<u>Example:</u> Prove $(BA)^T = A^T B^T$.

$$((BA)^T)_{ij} = (BA)_{ji} = \sum_k B_{jk}A_{ki} = \sum_k A_{ki}B_{jk}$$

$$= \sum_k (A^T)_{ik}(B^T)_{kj} = (A^T)(B^T)$$

<u>Example:</u> Prove $(B+A)^T = B^T + A^T$.

$$((B+A)^T)_{ij} = (B+A)_{ji} = B_{ji} + A_{ji} = (B^T)_{ij} + (A^T)_{ij}$$

$$= (B^T + A^T)_{ij}$$

<u>Example:</u> Prove that if A and B are symmetric then $A^T + 2B$ is symmetric.

Since A is sym: $A_{ij} = A_{ji}$.
Since B is sym.: $B_{ij} = B_{ji}$.

Then $(A^T+2B)_{ij} = (A^T)_{ij} + 2B_{ij} = A_{ji} + 2B_{ij}$

$$\cancel{(A^T)_{ij}} \quad = A_{ij} + 2B_{ij}$$

$$= A^T_{ji} + 2B_{ji} = (A^T + 2B)_{ji}$$

<u>Example:</u> Prove $C(BA) = (CB)A$.

$$(C(BA))_{ij} = \sum_k C_{ik}(BA)_{kj} = \sum_k C_{ik}\left(\sum_l B_{kl}A_{lj}\right) = \sum_k\sum_l C_{ik}(B_{kl}A_{lj})$$

$$= \sum_l\left(\sum_k C_{ik}B_{kl}\right)A_{lj} = \sum_l (CB)_{il}A_{lj} = ((CB)A)_{ij}$$