CSC2515: Assignment 1

Due on Friday, February 23, 2018

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Problem 0

Introductory information and readme instructions

Attached, you will find the required submissions *digits.py, faces.py, deepfaces.py, deepnn.tex & faces.pdf* in addition to two folders - both containing an appropriate amount of pictures for each actor/actress. Comments about the code are located in each of the source files. Code was written in Python 3.5 with the Anaconda environment - the packages used are outlined at the beginning of the file.

Problem 1

Dataset description

TALK ABOUT MNIST DATASET HERE - make some BS up, who cares

Problem 2

Computation of the provided Network.

The source code to compute the assigned network is provided below.

```
def compute(X, W, b):
   hypothesis = np.matmul(W.T, X)
   hypothesis = hypothesis + b
   return hypothesis
```

Problem 3

Binary Classification between Steve Carrel and Alec Baldwin

As outlined in the assignment documentation, linear regression was used to create a binary classifier. The cost function that was minimized was the quadratic cost function, also known as the least-squares cost function as outlined below.

$$J(\theta) = \sum_{i=1}^{n}(h_\theta(x^{(i)} - y^{(i)})^2$$

Using the Gradient-Descent algorithm, a training accuracy of 100% was obtained on the training data compared to a validation accuracy of 83%. The following code was used to output the results of the hypothesis from the binary classifier.

```
#Input - the training_hypothesis matrix, of shape m x 1, where m is the number of
    examples
   i = 0
   while i < training_hypothesis.shape[0]:
      if training_hypothesis[i] > 0.5:
         training_hypothesis[i] = 1
      else:
         training_hypothesis[i] = 0
      i+=1
```

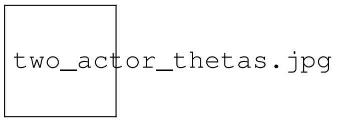
The Gradident Descent algorithm was modified for the purposes for this assignment was taken from the Galaxy code posted on the CSC2515 website. In order to get the system to work, a suitable number of maximum iterations was performed, in addition to modifying the learning rate, α. I did not formalize my selection of the two parameters, but tried experimenting with different values. I noticed that as α was increased by a factor of 10 (e.g. 10^{-4}), there were errors in the matrix operations being performed, resulting in numerical overflow issues. As α decreased, (e.g. 10^{-6}) it was observed that both the training and validation sets exhibited lower accuracy in correctly identifying the individuals - perhaps indicating a local minimum. However, this could certainly be investigated by modifying the number of maximum iterations (but in the cases mentioned, testing was completed on 10000 iterations). Furthermore, the above loop could further be optimized using a vectorization technique or by implementing one-hot initialization (e.g. as in the Tensorflow package).

Problem 4

Visualizing Binary $\theta's$

(A) The two figures below show the visualized $\theta's$ for training on the complete set of Baldwin and Carell and of two images respectively.

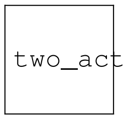
(a) $\theta's$ with full training set.



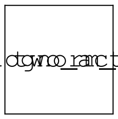
(b) $\theta's$ with training size 2.

Figure 1: Visualized $\theta's$

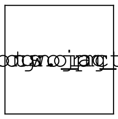
(B) Experiments were run by modifying both the initializations of θ and the number of iterations - however, it was noted that the experiments with an initialized θ matrix of 1's most closely resembled a face, indicating the the initialization of θ may be hindering the appearance of "face-like" θ values.



