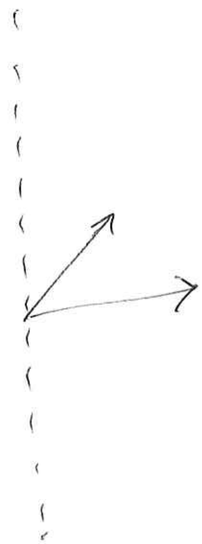
2018-10-16

d all vectors orthogonal to $\begin{pmatrix}-1\\-3\\2\end{pmatrix}$ and $\begin{pmatrix}0\\1\\1\end{pmatrix}$.

i.e. find x_1, x_2, x_3 s.t.:

$$\begin{pmatrix}-1\\-3\\2\end{pmatrix}\cdot\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=0 \quad \text{and} \quad \begin{pmatrix}0\\1\\1\end{pmatrix}\cdot\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=0.$$



$$\left.\begin{aligned}-x_1-3x_2+2x_3&=0\\0x_1+x_2+x_3&=0\end{aligned}\right\} \left(\begin{array}{ccc|c}-1&-3&2&0\\0&1&1&0\end{array}\right)$$

$$\to\left(\begin{array}{ccc|c}1&3&-2&0\\0&1&1&0\end{array}\right)\to\left(\begin{array}{ccc|c}1&0&-5&0\\0&1&1&0\end{array}\right)$$

$x_3=s$, $x_1=5s$, $x_2=-s$.

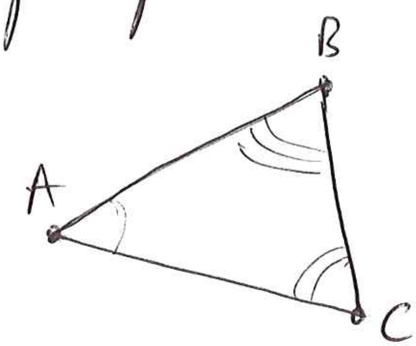
So $s\begin{pmatrix}5\\-1\\1\end{pmatrix}$ are orthogonal to both.

$\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix}$ $\mathbb{R}^n$

$\begin{pmatrix}1\\ \vdots\\ 1\end{pmatrix}$ $\begin{pmatrix}3\\5\\2\\ \vdots\\4\end{pmatrix}$

Are the following vertices of a right angled Δ?

$$\begin{pmatrix}4\\-7\\9\end{pmatrix}, \begin{pmatrix}6\\4\\4\end{pmatrix}, \begin{pmatrix}7\\10\\-6\end{pmatrix}$$
$$\quad A \qquad\quad B \qquad\quad C$$



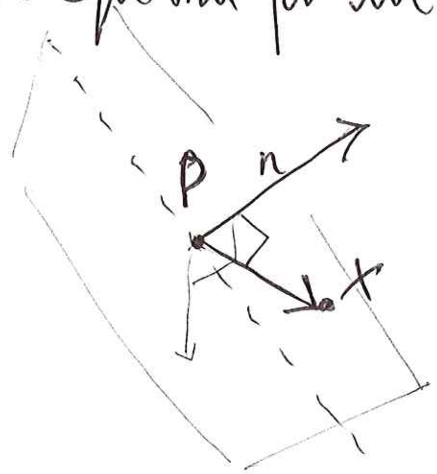
① $(B-A)\cdot(C-A) = \begin{pmatrix}2\\11\\-5\end{pmatrix}\cdot\begin{pmatrix}3\\17\\-15\end{pmatrix} = 6+187+75 \neq 0$

So no right angle at A.

② $(A-B)\cdot(C-B) = \begin{pmatrix}-2\\-11\\5\end{pmatrix}\cdot\begin{pmatrix}1\\6\\-10\end{pmatrix} = -2-66-50 \neq 0.$

③ $(A-C)\cdot(B-C) = \begin{pmatrix}-3\\-17\\15\end{pmatrix}\cdot\begin{pmatrix}-1\\-6\\10\end{pmatrix} = 3+102+150 \neq 0.$

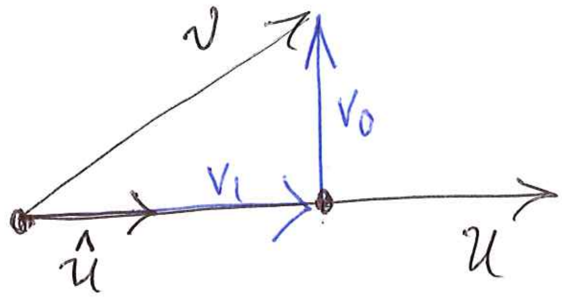
Find an equation for the plane containing $P=\begin{pmatrix}1\\-1\\0\end{pmatrix}$ and $\perp \begin{pmatrix}-3\\5\\2\end{pmatrix}=n$.



$$\begin{pmatrix}-3\\5\\2\end{pmatrix}\cdot\left(\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}-\begin{pmatrix}1\\-1\\0\end{pmatrix}\right) = 0.$$

Decompose $v = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$ into $v = v_0 + v_1$

such that v_0 is orthogonal to $\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}^{u}$ and v_1 is parallel to $\overset{u}{\overset{\shortparallel}{\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}}}$.



$$\hat{u} = \frac{1}{\|u\|} u = \frac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

So $v_1 = (v \cdot \hat{u}) \hat{u} = \left[\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \cdot \frac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \right] \frac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$

$$= \frac{1}{11} \left[6 - 1 + 0 \right] \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \frac{5}{11} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}.$$

$$v_0 = v - v_1 = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} - 5/11 \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{22}{11} - \frac{15}{11} \\ \frac{-11}{11} - 5/11 \\ 5/11 \end{pmatrix}$$

$$= \frac{1}{11} \begin{pmatrix} 7 \\ -16 \\ 5 \end{pmatrix}.$$