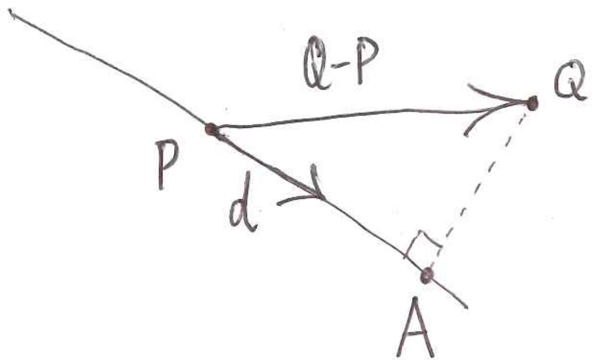
Math 211 2018-10-18

Find the closest point to $\overset{Q}{\overset{\shortparallel}{\begin{pmatrix}3\\2\\-1\end{pmatrix}}}$ on the line $\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \overset{P}{\overset{\shortparallel}{\begin{pmatrix}2\\1\\3\end{pmatrix}}} + s\,\overset{d}{\overset{\shortparallel}{\begin{pmatrix}3\\-1\\-2\end{pmatrix}}}$.



$$\overrightarrow{PA} = \left[\frac{d}{\|d\|}\cdot(Q-P)\right]\frac{d}{\|d\|}$$

First: $\dfrac{d}{\|d\|} = \dfrac{1}{\sqrt{9+1+4}}\begin{pmatrix}3\\-1\\-2\end{pmatrix} = \dfrac{1}{\sqrt{14}}\begin{pmatrix}3\\-1\\-2\end{pmatrix}$.

So: $\overrightarrow{PA} = \dfrac{1}{\sqrt{14}}\begin{pmatrix}3\\-1\\-2\end{pmatrix}\cdot\left[\begin{pmatrix}3\\2\\-1\end{pmatrix}-\begin{pmatrix}2\\1\\3\end{pmatrix}\right]\dfrac{1}{\sqrt{14}}\begin{pmatrix}3\\-1\\-2\end{pmatrix}$

$$= \frac{1}{14}\left[\begin{pmatrix}3\\-1\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\1\\-4\end{pmatrix}\right]\begin{pmatrix}3\\-1\\-2\end{pmatrix} = \frac{5}{7}\begin{pmatrix}3\\-1\\-2\end{pmatrix}$$

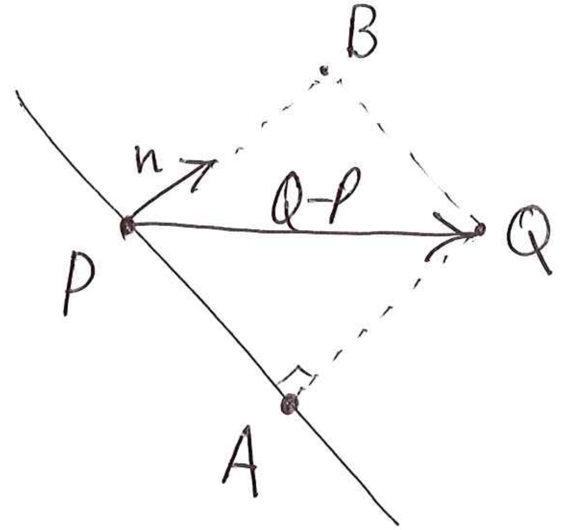
And so $A = P + \overrightarrow{PA} = \begin{pmatrix}2\\1\\3\end{pmatrix} + \dfrac{5}{7}\begin{pmatrix}3\\-1\\-2\end{pmatrix} = \dfrac{1}{7}\left[\begin{pmatrix}14\\7\\21\end{pmatrix}+\begin{pmatrix}15\\-5\\-10\end{pmatrix}\right] = \dfrac{1}{7}\begin{pmatrix}29\\2\\11\end{pmatrix}$

Find the closest point to $\begin{pmatrix}2\\3\end{pmatrix} = Q$ on the line $4x+3y=-3$.