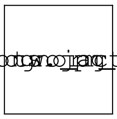
(a) 2500 iterations



(b) 5000 iterations

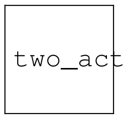


(c) 7500 iterations

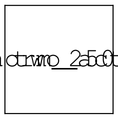


(d) 10000 iterations

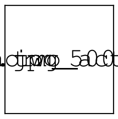
Figure 2: θ's after log-normal initialization of θ with $\mu = 0$ and $\sigma = 0.2$



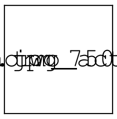
(a) 2500 iterations



(b) 5000 iterations

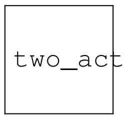


(c) 7500 iterations

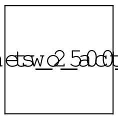


(d) 10000 iterations

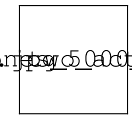
Figure 3: θ's after Normal initialization of θ with $\mu = 0$ and $\sigma = 0.2$



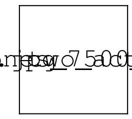
(a) 2500 iterations



(b) 5000 iterations



(c) 7500 iterations



(d) 10000 iterations

Figure 4: θ's 1 initialization

Problem 5

Investigating overfitting

The results from varying the training size on the training and validation accuracies are shown below.

