MCV4U : Calculus and Vectors
MCV4UR : Advanced Placement Calculus and Vectors

Assignment #3

Reference Declaration

Complete the Reference Declaration section below in order for your assigment to be graded.

If you used any references beyond the course text and lectures (such as other texts, discussions with colleagues or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

Be sure to cite appropriate theorems throughout your work. You may use shorthand for well-known theorems like the FT, RRT, MVT, IVT, etc.

Note: Your submitted work must be **your original work**.

Family Name:
First Name:

Declared References:

Instructions

1. Organize and express complete, effective and concise responses to each problem.
2. Use appropriate mathematical conventions and notation wherever possible.
3. Provide logical reasoning for your arguments and cite any relevant theorems.
4. The quality of your communication will be heavily weighted.

Evaluation

- Students will demonstrate an understanding of derivative rules and use them to solve problems.

Criteria	**Level 1**	**Level 2**	**Level 3**	**Level 4**
Understanding of Mathematical Concepts	Demonstrates limited understanding	Demonstrates some understanding	Demonstrates considerable understanding	Demonstrates thorough understanding of concepts
Selecting Tools and Strategies	Selects and applies appropriate tools and strategies, with major errors, omissions, or mis-sequencing	Selects and applies appropriate tools and strategies, with minor errors, omissions, or mis-sequencing	Selects and applies appropriate tools and strategies accurately, and in a logical sequence	Selects and applies appropriate and efficient tools and strategies accurately to create mathematically elegant solutions
Reasoning and Proving	Inconsistently or erroneously employs logic to develop and defend statements	Statements are developed and defended with some omissions or leaps in logic	Frequently develops and defends statements with reasonable logical justification	Consistently develops and defends statements with sophisticated and/or complete logical justification
Communicating	Expresses and organizes mathematical thinking with limited effectiveness	Expresses and organizes mathematical thinking with some effectiveness	Expresses and organizes mathematical thinking with considerable effectiveness	Expresses and organizes mathematical thinking with a high degree of effectiveness

1. Determine whether the following statements are True (T) or False (F).

   (a) If $\vec{u} \cdot \vec{v} = 0$ then either $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$.

   (b) $(\vec{u} \cdot \vec{v})\vec{u} \times (\vec{u} \times \vec{v})$ is a meaningful expression.

   (c) Force and velocity are scalar quantities.

   (d) If $|\vec{a}| = 1$ then $\vec{a}$ is a unit vector.

   (e) If $\vec{a} = \vec{b} \times \vec{c}$ and $\vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$, then $\vec{b}$ and $\vec{c}$ are collinear.

   (f) $\left\{\vec{i}, \left(\begin{smallmatrix} 0 \\ 1 \\ 0 \end{smallmatrix}\right), \vec{i} \times \vec{j}\right\}$ is a spanning set of $\mathbb{R}^3$.

   (g) Given the standard basis vectors $\vec{i}, \vec{j}, \vec{k} \in \mathbb{R}^3$, we can express $\vec{k}$ as a linear combination of $\vec{i}$ and $\vec{j}$.

   (h) If $\vec{a}$ and $\vec{b}$ are opposite vectors and $|\vec{a}| = n$ then $|\vec{b}| = -n$.

   (i) $\forall \vec{a}, \vec{b} \in \mathbb{R}^3$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

   (j) $\exists \vec{a}, \vec{b} \in \mathbb{R}^3$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

   (k) If $\forall m, n \in \mathbb{R}$, $|m\vec{a}| = |n\vec{a}|$ then $\vec{a} = \vec{0}$.

   (l) Given $\vec{u}, \vec{v} \in \mathbb{R}^3$, if $|\vec{u}| = |\vec{v}| = 1$ then $|\vec{u} \times \vec{v}| = 1$.

   (m) $\forall \vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$, $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$.

   (n) $\forall \vec{x}, \vec{y} \in \mathbb{R}^3$, $|\vec{x} \times \vec{y}| = |\vec{y} \times \vec{x}|$.

   (o) Given two lines in $\mathbb{R}^3$, ℓ_1 and ℓ_2, either $\ell_1 \parallel \ell_2$ or ℓ_1 intersects ℓ_2 at some point in $\mathbb{R}^3$.

   (p) If $\vec{x}, \vec{y} \in \mathbb{R}^3$ then $\text{Span}\{\vec{x}, \vec{y}\}$ is a plane in $\mathbb{R}^3$.

   (q) If $\{\vec{a}, \vec{b}\}$ spans $\mathbb{R}^2$ then $\{\vec{a}, \vec{a} + \vec{b}\}$ spans $\mathbb{R}^2$.

