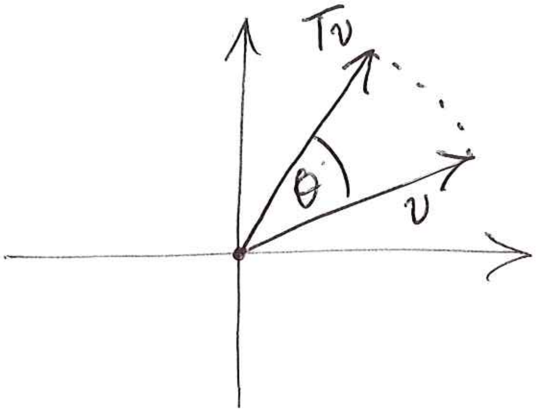
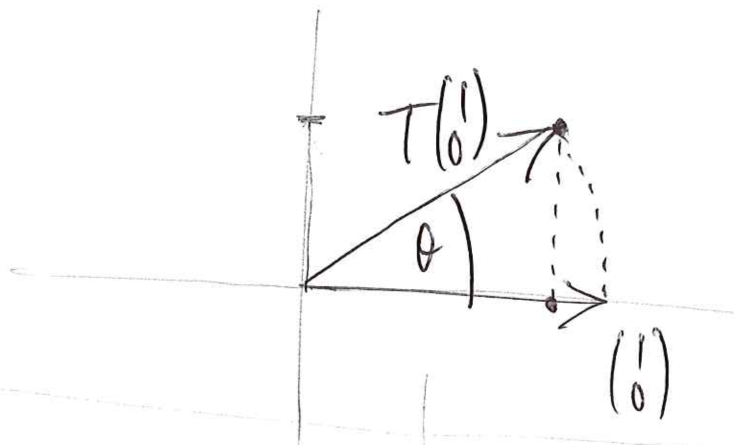
Math 211 2018-11-01:

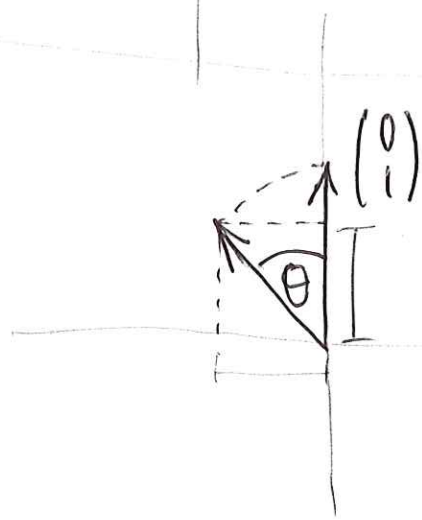
- Matrix for rotation by θ in $\mathbb{R}^2$.





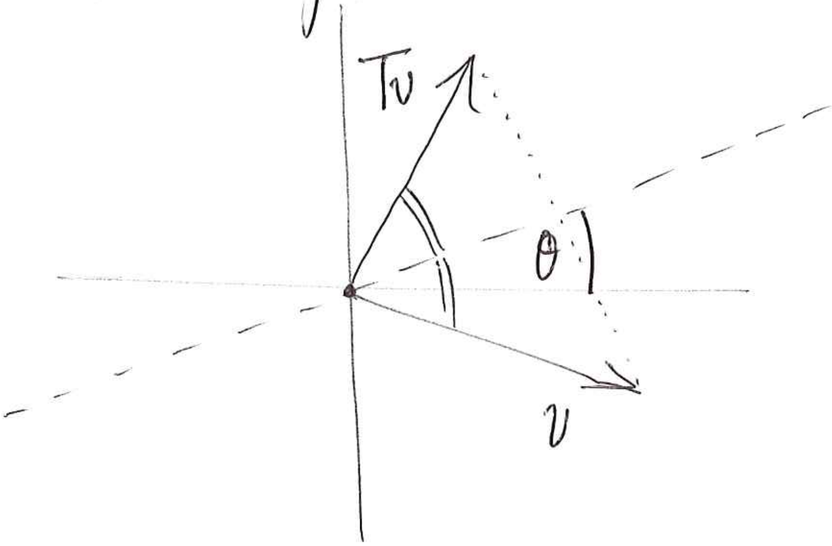
First col^n: $T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$.

