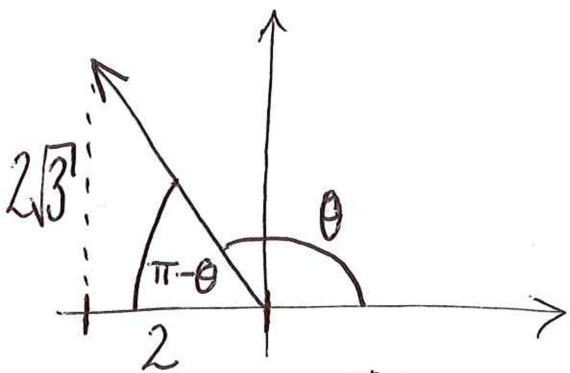
Math 211 2018-11-20:

$-2+2\sqrt{3}\,i$, $3i$, $-1-i$, $\sqrt{3}+3i$

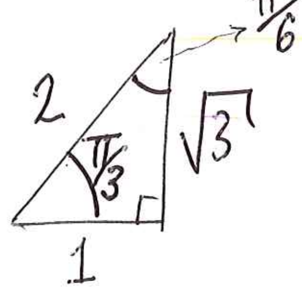
$2e^{\frac{2\pi i}{3}}$, $3e^{-i\pi}$

$-2+2\sqrt{3}\,i$: $R=\sqrt{4+12}=4$



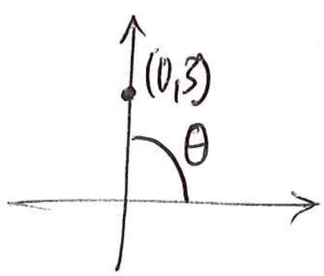
$$\pi-\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{2\pi}{3}$$



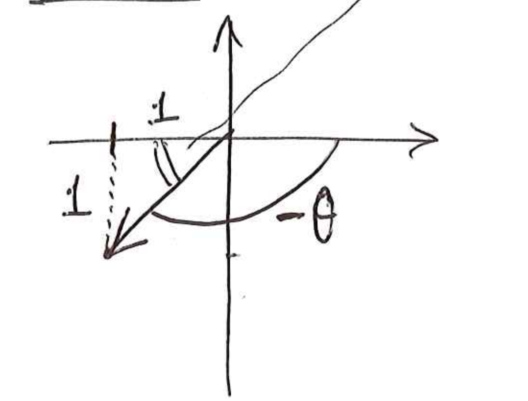
So: $-2+2\sqrt{3}\,i = 4e^{\frac{2i\pi}{3}}$.

$3i$:



$R=3$ $\theta=\frac{\pi}{2}$ So: $3i = 3e^{\frac{i\pi}{2}}$.

$-1-i$:

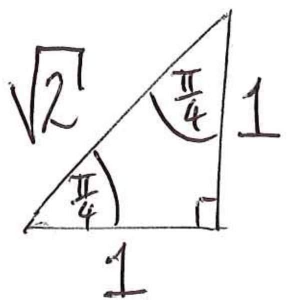


$$R=\sqrt{1+1}=\sqrt{2}$$

$$\pi+\theta = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

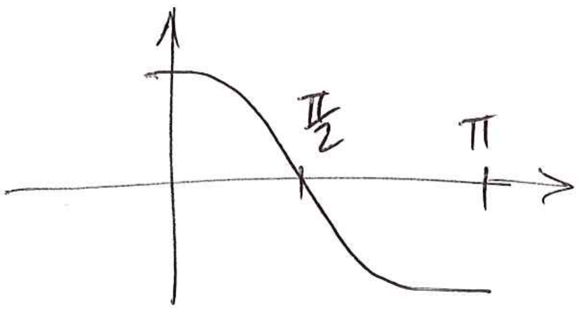
$$\Rightarrow \theta = \frac{-3\pi}{4}$$

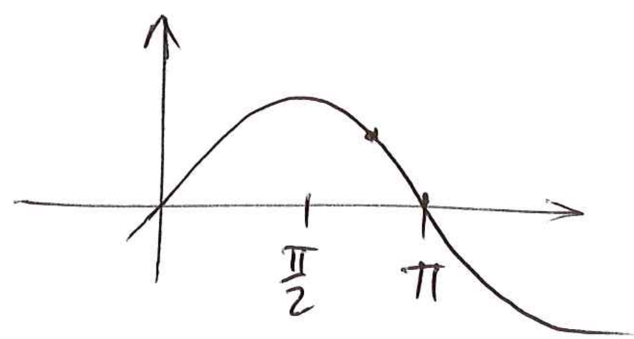
So: $-1-i = \sqrt{2}\,e^{\frac{-3i\pi}{4}}$



Recall $e^{i\theta} = \cos\theta + i\sin\theta$.