   (r) If $\vec{p}$ is a scalar multiple of $\vec{q}$ then $\vec{x} = t\vec{p} + \vec{q}$, $t \in \mathbb{R}$, is a line through the origin.

   (s) Given $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^4$, if $\vec{a} \cdot \vec{b} = 0$ and $\vec{b} \cdot \vec{c} = 0$ then $\vec{a} \cdot \vec{c} = 0$.

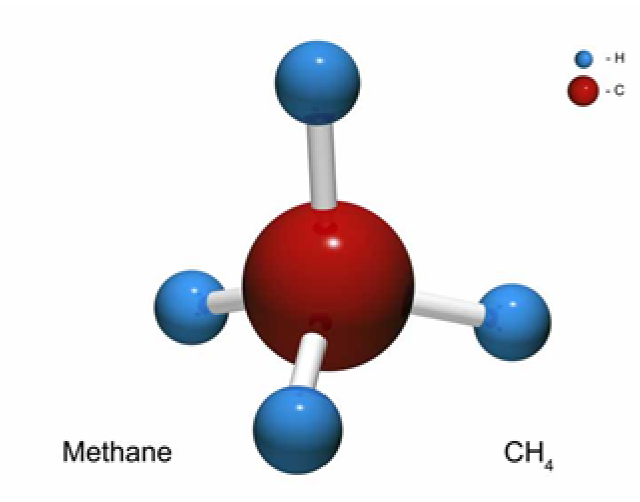
   (t) If $\left(\begin{smallmatrix} 1 \\ 1 \\ 1 \end{smallmatrix}\right)$ is a solution to a system of linear equations then $\left(\begin{smallmatrix} 2 \\ 2 \\ 2 \end{smallmatrix}\right)$ is also a solution.

True-False Answers

(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)
(m)
(n)
(o)
(p)
(q)
(r)
(s)
(t)

2. An **orthogonal spanning set of** $\mathbb{R}^3$ is a set of 3 non-zero vectors, all of which are orthogonal to each other and span $\mathbb{R}^3$. Given $\vec{a} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, **determine** vectors $\vec{b}$ and $\vec{c}$ such that $\{\vec{a}, \vec{b}, \vec{c}\}$ is an orthogonal spanning set of $\mathbb{R}^3$. **Express** the vector $\vec{u} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ as a linear combination of the vectors in this spanning set.

3. In a methane molecule (CH_4), a carbon atom is surrounded by four hydrogen atoms. Assume that the hydrogen atoms are located in space at $A(0,0,0), B(1,1,0), C(1,0,1)$ and $D(0,1,1)$ and that the carbon atom is located at $E\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$. **Determine** the *bond angle*, the angle from some hydrogen atom to the carbon atom, to another hydrogen atom.



4. If θ is the acute angle between $\vec{a}$ and $\vec{b}$ then **prove** $proj_{\vec{a}}\vec{b} \cdot proj_{\vec{b}}\vec{a} = (\vec{a} \cdot \vec{b}) \cos^2(\theta)$.

5. **Determine** the equation of a line through the point $A(1, 2, -3)$ and lying within the plane $x - 2y + 2z + 9 = 0$.

6. **Determine** the equation of a plane through the origin and the points $P(3, 1, 2)$ and $Q(-1, 2, -3)$.

7. Suppose that $\vec{v_1}$ and $\vec{v_2}$ are vectors such that $|\vec{v_1}| = 2$ and $|\vec{v_2}| = 3$, and $\vec{v_1} \cdot \vec{v_2} = 5$. Let $\vec{v_3} = proj_{\vec{v_1}} \vec{v_2}$, $\vec{v_4} = proj_{\vec{v_2}} \vec{v_3}$ and so on so that $\vec{v_{k+1}} = proj_{\vec{v_{k-1}}} \vec{v_k}$. **Evaluate** $\sum\limits_{i=1}^{\infty} |\vec{v_i}|$.

8. **Determine** and **classify** the nature of the intersection between the following planes:

$$\Pi_1 : x - 5y + 2z = 10$$

$$\Pi_2 : x + 7y - 2z + 6 = 0$$

$$\Pi_3 : 8x + 5y + z = 20$$

LaTeX note: You may find the *amatrix* custom environment included in the code useful for creating the augmented matrix below.

$$\left(\begin{array}{ccc|c} 1 & -5 & 2 & 10 \\ 1 & 7 & -2 & -6 \\ 8 & 5 & 1 & 20 \end{array}\right)$$