Supplementary Appendix To:

On Shakeouts and Staggered Exit: An R&D Race with Moral Hazard and Multiple Prizes.

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Contents

C Proofs for Section 4

The proofs make repeated use of the following auxiliary lemma.

Lemma C.1. *Fix a state* $s = (\sigma, N, \xi)$ *and, if* $|N| \geq 2$*, constants* $\mathbf{V} = \{\mathbf{V}_k\}_{k \in \{1,...,|N|-1\}}$ *and* $\mathbf{U} = \{\mathbf{U}_k\}_{k \in \{1,...,|N|-1\}}$ *such that the continuation payoffs in* s *are anonymous in the sense of Definition 4. Suppose the elements of the corresponding sequence* $\{G_\kappa\}_{\kappa=1}^{|N|}$ *are distinct and that it is* U*-shaped with lowest element* $G_{\overline{k}}$*. Further, let* $\underline{k}$ *be the lowest index* $\kappa \leq \overline{k}$ *of the elements satisfying* $G_\kappa < G_{|N|}$ *(setting* $\underline{k} = |N|$ *if there is no such element). Then, an index set* $\{\kappa_1, \kappa_2, ..., \kappa_\ell\}$ *with* $\kappa_\ell = |N|$ *is an element of* $\mathcal{G}_s$ *if and only if: (a)* $\kappa_q = q$ *for all* $q < \ell$ *and (b)* $\ell \in \{\underline{k}, \underline{k}+1, ..., \overline{k}\}$*.*

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Proof of Lemma C.1. The claim follows from the following four observations:

1. Whenever some index κ satisfying $2 < \kappa < \overline{k}$ is part of an index set in $\mathcal{G}_s$ then so must be $\kappa - 1$.

   (Proof: By assumption, the sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$ is strictly decreasing until $\kappa = \overline{k}$. The observation then follows from (a) in Definition 6.)

2. For any index set $\{\kappa_1, \kappa_2, ..., \kappa_\ell\} \in \mathcal{G}_s$ with $\ell \geq 2$ it must hold that $\kappa_{\ell-1} < \overline{k}$.

   (Proof: Suppose to the contrary that $\kappa_{\ell-1} \geq \overline{k}$. Then, $\overline{k} < |N|$ because $\kappa_{\ell-1} < \kappa_\ell = |N|$. Point (b) in Definition 6 implies $G_{\kappa_{\ell-1}} > G_{\kappa_{\ell-1}+1}$, which contradicts the fact that G_κ is increasing for $\kappa \in \{\overline{k}, \overline{k}+1, ..., |N|\}$.)

3. For any index set $\{\kappa_1, \kappa_2, ..., \kappa_\ell\} \in \mathcal{G}_s$ with $\ell \geq 2$ it must hold that $\kappa_{\ell-1} \geq \underline{k} - 1$.

   (Proof: If $\underline{k} = 1$, then the claim is evidentially true. As regards $\underline{k} \geq 2$, suppose to the contrary that $\kappa_{\ell-1} < \underline{k} - 1$. Then (a) in Definition 6 implies $G_{\kappa_{\ell-1}+1} \leq G_{|N|}$. But the definition of $\underline{k}$ gives $G_{\kappa_{\ell-1}+1} > G_{|N|}$, a contradiction.)

4. For every $\kappa \in \{\underline{k}, ..., \overline{k}\}$ there is an index set $\{1, 2, ..., \kappa - 1, |N|\} \in \mathcal{G}_s$ (which is taken to be $\{|N|\}$ if $\kappa = 1$).

   (Proof: This is a straightforward consequence of the facts that $\{G_\kappa\}_{\kappa=1}^{|N|}$ is strictly decreasing until $\kappa = \overline{k}$ and that for all $\kappa \in \{\underline{k}, ..., \overline{k}\}$ we have $G_\kappa < G_{|N|}$. Consequently, any index set $\{1, 2, ..., \kappa - 1, |N|\}$ satisfies conditions (a) and (b) in Definition 6.)

Observation 1 gives that any element in $\{\kappa_1, \kappa_2, ..., \kappa_\ell\} \in \mathcal{G}_s$ has $\kappa_1 = 1$ and – up to the $(\ell - 1)$-th element — consists of consecutive elements only. Observation 2 provides an upper bound on the length of any element in $\mathcal{G}_s$ and Observation 3 provides a lower bound. Finally, Observation 4 establishes that these bounds are tight. $\square$

Proof of Proposition 3. From (16), the sign of $G_\kappa - G_{\kappa+1}$ is equal to the sign of

$$\left[\Pi_\sigma - \frac{c+\theta}{\lambda}\right]\left[\kappa \mathbf{U}_\kappa - (\kappa-1)\mathbf{U}_{\kappa-1}\right] \\ + (\kappa-1)\left[\mathbf{V}_{\kappa-1} + \mathbf{U}_{\kappa-1}\right]\left[\frac{\phi}{\lambda} + \kappa \mathbf{U}_\kappa\right] \\ - \kappa\left[\mathbf{V}_\kappa + \mathbf{U}_\kappa\right]\left[\frac{\phi}{\lambda} + (\kappa-1)\mathbf{U}_{\kappa-1}\right]. \quad \text{(C.1)}$$

In the following I seek to characterize the behavior of the continuation payoffs $\mathbf{U}_\kappa$, $\mathbf{U}_{\kappa-1}$, $\mathbf{V}_\kappa$, and $\mathbf{V}_{\kappa-1}$ in equilibrium as ρ diverges to infinity. In view of Definition 4, I thus determine the limits of the expected firm and investor payoffs in equilibrium for any state s.

First, I show that, from Lemma A.1, it follows for any payoff-anonymous equilibrium profile C^* in full-R&D contracts and any state s that (i) $\lim_{\rho\to\infty} E[U_{is}(0; a_i, C^*)] = 0$, and (ii) $\lim_{\rho\to\infty} \rho E[U_{is}(0; a_i, C^*)] = \phi$.

- To establish (i), I observe that the claim for any state $s = (\sigma, N, \xi)$ with $\sigma = m$ follows directly from equation (A.2), because for $\sigma = m$ the continuation utility is always zero by construction; i.e., $\mathbf{U}_\kappa = 0$, for all $\kappa < |N|$. This gives $\lim_{\rho\to\infty} E[U_{is}(0; a_i, C^*)] = 0$ for all states s with $\sigma = m$, which in turn implies that the continuation payoffs vanish for any state s with $\sigma = m-1$. But then, also $\lim_{\rho\to\infty} E[U_{is}(0; a_i, C^*)] = 0$ for all such states. Repeating the argument by moving backwards through the spots σ until the states s with $\sigma = 1$, then gives the claim for all s.

