Pre-Read Notes

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Contents

1 Chapter 1: Function Characts. and Props.

1.1 Functions

- A relationship is a function if all values on the domain have less than or equal to 1 value on the range
- circular motion is represented by sinosoidal functions
- functions can be represented in many ways

1.2 Absolute value

- $f(x) = |x|$ describes values ≥ 0

1.3 Properties

- Each function has a unique mixture of elements, usually most visually apparent on a graph
- This can be used to distinguish them

1.4 Sketching Graphs

- Do transformations in steps
- Do translations last when listing transformations
- general formula: $y = af(k(x - d)) + c$

1.5 Inverse

- Inverse is done by swapping x and y variables
- graphically a reflection about x and y axis (along $y = x$)
- denoted by $f^{-1}(x)$
- not all inverses are functions

1.6 Piecewise

- A function with multiple rules
- Related to specific intervals in the domain
- filled circle for inclusive, empty circle for exclusive
- Does not have to be continuous

1.7 Operations within

- If functions have overlapping domains they can be combined
- By combining the dependant variable in some way
- Properties carry onwards

2 Chapter 3: Polynomial Functions

2.1 Polynomial Functions

- A polynomial arranged in this formula
- $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$
- where n are whole numbers and a are real numbers
- most simplified form
- the "degree" is the highest exponent in the polynomial
- degree is proportional to the number of "lines/curves" in the graph

2.2 Properties

- P. function's degree can indicate a lot:
- shape, turning points, zeroes, and end behavior
- odd degree $\rightarrow$ opposite end dir., even degree $\rightarrow$ same end dir.
- if even
- if leading coefficient is pos $\rightarrow$ goes positive to negative
- if leading coefficient is neg $\rightarrow$ goes negative to positive

- if odd
- neg $\rightarrow$ face negative, pos $\rightarrow$ face positive
- turning points proportional to n - 1
- y axis symmetrical $\rightarrow$ even function, rotational symetry $\rightarrow$ odd function

2.3 Factored Form

- Polynomial function family $\rightarrow$ P. functions of similar properties
- zeroes of a P. function are same as roots of related P equation (when factored?)
- Factored form gives roots, factored form at 0 gives zeroes
- Use zeroes and a point to get equation from $f(x) = a(x-b)(x-b)\cdots$ where a is solved using the extra point and b, c, $\cdots$ are zeroes
- if root is exponent 1 $\rightarrow$ passes through as if linear
- if root is exponent 2 $\rightarrow$ glances off like quad vertex
- if root is exponent 3 $\rightarrow$ passes flat before going through, like parent root function

2.4 Transformations

- Like any other function

2.5 Dividing

- Polynomials can be divided in similar manner to numbers
- Like with long division
- remainders are added to the end of the equation, rest becomes factors

2.6 Factoring

- Remainder theorum: $\frac{f(x)}{x-1} = f(a)$
- Factor theorum: x - a is a factor if $f(a) = 0$
- To factor:
  1. use factor theorum to determine factor
  2. divide by factor

2.7 Factoring Sum or Difference

- Expressions with two perfect cubes
- $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$
- $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

3 Chapter 4: Polynomial Equ. and Ineq.

3.1 Solving

- Solutios of f(x) = 0 are zeroes
- sometimes you need to ignore the values outside of the defined intervals

3.2 Solving Linear Inequalities

- Solve linear inequalities by rearranging, like solving linear equations
- If you multiply or divide by a negative number, flip over the inequality sign

3.3 Solving Polynomial Inequalities

- To solve:
  1. Solve for main points, like roots
  2. Plot on some sort of line system
  3. This will give you your solution ranges

3.4 Rates of Change in Polynomials

- Rate of change is $\frac{change\ in\ range}{change\ in\ domain}$
- On interval $x_1 \leq x \leq x_2$ is $\frac{f(x_2)-f(x_1)}{x_2-x_1}$
- When x is very small, $roc = \frac{f(x+h)-f(x)}{h}$
- On any “indexes” the roc is near 0

4 Chapter 5: Rational funcs., eqs., ineqs.

4.1 Graph of Reciprocals

- Reciprocals of linear and quadratic functions follow a similar general graphed form
- Take characteristics from original to graph Reciprocals
- Y coordinates mostly the same to the original
- original's zeroes determine vertical asymtotes
- Reciprocals always start with a asymtote on $y = 0$ unless translated

