MATH211: Linear Methods I

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Thursday 6$^{\text{th}}$ December, 2018

Lecture on Thursday 6th December, 2018

Small examples

Long examples

Small examples

Examples

Example

Construct an example of a 2×2 matrix with one eigenvalue equal to 3 that is not diagonal, is not invertible but is diagonalisable.

Example

Find a matrix that is not the identity or the zero matrix such that

$$A^2 = A$$

Long examples

Google PageRank theory

Imagine modelling the internet following way:-

▶ Each page consists of links to other pages.

▶ We travel from one page to another by clicking a link at random.

▶ Therefore the probability of going from page j to page i is

$$\frac{L_{ji}}{Out(j)}$$

where L_{ji} is the number of links from page j to page i and $Out(j)$ is the total number of links from page j.

▶ A steady state vector for this Markov chain (if it exists) gives the PageRank of the pages.

Google PageRank Example

Example (Simplified PageRank)

Suppose that the internet consists of three(!) pages P_1, P_2 and P_3. Suppose further that:-

- ▶ there are two links from P_1 to P_2
- ▶ there is one link from P_1 to P_3
- ▶ there is one link from P_2 to P_1
- ▶ there is one link from P_2 to P_3
- ▶ there is one link from P_3 to P_2

Then calculate the (relative) PageRanks for P_1, P_2 and P_3.

Examples

Example

Find the distance d between the lines

$$L_1 : \vec{x} = \begin{bmatrix} -6 \\ -7 \\ 7 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \text{ and } L_2 : \vec{x} = \begin{bmatrix} -7 \\ -14 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

and find a points A on L_1 and B on L_2 such that $d(A, B) = d$.

Examples

Example

Suppose that the sequence x_0, x_1, x_2 ... is defined by $x_0 = 2$, $x_1 = 1 + i$ and $x_{k+2} = (1 + i)x_{k+1} - ix_k$. Find a formula for x_k.

Examples

Example

A draconian country organises its education system as follows:-

- ▶ At age 11 all children take an exam.
- ▶ The children who pass attend a school of type A and those who fail attend a school of type B.
- ▶ Both schools administer weekly tests.
- ▶ Consider the fate of a certain individual Adam:-
    - ▶ Adam has probability $\frac{2}{3}$ of passing the first exam at age 11.
    - ▶ At school type A Adam has a $\frac{2}{5}$ chance of passing every weekly exam.
    - ▶ At school type B Adam has a $\frac{4}{5}$ chance of passing every weekly exam.

Model Adam's progress as a Markov chain and find all steady state vectors.

Examples

Example

Find equations for the lines through

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

that meet the line

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

at the two points at distance 3 from

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Example

Example

If possible diagonalise

$$\begin{bmatrix} 3 & -4 & 2 \\ 1 & -2 & 2 \\ 1 & -5 & 5 \end{bmatrix}$$