MCV4U : Calculus and Vectors
MCV4UR : Advanced Placement Calculus and Vectors

Assignment #2

Reference Declaration

Complete the Reference Declaration section below in order for your assigment to be graded.

If you used any references beyond the course text and lectures (such as other texts, discussions with colleagues or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

Be sure to cite appropriate theorems throughout your work. You may use shorthand for well-known theorems like the FT, RRT, MVT, IVT, etc.

Note: Your submitted work must be **your original work**.

Family Name:
First Name:

Declared References:

Instructions

1. Organize and express complete, effective and concise responses to each problem.
2. Use appropriate mathematical conventions and notation wherever possible.
3. Provide logical reasoning for your arguments and cite any relevant theorems.
4. The quality of your communication will be heavily weighted.

Evaluation

- Students will demonstrate an understanding of derivative rules and use them to solve problems.

Criteria	Level 1	Level 2	Level 3	Level 4
Understanding of Mathematical Concepts	Demonstrates limited understanding	Demonstrates some understanding	Demonstrates considerable understanding	Demonstrates thorough understanding of concepts
Selecting Tools and Strategies	Selects and applies appropriate tools and strategies, with major errors, omissions, or mis-sequencing	Selects and applies appropriate tools and strategies, with minor errors, omissions, or mis-sequencing	Selects and applies appropriate tools and strategies accurately, and in a logical sequence	Selects and applies appropriate and efficient tools and strategies accurately to create mathematically elegant solutions
Reasoning and Proving	Inconsistently or erroneously employs logic to develop and defend statements	Statements are developed and defended with some omissions or leaps in logic	Frequently develops and defends statements with reasonable logical justification	Consistently develops and defends statements with sophisticated and/or complete logical justification
Communicating	Expresses and organizes mathematical thinking with limited effectiveness	Expresses and organizes mathematical thinking with some effectiveness	Expresses and organizes mathematical thinking with considerable effectiveness	Expresses and organizes mathematical thinking with a high degree of effectiveness

Instructions: Answer True or False to the following statements.

1. If f and g are differentiable then $\frac{d}{dx}(f+g) = f' + g'$.

2. If f and g are differentiable then $\frac{d}{dx}(fg) = f'g + 2f'g' + fg'$.

3. If f and g are differentiable then $\frac{d}{dx}(f(g(x)) = f'(g(x)) \cdot g'(x)$.

4. If f is differentiable then $\frac{d}{dx}(\sqrt{f(x)}) = \frac{f'(x)}{2\sqrt{x}}$.

5. The derivative of a polynomial function is a polynomial function.

6. The derivative of a rational function is a rational function.

7. The derivative of an exponential function is an exponential function.

8. The derivative of a sinusoidal function is a sinusoidal function.

9. The derivative of a logarithmic function is a logarithmic function.

10. If $f(x) = (x^p - 1)^q$ where $p, q \in \mathbb{Z}^+$ then $-1 \le f^{(pq+1)}(x) \le +1$.

11. If $f'(c) = 0$, then f has a local maximum or minimum at $x = c$.

12. If f has an absolute minimum at $x = c$ then $f'(c) = 0$.

13. If f and g are increasing on an interval I then $f + g$ is also increasing on I.

14. If f and g are increasing on an interval I then fg is also increasing on I.

15. If f is periodic then f' is periodic.

16. If f is even then f' is odd.

17. There exists a function g such that $g(x) > 0$, $g'(x) < 0$, and $g''(x) > 0$ for all $x \in \mathbb{R}$.

18. There exists a function h such that $h(x) < 0$, $h'(x) < 0$, and $h''(x) > 0$ for all $x \in \mathbb{R}$.

19. If $f(x) = \sin(x)$ then the least value of $n \in \mathbb{Z}^+$ such that $f^{(n)}(x) = f(x)$ is $n = 4$.

20. If $f'(x) = g'(x)$ for $0 < x < 1$ then $f(x) = g(x)$ for $0 < x < 1$.

True-False Answers

1. true
2. false
3. true
4. false
5. true
6. true
7. false
8. false
9. false
10. true
11. false
12. false
13. true
14. true
15. true
16. true
17. false
18. true
19. true
20. false

Fill in your responses to the True-False questions above. Be sure to use only capital T or F.

21. **Prove** that the tangent lines to the graph of the function $y = ax^2 + bx + c$, where $a, b, c \in \mathbb{R}, a \neq 0$, at any two points with x-coordinates p and q, where $p, q \in \mathbb{R}$, must intersect at a point whose x-coordinate is $\dfrac{p+q}{2}$.

22. **Determine** the derivative of the function $f(x) = \ln\left(\dfrac{xe^{-x} + \sin(4x)}{1 + 6\sqrt{2x}}\right)$. Go one step at a time and justify each step. Do *not* simplify your answer.

$$LHS = RHS \qquad\qquad\qquad \text{Justification of Line 1}$$
$$LHS + 1 = RHS + 1 \qquad\qquad \text{Justificaiton of Line 2}$$

Consider using an align* environment like this one above that will let you organize your work.

23. **Prove** that if $f(x)$ is a cubic polynomial function with zeros a, b, c then the x-coordinate of the inflection point of $f(x)$ is $\dfrac{a+b+c}{3}$.

24. Determine the maximum area of a rectangle inscribed within an equilateral triangle that has a side length of a where $a \in \mathbb{R}^+$.