Fear of Hiking?
Monetary Policy and Sovereign Risk

Martin Wolf and Leopold Zessner-Spitzenberg*

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Abstract

What are the implications of a rise in interest rates by the central bank of a monetary union for sovereign borrowing decisions and sovereign default risk in a union member? We study this question in a quantitative sovereign default model and obtain two results. First, the sovereign's incentives to borrow following a monetary tightening are shaped by two competing effects, a positive income and a negative substitution effect. Second, a critical threshold for debt to GDP exists above which the income effect is dominant, implying that a monetary tightening increases debt levels and the risk of a sovereign default. We quantify this "Fear of Hiking" zone and study its business cycle properties in an application of the model to the euro area.

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Keywords: monetary policy, sovereign debt, sovereign debt crisis, sovereign default, spreads, currency union, monetary-fiscal interaction

*Martin Wolf: University of St. Gallen and CEPR; martin.wolf@unisg.ch. Leopold Zessner-Spitzenberg: Humboldt University of Berlin; leopold.zessner-spitzenberg@hu-berlin.de. We thank Javier Bianchi, Guido Cozzi, Sergio de Ferra, Winfried Koeniger and Paul Pichler for very useful comments. We also thank conference participants at the D-A-CH 2021 workshop "Towards a micro-funded theory of monetary policy", and seminar participants at Humboldt University of Berlin, University of St. Gallen, University of Konstanz and University of Vienna for useful comments. Leopold Zessner-Spitzenberg gratefully acknowledges financial support by the Austrian Science Fund (FWF) Grant Nr. I 3840-G27.

"*As former central bankers and as European citizens, we are witnessing the ECB's ongoing crisis mode with growing concern. [...] In contrast, the suspicion that behind this measure lies an intent to protect heavily indebted governments from a rise in interest rates is becoming increasingly well founded.*" Open letter to the ECB in October 2019—see footnote 1.

1 Introduction

The aftermath of the Great Recession has been characterized by high levels of public debt and volatile spreads in several euro members. The ongoing Covid crisis accompanied by unprecedented public support measures has reinforced these trends. In this environment, if the ECB raises key interest rates, what will be the implications for debt accumulation and sovereign risk (the risk of a sovereign default)? As exemplified by the above quote, this question is at the forefront of a heated debate.[1]

In this paper, we put forward a framework to study systematically the implications of a monetary tightening in a currency union for sovereign borrowing decisions and default incentives in a small union member. The core of our model is the standard sovereign default framework originally due to Eaton and Gersovitz (1981). We show analytically that, when the union-wide central bank raises interest rates, the implications for debt accumulation in the small union member are shaped by two opposing effects, a positive income and a negative substitution effect. Our main insight is that a threshold level for external public debt to GDP exists above which the income effect is dominant, implying that a rate rise increases debt accumulation and sovereign risk.[2] We quantify this "Fear of Hiking" zone by calibrating our model to Italy. We find that, on average, the economy is in the Fear of Hiking zone when external debt to GDP exceeds 50%, and that visiting the Fear of Hiking zone becomes more likely in a recession.

We construct a model of a small open economy within a currency union. The domestic government borrows without commitment, and its debt is held by risk-neutral investors inside the union but outside the domestic country. When the central bank changes interest rates, the outside option of investors changes. Through this channel, a monetary tightening has implications for government finances in the small union member in our framework. As in Chatterjee and Eyigungor (2012), the government issues long-term debt. Moreover, as in recent work by Bianchi and Mondragon (2018), Bianchi et al. (2019) and others, we consider a sticky-wage environment, with two implications. First, wage stickiness implies also price stickiness, hence a monetary tightening changes investors' outside option also in real terms. Second, a monetary tightening also changes output and therefore incomes in the domestic country, as it shifts aggregate demand for goods which are produced

[1] In October 2019, several former senior European central bankers formulated an open letter to the ECB, criticizing then-President Mario Draghi for the ECB's ultra-loose monetary stance. Only two weeks later, former-ECB president Jean-Claude Trichet formulated a response letter, criticizing in turn the open letter. The open letter can be found here: https://www.ft.com/content/71f90f42-e68f-11e9-b112-9624ec9edc59; and Trichet's response letter can be found here: http://prod-upp-image-read.ft.com/c8b98512-ec31-11e9-85f4-d00e5018f061.

[2] As is standard in the Eaton and Gersovitz (1981) framework, all public debt in our model is external, that is, held by foreign investors.

domestically.

We start by showing that the effects on sovereign borrowing following a *temporary* monetary tightening can be framed in terms of two familiar forces which push the incentives to take on debt in opposite directions: a substitution and an income effect. The substitution effect is that borrowing becomes more expensive, implying that borrowing declines. The income effect is that the domestic country becomes poorer, implying that borrowing rises.[3] We then provide a characterization of the conditions under which either effect is dominant. In particular, we show that a threshold for debt to GDP exists above (below) which the income (substitution) effect is dominant. Hence a monetary tightening raises debt accumulation, and by implication the risk of a sovereign default in the future (sovereign risk), whenever debt to GDP is high enough.[4,5]

The threshold has an intuitive interpretation and depends on three parameters i) the elasticity of intertemporal substitution (EIS) ii) the fraction of debt which needs to be refinanced (coupon payment) in a given period and iii) the fraction of domestic demand that falls on domestically-produced goods (home bias). Intuitively, the EIS governs the strength of the substitution effect, hence a higher EIS implies that the threshold rises (i.e., the region where the substitution effect dominates becomes larger). In turn, a larger coupon payment means that a larger share of debt becomes exposed to the rise in interest rates, which strengthens the income effect and thus reduces the threshold. Last, home bias determines to what extent changes in domestic demand feed into domestic incomes. If this feedback is large, it becomes more costly for the domestic government to reduce borrowing following a rise in interest rates, as this also reduces domestic incomes. Hence a larger home bias acts like a strengthening of the income effect, reducing the threshold. In addition to parameters, the threshold also depends on the state of the business cycle. In particular, we show that the threshold inherits the business cycle properties of the trade balance to GDP ratio. Everything else equal, a more negative trade balance reduces the threshold, making it more likely that a monetary tightening raises borrowing and sovereign risk. Intuitively, this happens because a negative trade balance reflects net capital inflows. Hence a larger part of the debt becomes exposed to the rise in interest rates, strengthening the income effect.

We next discuss the case of a *persistent* monetary tightening. We show that adding persistence tends to increase the threshold characterizing the behavior of sovereign borrowing, as it implies a strengthening of the substitution effect. Hence a monetary tightening is more likely to reduce sovereign borrowing when the rate rise is persistent. However, we also show that adding persistence decouples borrowing decisions from sovereign risk; following a monetary tightening, sovereign risk may actually *rise* even though sovereign borrowing *declines*. In this case, therefore, two separate

[3] The income and substitution effects of changes in interest rates are familiar from the two-period consumption model that is typically taught to undergraduates. We explain how our results are linked to the two-period consumption model in the paper.

[4] Of course, a monetary tightening may also trigger an *immediate* default, a case which we also discuss in the paper. However, the main focus of our paper is on how monetary policy influences borrowing decisions and the risk of a sovereign default going forward, i.e. conditional on repayment in the current period.

[5] When the monetary tightening is temporary, then debt levels and the risk of a sovereign default in the future always move in the same direction. When the monetary tightening is persistent, this may no longer be the case, as we discuss below.

thresholds for debt to GDP may exist. The first threshold defines the level of debt to GDP above which sovereign borrowing rises following a rise in interest rates. The second threshold - necessarily below the first one - defines the level of debt to GDP above which sovereign risk rises following a rise in interest rates.

To quantify these effects we turn to an application of the model to countries in the euro area. We first conduct a back-of-the envelope calculation, contrasting model-implied thresholds for various countries. For example, we find that the threshold is currently around 50% in Italy.[6,7] We then solve the model numerically by calibrating it to Italy, matching some key moments such as mean external debt and the counter-cyclical trade balance. In the calibrated model, both debt to GDP and the threshold are equilibrium objects, depending on the economy's state variables. We identify the set of states which imply that debt to GDP lies above the threshold in the equilibrium of the model, and call this the "Fear of Hiking" zone. We then describe the properties of the Fear of Hiking zone over the business cycle. We find that the economy is in the Fear of Hiking zone about 71% of the time. We also show that visiting the Fear of Hiking zone correlates negatively with the business cycle. As a result, in the calibrated model, a monetary tightening is more likely to raise sovereign borrowing and sovereign risk when the economy is in a recession.

Our paper adds to the literature at the intersection of monetary policy and sovereign default. Within this literature, closest to us are contributions with an explicit focus on monetary unions, as well as papers which construct small open economy models and apply them to the euro zone context. For instance, de Ferra and Romei (2020) build a model of a monetary union and study the interaction between monetary policy and sovereign default in some member countries. While in our framework monetary policy is effectively exogenous, they also study optimal monetary policy. Among other results, they find that the possibility of default tends to make monetary policy more expansionary. In their framework, the central bank affects the economy through the relative price of non-tradables, while it has no effect on the safe interest rate which enters the price of debt. Our analysis is therefore complementary, because in our model the central bank affects the economy exclusively through the price of debt.[8] Among the group of papers studying small open economies, Na et al. (2018) show that a sovereign default has negative effects on employment when a country cannot devalue the exchange rate due to lack of an independent monetary policy (such as membership in a monetary union); Bianchi and Mondragon (2018) focus on rollover crises, showing that joining a monetary union leaves countries more vulnerable to such crises; Kriwoluzky et al. (2019) show that a sovereign debt crisis is reinforced when there is a break-up risk of the monetary union.[9] To the best of our knowledge, we are the first paper in this literature to study

[6] Recall that this number refers to *external* debt to GDP - not total debt to GDP. At the time of writing (end 2021), external debt to GDP is slightly above 50% in Italy, while total debt to GDP is above 150%.

[7] Here we discuss the threshold that applies to a temporary monetary tightening. Recall that in this case, the thresholds describing the behavior of sovereign borrowing and the behavior of sovereign risk are identical. In the paper, we also discuss how the quantitative results change when looking at a persistent monetary tightening.

[8] Another paper studying the link between monetary policy and sovereign default in a monetary union is Aguiar et al. (2015). However, their focus is non non-fundamental equilibria (rollover crises). In our model, we focus our attention on default due to bad fundamentals.

[9] Other papers at the intersection of monetary policy and sovereign default are Corsetti and Dedola (2016),

systematically the comparative statics of sovereign borrowing and default risk with respect to the safe interest rate. In our framework, understanding this link is important because the safe rate is the policy instrument set by the central bank. More generally, our findings may also inform other frameworks where an important driver of the business cycle is exogenous variation in the safe interest rate.[10]

In terms of results, we also add to a second literature which highlights the presence of non-linearities when public debt levels are high enough. For instance, the literature on "fiscal limits" highlights that when debt levels are high enough, countries are no longer able to stabilize their stock of debt through higher taxes which may trigger expectations of inflation (Davig et al., 2011). In models with self-fulfilling default, the "Crisis zone" is typically reached when debt levels are high enough, whereas self-fulfilling default may not occur for low debt level (Cole and Kehoe, 2000; Bianchi and Mondragon, 2018). Relatedly, a "gambling for redemption" motive arises when debt levels are high enough (Conesa and Kehoe, 2017). In contrast to these papers, in our analysis high debt levels change how monetary policy influences borrowing decisions and thereby sovereign risk, because a high debt stock implies that the income effect of bond price changes becomes dominant. When this happens, a monetary tightening leads to even higher borrowing by the sovereign and a higher risk of a sovereign default.

The rest of the paper is structured as follows. Section 2 introduces the model. Section 3 presents our main analytical insights, notably the existence of a threshold for debt to GDP above which the effects of monetary policy flip. In Section 4 we quantify the threshold and introduce the "Fear of Hiking" zone. Section 5 discusses some implications of our results for monetary policy and makes concluding remarks. An accompanying Appendix collects the proofs of all propositions as well as additional materials.

2 The model

In this section, we develop a model of a small open economy inside a monetary union. The economy is small, because domestic developments have no repercussions on the rest of the monetary union. The economy issues defaultable debt which is held by investors in the rest of the monetary union. A union-wide central bank sets short-term nominal interest rates. We assume the existence of nominal rigidities which implies that monetary policy has real effects. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$.

Bacchetta et al. (2018) and Arellano et al. (2020), among others. Corsetti and Dedola (2016) focus on the effects of unconventional monetary policy, showing that purchases of risky debt by the central bank can rule out self-fulfilling default. Bacchetta et al. (2018) show that by generating inflation, a central bank may rule out slow-moving debt crises (in the sense of Lorenzoni and Werning 2019). Arellano et al. (2020) develop a New Keynesian model with sovereign default risk ("NK-Default"). They show that in presence of default risk, inflation volatility increases.

[10] For instance, Johri et al. (2020) study the dependence of emerging market spreads on fluctuations in the U.S. treasury bill rate. They find the safe rate to be an important driver of emerging market borrowing decisions and spreads.

2.1 Households

The domestic economy is populated by a large number of identical households. Each household supplies one unit of labor in-elastically and there is no dis-utility from working. However, due to the presence of nominal wage rigidities to be described below, households may be able to sell less than their endowment on the labor market. For future reference, we say that the economy operates below potential / in a slack labor market whenever equilibrium employment is below households' unit-endowment. Households' preferences over consumption are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}, \qquad 0 < \beta < 1, \quad \sigma > 0, \tag{1}$$

where C_t is a composite of tradable goods produced domestically $C_{h,t}$ and tradable goods produced in the rest of the monetary union $C_{f,t}$

$$C_t = \zeta C_{h,t}^{1-\gamma} C_{f,t}^{\gamma}, \qquad 0 < \gamma < 1, \tag{2}$$

where $\zeta \equiv (1-\gamma)^{-(1-\gamma)}\gamma^{-\gamma}$. The parameter β captures impatience, the parameter σ measures risk aversion (equivalently, the inverse of the elasticity of intertemporal substitution), and the parameter $1-\gamma$ captures home bias in consumption. Note that we assume the elasticity of substitution between domestically produced and foreign tradable goods (the trade elasticity) is equal to one. This is without loss of generality, because the relative price between the two goods is going to be constant in equilibrium.

Each households' budget constraint is given by

$$P_{h,t}C_{h,t} + P_{f,t}C_{f,t} = W_t L_t + P_{h,t}T_t, \tag{3}$$

where $W_t L_t$ is labor income and where T_t is a lump-sum transfer payment from the government (tax if negative). In the budget constraint, the prices of domestic and foreign consumption goods are denoted $P_{h,t}$ and $P_{f,t}$, respectively. Cost minimization then leads to the demand functions for domestically produced and foreign goods

$$C_{h,t} = (1-\gamma)\frac{P_t}{P_{h,t}}C_t \tag{4}$$

$$C_{f,t} = \gamma \frac{P_t}{P_{f,t}}C_t, \tag{5}$$

with the cost-minimizing price index (the consumer price index) $P_t = P_{h,t}^{1-\gamma}P_{f,t}^{\gamma}$.

