}{\varpi}\right)^{\frac{1-\gamma}{\gamma}}\beta + \phi - 1\right) = \delta.$$

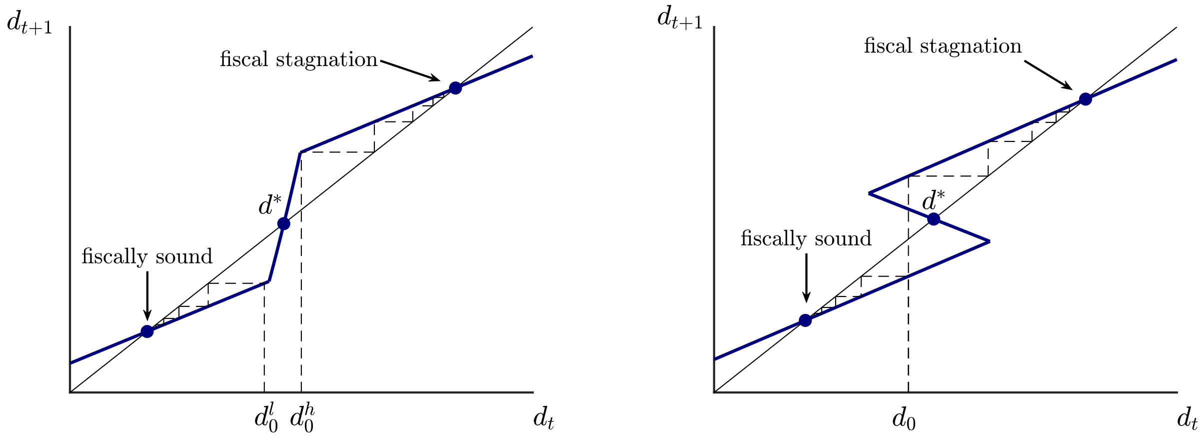

Figure 4: Dynamics with linear fiscal rule. Left panel: unique equilibrium. Right panel: multiple equilibria.

force exerted by the growth effect. To see this point most clearly, imagine that the economy starts with an initial level of debt slightly higher than d^*, say point d_0^h in Figure 4. To satisfy its budget, the government will be forced to increase public borrowing. But the associated rise in future taxes depresses investment and growth, boosting the debt-to-GDP ratio. Eventually, public debt gets so high relative to GDP that distortionary taxes drive investment and growth to zero, and the economy converges to the fiscal stagnation steady state. If instead initial debt is slightly lower than d^*, for instance point d_0^l in Figure 4, a virtuous circle of lower debt and higher growth leads the economy to the fiscally sound steady state.

What if condition (33) is violated? In this case, for some value of d_{t+1} the law of motion for debt becomes backward bending. Intuitively, the growth effect is so strong that higher legacy debt can be sustained only by reducing the future debt-to-GDP ratio. The reason is that the associated increase in investment and growth is sufficiently strong to cover all the budget needs associated with higher legacy debt. When this occurs, multiple equilibria are possible, and economic outcomes are determined by animal spirits.

The right panel of Figure 4 provides an example of an economy with multiple equilibria, in which for intermediate values of initial debt the economy can converge either to the fiscally sound or to the fiscal stagnation steady state.[23] The logic is similar to the one outlined in Section 3.1. If agents are pessimistic, they anticipate that the economy will enter a trajectory characterized by growing debt-to-GDP and rising distortionary taxes. These pessimistic expectations induce a cut in investment and growth, indeed forcing the government to resort to distortionary taxes to balance the budget. As a result, the economy will move to the fiscal stagnation steady state, validating agents' initial expectations. The opposite occurs if agents are optimistic, in which case

[23]To be precise, when condition (33) is violated, the law of motion for debt is backward bending close to the point $d_{t+1} = \frac{\delta+\bar{s}+\varpi(1-\bar{g}^{-\gamma})}{\phi}$. This is sufficient for equilibrium multiplicity, but not for expectations to move the economy between two steady states. Expectations can move the economy between the two steady states when the kink associated with the point $d_{t+1} = \frac{\delta+\bar{s}+\varpi(1-\bar{g}^{-\gamma})}{\phi}$ lies to the left of the point d^*, which is the case illustrated by Figure 4.

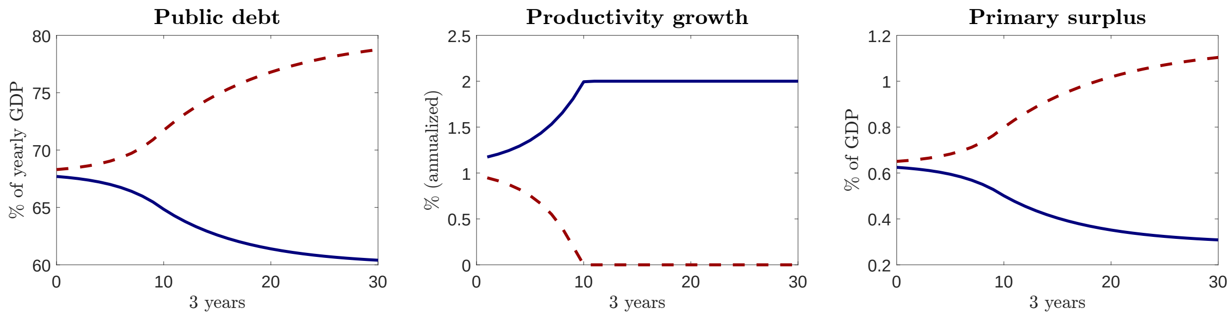

Figure 5: Fiscal hysteresis.

the economy will converge to the fiscally sound steady state. This type of equilibrium multiplicity is possible if the economy starts with intermediate values of debt.

Proposition 3 *Assume that conditions (29) and $\phi > 1 - \beta$ both hold, so that the fiscally sound and stagnation steady states coexist and are locally stable. Let also $c_t \approx 1$. Assume that γ is large enough such that condition (33) holds. Then, for $d_0 < d^*$, where d^* is a tipping point, the economy transits to the fiscally sound steady state, while for $d_0 > d^*$, it transits to the fiscal stagnation steady state. When γ is sufficiently small, in contrast, an interval $[d', d'']$, $d' < d^* < d''$, exists such that, for $d_0 \in [d', d'']$, animal spirits dictate if the economy converges to the fiscally sound or the fiscal stagnation steady state.*

A numerical example. We conclude this Section with a numerical example.[24] Figure 5 compares the evolution of two identical economies, except for a small difference in their initial public debt-to-GDP ratios. The economy depicted by the solid blue lines starts with an initial debt slightly below the threshold d^*. This economy converges to the fiscally sound steady state, by embarking in a virtuous cycle of reductions in distortionary taxes and increases in investment and growth, leading to a gradual drop in the debt-to-GDP ratio. The economy captured by the dashed red lines, instead, begins with a level of debt slightly above d^*. This economy converges to the fiscal stagnation steady state, through a vicious cycle of increases in distortionary taxation and drops in investment and growth. This numerical example illustrates well how the interaction between fiscal policy and productivity growth can greatly amplify the impact of small differences in initial conditions on long-run outcomes.

