CSC2515: Assignment 2 - Deep NN

Due on Friday, February 23, 2018

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Problem 0

Introductory information and readme instructions

This submissions containts the required files: *digits.py, faces.py, deepfaces.py, deepnn.tex & faces.pdf* in addition to two folders - both containing an previously downloaded pictures for each actor/actress. The downloading code can be found in the source files. Comments about the code are located in each of the source files. Code was written in Python 3.5 with the Anaconda environment - the packages used are outlined at the beginning of the file.

Problem 1

Dataset description

In parts 1 through , the MNIST dataset of handwritten digits will be used: *mnist_all.mat*. The data is loaded in the form of a python dictionary. It is already split into training and testing sets - with 60 000 digits in the training and 10 000 in testing, and is also split by digit which can be accessed with *train[digit]*. The number of of each digit in test/train is approximately equal although there are variations between digits. The digit data is in the form of a 784 integer array, representing the flattened pixel intensities of 28 by 28 pixel images.

10 examples of each digit are shown below:

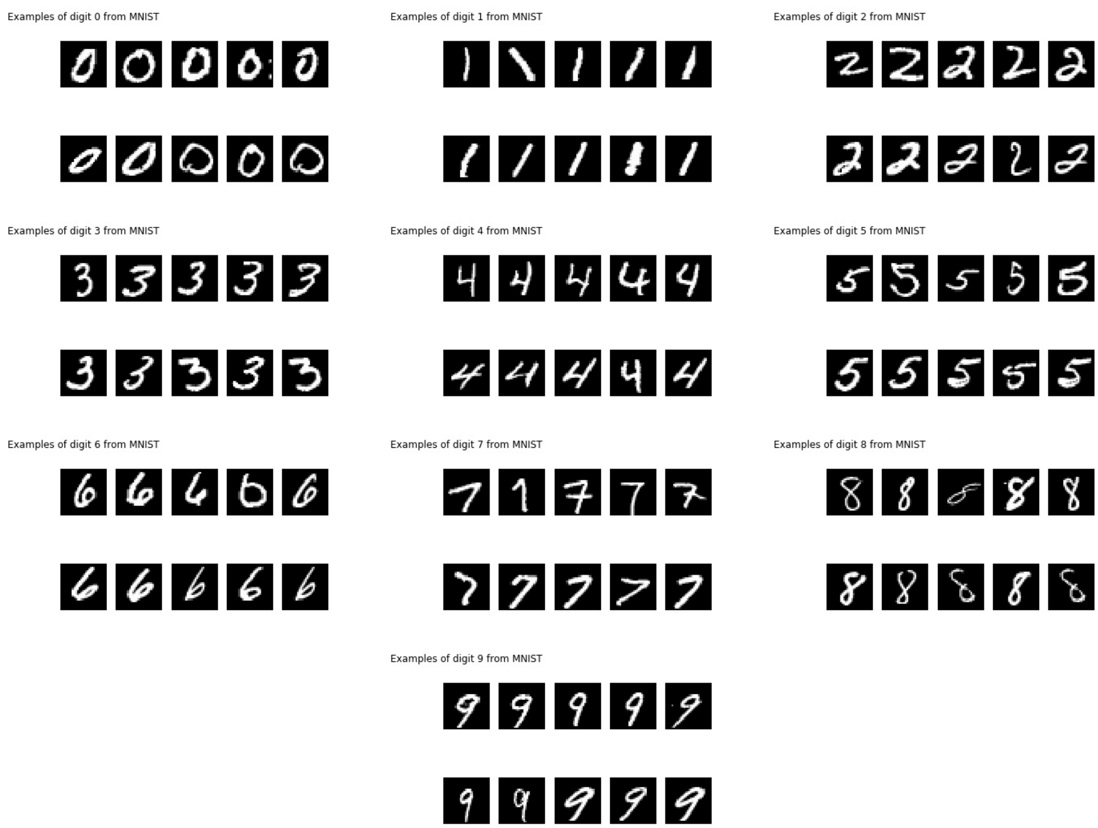
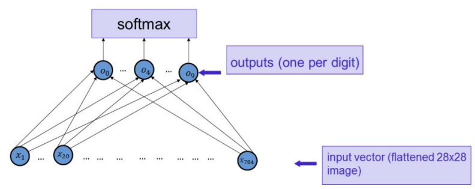


Figure 1: Digits from MNIST

Problem 2

Computation of the provided Network.