As is standard in the sovereign default literature, we assume households have no direct access to international financial markets. Rather, the decision to borrow or save is taken by the government, and households receive the proceeds from these transactions in the form of transfer payments T_t.

2.2 Firms

The economy is populated by a large number of identical firms which are owned by the households. Firms produce the domestic consumption good by using the technology $Y_t = L_t$, where L_t is labor demand. Firms' profits are $P_{h,t}Y_t - W_tL_t$. The first order condition is

$$P_{h,t} = W_t. \tag{6}$$

Firms make zero profits in equilibrium.

2.3 Nominal rigidities

We consider an economy with frictions in the adjustment of nominal wages.[11] The presence of nominal wage rigidities plays two roles in the model. First, it creates the possibility of involuntary unemployment. Second, it implies that monetary policy has real effects. Indeed, as we will see, prices inherit the wage stickiness, so that the central bank can change the real interest rate of the monetary union through movements in the nominal interest rate.

Throughout the paper, we assume a particularly simple friction in the adjustment of nominal wages, namely that

$$W_t = \bar{W}. \tag{7}$$

By combining (7) with labor demand (6), this implies that $P_{h,t} = \bar{W}$, that is, the price of domestically produced goods is constant as well. We make symmetric assumptions in the rest of the monetary union, implying that also $P_{f,t} = \bar{W}$. This implies that the consumer price index is constant as well, $P_t = \bar{W}$. Hence, all prices in the monetary union are constant in equilibrium. For simplicity, we normalize $\bar{W} = 1$.

A discussion of the assumption of zero inflation can be found in Section 2.8.

2.4 Domestic government

The government collects taxes $-T_t$ from households and issues non-contingent, defaultable long-term debt B_t which is held by foreign investors. To model long-term debt, we follow Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) and assume that debt pays geometrically declining coupons. In particular, a bond issued in period t promises to pay $\tilde{\mu}(1-\mu)^{j-1}$ in period $t+j$ for all $j \geq 1$, where $\tilde{\mu} \equiv \mu + \iota$ and ι is the nominal interest rate in steady state.[12] At the start of each period, the government can choose to default on its debt. Conditional on repayment, the

[11] A growing body of evidence emphasizes how nominal wage rigidities represent an important transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by Olivei and Tenreyro (2007), who show that monetary policy shocks in the US have a bigger impact on output in the aftermath of the season in which wages are adjusted. Micro-level evidence on the importance of nominal wage rigidities is provided by Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2014) and Fabiani et al. (2010).

[12] We normalize the debt service payment of the bond to $\mu + \iota$, so that the default-free bond price in steady state equals 1. This allows us to interpret B_{t-1}/Y_t as debt to GDP, as we do in the rest of the paper.

budget constraint of the government is

$$\tilde{\mu} B_{t-1} + T_t = q_t(B_t - (1-\mu)B_{t-1}), \tag{8}$$

where we denote q_t the price of debt. The left-hand side captures the government's expenditure, consisting of coupon payments plus transfer payments. In turn, the right-hand side captures the government's income, given by the net issuance of new debt.

In case the government defaults, the domestic country is excluded from credit markets for a random number of periods. Moreover in this case, a utility loss $\kappa(Y_t, \xi_t)$ materializes, which depends non-negatively on output Y_t and is subject to exogenous shifts through the variable ξ_t. The utility loss captures various un-modeled costs of default, such as a reputation loss or turmoil in domestic financial markets. As is well known, without the utility loss from default, plausible levels of debt can not be generated in equilibrium.[13] The dependence of the utility loss on Y_t implies that the government' incentives to default are higher in a recession. Finally, the fact that κ can fluctuate through the variable ξ_t implies that the default decision becomes (somewhat) detached from the business cycle. The fact that default cannot be perfectly explained by fundamentals is a key feature in emerging market data (Aguiar et al., 2016) and it helps us to calibrate the model to observed spread levels in the quantitative application of the model in Section 4.

2.5 Foreign lenders

Debt issued by the domestic government is held by foreign investors. Foreign investors understand the governments' incentives to default, inducing them the charge an ex-ante lower price (a spread over the risk free rate). Assuming risk neutral foreign lenders, the price of debt is given by

$$q_t = \frac{\mathbb{E}_t(1-\delta_{t+1})(\tilde{\mu} + (1-\mu)q_{t+1})}{1+i_t}, \tag{9}$$

where δ_{t+1} is an indicator variable that takes the value of 1 if the government defaults in period $t+1$.

The opportunity cost of funds for foreign investors is the nominal interest rate i_t. It is the nominal interest rate set by the union-wide monetary authority. The fact that investors' opportunity cost is i_t reflects that investors are also members of the monetary union. Indeed, we assume that all debt issued by the domestic economy is held inside the monetary union, and that the monetary union as a whole is a closed economy.[14]

[13] In the original work of Eaton and Gersovitz (1981) and Arellano (2008), rather than assuming a utility loss, the authors assumed an output loss that materializes in the event of default. Both versions of the model have been extensively used and produce very similar results. We follow Bianchi and Mondragon (2018) and assume that the loss is in terms of utility.

[14] Without this assumption, investors from outside the monetary union would rush to buy assets inside the monetary union in case their opportunity cost of funds lies below the rate set by the union-wide monetary authority. A model of Eurozone debt dynamics, where capital flows are possible both inside the monetary union and between the monetary union and investors outside the monetary union, is provided by de Ferra (2020).

2.6 Market clearing

Domestic goods market clearing is given by

$$Y_t = (1-\gamma)C_t + X_t. \tag{10}$$

It implies that domestic output is equal to domestic plus foreign demand for domestically produced goods. Domestic demand equals the home bias coefficient times total domestic consumption, from equation (4). In turn, foreign demand is given by X_t, and is treated as an exogenous variable. In the quantitative application of the model in Section 4, X_t will be the main driver of the domestic business cycle. In what follows, we assume that X_t takes values in the interval $[X_{\min}, X_{\max}]$. Moreover, we assume that $X_{\max} \leq \gamma$, implying that the domestic economy suffers from a negative output gap in the absence of trade deficits, see Section 2.8 for an explanation. Notice that equation (10) can be seen as the equation *determining* Y_t: due to the assumption of nominal rigidities, output is endogenous and driven by (internal and external) aggregate demand. Firms satisfy this demand by hiring the appropriate number of hours, which households supply as long as labor demand is below their unit-endowment.

The second equilibrium condition is the domestic resource constraint. Combining equations (3) and (8), and using that $C_{h,t} + C_{f,t} = C_t$ (this can be easily seen by combining equations (4)-(5), after using that prices are constant), we obtain

$$C_t = Y_t - \tilde{\mu}B_{t-1} + q_t(B_t - (1-\mu)B_{t-1}), \tag{11}$$

which holds in periods where the government chooses to repay. For future reference, we note that the resource constraint can also be written in terms of the trade balance. By using equations (5) and (10), we can see that $C_t = Y_t + C_{f,t} - X_t$, implying that

$$X_t - C_{f,t} = \tilde{\mu}B_{t-1} - q_t(B_t - (1-\mu)B_{t-1}). \tag{12}$$

The left-hand side is the trade balance, the difference between exports and imports. The right-hand side captures how the trade balance is financed. For instance, a trade balance deficit is financed by issuing new debt in excess of the coupon payment.

Finally, conditional on default, the resource constraint is simply

$$C_t = Y_t, \tag{13}$$

and the trade balance is equal to zero.

2.7 Recursive government problem and Markov equilibrium

We study the optimal policy of a benevolent government that chooses sequentially without commitment. In each period, the government decides whether to default on its debt. The equilibrium concept is therefore the one of Markov perfect equilibrium. Intuitively, in each period the government takes as given its own action in the following period, while internalizing that its current action influences its action in the future through endogenous state variables. In a rational expectations equilibrium, the government's current and expected future actions, conditional on state variables, are compatible.

As is standard in the literature, we state the government's problem using recursive notation, that is, we henceforth omit time subscript t. Denote $s = (X, \xi)$ the exogenous state. The other state variables are B, the current level of debt, and i, the nominal interest rate set by the central bank. When the government has access to financial markets, at the start of the period, it compares the value of repayment and the value of default, denoted by respectively $V^r(B,i,s)$ and $V^\delta(i,s)$

$$V(B,i,s) = \max_{\delta\in\{0,1\}} \left\{(1-\delta)V^r(B,i,s) + \delta V^\delta(i,s)\right\}. \tag{14}$$

Let $U(C)$ denote the utility flow. The value of repayment is

$$V^r(B,i,s) = \max_{B',C} \left\{U(C) + \beta\mathbb{E}V(B',i',s')\right\} \tag{15}$$

subject to the set of constraints

$$\begin{aligned}
&i) && C = Y - \tilde{\mu}B + q(B',i,s)(B' - (1-\mu)B)\\
&ii) && Y = (1-\gamma)C + X\\
&iii) && Y \leq 1.
\end{aligned}$$

In the set of constraints, the price of debt q is taken as given by the government, but the government understands that q is a function of its borrowing decision B'. The last constraint captures that labor demand cannot be larger than households' endowment.

In turn, the value of default is

$$V^\delta(i,s) = U\left(\frac{X}{\gamma}\right) - \kappa\left(\frac{X}{\gamma},\xi\right) + \beta\mathbb{E}\left(pV(0,i',s') + (1-p)V^\delta(i',s')\right), \tag{16}$$

where we denote p the probability of re-entering credit markets in the next period. Here we have combined the goods market clearing condition (10) and the resource constraint under financial autarky (13) to obtain $Y = C = X/\gamma$, that is, equilibrium consumption and output become fully determined by foreign demand.[15]

Definition 1 (Markov-perfect equilibrium) *For a given law of motion governing i and s, a Markov-perfect equilibrium is a set of value functions $\{V(B,i,s), V^r(B,i,s), V^\delta(i,s)\}$, a set of policy functions $\{\delta(B,i,s), B'(B,i,s), C(B,i,s)\}$ and a pricing function $q(B',i,s)$ such that*

1. *Given the bond price schedule, the value and policy functions solve equations* (14)-(16)

[15] Note that $Y \leq 1$ holds under financial autarky, because we have assumed that X can never be larger than γ. Hence labor demand is always at most households' endowment.

2. The bond price schedule satisfies

$$q(B', i, s) = \frac{\mathbb{E}(1 - \delta(B', i', s'))(\tilde{\mu} + (1 - \mu)q(B'', i', s'))}{1 + i},$$

where $B'' = B'(B', i', s')$.

2.8 Discussion of economic environment

In what follows we try to understand how changes in the interest rate set by the union-wide monetary authority affect debt accumulation and default incentives in the small union member. We study this issue in light of recent events in the euro area. We focus on the time after the 2008 Global Financial Crisis, where debt levels have been elevated and spreads have been volatile in several euro countries (notably in Spain, Italy, Portugal and Greece).

To be able to speak to developments in these countries, we have made two assumptions to try to capture these countries' experiences, which we add to an otherwise-standard model of sovereign default with long-term debt.

The first assumption is that inflation is low and stable due to nominal rigidities. While these countries have seen high levels of (wage) inflation before 2008, inflation rates have been low and stable after 2008, in spite of high unemployment. This finding has triggered interests in the macro effects of downward nominal wage rigidity during the European crisis (see for instance Schmitt-Grohé and Uribe (2016), Na et al. (2018), Bianchi et al. (2019) and Wolf (2020)). Because we only seek to capture the post-2008 period, here we take a short-cut and assume that nominal rigidity binds in all states of the business cycle.[16,17]

The second assumption is that the economy has a tendency to operate in a slack labor market due to insufficient demand. To see this feature of the model, assume that the trade balance is zero, implying that $C_t = Y_t$. Combine this with the goods market clearing condition (10) to see that $Y_t = X_t/\gamma$. Recalling that we assumed $X_{\max} \leq \gamma$, this shows that $Y_t < 1$ under a zero trade balance for typical realizations of X_t. The economy suffers a demand shortage, in the sense that demand is not strong enough to maintain the economy at full employment when the trade balance is zero. How can the government respond to the demand shortage? Combine again equations (10), (11) and (12), this time for an arbitrary trade balance, to obtain

$$Y_t = -\frac{1 - \gamma}{\gamma}(X_t - C_t^f) + \frac{X_t}{\gamma}. \tag{17}$$

As this equation shows, the government can fight the demand shortage by running a trade balance deficit ($X_t - C_t^f < 0$). A similar "demand channel" operates in the model by Bianchi et al. (2019), who focus explicitly on the experience of Spain. In their model, the government faces a trade-off between stabilizing domestic incomes through higher government spending and incurring higher

[16] All our results would be unchanged if we had assumed a constant rate of trend inflation in the monetary union.

[17] At a conceptual level, abstracting from inflation is also convenient as the relationship between sovereign debt and inflation is already the subject of a vast literature. This literature is mentioned in the related literature section of the introduction.

spreads. The trade-off arises, because government spending triggers a trade balance deficit, that is, it is financed with foreign debt. In our model, the government can raise consumption demand and domestic incomes by running a trade balance deficit, but it also faces the trade-off that this implies more borrowing and thus potentially higher spreads.

Our model also nests the more common framework in the sovereign default literature where output is exogenous as a special case. To see this, note that the strength of the demand channel is governed by the parameter γ and it disappears when γ approaches 1 (zero home bias). In this case, as equation (17) reveals, output is fully exogenous with respect to the trade balance as it is exclusively driven by foreign demand. Intuitively, when home bias is zero, raising domestic consumption will not raise domestic incomes, as all of households' demand falls on foreign goods. In this case, our model becomes isomorphic to the model in Chatterjee and Eyigungor (2012), as output behaves like a stochastic endowment.[18]

In sum, we believe that our environment of stable inflation, demand shortage and slack in the labor market, high debt levels and spreads characterizes well several euro countries' experiences in the post-2008 period. This makes our model a good laboratory to ask about the effects of monetary interventions on borrowing and the risk of default in these countries.

3 Monetary policy, sovereign borrowing and sovereign risk

In this section, we study analytically how the domestic government's incentives to borrow and default are shaped by the union-wide monetary policy. We show that a monetary tightening can either raise or reduce borrowing and the risk of a sovereign default. Our main insight is that a threshold for debt to GDP exists above which the effects of monetary policy flip. When debt to GDP is below the threshold, a monetary tightening reduces borrowing and the risk of default going forward, whereas the opposite happens when debt to GDP is above the threshold.