[24]While our model is too simple to perform a careful quantitative analysis, we try to pick reasonable values for the parameters. We set the length of a period to 3 years, to capture the fact that it takes time for investments in innovation to affect productivity. We set $\varpi = .1$, so that 10% of GDP goes to profits, and $\gamma = .5$ for illustrative purposes. We then set $\chi = 10.8$ and $\beta = .956$ so that in the fiscally sound steady state yearly productivity growth is equal to 2%, while the yearly interest rate is 2.5%. Turning to the fiscal variables, we set $\bar{s}$ so that the primary surplus that can be financed with lump-sum taxes is equal to 0.5% of GDP, while we set $\delta = .023$ and $\phi = .13$ so that the debt-to-GDP ratio in the fiscally sound steady state is 60%, while in the fiscal stagnation stagnation steady state it is 80%.

4 Exiting fiscal stagnation

Being in fiscal stagnation is highly detrimental for welfare, given its depressive impact on growth and consumption. To see this point, imagine that the government could fully finance itself with non-distortionary taxes. The optimal fiscal policy would then consist in setting profit taxes to zero, and the economy would immediately jump on the fiscally sound steady state with $g_{t+1} = \bar{g}$. The reason is that the classic knowledge spillovers characterizing endogenous growth frameworks depress private investment in innovation below its first best level.[25] Fiscal distortions compound these inefficiencies, moving the economy further away from the first best allocation.

But what is the best way to get out of fiscal stagnation? In our stylized model, fiscal stagnation can be escaped through a combination of austerity and pro-growth fiscal policies. The basic intuition goes back to the government budget constraint

$$d_{t+1} = \frac{d_t - s_t}{\beta g_{t+1}^{1-\gamma}},$$

where for simplicity we have used the approximation $c_t \approx 1$. By increasing its primary surplus in the present s_t, the government reduces its future debt-to-GDP ratio, thus decreasing the future surpluses required to balance the public budget. This is the austerity approach to exit fiscal stagnation. On the other hand, committing to a lower future surplus s_{t+1} fosters private investment and growth g_{t+1}, which leads to a drop in the future debt-to-GDP ratio. This is the pro-growth approach to exiting fiscal stagnation.[26]

As we will see, the optimal mix between austerity and pro-growth policies depends on the specific circumstances faced by the economy. However, one robust result stands out: governments lacking commitment rely too heavily on austerity policies, due to a time consistency problem. In a nutshell, governments have an incentive to promise future pro-growth policies to foster investment and growth. But ex-post, once investments have been made, they have an incentive to renege on these promises.

4.1 Optimal fiscal policy under discretion

We start the analysis by deriving the optimal policy under discretion. We thus consider a benevolent government, that sets fiscal policy to maximize households' welfare, but that lacks the ability to commit to its future actions.

We simplify the optimal policy problem by assuming that in period $t = 1$ the economy reaches a steady state. In this steady state, the government sets the surplus to maintain the debt-to-GDP ratio constant. The model then effectively collapses to a two-period economy, with period 0

[25] Indeed, if we allowed for investment subsidies, the government would use them to raise private investment to its first best level.

[26] In reality, governments may consider other policy tools to exit fiscal stagnation. For instance, renegotiating the repayment of legacy debt may be an interesting option (Krugman, 1988). We rule out this possibility by assuming that default is infinitely costly, but it would be interesting to relax this assumption in future work.

capturing the transition toward the steady state.[27] Starting from this simple case is useful, because it makes the analysis transparent. We discuss the full infinite horizon model later on.

We consider the following timing of actions within each period. First, the private sector forms its expectations about fiscal policy and invests. Second, taking investment decisions as given, the government sets fiscal policy, i.e. the primary surplus. Of course, in equilibrium the expectations of the private sector have to be consistent with the government's behavior.

Let us start by describing the steady state reached in period $t = 1$. As we showed in Section 3.1, the shape of the equilibrium depends on the inherited stock of debt relative to GDP, which we denote by d. In particular, if $d > \bar{s}/(1 - \beta\bar{g}^{1-\gamma})$ the only steady state consistent with rational expectations is the fiscal stagnation one. Conversely, if $d < (\bar{s} + \varpi(1 - \bar{g}^{-\gamma}))/(1 - \beta)$ the economy lands on the fiscally sound steady state. For intermediate values of d, both steady states are possible.

To put some structure on households' belief formation, we assume that if $d \le \bar{d}$ households expect future fiscal distortions to be low. Because this leads to high growth, the economy ends up in the fiscally sound steady state. Conversely, when $d > \bar{d}$, households expect large fiscal distortions, depressing growth and putting the economy on the fiscal stagnation steady state. The parameter $\bar{d}$ can be thus taken as a measure of households' animal spirits. A lower $\bar{d}$ is associated with more pessimistic households, since it implies that a lower debt-to-GDP ratio is needed to reach the fiscally sound steady state. To be consistent with rational expectations, the threshold $\bar{d}$ needs to satisfy $(\bar{s} + \varpi(1 - \bar{g}^{-\gamma}))/(1 - \beta) \le \bar{d} \le \bar{s}/(1 - \beta\bar{g}^{1-\gamma})$.

Now let us define $V(d)$ as the function describing how households' welfare in the final steady state depends on the inherited stock of debt d. It is easy to show that[28]

$$V(d) = \begin{cases} \frac{\left(1-\frac{\bar{g}-1}{\chi}\right)^{1-\gamma}}{(1-\gamma)(1-\beta\bar{g}^{1-\gamma})} & \text{if } d \le \bar{d} \\ \frac{1}{(1-\gamma)(1-\beta)} & \text{if } d > \bar{d}. \end{cases} \tag{34}$$

Households' welfare in the final steady state thus experiences an upward jump once d drops below $\bar{d}$, because the fiscally sound steady state welfare dominates the fiscal stagnation one.

We now characterize the optimal policy in period $t = 0$. It is useful to clarify the intra-period timing. The economy starts with an inherited stock of public debt relative to GDP equal to d_0. To make the problem interesting, we assume that $d_0 > \bar{d}$. Then households form expectations about

[27] Lorenzoni and Werning (2019) use a similar assumption when studying equilibrium multiplicity driven by default risk.

[28] To derive this expression, consider that households' welfare is equal to

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma} = \sum_{t=0}^{\infty} \beta^t \frac{(A_t c_t)^{1-\gamma} - 1}{1-\gamma}.$$

In steady state, and ignoring terms that do not depend on fiscal policy, this equation becomes

$$\sum_{t=0}^{\infty} \left(\beta g^{1-\gamma}\right)^t \frac{(A_0 c)^{1-\gamma}}{1-\gamma} = A_0^{1-\gamma} V(d),$$

where we have used the fact that $g = \bar{g}$ if $d \le \bar{d}$ and $g = 1$ otherwise, and that $c = 1 - (g - 1)/\chi$.

the final steady state and take their investment decisions, which determine g_1 and c_0. Following Lorenzoni and Werning (2019), we also assume that the bonds price, i.e. the interest rate R_0, is set by households before the government chooses its fiscal stance. These timing assumptions imply that when setting s_0 the government takes g_1, c_0 and R_0 as given, capturing the notion that a government lacking commitment cannot affect households' expectations.