The network provided in Part 2 is shown below:



The source code to compute the assigned network is provided below.

```
def compute(X, W, b):
   hypothesis = np.matmul(W.T, X)
   hypothesis = hypothesis + b
   return hypothesis
```

The source code running a test case of the assigned network is provided below, followed by the output. M is the MNIST dataset provided, W and b are randomly initialized numpy variables.

```
def testPart2():
   # Load Data
   M = loadData()
   train = loadTrain(M)
   # Testing computation function
   # Initialize weights and bias to random normal variables
   W = np.random.normal(0.0, 0.1, (784, 10))
   b = np.random.normal(0, 0.1, (10, 1))
   hypothesis = compute(train, W, b)
   print(softmax(hypothesis))
```

Output (after applying softmax) from the test case:

```
[[ 0.02681158 0.0532326 0.04155301 ..., 0.02401518 0.05461363
  0.09298066]
 [ 0.02934882 0.02945978 0.00866719 ..., 0.02812303 0.05160524
  0.06640991]
 [ 0.07800929 0.14095711 0.05324094 ..., 0.08344358 0.071218
  0.03403779]
 ...,
 [ 0.06811845 0.09068319 0.11408343 ..., 0.09293973 0.23242635
  0.24750088]
 [ 0.42558937 0.28527524 0.36294473 ..., 0.32821263 0.12923604
  0.11150445]
 [ 0.04023265 0.03443131 0.05201543 ..., 0.0454216 0.06806685
  0.08509415]]
```

Problem 3

Gradient derivation and vectorized format

(A) For the network described above, the cost function used will be the sum of negative log-probabilities. This cost function, summed over training examples is:
$C = -\sum_j y_j log p_j$
Where y is the actual target and p is the predicted value.
The output p_i is the calculated the softmax function to o_i,:
$p_i = \frac{e^{o_i}}{\sum_j e^{o_j}}, \quad o_i = \sum_j x_j w_{ij}$
We use the chain rule to find $\frac{dC}{dw_{ij}}$:
$\frac{dC}{dw_{ij}} = \frac{dC}{dp_i}\frac{dp_i}{do_i}\frac{do_i}{dw_{ij}}$
$\frac{dp_i}{do_i} = \frac{e^{o_i}}{\sum_j e^{o_i}} - \left(\frac{e^{o_i}}{\sum_j e^{o_i}}\right)^2 = p_i(1-p_i)$
$\frac{do_i}{dw_{ij}} = x_j$
$\frac{dC}{do_i} = \sum_j \frac{dC}{dp_j}\frac{dp_j}{do_i}$

(B) The source code for computing the gradient is shown below.

```
def grad_NLL_W(y, o, layer):
    p = softmax(o)
    grad = p - y
    grad = np.matmul(grad, np.transpose(layer))
    return np.transpose(grad)


# Computes the gradient of negative log-loss function for biases only
def grad_NLL_b(y, o):
    p = softmax(o)
    grad = p - y
    grad = np.sum(grad, axis=1, keepdims=True)
    return grad
```