3.1 Income and substitution effect

To illustrate the forces shaping debt accumulation, we focus on the government's optimality condition in a period where it has access to financial markets. By combining constraints *i*) and *ii*) and inserting them in the value function (15), we obtain

$$V^r(B,i,s) = \max_{B'} \left\{ U\left(\frac{1}{\gamma}(X - \tilde{\mu}B + q(B',i,s)(B' - (1-\mu)B))\right) + \beta \mathbb{E} V(B',i',s') \right\}$$

[18] To be clear, even when home bias is zero output is demand-determined in our model. However, it is determined only by foreign demand, which is exogenous and stochastic. Hence the model behaves like a model where output is a stochastic endowment.

subject to the full-employment constraint $Y \leq 1$. We consider a state where the full employment constraint is slack.[19] Then the first order condition is[20]

$$\frac{U'(C)}{\gamma}\left(q(B',i,s)+\frac{\partial q(B',i,s)}{\partial B'}(B'-(1-\mu)B)\right)+\beta\frac{\partial}{\partial B'}\mathbb{E}V(B',i',s')=0, \tag{18}$$

which relates the marginal utility of consumption $U'(C)$ to the bond price schedule $q(B',i,s)$ and the continuation value $V(B',i',s')$. As we can see, the government borrows according to a standard Euler equation, but subject to an endogenous borrowing limit as the price of debt is endogenous. Specifically, a large (negative) derivative $\partial q(B',i,s)/\partial B'$ indicates a steep price curve which, everything else equal, discourages further borrowing.

In what follows, we study the effects of a temporary monetary tightening, that is, of a rise in i which leaves agents' expectations about the future path of monetary policy unchanged. Stated differently, the future interest rate i' becomes independent of the current interest rate i. In this case, a change in i has no *direct* effect on future variables, but only *indirect* ones through its effect on borrowing B' (the endogenous state variable). This makes the problem tractable. We study persistent interventions below, in Section 3.6.

The following proposition establishes that a rise in i can either raise or reduce B', due to an income and a substitution effect. Moreover, we provide a clean characterization of the conditions under which either of the two effects is dominant. Because it is instructive, we provide a sketch of the proof in the main text. A full proof can be found in Appendix A.1.

Proposition 1 *Assume that the government takes borrowing decisions according to the Euler equation* (18)*. Consider a temporary monetary policy change. Then*

$$\frac{\partial B'}{\partial i}>0 \iff \frac{\gamma}{\sigma}<\frac{q(B',i,s)(B'-(1-\mu)B)}{C}. \tag{19}$$

Proof (sketch). Start by extracting the interest rate $1+i$ in equation (18)

$$\frac{1}{\gamma}U'\left(\frac{1}{\gamma}\left(X-\tilde{\mu}B+\frac{\tilde{q}(B',s)}{1+i}(B'-(1-\mu)B)\right)\right)\left(\tilde{q}(B',s)+\frac{\partial\tilde{q}(B',s)}{\partial B'}(B'-(1-\mu)B)\right)$$
$$+(1+i)\beta\frac{\partial}{\partial B'}\mathbb{E}V(B',i',s')=0,$$

where we define the bond price exclusive of the interest rate $\tilde{q}(B',s)=q(B',i,s)(1+i)$. Because the previous equation must hold in all states (B,i,s), it can be viewed as defining borrowing B' as a function of i implicitly. It reveals that changes in i affect B' through two competing channels. First, a rise in i reduces the amount of resources that can be raised for a given amount of new debt

[19] In a state where the full employment constraint binds, borrowing decisions are not determined by an optimality condition, but are the mechanical outcome of combining constraints $i)-ii)$ and $Y=1$.

[20] To derive the first order condition, we implicitly assume that all functions in the government's problem are differentiable. We do not make these assumptions in Section 4 where we solve the model numerically, as we solve the model using value function iteration. This being said, we verify in the numerical solution of the model that all conditions derived in this section describe well the equilibrium of the model; that is, we find that the Euler equation does in fact describe well the choices made by the sovereign.

$B' - (1-\mu)B$, the term inside the marginal utility of consumption. This corresponds to an *income effect* of higher interest rates. Second, the interest rate i multiplies the time preference rate β, thus changing the relative price of consumption across periods. This corresponds to a *substitution effect* of higher interest rates. There is also an interaction with the fact that output is endogenous in our model, which we called the demand channel in Section 2. Indeed, borrowing B' raises consumption by a factor $1/\gamma$, reflecting that domestic incomes increase when domestic consumption increases (recall equation (10)).

By differentiating with respect to i, the income effect appears as

$$\mathcal{I} \equiv \frac{U''(C)}{\gamma^2}\left(-\frac{\tilde{q}(B',s)}{(1+i)^2}(B'-(1-\mu)B)\right)\left(\tilde{q}(B',s)+\frac{\partial \tilde{q}(B',s)}{\partial B'}(B'-(1-\mu)B)\right). \tag{20}$$

In turn, the substitution effect is simply $\beta(\partial/\partial B')\mathbb{E}V(B',i',s')$. By using the Euler equation, this can also be written as

$$\mathcal{S} \equiv -\frac{1}{1+i}\frac{U'(C)}{\gamma}\left(\tilde{q}(B',s)+\frac{\partial \tilde{q}(B',s)}{\partial B'}(B'-(1-\mu)B)\right). \tag{21}$$

When the government is a net borrower in the current period ($B'-(1-\mu)B>0$), the income effect is strictly positive ($\mathcal{I}>0$).[21] In contrast, the substitution effect is unambiguously negative ($\mathcal{S}<0$). Which effect dominates? Using that $\sigma = -U''(C)(C/U'(C))$ and factorizing terms, the income effect dominates ($\mathcal{I}+\mathcal{S}>0$) if and only if

$$\frac{U'(C)}{\gamma}\left(q(B',i,s)+\frac{\partial q(B',i,s)}{\partial B'}(B'-(1-\mu)B)\right)\left(\frac{\sigma}{\gamma}\frac{q(B',i,s)(B'-(1-\mu)B)}{C}-1\right)>0.$$

Because the first two factors are positive, we can write

$$\frac{\partial B'}{\partial i}>0 \iff \frac{\gamma}{\sigma}<\frac{q(B',i,s)(B'-(1-\mu)B)}{C},$$

which is the final expression.[22] ■

3.2 A threshold for debt to GDP

The interesting aspect of equation (19) is that it implies a threshold for debt to GDP above which the effects of monetary policy flip. Specifically, the substitution effect is dominant on one side of the threshold, whereas the income effect is dominant on the other side. In this section we explore this result in depth.

[21] To see this result, note that $U''(C)<0$. Note moreover that

$$\tilde{q}(B',s)+\frac{\partial \tilde{q}(B',s)}{\partial B'}(B'-(1-\mu)B)>0.$$

This follows because in equation (18), the (expected) continuation value is declining in B'.

[22] The careful reader will have noticed that in this sketch of proof, we have omitted indirect derivatives due to the fact that B' is a function of i in equilibrium. As it turns out, due to an Envelope-type of condition, these indirect derivatives are irrelevant for the argument. Note moreover that on two occasions, we have used that i' is independent of i, without mentioning it explicitly. See the full proof in Appendix A.1 for all these details.

Corollary 1 *Assume that the domestic country runs a zero trade balance in the current period, $X - C_f = 0$. Then the threshold for debt to GDP above which the effects of monetary policy flip is*

$$\mathcal{T} = \frac{\gamma}{\tilde{\mu}\sigma}. \tag{22}$$

Whenever $B/Y < \mathcal{T}$, a monetary tightening reduces borrowing B' (the substitution effect is dominant). Whenever $B/Y > \mathcal{T}$, a monetary tightening increases borrowing B' (the income effect is dominant).

We start with a special case which is particularly instructive. Assume that the trade balance in the current period is equal to zero, $X - C_f = 0$. From the resource constraint (11), this is the case whenever new debt issuance by the government $q(B' - (1-\mu)B)$ exactly covers the coupon payment $\tilde{\mu}B$. Intuitively, this corresponds to a tranquil state of the world in which the government intends to maintain a constant stock of debt going forward.[23] By inserting $q(B' - (1-\mu)B) = \tilde{\mu}B$ in equation (19), and using that $C = Y$ when the trade balance is zero (from the resource constraint (11)), we obtain the condition $\gamma/\sigma < \tilde{\mu}B/Y$. This yields the threshold (22) from Corollary 1.

The threshold (22) is determined by three parameters. We explain the influence of each of these parameters in turn.

The elasticity of intertemporal substitution. The first parameter is the EIS, given by $1/\sigma$. The influence of the EIS is easy to understand. The higher the EIS, the more the government is willing to shift consumption across periods. This parameter therefore determines the strength of the substitution effect. The higher the EIS, the stronger the substitution effect, making it more likely that the substitution effect dominates the income effect. This implies that the threshold $\mathcal{T}$ increases in the EIS.

Coupon payment. In our model $\tilde{\mu}$ captures the coupon payment, which equals the fraction of debt which is re-financed in a period where the trade balance is equal to zero. When a small part of the debt is re-financed, then a rise in interest rates only affects this small part of debt, making the income effect weaker. It follows that a low $\tilde{\mu}$ increases the threshold $\mathcal{T}$ above which the income effect is dominant. As pointed out by Chatterjee and Eyigungor (2012), the fact that debt becomes insulated from changes in the bond price q is what enables models with long-term debt to explain large equilibrium amounts of debt. For the threshold that we derived, what matters is that long-term debt insulates the government from changes in the safe interest rate i.

Demand channel. The last parameter is the home bias coefficient $1 - \gamma$, which governs the degree to which changes in domestic consumption change domestic income. Recall from equation (10) that the lower γ, the more consumption matters for domestic income due to a Keynesian-cross type of effect. This feedback of consumption into output enters the threshold $\mathcal{T}$ as strengthening the income effect (hence reducing the threshold). Intuitively, a rise in interest rates lowers current consumption, which is unambiguous as in the two-period consumption model. As the drop in

[23] By following this policy, the stock of debt remains not exactly constant but the part $\tilde{\mu}B$ that is refinanced grows at the rate $1/q$.

consumption drives down domestic incomes, this makes it more attractive for the government to borrow in order to cushion the drop in incomes and consumption. Hence it is more likely that the government increases borrowing in response to a rise in interest rates, which is *as if* the income effect of the interest rate change became stronger.

Corollary 2 *If the trade balance $X - C_f$ is not equal to zero, the threshold for debt to GDP above which the effects of monetary policy flip is*

$$\mathcal{T} = \frac{\gamma}{\tilde{\mu}\sigma}\left(1 + \frac{X - C_f}{Y}\left(\frac{\sigma}{\gamma} - 1\right)\right) \tag{23}$$

Whenever $B/Y < \mathcal{T}$, a monetary tightening reduces borrowing B' (the substitution effect is dominant). Whenever $B/Y > \mathcal{T}$, a monetary tightening increases borrowing B' (the income effect is dominant).

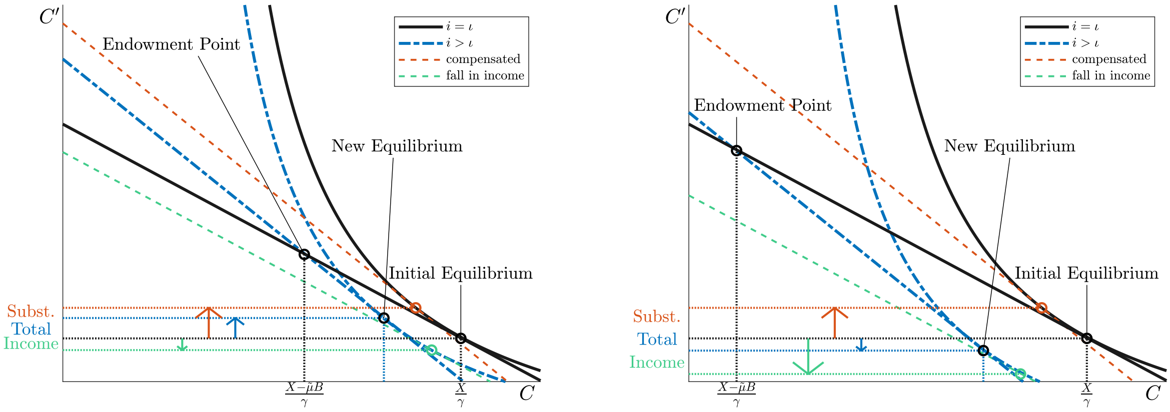
We next generalize the threshold to the case where the trade balance is non-zero in the current period. Again using the resource constraints (11) and (12), we can see that condition (19) can be written as $\gamma/\sigma < (\tilde{\mu}B - (X - C_f))/(Y - (X - C_f))$. Rearranging terms yields the threshold in equation (23).

Equation (23) shows that in the general case, the threshold is *itself* a function of the state of the economy. In particular, the threshold depends on the trade balance to GDP ratio (positively so in the plausible case that $\sigma \geq 1$, that is, assuming an EIS smaller than 1). The intuition is the following. In a period where the trade balance is negative, the domestic country experiences capital inflows in the form of fresh government debt (in excess of those flows which re-finance the coupon payment). Hence the government is exposed more strongly to changes in interest rates in such a period, making the income effect stronger. This explains that the threshold becomes *smaller* in a period of capital inflows (negative trade balance).

Because the trade balance is endogenous in the equilibrium of the model, it is hard to characterize the properties of the threshold further from equation (23). However, what becomes clear is that it inherits the business cycle properties of the trade balance to GDP ratio. For instance, if the trade balance is counter-cyclical the threshold becomes larger in a recession. We explore the implications of this dependence on the cycle in our quantitative application in Section 4.

3.3 Link to the two-period consumption model

The fact that interest rate changes have contrasting effects on borrowing should not come as a surprise. Indeed this result squares with undergraduate teaching of the effects of interest rate changes in the two-period consumption model. In this model, the substitution and income effects are typically explained in terms of consumption. The substitution effect of higher interest rates means that first-period consumption becomes more expensive relative to second-period consumption. Hence first-period consumption falls, whereas second-period consumption rises. In turn, for a *borrowing* agent, the income effect implies that the agent becomes poorer, which reduces consumption in



(a) Low B: Substitution effect dominates.

(b) High B: Income effect dominates.

Figure 1: Illustration of the effects of interest rate changes. Shown are the effects on current and future consumption of a rise in current-period interest rates i. We distinguish a case of low incoming debt (left panel), and high incoming debt (right panel). The red and green dashed lines furthermore decompose the overall effect into a compensated effect and the effect due to a lower present value of income. A full Slutsky decomposition of the effects of changes in i on C' can be found in Appendix A.3.

both periods. The overall effect on *second-period* consumption is therefore ambiguous. Because borrowing and second-period consumption are directly related (second-period consumption equals second-period income net of debt repayment), the ambiguous response of second-period consumption maps into an ambiguous response of debt repayment. In addition, the income effect clearly scales with the level of debt: the higher the debt stock, the stronger the income effect. In contrast, the substitution effect is inherently a marginal effect, and operates even when the debt stock is zero. This makes it clear that the income effect may start to dominate the substitution effect when the stock of debt is sufficiently high.