The government thus chooses s_0 to maximize households' welfare[29]

$$\max_{0 \leq s_0 \leq s^{max}} \frac{c_0^{1-\gamma}}{1-\gamma} + g_1^{1-\gamma} \beta V(d) \tag{35}$$

$$\text{s.t.} \quad d\beta g_1 / R_0 = d_0 - s_0. \tag{36}$$

The first term of the maximand captures the utility from consumption in period $t = 0$, while the second one is the continuation value in the final steady state. Though this is not crucial, we assume that there is an upper bound $s^{\max}$ on the primary surplus that the government can run.[30] Constraint (36) captures how fiscal policy affects the path of public debt. Recall that when setting s_0 the government takes R_0 and g_1 as given. Equation (36) then implies that, from the government's perspective, a higher surplus in period 0 leads to a lower debt-to-GDP ratio in the final steady state.

The optimal choice of s_0 can be described by

$$s_0 = \min\left(d_0 - \bar{d}\beta g^{1-\gamma}, s^{max}\right). \tag{37}$$

Intuitively, lowering debt until the threshold $\bar{d}$ has been reached and the fiscally sound steady state is attained improves welfare, because it maximizes $V(d)$. This is the benefit of increasing s_0. What about the cost? Since s_0 is set after investment has been done, there is no cost associated with setting a high ex-post tax on profits. Hence, it is optimal for the government to increase the surplus up to its maximum s^{max}, in an attempt to reach the fiscally sound steady state.[31]

The private sector's behavior is characterized by

$$g_1 = \begin{cases} \bar{g} & \text{if } d \leq \bar{d} \\ 1 & \text{if } d > \bar{d} \end{cases} \tag{38}$$

[29]To derive this objective function, consider that in period $t = 0$ households' welfare is given by

$$\frac{C_0^{1-\gamma}}{1-\gamma} + A_1^{1-\gamma}\beta V(d) = A_0^{1-\gamma}\left(\frac{c_0^{1-\gamma}}{1-\gamma} + g_1^{1-\gamma}\beta V(d)\right).$$

[30]This limit arises naturally from the fact that the government cannot impose a tax higher than 100% on profits. But we allow for even tighter upper bounds on s_0, capturing the fact that the government may not be able to fully tax profits.

[31]While equation (37) describes one possible implementation of the optimal policy, sometimes other choices of s_0 are also compatible with optimality. For instance, imagine that d_0 is so large that the fiscally sound steady state cannot be reached even if $s_0 = s^{max}$. In this case, the government is indifferent between all values of $0 \leq s_0 \leq s^{max}$. In (37), we assume that this indifference is resolved by the government choosing $s_0 = s^{max}$. We do so because this policy rule captures the spirit of the optimal policy in an infinite horizon model.

$$c_0 = 1 - \frac{g_1 - 1}{\chi} \tag{39}$$

$$R_0 = \frac{g_1^{\gamma}}{\beta}. \tag{40}$$

Expression (38) captures how households' investment shapes productivity growth between periods 0 and 1. Intuitively, households anticipate zero profits taxes if they believe that the fiscally sound steady state will be attained, explaining why $g_1 = \bar{g}$ if $d \leq \bar{d}$. Instead, they anticipate that taxes will be high enough to drive investment to zero if they believe that the economy will land in the fiscal stagnation steady state. Hence, $g_1 = 1$ if $d > \bar{d}$.[32] Equation (39) is the resource constraint of the economy, while equation (40) gives the equilibrium interest rate.

Combining expressions (36), (37) (38) and (40), and imposing rational expectations, gives that

$$d = \begin{cases} \bar{d} & \text{if } d_0 \leq s^{max} + \bar{d}\beta \\ \left\{\bar{d}, (d_0 - s^{max})/\beta\right\} & \text{if } s^{max} + \bar{d}\beta < d_0 < s^{max} + \bar{d}\beta\bar{g}^{1-\gamma} \\ (d_0 - s^{max})/\beta & \text{if } d_0 \geq s^{max} + \bar{d}\beta\bar{g}^{1-\gamma}. \end{cases} \tag{41}$$

This expression shows that households' expectations are a key determinant of economic outcomes when the government lacks the ability to commit. Indeed, the fiscally sound steady state is harder to attain if agents have pessimistic expectations. Naturally, a lower $\bar{d}$ makes it harder for the economy to escape fiscal stagnation. Moreover, for intermediate values of d_0, animal spirits determine economic outcomes even holding constant $\bar{d}$.

Figure 6 illustrates graphically these results. The figure compares two economies starting with the same debt-to-GDP ratio d_0, and characterized by identical fundamentals. The only difference is that agents are more optimistic, i.e. $\bar{d}$ is higher, in the economy depicted by the solid lines, compared to the dashed lines one.[33]

Even though they have identical fundamentals, the optimistic economy reaches the fiscally sound steady state, while the pessimistic one does not manage to escape fiscal stagnation. This result directly follows from the fact that, when households are optimistic, it takes a smaller reduction in public debt to attain the fiscally sound steady state.

Moreover, the public debt-to-GDP ratio drops by more in the optimistic economy. This happens in spite of the fact that the government implements a harsher fiscal tightening, i.e. a larger increase in s_0, in the pessimistic economy. The reason is that households' optimism about the future boosts investment and growth, thus reducing public debt relative to output. This result highlights the

[32] To derive expression (38), guess and verify that g_1 is equal to its value in the final steady state. Then equation (39) implies that c_0 is also equal to its value in the final steady state. Using expression (20), we can then verify that indeed the initial guess was correct, and that g_1 is given by expression (38). A similar reasoning delivers equation (40).

[33] To derive Figure 6, we use the same parametrization described in footnote 24. In addition, we set d_0 so that in the initial steady state public debt is around 105% of yearly GDP, and $s^{\max}$ so that the maximum primary surplus that can be attained is 2.5% of GDP. The solid lines refer to a value of $\bar{d}$ so that the fiscally sound steady state is attained if debt does not exceed 100.6% of yearly GDP, while the dashed lines refer to a case in which to attain the fiscally sound steady state public debt should not exceed 80% of yearly GDP.

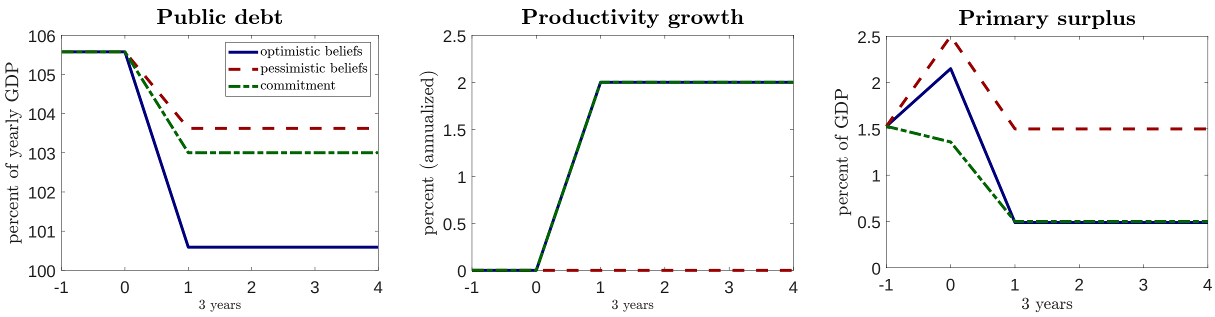

Figure 6: Optimal fiscal policy in a two-periods economy.

importance of households' expectations in determining the impact of a fiscal adjustment on the future public debt-to-GDP ratio.