The source for our test cases, along with the outputs, are also shown below.

```
def testPart3():
    # Test gradient functionality
    y = np.zeros((10, 1))
    y[1, :] = 1

    # Create a test matrix
    M = loadData()
    test0 = ((M["train1"][130].T) / 255.0).reshape((784, 1))
    W = np.random.normal(0, 0.2, (784, 10))
    b = np.random.normal(0, 0.2, (10, 1))
    # print(np.where(test0 != 0))

    # Create a finite difference
    h = 0.00001

    # Weight testing
```

```
finite_W = np.zeros((784, 10))
finite_W[542, 0] = h
finite_d = (NLL(y, compute(test0, W + finite_W, b)) - NLL(y, compute(test0, W,
    b))) / (h)
print("Cost for row 542, column 0: " + str(finite_d))
gradient = grad_NLL_W(y, compute(test0, W, b), test0)
print("Gradient for row 542, column 0: " + str(gradient[542, 0]))

# Bias testing
finite_b = np.zeros((10, 1))
finite_b[1, :] = h
finite_d = (NLL(y, compute(test0, W, b + finite_b)) - NLL(y, compute(test0, W,
    b))) / (h)
print("Cost for second element in bias: " + str(finite_d))
gradient = grad_NLL_b(y, compute(test0, W, b))
print("Gradient matrix: " + str(gradient))

# Reinitialize test variables for another test
finite_W = np.zeros((784, 10))
finite_b = np.zeros((10, 1))
y = np.zeros((10, 1))
test1 = ((M["train9"][130].T) / 255.0).reshape((784, 1))
y[9, :] = 1

# Weight testing
finite_W[300, 4] = h
finite_d = (NLL(y, compute(test1, W + finite_W, b)) - NLL(y, compute(test1, W,
    b))) / (h)
print("Cost for row 300, column 4: " + str(finite_d))
gradient = grad_NLL_W(y, compute(test1, W, b), test1)
print("Gradient for row 300, column 4: " + str(gradient[300, 4]))

# Bias testing
finite_b[4, :] = h
finite_d = (NLL(y, compute(test1, W, b + finite_b)) - NLL(y, compute(test1, W,
    b))) / (h)
print("Cost for fifth element in bias: " + str(finite_d))
gradient = grad_NLL_b(y, compute(test1, W, b))
print("Gradient matrix: " + str(gradient))
```

```
Cost for row 542, column 0: 0.0424078520744
Gradient for row 542, column 0: 0.0424076964593
Cost for second element in bias: -0.996437311773
Gradient matrix: [[ 0.05461597]
 [-0.99643733]
 [ 0.03592502]
 [ 0.32774037]
 [ 0.04444944]
 [ 0.05179153]
 [ 0.08930539]
 [ 0.14572631]
 [ 0.19441422]
 [ 0.05246907]]
```

```
Cost for row 300, column 4: 0.000106382902487
Gradient for row 300, column 4: 0.000106382357955
Cost for fifth element in bias: 0.00010680176743
Gradient matrix: [[ 3.36017608e-03]
 [ 1.57850465e-02]
 [ 1.07051677e-02]
 [ 1.06504072e-01]
 [ 1.06801186e-04]
 [ 6.75277789e-02]
 [ 9.15366897e-02]
 [ 8.16717644e-02]
 [ 6.16811862e-01]
 [ -9.94009358e-01]]
```

Problem 4

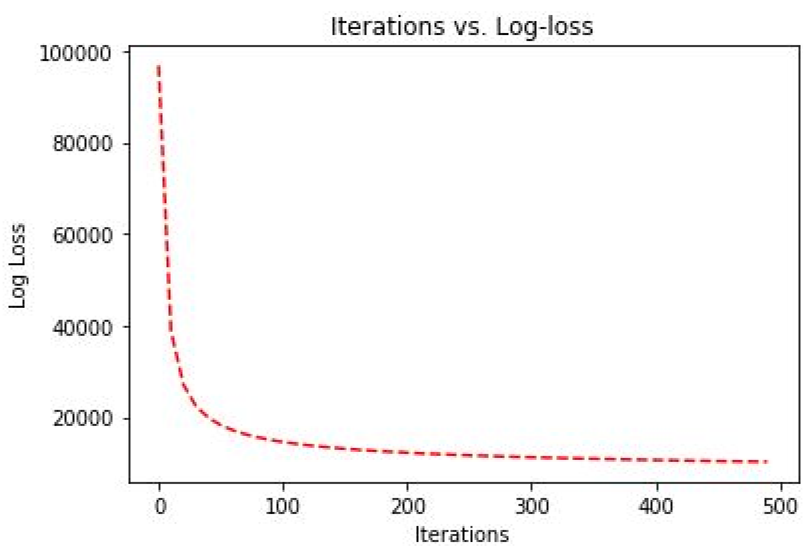
Gradient Descent Training

For the Vanilla Gradient Descent algorithm, the weights and biases were sampled from the normal distribution around a mean of 0 and standard deviation of 0.2, using *numpy.random.normal*. The weights and biases were initialized with shapes (784, 10) and (10, 1) respectively.