- Multiplying the right side of (A.2) with ρ and in view of (i) above, it becomes clear that to establish (ii), it is sufficient to show that in any payoff-anonymous equilibrium and for any optimal deadline T^*_{is} it must hold $\lim_{\rho\to\infty} T^*_{is} > 0$. So, suppose T^*_{is} is the shortest deadline among all $|N|$ deadlines. If T^*_{is} is optimal, then it does not pay investor i to marginally increase the deadline. Specifically, suppose there are $0 \le \kappa < |N|-1$ dyads choosing a strictly higher deadline than T^*_{is}; then Lemma 4 gives

$$\left[\lambda\Pi_\sigma - (c+\theta) + \lambda\kappa\left[\mathbf{V}_\kappa + \mathbf{U}_\kappa\right]\right] e^{-\lambda T^*_{is}} \le \phi + \lambda\kappa \mathbf{U}_\kappa.$$

Because the above inequality must hold for all $0 \leq \kappa < |N| - 1$, we have for any state $s = (\sigma, N, \xi)$ and any player $i \in N$,

$$T_{is}^* \geq \min_{\kappa \in \{1,\ldots,|N|\}} \frac{1}{\lambda} \ln (G_\kappa) . \tag{C.2}$$

Now, from claim (i) above we know that $\lim_{\rho\to\infty} E[U_{is}(0; a_i, C^*)] = 0$ for any state s. Because $E[V_{is}(0; C^*)] \geq 0$ for all $\rho \geq 0$, we see from (16) that $\lim_{\rho\to\infty} G_\kappa \geq G_1 > 0$ for all κ, where the strict inequality follows from Assumption (A). But this gives $\lim_{\rho\to\infty} T_{is}^* > 0$, as desired. Hence, $\lim_{\rho\to\infty} \rho U_{is}(0; a_i, C^*) = \phi$ for any s, giving us the claim.

To continue, I show that Lemma A.2 implies for any payoff-anonymous equilibrium profile C^* in full-R&D contracts and any state s that (iii) $\lim_{\rho\to\infty} E[V_{is}(0; C^*)] = 0$, and (iv) $\lim_{\rho\to\infty} \rho E[V_{is}(0; C^*)] = \lambda \Pi_\sigma - (c + \theta + \phi)$.

- To establish (iii) observe first that from the arguments establishing (i) above it also follows $\lim_{\rho\to\infty} U_{is}(\tau; a_i, C^*) = 0$ for all $\tau > 0$. Moreover, limited liability implies that the continuation utility of any investor is bounded, $E[V_{is}(0; C^*)] \leq \max_{\hat{\sigma}\geq\sigma} \Pi_{\hat{\sigma}}$. And last, observe that the optimal deadlines are bounded when $\rho \to \infty$: Suppose T_{is}^* is the longest deadline among all $|N|$ deadlines. If T_{is}^* is optimal, then it does not pay investor i to marginally decrease the deadline. Specifically, suppose there are $0 \leq \kappa < |N| - 1$ dyads choosing a strictly lower deadline than T_{is}^*; then Lemma 4 gives

  $$[\lambda \Pi_\sigma - (c + \theta) + \lambda \kappa [\mathbf{V}_\kappa + \mathbf{U}_\kappa]] e^{-\lambda T_{is}^*} \geq \phi + \lambda \kappa \mathbf{U}_\kappa .$$

  Because the above inequality must hold for all $0 \leq \kappa < |N| - 1$, we have for any state $s = (\sigma, N, \xi)$ and any player $i \in N$,

  $$T_{is}^* \leq \max_{\kappa \in \{1,\ldots,|N|\}} \frac{1}{\lambda} \ln (G_\kappa) . \tag{C.3}$$

  Now, from (16) together with (i) above and the fact that $E[V_{is}(0; C^*)] \leq \max_{\hat{\sigma}\geq\sigma} \Pi_{\hat{\sigma}}$, we see that, in equilibrium, the numerator of any G_κ is strictly bounded away from infinity when $\rho \to \infty$ and the denominator is strictly bounded away from zero when $\rho \to \infty$. This gives that $\lim_{\rho\to\infty} T^*_{is} < \infty$, as desired.

With these preliminary observations, it is now possible to show that all integrals appearing in (A.5) – (A.8) converge to zero, thus establishing claim (iii). Consider the integral in (A.7) (the argument for (A.8) is analogous). Observe that it is bounded above by

$$\frac{\lambda}{(\lambda\ell+\rho)}\left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (\ell-1)\max_{\hat{\sigma}\geq\sigma}\Pi_{\hat{\sigma}}\right]\left[e^{-(\lambda\ell+\rho)T_\ell} - e^{-(\lambda\ell+\rho)T_{\ell-1}}\right],$$

which vanishes as $\rho \to \infty$. Next, consider the integral in (A.5) (the argument for (A.6) is analogous), which is bounded above by

$$\frac{\lambda}{(\lambda|N|+\rho)}\left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (|N|-1)\max_{\hat{\sigma}\geq\sigma}\Pi_{\hat{\sigma}}\right]\left[1 - e^{-(\lambda|N|+\rho)T_{is}}\right].$$

Again, this bound approaches zero when $\rho \to \infty$, thus yielding the claim.

- To establish (iv) I begin by showing that both the product of ρ with the integral in (A.7) and the product of ρ with the integral in (A.8) vanish as $\rho \to \infty$. Consider the integral in (A.7), multiplied by ρ (the argument for (A.8) is analogous). Observe that it is bounded above by

$$\frac{\rho\lambda}{(\lambda\ell+\rho)}\left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (\ell-1)\max_{\hat{\sigma}\geq\sigma}\Pi_{\hat{\sigma}}\right]\left[e^{-(\lambda\ell+\rho)T_\ell} - e^{-(\lambda\ell+\rho)T_{\ell-1}}\right].$$

Again, this bound approaches zero when $\rho \to \infty$. It remains to show that both the product of ρ with the integral in (A.5) and the product of ρ with the integral in (A.6) approach $\lambda\Pi_\sigma - (c+\theta+\phi)$ as $\rho \to \infty$. Consider the integral in (A.5), multiplied by ρ (the argument for (A.6) is analogous), which we can rewrite as

$$\frac{\lambda\rho}{(\lambda\ell+\rho)}\left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (|N|-1)\mathbf{V}_{|N|-1}\right]\left[1 - e^{-(\lambda|N|+\rho)T_{is}}\right] - \int_0^{T_{is}} U_{is}(\tau; a_i, C^*)\rho e^{-(\lambda|N|+\rho)\tau}\mathrm{d}\tau.$$