Optimal fiscal policy in the infinite horizon model. In Appendix B, we derive the optimal fiscal policy under discretion in the full infinite-horizon model. There we show that it is optimal for the government to rely on fiscal austerity, i.e. on setting $s_t = s^{max}$, to decrease the public debt-to-GDP ratio until the fiscally sound steady state has been reached. The intuition is the same as in the two-periods version of the model. Until the fiscally sound steady state is attained, running down debt increases future welfare, by reducing the fiscal distortions that the government will impose in the future. Moreover, from the perspective of a government lacking commitment, taxing current profits at a high rate does not generate distortions, since fiscal policy is set after investment decisions have been made.

The first implication is that the transition to the final steady state is characterized by high fiscal distortions and low growth. Moreover, pessimistic expectations lead to a drop in welfare, because they call for a longer period of high fiscal distortions and low growth to reach the fiscally sound steady state. As in the two-periods model, in fact, animal spirits determine the public debt-to-GDP ratio at which the economy switches to fiscal soundness.

4.2 On the gains from credibility

We now consider a government that can commit to its promises about future fiscal policy. The shape of the optimal fiscal policy under commitment depends on the precise scenario, that is on the initial conditions and parameter values, that one wishes to consider. Rather than fully characterizing it, we will show that the ability to commit allows the government to exit stagnation through pro-growth policies, thus improving welfare.

Imagine that the economy has reached the debt-to-GDP ratio $d_t = \frac{\bar{s}}{(1-\beta\bar{g}^{1-\gamma})}$. As we have shown in the previous section, under discretion the debt-to-GDP ratio will have to fall further to achieve the fiscally sound steady state.[34] Under commitment, instead, the government can rely on pro-growth policies. That is, it can credibly promise that from then on it will refrain from using distortionary taxes by setting $s_t = \bar{s}$. The private sector will react by investing to bring

[34]The exception is the knife-edge case in which agents' expectations coordinate on $\bar{d} = \frac{\bar{s}}{(1-\beta\bar{g}^{1-\gamma})}$.

growth equal to its undistorted value $\bar{g}$. High growth will make debt sustainable, and the economy will immediately enter the fiscally sound steady state. The ability to commit to a future path for fiscal policy thus rules out the source of multiplicity that we described in the previous section, by coordinating agents' expectations on the equilibrium with the highest welfare.

The dashed-dotted lines in Figure 6 illustrate this result in the two-periods economy. A credible government can attain the fiscally sound steady state with a modest reduction in the debt-to-GDP ratio. Moreover, in the example considered in Figure 6, the economy exits fiscal stagnation even though the primary surplus s_0 drops compared to its value in the initial steady state. This happens, of course, because the expected drop in distortionary taxes fosters investment and growth, which helps to reduce the public debt-to-GDP ratio.

Why does the government need credibility to fully exploit pro-growth policies? The reason is that fiscal policy is subject to a time-consistency issue, since profits are taxed after firms have invested. Intuitively, the government may wish to promise low future taxes to foster investment and growth. However, once firms have invested and the tax base has been set, it may be optimal for the government to tax profits at a higher rate than promised.[35]

To see this point, suppose again that - once the economy has reached the level of debt $d_t = \frac{\bar{s}}{(1-\beta\bar{g}^{1-\gamma})}$ - the government announces that it will set $s_t = \bar{s}$ from then on. But now imagine that private agents do not believe this promise, and instead anticipate that the government will have to tax profits to make public debt sustainable. Productivity growth will then fall short of its undistorted value $\bar{g}$, and sticking to the original fiscal plan would lead to an increase in the public debt-to-GDP ratio. The government will then have to increase the primary surplus and tax profits to make public debt sustainable, validating the expectations by the private sector. The ability to commit to a future fiscal policy breaks this vicious cycle, and allows the government to coordinate expectations on the best equilibrium.

Taking stock, our model implies that credibility is crucial to exit fiscal stagnation efficiently. First, a credible government can boost investment and growth by promising low future taxes, thus reducing the public debt-to-GDP ratio through pro-growth policies. Moreover, a credible government can shield the economy from the risk of self-fulfilling episodes of fiscal stagnation, by re-anchoring expectations on the fiscally sound steady state.

5 Conclusion

This paper argues that taking into account the impact of fiscal policy on productivity growth is crucial to understand debt sustainability. Our model has two key features: i) financing large primary surpluses entails fiscal distortions that depress investment and growth, ii) low growth increases the primary surpluses needed to stabilize the public debt-to-GDP ratio. Negative shocks to fundamentals or pessimistic animal spirits may drive the economy into a state of fiscal stagnation,

[35] This is the classic time-consistency issue associated with capital levies. See Chapter 12 of Persson and Tabellini (2002) for a textbook treatment.

characterized by high public debt, large fiscal distortions and low productivity growth. We also show that government credibility is crucial to avoid/escape fiscal stagnation.

We conclude with some thoughts about future research. We have voluntarily kept the model of this paper abstract and stylized, in order to derive general insights. A natural next step of this research program would be the development of realistic quantitative frameworks, to study public debt sustainability taking into account the impact of fiscal policy on productivity growth. Doing so requires a careful modeling of the impact on firms' investment and innovation activities of different fiscal measures, in the spirit of the quantitive endogenous growth frameworks developed by Atkeson and Burstein (2019) and Akcigit et al. (2022). The payoff from developing this class of frameworks is likely to be large, because they could turn out to be crucial in helping policymakers to design their fiscal strategies.

Appendix

A Proofs of all propositions

A.1 Proof of Proposition 1

Proposition 1 *Suppose that* $1 < \bar{g} < \beta^{\frac{1}{\gamma-1}}$ *and that condition* (23) *holds. Then, there exists a unique fiscally sound steady state with* $s < \bar{s}$*. The fiscally sound steady state is characterized by positive growth* $g = \bar{g} > 1$*.*

Proof. We start by proving existence. A fiscally sound steady state is characterized by a set of values (s, g, c) such that conditions (FF), (MK), as well as $g = \bar{g}$ and $s < \bar{s}$ all hold. Putting $g = \bar{g}$ into (FF) yields

$$s = d(1 - \beta\bar{g}^{1-\gamma}). \tag{A.1}$$

Condition (23) then ensures that $s < \bar{s}$. Plugging $g = \bar{g}$ into (MK) yields

$$c = 1 - \frac{\bar{g} - 1}{\chi}, \tag{A.2}$$

which is positive by the condition stated in footnote (13). The condition $\bar{g} < \beta^{\frac{1}{\gamma-1}}$ ensures that households' utility is finite.

To prove uniqueness, consider that only one level of s is compatible with $g = \bar{g}$, and is given by (A.1). Equivalently, only one level of c is compatible with $g = \bar{g}$, and is given by (A.2). Hence, the fiscally sound steady state is unique. ■

A.2 Proof of Proposition 2

Proposition 2 *Suppose that condition* (24) *holds. Then a fiscal stagnation steady state exists, and it is characterized by zero growth* $g = 1$ *and high fiscal distortions* $s > \bar{s} + \varpi\,(1 - \bar{g}^{-\gamma})$*.*

Proof. We start by proving existence. A fiscal stagnation steady state is characterized by a set of values (s, g, c) such that conditions (FF), (MK), as well as $g = 1$ and $s > \bar{s} + \varpi(1 - \bar{g}^{-\gamma})$ all hold.

