Math 211 2018-11-06

$\left(T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ y \end{pmatrix} \text{ invert} \right)$

$S\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$, $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$

Find $[S \circ T]$. $(= [S][T])$

First col^n: $(S\circ T)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = S\left(T\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = S\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$.

Second col^n: $(S\circ T)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = S\left(T\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = S\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$.

$$[S\circ T] = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}.$$

Invert the transformation: $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ y \end{pmatrix}$

We know $[T^{-1}] = [T]^{-1}$. So first: what is $[T]$?

First col^n: $T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Second col^n: $T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

$$[T] = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \text{ so } [T]^{-1} = \frac{1}{1}\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}.$$

$$T^{-1}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-y \\ y \end{pmatrix}$$

$(-3+6i)+(5-i) = 2+5i$

$(4-7i)+(6-2i) = 10-9i$

$(-3+6i)-(5-i) = -8+7i$

$(4-7i)-(6-2i) = -2-5i$

$$(2-3i)\cdot(-3+4i) = -6+8i+9i-12i^2$$
$$= (-6+12)+17i = 6+17i$$

Solve $z^2 = -3+4i$

Let $z = a+ib \Rightarrow (a+bi)^2 = -3+4i$

$$a^2+abi+abi+b^2i^2 = -3+4i$$
$$(a^2-b^2)+2abi = -3+4i$$

$$\begin{pmatrix} a^2-b^2 \\ 2ab \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

① $a^2-b^2 = -3$

② $2ab = 4$

First, b cannot be 0 because then $0=4$ ※.

② $\Rightarrow a = \frac{2}{b}$ so ① $\Rightarrow \left(\frac{2}{b}\right)^2 - b^2 = -3$

$4 - b^4 = -3b^2$; $b^4 - 3b^2 - 4 = 0$

$(b^2+1)(b^2-4) = 0$

$\Rightarrow b = \pm 2$ because b is real.

~~$\Rightarrow$~~ when $b=2$ $a=1$

when $b=-2$ $a=-1$

Two sol'ns: $1+2i$ and $-1-2i$.

$\frac{1}{i}$, $\frac{2-i}{3+4i}$

Write $\frac{a+bi}{c+di}$ (where at least one of c, d is non-zero.)

in the form $e+fi$.

$$\frac{a+bi}{c+di} \cdot \frac{c-di}{c-di} = \frac{ac - adi + bci - bdi^2}{c^2 - cdi + cdi - d^2i^2}$$

$$= \frac{(ac+bd) + (bc-ad)i}{c^2+d^2} = \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right)i$$

e.g. $\frac{1}{i} = \frac{1}{i} \cdot \frac{-i}{-i} = \frac{-i}{-i^2} = -i$

e.g. $\frac{2-i}{3+4i} = \frac{2-i}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{6-8i-3i+4i^2}{25}$

$= \frac{2}{25} - \frac{11}{25} i$

$z = 2+6i$

$\Rightarrow \quad z^{-1} = \frac{1}{2+6i} = \frac{1}{2+6i} \cdot \frac{2-6i}{2-6i} = \frac{2-6i}{40}$

$= \frac{1}{20} - \frac{3}{20} i$