4.2 Quotients of Polynomials

- Rational function is $f(x) = \frac{g)x()}{h(x)}$ where $f(x)$ and $g(x)$ are Polynomials
- Has breaks or gaps where denominator is zero, must be restricted
- vertical asymtotes and gaps determine restrictions on the domain
- end behavior determined by vertical or oblique asymtotes
- oblique asymtote: slanted asymtote
- Horizontal asymtote $\rightarrow g(x)$ degree is less than or equal to $h(x)$
- Otherwise, if greater, slanted asymtote

4.3 graph in form $\frac{ax+b}{cx+d}$

- Most have vertical and horizontal asymtotes
- Determine vertical asymtote by finding what creates 0 on the denominator
- Determine horizontal asymtote by comparing the ratio between numerator and denominator leading coefficient
- in form $\frac{b}{cx+d}$ vetical asymtote at $x = -\frac{d}{c}$ and horizontal asymtote at $y = 0$
- If numerator and denominator have a common linear factor, line has hole where the zero of the common factor occurs

4.4 Solving Rational Equations

- Solve algebraicly
- zeroes in rational function are zeroes of the numerator
- Make sure to check for extraneous answers

4.5 Solving Rational Inequalities

- Find all values that satisfy inequality
- Use roots and an inequality table

4.6 Rates of Change

- Use previous methods to calculate rates of change
- Cannot calculate where there is a hole
- roc at vertical gaps or asymtotes are undefined
- roc $\rightarrow$ horizontal asymtotes approach zero

5 Chapter 6: Trigonometric functions

5.1 Radians

- Radians is defined as the angle formed when 2r is equal to the arc (c between the 2 r's)
- 2π is equivalent to 180 degrees
- Gives exact numbers without units

5.2 Radians on Cartesian Plane

- Special trangle angles can be expressed with Radians
- Otherwise, same operations as with degrees
- Unit circle stuff

5.3 Graphs of primary trig functions

- A recap on sin and cosine functions
- New: Tangent functions $\rightarrow$ period is from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

5.4 Transformations

- transformed from their parent functions
- to sketch either take information from the equation and sketch or take key points, apply transformations and then sketch
- $g(x) = af(k(x-d)) + c$ where $f(x)$ is sin, cos or tan
- $|a|$ gives amplitude and v. stretch, compression and a gives reflection
- $\frac{1}{|k|}$ gives h. stretch/compression and k gives reflection. $\frac{2\pi}{|k|}$ gives period

5.5 Reciprocal Graphs

- The reciprocal function is closely related to the original function
- Vertical asymtotes at zeroes of original
- Same positive negative intervals
- increase points on the original or the inverse on the Reciprocals

5.6 Modelling

- Casn be used for application

5.7 Rates of Change

- Like any other
- Waiting for derivatives :/
- roc at midle line in tangent functions is zero

6 Chapter 7: Identities and Equations

6.1 Equivalent Trig Functions

- Different trig functions can produce the same result/relationship
- Can use h. translations
- Sin and Tan seperately have rotational symetry with themselves

6.2 Compound Angle Formulas

Addition Formulas	**Subtraction Formulas**
$\sin(a+b) = \sin a \cos b + \cos a \sin b$	$\sin(a-b) = \sin a \cos b - \cos a \sin b$
$\cos(a+b) = \cos a \cos b - \sin a \sin b$	$\cos(a-b) = \cos a \cos b + \sin a \sin b$
$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$	$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$

6.3 Double angels

- Derived from compound angle Formulas

Double Angle for Sine

$\sin 2\theta = 2 \sin\theta \cos\theta$

Double Angle for Cosine

$\cos 2\theta = \cos^2\theta - sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