Perhaps the best way to make this link to the two-period consumption model is by using a diagram, which we do in Figure 1. To make the analogy as clear as possible, in the figure we make assumptions to reduce our model effectively to a two-period setup. We do so by assuming perfect foresight (implying in particular no default risk), and by assuming that the interest rate equals the steady state value $\iota = 1/\beta - 1$ from next period onward. By implication, next-period consumption is $C' = Y' - \iota B' = (X' - \iota B')/\gamma$, where we have used equation (10) to replace Y'. In the current period, instead, consumption is given by $C = (X - \tilde{\mu}B + q(B' - (1-\mu)B))/\gamma$, where $q = (1+\iota)/(1+i)$. Combining both, we obtain the usual budget line in (C, C') space, connecting different levels of B'. Last, by using an indifference curve, we also plot the consumption bundle in the initial equilibrium. For simplicity, in the figure we focus on the case of zero trade balance (see Corollary 1), in which case current consumption in the initial equilibrium is $C = X/\gamma$.

Now turn to the effects of a rise in current-period interest rates i. As it is standard, the budget line tilts around the endowment point. In our model, however, the endowment point is a function of the model's endogenous state variable B. Precisely, it is given by $C = (X - \tilde{\mu}B)/\gamma$ (as this implies that $q(B' - (1-\mu)B) = 0$). The endowment point is necessarily to the left of the initial equilibrium, and the more so, the higher the level of B. We thus distinguish between a low level of B (left panel) and a high level of B (right panel). When interest rates rise, from the perspective of the initial equilibrium, the budget line not only tilts but it also *shifts*. Moreover, the shift is larger, the larger is the level of B. This implies that, depending on the level of B, the new equilibrium can be characterized either by a higher level of C' (the tilt dominates, left panel), or by a lower level of C' (the shift dominates, right panel). This is just the graphical representation of the substitution and income effects discussed before.[24] Last, recalling that $C' = (X' - \iota B')/\gamma$, the ambiguous response of C' translates into an ambiguous response of B' in the new equilibrium. The figure thus makes it clear that a monetary tightening *reduces* B' when current debt B is low, whereas it *raises* B' when current debt B is high.

In sum, there is a clear link between our findings and the conventional two-period consumption model. However, there is also a key difference. In our model, the endowment point around which the budget line tilts is *endogenous*, as it depends on the endogenous state variable B. This implies that - depending on the current debt stock - either the substitution effect or the income effect of a change in interest rates may be dominant.[25,26]

3.4 Implications for the risk of default and spreads

We have shown that monetary policy has ambiguous effects on borrowing B'. Moreover, we saw that the current stock of debt to GDP can be used in order to tease out this ambiguity. What are the implications of this finding for the risk of a sovereign default?

A conventional measure of sovereign default risk is the spread, defined as the yield-to-maturity of the bond with price q relative to a bond of equivalent maturity that does not carry a risk of default (see for instance Hatchondo and Martinez, 2009). As it can be easily shown, the spread in our model solves the expression[27]

$$spread = (\mu + \iota)\left(\frac{1}{q} - \frac{1}{q^f}\right),$$

[24] In the figure, we also decompose the overall response of the equilibrium consumption point into the compensated (Hicksian) response plus the response triggered by the fall in (the present value of) income. We refer the interested reader to Appendix A.3, where we provide the full Slutsky decomposition of the rise in interest rates on future consumption C'.

[25] The influence of the three parameters $1/\sigma$, $\tilde{\mu}$ and γ discussed in the last section can also be understood by looking at Figure 1. A larger EIS $1/\sigma$ implies less-curved indifference curves, hence when the budget line tilts there is a larger effect on C'. A larger coupon $\tilde{\mu}$ increases the distance between the initial equilibrium and the endowment point, implying a larger shift when the budget line tilts. Finally, a larger γ reduces the distance between the initial equilibrium and the endowment point, implying a smaller shift when the budget line tilts.

[26] What about the case of a non-zero trade balance (see Corollary 2)? Going back to Figure 1, a negative trade balance implies that $C > X/\gamma$ in the initial equilibrium. Hence the distance between the initial equilibrium and the endowment point becomes larger, implying a larger shift when the budget line tilts. This makes it clear that the income effect becomes stronger (the threshold falls) in a period where the trade balance is in deficit.

[27] The yield to maturity is defined as the interest rate y_t which solves:

$$q_t = \sum_{s=1}^{\infty}(1 + y_t)^{-s}(1 + \mu)^{s-1}(\mu + \iota).$$

Inserting q_t and q_t^f, where q_t is defined in equation (9) and where q_t^f is defined in the text, we obtain the spread by using the equation $spread_t = y_t - y_t^f$.

where q^f is the price of a risk-free bond which solves the equation $q^f = (\mu + \iota + (1-\mu) q^{f\prime})/(1+i)$. However, the spread is not a good measure to assess the effects of monetary policy on default risk in the context of our analysis. To understand this point, consider the special case of the model where debt maturity is one period ($\mu = 1$). In this case, the spread becomes

$$spread = (1+\iota)\left(\frac{1+i}{\mathbb{E}(1-\delta')(1+\iota)} - \frac{1+i}{1+\iota}\right) = (1+i)\left(\frac{1}{\mathbb{E}(1-\delta')} - 1\right).$$

As this equation shows, the spread is increasing in $1+i$. This implies that a monetary tightening increases the spread, even when expected default $\mathbb{E}\delta'$ is completely unaffected by monetary policy. This effect is well known and is explained in for instance Arora and Cerisola (2001). In intuitive terms, when there is a risk of default, investors receive the safe rate only in states of the world in which default does not actually occur. To compensate, when the safe rate rises, the rate on the risky bond must increase more than one-by-one, which shows up as a rise in the spread.

To assess the impact of monetary policy on the risk of default, we therefore use an alternative measure than the spread. This measure is given by

$$S \equiv (\mu + \iota)\left(\frac{1}{\tilde{q}} - 1\right) \tag{24}$$

where $\tilde{q}$ is given by

$$\tilde{q} = \frac{\mathbb{E}(1-\delta')(\mu + \iota + (1-\mu)\tilde{q}')}{1+\iota}. \tag{25}$$

The difference to before is that the numerator in the bond price is $1+\iota$ (the steady state interest rate), rather than the actual interest rate $1+i$. Intuitively, this modification isolates the change in the spread following a monetary tightening that is due to a change in default risk.[28] In the following, we say that a monetary tightening increases sovereign risk (the risk of a sovereign default in the future) whenever it entails a rise in S.

We next argue that the ambiguous response of borrowing B' documented before maps directly into an ambiguous response of S when monetary interventions are temporary. To understand this point, consider the probability of repayment next period

$$\mathbb{E}(1 - \delta(B', i', s')) = probability\ (V^{\delta}(i', s') < V^{r}(B', i', s')).$$

The two values V^{δ} and V^{r} depend on i', but they do not depend directly on i. Because we study a case where i' is independent of i, a change in i impacts the repayment probability only through changes in the endogenous state variable B'. In turn, the probability of repayment falls in B', because the value of repayment V^r falls in B'. It follows that a monetary tightening reduces the probability of a default in the next period whenever it induces a decline in B', whereas it raises the default probability whenever it induces a rise in B'. The same argument applies to the probability of default two periods ahead, three periods ahead, and so forth in all periods after.[29]

[28] Note that $q^f = 1$ by using this modification, hence no q^f appears in the spread as defined by equation (24)

[29] Formally, the following corollary assumes that $\tilde{q}'$ is downward sloping in B'. That this property is satisfied in

Corollary 3 *Continue to assume temporary monetary interventions. Then the threshold in equation (23) applies not just to borrowing B', but equivalently to sovereign risk S. Whenever $B/Y < \mathcal{T}$, a monetary tightening reduces sovereign risk S. Whenever $B/Y > \mathcal{T}$, a monetary tightening increases sovereign risk S.*

In sum, when monetary interventions are temporary, the threshold in equation (23) describes not just the behavior of borrowing, but equivalently the reaction of sovereign risk.

3.5 Immediate default

To this point, we have taken it for granted that the government chooses to repay in the current period, such that it maintains access to the credit market from this period to the next. However, of course the government may also choose to default in the current period, and this decision will be influenced by monetary policy as well. We discuss this issue briefly in this section.

The government chooses to default in the current period whenever the value of default is higher than the value of repayment, that is if

$$V^{\delta}(s) < V^{r}(B, i, s). \tag{26}$$

Here we have focused on temporary monetary interventions, such that the value of default is not directly a function of the interest rate i (see equation (16)). In a period where the government is a net borrower ($B' - (1-\mu)B > 0$), any rise in interest rates i reduces the value of repayment V^r.[30] Hence a sufficiently large monetary tightening may imply that the value of repayment falls below the value of default. Conversely, a cut in the interest rate by the central bank may help prevent an imminent default.

In what follows, we focus on scenarios in which monetary policy does not alter the current default decision of the government. Even in this case, the fact that monetary policy may directly affect the government's incentives to default will be relevant for understanding the implications of persistent monetary interventions, which we discuss in the next section.

3.6 The effects of persistence

We now relax the assumption that the monetary intervention is temporary. Instead, we assume that the intervention is persistent with persistence parameter $0 \leq \rho \leq 1$. Because a persistent intervention can be understood as the sum of a temporary intervention (analyzed before) plus an anticipated future intervention, we next discuss the effects of anticipated future interventions on borrowing B' and on sovereign risk S.

models with long-term debt such as the one studied here has been shown formally in Chatterjee and Eyigungor (2012).

[30] This follows because a rise in i strictly reduces the choice set by the government in equation (15), which must be welfare reducing. In the calibrated model in Section 4, we find that the government is always a net borrower in equilibrium.

The effects of anticipated future interventions can be understood as the sum of two channels. First, a future monetary tightening implies that borrowing decisions in the future change, which because the government is forward looking, changes borrowing decisions already today. This effect would even be present in the absence of uncertainty, that is, if X and ξ were not random variables implying that the risk of default would be zero. Second, in the case of persistent interventions, monetary policy has a *direct* effect on future default risk that is independent of the response of borrowing. This follows from the arguments made in Section 3.5. A future monetary tightening raises the government's future incentives to default, which raises sovereign risk S today even when debt levels are unaffected.[31] Of course, this feeds back into the response of borrowing as well, because the government's incentives to borrow change following a change in the risk of a future default.

Because of the interaction of these effects, the problem becomes analytically intractable. However, we can obtain a partial characterization by isolating the implications of the first channel, the fact that a future monetary tightening changes current borrowing even in the absence of uncertainty. This allows us to generalize the threshold (23) to the case where monetary interventions are persistent.

Proposition 2 *Assume that X and ξ are constant. Assume that $\iota = 1/\beta - 1$, implying that the model without uncertainty is stationary. Consider a persistent monetary tightening, with persistence parameter $0 \leq \rho \leq 1$. Define the auxiliary parameter $\tilde{\rho}$ such that*

$$\tilde{\rho} \equiv \frac{1 + \iota - (1 - \mu)\rho}{1 + \iota - \rho} \geq 1.$$

Then the threshold in equation (23) can be generalized to

$$\mathcal{T} = \frac{\gamma\tilde{\rho}}{(\tilde{\mu} + \iota(1 - \tilde{\rho}))\sigma} \left(1 + \frac{X - C_f}{Y} \left(\frac{\sigma}{\gamma\tilde{\rho}} - 1\right)\right). \tag{27}$$

Whenever $B/Y < \mathcal{T}$, a monetary tightening reduces borrowing B'. Whenever $B/Y > \mathcal{T}$, a monetary tightening increases borrowing B'.

Proof. In the Appendix A.2. ∎

The threshold in equation (27) is a generalization of the threshold in equation (23), in the sense that the latter is nested when persistence becomes equal to zero $\rho = 0$.[32,33]

As before, the threshold in equation (27) equals a constant plus a term that depends on the trade balance. By inspecting the former, we note that the effect of persistence is to *raise* the threshold $\mathcal{T}$. This is because $\tilde{\rho}$ is increasing in the persistence parameter ρ. Hence, adding persistence to the

[31] Moreover, a future monetary tightening may also trigger an immediate default. We proceed under the assumption that this is not the case (see Section 3.5).

[32] To see this, note that $\tilde{\rho} = 1$ when $\rho = 0$.

[33] This shows that the threshold in equation (23) would hold also in a model without uncertainty, implying that default *per se* does not matter for the threshold. Default only matters for the threshold insofar, as it changes the business cycle properties of the trade balance. See Section 4 for a discussion.

monetary intervention acts like a strengthening of the substitution effect, making it more likely that a monetary tightening reduces borrowing. What is the intuition for this effect? A monetary tightening in the future reduces future consumption unambiguously.[34] The government reacts by reducing its borrowing today, in order to smooth consumption. Hence persistence makes it more likely that the government reduces its borrowing today following a monetary tightening, which appears as a strengthening of the substitution effect.

However, as we noted above, the threshold in equation (27) represents only a partial characterization of the effects of persistent interventions on borrowing, as it abstracts from the direct effect of monetary policy on future default risk that may matter for current borrowing decisions as well. Taking into account this effect may shift the threshold in equation (27), implying that it becomes only an approximation.[35]

What about the effect of persistent monetary interventions on sovereign risk S? As we noted above, when monetary interventions are persistent, a monetary tightening can raise S even when borrowing is unchanged. Unlike in the case of temporary interventions, the threshold in equation (27) may thus not be used to predict the behavior of sovereign risk. We take up this issue numerically in the calibrated model in Section 4. To anticipate, we find that a threshold for debt to GDP describing the behavior of sovereign risk still exists. Hence when debt to GDP is low enough, a monetary tightening reduces sovereign risk even when the monetary tightening is persistent. We also find that this threshold does not overlap with the threshold that describes the behavior of borrowing B', but instead is strictly *lower*. This result squares with the intuition developed in this section, as it implies the existence of states (intermediate levels of debt to GDP) where a monetary tightening *raises* sovereign risk S, but it *reduces* borrowing B'.

4 Quantitative results

We have shown that the effects of a monetary tightening on borrowing B' and on sovereign risk S in the small union member are ambiguous. We have also shown that a threshold for debt to GDP exists above which the effects of monetary policy flip. This begs the question how *large* is the threshold in a reasonable calibration, and what are its business cycle properties. We take up this issue in this section.

In the numerical solution of the model, both debt to GDP and the threshold are equilibrium objects, depending on the economy's current state. We trace the set of states where debt to GDP exceeds the threshold in the equilibrium of the model, and call this the "Fear of Hiking" zone. When we calibrate our model to Italy, on average the economy is in the Fear of Hiking zone when

[34] Going back to the two-period consumption model, we noted that the effect of a rise in interest rates for *current* consumption are unambiguous for a borrowing agent. Hence a monetary tightening in period $t+k$ unambiguously reduces consumption in this period.