First: $\begin{pmatrix}4\\3\end{pmatrix}\cdot\begin{pmatrix}x\\y\end{pmatrix} = -3$, $\begin{pmatrix}4\\3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + 3 = 0$.

$$\begin{pmatrix}4\\3\end{pmatrix}\cdot\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}4\\3\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix} = 0$$

$$\underbrace{\begin{pmatrix}4\\3\end{pmatrix}}_{=\,n}\cdot\left[\begin{pmatrix}x\\y\end{pmatrix} - \underbrace{\begin{pmatrix}0\\-1\end{pmatrix}}_{=\,P}\right] = 0$$



$$\overrightarrow{PB} = \left[\frac{n}{\|n\|}\cdot(Q-P)\right]\frac{n}{\|n\|}, \quad \frac{n}{\|n\|} = \frac{1}{5}\begin{pmatrix}4\\3\end{pmatrix}.$$

So $\overrightarrow{PB} = \frac{1}{5}\begin{pmatrix}4\\3\end{pmatrix}\cdot\left[\begin{pmatrix}2\\3\end{pmatrix}-\begin{pmatrix}0\\-1\end{pmatrix}\right]\frac{1}{5}\begin{pmatrix}4\\3\end{pmatrix} = \frac{1}{25}\left[\begin{pmatrix}4\\3\end{pmatrix}\cdot\begin{pmatrix}2\\4\end{pmatrix}\right]\begin{pmatrix}4\\3\end{pmatrix}$

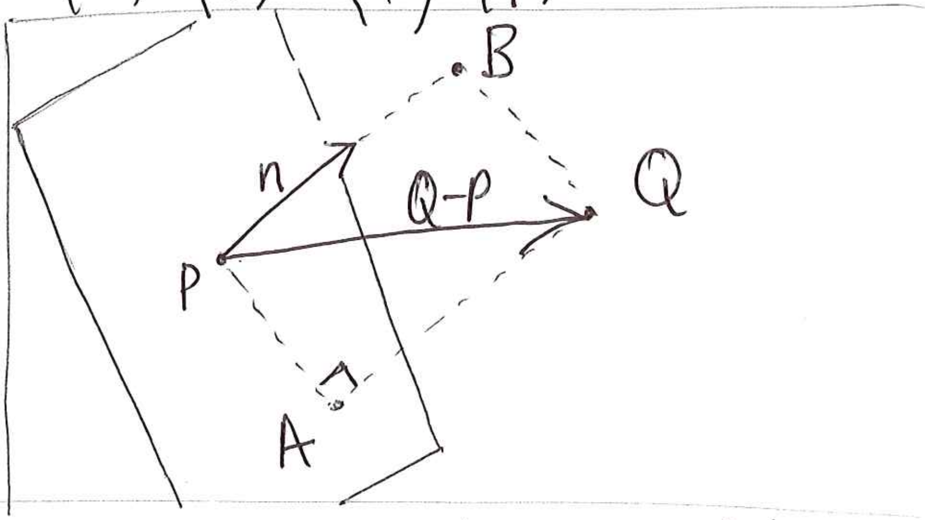
$$= \frac{1}{25}\left[8+12\right]\begin{pmatrix}4\\3\end{pmatrix} = \frac{4}{5}\begin{pmatrix}4\\3\end{pmatrix} = \frac{1}{5}\begin{pmatrix}16\\12\end{pmatrix}.$$

So $A = Q - \overrightarrow{PB} = \begin{pmatrix}2\\3\end{pmatrix} - \frac{1}{5}\begin{pmatrix}16\\12\end{pmatrix} = \frac{1}{5}\begin{pmatrix}10-16\\15-12\end{pmatrix} = \frac{1}{5}\begin{pmatrix}-6\\3\end{pmatrix}$

Find the closest point to $\begin{pmatrix}2\\3\\0\end{pmatrix} = Q$ on the plane $5x+y+z=-1$.

First $\begin{pmatrix}5\\1\\1\end{pmatrix} \cdot \begin{pmatrix}x\\y\\z\end{pmatrix} + 1 = 0$, $\begin{pmatrix}5\\1\\1\end{pmatrix} \cdot \begin{pmatrix}x\\y\\z\end{pmatrix} + \begin{pmatrix}5\\1\\1\end{pmatrix} \cdot \begin{pmatrix}0\\0\\1\end{pmatrix} = 0$

$$\underbrace{\begin{pmatrix}5\\1\\1\end{pmatrix}}_{n} \cdot \left[\begin{pmatrix}x\\y\\z\end{pmatrix} - \underbrace{\begin{pmatrix}0\\0\\-1\end{pmatrix}}_{P} \right] = 0$$



$$\overrightarrow{PB} = \left[\frac{n}{\|n\|} \cdot (Q-P) \right] \frac{n}{\|n\|}, \quad \frac{n}{\|n\|} = \frac{1}{\sqrt{25+1+1}} \begin{pmatrix}5\\1\\1\end{pmatrix} = \frac{1}{\sqrt{27}} \begin{pmatrix}5\\1\\1\end{pmatrix}.$$