From the facts that, in equilibrium, $T_{is} > 0$ for any $\rho \geq 0$ as observed above and that $\mathbf{V}_{|N|-1} \to 0$ when $\rho \to \infty$ from (iii), the first term in the above expression approaches $\lambda \Pi_\sigma - (c + \theta + \phi)$ as $\rho \to \infty$. As regards the second term, observe that

$$\int_0^{T_{is}} U_{is}(\tau; a_i, C^*) \rho e^{-(\lambda|N|+\rho)\tau} \mathrm{d}\tau \leq \left[\max_{\hat{\tau} \in [0, T_{is}]} U_{is}(\hat{\tau}; a_{is}, C^*) \right] \cdot \frac{\rho}{\lambda|N| + \rho} \left[1 - e^{-(\lambda|N|+\rho)T_{is}} \right],$$

where the above observations give us that the first term in the product on the right side goes to zero as $\rho \to \infty$ while the other one remains bounded, thus yielding the claim.

To finish the proof, we may multiply (C.1) with ρ and use the observations (i)–(iv) above together with the definition of $\mathbf{V}_\kappa$ and $\mathbf{U}_\kappa$ in Definition 4 to obtain that, as $\rho \to \infty$, we have $\operatorname{sgn}(G_\kappa - G_{\kappa+1}) = 1$ if and only if $\Pi_\sigma - \Pi_{\sigma+1} > 0$ as well as $\operatorname{sgn}(G_\kappa - G_{\kappa+1}) = -1$ if and only if $\Pi_\sigma - \Pi_{\sigma+1} < 0$. This, together with points (b) and (c) of Corollary 1 then gives us the claim. □

Proof of Proposition 4. First observe that when the current prize has value Π_σ then $\max_{\hat{\sigma} \geq \sigma} \Pi_{\hat{\sigma}}$ is an upper bound on the dyad continuation payoff. Hence, it follows from (C.3) in the proof of Proposition 3 that in any state s the mutually optimal deadlines are bounded above by

$$\overline{T}^* = \max_{\kappa \in \{1, \ldots, |N|\}} \frac{1}{\lambda} \ln \left(\frac{\lambda \Pi_\sigma - (c + \theta) + \lambda(\kappa - 1) \max_{\hat{\sigma} \geq \sigma} \Pi_{\hat{\sigma}}}{\phi} \right), \tag{C.4}$$

which, using l'Hopital's rule, is readily confirmed to converge to zero as λ diverges to infinity. Moreover, it is clear that $\lim_{\lambda \to \infty} \lambda \overline{T}^* = \infty$. Consequently, we have $\lim_{\lambda \to \infty} e^{-\overline{T}^*} = 1$ and $\lim_{\lambda \to \infty} e^{-\lambda \overline{T}^*} = 0$.

In order to determine the limit of the sign of $G_\kappa - G_{\kappa'}$, I first make some preliminary observations on the limits both of expected investor and expected firm payoffs in equilibrium. Throughout the following, I fix a state $s = (\sigma, N, \xi)$ and suppose C^* is an payoff-anonymous equilibrium profile.

First, we consider the limit $\lim_{\lambda \to \infty} U_{is}(0; a_i, C^*)$. When $|N| = 1$, then (A.2) gives $U_{is}(0; a_i, C^*) = \frac{\phi}{\rho}[1 - e^{-\rho T^*_{is}}]$. From the facts that $[1 - e^{-\rho T^*_{is}}] \leq [1 - e^{-\rho \overline{T}^*}]$ and

$\lim_{\lambda\to\infty} \overline{T}^* = 0$ we obtain that $\lim_{\lambda\to\infty} U_{is}(0; a_i, C^*) = 0$. Consequently,

$$|N| = 1 \implies \lim_{\lambda\to\infty} E[U_{is}(0; a_i, C^*)] = 0. \tag{C.5}$$

On the other hand, if $|N| \geq 2$ but $\sigma = m$, then, because $\mathbf{U}_\kappa = 0$ for all $\kappa \leq |N| - 1$ in such a state, (A.2) gives $\lim_{\lambda\to\infty} U_{is}(0; a_i, C^*) = 0$ directly. Consequently,

$$\sigma = m \implies \lim_{\lambda\to\infty} E[U_{is}(0; a_i, C^*)] = 0. \tag{C.6}$$

Finally, when $\sigma \leq m - 1$ and $|N| \geq 2$, then the first case in (A.2) gives that for any $T_{is} \in [T_\kappa, T_{\kappa-1})$ it holds

$$0 \leq \lim_{\lambda\to\infty} U_{is}(T_\kappa; a_i, C^*) \leq \lim_{\lambda\to\infty} \mathbf{U}_{\kappa-1}.$$

Then, from the second case in (A.2) we obtain

$$0 \leq \lim_{\lambda\to\infty} U_{is}(T_{\kappa+1}; a_i, C^*) \leq \lim_{\lambda\to\infty} \mathbf{U}_{\kappa-1} + \lim_{\lambda\to\infty} \mathbf{U}_\kappa.$$

More generally, using the second case in (A.2) repeatedly, we get

$$0 \leq \lim_{\lambda\to\infty} U_{is}(0; a_i, C^*) \leq \sum_{j=\kappa-1}^{|N|-1} \lim_{\lambda\to\infty} \mathbf{U}_j.$$

Consequently, firm utility is invariant in permutations of the deadline profile, and we have

$$0 \leq \lim_{\lambda\to\infty} E[U_{is}(0; a_i, C^*)] \leq \sum_{j=\kappa-1}^{|N|-1} \lim_{\lambda\to\infty} \mathbf{U}_j. \tag{C.7}$$

Because the inequalities in (C.7) must hold for every state s, we can use them recursively together with Definition 4 and the boundary conditions (C.5) and (C.6) to conclude

$$\lim_{\lambda\to\infty} E[U_{is}(0; a_i, C^*)] = 0 \text{ for any state } s. \tag{C.8}$$

Next, recall from (C.2) in the proof to Proposition 3 that all mutually optimal deadlines are bounded below by $\underline{T}^*$ where

$$\underline{T}^* \equiv \min_{\kappa} \frac{1}{\lambda} \ln\left(G_\kappa\right).$$

