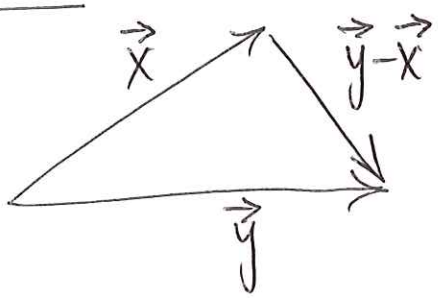
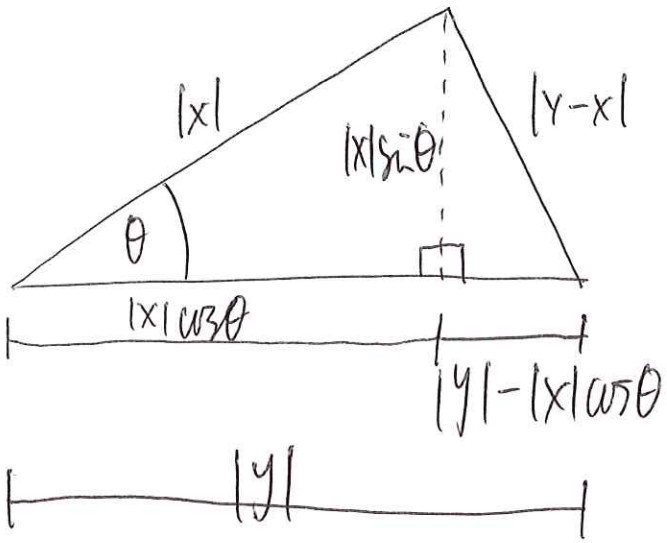
Proof that the trig. and algebraic definitions coincide.
(Non-essential; just for fun.)

Picture:



Picture in terms of lengths:



On the one hand (using algebraic properties):

$$|y-x|^2 = (y-x)\cdot(y-x) = y\cdot y - 2y\cdot x + x\cdot x = |x|^2 - 2x\cdot y + |y|^2$$

On the other hand (using trig.):

$$\begin{aligned} |y-x|^2 &= |x|^2\sin^2\theta + (|y|-|x|\cos\theta)^2 \\ &= |x|^2\sin^2\theta + |y|^2 - 2|y||x|\cos\theta + |x|^2\cos^2\theta \\ &= |x|^2 - 2|x||y|\cos\theta + |y|^2 \end{aligned}$$

So cancelling the $|x|^2$ and $|y|^2$ terms:

$$x\cdot y = |x||y|\cos\theta.$$