Advanced Functions

Assignment #1

Reference Declaration

Complete the Reference Declaration section below in order for your assigment to be graded.

If you used any references beyond the course text and class notes (such as other texts, discussions with peers or online resources), indicate this information in the space below. If you did not use any aids then explicitly state this in the space provided.

Be sure to cite appropriate theorems throughout your work. You may use shorthand for well-known theorems like the FT (Factor Theorem), RRT (Rational Root Theorem), etc.

Note: Your submitted work must be **your original work**.

Family Name: WongFirst Name: Max

Declared References:

Used this forum answer to number the an element within align (Question 2):
stackexchange align* but show one equation number at the end

Instructions

1. Organize and express complete, effective and concise responses to each problem.
2. Use appropriate mathematical conventions and notation wherever possible.
3. Provide logical reasoning for your arguments and cite any relevant theorems.
4. Ask your teacher questions if you need any clarification.

Evaluation

D3 Students will compare the characteristics of functions, and solve problems by modelling and reasoning with functions.

Criteria	Level 1	Level 2	Level 3	Level 4
Understanding of Mathematical Concepts	Demonstrates limited understanding	Demonstrates some understanding	Demonstrates considerable understanding	Demonstrates thorough understanding of concepts
Selecting Tools and Strategies	Selects and applies appropriate tools and strategies, with major errors, omissions, or mis-sequencing	Selects and applies appropriate tools and strategies, with minor errors, omissions, or mis-sequencing	Selects and applies appropriate tools and strategies accurately, and in a logical sequence	Selects and applies appropriate and efficient tools and strategies accurately to create mathematically elegant solutions
Reasoning and Proving	Inconsistently or erroneously employs logic to develop and defend statements	Statements are developed and defended with some omissions or leaps in logic	Frequently develops and defends statements with reasonable logical justification	Consistently develops and defends statements with sophisticated and/or complete logical justification
Communicating	Expresses and organizes mathematical thinking with limited effectiveness	Expresses and organizes mathematical thinking with some effectiveness	Expresses and organizes mathematical thinking with considerable effectiveness	Expresses and organizes mathematical thinking with a high degree of effectiveness

1. **Describe** the characteristics of the function $f(x) = -2|x-3|+2$ by filling in the table given below. **Write** a paragraph briefly explaining how you determined each characteristic.

Characteristic	
domain	$\{x \mid x \in \mathbb{R}\}$
range	$\{y \mid y \in \mathbb{R}, y < 2\}$
zero(s)	x = 4, 2
y-intercept	(0,-4)
interval(s) of increase	none
interval(s) of decrease	$f(x)$ decreasing on $(-\infty, 3)$ $f(x)$ decreasing on $[3, \infty)$
discontinuities	none
symmetry	even symmetry along vertical line $x = 3$
end behaviours	as $x \to \infty, f(x) \to -\infty$ as $x \to -\infty, f(x) \to -\infty$

The domain is all real whole numbers because of the key characteristic of an absolute value relationship. The range for the parent function of an absolute value relationship is all non negative values but since f(x) is reflected vertically and vertically translted 2 up the range becomes all real values below and equal to 2. Zeroes are found by solving for x=0. The y intercept is found by solving for x = 0 or simply using the vertical translation. In the parent function of f(x) there are 2 positive intervals but since the functions is reflected vertically and translated 3 to the right, there are 2 decreasing intervals to the left and right of x=3. This function does not have any restrictions or characteristics that cause discontinuities. Doing the even, odd and neither symmetry test with f(x), -f(x), f(-x) and -f(-x) I determined that there is even symmetry and due to the horizontal translation mentioned before the line of symmetry is at x=3. Referencing the intervals both ends of the function are decreasing.

2. **Solve** both inequalities.

   (a) Solve $|3x - 5| \leq 2$

   (b) Solve $-|-2x - 1| < -4$

Solution for (a):

$$|3x - 5| \leq 2$$

$$|3(x - \frac{5}{3})| \leq 2 \qquad \text{Factor by 3}$$

$$|3||x - \frac{5}{3}| \leq 2 \qquad \text{1st property of absolute value}$$

$$3|x - \frac{5}{3}| \leq 2 \qquad \text{Since } —3— = 3$$

$$|x - \frac{5}{3}| \leq \frac{2}{3} \qquad \text{divide by 3} \qquad (1)$$

$$1 \leq x \leq \frac{7}{3} \qquad k - c \leq x \leq k + c \Longleftrightarrow |x - k| \leq c$$