Because $E[U_{is}(0; a_i, C^*)]$ vanishes by (C.8), applying l'Hopital's rule gives that $\underline{T}^*$ converges to zero as λ diverges to infinity. Moreover, it is clear that $\lim_{\lambda\to\infty} \lambda\underline{T}^* = \infty$. Because both $\underline{T}^*$ and $\overline{T}^*$ converge to zero, it must hold that any equilibrium deadline converges to zero as $\lambda \to \infty$. Further, because both $\lambda\underline{T}^*$ and $\lambda\overline{T}^*$ diverge to infinity, the same must hold for the product of λ and any equilibrium deadline. For any equilibrium deadline T_{is}^* it thus holds

$$\lim_{\lambda\to\infty} e^{-\lambda T_{is}^*} = 0 \text{ and } \lim_{\lambda\to\infty} e^{-T_{is}^*} = 1. \tag{C.9}$$

Next, I consider the limit $\lim_{\lambda\to\infty} \frac{\lambda}{\phi} E[U_{is}(0; a_i, C^*)]$. To this end define $\hat{U}_{is}(0; a_i, C^*) \equiv \frac{\lambda}{\phi} U_{is}(0; a_i, C^*)$ and, accordingly, $\hat{\mathbf{U}}_\kappa = \frac{\lambda}{\phi}\mathbf{U}_\kappa$. Let $\tilde{T}$ be lowest of all opponent deadlines. Then, from (A.2), $\hat{U}_{is}(0; a_i, C^*)$ is given by

$$\hat{U}_{is}(0; a_i, C^*) = \begin{cases} \dfrac{1 + (|N|-1)\hat{\mathbf{U}}_{|N|-1}}{|N|-1+\rho/\lambda} \times \left[1 - e^{-(\lambda(|N|-1)+\rho)T_{is}^*}\right] & \text{if } T_{is}^* \leq \tilde{T} \\ \hat{U}_{is}(\tilde{T}; a_{is}, C^*) e^{-((|N|-1)\lambda+\rho)\tilde{T}} & \\ \quad + \dfrac{1 + (|N|-1)\hat{\mathbf{U}}_{|N|-1}}{|N|-1+\rho/\lambda} \times \left[1 - e^{-(\lambda(|N|-1)+\rho)\tilde{T}}\right] & \text{if } T_{is}^* > \tilde{T}. \end{cases} \tag{C.10}$$

To determine the value of $\lim_{\lambda\to\infty} \hat{U}_{is}(0; a_i, C^*)$ we need to distinguish three cases. First, if $|N| = 1$, then only the first case above applies — as $\tilde{T} = \infty$ in that case — and we obtain $\lim_{\lambda\to\infty} \hat{U}_{is}(0; a_i, C^*) = \infty$. To see this, observe that $\overline{T}^* = \underline{T}^* = \frac{1}{\lambda} \ln\left(\frac{\lambda\Pi_\sigma - (c+\theta)}{\phi}\right)$ when $|N| = 1$. Consequently, it holds

$$\hat{U}_{is}(0; a_i, C^*) = \frac{\lambda}{\rho}\left[1 - \exp\left(-\frac{\rho}{\lambda}\ln\left(\frac{\lambda\Pi_\sigma - (c+\theta)}{\phi}\right)\right)\right].$$

Applying L'Hôpital's rule, we have

$$
\begin{aligned}
&\lim_{\lambda\to\infty} \hat{U}_{is}(0; a_i, C^*) \\
&\quad = \lim_{\lambda\to\infty} \frac{\exp\left(-\frac{\rho}{\lambda}\ln\left(\frac{\lambda\Pi_\sigma-(c+\theta)}{\phi}\right)\right)\left[\frac{\rho}{\lambda^2}\ln\left(\frac{\lambda\Pi_\sigma-(c+\theta)}{\phi}\right)-\frac{\rho}{\lambda}\frac{\phi\Pi_\sigma}{\lambda\Pi_\sigma-(c+\theta)}\right]}{\rho\lambda^{-2}} \\
&\quad = \lim_{\lambda\to\infty} \exp\left(-\frac{\rho}{\lambda}\ln\left(\frac{\lambda\Pi_\sigma-(c+\theta)}{\phi}\right)\right)\left[\ln\left(\frac{\lambda\Pi_\sigma-(c+\theta)}{\phi}\right)-\frac{\phi\lambda\Pi_\sigma}{\lambda\Pi_\sigma-(c+\theta)}\right].
\end{aligned}
$$

From (C.9), the limit of the term before the square brackets is one. The difference in the square brackets diverges to infinity.

The second case is when $\sigma = m$ yet $|N| \geq 2$. Because in that case it holds $\hat{\mathbf{U}}_\kappa = 0$ for all $\kappa \leq |N|-1$ it follows from (C.9) that $\lim_{\lambda\to\infty}\hat{U}_{is}(0; a_i, C^*) = \frac{1}{|N|-1}$ (irrespective of which case we look at in (C.10)). Last, if $\sigma \leq m-1$ and $|N| \geq 2$, then we obtain for either case in (C.10) that $\lim_{\lambda\to\infty}\hat{U}_{is}(0; a_i, C^*) = \frac{1}{|N|-1} + \lim_{\lambda\to\infty}\hat{\mathbf{U}}_{|N|-1}$.

Together with Definition 4, the observations in above paragraphs allow us to conclude that, for any state s,

$$
\lim_{\lambda\to\infty} E[\hat{U}_{is}(0; a_i, C^*)] = \begin{cases} \infty & \text{if } |N|-1 \leq m-\sigma \\ \sum_{j=1}^{m-\sigma+1} \dfrac{1}{|N|-j} & \text{if } |N|-1 > m-\sigma. \end{cases}
$$

From this it follows for any state $s = (\sigma, N, \xi)$ with $\sigma < m$ that, for $\kappa < |N|$,

$$
\lim_{\lambda\to\infty} \hat{\mathbf{U}}_\kappa = \begin{cases} \infty & \text{if } \kappa \leq m-\sigma \\ \sum_{j=1}^{m-\sigma} \dfrac{1}{\kappa-j} & \text{if } \kappa > m-\sigma. \end{cases} \tag{C.11}
$$

Further, I want to argue that from Lemma A.2 and the fact that $\lim_{\lambda\to\infty} U_{is}(t; a_i, C^*) = 0$ for all $t \geq 0$ (which follows from an analogous argument as the one used to derive (C.8) above) we obtain, for all $s = (\sigma, N, \xi)$ with $\sigma < m$, that for any $\kappa \leq |N|-1$ it holds

$$
\lim_{\lambda\to\infty} \kappa\mathbf{V}_\kappa = \sum_{j=0}^{\substack{\min\{\kappa-1,\\ m-\sigma-1\}}} \Pi_{\sigma+1+j}. \tag{C.12}
$$

To see this, we first establish that the intervals appearing in (A.7)–(A.8) vanish as $\lambda \to \infty$. Consider the integral in (A.7), which (with the relevant $\ell \geq 1$) is bounded above by

$$\left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (\ell-1)\max_{\hat\sigma\geq\sigma}\Pi_{\hat\sigma}\right]\int_{T_\ell}^{T_{\ell-1}} \lambda e^{-(\lambda\ell+\rho)\tau}\mathrm{d}\tau$$
$$= \frac{\lambda}{\lambda\ell+\rho}\left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (\ell-1)\max_{\hat\sigma\geq\sigma}\Pi_{\hat\sigma}\right]\left[e^{-(\lambda\ell+\rho)T_\ell} - e^{-(\lambda\ell+\rho)T_{\ell-1}}\right].$$

This bound vanishes because, in equilibrium, $\lambda T_\ell, \lambda T_{\ell-1} \to \infty$, as observed above. The argument for the integral in (A.8) is analogous.