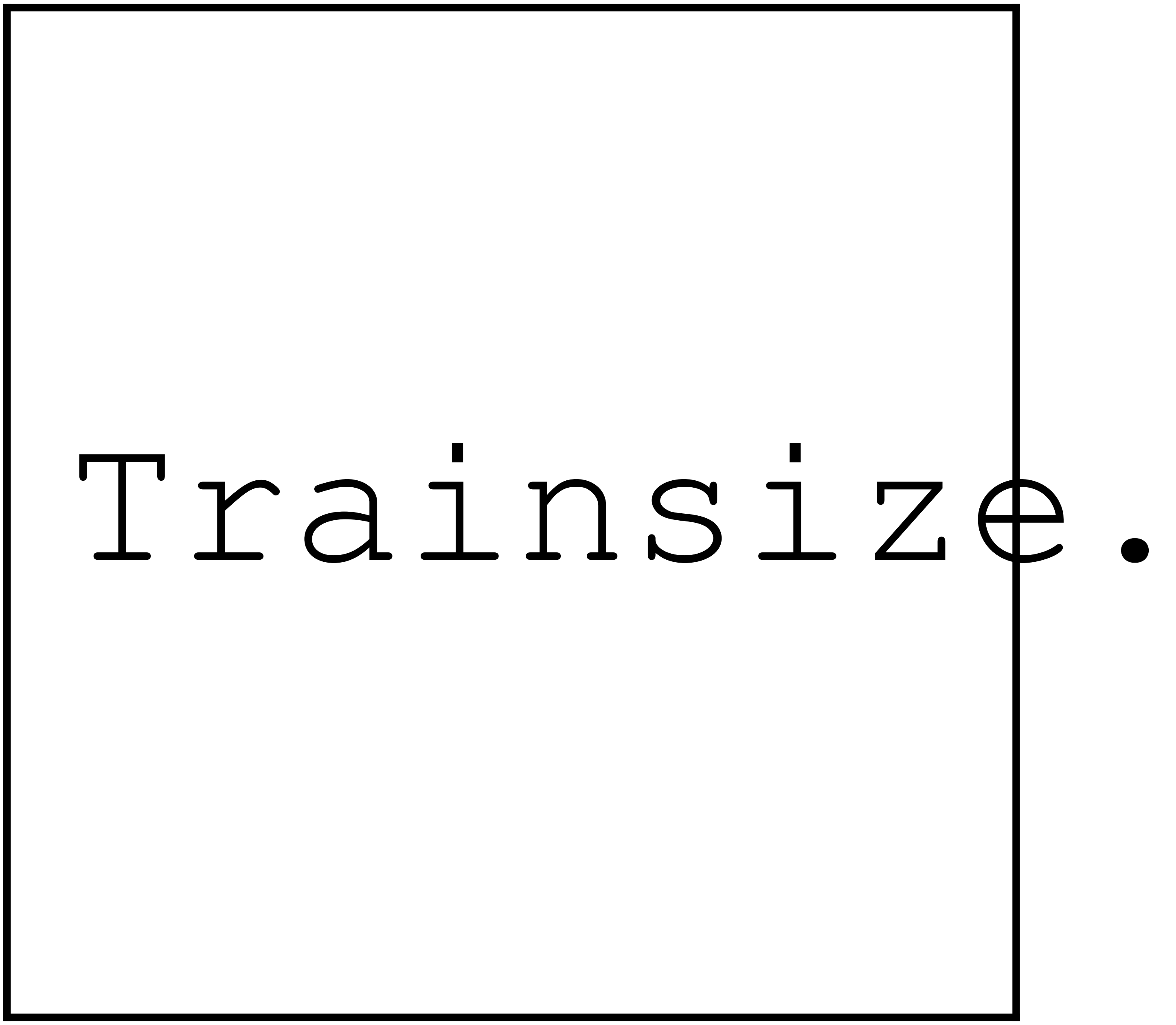


Figure 5: Graph comparing size of training set size vs. training accuracy and validation in increments of 50 images

A potential explanation for the results in the figure may be that the training and validation sets, when split amongst the individual actors, may be favouring some actors over others. There is an intersection at 100 training examples, and it appears the the accuracy between the two sets is almost identical. The fact that there is a large discrepancy between the accuracies between a training sample size of 180 to 300 (e.g. validation accuracy stays relatively constant) may reveal support for the above claim that actors are not being represented properly, presumably due to a lack of rigour in the splitting between data sets for the individual actors. Upon testing on the other actors and actresses not included in *act*, the validation accuracy obtained was 69%. As outlined in the **Post-Mortem Addendum** in Section 2, the selection of data from the images may also have played a role in this behaviour.

Problem 6

Mathematical Derivations of Gradient Descent

(A) We can compute $\frac{\partial J}{\partial \theta_{pq}}$ by expanding the expression as outlined in the assignment documentation as follows:

$$J(\theta) = \sum_i \left((\theta^T x^{(i)} - y^{(i)})_1^2 + (\theta^T x^{(i)} - y^{(i)})_2^2 + \ldots + (\theta^T x^{(i)} - y^{(i)})_j^1 \right)$$
$$J(\theta) = (\theta^T x^{(1)} - y^{(1)})_1^2 + (\theta^T x^{(1)} - y^{(1)})_2^2 + \ldots (\theta^T x^{(1)} - y^{(1)})_j^1 + (\theta^T x^{(2)} - y^{2)})_1^2 + \ldots + (\theta^T x^{(i)} - y^{(i)})_j^2$$

Therefore, it appears we can model this double summation as a summation over the p^{th} column and q^{th} row. Therefore, the partial derivative of a specific element, say, element pq can be expressed as:

$$\boxed{\frac{\partial J}{\partial \theta_{pq}} = 2x_q^p(\theta^T x^p - y^p)_q}$$

(B) Based on the assignment specification, let X be a matrix of size $n \times m$, where n is the number of features and m is the number of training examples. Let θ be a matrix of size $k \times n$, where k is the number of features. Let Y be a $k \times m$ matrix. The provided expression taken from the assignment specification, with matrices substituted into their respective variables, gives the following expression:

$$2\begin{bmatrix} x_{11} & \ldots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \ldots & x_{nk} \end{bmatrix} \times \left[\begin{bmatrix} \theta_{11} & \ldots & \theta_{1n} \\ \vdots & \ddots & \vdots \\ \theta_{k1} & \ldots & \theta_{kn} \end{bmatrix}^T \times \begin{bmatrix} x_{11} & \ldots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \ldots & x_{nm} \end{bmatrix} - \begin{bmatrix} y_{11} & \ldots & y_{1m} \\ \vdots & \ddots & \vdots \\ y_{k1} & \ldots & y_{km} \end{bmatrix} \right]^T$$