Putting $g = 1$ into (FF) yields

$$s = d(1 - \beta). \tag{A.3}$$

Condition (24) then ensures that $s > \bar{s} + \varpi\,(1 - \bar{g}^{-\gamma})$. Notice also that $s < 1$, by the condition stated in footnote (14).

Plugging $g = 1$ into (MK) yields

$$c = 1.$$

To prove uniqueness, consider that only one level of s is compatible with $g = 1$, and is given by (A.3). Equivalently, only one level of c is compatible with $g = 1$, and is given by $c = 1$. Hence, the fiscal stagnation steady state is unique. ■

A.3 Proof of Proposition 3

Proposition 3 *Assume that conditions (29) and $\phi > 1 - \beta$ both hold, so that the fiscally sound and stagnation steady states coexist and are locally stable. Let also $c_t \approx 1$. Assume that γ is large enough such that condition (33) holds. Then, for $d_0 < d^*$, where d^* is a tipping point, the economy transits to the fiscally sound steady state, while for $d_0 > d^*$, it transits to the fiscal stagnation steady state. When γ is sufficiently small, in contrast, an interval $[d', d'']$, $d' < d^* < d''$, exists such that, for $d_0 \in [d', d'']$, animal spirits dictate if the economy converges to the fiscally sound or the fiscal stagnation steady state.*

Proof. We start by establishing that condition (29) ensures the coexistence of the fiscally sound and stagnation steady states.

The fiscally sound steady state is characterized by a vector (d, s, g) such that $s \leq \bar{s}$, (FF), $g = \bar{g}$ and $s = -\delta + \phi d$ are all satisfied. Plugging the last two conditions into (FF)

$$s = \frac{s + \delta}{\phi}(1 - \beta\bar{g}^{1-\gamma}).$$

Rearranging and imposing that $s \leq \bar{s}$

$$\phi \geq \frac{\bar{s} + \delta}{\bar{s}}(1 - \beta\bar{g}^{1-\gamma}). \tag{A.4}$$

Hence, this steady state exists if ϕ is large enough.

The fiscal stagnation steady state is characterized by a set of values (d, s, g) such that $s \geq \bar{s} + \varpi(1 - \bar{g}^{-\gamma})$, (FF), $g = 1$ and $s = -\delta + \phi d$ are all satisfied. Plugging the last two conditions into (FF)

$$s = \frac{s + \delta}{\phi}(1 - \beta).$$

Imposing that $s \geq \bar{s} + \varpi(1 - \bar{g}^{-\gamma})$ and rearranging for ϕ yields that

$$\phi \leq \frac{\bar{s} + \varpi(1 - \bar{g}^{-\gamma}) + \delta}{\bar{s} + \varpi(1 - \bar{g}^{-\gamma})}(1 - \beta). \tag{A.5}$$

Hence, this steady state exists if ϕ is small enough.

Putting the two conditions together shows that, when condition (29) holds, both steady states coexist for intermediate values of ϕ.

Before we move on, we spend a few words on the relationship between conditions (25) and (29). Recall condition (25) implies the coexistence of the fiscally sound and stagnation steady states for some intermediate levels of debt d under the constant-debt fiscal policy, analysed in Section 3.1. In turn, condition (29) implies the coexistence of the two steady states for some intermediate levels of ϕ under the gradual adjustment fiscal policy.

It turns out that condition (25) already implies that condition (29) holds. To see this, rewrite condition (25) as

$$\frac{\bar{s}}{1 - \beta\bar{g}^{1-\gamma}} > \frac{\bar{s} + \varpi(1 - \bar{g}^{-\gamma})}{1 - \beta}. \tag{A.6}$$

and condition (29) as

$$\beta(1 - \bar{g}^{1-\gamma}) + \delta\left(\frac{1 - \beta\bar{g}^{1-\gamma}}{\bar{s}} - \frac{1 - \beta}{\bar{s} + \varpi(1 - \bar{g}^{-\gamma})}\right) < 0. \tag{A.7}$$

When (A.6) holds, then both terms in (A.7) are negative and hence the inequality is satisfied. In words, once parameters are such that the two steady states coexist for some levels of d under the constant-debt fiscal policy, then for any δ, one can always find a ϕ such the two steady states also coexist with the gradual adjustment fiscal policy.

We now move on with the proposition. Under the assumption $c_t \approx 1$, debt dynamics are given by equations (30)-(31), which we repeat here for convenience:

$$d_{t+1}g_{t+1}^{1-\gamma}\beta = \delta + (1 - \phi)d_t \tag{A.8}$$

$$g_{t+1} = \begin{cases} \bar{g} & \text{if } d_{t+1} \leq \frac{\delta+\bar{s}}{\phi} \\ \bar{g}\left(1 - \frac{\phi d_{t+1} - \delta - \bar{s}}{\varpi}\right)^{1/\gamma} & \text{if } \frac{\delta+\bar{s}}{\phi} < d_{t+1} \leq \frac{\delta+\bar{s}+\varpi\left(1-\bar{g}^{-\gamma}\right)}{\phi} \\ 1 & \text{if } \frac{\delta+\bar{s}+\varpi\left(1-\bar{g}^{-\gamma}\right)}{\phi} < d_{t+1}. \end{cases} \tag{A.9}$$

When $d_{t+1} \leq \frac{\delta+\bar{s}}{\phi}$, then $g_{t+1} = \bar{g}$ and debt dynamics are simply

$$d_{t+1}\bar{g}^{1-\gamma}\beta = \delta + (1 - \phi)d_t, \tag{A.10}$$

with standard local dynamics. In particular, because we assume $\phi > 1 - \beta$, this equation has a locally stable fixed point, given by the fiscally sound steady state.

When $d_{t+1} \geq \frac{\delta+\bar{s}+\varpi\left(1-\bar{g}^{-\gamma}\right)}{\phi}$, then $g_{t+1}=1$ and debt dynamics are

$$d_{t+1}\beta = \delta + (1-\phi)d_t, \tag{A.11}$$

again with standard dynamics. In particular, because we assume $\phi > 1-\beta$, this equation has a locally stable fixed point, given by the fiscal stagnation steady state.

Consider now the two boundary points $d_{t+1} = \frac{\delta+\bar{s}}{\phi}$ and $d_{t+1} = \frac{\delta+\bar{s}+\varpi\left(1-\bar{g}^{-\gamma}\right)}{\phi}$. It is easy to see that, once condition (A.4) holds, equation (A.10) implies that $d_{t+1} < d_t$ at the point $d_{t+1} = \frac{\delta+\bar{s}}{\phi}$. In turn, once condition (A.5) holds, equation (A.11) implies that $d_{t+1} > d_t$ at the point $d_{t+1} = \frac{\delta+\bar{s}+\varpi\left(1-\bar{g}^{-\gamma}\right)}{\phi}$. In words, as the economy moves from $d_{t+1} = \frac{\delta+\bar{s}}{\phi}$ to $d_{t+1} = \frac{\delta+\bar{s}+\varpi\left(1-\bar{g}^{-\gamma}\right)}{\phi}$, it moves from below to above the 45 degree line in a (d_{t+1}, d_t) diagram.

In the middle, it intersects the 45 degree line, giving another (unstable) steady state. This is simply the constant solution of (A.8)-(A.9) in the intermediate region

$$d^* \left(\bar{g}^{1-\gamma}\left(1-\frac{\phi d^*-\delta-\bar{s}}{\varpi}\right)^{\frac{1-\gamma}{\gamma}}\beta+\phi-1\right)=\delta. \tag{A.12}$$

This is the expression in footnote 22.