Next, I show that the intervals appearing in (A.5)–(A.6) approach $|N|^{-1}[\Pi_\sigma + (|N|-1)\lim_{\lambda\to\infty}\mathbf{V}_{|N|-1}]$ as $\lambda \to \infty$. Consider the integral appearing in (A.5), which can be rewritten as

$$\frac{\lambda}{\lambda|N|+\rho}\left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (|N|-1)\mathbf{V}_{|N|-1}\right]\left[1 - e^{-(\lambda|N|+\rho)T_{is}}\right]$$
$$- \int_0^{T_{is}} U_{is}(\tau; a_i, C^*)\lambda e^{-(\lambda|N|+\rho)\tau}\mathrm{d}\tau.$$

Because $\lambda T_{is} \to \infty$ for any equilibrium deadline, the first term above approaches

$$\frac{1}{|N|}\left[\Pi_\sigma + (|N|-1)\lim_{\lambda\to\infty}\mathbf{V}_{|N|-1}\right].$$

As regards the second term, observe that

$$\int_0^{T_{is}} U_{is}(\tau; a_i, C^*)\lambda e^{-(\lambda|N|+\rho)\tau}\mathrm{d}\tau \leq \left[\max_{\hat\tau\in[0,T_{is}]} U_{is}(\hat\tau; a_i, C^*)\right]\cdot\int_0^{T_{is}} \lambda e^{-(\lambda|N|+\rho)\tau}\mathrm{d}\tau,$$

where both terms in the product on the right side go to zero as $\lambda \to \infty$. The same argument can be applied to the integral in (A.6).

Together with above observations this gives us $\lim_{\lambda\to\infty} V_{is}(0; C^*) = |N|^{-1}[\Pi_\sigma +$

$(|N|-1)\lim_{\lambda\to\infty}\mathbf{V}_{|N|-1}]$. But this implies that for any state $s=(\sigma,N,\xi)$ we have

$$|N|\lim_{\lambda\to\infty}E[V_{is}(0;C^*)]=\Pi_\sigma+(|N|-1)\lim_{\lambda\to\infty}\mathbf{V}_{|N|-1}.$$

Using Definition 4 together with the boundary conditions for states s with $|N|=1$, $\mathbf{V}_0=0$, and for state s with $\sigma=m$, $\mathbf{V}_{|N|-1}=0$, this can be written

$$|N|\cdot\lim_{\lambda\to\infty}E[V_{is}(0;C^*)]=\sum_{j=0}^{\substack{\min\{|N|-1,\\ m-\sigma\}}}\Pi_{\sigma+j}.$$

Using Definition 4 again, this then finally yields (C.12).

Now, multiplying the right side of (16) with λ/ϕ we obtain that the sign of $G_\kappa-G_{\kappa'}$ for $\kappa'>\kappa$ is equal to the sign of

$$\begin{aligned}\left[\Pi_\sigma-\frac{c+\theta}{\lambda}\right]&\left[(\kappa'-1)\hat{\mathbf{U}}_{\kappa'-1}-(\kappa-1)\hat{\mathbf{U}}_{\kappa-1}\right]\\&+(\kappa-1)\left[\mathbf{V}_{\kappa-1}+\mathbf{U}_{\kappa-1}\right]\left[1+(\kappa'-1)\hat{\mathbf{U}}_{\kappa'-1}\right]\\&-(\kappa'-1)\left[\mathbf{V}_{\kappa'-1}+\mathbf{U}_{\kappa'-1}\right]\left[1+(\kappa-1)\hat{\mathbf{U}}_{\kappa-1}\right].\end{aligned}\tag{C.13}$$

To determine the sign of above expression when $\lambda\to\infty$, we need to distinguish three cases:

1. $\kappa=1$. In this case, (C.13) boils down to

$$\left[\Pi_\sigma-\frac{c+\theta}{\lambda}\right](\kappa'-1)\hat{\mathbf{U}}_{\kappa'-1}-(\kappa'-1)\left[\mathbf{V}_{\kappa'-1}+\mathbf{U}_{\kappa'-1}\right].$$

   We have to further distinguish:

   (a) $\kappa'-1\leq m-\sigma$. From (C.11) we obtain that the first term in above difference diverges to infinity, while (C.8) and (C.12) give that the second term remains bounded. Hence, $G_1>G_{\kappa'}$ in this case.

   (b) $\kappa'-1>m-\sigma$. In this case, (C.11) gives that the first term remains bounded, too, and we obtain from (C.11) and (C.12) that, in the limit

$\lambda \to \infty$, $G_1 > G_{\kappa'}$ if and only if

$$\Pi_\sigma(\kappa'-1)\sum_{j=1}^{m-\sigma}\frac{1}{\kappa'-1-j} - \sum_{j=0}^{m-\sigma-1}\Pi_{\sigma+1+j} > 0.$$

Observe that

$$(\kappa'-1)\sum_{j=1}^{m-\sigma}\frac{1}{\kappa'-1-j} > (\kappa'-1)\frac{m-\sigma}{\kappa'-1} = m-\sigma,$$

implying that a sufficient condition for $G_1 > G_{\kappa'}$ to hold for $\kappa' > m-\sigma+1$ is

$$\Pi_\sigma(m-\sigma) - \sum_{j=1}^{m-\sigma}\Pi_{\sigma+j} > 0,$$

which holds because the values of the spots, Π_σ, strictly decrease in σ. This gives us $G_1 > G_{\kappa'}$ in this case, too.