[35] This being said, in our numerical analysis of the model, we find that the threshold (27) predicts perfectly the behavior of borrowing even once we allow for uncertainty. Without proving it, we therefore suspect that the response of default risk only affects the magnitude of the change in debt, but not its sign (i.e., whether borrowing rises or declines).

debt to GDP is higher than 50%. Moreover, we find that visiting the Fear of Hiking zone is negatively correlated with the business cycle, implying that a monetary tightening is more likely to raise borrowing and sovereign risk in a recession.

4.1 Back-of-the-envelope calculation

Before solving the model, we first conduct a back-of-the-envelope calculation of the two thresholds (22)-(23), as they depend only on parameters and on a variable which is observable (trade balance to GDP). Consider the threshold (22) first, abstracting from trade imbalances. To repeat, the threshold is $\mathcal{T} = \gamma/(\tilde{\mu}\sigma)$, where γ captures the demand channel, $\tilde{\mu}$ is the coupon payment and $1/\sigma$ is the EIS. We discuss next how we obtain the three parameters.

Using an annual calibration, we assume that the nominal interest rate is 2% in steady state, implying $\iota = 0.02$.[36] The average maturity of debt $1/\mu$ is observable in Eurozone countries. For instance, it is 7 years in Italy, implying that $\mu = 0.14$ and so $\tilde{\mu} = \mu + \iota = 0.16$. Next, equation (17) establishes a link between γ and gross exports to GDP and the trade balance to GDP ratio in our economy. We can measure the evolution of both for Italy over the last 20 years, and take an average in order to obtain a plausible value for γ.[37] This yields $\gamma = 0.27$, implying that about 75% of the consumption basket are domestic goods. The last parameter is the EIS. The evidence suggests that this parameter is quite close to zero.[38] For instance, a recent estimate for the EIS by Best et al. (2019) is 0.1. Conducting a meta analysis, Havránek (2015) finds that estimates for the EIS are clustered around 0.3-0.4, after accounting for publication bias. We proceed by placing ourselves in the middle of these estimates, by using $1/\sigma = 0.25$. By using those values for $\tilde{\mu}, \gamma$ and $1/\sigma$, we obtain a threshold of 41% for Italy.

We can compute the threshold in the above way not just for Italy, but also for Spain, Greece and Portugal, all countries in which debt levels are currently quite high. We may then contrast the thresholds with actual debt levels. When making this comparison, it is important to bear in mind that in our model, all debt is held by foreign creditors. The relevant statistic to consider is thus external debt - debt held by non-residents, plus potentially debt held by foreign public creditors (such as the ECB).[39]

We make this comparison in Table 1. We find that, in 2021, external debt is above the threshold in all countries considered, even more so when including holdings by the ECB. In the case of Italy,

[36] Recall that we abstract from inflation, hence the real and nominal interest rate coincide. In turn, a two-percent yearly real rate is a standard target in many studies.

[37] Solving equation (17) for γ, we obtain

$$\gamma = \left(1 - \frac{X_t - C_{f,t}}{Y_t}\right)^{-1} \left(\frac{X_t}{Y_t} - \frac{X_t - C_{f,t}}{Y_t}\right).$$

In a state where the trade balance is equal to zero ($X_t - C_{f,t} = 0$), the parameter γ therefore measures gross exports to GDP.

[38] See Hall (1988) for early evidence that the EIS is not much different from zero.

[39] Bocola et al. (2019) develop a model of the European debt crisis in which the total stock of debt, rather than external debt, is the relevant state variable for the government's incentives to default. We suspect that a threshold for debt to GDP separating the effects of monetary policy exists also in their model. Hence the relevant statistic may also be the total stock of debt, which is considerably higher than external debt in all economies considered.

	Italy	Spain	Greece	Portugal
Debt to GDP (non-residents)	55%	56%	89%	59%
Debt to GDP (non-residents + ECB)	86%	80%	102%	87%
Threshold $\mathcal{T}$	41%	47%	57%	48%

Table 1: Public debt and thresholds in 2021. Shown are external public debt to GDP, total and according to holdings, in Italy, Spain, Greece and Portugal. The table also shows the model-implied threshold $\mathcal{T}$, defined in equation (22). Data sources are in Appendix B.

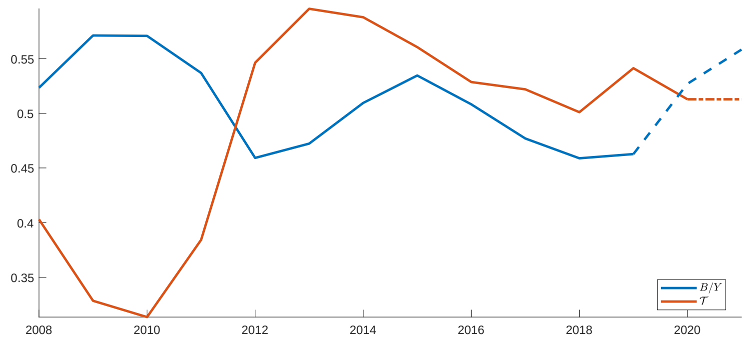


Figure 2: Debt to GDP and threshold for Italy, 2008-2021. Data Sources are in Appendix B. As explained there, we approximate debt to GDP held by non-residents in the years 2020 and 2021, as the data is not available. This is indicated in the figure by using a broken line. We construct the threshold using equation (23), by using data on the trade balance to GDP ratio in Italy as well as the parameters $1/\sigma = 0.25$, $\gamma = 0.27$ and $\bar{\mu} = 0.16$.

Spain and Portugal, external debt is not too far above the threshold. However, for Greece external debt is far above the threshold. This simple analysis therefore suggests that a rate rise by the ECB is likely to increase debt levels and the risk of a sovereign default in these countries, and most so for Greece.

The threshold in equation (22) abstracts from fluctuations in the trade balance. Taking account of this effect by using equation (23), we can study how the threshold has evolved over time. Figure 2 shows the example of Italy, from 2008-2021. In the figure, we contrast the threshold with debt to GDP held by non-residents (excluding holdings by the ECB). We find that debt to GDP has crossed the threshold twice since 2008. In 2012, Italian debt to GDP fell below the threshold, driven mainly by a rise in the trade balance. However, following the Covid crisis which started in 2020 and which was accompanied by large public support measures, debt to GDP has climbed above the threshold again. Of course, this exercise is only suggestive and the numbers should not be taken too seriously. However, it illustrates how our results can be used to make predictions regarding the effects of monetary policy on borrowing and sovereign risk in practice.

To obtain further insights, we next solve the model numerically. In the solution of the model, both the threshold and debt to GDP will be equilibrium objects, and their interaction will be critical for the effects of monetary policy.

4.2 Calibration and numerical algorithm

We proceed with the case of Italy. Following Section 4.1, we set the parameter values $\iota = 0.02$, $\mu = 0.14$, $\gamma = 0.27$ as well as $1/\sigma = 0.25$. Furthermore, following Bianchi et al. (2019), we set the annual probability of reentry after default $p = 0.18$.

All remaining parameters are set to match key moments in the data through simulation. Details on the construction of moments are provided in Appendix B.3. We calibrate the discount factor β to match the average level of external debt to GDP (non-residents) between 2000 and 2019, given by 49.4%. The logarithm of foreign demand follows an AR(1) process

$$\log(X_t/\mu_X) = \rho_X \log(X_{t-1}/\mu_X) + \sigma_X \varepsilon_t$$

with $\varepsilon_t \sim_{i.i.d.} \mathcal{N}(0,1)$.[40] The parameters μ_X, ρ_X and σ_X are set to match mean unemployment (our measure for the output gap) as well as the standard deviation and autocorrelation of detrended GDP. For the utility loss in default, we follow Bianchi et al. (2019) and use

$$\kappa(Y_t, \xi_t) = \max(0, L_0 + L_1 \log(Y_t)) + \xi_t.$$

As explained in Section 2, we assume that the cost of default is subject to shocks through the variable ξ_t. This shock plays a dual role in our analysis. First, it partly delinks the default decision from the economy's current state, reducing the sensitivity of the spread to economic fundamentals. This reduces the disciplining effect of spreads on borrowing which means higher spread levels are reached in equilibrium.[41] Second, as is well known, this shock improves the convergence properties of the solution algorithm in models with long-term debt.[42] Following this literature, we assume that ξ_t is i.i.d. and follows a logistic distribution with location parameter 0 and scale parameter s_x.

We set s_x to match the Italian spread over German bonds during 2006-2021. We calibrate the parameters L_0 and L_1 to match the standard deviation of the spread as well as the correlation of the trade balance with GDP. To understand the last point, note that the emergence of spreads tends to make the trade balance counter-cyclical, a well-known feature of sovereign default models. Hence the loss function κ which matters for spreads can be used to discipline the cyclicality of the trade balance. Matching this cyclicality is important for us, because the cyclicality of the trade balance maps directly into the cyclicality of the threshold, from equation (23).[43]

To solve the model numerically, we discretize the state space for B and X. We approximate the process of X by using the Tauchen (1986) algorithm. In our calibration, the highest level of X is smaller than γ, in line with our earlier assumptions. We do not need a grid for ξ because

[40] As described below, we approximate this process discretely in our numerical solution which ensures $X_{\max} \leq \gamma$.

[41] As discussed in Aguiar et al. (2016), the spread tends to react very steeply to borrowing in the absence of this shock. As a result, optimal debt levels tend to lie just below this steep part, at a low spread level.

[42] See Chatterjee and Eyigungor (2012) for an explanation. See Gordon (2019), Dvorkin et al. (2021) and Arellano et al. (2020) for recent applications of this method to sovereign default models.

[43] It should be noted that, while we state individual targets for each parameter, all parameters influence all moments to some extent and so we calibrate them jointly.

Parameter	Value	Target	Data	Model
ι	0.02	Risk free rate	-	-
σ	4	EIS	-	-
γ	0.27	Home bias	-	-
μ	0.14	Debt maturity	-	-
p	0.18	Exclusion period	-	-
β	0.945	mean(B_t/Y_t)	0.499	0.529
μ_X	0.257	mean($1 - L_t$)	0.094	0.088
σ_X	0.022	std(Y_t)	0.023	0.021
ρ_X	0.65	corr(Y_t, Y_{t-1})	0.064	0.061
s_ξ	0.66	mean($spread_t$)	0.014	0.014
L_0	2.262	corr($X_t - C_{f,t}$, Y_t)	-0.170	-0.140
L_1	20	std($spread_t$)	0.011	0.006

Table 2: Parameters. Details on the calibration can be found in Appendix B.

this shock does not matter for the solution of the model conditional on repayment in the current period. In turn, we solve the model only at the steady state for the interest rate, $i = \iota$. To study monetary policy shocks, we use "MIT shocks", shocks that are zero-probability events. To study such shocks, it is not necessary to solve for policy functions outside of the steady state value for i. This implies that in all policy functions that follow, we omit the two arguments $i = \iota$ and ξ for better readability.

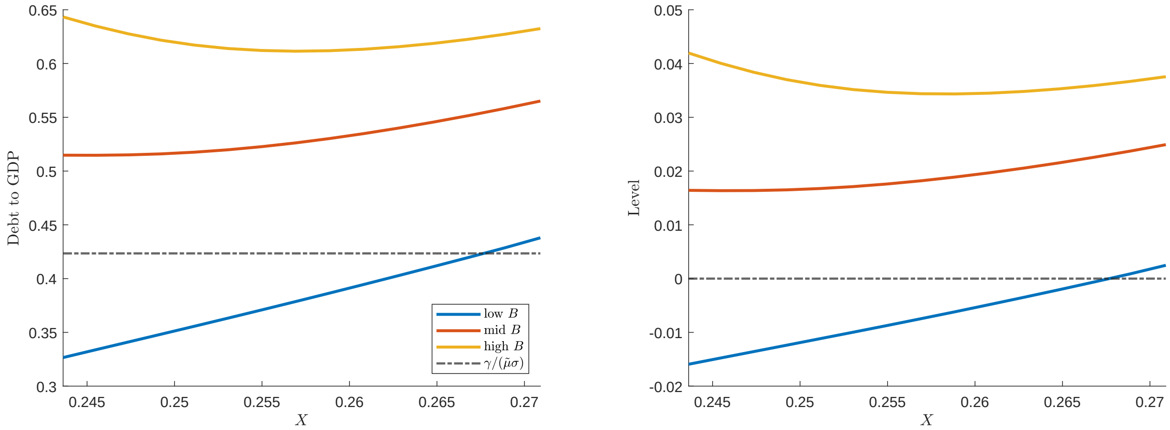
More details on the numerical algorithm can be found in Appendix C.

4.3 The Fear of Hiking zone

In the equilibrium of the model, the threshold is a function of the state variables and hence there exists a policy function $\mathcal{T}(B, X)$. To obtain this policy function, we evaluate equation (23) at the policy function of the trade balance to GDP ratio. The result is in Figure 3. In the left panel, we plot the equilibrium threshold against the foreign demand variable X, by contrasting three different levels of current indebtedness B. In this figure, we also plot the baseline threshold $\mathcal{T} = \gamma/(\tilde{\mu}\sigma)$ (see equation (22)), which applies in states of the world where the trade balance is equal to zero (the dashed-dotted line). In turn, in the right panel we show the trade balance to GDP ratio in the equilibrium of the model.[44]

The first thing the figure shows is that the threshold may depart substantially from its baseline value, reflecting that the trade balance may be substantially different from zero. Turning to the comparative statics, for a given level of foreign demand X, a higher level of debt B implies a higher trade balance and thereby a higher threshold. The trade balance response reflects that the

[44] We verify numerically that in all points shown in the figure, the government chooses to repay in the current period (no immediate default). More precisely, the government chooses to repay unless the utility loss shock ξ is very negative.



(a) Threshold. **(b) Trade balance to GDP.**

Figure 3: Policy functions for threshold and trade balance. Shown are policy functions $\mathcal{T}(B, X)$ (by using equation (23)) and $(X - C_f(B, X))/Y(B, X)$, both drawn against X on the horizontal axis for three different values of B.

country starts to repay when its current debt stock is high. But capital outflows imply that the income effect becomes weaker, hence that the threshold rises (recall Section 3.2). The comparative statics with respect to X (keeping fixed B) are more involved as they depend on the level of B itself. Specifically, when current debt levels are low, then a higher X raises the trade balance and therefore the threshold. However, when current debt levels are high, then a higher X *reduces* the trade balance and therefore the threshold. This behavior of the trade balance (and by implication of the threshold) highlights the role of sovereign default in our model. When debt levels are low, spreads are low and the country borrows in a recession for consumption-smoothing purposes, which explains that both variables are pro-cyclical.[45] In contrast, when debt levels are high then spreads start to climb in a recession which creates incentives for the government to repay. Hence the trade balance becomes larger in a recession, making both the trade balance and the threshold counter-cyclical.