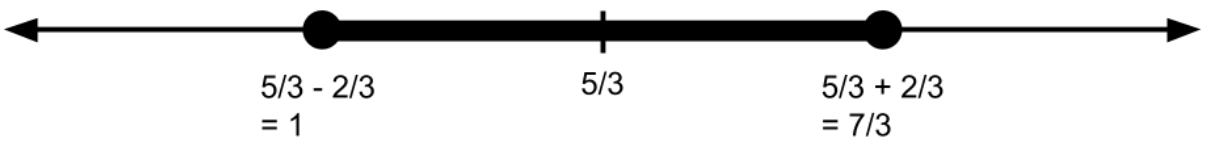
The solutiont to the inequality$|3x - 5| \leq 2$ is $\boxed{1 \leq x \leq \frac{7}{3}}$

Checked answer by using the following line diagram and math starting from (1):

$$|x - \frac{5}{3}| = \frac{2}{3}$$

$$x - \frac{5}{3} = \pm\frac{2}{3} \qquad |x| = c \longleftrightarrow x = \pm c$$

$$x = \frac{5}{3} \pm \frac{2}{3} \qquad \text{add } \frac{5}{3} \text{ to both sides}$$



Solution for (b):

	multiply all by -1, flip inequality sign
$-\lvert -2x-1\rvert < -4$	3rd property of inequalities
$\lvert -2x-1\rvert > 4$	
$\lvert -2(x+\frac{1}{2})\rvert > 4$	factor -2
$\lvert -2\rvert\lvert x+\frac{1}{2}\rvert > 4$	1st properoty of absolute values
$2\lvert x+\frac{1}{2}\rvert > 4$	$\lvert -2\rvert = 2$
$\lvert x+\frac{1}{2}\rvert > 2$	divide all by 2 (2)
$x < -\frac{5}{2},\ \frac{3}{2} < x$	$\lvert x-k\rvert > c \Longleftrightarrow x < k-c$ or $x > k+c$

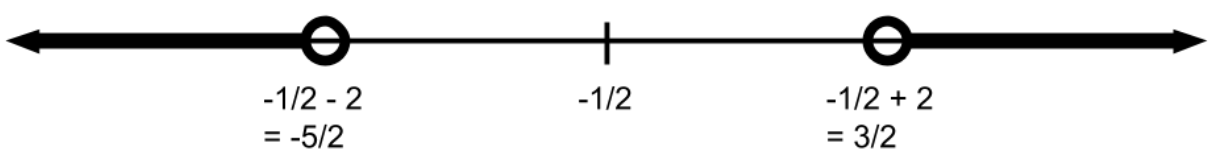
The solutiont to the inequality $-\lvert -2x-1\rvert < -4$ is $\boxed{-\frac{5}{2},\ \frac{3}{2} < x}$

Checked answer by using the following line diagram and math starting from (2):

$$\lvert x+\frac{1}{2}\rvert = 2$$

$$x+\frac{1}{2} = \pm 2 \qquad \lvert x\rvert = c \longleftrightarrow x = \pm c$$

$$x = -\frac{1}{2} \pm 2 \qquad \text{subtract } \frac{1}{2}$$



3. A 10 foot long stem of bamboo is broken in such a way that its tip touches the ground 3 feet away from the base of the stem. **Determine** the height of the break.

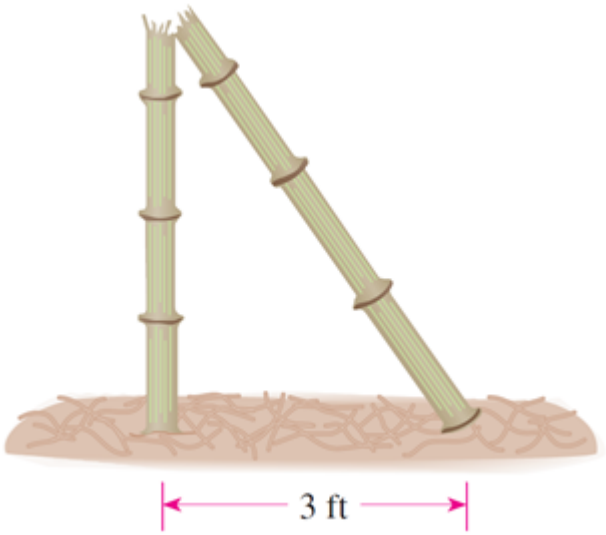


Figure 1: Diagram from Problem 3

4. **Define** (rewrite) $f(x) = |x^3 - x|$ as a piecewise function not including any expressions involving absolute value.

5. **Determine** the inverse of the function $g(x) = \dfrac{-2}{x-1} + 4$. **Prove** that g and g^{-1} satisfy the expression $g(g^{-1}(x)) = x$.