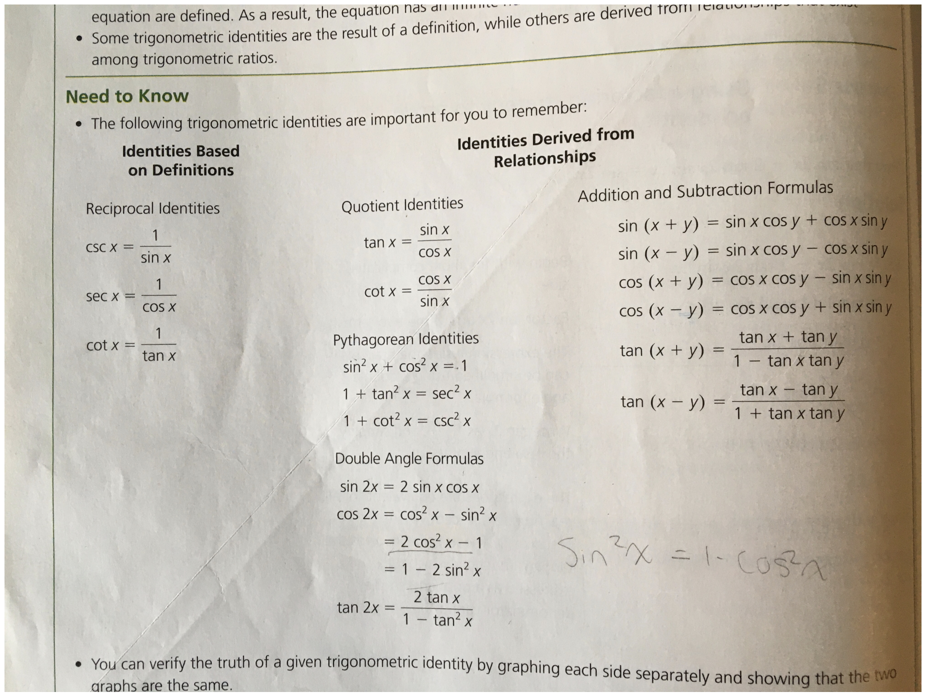
Double Angle for Tangent

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

6.4 Prove Identities

- Rations are identities if both sides are proven equal

- Use other proven identities to prove an identity or by applying zero sum and 1 product rules


equation are defined. As a result, the equation has an infinite ...

- Some trigonometric identities are the result of a definition, while others are derived from relationships that exist among trigonometric ratios.

Need to Know

- The following trigonometric identities are important for you to remember:

Identities Based on Definitions

Reciprocal Identities

$\csc x = \frac{1}{\sin x}$

$\sec x = \frac{1}{\cos x}$

$\cot x = \frac{1}{\tan x}$

Identities Derived from Relationships

Quotient Identities

$\tan x = \frac{\sin x}{\cos x}$

$\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Double Angle Formulas

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

Addition and Subtraction Formulas

$\sin (x + y) = \sin x \cos y + \cos x \sin y$

$\sin (x - y) = \sin x \cos y - \cos x \sin y$

$\cos (x + y) = \cos x \cos y - \sin x \sin y$

$\cos (x - y) = \cos x \cos y + \sin x \sin y$

$\tan (x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan (x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\sin^2 x = 1 - \cos^2 x$

- You can verify the truth of a given trigonometric identity by graphing each side separately and showing that the two graphs are the same.

6.5 Solve Linear Trig Equations

- Solve linear trig equations like regular linear Equations
- Be wary of the periodic nature of the relationship, can have multiple solutions within the interval given

