Math 211 - 2018-09-18:

Find the (2,3)-enty of

$$\overset{B}{\begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix}} \overset{A}{\begin{pmatrix} -1 & 1 & 2 \\ 0 & -2 & 4 \\ 1 & 0 & 0 \end{pmatrix}}$$

$$(BA)_{23} = \sum_{k=1}^{3} b_{2k} a_{k3} = \begin{pmatrix} 2 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$$

$$= b_{21} a_{13} + b_{22} a_{23} + b_{23} a_{33}$$

$$= 2 \times 2 + (-1) \times 4 + 1 \times 0 = 0$$

Find the (3,1)-enty of

$$\begin{pmatrix} -1 & 0 & 3 \\ 0 & 0 & 1 \\ 5 & 4 & 9 \end{pmatrix} \begin{pmatrix} 12 & 1 & 2 \\ 4 & 0 & 4 \\ -2 & 0 & 0 \end{pmatrix} ; \quad (BA)_{31} = \sum_{k=1}^{3} b_{3k} a_{k1}$$

$$= \begin{pmatrix} 5 & 4 & 9 \end{pmatrix} \begin{pmatrix} 12 \\ 4 \\ -2 \end{pmatrix} = 60 + 16 - 18 = 58.$$

$$((CB)A)_{ij} = \sum_k (CB)_{ik} A_{kj} = \sum_k \left(\sum_l C_{il} B_{lk}\right) A_{kj}$$

$$= \sum_k \sum_l (C_{il} B_{lk}) A_{kj}$$

$$C(A+B)_{ij} = \sum_{k=1}^{m} C_{ik} (A+B)_{kj} = \sum_k C_{ik}(A_{kj} + B_{kj}).$$

where m is: - the no. of col^{n}s of C

- the no. of rows of A
- the no. of rows of B.

Prove: $C(B+A) = CB + CA$

$$(C(A+B))_{ij} = \sum_k C_{ik} (A+B)_{kj} = \sum_k C_{ik}(A_{kj} + B_{kj})$$

$$= \sum_k \left[C_{ik} A_{kj} + C_{ik} B_{kj}\right]$$

$$= \sum_k C_{ik} A_{kj} + \sum_k C_{ik} B_{kj}$$

$$= \sum_k C_{ik} B_{kj} + \sum_k C_{ik} A_{kj}$$

$$= (CB)_{ij} + (CA)_{ij} = (CB + CA)_{ij}$$

$$\begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}a_{11}\\a_{21}\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix} \quad \& \quad \begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}a_{12}\\a_{22}\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$$

$$\Updownarrow$$

$$\left(\begin{array}{cc|c}1&2&1\\3&4&0\end{array}\right)$$

$R_2 \leftarrow R_2 - 3R_1$

$$\left(\begin{array}{cc|c}1&2&1\\0&-2&-3\end{array}\right)$$

$R_2 \leftarrow -\tfrac{1}{2}R_2$

$$\left(\begin{array}{cc|c}1&2&1\\0&1&3/2\end{array}\right)$$

$R_1 \leftarrow R_1 - 2R_2$

$$\left(\begin{array}{cc|c}1&0&-2\\0&1&3/2\end{array}\right)$$

$\therefore \left.\begin{array}{l}a_{11}=-2\\a_{21}=3/2\end{array}\right\}$ first col^n of inverse.