To characterize the behavior of the economy in the intermediate region, we compute the derivative of d_{t+1} with respect to d_t explicitly. It is (see equation (32) in the main text)

$$\frac{\partial d_{t+1}}{\partial d_t} = \frac{1-\phi}{g_{t+1}^{1-\gamma}\beta}\left(1+(1-\gamma)\frac{d_{t+1}}{g_{t+1}}\frac{\partial g_{t+1}}{\partial d_{t+1}}\right)^{-1}, \tag{A.13}$$

where the amplification term in brackets captures the growth effect. Using (A.9), we can compute the derivative of growth with respect to debt

$$\frac{\partial g_{t+1}}{\partial d_{t+1}} = -g_{t+1}\frac{\phi/\gamma}{\delta+\bar{s}+\varpi-\phi d_{t+1}}. \tag{A.14}$$

Inserting (A.14) into (A.13) yields

$$\frac{\partial d_{t+1}}{\partial d_t} = \left(\frac{\varpi+\bar{s}+\delta-\phi d_{t+1}}{\varpi+\bar{s}+\delta-\phi\frac{d_{t+1}}{\gamma}}\right)\frac{(1-\phi)}{g_{t+1}^{1-\gamma}\beta}. \tag{A.15}$$

We now state a condition which ensures that (A.15) is everywhere positive. Notice first the numerator in the round brackets is always positive. This happens because $d_{t+1} \leq \frac{\delta+\bar{s}+\varpi\left(1-\bar{g}^{-\gamma}\right)}{\phi}$ in the intermediate region for debt, which is a stricter condition than $d_{t+1} < \frac{\delta+\bar{s}+\varpi}{\phi}$. To rule out that the denominator is negative, it must be that $\phi d_{t+1} < \gamma(\varpi+\bar{s}+\delta)$ for all d_{t+1} in the intermediate region for debt. Inserting the highest possible point for d_{t+1}, given by $d_{t+1} = \frac{\delta+\bar{s}+\varpi\left(1-\bar{g}^{-\gamma}\right)}{\phi}$, yields

$$\phi\frac{\frac{\delta+\bar{s}+\varpi\left(1-\bar{g}^{-\gamma}\right)}{\phi}}{\gamma} < \delta+\bar{s}+\varpi.$$

Rearranging gives condition (33) from the main text:

$$\gamma > \frac{\delta + \bar{s} + \varpi(1 - \bar{g}^{-\gamma})}{\delta + \bar{s} + \varpi}.$$

When this condition holds, d_{t+1} is increasing in d_t everywhere, the case depicted graphically in Figure 4, the left panel. Notice that in this case, whenever debt lies initially below the tipping point ($d_0 < d^*$), the economy converges to the fiscally sound steady state over time, whereas when $d_0 > d^*$, it converges to the stagnation steady state over time.

We can also state a condition which ensures that d_{t+1} is declining in d_t everywhere in the intermediate region, the case depicted in Figure 4, the right panel. Following the same logic as above, this happens once $\phi d_{t+1} > \gamma(\varpi + \bar{s} + \delta)$ everywhere in the intermediate region. Inserting the lowest possible point, given by $d_{t+1} = \frac{\delta+\bar{s}}{\phi}$, we get

$$\gamma < \frac{\delta + \bar{s}}{\delta + \bar{s} + \varpi}. \tag{A.16}$$

As can be seen in the figure, now an interval $[d', d'']$ exists such that, for d_0 inside the interval, the economy may converge either to the fiscally sound or the stagnation steady state, depending on animal spirits. The boundaries of the set are given by the d_t which implies $d_{t+1} = \frac{\delta+\bar{s}+\varpi(1-\bar{g}^{-\gamma})}{\phi}$ in the region with low growth:

$$d' = \frac{\frac{\delta+\bar{s}+\varpi(1-\bar{g}^{-\gamma})}{\phi}\beta - \delta}{1-\phi},$$

where we used equation (A.9), and by the d_t which implies $d_{t+1} = \frac{\delta+\bar{s}}{\phi}$ in the region with high growth:

$$d'' = \frac{\frac{\delta+\bar{s}}{\phi}\bar{g}^{1-\gamma}\beta - \delta}{1-\phi},$$

where we used equation (A.8). Condition (A.16) ensures that $d' < d''$.[36] ■

B Optimal fiscal policy in the infinite horizon model

In this Appendix, we derive the optimal fiscal policy under discretion in the full infinite horizon model. Compared to the analysis in the main text, therefore, we remove the restriction that the government needs to keep the surplus constant from period $t = 1$ on.

The intra-period timing is the same as in the two-periods example. The economy inherits from the past the stock of public debt d_t. At the start of each period, the private sector forms its expectations about fiscal policy and invests, which determines g_{t+1}, c_t and R_t. The government then sets fiscal policy. This timing implies that the government takes g_{t+1}, c_t and R_t as given when

[36] When γ lies in between conditions (33) and (A.16), the derivative $\frac{\partial d_{t+1}}{\partial d_t}$ is negative at the point $d_{t+1} = \frac{\delta+\bar{s}+\varpi(1-\bar{g}^{-\gamma})}{\phi}$, but positive at the point $d_{t+1} = \frac{\delta+\bar{s}}{\phi}$. In this case, multiple equilibria exist, but they may arise (depending on the value of γ) to the right of the tipping point d^*. For $d_0 < d^*$, pessimism will then not be enough to push the economy on a path to fiscal stagnation.

choosing s_t. In equilibrium, households' beliefs must be consistent with the government's actions.

Throughout we will restrict attention to Markov equilibria, that is deterministic equilibria in which the state of the economy is fully summarized by the inherited public debt-to-GDP ratio. Formally, we focus on Markov-stationary policy rules that are functions of the payoff-relevant state variable d_t only. Since the government operates under discretion, it chooses its policy rules in any given period taking as given the policy rules associated with future government's decisions. A Markov-perfect equilibrium is then characterized by a fixed point in these policy rules. At this fixed point the current government does not have an incentive to deviate from future governments' policy rules, so that these rules are time consistent.

We define by $V(d_t)$ the government's value function. In each period t, the government thus sets s_t to maximize[37]

$$V(d_t) = \max_{0 \leq s_t \leq s^{max}} \frac{c_t^{1-\gamma}}{1-\gamma} + \beta g_{t+1}^{1-\gamma} \beta V(d_{t+1}) \tag{B.1}$$

subject to

$$s_t = d_t - d_{t+1} \frac{g_{t+1}}{R_t}, \tag{B.2}$$

given the initial stock of debt d_t. Constraint (B.2) encapsulates the dynamics of public debt. The constraint $s_t \leq s^{max}$ captures the fact that there is a limit on how much income the government can tax away from the private sector. For sure s_t cannot exceed $\bar{s} + \varpi$, since the government cannot tax profits at a rate higher than 100%. Throughout we allow for a tighter limit on taxation, by assuming that $\bar{s} < s^{\max} \leq \bar{s} + \varpi$.

