Lecture 1

Thursday 6th September, 2018

Lecture 1

Admin

One/Two variables

Three variables

Reduced row echelon form

Admin

Office hours and contact details

office MS 448

office hours 11:00-12:00 and 15:30-16:30 Tuesday and Thursday

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Notes and textbook

d2l > Content > Additional Course Material

Class notes L02/L03 Notes from these lectures

General lecture notes Slides describing the course in detail

Additional practice problems for the whole course

External resources A curated list of videos, books and Q&A forums

Free online textbook 'A first course in linear algebra' by Kuttler

Course structure

d2l > Content > Course Information > Course Schedule

d2l > Content > Course Information > How to Access Lyryx

- 10 Lyryx assignments
  - Don't worry about 1-2 marks; use to increase marks in final
  - First due on 23 September 11:59 PM
- ONE midterm
  - Friday 26 October
- Final Examination
  - Not yet scheduled: between 10-20 December

More support

d2l > Content > Course Information > (various)

- Labs
- Continuous tutorials
- Student success centre
- Engineering academic success centre

General advice

- Figure out which resources work for you sooner rather than later.
  - Probably far too many to actually use

- My advice: emphasise practising typical questions. E.g.
  - attend labs
  - go to continuous tutorials with specific questions
  - use Lyryx effectively

Linear equations in one variable

Find all real numbers x such that

$$ax = b$$

where a and b are real numbers.

Linear equation in two variables

Find all real numbers x and y such that both

$$x + 2y = 1$$
$$3x + 4y = 0$$

hold.

Questions?

Elementary row operations

But what operations are we allowed to do on the rows?

The following operations will not change the solutions:-

1. Swap two rows.
2. Add a multiple of one row to another row.
3. Multiply a row by a scalar.

See pictures on Jupyter notebook.

Inconsistent equations

Find all real numbers x and y such that both

$$x + 2y = 1$$
$$5x + 10y = 2$$

hold.

Infinitely many solutions

Find all real numbers x and y such that both

$$3x + 12y = 18$$
$$4x + 16y = 24$$

hold.

Questions?

Find all x, y and z such that

$$\begin{aligned} x + 2y + 3z &= 4 \\ 5x + 6y + 7z &= 8 \\ 9x + 10y + 11z &= 12 \end{aligned}$$

hold.

Find all x, y and z such that

$$\begin{aligned} x + y + 2z &= -1 \\ 2x + y + 3z &= 0 \\ 0x + -2y + 1z &= 2 \end{aligned}$$

hold.

Find all x, y and z such that

$$\begin{aligned} x + 2y + 3z &= 4 \\ 5x + 6y + 7z &= 8 \\ 3x + 4y + 5z &= 1 \end{aligned}$$

hold.

Find all x, y and z such that

$$\begin{aligned} 6x + 4y + 2z &= 4 \\ 3x + 2y + 1z &= 2 \\ 9x + 6y + 3z &= 6 \end{aligned}$$

hold.

Questions?

Row echelon form

A matrix is in *row echelon form* iff:-

- All rows consisting entirely of zeros are at the bottom.
- The first nonzero entry in each nonzero row is a 1 (called the **leading 1** for that row).
- Each leading 1 is to the right of all leading 1's in rows above it.

For instance:

$$\begin{bmatrix} 0 & 1 & * & * & * & * & * & * \\ 0 & 0 & 0 & 1 & * & * & * & * \\ 0 & 0 & 0 & 0 & 1 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where $*$ can be any number.

Reduced row echelon form

A matrix is in *reduced row echelon form* iff:-

- It is a row-echelon matrix.
- Each leading 1 is the only nonzero entry in its column.

$$\begin{bmatrix} 0 & 1 & * & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 1 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 1 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where $*$ can be any number.

In short:

- row echelon form $\leftrightarrow$ ready for back-substitution
- reduced row echelon form $\leftrightarrow$ read off solutions

another advantage of reduced row echelon form is that:

Theorem

Reduced row echelon form is unique.

Reading off solutions

The matrix

$$\begin{bmatrix} 0 & 1 & 0 & 5 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

is in reduced row echelon form.