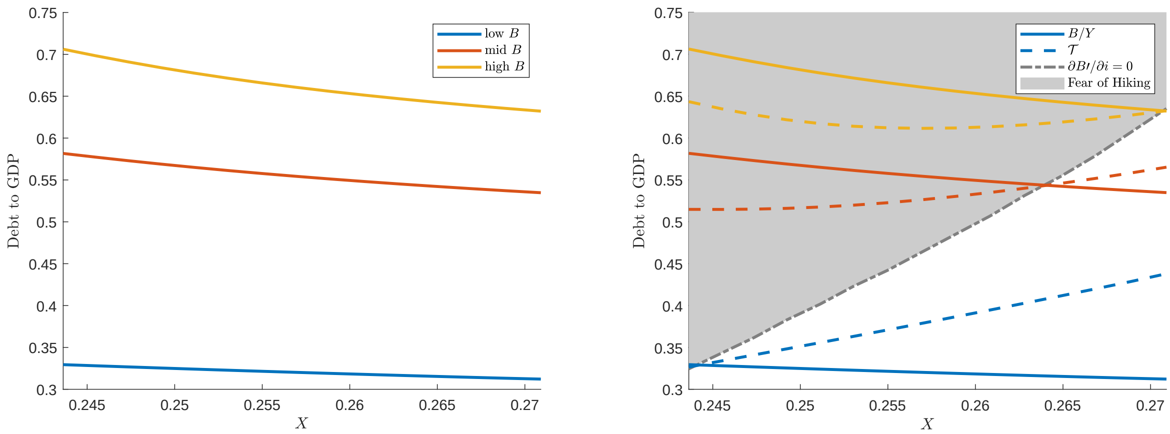
We also study unconditional statistics, in Table 3. The first row shows the mean of the stationary distribution of $\mathcal{T}$. The mean threshold is 0.51 and is thus 9 percentage points higher than the baseline threshold (22), which was 0.42. This reflects that on average, the country is running trade balance surpluses. The second row shows the unconditional correlation between the threshold and GDP. The correlation is negative, hence the threshold co-moves negatively with the cycle. Looking at equation (23), it is clear that the threshold inherits this property directly from the trade balance to GDP ratio (where a negative correlation with output was a calibration target). In turn, the last two rows in Table 3 are explained below.

From Figure 3 alone, one cannot determine the states of the world in which the country is on either side of the threshold. To do so, one must compare the threshold with actual levels of debt

[45] We are not showing it, but the policy function for output Y shows the expected pattern: it is increasing in foreign demand X, as it is intuitive from equation (10). Hence the country is in a recession when X is low, and we may infer pro-cyclicality and counter-cyclicality from looking at Figure 2.

Statistic	Value
mean($\mathcal{T}$)	0.5116
corr($\mathcal{T}, Y$)	-0.2122
mean($\mathbb{1}_{\mathcal{I}}$)	0.7056
corr($\mathbb{1}_{\mathcal{I}}, Y$)	-0.6781

Table 3: Unconditional statistics. Shown are unconditional statistics of the threshold (23) and the indicator variable (28). The indicator variable equals 1 in states where the income effect is dominant, i.e. where the economy is in the Fear of Hiking zone.



(a) Debt to GDP. **(b) Debt to GDP and threshold.**

Figure 4: Contrasting debt to GDP and the threshold. Shown are policy functions $B/Y(B, X)$ in the left panel, and $B/Y(B, X)$ as well as $\mathcal{T}(B, X)$ in the right panel, all drawn against X on the horizontal axis for three different values of B. In the right panel, the intersections of debt to GDP and the threshold indicate the set of points where $\partial B'/\partial i = 0$. Moreover, the Fear of Hiking zone indicates the set of points where the income effect is dominant, hence where $\partial B'/\partial i > 0$.

to GDP, which are also determined in the equilibrium of the model. The policy function for debt to GDP is shown in the left panel of Figure 4. The figure is constructed in the same way as Figure 3. We see that a higher B implies that debt to GDP rises. Moreover, we see that for a given B, a lower X implies that debt to GDP rises. This happens because a lower X implies that also output Y is lower, hence debt to GDP must increase mechanically.

The interesting aspect is to compare the equilibrium for debt to GDP with the equilibrium for the threshold, which is done in the right panel of Figure 4. To create this figure, we simply draw the left panels of Figures 3-4 on top of each other. Let us again go through the comparative statics. For a given level of B (movement along the lines), debt to GDP tends to lie above the threshold for low levels of X, but below the threshold for high levels of X. At the intersections of the curves, debt to GDP and the threshold coincide. In the figure, we connect these intersections through a gray dashed-dotted line. Hence, one can find the region where the income effect dominates (implying that a rate rise raises borrowing and sovereign risk) to the left of the gray dashed-dotted line. We call this part of the state space the "Fear of Hiking" zone, and make it gray in the figure for better visibility. Because output Y is low when X is low, we see that the economy is more likely to be in the Fear of Hiking zone when the economy is in a recession. In turn, a higher level of B also makes it more likely that the economy is in the Fear of Hiking zone.

We can also make the point that the economy is more likely to visit the Fear of Hiking zone in a recession by looking at unconditional statistics. To do so, we define an indicator variable $\mathbb{1}_{\mathcal{I}}$ which takes the value of 1 in states of the world where the income effect is dominant (the economy is in the Fear of Hiking zone)

$$\mathbb{1}_{\mathcal{I}}(B, X) \equiv \mathbb{1}\left\{\frac{B}{Y(B, X)} > \mathcal{T}(B, X)\right\}. \tag{28}$$

We study the properties of $\mathbb{1}_{\mathcal{I}}$ in the last two rows in Table 3. The mean value of $\mathbb{1}_{\mathcal{I}}$ in the stationary distribution of the model is 0.7059, implying that 71 percent of the time, the economy finds itself in the Fear of Hiking zone. The last row shows the correlation between $\mathbb{1}_{\mathcal{I}}$ and output. The correlation is -0.68, confirming that the substitution effect tends to dominate in good times, whereas the income effect tends to dominate in a recession.[46]

We conclude this section with two additional remarks. The first remark is on the construction of the gray dashed-dotted line in the right panel of Figure 4. As we explained above, we obtain this line by using the policy function for $\mathcal{T}$, from equation (23). An alternative - and more direct - way of obtaining this line is by using numerical differentiation. Indeed, one may compute the derivative $\partial B'/\partial i$ in the solution of the model and check if the derivative is positive, zero, or negative. We verified that the set of points where the derivative is zero equals exactly the gray dashed-dotted line in the figure. This testifies to the accuracy of our numerical algorithm, and it also confirms that equation (23) provides a correct description of the equilibrium of the model. A subtle issue is that by using the numerical approach, one can obtain *only* the gray dashed-dotted line, but not the rest of the policy function of the threshold. This is because this method can only find the threshold points which are attained by the model in equilibrium. Still, knowing this line is enough to be able to find the Fear of Hiking zone in the equilibrium of the model. We exploit this fact below, in Section 4.4.

Our second remark is to mention a perhaps paradoxical prediction of the model when debt levels and spreads are high. As one can guess by looking at the right panel in Figure 4, by studying even higher debt levels than the yellow (high B) line, a second intersection between debt to GDP and the threshold may occur when X is very low. Intuitively, this intersection can arise because the threshold becomes very steep when X is low, due to the fact that the trade balance to GDP ratio becomes very steep (driven by the reaction of spreads, as we explained above). This second

[46]It might be confusing to the reader that we found $\mathcal{T}$ to be counter-cyclical (second row in Table 3), but $\mathbb{1}_{\mathcal{I}}$ to be counter-cyclical as well (fourth row). After all, a counter-cyclical $\mathcal{T}$ implies that the threshold is higher in a recession, hence that the income effect should be less likely to be dominant in a recession (implying a pro-cyclical $\mathbb{1}_{\mathcal{I}}$). What this argument ignores is that not only $\mathcal{T}$ is counter-cyclical, but also debt to GDP. Indeed as shown in Figure 4 right panel, debt to GDP is even *more* counter-cyclical than $\mathcal{T}$. Thus, while $\mathcal{T}$ tends to rise in a recession, debt to GDP tends to rise by *even more*, implying that debt to GDP tends to exceed the threshold in a recession. This explains why we find $\mathbb{1}_{\mathcal{I}}$ to be counter-cyclical.

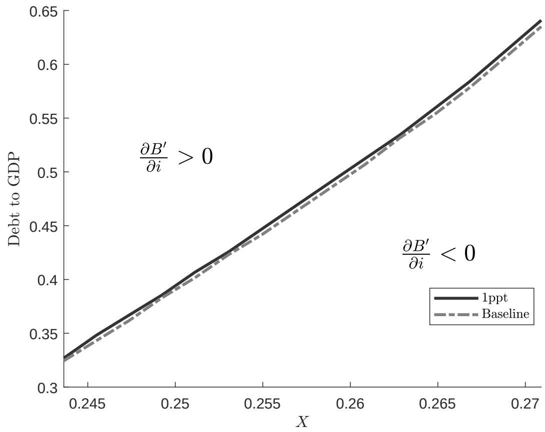


Figure 5: Large interventions. The figure shows the set of points where $\Delta B'/\Delta i = 0$, where Δi equals one percentage point.

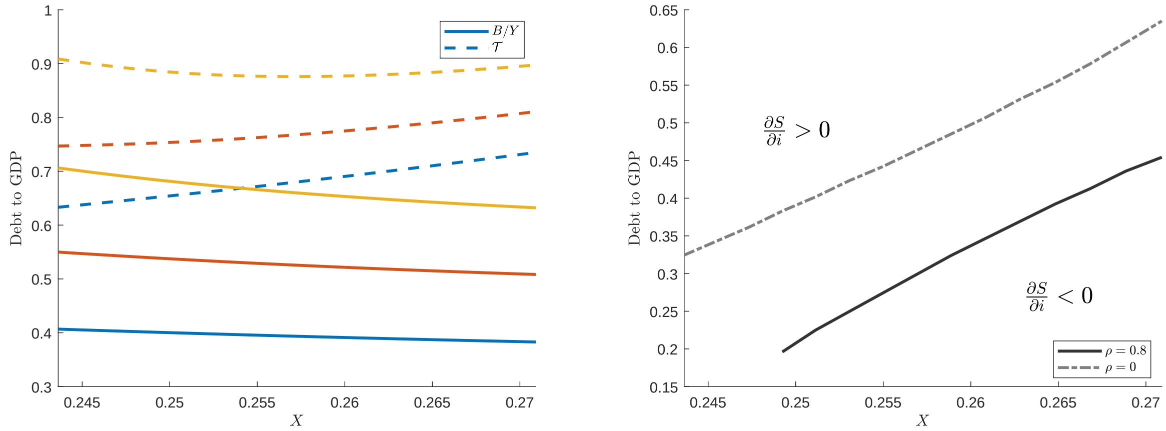
intersection has tipsy-turvy comparative statics. When X is reduced further (a deeper recession), the substitution effect starts to dominate because the threshold climbs even faster than debt to GDP. However, we find that this region is not important in the equilibrium of the model. In the stationary distribution, these states of the world precede those states where a default happens, hence they are visited only briefly and infrequently (if at all).

4.4 Robustness: large interventions and persistent interventions

In this section we discuss two robustness checks to our main numerical results from the last section. First, we discuss how the threshold is affected by *large* monetary interventions. Second, we discuss how the results change if monetary interventions are *persistent* (going back to Section 3.6).

The threshold in equation (23) holds for a marginal change in the interest rate. This begs the question how accurate this equation is when considering more realistic changes in interest rates. In the case of large monetary interventions, we do not have the analogue of equation (23), hence we cannot conduct the same type of analysis that is based on policy functions as in the last section. However, as explained at the end of the last section, we can still obtain the regions of the state space where the substitution/income effects dominate (i.e., the gray dashed-dotted line in Figure 4) by computing derivatives numerically. Specifically, we search for the set of points in the equilibrium of the model where $\Delta B'/\Delta i = 0$, where Δi measures a discrete jump in the interest rate. The result is in Figure 5. It repeats the gray dashed-dotted line from Figure 4 right panel, and contrasts it with the line obtained under a one-percentage-point monetary tightening. As we can see, large interventions shift the curve up, making the Fear of Hiking zone smaller . However, quantitatively the two lines are very close to each other. Hence, our equation capturing the marginal effect provides a good description of the model even when monetary interventions are large.[47]

[47] Again, we have verified numerically that in all points in the figure, the monetary tightening does not trigger an immediate default.



(a) Effect on borrowing. **(b) Effect on sovereign risk.**

Figure 6: Persistent interventions. The left panel shows policy functions for debt to GDP and the threshold (using equation (27)), as in the right panel of Figure 4. The right panel shows the set of points where $\partial S/\partial i = 0$, by contrasting results with those obtained under temporary monetary interventions ($\rho = 0$).

We next turn to the case of persistent interventions. As we discussed in Section 3.6, adding persistence has two effects. First, persistence shifts the threshold for debt to GDP which describes the behavior of borrowing B'. Second, persistence decouples borrowing decisions B' and sovereign risk S. To illustrate, we use a very high degree of persistence by using $\rho = 0.8$, implying a quarterly autocorrelation of the monetary shock of 95%.

The result is in Figure 6. Consider the left panel first, which shows the behavior of borrowing. We repeat the analysis from the right panel in Figure 4, that is, we contrast the equilibrium threshold obtained using equation (27) and the equilibrium for debt to GDP. As equation (27) predicts, the threshold curves have shifted up compared to the case of temporary monetary interventions. Indeed, they have shifted up by so much that they no longer intersect with the curves for debt to GDP. This implies that the substitution effect is now *always* dominant, implying that a monetary tightening always reduces borrowing B'.[48,49]

What about the response of sovereign risk? Because equation (27) does not describe the reaction of sovereign risk when monetary interventions are persistent, we proceed by using numerical differentiation. Specifically, we search for the points in the state space where $\partial S/\partial i = 0$ in the equilibrium of the model, and connect these points through a gray dashed-dotted line. The result is in the right panel of Figure 6. For transparency, we also repeat the case of temporary monetary interventions in the figure (the gray dashed-dotted line from Figure 4, the right panel). The first result is that a line separating the income and substitution effect regions still exists. Unlike for the case of borrowing (left panel), a monetary tightening may thus also increase sovereign risk (in the region above the gray dashed-dotted line). The second result is that the line has shifted

[48] We confirm this result by using numerical differentiation. We find that $\partial B'/\partial i < 0$ holds everywhere in the state space in the equilibrium of the model.

[49] As it should be clear, the fact that no intersection exists is not a general feature of high persistence, but depends on the overall calibration of the model.

downward relative to the case of temporary interventions, implying that a monetary tightening is now more likely to increase sovereign risk. Interestingly, we obtain this result despite the fact that borrowing B' necessarily declines following a persistent monetary tightening (which by itself tends to reduce sovereign risk). As explained in Section 3.6, persistent monetary interventions may therefore reduce borrowing, while at the same time increasing sovereign risk.