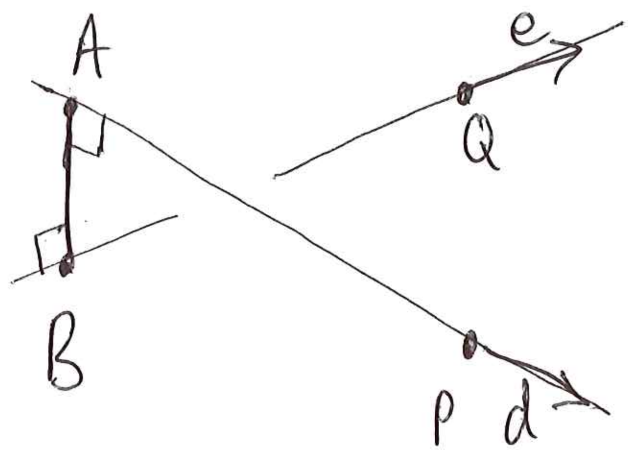
$$\overrightarrow{PB} = \frac{1}{\sqrt{27}} \begin{pmatrix}5\\1\\1\end{pmatrix} \cdot \left[\begin{pmatrix}2\\3\\0\end{pmatrix} - \begin{pmatrix}0\\0\\-1\end{pmatrix} \right] \frac{1}{\sqrt{27}} \begin{pmatrix}5\\1\\1\end{pmatrix} = \frac{1}{27} \left[\begin{pmatrix}5\\1\\1\end{pmatrix} \cdot \begin{pmatrix}2\\3\\1\end{pmatrix} \right] \begin{pmatrix}5\\1\\1\end{pmatrix}$$

$$= \frac{1}{27} [10+3+1] \begin{pmatrix}5\\1\\1\end{pmatrix} = \frac{1}{27} \begin{pmatrix}70\\14\\14\end{pmatrix}.$$

$$A = Q - \overrightarrow{PB} = \begin{pmatrix}2\\3\\0\end{pmatrix} - \frac{1}{27} \begin{pmatrix}70\\14\\14\end{pmatrix} = \frac{1}{27} \left[\begin{pmatrix}54\\81\\0\end{pmatrix} - \begin{pmatrix}70\\14\\14\end{pmatrix} \right] = \frac{1}{27} \begin{pmatrix}-16\\67\\-14\end{pmatrix}$$

Given the two lines:

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \overset{=P}{\begin{pmatrix}3\\1\\-1\end{pmatrix}} + s\overset{=d}{\begin{pmatrix}1\\1\\-1\end{pmatrix}}, \quad \begin{pmatrix}x\\y\\z\end{pmatrix} = \overset{Q}{\begin{pmatrix}1\\2\\0\end{pmatrix}} + t\overset{e}{\begin{pmatrix}1\\0\\2\end{pmatrix}}$$



① $(B-A)\cdot e = 0$

② $(B-A)\cdot d = 0.$

$A = P + sd$; $B = Q + te$

$$A = \begin{pmatrix}3\\1\\-1\end{pmatrix} + s\begin{pmatrix}1\\1\\-1\end{pmatrix} = \begin{pmatrix}3+s\\1+s\\-1-s\end{pmatrix}, \quad B = \begin{pmatrix}1+t\\2\\2t\end{pmatrix}.$$

$$B-A = \begin{pmatrix}1+t-3-s\\2-1\cancel{\ }-s\\2t+1+s\end{pmatrix} = \begin{pmatrix}t-s-2\\1-s\\2t+s+1\end{pmatrix}$$

①
$$0 = \begin{pmatrix}t-s-2\\1-s\\2t+s+1\end{pmatrix}\cdot\begin{pmatrix}1\\0\\2\end{pmatrix} = t-s-2+4t+2s+2$$
$$= 5t+s$$

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$$0 = \begin{pmatrix} t - s - 2 \\ 1 - s \\ 2t + s + 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \cancel{t} \cancel{-s} - 2 + 1 \cancel{-s} \cancel{-2t} \cancel{-s} - 1$$

$$= -t - 3s - 2$$

So solve: $0 = 5t + s$ } $s + 5t = 0$

$0 = -t - 3s - 2$ } $-3s - t = +2$