The optimization procedure for gradient descent involved testing: the type of distribution used in weight and bias initializiation, the learning rate, and the number of iterations. The reported results proved to be optimial. The network converges relatively quickly at 500 iterations - although itwas tested up to approximately 5000 iterations, however this had little accuracy on validation performance. The learning curve is shown below.

By trial and error, a learning rate of 0.00001 was selected. The learning rate was chosen to optimize convergence time - a low learning rate resulted in slow convergence time, and a high learning rate resulted in fluctuations and hindered convergence.

The log loss is plotted against the number of iterations (at this stage different methods are being compared for the same number of training examples so it was not necessary to divide loss by number of examples).



Problem 5

Gradient Descent With Momentum

Momentum is used in gradient descent to dampen oscillation and include an directional accumulation term to increase speed:
The paramter update rule for momentum, as provided in lecture notes is:

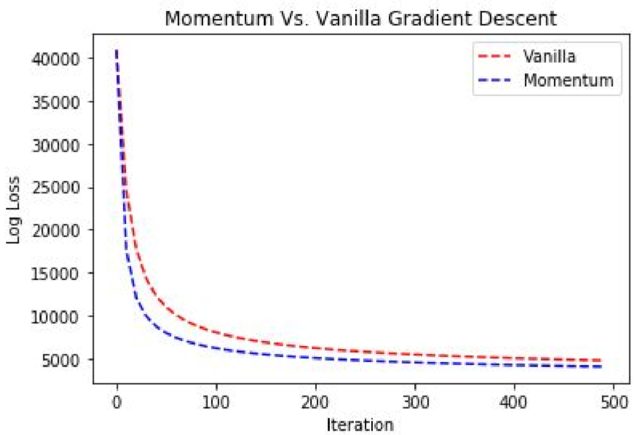
$\nu \leftarrow \gamma\nu + \alpha\frac{dC}{dW}$
$W \leftarrow W - \nu$

Where γ is the momentum term and α is the learning rate. C is the cost, and W is the parameter (weight) being updated. The momentum rate was set to 0.5 based on some initial tests, however the number could be optimized further for better/faster performance. This momentum rate was sufficient to demonstrate its effect.
The function for vectorized momentum is shown below.

```
\# MOMENTUM GRADIENT DESCENT

def grad_descent_w(NLL, grad_NLL_W, grad_NLL_b, x, y, init_w, init_b, alpha,
    iterations):
   iters = list()
   costy = list()

   EPS = 1e-5
   prev_w = init_w - 10 * EPS
   prev_b = init_b - 10 * EPS
   w = init_w.copy()
   b = init_b.copy()
   max_iter = iterations
   iter = 0

   v = np.zeros((784, 10))
   v[:, :] = 0.000001 # Initialize
   momentum_rate = 0.5

   while np.linalg.norm(w - prev_w) > EPS and iter < max_iter and np.linalg.norm(b -
       prev_b) > EPS:
      prev_w = w.copy()
      prev_b = b.copy()
      grad_desc = alpha * (grad_NLL_W(y, compute(x, w, b), x))
      mv = np.multiply(momentum_rate, v)
      v = np.add(mv, grad_desc)
      w -= v
      b -= alpha * (grad_NLL_b(y, compute(x, w, b)))
      if iter % 10 == 0:
         print("Iteration: " + str(iter))
         print("Cost: " + str(NLL(y, compute(x, w, b))))
         cost_i = NLL(y, compute(x, w, b))
```

```
        iters.append(iter)
        costy.append(cost_i)
    iter += 1
```

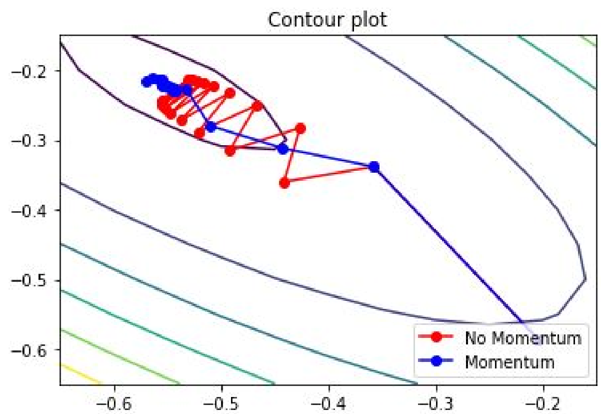
The figure below shows the learning curve on train data for gradient descent with and without momentum. It can be observed that momentum increases convergence speed.



