

MCV4U : Calculus and Vectors
MCV4UR : Advanced Placement Calculus and Vectors
Assignment #1

Reference Declaration

Complete the Reference Declaration section below in order for your assignment to be graded.

If you used any references beyond the course text and lectures (such as other texts, discussions with colleagues or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

Be sure to cite appropriate theorems throughout your work. You may use shorthand for well-known theorems like the FT, RRT, MVT, IVT, etc.

Note: Your submitted work must be **your original work**.

Family Name: Wong

First Name: Max

Declared References:

Instructions

1. Organize and express complete, effective and concise responses to each problem.
2. Use appropriate mathematical conventions and notation wherever possible.
3. Provide logical reasoning for your arguments and cite any relevant theorems.
4. The quality of your communication will be heavily weighted.

Evaluation

- Students will demonstrate an understanding of limits and apply them to solve problems.

Criteria	Level 1	Level 2	Level 3	Level 4
<i>Understanding of Mathematical Concepts</i>	Demonstrates limited understanding	Demonstrates some understanding	Demonstrates considerable understanding	Demonstrates thorough understanding of concepts
<i>Selecting Tools and Strategies</i>	Selects and applies appropriate tools and strategies, with major errors, omissions, or mis-sequencing	Selects and applies appropriate tools and strategies, with minor errors, omissions, or mis-sequencing	Selects and applies appropriate tools and strategies accurately, and in a logical sequence	Selects and applies appropriate and efficient tools and strategies accurately to create mathematically elegant solutions
<i>Reasoning and Proving</i>	Inconsistently or erroneously employs logic to develop and defend statements	Statements are developed and defended with some omissions or leaps in logic	Frequently develops and defends statements with reasonable logical justification	Consistently develops and defends statements with sophisticated and/or complete logical justification
<i>Communicating</i>	Expresses and organizes mathematical thinking with limited effectiveness	Expresses and organizes mathematical thinking with some effectiveness	Expresses and organizes mathematical thinking with considerable effectiveness	Expresses and organizes mathematical thinking with a high degree of effectiveness

1. The sum $3 - \frac{3}{10} + \frac{3}{100} - \frac{3}{1000} + \cdots + \frac{(-1)^k 3}{10^k} + \cdots$ is closest to:
 - (a) $\frac{2}{e}$
 - (b) 2
 - (c) e
 - (d) e^2
2. The value of $\lim_{x \rightarrow a} \frac{x - a}{\sqrt{x} - \sqrt{a}}$ is:
 - (a) $2a$
 - (b) $2\sqrt{a}$
 - (c) $\sqrt{2a}$
 - (d) a
3. If $\lim_{x \rightarrow c} f(x) = L$ where $L \in \mathbb{R}$ then which of the following must be true?
 - (a) f is defined at $x = c$
 - (b) f is continuous at $x = c$
 - (c) $f(c) = L$
 - (d) None of the above
4. Given f is continuous on $[-2, 6]$, if $f(-2) = 7$ and $f(6) = -1$ then the Intermediate Value Theorem guarantees that:
 - (a) $f(0) = 0$
 - (b) $f(c) = 2$ for at least one value of $c \in [-2, 6]$
 - (c) $f(c) = 0$ for at least one value of $c \in [-1, 7]$
 - (d) $\forall x \in [-2, 6], -1 \leq f(x) \leq 7$
5. The value of $\lim_{x \rightarrow 0} \frac{\sin(4x)}{2x}$ is:
 - (a) 0
 - (b) 0.5
 - (c) 1
 - (d) 2
6. The value of $\lim_{x \rightarrow -\infty} \frac{\sqrt{8x^2 - 4x}}{x + 2}$ is:
 - (a) $-\infty$
 - (b) $-2\sqrt{2}$
 - (c) 4
 - (d) $2\sqrt{2}$

7. The value of k such that $f(x) = \begin{cases} x^2 + 2 & : x \leq -1 \\ kx + 4 & : x > -1 \end{cases}$ is continuous on \mathbb{R} is:

- (a) -3
- (b) -1
- (c) 1
- (d) 3

8. For what value of a is the function $f(x) = \frac{x^2 - 3x}{x - a}$ continuous on \mathbb{R} ?

- (a) -3
- (b) -1
- (c) 3
- (d) There does not exist a value of a such that f is continuous on \mathbb{R} .

9. The value of $\lim_{x \rightarrow 0} \frac{e^{ax} \sin(bx) \tan(cx)}{x^2}$, where $a, b, c \in \mathbb{R} \setminus \{0\}$ is:

- (a) $\frac{a}{bc}$
- (b) bc
- (c) $\frac{ab}{c}$
- (d) ab

10. The value of $\lim_{x \rightarrow 0} x^3 \sin\left(\frac{1}{x}\right)$ is:

- (a) -1
- (b) 0
- (c) 1
- (d) nonexistent

Multiple Choice Answers

1. C
2. B
3. D
4. B
5. D
6. D
7. C
8. C
9. B
10. B

Fill in your responses to the multiple choice questions above. Be sure to use capital letters.

11. If $f(x) = 3x^3 - 2x + 1$ then **determine** $f'(x)$ from *First Principles* by using the definition of the derivative.