5 Implications for monetary policy and concluding remarks

When the central bank of a currency union raises its policy rate, what are the implications for sovereign borrowing decisions and the risk of a sovereign default in a union member? Motivated by this question, in this paper we have constructed a quantitative sovereign default model of a small member of a monetary union. We have found that a rise in interest rates by the union-wide monetary authority may either raise or reduce sovereign borrowing and the risk of a sovereign default and that this depends on the current stock of debt to GDP in the small union member. We have also quantified this threshold level of debt to GDP in an application of the model to countries in the euro area.

We view this paper as a first step in a broader research agenda. For example, we have studied how a given (exogenous) change in monetary policy impacts a union member in the presence of default risk, but we have not asked the reverse question, how default risk in a union member matters for the conduct of monetary policy.[50,51] We plan to address this normative question in future work. This being said, even the present analysis allows to think about the implications for monetary policy, and we touch on this in the remainder of this section.

First of all, we have shown that both a rise or a cut in the interest rate may be required to reduce the likelihood of a sovereign default of a union member in the future. Importantly, the rise in the interest rate will only work if debt to GDP is low enough to begin with, that is as long as the economy is not in the Fear of Hiking zone. Conversely, what is needed to make a default less likely when debt levels are already high - or even to prevent an imminent default - is a cut in the interest rate. This finding is therefore in line with a current narrative in the euro area, according to which a hike in interest rates may make a sovereign default in some high-debt member states more likely.[52]

A second point that comes out in our analysis is that expectations matter for the effects of

[50] de Ferra and Romei (2020) study the two-way interaction between sovereign default in member states and union-wide monetary policy. However, as we discuss in the related literature section in the introduction, they focus on a different economic mechanism than the one we put forward in this paper. Arellano et al. (2020) study the interaction between sovereign default risk and monetary policy in a small open economy which has an independent monetary policy.

[51] Of course, a central bank's mandate is to preserve price stability, not per se financial stability (such as preventing a sovereign default). However, its imprint on sovereign default risk should still be of interest for a central bank. Moreover, a sovereign default has also feedback effects on price stability. As argued by de Ferra and Romei (2020), a sovereign default tends to be deflationary, and as shown by Na et al. (2018), a sovereign default in a country with a fixed nominal exchange rate and nominal rigidities tends to entail a negative output gap.

[52] What if member states are heterogeneous in terms of their current levels of debt to GDP? In this case, a one-size-fits-all monetary policy may not be successful at mitigating the risk of sovereign default in all countries simultaneously.

monetary policy. In particular, we have shown that expectations about persistence of the monetary tightening make it more likely that debt levels will decline. On the other hand, for a given level of debt, higher interest rates also raise the probability of a sovereign default. The right combination of a current monetary tightening and forward guidance about the future path of interest rates may thus be required in order to minimize the risk of a sovereign default.[53]

We hope that our framework will turn out useful and flexible enough in order to address such questions in the future.

Appendix

A Analytical derivations

A.1 Proof of Proposition 1

Denote $\tilde{q}(B', i, s) \equiv q(B', i, s)(1+i)$ the bond price exclusive of the interest rate. By equation (9), it is given by

$$\tilde{q}(B', i, s) = \mathbb{E}(1 - \delta(B', i', s'))(\tilde{\mu} + (1-\mu)q(B'(B', i', s'), i', s')).$$

Because i' is by assumption independent of i, this equation makes it clear that $\tilde{q}$ is not a function of i. Hence we can write $\tilde{q}(B', s)$. Furthermore we have

$$\frac{\partial q(B', i, s)}{\partial B'} = \frac{\partial}{\partial B'} \frac{\tilde{q}(B', s)}{1+i} = \frac{1}{1+i} \frac{\partial \tilde{q}(B', s)}{\partial B'}.$$

Using both in the Euler equation (18), we can write

$$\frac{1}{\gamma} U' \left(\frac{1}{\gamma} \left(X - \tilde{\mu} B + \frac{\tilde{q}(B', s)}{1+i} (B' - (1-\mu)B) \right) \right) \left(\tilde{q}(B', s) + \frac{\partial \tilde{q}(B', s)}{\partial B'} (B' - (1-\mu)B) \right)$$
$$+ (1+i)\beta \frac{\partial}{\partial B'} \mathbb{E} V(B', i', s') = 0.$$

We can write this as $\Lambda(B', B, i, i', s, s') = 0$, which must hold, in particular, for all levels of i. Hence, this equation defines a mapping of i into B', which is implicitly defined $\Lambda(B'(i), B, i, i', s, s') \equiv 0$. We differentiate Λ with respect to i

$$\mathcal{I} + \mathcal{S} + \frac{\partial \Lambda(B', B, i, i', s, s')}{\partial B'} \frac{\partial B'(i)}{\partial i} = 0,$$

where the *direct* derivatives $\mathcal{I}$ and $\mathcal{S}$ have been defined in equations (20) and (21) and where the last summand captures the indirect derivatives (the fact that B' depends on i in equilibrium).

[53] In addition, the announcement of the central bank must also be credible. Ex post, the central bank has an incentive to cut the interest in order to prevent an imminent default. Anticipating this, investors may not believe the central bank's announcements, i.e. its path of forward guidance.

Notice that, in the computation of $\mathcal{S}$, we have also used that $\mathbb{E}V(B', i', s')$ is not dependent on i, because we assumed that i' is independent of i.

The last step is to pin down the indirect derivatives. As it turns out, we do not need to compute these derivatives in order to pin down their sign. This is because, by assumption, we study the Euler equation at a local maximum, hence the second order condition at the equilibrium point is negative:

$$\frac{\partial}{\partial B'}\Lambda(B', B, i, i', s, s') < 0.$$

Hence $\partial B'/\partial i > 0$ if and only if $\mathcal{I} + \mathcal{S} > 0$, which yields the statement in the proposition.

A.2 Proof of Proposition 2

In what follows we use sequence (rather than recursive) notation as this makes the proof of the proposition easier. As explained in the main text, we consider an economy where all uncertainty is resolved in period t and hence $X_{t+s} = X$ is constant for all $s \geq 0$. To rule out explosive paths, we also assume $\lim_{s\to\infty} \beta(1 + i_{t+s}) = \beta(1 + \iota) = 1$.

The proof proceeds in two steps. We start by characterizing the effects of a one-time shock in period $t + s$ that is perfectly anticipated in period t. Thereafter, we take the sum over the effects of perfectly anticipated shocks in all future periods $t + s$, for $s \geq 0$, by weighting them with the persistence term ρ^s. Hence we study the case of AR(1) shocks as a perfectly foreseen path of future temporary shocks.

Future one-time shock. The strategy to determine the effect of an expected future interest change on borrowing is to solve for the derivatives of the bond price and consumption with respect to the interest rate analytically. These can be combined to find the derivative of borrowing with respect to the interest rate change.

Iterating forward the bond pricing equation (9), the price of the sovereign bond as a function of future interest rates is

$$q_t = \sum_{s=1}^{\infty} \left(\Pi_{r=0}^{s-1}(1 + i_{t+r})\right)^{-1} (\tilde{\mu})(1 - \mu)^{s-1}.$$

Notice that no expectation and no default premium enter here, as we have assumed that there is no more uncertainty after period t. We can directly take the derivative with respect to a future i_{t+s}

$$\frac{\partial q_t}{\partial i_{t+s}} = -\frac{\tilde{\mu}}{1+\iota}\sum_{p=s}^{\infty}(1 + \iota)^{-p-1}(1 - \mu)^p = -\frac{\tilde{\mu}}{(1 + \iota)^2}\left(\frac{1-\mu}{1+\iota}\right)^s \frac{1}{1 - \frac{1-\mu}{1+\iota}}.$$

where the derivative (as all other derivatives in this section) is evaluated at the steady state interest rate $i_{t+s} = \iota$ for all $s \geq 0$. For readability, we do not make this explicit. The expression above simplifies to

$$\frac{\partial q_t}{\partial i_{t+s}} = -\frac{(1 - \mu)^s}{(1 + \iota)^{s+1}}. \tag{A.1}$$

Next, we solve for consumption as a function of future interest rates. The Euler equation (18) can be iterated in absence of uncertainty to yield

$$C_{t+s} = \left(\Pi_{r=0}^{s-1}(1+i_{t+r})\beta^s\right)^{\frac{1}{\sigma}} C_t \tag{A.2}$$

for all $s \geq 1$. Similarly, iterating forward the resource constraint (11) yields the inter-temporal resource constraint[54]

$$X - \gamma C_t + \sum_{s=1}^{\infty} \left(\Pi_{r=0}^{s-1}(1+i_{t+r})\right)^{-1}(X - \gamma C_{t+s}) = ((1-\mu)q_t + \tilde{\mu})B_{t-1}.$$

Using equation (A.2), the intertemporal resource constraint can also be written as

$$\begin{aligned}\gamma C_t + \sum_{s=1}^{\infty} &\left(\Pi_{r=0}^{s-1}(1+i_{t+r})\right)^{\frac{1-\sigma}{\sigma}} \beta^{\frac{s}{\sigma}} \gamma C_t \\ &= X + \sum_{s=1}^{\infty} \left(\Pi_{r=0}^{s-1}(1+i_{t+r})\right)^{-1} X - ((1-\mu)q_t + \tilde{\mu})B_{t-1}.\end{aligned} \tag{A.3}$$

Notice that, evaluated in steady state $i_{t+s} = \iota$ for all $s \geq 0$, this implies the simple expression for current consumption

$$C_t = \frac{1}{\gamma}\left(X - \iota B_{t-1}\right). \tag{A.4}$$

The derivative of consumption with respect to a future interest change can be calculated from equation (A.3) and is given by

$$\frac{\partial C_t}{\partial i_{t+s}} = \frac{\iota}{\gamma}\left(-\frac{1}{\sigma\iota(1+\iota)^{s+2}}X + \frac{(1-\mu)^{s+1} + \frac{1-\sigma}{\sigma}}{(1+\iota)^{s+2}}B_{t-1}\right). \tag{A.5}$$

We now have all preliminaries to calculate the derivative of B_t with respect to i_{t+s}. First, we write the choice of debt as a function of other variables

$$B_t = \frac{1}{q_t}(\gamma C_t - X + \tilde{\mu}B_{t-1}) + (1-\mu)B_{t-1}.$$

Taking the derivative gives

$$\frac{\partial B_t}{\partial i_{t+s}} = -\frac{1}{q_t^2}\frac{\partial q_t}{\partial i_{t+1}}(\gamma C_t - X + \tilde{\mu}B_{t-1}) + \frac{\gamma}{q_t}\frac{\partial C_t}{\partial i_{t+s}}.$$

Plugging in the results (A.1), (A.4) and (A.5) and rewriting, we get the following final expression for a shock at a general time $t+s$ on current borrowing B_t

$$\frac{\partial B_t}{\partial i_{t+s}} = \frac{1}{(1+\iota)^{s+2}}\left((1-\mu)^s\tilde{\mu}B_{t-1} - \frac{1}{\sigma}X + \frac{1-\sigma}{\sigma}\iota B_{t-1}\right). \tag{A.6}$$

[54] This involves solving the period $t+1$ budget constraint for $q_t B_t$ and then using the fact that $\frac{q_t}{q_{t+1}(1-\mu)+\tilde{\mu}} = \frac{1}{1+i_t}$.

Persistent shock. To compute the derivative of B_t with respect to a shock at time t which follows an AR(1) process with persistence parameter ρ, we take a weighted sum over the effects on B_t of future expected temporary shocks. By using equation (A.6), we can write

$$\begin{aligned}
\frac{\partial B_t}{\partial i_t^{\mathrm{AR}(1)}} &= \sum_{s=0}^{\infty} \rho^s \frac{\partial B_t}{\partial i_{t+s}} = \sum_{s=0}^{\infty} \frac{\rho^s}{(1+\iota)^{s+2}} \left((1-\mu)^s \tilde{\mu} B_{t-1} - \frac{1}{\sigma} X + \frac{1-\sigma}{\sigma} \iota B_{t-1} \right) \\
&= \frac{1}{(1+\iota)} \left(\frac{1}{1+\iota-(1-\mu)\rho} \tilde{\mu} B_{t-1} - \frac{1}{1+\iota-\rho} \frac{1}{\sigma} \left(X + (\sigma-1)\iota B_{t-1} \right) \right) \\
&= \frac{1}{(1+\iota)} \left(\frac{1}{1+\iota-(1-\mu)\rho} (\tilde{\mu} B_{t-1} + X - C_{f,t} + \iota B_{t-1}) - \frac{1}{1+\iota-\rho} \left(\frac{\gamma}{\sigma} (Y - (X - C_{f,t})) + \iota B_{t-1} \right) \right),
\end{aligned}$$

where we have used $\iota B_{t-1} = X - C_{f,t}$ and $X - \iota B_{t-1} = \gamma(Y - (X - C_{f,t}))$ in the last equality. This implies that

$$\frac{\partial B_t}{\partial i_t^{\mathrm{AR}(1)}} > 0 \iff (\tilde{\mu} + \iota(1-\tilde{\rho}))B_{t-1} \leq \tilde{\rho} \left(\frac{\gamma}{\sigma} (Y - (X - C_t^f)) \right) + (X - C_t^f),$$

where we define the auxiliary parameter $\tilde{\rho} = \frac{1+\iota-(1-\mu)\rho}{1+\iota-\rho}$. Dividing both sides by Y and rearranging yields the threshold stated in the main text.

A.3 Slutsky decomposition in Figure 1

We consider the same economy as in the proof of Proposition 2 in Appendix A.2, that is, a perfect foresight economy in which interest rates equal $\iota = 1/\beta - 1$ in the long run. In fact, in what follows we assume that the interest rate is equal to the steady state value ι in all future periods after the current period.

The objective is to decompose the effect of changes in the current interest rate i on future consumption C' into an income and a substitution effect via a Slutsky decomposition. To economize on notation, we write all variables as functions of the current interest rate only. For example, we write $C'(i)$.

Since C is constant for all periods after the current period t, lifetime utility is

$$U(i) = \frac{C(i)^{1-\sigma}}{1-\sigma} + \sum_{t=1}^{\infty} \beta^t \frac{C'(i)^{1-\sigma}}{1-\sigma} = \frac{C(i)^{1-\sigma}}{1-\sigma} + \frac{1}{\iota} \frac{C'(i)^{1-\sigma}}{1-\sigma} \tag{A.7}$$

Using $\beta = \frac{1}{1+\iota}$ the Euler equation becomes

$$C(i) = C'(i) \left(\frac{1+\iota}{1+i} \right)^{\frac{1}{\sigma}}.$$

Substituting for $C(i)$ in equation (A.7) and solving for $C'(i)$, we can write

$$C'(i, U(i)) = \left(\frac{U(i)(1-\sigma)}{\frac{1}{\iota} + \left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}} \right)^{\frac{1}{1-\sigma}}. \tag{A.8}$$

This expression allows us to decompose the effects of an interest rate change into an income effect (green) and a substitution effect (red)

$$\frac{dC'(i,U)}{di} = \frac{\partial C'(q,U)}{\partial U}\frac{\partial U}{\partial i} + \frac{\partial C'(i,U)}{\partial i} \tag{A.9}$$

The second term is the *compensated* effect of the interest rate on future consumption, i.e. how future consumption changes with the interest rate holding the level of lifetime utility (the present value of income) constant. Graphically, the income effect corresponds to the shifting of the green line and the substitution effect corresponds to the tilting of the red line in Figure 1. In the following, we determine the values of the derivatives in Equation (A.9).

