

MATH211: Linear Methods I

Lecture 1

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Lecture 1

One/Two variables

Three variables

Reduced row echelon form

One/Two variables

Linear equations in one variable

Find all real numbers x such that

$$ax = b$$

where a and b are real numbers.

Linear equation in two variables

Find all real numbers x and y such that both

$$x + 2y = 1$$

$$3x + 4y = 0$$

hold.

Elementary row operations

But what operations are we allowed to do on the rows?

The following operations will not change the solutions:-

1. Swap two rows.
2. Add a multiple of one row to another row.
3. Multiply a row by a *non-zero* scalar.

See pictures on Jupyter notebook.

Questions?

Inconsistent equations

Find all real numbers x and y such that both

$$x + 2y = 1$$

$$5x + 10y = 42$$

hold.

Infinitely many solutions

Find all real numbers x and y such that both

$$3x + 12y = 18$$

$$4x + 16y = 24$$

hold.

Questions?

Three variables

Find all x , y and z such that

$$x + 2y + 3z = 4$$

$$5x + 6y + 7z = 8$$

$$9x + 10y + 11z = 12$$

hold.

Find all x , y and z such that

$$x + y + 2z = -1$$

$$2x + y + 3z = 0$$

$$0x + -2y + 1z = 2$$

hold.

Find all x , y and z such that

$$x + 2y + 3z = 4$$

$$5x + 6y + 7z = 8$$

$$3x + 4y + 5z = 1$$

hold.

Find all x , y and z such that

$$6x + 4y + 2z = 4$$

$$3x + 2y + 1z = 2$$

$$9x + 6y + 3z = 6$$

hold.

Questions?

Reduced row echelon form

Row echelon form

A matrix is in *row echelon form* iff:-

- ▶ All rows consisting entirely of zeros are at the bottom.
- ▶ The first nonzero entry in each nonzero row is a 1 (called the **leading 1** for that row).
- ▶ Each leading 1 is to the right of all leading 1's in rows above it.

For instance:

$$\begin{bmatrix} 0 & 1 & * & * & * & * & * & * \\ 0 & 0 & 0 & 1 & * & * & * & * \\ 0 & 0 & 0 & 0 & 1 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where * can be any number.

Reduced row echelon form

A matrix is in *reduced row echelon form* iff:-

- ▶ It is a row-echelon matrix.
- ▶ Each leading 1 is the only nonzero entry in its column.

$$\begin{bmatrix} 0 & 1 & * & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 1 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 1 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where * can be any number.

Advantages of reduced row echelon form

- ▶ Row echelon form \leftrightarrow ready for back-substitution.
- ▶ Reduced row echelon form \leftrightarrow read off solutions.

Another advantage of reduced row echelon form is that:

Theorem

Reduced row echelon form is unique.

Reading off solutions

- ▶ The augmented matrix

$$\left[\begin{array}{ccccc|c} 1 & 0 & 5 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

is in reduced row echelon form.

- ▶ If there are any rows of the form $(0 \ 0 \dots 0 | 1)$ then we know immediately that there are no solutions.

Identify leading 1s and parameter blocks

- ▶ The leading 1s are shown in green

$$\left[\begin{array}{ccccc|c} 1 & 0 & 5 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

and the other non-zero numbers are shown in blue and red.

- ▶ Variables associated to the leading 1s are the *leading variables*.
- ▶ In this case x_1 , x_2 and x_4 are leading and x_3 and x_5 are not.

The non-leading variables can take any value so we assign parameters to them.

In this case:

$$x_3 = s$$

$$x_5 = t$$

and then we solve for the leading variables in terms of these parameters:

$$x_1 = 1 - 5s - t$$

$$x_2 = -s - 2t$$

$$x_4 = -4t$$

Questions?