Solution To Question 11:

Given the function $f(x) = 3x^3 - 2x + 1$, we can solve for $f'(x)$ using the first principal definition of the derivative.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{First Principle Definition}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(3(x+h)^3 - 2(x+h) + 1) - (3x^3 - 2x + 1)}{h} \quad \text{Implement f(x) with x+h and x}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3(x^3 + 3x^2h + 3xh^2 + h^3) - (2x + 2h) + 1 - 3x^3 + 2x - 1}{h} \quad \text{Expand and Simplify}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x^3 + 9x^2h + 9xh^2 + 3h^3 - 2x - 2h + 1 - 3x^3 + 2x - 1}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{9x^2h + 9xh^2 + 3h^3 - 2h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(9x^2 + 9xh + 3h^2 - 2)}{h} \quad \text{Factor h}$$

$$f'(x) = \lim_{h \rightarrow 0} 9x^2 + 9xh + 3h^2 - 2 \quad \text{Simplify, Cross out h}$$

$$f'(x) = 9x^2 + 9x(0) + 3(0)^2 - 2 \quad \text{Apply Limit}$$

$$f'(x) = 9x^2 - 2$$

Therefore, using the first principles we find that the derivative of $f(x)$ is $9x^2 - 2$.

12. **Evaluate** $\lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$ or assert that it does not exist and provide evidence to support your conclusion.

Solution To Question 12:

When you look at the function within the limit, the first thing that might come to mind is the square root being subtracted to something. If we used 1 rule in order to rationalize, we can eliminate the square root and unlock the possibility for further simplification.

$$\begin{aligned}
 & \lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) \\
 &= \lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{\sqrt{1+t}}{t\sqrt{1+t}} \right) && \text{Crossmultiply, Get Same Denominators} \\
 &= \lim_{t \rightarrow 0} \left(\frac{1 - \sqrt{1+t}}{t\sqrt{1+t}} \right) && \text{Simplify, Add Fractions Together} \\
 &= \lim_{t \rightarrow 0} \left(\frac{1 - \sqrt{1+t}}{t\sqrt{1+t}} \cdot \frac{1 + \sqrt{1+t}}{1 + \sqrt{1+t}} \right) && \text{Use 1 Rule, Rationalize} \\
 &= \lim_{t \rightarrow 0} \left(\frac{1 - (1+t)}{(t\sqrt{1+t})(1 + \sqrt{1+t})} \right) && \text{Simplify, } (a+b)(a-b) = a^2 - b^2 \\
 &= \lim_{t \rightarrow 0} \left(\frac{-t}{(t\sqrt{1+t})(1 + \sqrt{1+t})} \right) && \text{Simplify} \\
 &= \lim_{t \rightarrow 0} \left(\frac{-1}{(\sqrt{1+t})(1 + \sqrt{1+t})} \right) && \text{Factor t out} \\
 &= \frac{-1}{(\sqrt{1+0})(1 + \sqrt{1+0})} && \text{Apply Limit} \\
 &= \frac{-1}{(1)(2)} && \text{Simplify} \\
 &= -\frac{1}{2}
 \end{aligned}$$

Therefore, the limit, when evaluated, is $-\frac{1}{2}$

13. **Evaluate** $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+cx} - 1}{x}$ where $c \in \mathbb{R}^*$.

Solution To Question 13:

Just like the previous question, we can see that there is some sort of root (cubed this time) which is being subtracted. If we could rationalize this, possibly with a cubic identity, we could potentially gain opportunities to simplify. Given the cubic identity $(a - b)(a^2 + ab + b^2) = a^3 - b^3$, we can possibly use this to eliminate the cube root.

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+cx} - 1}{x} \\
 &= \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+cx} - 1}{x} \cdot \frac{(\sqrt[3]{1+cx})^2 + 1(\sqrt[3]{1+cx}) + 1^2}{(\sqrt[3]{1+cx})^2 + 1(\sqrt[3]{1+cx}) + 1^2} && \text{Use 1 rule \& cubic identity, rationalize} \\
 &= \lim_{x \rightarrow 0} \frac{(\sqrt[3]{1+cx} - 1)((\sqrt[3]{1+cx})^2 + 1(\sqrt[3]{1+cx}) + 1^2)}{(x)((\sqrt[3]{1+cx})^2 + 1(\sqrt[3]{1+cx}) + 1^2)} && \text{Simplify} \\
 &= \lim_{x \rightarrow 0} \frac{(\sqrt[3]{1+cx})^3 - 1^3}{(x)((\sqrt[3]{1+cx})^2 + 1(\sqrt[3]{1+cx}) + 1^2)} && \text{Apply Cubic identity where } a = \sqrt[3]{1+cx}, b = 1 \\
 &= \lim_{x \rightarrow 0} \frac{1 + cx - 1}{(x)((\sqrt[3]{1+cx})^2 + 1(\sqrt[3]{1+cx}) + 1^2)} && \text{Simplify} \\
 &= \lim_{x \rightarrow 0} \frac{cx}{(x)((\sqrt[3]{1+cx})^2 + 1(\sqrt[3]{1+cx}) + 1^2)} \\
 &= \lim_{x \rightarrow 0} \frac{c}{(\sqrt[3]{1+cx})^2 + 1(\sqrt[3]{1+cx}) + 1^2} && \text{Cancel out x} \\
 &= \frac{c}{(\sqrt[3]{1+c(0)})^2 + 1(\sqrt[3]{1+c(0)}) + 1^2} && \text{Apply Limit} \\
 &= \frac{c}{(1)^2 + 1(1) + 1} && \text{Simplify} \\
 &= \frac{c}{3}
 \end{aligned}$$

Therefore, when evaluated the value of the limit is $\frac{c}{3}$ For $c \in \mathbb{R}^*$

14. The figure shows a point P on the parabola $y = x^2$ and the point Q where the perpendicular bisector of OP intersects the y-axis. As P approaches the origin along the parabola, what happens to Q ? Does it have a limiting position? If so, **determine** the limiting position of Q .

Solution To Question 14:

Your solution starts here.

