Lecture 2

Chapters R.2-R.4

R.2 Discrete Random Variables and Expectations

Random variable – any variable whose value cannot be predicted exactly.

Discrete random variable – one that has a specific set of possible values.

Continuous random variable – one that can take on a continuous range of values.

Exercise 1: Give 3 examples of discrete random variables and 3 examples of continuous random variables.

Discrete	Continuous
1.	1.
2.	2.
3.	3.

Experiment – any process of observation or measurement.

Outcomes – the mutually exclusive different ways that an experiment can turn out.

Value – the result of an experimental outcome.

Population – the set of all unique values for the random variable.

Probability – the chance with which a specific outcome occurs.

Exercise 2: suppose you are playing a game of monopoly. You roll two dice, and the sum tells you how far to move. Fill in the table of possible outcomes.

Exercise 3: From the table of outcomes, fill each value, its frequency, and its probability.

Value of X:											
Frequency:											
Probability:											

Expected value of a discrete random variable – the weighted average of all its possible value.

Formula – Expected value of a discrete random variable.

Exercise 4: From the dice experiment above, what is the expected value?

The expected value of a function of a discrete random variable we simply calculate the function, then the expected value.

Exercise 5: What is the expected value of X^2 from the dice throw?

Rules of Expected Value:

1. $E(X+Y) = E(X) + E(Y)$
2. $E(bX) = bE(X)$, where b is a constant.
3. $E(b) = b$, where b is a constant.

Exercise 6: Suppose Y = 2+3X. Let X be the sum of dice from our original example (Exercise 2 and 3). What is the expected value of Y?

Variance – the measure of distance of dispersion of a random variable's probability distribution.

Formula – Population variance.

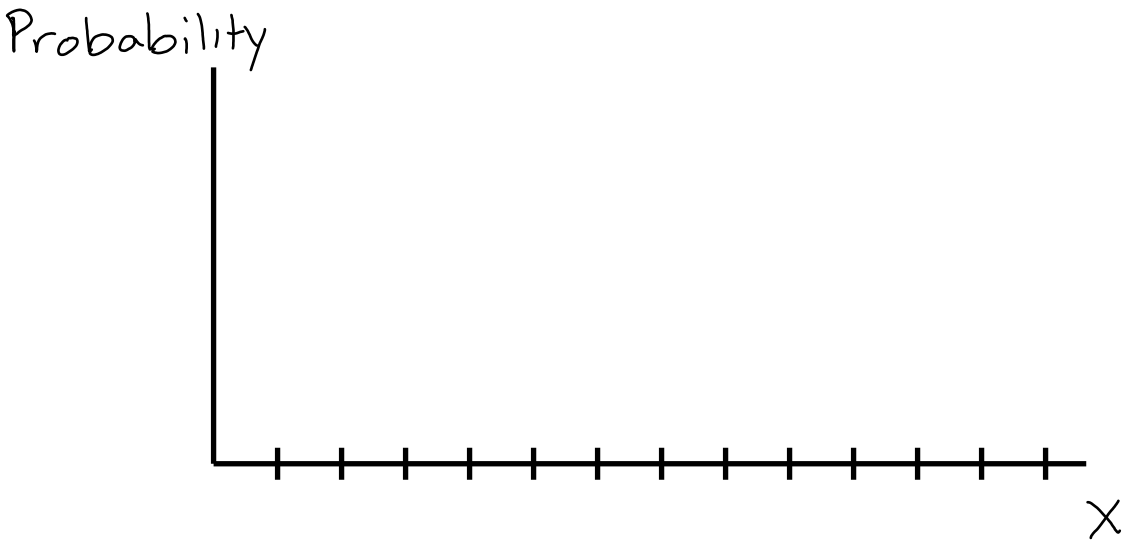
Exercise 7: What is the population variance of the dice throw from the previous exercises?

Formula – population variance

Exercise 8: Use the above formula to re-solve for the population variance of the dice throw.

R.3 Continuous Random Variables

Exercise 9: Draw a histogram of the probabilities from the dice throw.



Exercise 10: Suppose we want to guess what the temperature in a classroom will be tomorrow. If the classroom is always between 55 and 75 with equal probability, what is the probability that it will be exactly 69?

Exercise 11: What is the probability of it being between 59 and 60?

Exercise 12: What is the probability of it being between 65 and 70?

Probability density – The height at any point on the graph of probabilities.

Probability density function – A function that represents the probability of a continuous random variable, X/

Rules for probability density functions:

1. The probability of any point must be between 0 and 1.
2. The area of the probability density function must equal 1.

Exercise 13: Draw the probability density and write the probability density function for exercise 12.

R.4 Population Covariance and Correlation

Population covariance – expected value of the product of the deviation of two random variables from their means.

Formula – Population covariance:

Exercise 14: Plot the points of the random variables and find their covariance.

X	Y
1	1
2	2
3	3

Exercise 15: Plot the following points and find the covariance.

X	Y
1	3
2	2
3	1

Exercise 16: Find the covariance of the following points.

X	Y
1	300
2	200
3	100

Correlation coefficient – a unitless measure of the association of two variables.

Formula – correlation coefficient

Exercise 17: Find the correlation coefficient from exercise 15.

Exercise 18: Find the correlation coefficient from exercise 16.

Exercise 19: Label the following plots with their correlation coefficient.

$(\rho = -1,\ -1 < \rho < 0,\ \rho = 0,\ 0 < \rho < 1,\ \rho = 1)$