Problem 6

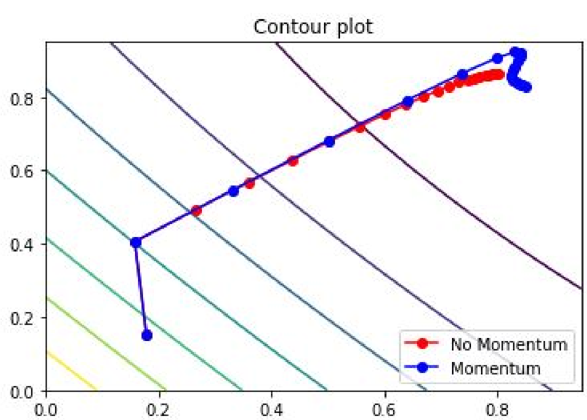
Contours of Gradient Descent

In this section, the effectiveness of gradient descent moving towards a minima will be demonstrated on contour plots. The weights chosen were at $init\_w[490, 3]$ and $init\_w[490, 3]$. These weights were chosen because by checking the corresponding values on the digits, to ensure that they were not all 0s would not have an effect on the output. When the weights were allowed to vary around the vales obtained in Part 5, without gradient descent, they did not move towards the local minima.
The weights were moved $\pm$ 0.15 from their trained values. The learning rate was increased to 0.017, and 20 iterations were run.



As shown in the figure momentum results in a faster path towards the minimum with fewer oscillations. The momentum term in the update introduces memory and allows the learning process to accelerate in consistent directions.

An example of momentum not being effective, would be as stated above, where the weights correspond to pixels of 0 or close to 0. In the example below, the weights chosen were changed to at $init\_w[10, 3]$ and $init\_w[11, 3]$, which may have been closer to the sides. It can be seen from the contours that they are more linear and less effected by the weights. Momentum could be less effective, or very similar to vanilla gradient descent, when there is already a 'smooth' path towards the minima.



Problem 7

Running the linear regression classifier results in a training accuracy of 89% and a validation accuracy of 56%. Both the α and number of iterations were modified, to varying degrees of success - however, it is worth noting that the validation accuracy is significantly lower than the training accuracy which may be a symptom of overfitting.

Two loops were written in order to obtain the training hypothesis and validation hypothesis labels. The steps are written below.

1. A matrix of 0's was initialized to the same shape as that of the label matrix
2. Each column in the label matrix was scanned for it's maximum value and the index of that value was stored in a new variable, *max*
3. Starting at the 0^{th} column, a 1 was stored into the 0's matrix for each index in *max*, and iterated through until the last column was reached.
4. The resultant matrix now had only one value of 1 in each column and was compared to the label matrix.

An example of this for the training data is shown below.

```
#Training hypothesis
ones_t = np.ones((1,training.shape[1]))
training_with_bias = vstack((ones_t,training))
training_hypothesis = np.matmul(theta.T,training_with_bias)
max = training_hypothesis.argmax(axis=0)
max = np.array(max)
training_hypothesis_labels = np.zeros((training_hypothesis.shape))
print(training_hypothesis_labels.shape)
i = 0
while i < training_hypothesis_labels.shape[1]:
   index = max[i]
   training_hypothesis_labels[index,i]=1
   i+=1

correct = 0
i = 0
while i < training_hypothesis_labels.shape[1]:
   if np.array_equal(training_hypothesis_labels[:,i],training_labels[:,i]) is True:
      correct += 1
   i+=1
print("Training accuracy is: " + str(correct/600.0))
```

Problem 8

Training a single-hidden-layer fully connected network

The provided source was modified to train a single-hidden layer, fully-connected neural network to classify actors and actresses from Project 1, assuming the original list of actors and actresses were the following:

```
act =['Lorraine Bracco', 'Peri Gilpin', 'Angie Harmon', 'Alec Baldwin', 'Bill Hader',
    'Steve Carell']
```

The neural network architecture involved a fully connected layer (Linear), followed by ReLU activation (activation), followed by another fully connected layer (Linear) with input dimensions of 32*32*3 (e.g. $32 \times 32 \times 3$ representing the pixel areas and colour channels), 20 hidden neurons, and finally a output layer consisting of 6 output units. The Cross Entropy cost was minimized as the cost function. Furthermore, experimentation involving the learning rate, α, number of iterations, as well as batch size was conducted, leading to an optimal learning rate of 0.0001, 5000 iterations, and a batch size of approximately 200 (out of a total 450 training examples). After training, the accuracy for the training set, validation set and test set were 87.3%, 84.9% and 79.2% respectively. A plot of the training & validation accuracies throughout the training are shown below.