Simplification:

$$\begin{bmatrix} 2x_{11} & \ldots & 2x_{1m} \\ \vdots & \ddots & \vdots \\ 2x_{n1} & \ldots & 2x_{nk} \end{bmatrix} \times \begin{bmatrix} \theta_{11}x_{11} + \ldots + \theta_{k1}x_{n1} - y_{11} & \ldots \\ \vdots & \vdots \\ \ldots & \theta_{1n}x_{1m} + \ldots + \theta_{kn}x_{nm} - y_{km} \end{bmatrix}^T$$

If the above matrix computation is carried out, we can observe, from part A, that the pq term in the derivative of the cost function refers to an element within this gradient matrix. For example, the top left hand side derivative, e.g. $\frac{\partial J}{\partial \theta_{11}} = 2x_{11}(\theta_{11}x_{11} + \ldots + \theta_{k1}x_{n1} - y_{11}$. **Note:** The top left hand corner element of the gradient was selected for ease of computation, but any element within this gradient matrix would be valid.

```
#Cost Function
def f2(x,y,theta):
    ones = np.ones((1,x.shape[1]))
    x = np.vstack((ones,x))
    hypothesis = np.matmul(theta.T,x)
    loss = np.power((hypothesis-y),2)
    return np.sum(loss)
```

```
#Gradient Function
def df2(x,y,theta):
    ones = np.ones((1,x.shape[1]))
    x = vstack((ones,x))
    hypothesis = np.matmul(np.transpose(theta),x)-y
    hypothesis = np.transpose(hypothesis)
    gradient = 2*np.matmul(x,hypothesis)
    return gradient
```

(C) The code for testing the implementation of the cost and gradient is shown below.

```
#Testing the cost and gradient functionality
def part6():
    #Testing the loss function
    x = np.random.normal(0,0.6,(20,15))
    y = np.random.normal(0.2,0.4,(5,15))
    theta = np.random.normal(-0.1,0.3,(21,5))
    h = 0.00001
    #Cost of individual component
    testarr1 = np.zeros(theta.shape)
    testarr1[3,4] = h
    print("Cost is:")
    print((f2(x,y,theta+testarr1)-f2(x,y,theta-testarr1))/(2*h))
    print("Gradient is:")
    print(df2(x,y,theta)[3,4])
    testarr2 = np.zeros(theta.shape)
    testarr2[1,4] = h
    print("Cost is:")
    print((f2(x,y,theta+testarr2)-f2(x,y,theta-testarr2))/(2*h))
    print("Gradient is:")
    print(df2(x,y,theta)[1,4])
    testarr3 = np.zeros(theta.shape)
    testarr3[2,3] = h
    print("Cost is:")
    print((f2(x,y,theta+testarr3)-f2(x,y,theta-testarr3))/(2*h))
    print("Gradient is:")
    print(df2(x,y,theta)[2,3])
    testarr4 = np.zeros(theta.shape)
    testarr4[8,4] = h
    print("Cost is:")
    print((f2(x,y,theta+testarr4)-f2(x,y,theta-testarr4))/(2*h))
    print("Gradient is:")
    print(df2(x,y,theta)[8,4])
    testarr5 = np.zeros(theta.shape)
    testarr5[10,1] = h
    print("Cost is:")
    print((f2(x,y,theta+testarr5)-f2(x,y,theta-testarr5))/(2*h))
    print("Gradient is:")
    print(df2(x,y,theta)[10,1])
```

The output that results from execution of this code is below:

```
Cost is:
-8.507892229658864
Gradient is:
-8.5078922306346
Cost is:
```

```
-5.070817460506305
Gradient is:
-5.070817460432679
Cost is:
-11.655092452400593
Gradient is:
-11.655092451975934
Cost is:
-3.5024079892309596
Gradient is:
-3.5024079896724483
Cost is:
-0.24353752436923057
Gradient is:
-0.2435375237734189
```

The h was selected so as to try to compute a small difference, similar to the definition of the derivative - as h further decreases, you would observe that the gradient and incremental difference in cost appear to converge.