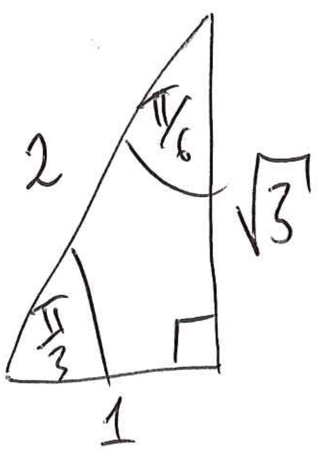
$$2e^{\frac{2\pi i}{3}} = 2\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$$





$$= 2\left(-\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$$

$$= 2\left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right) = -1+\sqrt{3}\,i$$



$$3e^{-i\pi} = -3$$

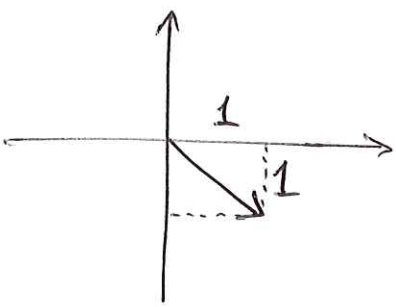
$$\parallel \qquad \parallel$$

$$3(\cos\pi + i\sin\pi)$$

$$\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)^{17}$$

$$(1-i)^6(\sqrt{3}+i)^3 = ?$$

<u>$1-i$:</u>

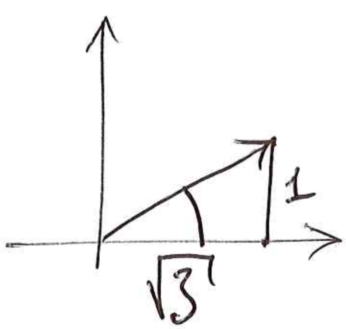


$R = \sqrt{1+1} = \sqrt{2}.$

$$\theta = \tan^{-1}\left(\frac{-1}{1}\right) = -\tan^{-1}(1) = \frac{-\pi}{4}.$$

So $1-i = \sqrt{2}\, e^{\frac{-i\pi}{4}}$

<u>$\sqrt{3}+i$:</u>



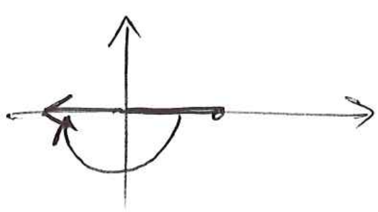
$R = \sqrt{3+1} = 2$

$\theta = \frac{\pi}{6}$

So: $\sqrt{3}+i = 2e^{\frac{i\pi}{6}}$

And: $(1-i)^6(\sqrt{3}+i)^3 = \left(\sqrt{2}\, e^{\frac{-i\pi}{4}}\right)^6 \left(2e^{\frac{i\pi}{6}}\right)^3$

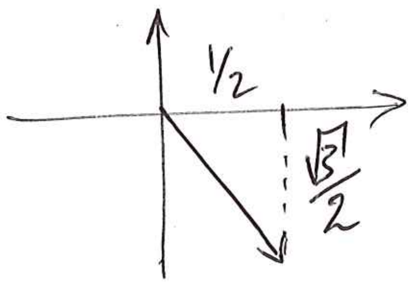
$$= (\sqrt{2})^6 \cdot e^{\frac{-3i\pi}{2}} \cdot 8 \cdot e^{\frac{i\pi}{2}} = 2^3 \cdot 8 \cdot \underbrace{e^{-i\pi}}_{-1} = -64$$



$$e^{-i\pi} = \cos(-\pi) + i\sin(-\pi) = -1$$

$$\left(\frac{1}{2} + -\frac{\sqrt{3}}{2}i\right)^{17} = ?$$

$\frac{1}{2} - \frac{\sqrt{3}}{2}i$: $\quad R = \sqrt{1/4 + 3/4} = 1$



$$\theta = \tan^{-1}\left(\frac{\left(\frac{-\sqrt{3}}{2}\right)}{(1/2)}\right) = \tan^{-1}(-\sqrt{3})$$

$$= -\tan^{-1}(\sqrt{3}) = \frac{-\pi}{3}.$$

So $\frac{1}{2} - \frac{\sqrt{3}}{2}i = e^{\frac{-i\pi}{3}}$.

$$\therefore \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{17} = \left(e^{\frac{-\pi i}{3}}\right)^{17} = e^{\frac{-17i\pi}{3}} = e^{\frac{-5i\pi}{3}}$$

$$= e^{\frac{i\pi}{3}} = \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}i$$