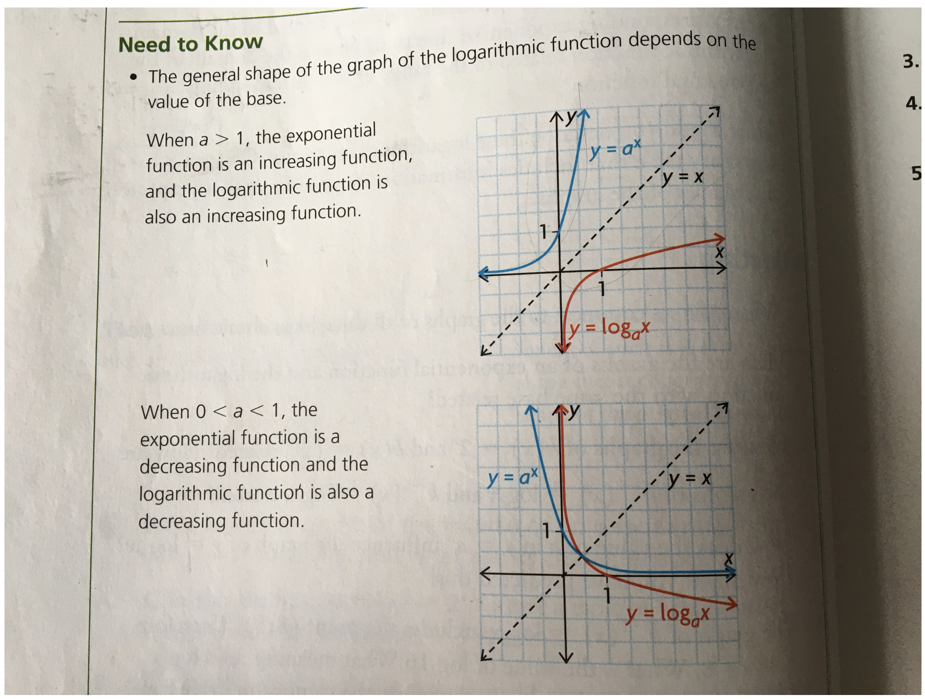
6.6 Solve Quadratic Trig Equations

- WARNING: WEAK IN THIS AREA
- Can be solved initially algebraicly
- Can have multiple solutions, some are extraneous
- Usually factor first before solving

7 Chapter 8: Logarithmic Functions

7.1 Exploring Logarithms

- Logarithms is the inverse of exponents
- the logarithmic form of $y = a^x$ is $x = \log_a y$
- The y axis acts as asymtotes
- x and y intercept is 1
- D:$\{x \in R \mid x > 0\}$, R:$\{y \in R\}$



7.2 Transformations

- Like exponential transformations
- $f(x) = a \log_{base}(k(x - d)) + c$
- **transformations of parent functions**
  - $|a|$ gives vertical stretch or compression, a gives reflection

- $\frac{1}{|k|}$ gives horizontal stretch or compression, k gives reflection
- d and c give h and v translations

7.3 Evaluating Logs

- Can be solved in many ways
- Can rewrite in exponents
- Rewrite in log form and simplify
- Log of negatives does not exist
- properties:
  - $log_a 1 = 0$
  - $log_a a^x = x$
  - $a^{log_a x} = x$

7.4 Laws of Logarithms

- Directly related to exponent laws
- When a, x, y > 0 and a ≠ 1
  - product law: $\log_a xy = \log_a x + \log_a y$
  - quotient law: $\log_a \frac{x}{y} = \log_a x - \log_a y$
  - power law: $\log_a x^r = r \log_a x$

7.5 Solving Exponential Equations

- if $a^m = a^n, m = n$ where $a > 0, a \neq 1 andm, n \in R$
- if $\log_a M = \log_a N, M = N$ where $M, N > 0, a > 0, a \neq 1$
- "To solve the exponential equation algebraically, take base 10 logs of both sides and use power rule to simplify before solving for unknown var"

7.6 Solvng Logarithmic Equations

- Can be expressed in exponential form
- Can be simplified with log/exp laws
- Dont forget to check for inadmissable solutions

7.7 Application

- Log 10 helps to compare a scale that accelerates rapidly

7.8 Rates of Change

- ROC is not constant
- instantaneous roc can be take using estimate linear tangents
- estimate using greatest and least roc

8 Chapter 9: Combinations of Functions

8.1 Exploring

- New functions can be made by combining simple functions
- Done by regular operations: add, subtract, multiply, divide
- New characteristics are a combination of prev parent characteristics

8.2 Sum and Difference

- When f(x) and g(x) are combined, they become the sum $(f + g)(x)$ or difference $(f - g)(x)$ of f and g
- Y (output) values are combined
- $(f + g)(x) = f(x) + g(x)$

8.3 Product

- $f(x) \times g(x)$ creates a product formula: $(f \times g)(x)$
- Graph result by multiplying y value

8.4 Quotient

- Same thing, just divide functions and y results

8.5 Composition

- Composite functions is a where one function lies within another function
- $f(g(x)) = (f \circ g)(x)$

8.6 Solving

- Can be solved by finding the unknown value
- try to rearange to equal zeroe
- Be careful of $>$ vs $\geq$

8.7 Modelling

- Use a model in order to give an approximate representation of a real life relationship
- Increasing data increases points, increases accuracy
- Function chosen should make sense in the situations

9 Chapter 2: Rates of Change

9.1 Determining Average ROC

- essentielly change in y / change in x
- $\frac{f(x_2)-f(x_1)}{x_2-x_1}$
- The slope of the line passing through the 2 points

9.2 Instantaneous ROC

- Average raote of change at close to a single point, a Tangent
- Calculate by getting 2 points that are very close or equal to the point chosen

9.3 Istantaneous ROC Graphs

- Slope = ROC

9.4 Creating Graphical Models

- Can be used to express distance over time $\rightarrow$ speed

9.5 Solving Problems

- Zero at the peaks and troughs of curves
- if negative or positive, will trend downwards or upwards