2. $0 < \kappa - 1 \le m - \sigma$. In this case, too, the limit of (C.13) depends on which of the following two cases holds:

   (a) $\kappa' - 1 \le m - \sigma$. In this case, the terms involving $\hat{\mathbf{U}}_{\kappa-1}$ and $\hat{\mathbf{U}}_{\kappa'-1}$ diverge to infinity by (C.11). They all do so at the same rate, so that from (C.11) and (C.12) we have $G_\kappa > G_{\kappa'}$ if and only if

   $$\Pi_\sigma(\kappa'-\kappa) + (\kappa'-1)\sum_{j=0}^{\kappa-2}\Pi_{\sigma+1+j} - (\kappa-1)\sum_{j=0}^{\kappa'-2}\Pi_{\sigma+1+j} > 0.$$

   For $\kappa' = \kappa + 1$, this condition becomes

   $$\Pi_\sigma + \kappa\sum_{j=0}^{\kappa-2}\Pi_{\sigma+1+j} - (\kappa-1)\sum_{j=0}^{\kappa-1}\Pi_{\sigma+1+j} > 0.$$

   This is equivalent to

   $$\Pi_\sigma + \sum_{j=1}^{\kappa}\Pi_{\sigma+j} - \kappa\Pi_{\sigma+\kappa} > 0,$$

which holds because the values of the spots are decreasing. This gives $G_\kappa > G_{\kappa'}$ for all $1 < \kappa < \kappa' \leq m - \sigma + 1$.

(b) $\kappa' - 1 > m - \sigma$. In this case, the terms involving $\hat{\mathbf{U}}_{\kappa-1}$ diverge to infinity (cf. (C.11)) while the other terms remain bounded, such that (C.13) diverges to minus infinity. Consequently, $G_\kappa < G_{\kappa'}$ in this case.

3. $m - \sigma < \kappa - 1$. In this case, we obtain from (C.8), (C.11), and (C.12) that (C.13) converges to

$$\begin{aligned}\Pi_\sigma &\left[(\kappa' - 1)\sum_{j=1}^{m-\sigma}\frac{1}{\kappa' - 1 - j} - (\kappa - 1)\sum_{j=1}^{m-\sigma}\frac{1}{\kappa - 1 - j}\right] \\ &\qquad + \sum_{j=0}^{m-\sigma-1}\Pi_{\sigma+1+j}\left[1 + (\kappa' - 1)\sum_{j=1}^{m-\sigma}\frac{1}{\kappa' - 1 - j}\right] \\ &\qquad\qquad - \sum_{j=0}^{m-\sigma-1}\Pi_{\sigma+1+j}\left[1 + (\kappa - 1)\sum_{j=1}^{m-\sigma}\frac{1}{\kappa - 1 - j}\right].\end{aligned}$$

Now, for $\kappa' = \kappa + 1$, we can rewrite these terms as

$$\sum_{j=0}^{m-\sigma}\Pi_{\sigma+j}\left[\kappa\sum_{j=1}^{m-\sigma}\frac{1}{\kappa - j} - (\kappa - 1)\sum_{j=1}^{m-\sigma}\frac{1}{\kappa - 1 - j}\right]$$

which is equal to

$$\sum_{j=0}^{m-\sigma}\Pi_{\sigma+j}\left[\kappa\left[\frac{1}{\kappa - 1} - \frac{1}{\kappa - 1 - (m - \sigma)}\right] + \sum_{j=2}^{m-\sigma+1}\frac{1}{\kappa - j}\right],$$

Observe that the sign of the expression in the outer brackets above is equal to that of

$$\begin{aligned}\frac{1}{\kappa}\sum_{j=2}^{m-\sigma+1}\frac{1}{\kappa - j} &- \frac{1}{\kappa - 1}\frac{m - \sigma}{\kappa - 1 - (m - \sigma)} \\ &< \frac{1}{\kappa}\frac{m - \sigma}{\kappa - 1 - (m - \sigma)} - \frac{1}{\kappa - 1}\frac{m - \sigma}{\kappa - 1 - (m - \sigma)} \\ &= \frac{m - \sigma}{\kappa - 1 - (m - \sigma)}\left[\frac{1}{\kappa} - \frac{1}{\kappa - 1}\right] < 0.\end{aligned}$$

Consequently, we have $G_\kappa < G_{\kappa'}$ whenever $m - \sigma + 1 < \kappa < \kappa'$.

Letting $\underline{k} = 2$ and $\overline{k} = \min\{|N|, m - \sigma + 1\}$ above observations give us that $G_1 > G_\kappa$ for all $\kappa \geq 2$ (points 1.a and 1.b above), $G_\kappa < G_{|N|}$ for all $\kappa \geq \underline{k}$ (points 2.b and 3) and that $\{G_\kappa\}_{\kappa=1}^{|N|}$ is U-shaped with $G_{\overline{k}}$ the lowest of all elements (points 2.a, 2.b, and 3). Moreover, it is then a consequence of ponts 1 and 2.b) that all elements in the sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$ are distinct (i.e., $G_\kappa > G_{\kappa+1}$ for all $\kappa < \min\{|N|, m - \sigma + 1\}$, and, if $|N| > m - \sigma + 1$, then $G_\kappa < G_{\kappa+1}$ for all $\kappa \geq m - \sigma + 1$ where, crucially, $G_2 < G_\kappa < G_1$ for all $\kappa > m - \sigma + 1$). Consequently, the claim follows from Lemma C.1 and Proposition 1. □

Proof of Proposition 5. To begin, observe that for $\sigma = m - 1$ we have

$$\lim_{\phi \to 0} \phi \cdot G_\kappa = \frac{\lambda \Pi_{m-1} - (c + \theta) + \dfrac{\lambda(\kappa - 1)}{\lambda(\kappa - 1) + \rho} [\lambda \Pi_m - (c + \theta)]}{1 + \dfrac{\lambda(\kappa - 1)}{\lambda(\kappa - 2) + \rho}} \equiv \bar{G}_\kappa. \tag{C.14}$$

This follows from expanding the fraction in (16) by λ and appreciating

$$\lim_{\phi \to 0} [\mathbf{V}_{\kappa-1} + \mathbf{U}_{\kappa-1}] = \frac{[\lambda \Pi_m - (c + \theta)]}{\lambda(\kappa - 1) + \rho}$$

and

$$\lim_{\phi \to 0} \frac{\mathbf{U}_{\kappa-1}}{\phi} = \frac{1}{\lambda(\kappa - 2) + \rho}.$$

The first limit corresponds to expected total dyad welfare when there are $\kappa - 1$ players racing for the last spot. The second limit can be derived from Part (b) in Lemma A.1, appreciating that the continuation payoff in any state s with $\sigma = m$ is zero.