Problem 7

Linear Regression with Multiple Actors

Running the linear regression classifier results in a training accuracy of 89% and a validation accuracy of 56%. Both the α and number of iterations were modified, to varying degrees of success - however, it is worth noting that the validation accuracy is significantly lower than the training accuracy which may be a symptom of overfitting.

Two loops were written in order to obtain the training hypothesis and validation hypothesis labels. The steps are written below.

1. A matrix of 0's was initialized to the same shape as that of the label matrix
2. Each column in the label matrix was scanned for it's maximum value and the index of that value was stored in a new variable, *max*
3. Starting at the 0^{th} column, a 1 was stored into the 0's matrix for each index in *max*, and iterated through until the last column was reached.
4. The resultant matrix now had only one value of 1 in each column and was compared to the label matrix.

An example of this for the training data is shown below.

```python
#Training hypothesis
ones_t = np.ones((1,training.shape[1]))
training_with_bias = vstack((ones_t,training))
training_hypothesis = np.matmul(theta.T,training_with_bias)
max = training_hypothesis.argmax(axis=0)
max = np.array(max)
training_hypothesis_labels = np.zeros((training_hypothesis.shape))
print(training_hypothesis_labels.shape)
i = 0
while i < training_hypothesis_labels.shape[1]:
   index = max[i]
   training_hypothesis_labels[index,i]=1
   i+=1

correct = 0
i = 0
while i < training_hypothesis_labels.shape[1]:
   if np.array_equal(training_hypothesis_labels[:,i],training_labels[:,i]) is True:
      correct += 1
   i+=1
print("Training accuracy is: " + str(correct/600.0))
```

Problem 8

Visualizing $\theta' s$ with multiple label matrix

The 6 figures below show the visualized $\theta' s$ for training on the full set of actors in *act* with the label matrix, as outlined in the assignment specification.

(a) Baldwin $\theta' s$

(b) Bracco $\theta' s$

(c) Carell $\theta' s$

(d) Gilpin $\theta' s$

(e) Hader $\theta' s$

(f) Harmon $\theta' s$

Figure 6: θ values for each individual actor

A potential explanation for the lack of ”face-like” appearance in $\theta' s$ obtained after training may be that the size of the training set is too large for each actor. As observed in part 4, with only two actors from the Baldwin and Carell sets, the $\theta' s$ visualized more resembled faces.

Problem 9

Visualizing $\theta's$ with multiple label matrix

The 6 figures below show the visualized $\theta's$ for training on the full set of actors in *act* with the label matrix, as outlined in the assignment specification.

(a) Baldwin $\theta's$

(b) Bracco $\theta's$

(c) Carell $\theta's$

(d) Gilpin $\theta's$

(e) Hader $\theta's$

(f) Harmon $\theta's$

Figure 7: θ values for each individual actor

A potential explanation for the lack of "face-like" appearance in $\theta's$ obtained after training may be that the size of the training set is too large for each actor. As observed in part 4, with only two actors from the Baldwin and Carell sets, the $\theta's$ visualized more resembled faces.

Problem 10

Visualizing $\theta' s$ with multiple label matrix

The 6 figures below show the visualized $\theta' s$ for training on the full set of actors in *act* with the label matrix, as outlined in the assignment specification.

thetas0.jpg

(a) Baldwin $\theta' s$

thetas1.jpg

(b) Bracco $\theta' s$

thetas2.jpg

(c) Carell $\theta' s$

thetas3.jpg

(d) Gilpin $\theta' s$

thetas4.jpg

(e) Hader $\theta' s$

thetas5.jpg

(f) Harmon $\theta' s$

Figure 8: θ values for each individual actor

A potential explanation for the lack of "face-like" appearance in $\theta' s$ obtained after training may be that the size of the training set is too large for each actor. As observed in part 4, with only two actors from the Baldwin and Carell sets, the $\theta' s$ visualized more resembled faces.