We start with the compensated (substitution) effect which is given by

$$\frac{\partial C'(i,U)}{\partial i} = \frac{1}{\sigma} C'(i)^{\sigma} \frac{U(i)(1-\sigma)}{\left(\frac{1}{\iota} + \left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}\right)^2} \left(\frac{1+i}{1+\iota}\right)^{-\frac{1}{\sigma}} \frac{1}{1+\iota}$$

Using $\frac{U(i)(1-\sigma)}{\frac{1}{\iota}+\left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}} = C'(i)^{1-\sigma}$ this is

$$\frac{\partial C'(i,U)}{\partial i} = \frac{1}{\sigma} \frac{C'(i)}{\left(\frac{1}{\iota} + \left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}\right)} \left(\frac{1+i}{1+\iota}\right)^{-\frac{1}{\sigma}} \frac{1}{1+\iota} \tag{A.10}$$

Next, we derive the partial derivative of C' with respect to U

$$\frac{\partial C'(i,U)}{\partial U} = C'(i)^{\sigma} \left(\frac{1}{\frac{1}{\iota} + \left(\frac{1+\iota}{1+i}\right)^{\frac{\sigma-1}{\sigma}}} \right) \tag{A.11}$$

It remains to determine the derivative of U with respect to i. As in the proof of Proposition 2, consumption in all future periods is given by permanent income

$$C'(i) = \frac{X - \iota B'(i)}{\gamma}. \tag{A.12}$$

Notice that $q(i) = \frac{1+\iota}{1+i}$ holds and we can write the budget constraint as

$$B'(i) = (1-\mu B) + \frac{1+i}{1+\iota}\left(C(i) + \frac{\tilde{\mu}B - X}{\gamma}\right).$$

This yields future consumption in terms of current consumption and the interest rate

$$C'(i) = \frac{X - \iota(1-\mu)B}{\gamma} - \iota\frac{1+i}{1+\iota}\left(C(i) + \frac{\tilde{\mu}B - X}{\gamma}\right).$$

Lifetime utility then is

$$U(i) = \frac{C(i)^{1-\sigma}}{1-\sigma} + \frac{1}{\iota}\frac{\left(\frac{X-\iota(1-\mu)B}{\gamma} - \iota\frac{1+i}{1+\iota}\left(C(i) + \frac{\tilde{\mu}B-X}{\gamma}\right)\right)^{1-\sigma}}{1-\sigma}.$$

Importantly, $U(i)$ is evaluated at the optimal choice $C(i)$. Therefore the envelope theorem applies and we only have to take the direct derivative with respect to i:

$$\frac{\partial U}{\partial i} = -\frac{1}{1+\iota}C'(i)^{-\sigma}\left(C(i) + \frac{\tilde{\mu}B - X}{\gamma}\right) \tag{A.13}$$

We substitute (A.10), (A.11) and (A.13) into (A.9) to obtain

$$\frac{dC'(i,U)}{di} = -\left(\frac{1}{\frac{1}{\iota} + \left(\frac{1+\iota}{1+i}\right)^{\frac{\sigma-1}{\sigma}}}\right)\frac{1}{1+\iota}\left(C(i) + \frac{\tilde{\mu}B - X}{\gamma}\right) + \frac{1}{\sigma}\frac{C'(i)}{\left(\frac{1}{\iota} + \left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}\right)}\left(\frac{1+i}{1+\iota}\right)^{-\frac{1}{\sigma}}\frac{1}{1+\iota}$$

Using $C(i) = C'(i)\left(\frac{1+\iota}{1+i}\right)^{\frac{1}{\sigma}}$ and evaluating at the steady state $i = \iota$

$$\frac{dC'(i,U)}{di} = -\frac{1}{\left(\frac{1}{\iota}+1\right)}\frac{1}{1+\iota}\left(C(i) + \frac{\tilde{\mu}B - X}{\gamma}\right) + \frac{1}{\sigma}C(i)\frac{1}{\left(\frac{1}{\iota}+1\right)}\frac{1}{1+\iota}$$

Collecting terms, we find

$$\frac{dC'(i,U)}{di} = \frac{\iota}{(1+\iota)^2}\left(-\left(C(i) + \frac{\tilde{\mu}B - X}{\gamma}\right) + \frac{1}{\sigma}C(i)\right) \tag{A.14}$$

Here we see that the income effect of an interest rate increase on future consumption is negative, while the substitution effect is positive. Equation (A.12) established a negative linear relationship between C' and B'. This implies that these terms have to be multiplied by $-\frac{\gamma}{\iota}$ to translate them into effects on B'.

Finally note that it is straightforward to see that equation (A.14) gives again the threshold $\mathcal{T}$ that we have found in the main text.

B Data Appendix

Our main source is the Eurostat database from which we get data on gross trade flows and public debt relative to GDP, as well as unemployment.[55] We complement this with data on debt maturity

[55] See ec.europa.eu/eurostat/en/web/main/data/database.

from the OECD (2021)[56], data on bond yields from the Bundesbank and Banca d'Italia and data on the sectoral decomposition of government debt holdings from the updated Bruegel database of sovereign bond holdings developed in Merler and Pisani-Ferry (2012).[57] In Table 4 we summarize the details on the data used along with the respective source.

B.1 Construction of Table 1

To compute debt to GDP according to holdings in Table1, we multiply total debt to GDP (obtained from Eurostat) with the shares of debt held by non-residents (and the central bank) in Q3 2019 by using the Bruegel database. The way we compute the threshold is explained in the main text.

B.2 Construction of Figure 2

In Figure 2, the threshold is computed from the trade balance to GDP ratio from 2008 to 2020, with the values for σ, γ and $\bar{\mu}$ given in the main text. Since the trade balance is not available yet for the year 2021, we extrapolate the threshold using the same value as in 2020. To compute the ratio of public debt held by foreigners to GDP we multiply total debt holdings with the shares held by non-residents for the years 2008 to 2019 (using the Bruegel database). Because the Bruegel database ends in Q3 2019, to compute debt holdings after this period, we extrapolate by using the most recent value. We use data on total public debt to GDP until the most recent observation in Q2 2021. We take averages to aggregate to a yearly frequency.

B.3 Calibration details

For the standard deviation and autocorrelation of GDP, as well as the correlation of the trade balance to GDP ratio with GDP, we use the values provided by Uribe and Schmitt-Grohé (2017). They compute these moment on annual Italian data from 1965 to 2010 after removing a log-quadratic trend. We compute the remaining moments by using our own data sources above. For the level of government debt to GDP and government debt holdings by sector we use quarterly observations from Q1 2000 to Q3 2019, the latest observations available. We compute holdings by foreigners as the product of total debt to GDP and the share of government debt held by non-residents.[58] We then compute the mean over the quarterly series. For unemployment we take the mean over the annual observations from 2000-2020. To compute the spread, we use data on 5 year bond yields (which is the closest to average maturity available) for Italy and Germany. We average over monthly observations to aggregate the data to yearly frequency. We then take the difference between the two series to obtain the spread. All data moments can be found in the second to last column of Table 2.

To compute model analogues of these moments, we simulate the model for 100 000 periods. We then exclude the first 1000 periods as well as any periods the economy spends in autarky

[56] See www.oecd.org/finance/oecdsovereignborrowingoutlook.htm.

[57] See www.bruegel.org/publications/datasets/sovereign-bond-holdings/.

[58] We do not include central bank holdings in the calibration.

and the first 25 periods after reentry.[59] We apply log-quadratic detrending to model generated GDP separately for each sub-sample. We report the weighted mean over the sub-samples for the standard deviation and autocorrelation. All model moments can be found in the last column of Table 2.

[59]Since no defaults occur in the data, there is no need to apply a similar procedure.

Name	Unit	Dates	Frequency	Source
Gross domestic product	Chain linked volumes (2010), million euro	2000-2020	annual	Eurostat
Exports of goods and services	Chain linked volumes (2010), million euro	2000-2020	annual	Eurostat
Imports of goods and services	Chain linked volumes (2010), million euro	2000-2020	annual	Eurostat
Unemployment	Percentage of population in the labour force	2000-2020	annual	Eurostat
Government consolidated gross debt	Percentage of gross domestic product	Q1 2000 - Q2 2021	quarterly	Eurostat
Share of government debt held by non residents	Percentage of total government debt	Q1 2000 - Q3 2019	quarterly	Bruegel database
Share of government debt held by central bank	Percentage of total government debt	Q1 2000 - Q3 2019	quarterly	Bruegel database
Average term-to-maturity of outstanding marketable debt	Years	2020	-	OECD
German Bund 5-year notes yield	ppt	9.2006-11.2021	monthly	German Bundesbank
Italy yield of 5 year benchmark BTP	ppt	9.2006-11.2021	monthly	Bank of Italy

Table 4: Data sources

C Numerical algorithm and accuracy

Our numerical solution relies on discrete choice value function iteration augmented with extreme value taste shocks. As discussed for example in Gordon (2019) and Dvorkin et al. (2021) these shocks smooth out kinks in policy functions and bond price schedules which otherwise lead to convergence problems. Our solution algorithm closely follows Arellano et al. (2020) where a full description can be found.

For borrowing B we choose a discrete grid over the interval [-0.05, 0.8] with 800 linearly spaced points. We denote the set of possible borrowing levels with $\mathcal{B}$. We discretize the autoregressive process for X using the Tauchen (1986) algorithm. We use 15 gridpoints and set the gridwidth to 1.88 standard deviations to ensure that the highest gridpoint $X_{\max}$ is smaller than γ.

The algorithm involves extending the exogenous state with a taste shock specific to each possible borrowing choice, so $s = (X, \xi, \{\psi_{B'}\}_{B' \in \mathcal{B}})$. The shocks $\psi_{B'}$ follow a Gumble (Extreme Value Type 1) distribution with standard deviation σ_ψ. The value function in repayment is then given by

$$V^r(B, i, s) = \max_{B' \in \mathcal{B}, C} \left\{ U(C) + \beta \mathbb{E} V(B', i', s') + \psi_{B'} \right\}. \tag{C.1}$$

Before taste shocks realize, the policy functions for borrowing B' and default δ are probability distributions over the possible choices at each grid point (B, X).[60] In all figures we report the mean over the borrowing policy function. We set the standard deviation of the borrowing taste shock to a very small value $\sigma_\psi = 10^{-4}$ which ensures that these shocks do not affect our quantitative results while the algorithm still convergences. Figure 7 shows the borrowing policy function at the gridpoint $(0.544, 0.255)$. It can be seen that nearly all mass lies on the three gridpoints in the narrow interval $[0.539, 0.541]$.

We iterate backward in time on the value function and bond price schedule until updating errors are smaller than 10^{-6}.

Interest rate changes are modeled as MIT shocks. To do so, we additionally add gridpoints with different interest rates. However, the probability of transitioning from the steady state to any point on the grid outside the steady state is set to zero when computing optimal choices. The grid and transition matrix we use for the interest rate depends on the experiment. For Figure 4, we use a grid of $i \in \{\iota, \iota + 10bps\}$ to compute the numerical derivative of borrowing with respect to the interest rate. Since the shock has no persistence, we set the probability of transitioning to the steady state interest rate in the following period to one. For Figure 5, we use a grid of $i \in \{\iota, \iota + 1ppt\}$, again with a probability of returning to steady state equal to one. For Figure 6, we again use the grid $i \in \{\iota, \iota + 10bps\}$. However, here the probability of remaining in the elevated interest state is equal to $\rho = 0.8$.

Our analytical analysis provides a natural test for the accuracy of our numerical solution. As pointed out in the main text, the points in the state space where $\partial B'/\partial i = 0$ can be computed in two ways. First, we can use the analytical threshold in equation (23). Second, we can numerically

[60] See Arellano et al. (2020) for details on how to compute the choice probabilities.

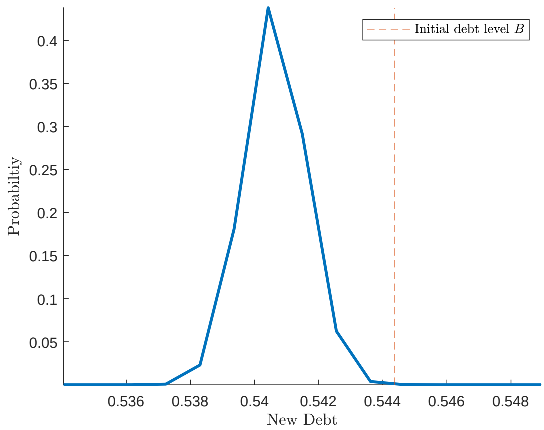


Figure 7: Borrowing policy function. The figure shows the choice probabilities for different values of B' before the realization of the taste shock at the gridpoint $(B, X) = (0.544, 0.255)$.

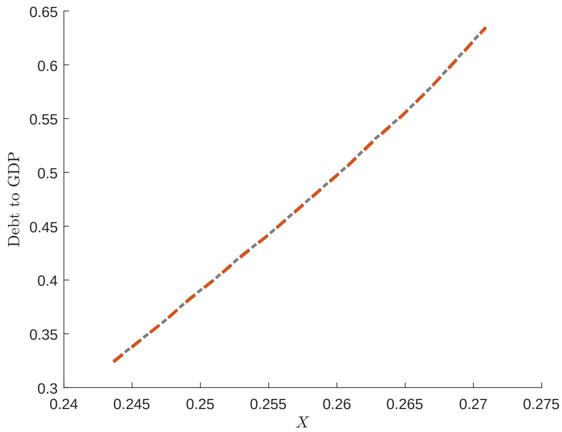


Figure 8: Theoretical and numerical threshold. The gray dashed-dotted line is identical to the line in the right panel of Figure 4, which is the analytical threshold based on equation (23). The red dashed line is the threshold given by the zeros of the numerical derivative $\frac{\partial B'}{\partial i} = 0$.

compute the derivative of the policy function and find the points where it is zero. In our numerical results, we find that these two approaches yield virtually identical results (see Figure 8). This gives us confidence in our results in two ways. First, it shows that the Euler equation does in fact describe well the choices made by the sovereign.[61] Second, it shows that our numerical solution is accurate in the sense that it matches the theoretical predictions of the Euler equation.

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[61] Due to the complexity of the model and possible non-convexities, this cannot be shown analytically.

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