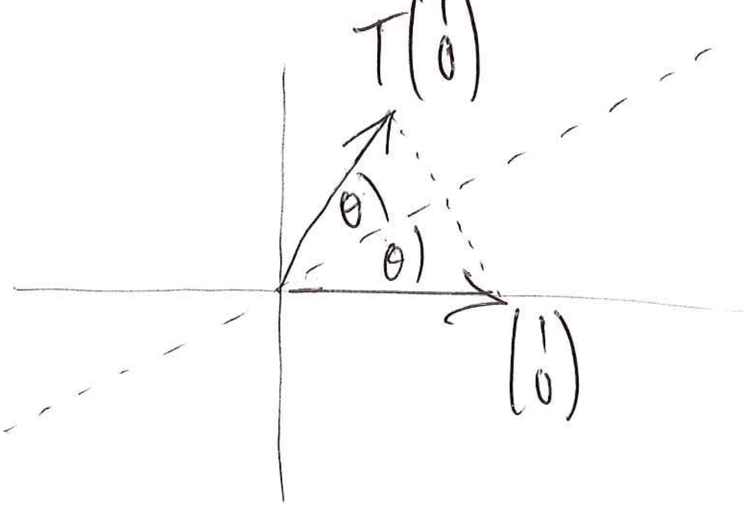
Second col^n: $T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}$



So $\boxed{R_\theta = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}.}$

- Matrix for a reflection in a line through the origin and at angle θ to the x-axis.

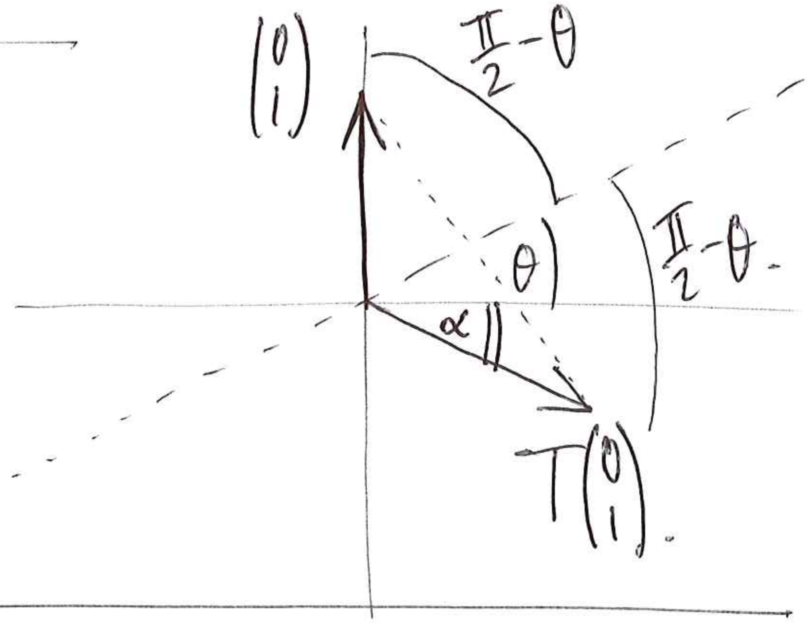




First col^n: $T\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}\cos 2\theta\\ \sin 2\theta\end{pmatrix}$

Second col^n: $T\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\cos\alpha\\ -\sin\alpha\end{pmatrix}$

$\alpha = \frac{\pi}{2} - 2\theta.$



So $T\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\cos\left(\frac{\pi}{2}-2\theta\right)\\ -\sin\left(\frac{\pi}{2}-2\theta\right)\end{pmatrix}$

$$=\begin{pmatrix}\sin(2\theta)\\ -\cos 2\theta\end{pmatrix}$$

$$\boxed{Q_\theta = \begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix}.}$$

• Matrix for $x \overset{\in \mathbb{R}^3}{} \xmapsto{T} v \times x$ where v is a fixed vector $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$.

First col^n: $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ v_3 \\ -v_2 \end{pmatrix}$ Second col^n: $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -v_3 \\ 0 \\ v_1 \end{pmatrix}$

Third col^n: $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} +v_2 \\ -v_1 \\ 0 \end{pmatrix}$

$$[T] = \begin{pmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{pmatrix}. \qquad [T]^T = -[T].$$

(Skew-symmetric)

$$\begin{bmatrix} \text{Rotation} \\ \text{OR} \\ \text{Reflection} \end{bmatrix} \circ \begin{bmatrix} \text{Rotation} \\ \text{OR} \\ \text{Reflection} \end{bmatrix} = \begin{bmatrix} \text{Rotation} \\ \text{OR} \\ \text{Reflection} \end{bmatrix}$$

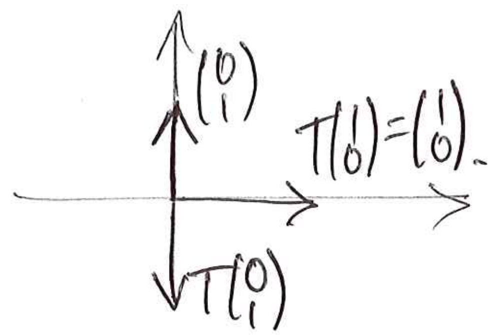
to tell which ... take the determinant.