By construction, we have $\text{sgn}(G_\kappa - G_{\kappa'}) = \text{sgn}(\bar{G}_\kappa - \bar{G}_{\kappa'})$ for all sufficiently low ϕ whenever $\text{sgn}(\bar{G}_\kappa - \bar{G}_{\kappa'}) \in \{-1, 1\}$. This allows me to use $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ rather than $\{G_\kappa\}_{\kappa=1}^{|N|}$ to construct $\mathcal{G}_s$ in order to draw conclusions on the properties of the profile of mutually optimal deadlines when ϕ becomes small.

Fix κ, κ' satisfying $\kappa' > \kappa \geq 2$. Then we have from (C.14) that $\bar{G}_2^{\kappa} > \bar{G}_2^{\kappa'}$ is equivalent to

$$\Phi\left[\frac{\lambda(\kappa'-1)}{\lambda(\kappa'-2)+\rho}-\frac{\lambda(\kappa-1)}{\lambda(\kappa-2)+\rho}\right] > \frac{\lambda(\kappa'-1)}{\lambda(\kappa'-1)+\rho}\left[1+\frac{\lambda(\kappa-1)}{\lambda(\kappa-2)+\rho}\right]-\frac{\lambda(\kappa-1)}{\lambda(\kappa-1)+\rho}\left[1+\frac{\lambda(\kappa'-1)}{\lambda(\kappa'-2)+\rho}\right],$$

where Φ is defined in (22). Straightforward yet tedious calculations reveal that this is equivalent to

$$\Phi\lambda(\rho-\lambda)[\lambda(\kappa-1)+\rho][\lambda(\kappa'-1)+\rho] > \lambda(\rho-\lambda)\left[[\lambda(\kappa-1)+\rho][\lambda(\kappa'-1)+\rho]-\rho\lambda\right]+2\lambda^4(\kappa-1)(\kappa'-1).$$

From this we get that $\bar{G}_\kappa > \bar{G}_{\kappa'}$ for $\kappa' > \kappa \geq 2$ is equivalent to $f(\kappa,\kappa') > 0$, where $f(\kappa,\kappa')$ is given by

$$f(\kappa,\kappa') = \lambda^2(\kappa-1)(\kappa'-1)\left[(\Phi-1)(\rho-\lambda)-2\lambda\right] + \rho(\rho-\lambda)\left[(\Phi-1)\left[\lambda(\kappa+\kappa'-2)+\rho\right]+\lambda\right]. \quad \text{(C.15)}$$

To continue, treat κ as a real number where necessary, define the binomial $g(\kappa) \equiv f(\kappa,\kappa+1)$ and observe that $g''(k) \geq 0$ with $\lim_{k\to\infty} g(k) = \infty$ when $\Phi \geq (\rho+\lambda)/(\rho-\lambda)$ and that $g''(\kappa) < 0$ with $\lim_{k\to\infty} g(k) = -\infty$ when $\Phi < (\rho+\lambda)/(\rho-\lambda)$. Moreover, for $\kappa = 2$ we have

$$g(2) = 2\lambda^2\left[(\Phi-1)(\rho-\lambda)-2\lambda\right]+\rho(\rho-\lambda)\left[(\Phi-1)(3\lambda+\rho)+\lambda\right],$$

which is increasing in Φ, $\partial g(2)/\partial\Phi > 0$.

Claim (a). If $\Phi < \rho/(\lambda+\rho)$, then $\Phi < (\rho+\lambda)/(\rho-\lambda)$, in which case we know that the binomial $g(\kappa)$ is strictly convex, and that $g(\kappa)$ diverges to minus infinity when κ grows large. Moreover, plugging in $\Phi = \rho/(\lambda+\rho)$ in the expression for $g(2)$ above, we obtain $g(2) < 0$, which together with the fact that $g(2)$ decreases in Φ implies that $g(2) < 0$ for all $\Phi \leq \rho/(\lambda+\rho)$. That is, we have $g(\kappa) < 0$ for all $\kappa \geq 2$, implying that $\bar{G}_\kappa$ is strictly increasing for all $\kappa \geq 2$. Finally, because $\Phi < \rho/(\lambda+\rho)$ implies

$\bar{G}_1 < \bar{G}_2$ we obtain that $\bar{G}_\kappa$ (and, hence, G_κ for all sufficiently small $\phi > 0$) is also strictly increasing for all $\kappa \geq 1$, together with Corollary 1 giving the claim.

Claim (b). Observe that for $\Phi = (\rho+\lambda)/(\rho-\lambda)$ it holds $g(2) > 0$. By an analogous argument as above, this implies that $\bar{G}_\kappa > \bar{G}_{\kappa+1}$ for all $\kappa \geq 2$ when $\Phi > (\rho+\lambda)/(\rho-\lambda)$. Moreover, recall that $\bar{G}_1 > \bar{G}_2$ iff $\Phi > \rho/(\rho+\lambda)$. Appreciating that $(\rho+\lambda)/(\rho-\lambda) \geq \rho/(\rho+\lambda)$ then yields that $\bar{G}_\kappa$ (and, hence, G_κ for all sufficiently small $\phi > 0$) is strictly decreasing for all $\kappa \geq 1$, together with Corollary 1 giving the claim.

Claim (c). It remains to analyze the case $\Phi \in (\rho/(\lambda+\rho), (\rho+\lambda)/(\rho-\lambda))$. Doing so, I suppose Π_{m-1} and Π_m are such that no two elements in the sequence $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ are equal, which holds for generic values of Π_{m-1} and Π_m, as is readily verified from (C.14).

I first argue for the existence of $\underline{k}$ and $\overline{k}$. We know from above that $\Phi > \rho/(\lambda+\rho)$ implies $\bar{G}_1 > \bar{G}_2$. Also, we know that, as a consequence of $\Phi < (\rho+\lambda)/(\rho-\lambda))$, the binomial $g(\kappa)$ is strictly convex and that $g(\kappa)$ diverges to minus infinity when κ grows large. Together with the assumption that all elements in $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ are distinct, the following cutoff is well defined:

$$\overline{k} = \min\{\kappa \in \{2, 3, ..., |N|\} : g(\kappa) < 0\},$$

with the usual convention that $\min\{\emptyset\} = |N|$. Because $g(\kappa) > 0$ for all natural $\kappa < \overline{k}$ and $g(\kappa) < 0$ for all natural $\kappa \geq \overline{k}$, together with $\bar{G}_1 > \bar{G}_2$, it thus holds $\bar{G}_\kappa > \bar{G}_{\kappa+1}$ for all $\kappa \in \{1, ..., \overline{k}-1\}$ and $\bar{G}_\kappa \leq \bar{G}_{\kappa+1}$ for all $\kappa \in \{\overline{k}, ..., |N|\}$. That is, the cutoff index $\overline{k}$ refers to the lowest element in the sequence $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$.

