Math 211.2

Solve $\left(\begin{array}{ccccc|c} 0 & 0 & 0 & -2 & -8 & 4 \\ -3 & 6 & -4 & 9 & 3 & -1 \\ -1 & 2 & -2 & -4 & -3 & 3 \\ 1 & -2 & 1 & 3 & -1 & 1 \end{array}\right)$

$R_1 \leftrightarrow R_4$

$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ -3 & 6 & -4 & 9 & 3 & -1 \\ -1 & 2 & -2 & -4 & -3 & 3 \\ 0 & 0 & 0 & -2 & -8 & 4 \end{array}\right)$

$R_2 \leftarrow R_2 + 3R_1$; $R_3 \leftarrow R_3 + R_1$

$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & -1 & 18 & 0 & 2 \\ 0 & 0 & -1 & -1 & -4 & 4 \\ 0 & 0 & 0 & -2 & -8 & 4 \end{array}\right)$

$R_2 \leftarrow -R_2$

$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & 1 & -18 & 0 & -2 \\ 0 & 0 & -1 & -1 & -4 & 4 \\ 0 & 0 & 0 & -2 & -8 & 4 \end{array}\right)$

$R_3 \leftarrow R_3 + R_2$

$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & 1 & -18 & 0 & -2 \\ 0 & 0 & 0 & -19 & -4 & 2 \\ 0 & 0 & 0 & -2 & -8 & 4 \end{array}\right)$

$R_3 \leftrightarrow R_4$

$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & 1 & -18 & 0 & -2 \\ 0 & 0 & 0 & -2 & -8 & 4 \\ 0 & 0 & 0 & -19 & -4 & 2 \end{array}\right)$

$R_3 \leftarrow \frac{-1}{2} R_3$

$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & 1 & -18 & 0 & -2 \\ 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & -19 & -4 & 2 \end{array}\right)$

$R_4 \leftarrow R_4 + 19R_3$

$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & 1 & -18 & 0 & -2 \\ 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 18 \cdot 4 = 72 & -36 \end{array}\right)$

$R_4 \leftarrow \frac{1}{72} R_4$

$$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & 1 & -18 & 0 & -2 \\ 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 1 & -1/2 \end{array}\right)$$

(This is in row echelon form.)

$R_1 \leftarrow R_1 - R_2$

$$\left(\begin{array}{ccccc|c} 1 & -2 & 0 & 21 & -1 & 3 \\ 0 & 0 & 1 & -18 & 0 & -2 \\ 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 1 & -1/2 \end{array}\right)$$

$R_1 \leftarrow R_1 - 21R_3$; $R_2 \leftarrow R_2 + 18R_3$

$$\left(\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & -85 & 45 \\ 0 & 0 & 1 & 0 & 72 & -38 \\ 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 1 & -1/2 \end{array}\right)$$

$R_1 \leftarrow R_1 + 85R_4$; $R_2 \leftarrow R_2 - 72R_4$; $R_3 \leftarrow R_3 - 4R_4$.

$$\left(\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 0 & 45 - \frac{85}{2} \\ 0 & 0 & 1 & 0 & 0 & -38 + \frac{72}{2} \\ 0 & 0 & 0 & 1 & 0 & -2 + 2 = 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 \end{array}\right)$$

Set $x_2 = s \rightsquigarrow$ param.

$x_1 = 2s + 45 - 85/2$

$x_3 = -38 + \frac{72}{2}$

$x_4 = 0$

$x_5 = -1/2$.

Sol: $x_1 = 2s + 5/2$

$x_2 = s$

$x_3 = -2$

$x_4 = 0$

$x_5 = -1/2$

Solve $\left(\begin{array}{cccc|c} \textcircled{1} & 0 & 0 & -4 & 0 \\ 0 & \textcircled{1} & 0 & 7 & 2 \\ 0 & 0 & \textcircled{1} & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

So: $x_4 = s$

$x_1 = 4s$

$x_2 = 2 - 7s$

$x_3 = -1 + 2s$