Math 211 . 2018-09-27

$$\det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1 \qquad \det\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 4$$

$$\det\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = 2 \qquad \det\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = -1$$

$$\det\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = \det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1.$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{vmatrix} := \det\begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{pmatrix} = 1 \times 5 \times 9 = 45$$

$R_2 \leftrightarrow R_1 \qquad R_2 \leftarrow R_2 + 3R_1 \,;\; R_3 \leftarrow R_3 - 2R_1$

$$\begin{vmatrix} -3 & 5 & -6 \\ 1 & -1 & 3 \\ 2 & -4 & 1 \end{vmatrix} = -\begin{vmatrix} 1 & -1 & 3 \\ -3 & 5 & -6 \\ 2 & -4 & 1 \end{vmatrix} = -\begin{vmatrix} 1 & -1 & 3 \\ 0 & 2 & 3 \\ 0 & -2 & -5 \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & -1 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & -2 \end{vmatrix} = -(1)\times(2)\times(-2) = 4$$

$R_3 \leftarrow R_3 + R_2$

$R_1 \leftrightarrow R_3 \qquad R_2 \leftarrow R_2 + R_1 ;\; R_3 \leftarrow R_3 - 3R_1 ;\; R_4 \leftarrow R_4 - R_1$

$$\begin{vmatrix} 3 & 1 & 2 & 4 \\ -1 & -3 & 8 & 0 \\ 1 & -1 & 5 & 5 \\ 1 & 1 & 2 & -1 \end{vmatrix} = -\begin{vmatrix} 1 & -1 & 5 & 5 \\ -1 & -3 & 8 & 0 \\ 3 & 1 & 2 & 4 \\ 1 & 1 & 2 & -1 \end{vmatrix} = -\begin{vmatrix} 1 & -1 & 5 & 5 \\ 0 & -4 & 13 & 5 \\ 0 & 4 & -13 & -11 \\ 0 & 2 & -3 & -6 \end{vmatrix}$$

$R_3 \leftarrow R_3 + R_2 \qquad R_2 \leftrightarrow R_4 \qquad R_4 \leftarrow R_4 + 2R_2$

$$= -\begin{vmatrix} 1 & -1 & 5 & 5 \\ 0 & -4 & 13 & 5 \\ 0 & 0 & 0 & -6 \\ 0 & 2 & -3 & -6 \end{vmatrix} = +\begin{vmatrix} 1 & -1 & 5 & 5 \\ 0 & 2 & -3 & -6 \\ 0 & 0 & 0 & -6 \\ 0 & -4 & 13 & 5 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 5 & 5 \\ 0 & 2 & -3 & -6 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 7 & -7 \end{vmatrix}$$

$R_3 \leftrightarrow R_4$

$$-\begin{vmatrix} 1 & -1 & 5 & 5 \\ 0 & 2 & -3 & -6 \\ 0 & 0 & 7 & -7 \\ 0 & 0 & 0 & -6 \end{vmatrix} = -(1 \times 2 \times 7 \times (-6)) = 84$$

The following examples are corrected from lectures.

The $(1,2)$-minor of $\begin{pmatrix} 1 & 1 & 3 \\ 2 & 4 & 1 \\ 5 & 2 & 6 \end{pmatrix}$ is $\begin{vmatrix} 2 & 1 \\ 5 & 6 \end{vmatrix} = 12 - 5 = 7$

The $(1,2)$-cofactor of $\begin{pmatrix} 1 & 1 & 3 \\ 2 & 4 & 1 \\ 5 & 2 & 6 \end{pmatrix}$ is $(-1)^{1+2} \text{minor}_{12}(A) = -7.$

The $(2,2)$-minor of $\begin{pmatrix} 1 & 5 & 2 & 5 \\ 2 & 6 & 7 & 1 \\ 9 & 9 & 8 & 2 \\ 0 & 8 & 6 & 0 \end{pmatrix}$

is $\begin{vmatrix} 1 & 2 & 5 \\ 9 & 8 & 2 \\ 0 & 6 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 5 \\ 0 & -10 & -43 \\ 0 & 6 & 0 \end{vmatrix} = - \begin{vmatrix} 1 & 2 & 5 \\ 0 & 6 & 0 \\ 0 & -10 & -43 \end{vmatrix}$

$$= - \begin{vmatrix} 1 & 2 & 5 \\ 0 & 6 & 0 \\ 0 & 0 & -43 \end{vmatrix} = (-1) \times (6) \times (-43) = 258.$$