[37] To derive this objective function, consider that expected utility at time t is given by

$$\sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{1-\gamma} - 1}{1-\gamma} = \sum_{j=0}^{\infty} \beta^j \frac{(A_{t+j} c_{t+j})^{1-\gamma} - 1}{1-\gamma}.$$

We scale this expression with $A_t^{1-\gamma}$, and leave out summands which are independent of policy, to define the value function

$$V(d_t) = \sum_{j=0}^{\infty} \beta^j \frac{\left(\frac{A_{t+j}}{A_t} c_{t+j}\right)^{1-\gamma}}{1-\gamma}.$$

This value function can be written recursively

$$V(d_t) = \frac{c_t^{1-\gamma}}{1-\gamma} + \sum_{j=1}^{\infty} \beta^j \frac{\left(\frac{A_{t+j}}{A_t} c_{t+j}\right)^{1-\gamma}}{1-\gamma} = \frac{c_t^{1-\gamma}}{1-\gamma} + \beta g_{t+1}^{1-\gamma} \underbrace{\sum_{j=0}^{\infty} \beta^j \frac{\left(\frac{A_{t+1+j}}{A_{t+1}} c_{t+1+j}\right)^{1-\gamma}}{1-\gamma}}_{=V(d_{t+1})}.$$

The private sector's behavior is described by

$$g_{t+1} = \begin{cases} \frac{c_t}{c_{t+1}}\bar{g} & \text{if } s_{t+1} \leq \bar{s} \\ \frac{c_t}{c_{t+1}}\bar{g}\left(1 - \frac{s_{t+1}-\bar{s}}{\varpi}\right)^{\frac{1}{\gamma}} & \text{if } \bar{s} < s_{t+1} \leq \bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\left(\frac{c_{t+1}}{c_t}\right)^{\gamma}\right) \\ 1 & \text{if } s_{t+1} > \bar{s} + \varpi\left(1 - \bar{g}^{-\gamma}\left(\frac{c_{t+1}}{c_t}\right)^{\gamma}\right) \end{cases} \tag{B.3}$$

$$R_t = \frac{1}{\beta}\left(g_{t+1}\frac{c_{t+1}}{c_t}\right)^{\gamma} \tag{B.4}$$

$$1 = c_t + \frac{g_{t+1} - 1}{\chi}. \tag{B.5}$$

As it is often the case, the shape of the optimal policy under discretion depends on the beliefs of the private sector. Rather than characterizing all the possible equilibria, we put some structure on households' beliefs formation. First, we assume that households expect the economy to settle on the fiscally sound steady state if $d_t \leq \bar{d}$, where $\bar{d} \leq \bar{s}/(1 - \beta\bar{g}^{1-\gamma})$. Hence, when $d_t \leq \bar{d}$ households set $g_{t+1} = \bar{g}$, $c_t = 1 - \frac{\bar{g}-1}{\chi}$ and $R_t = \bar{g}^{\gamma}/\beta$. Second, we assume that for values of $d_t > \bar{d}$ households' behavior is such that the value function $V(d_t)$ is decreasing in d_t. This captures the notion that, until the fiscally sound steady state has been reached, reducing debt increases welfare because it is associated with lower future fiscal distortions.[38]

We now show that the government's actions will validate these beliefs. First, notice that if $d_t \leq \bar{d}$ it is feasible for the government to maintain $d_{t+1} = d_t$ while setting $s_t \leq \bar{s}$. This follows directly from the restriction $\bar{d} \leq \bar{s}/(1 - \beta\bar{g}^{1-\gamma})$. Second, it turns out that it is optimal for the government to do so. Once the fiscally sound steady state has been attained, in fact, the government's value function collapses to

$$V(d_t \leq \bar{d}) = \frac{\left(1 - \frac{\bar{g}-1}{\chi}\right)^{1-\gamma}}{(1-\gamma)(1-\beta\bar{g}^{1-\gamma})}, \tag{B.6}$$

so it is flat in the level of debt d_t. Inspecting the optimal policy problem then makes clear that setting $d_{t+1} = d_t$ is optimal. Intuitively, there are no welfare gains from further reducing debt when the fiscally sound steady state has been reached.

Now turn to the range $d_t > \bar{d}$. Recall that we focus on equilibria in which, over this range, the value function $V(d_t)$ is decreasing in d_t. Since when setting s_t the government takes R_t and g_{t+1} as given, constraint (B.2) implies that from the government's perspective d_{t+1} is decreasing in s_t. Since the government takes c_t as given and $V(d_{t+1})$ is decreasing in d_{t+1}, it is optimal for the government to decrease d_{t+1} as much as possible, until the threshold $\bar{d}$ has been reached.

The government's optimal policy is therefore given by the following expression

$$s_t = \begin{cases} d_t(1 - \beta\bar{g}^{1-\gamma}) & \text{if } d_t \leq \bar{d} \\ \min\left(d_t - \bar{d}\frac{g_{t+1}}{R_t}, s^{max}\right) & \text{if } d_t > \bar{d}. \end{cases} \tag{B.7}$$

[38] We verified numerically that this property holds in all the simulations that we have performed.

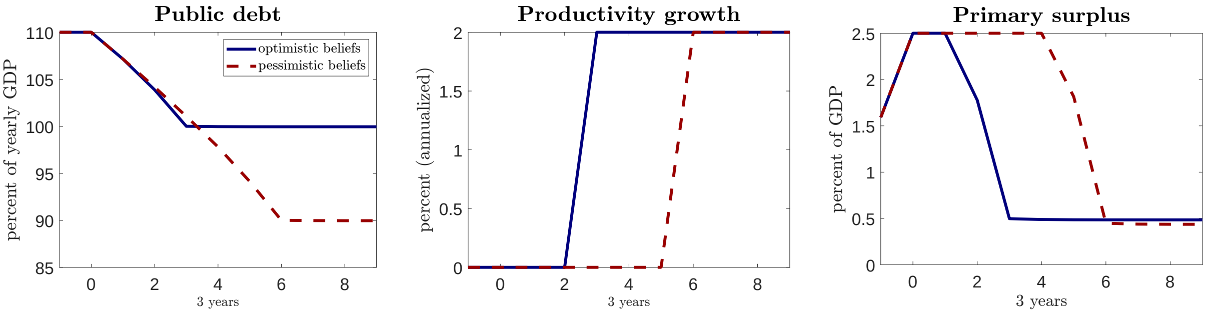

Figure 7: Optimal fiscal policy under discretion, infinite horizon.

This policy shows that i) the government keeps the debt ratio stable once $d_t \leq \bar{d}$, as this implies that the economy has reached the fiscally sound steady state ii) the government relies on austerity (high level of s_t) until debt has been run down to $\bar{d}$.

An equilibrium under optimal discretion is a sequence for $\{s_t, d_{t+1}, g_{t+1}, c_t, R_t\}$ satisfying equations (B.2), (B.3), (B.4), (B.5) and (B.7), for given initial d_0.

We illustrate the property of the equilibrium with a numerical example. We use the same calibration described in footnote 24. In addition, we assume that initial debt is given by 110% of GDP. Moreover, we contrast two different cases: an optimistic economy in which fiscal soundness is reached at a debt ratio of 100% of GDP, and a pessimistic one in which to reach fiscal soundness, the debt level must decline to 90% of GDP.

