

MHF4U : Advanced Functions

Assignment #2

Reference Declaration

Complete the Reference Declaration section below in order for your assignment to be graded.

If you used any references beyond the course text and lectures (such as other texts, discussions with colleagues or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

Be sure to cite appropriate theorems throughout your work. You may use shorthand for well-known theorems like the FT (Factor Theorem), RRT (Rational Root Theorem), etc.

Note: Your submitted work must be **your original work**.

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Declared References:

Used this forum answer to number the an element (Question 1):

stackexchange align* but show one equation number at the end

Used YouTube tutorial to do long division with polynomials (Question 1):

1. Given the volume of a rectangular box is $V(x) = x^3 + 6x^2 + 11x + 6$, if the length of the box is $x + 3$ cm, and its width is $x + 2$ cm then **determine** the height of the box. Also, **determine** the domain and range of the function V .

Solution to Question 1:

Considering $v(x)$ has a degree of 3. We are also given the length and width, $x + 3$ and $x + 2$ respectively. This means that:

$$(x + 3)(x + 2)(x + a) = v(x)$$

Where $a \in \mathbb{R}$. We know that there are 3 factors because of the degree. Since:

$$v(x) = x^3 + 6x^2 + 11x + 6$$

$$\therefore (x + 3)(x + 2)(x + a) = x^3 + 6x^2 + 11x + 6 \quad \text{Combining the previously listed equations} \quad (1)$$

Dividing both sides by $(x + 3)(x + 2)$:

$$(x + 3)(x + 2)(x + a) = x^3 + 6x^2 + 11x + 6$$

$$(x + a) = \frac{x^3 + 6x^2 + 11x + 6}{(x + 3)(x + 2)}$$

$$(x + a) = \frac{x^3 + 6x^2 + 11x + 6}{x^2 + 5x + 6} \quad \text{Expanding denominator bracket}$$

Now, using long division, find the final missing value factor, $(x + a)$:

$$\begin{array}{r}
 \overline{) x^3 + 6x^2 + 11x + 6} \\
 \underline{x^3 + 5x^2 + 6x} \\
 x^2 + 5x + 6 \\
 \underline{x^2 + 5x + 6} \\
 0
 \end{array}$$

$$\boxed{\therefore \text{the height of the box is } x+1}$$

Also since this relationship is of degree 3, it is a cubic function. Cubic functions have the properties of a domain and range of all real whole numbers.

$$\boxed{\therefore D \in \mathbb{R}, R \in \mathbb{R}}$$

Check work for question 1 by graphing and using a logical check: From before, we stated that $(x + 3)(x + 2)(x + a) = x^3 + 6x^2 + 11x + 6$. Now that we know the value of a (1), redo the operation to check the work.

$$x^3 + 6x^2 + 11x + 6 = (x + 3)(x + 2)(x + a) \quad \text{From (1)}$$

$$x^3 + 6x^2 + 11x + 6 = (x + 3)(x + 2)(x + 1) \quad a = 1$$

$$= (x^2 + 5x + 6)(x + 1) \quad \text{multiply first 2 factors and simplify}$$

$$= x^3 + 6x^2 + 11x + 6 \quad \text{expand and simplify}$$

$$LHS = RHS \quad \therefore \text{height is correct}$$

From the graph, we can also see that domain and range are all real whole numbers.



2. Consider the quartic polynomial function $f(x) = x^4 - 5x^3 + x^2 + 21x - 18$. Given that there is a local minimum on $f(x)$ at $(-1, -32)$, use a logical argument to **prove** that this must be the absolute minimum of $f(x)$ without graphing the function.

Recall that $x = c$, where $c \in \mathbb{R}$, corresponds to a *local minimum* of a function $f(x)$ on an interval $I = (a, b)$ which contains c provided that for every value of x on the interval I we have that $f(c) \leq f(x)$.

3. Recall that a prime number is defined as an integer $n > 1$ such that its only positive divisors are 1 and n . Let \mathbb{P} represent the set of prime numbers. Suppose that you know that when you divide $g(x) = x^3 + 2x^2 + cx + d$ by $x - 2$ you obtain a remainder of 14, **determine** the specific values of c and d given that $c \in \mathbb{Z}$ and $d \in \mathbb{P}$.

4. Solve $\frac{7}{x+2} + \frac{5}{x-2} = \frac{10x-2}{x^2-4}$.

5. Solve $\left| \frac{x-4}{x+5} \right| \leq 4$.