Moreover, because the sequence $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ is first decreasing, up to $\overline{k}$, and then increasing, the following cutoff is also well defined:

$$\underline{k} = \begin{cases} 1 & \text{if } \bar{G}_1 < \bar{G}_{|N|} \\ \min\{\kappa \in \{2, 3, ..., \overline{k}\} : f(\kappa, |N|) < 0\} & \text{if } \bar{G}_1 > \bar{G}_{|N|}, \end{cases}$$

again with the convention that $\min\{\emptyset\} = \overline{k}$. By construction, the index $\underline{k}$ refers to the earliest element in $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ that is below $\bar{G}_{m-1}^{|N|}$. Recalling that we have $\operatorname{sgn}(G_\kappa - G_{\kappa'}) = \operatorname{sgn}(\bar{G}_\kappa - \bar{G}_{\kappa'})$ for all sufficiently low ϕ whenever $\operatorname{sgn}(\bar{G}_\kappa - \bar{G}_{\kappa'}) \in \{-1, 1\}$, the first claim in (c) then follows from Lemma C.1 together with Proposition 1.

Next, I argue that both $\underline{k}$ and $\overline{k}$ are increasing ceteris paribus in Φ, which gives the claim w.r.t. Π_{m-1} and Π_m. Observe that both $f(\kappa, \kappa')$ and $g(\kappa)$ are increasing in Φ. It follows from above that $g(\kappa)$ is decreasing at $\kappa = \kappa^*$ solving $g(\kappa^*) = 0$. Consequently, κ^* is increasing in Φ, implying that the same holds true for $\overline{k}$. Moreover, by construction of $\underline{k}$ it hold $f(\underline{k} - 1, |N|) > 0$ and $f(\underline{k}, |N|) < 0$. Together with the observation that $f(.,.)$ increases in Φ we get that $\underline{k}$ cannot be decreasing.

Finally, I claim that for every sufficiently high ρ, there is a number of players, $|N|$, together with values for Φ such that $\underline{k} < \overline{k}$. I show this by example. Note that if Φ is close to 1 then $f(\kappa, \kappa')$ is approximately

$$f(\kappa, \kappa') \approx -2\lambda^2(\kappa - 1)(\kappa' - 1)\lambda + \rho(\rho - \lambda)\lambda. \tag{C.16}$$

The cutoff $\overline{k}$ satisfies $f(\overline{k}, \overline{k} + 1) < 0$ and $f(\overline{k} - 1, \overline{k}) > 0$ while the cutoff $\underline{k}$ satisfies $f(\underline{k}, |N|) < 0$ and $f(\underline{k} - 1, |N|) > 0$. From (C.16) we see that $\overline{k}$ is independent of $|N|$, while $\underline{k}$ decreases in $|N|$, because $f(\kappa, \kappa')$ decreases both in κ and κ'. Last, $f(\kappa, \kappa')$ increases in ρ. Hence, whenever ρ is sufficiently large but finite, then $\overline{k}$ is greater than three, and $|N|$ can be chosen such that $2 \leq \underline{k} < \overline{k}$. $\square$

D More on Vanishing Moral Hazard

This appendix treats the case $\rho \leq \lambda$. We have the following result:

Proposition D.1. *Take any state $s = (\sigma, N, \xi)$ with $\sigma = m - 1$ and $|N| \geq 2$ and suppose $\rho \leq \lambda$. For any equilibrium and every $\phi > 0$ sufficiently close to zero:*

(a) *If $\Phi < \rho/(\lambda + \rho)$, then $\{G_\kappa\}_{\kappa=1}^{|N|}$ is strictly increasing and, hence, any profile $\{T^*_\kappa\}_{\kappa=1}^{|N|} \in \mathcal{T}_s$ is completely symmetric.*

(b) *If $\Phi > (\lambda(|N| - 2) + \rho)/(\lambda(|N| - 1) + \rho)$, then all $\{T^*_\kappa\}_{\kappa=1}^{|N|} \in \mathcal{T}_s$ have $|N| - 1$ investors choosing the same deadline and one investor choosing a strictly longer deadline.*

(c) *For a.e. $\Phi \in (\rho/(\lambda + \rho), (\lambda(|N| - 2) + \rho)/(\lambda(|N| - 1) + \rho))$, both kind of profiles described in (a) and (b) above are mutually optimal.*

Proof of Proposition D.1. Consider $\bar{G}_\kappa$ defined in (C.14) from the proof of Proposition 5. While the numerator of $\bar{G}_\kappa$ strictly increases in κ, the denominator weakly decreases in $\kappa \geq 2$ if an only if $\rho \leq \lambda$. As a consequence, $\bar{G}_\kappa$ strictly increases in $\kappa \geq 2$ if $\rho \leq \lambda$. Further, note that $\bar{G}_1 < \bar{G}_\kappa$ holds if and only if $\frac{\lambda(\kappa-2)+\rho}{\lambda(\kappa-1)+\rho} < \Phi$, where Φ is defined in (22).

Consequently, in case (a) we have $\bar{G}_1 < \bar{G}_2$, giving that $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ is strictly increasing. But then, recalling from the proof of Proposition 5 that we have $\text{sgn}(G_\kappa - G_{\kappa'}) = \text{sgn}(\bar{G}_\kappa - \bar{G}_{\kappa'})$ for all sufficiently low ϕ whenever $\text{sgn}(\bar{G}_\kappa - \bar{G}_{\kappa'}) \in \{-1, 1\}$, the sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$ is strictly increasing for all $\phi > 0$ sufficiently small. The claim thus follows from Corollary 1.

In case (b) we have $\bar{G}_1 > \bar{G}_k$. Consequently, the elements of $\{G_\kappa\}_{\kappa=1}^{|N|}$ are distinct for all $\phi > 0$ sufficiently small. Letting $\underline{k} = \overline{k} = 2$, the claim then follows from Lemma C.1 together with Proposition 1.

Last, in case (c), we have $\bar{G}_{|N|} > \bar{G}_1 > \bar{G}_2$. Since $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ strictly increases in $\kappa \geq 2$, it might be that $\bar{G}_1$ is equal to some $\bar{G}_\kappa$, $\kappa \geq 3$. As can easily be verified from (C.14), this only happens for non-generic values of Π_{m-1} and Π_m. Consequently, for generic values of Φ, all elements of $\{G_\kappa\}_{\kappa=1}^{|N|}$ are distinct for all $\phi > 0$ sufficiently small. Letting $\underline{k} = 1$ and $\overline{k} = 2$, the claim then follows again from Lemma C.1 together with Proposition 1. □