---

① Reflection in x-axis in $\mathbb{R}^2$. $\rightsquigarrow$ call it T

② Rotation through $\frac{\pi}{2}$. $\rightsquigarrow$ call it S.

Method 1:

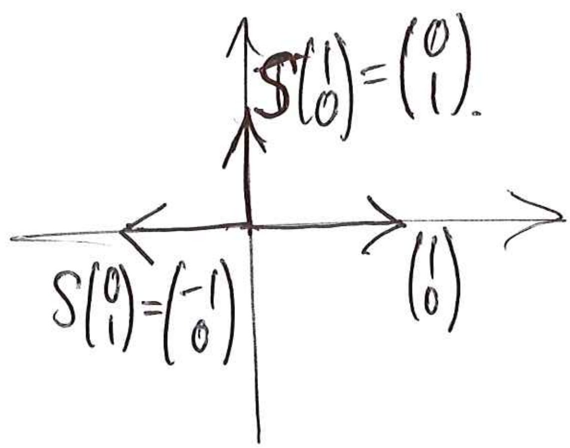


The matrix $[T]$: $T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

$T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix}$ So: $[T] = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$.

The matrix $[S]$: $S\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$

$S\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-1\\0\end{pmatrix}$



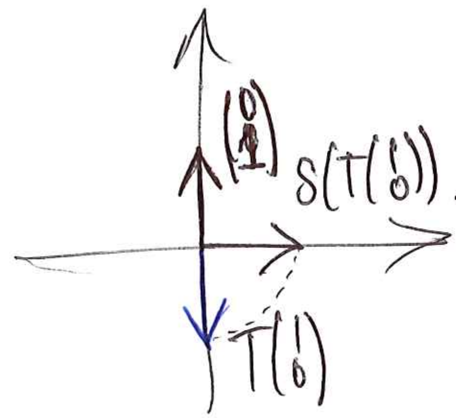
So $[S] = \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$.

$$[S \circ T] = [S]\cdot[T] = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

<u>Method 2:</u> Let $S \circ T$ act on standard basis.

$$(S\circ T)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = S\left(T\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = S\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

$$(S\circ T)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = S\left(T\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = S\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$



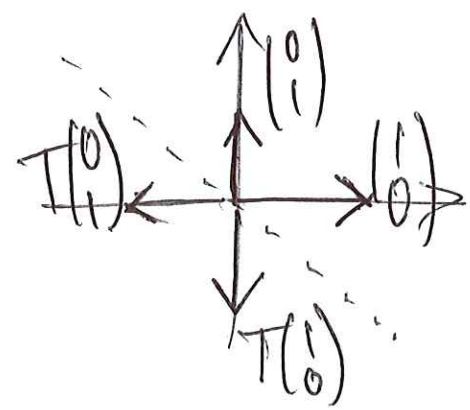
---

① reflection in line $y = -x$ ⇝ call it T

② reflection in y-axis. ⇝ call it S.

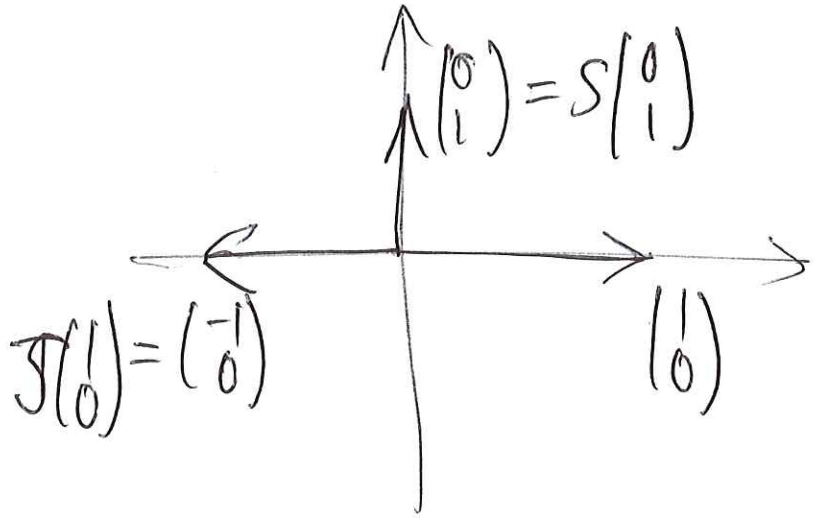
<u>Method 1:</u> $T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$



$$[T] = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}.$$

$$S\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}$$



$$S\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}.$$

$$[S]=\begin{pmatrix}-1&0\\0&1\end{pmatrix}$$

So $[S\circ T]=[S]\cdot[T]=\begin{pmatrix}-1&0\\0&1\end{pmatrix}\begin{pmatrix}0&-1\\-1&0\end{pmatrix}=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$

(<u>i.e.</u> a rotation...)