Figure 7 shows the result. As we argued above, the government relies on austerity, i.e. on high primary surpluses, to run down debt until the fiscally sound steady state has been reached. This implies that, during the transition toward the final steady state, distortionary taxes are high and productivity growth weak. Moreover, even though eventually both economies reach the fiscally sound steady state, animal spirits determine the length of the transition. If agents are pessimistic, indeed, it takes a long period of austerity to reduce public debt enough to attain the fiscally sound steady state. Just as in the two-periods case considered in the main text, when the government lacks the ability to commit, private sector's expectations are a key determinant of economic outcomes, and in particular of how fiscal policy affects the evolution of public debt.

C Data sources for Figure 1

The figure contrasts public debt (in percent of GDP), the primary surplus (in percent of GDP) and productivity growth (measured as real GDP per employment), in Italy versus a group of advanced countries. The group of advanced countries includes AUS, BEL, CAN, DNK, FIN, FRA, DEU, ITA, JPN, NLD, NOR, PRT, ESP, SWE, GBR and USA. We use real-GDP weights from 1980 to construct an average across these countries.

For public debt and the primary surplus, the data source is the IMF, the time series "Gross public debt, percent of GDP" and "Government primary balance, percent of GDP".

For productivity, the data source is the Penn World Tables, version 10. To construct real GDP per employment, we divide the time series “rgdpo” by the time series “emp”. We then take the log difference to obtain the growth rate of productivity. As indicated in the figure, we also smooth the productivity series by using a 5-year moving average.

The Penn World Tables data end in 2019, which explains why Figure 1 stops in this year.

References

Aghion, Philippe and Peter Howitt (1992) “A Model of Growth Through Creative Destruction,” *Econometrica*, Vol. 60, No. 2, pp. 323–351.

Akcigit, Ufuk, Douglas Hanley, and Stefanie Stantcheva (2022) “Optimal taxation and R&D policies,” *Econometrica*, Vol. 90, No. 2, pp. 645–684.

Angeletos, George-Marios, Chen Lian, and Christian K Wolf (2024) “Can Deficits Finance Themselves?” *Econometrica*, Vol. 92, No. 5, pp. 1351–1390.

Antolin-Diaz, Juan and Paolo Surico (2022) “The long-run effects of government spending,” *American Economic*.

Atkeson, Andrew and Ariel Burstein (2019) “Aggregate implications of innovation policy,” *Journal of Political Economy*, Vol. 127, No. 6, pp. 2625–2683.

Bansal, Ravi and Amir Yaron (2004) “Risks for the long run: A potential resolution of asset pricing puzzles,” *The Journal of Finance*, Vol. 59, No. 4, pp. 1481–1509.

Barro, Robert J (1990) “Government spending in a simple model of endogeneous growth,” *Journal of Political Economy*, Vol. 98, No. 5, Part 2, pp. S103–S125.

Bastasin, Carlo and Gianni Toniolo (2023) *The Rise and Fall of the Italian Economy*: Cambridge University Press.

Benigno, Gianluca and Luca Fornaro (2018) “Stagnation traps,” *Review of Economic Studies*, Vol. 85, No. 3, pp. 1425–1470.

Blanchard, Olivier (2023) *Fiscal Policy under Low Interest Rates*: The MIT Press.

Bohn, Henning (1998) “The Behavior of U. S. Public Debt and Deficits,” *The Quarterly Journal of Economics*, Vol. 113, No. 3, pp. 949–963.

Calvo, Guillermo A (1988) “Servicing the public debt: The role of expectations,” *The American Economic Review*, pp. 647–661.

Carroll, Christopher D., Jody Overland, and David N. Weil (2000) “Saving and Growth with Habit Formation,” *American Economic Review*, Vol. 90, No. 3, pp. 341–355.

Cloyne, James, Joseba Martinez, Haroon Mumtaz, and Paolo Surico (2022) “Short-term tax cuts, long-term stimulus,” NBER working paper.

Cole, Harold L and Timothy J Kehoe (2000) “Self-fulfilling debt crises,” *The Review of Economic Studies*, Vol. 67, No. 1, pp. 91–116.

Croce, M Max, Howard Kung, Thien T Nguyen, and Lukas Schmid (2012) “Fiscal policies and asset prices,” *The Review of Financial Studies*, Vol. 25, No. 9, pp. 2635–2672.

Croce, Mariano Massimiliano (2014) “Long-run productivity risk: A new hope for production-based asset pricing?” *Journal of Monetary Economics*, Vol. 66, pp. 13–31.

Croce, Mariano, Thien T Nguyen, and Steve Raymond (2021) “Persistent government debt and aggregate risk distribution,” *Journal of Financial Economics*, Vol. 140, No. 2, pp. 347–367.

Croce, Massimiliano Mariano, Thien T Nguyen, Steve Raymond, and Lukas Schmid (2019) “Government debt and the returns to innovation,” *Journal of Financial Economics*, Vol. 132, No. 3, pp. 205–225.

Eberhardt, Markus and Andrea F Presbitero (2015) “Public debt and growth: Heterogeneity and non-linearity,” *Journal of International Economics*, Vol. 97, No. 1, pp. 45–58.

Elfsbacka-Schmöller, Michaela and Nigel McClung (2024) “Can growth stabilize debt? A fiscal theory perspective,” Bank of Finland Research Discussion Papers.

Fatás, Antonio and Lawrence H Summers (2018) “The permanent effects of fiscal consolidations,” *Journal of International Economics*, Vol. 112, pp. 238–250.

Fieldhouse, Andrew J and Karel Mertens (2023) “The Returns to Government R&D: Evidence from US Appropriations Shocks,” Federal Reserve Bank of Dallas working paper.

Galli, Carlo (2021) “Self-fulfilling debt crises, fiscal policy and investment,” *Journal of International Economics*, Vol. 131, p. 103475.

Jaimovich, Nir and Sergio Rebelo (2017) “Nonlinear Effects of Taxation on Growth,” *Journal of Political Economy*, Vol. 125, No. 1, pp. 265–291.

Jiang, Zhengyang, Hanno Lustig, Stijn Van Nieuwerburgh, and Mindy Z Xiaolan (2024) “The US public debt valuation puzzle,” *Econometrica*, Vol. 92, No. 4, pp. 1309–1347.

Krugman, Paul (1988) “Financing vs. forgiving a debt overhang,” *Journal of Development Economics*, Vol. 29, No. 3, pp. 253–268.

Lamont, Owen (1995) “Corporate-debt overhang and macroeconomic expectations,” *The American Economic Review*, Vol. 85, No. 5, pp. 1106–1117.

Lorenzoni, Guido and Ivan Werning (2019) "Slow moving debt crises," *American Economic Review*, Vol. 109, No. 9, pp. 3229–3263.

Mehrotra, Neil R and Dmitriy Sergeyev (2021) "Debt sustainability in a low interest rate world," *Journal of Monetary Economics*, Vol. 124, pp. S1–S18.

Mian, Atif R, Ludwig Straub, and Amir Sufi (2022) "A goldilocks theory of fiscal deficits," NBER working paper.

Persson, Torsten and Guido Tabellini (2002) *Political economics: explaining economic policy*: MIT press.

Romer, Paul M (1990) "Endogenous Technological Change," *Journal of Political Economy*, Vol. 98, No. 5, pp. S71–S102.

Saint-Paul, Gilles (1992) "Fiscal policy in an endogenous growth model," *The Quarterly Journal of Economics*, Vol. 107, No. 4, pp. 1243–1259.