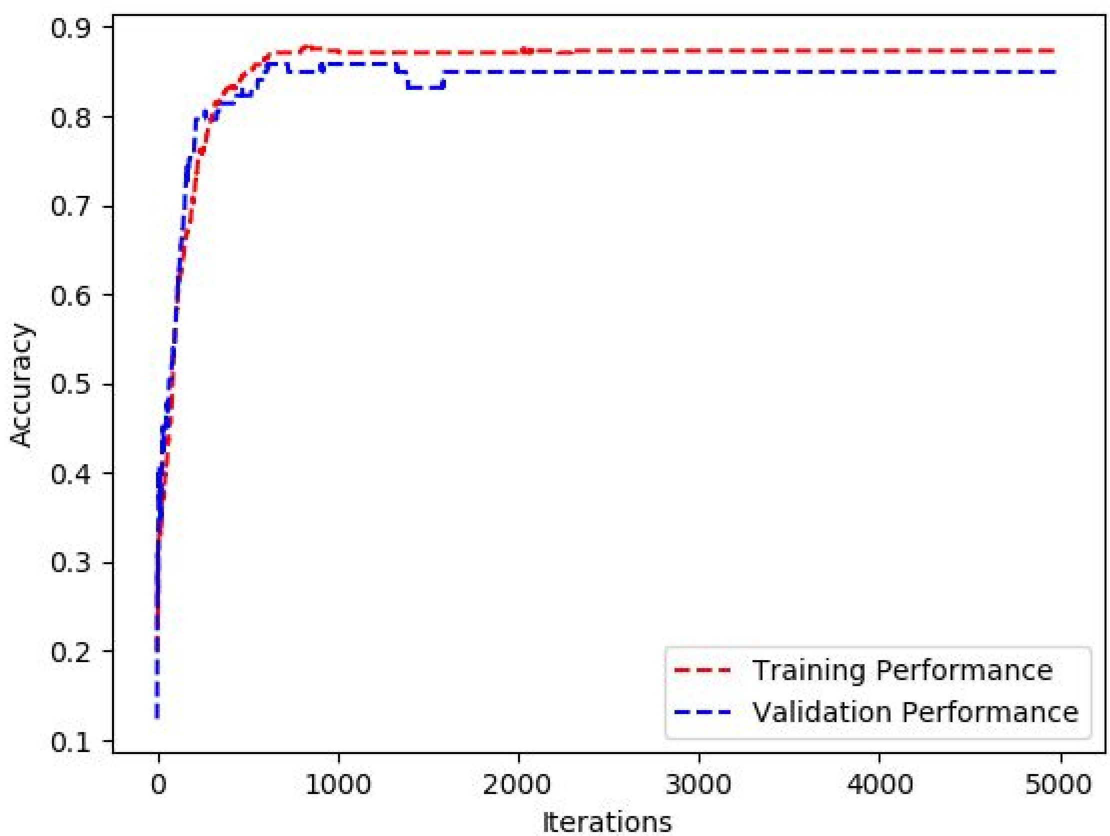


Figure 2: Training & Validation performance for Part 8

Problem 9

Visualizing $\theta's$ with multiple label matrix

The 6 figures below show the visualized $\theta's$ for training on the full set of actors in *act* with the label matrix, as outlined in the assignment specification.

A potential explanation for the lack of "face-like" appearance in $\theta's$ obtained after training may be that the size of the training set is too large for each actor. As observed in part 4, with only two actors from the Baldwin and Carell sets, the $\theta's$ visualized more resembled faces.

Problem 10

Visualizing $\theta's$ with multiple label matrix

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