Math 211 2018-11-27

① $z^2 - 14z + 58 = 0$

$$z_{1,2} = \frac{14 \pm \sqrt{14 \cdot 14 - 4 \cdot 58}}{2} = 7 \pm \sqrt{49-58} = 7 \pm \sqrt{-9}$$
$$= 7 \pm 3i.$$

② Recall that if $f(z)$ is a real quadratic and u, v are its roots

$u+v$ is real and uv is real

(because $(z-u)(z-v) = z^2 + (-u-v)z + uv$.)

$\Rightarrow$ EITHER u, v real

OR $u = \bar{v}$.

Therefore the other root is $\overline{5-2i} = 5+2i$.

③ $z^2 - (3-2i)z + (5-i) = 0$

$$z_{1,2} = \frac{3-2i \pm \sqrt{(3-2i)^2 - 4(5-i)}}{2} = \frac{3-2i \pm \sqrt{9-12i-4-20+4i}}{2}$$

$$= \frac{3-2i \pm \sqrt{-15-8i}}{2}$$

So find $a, b \in \mathbb{R}$ such that $-15-8i = (a+bi)^2 = (a^2-b^2) + 2abi$.

Since $b \neq 0 \Rightarrow a = \frac{-8}{2b} = \frac{-4}{b}$

$$\frac{16}{b^2} - b^2 = -15 \Rightarrow 0 = b^4 - 15b^2 - 16 = (b^2-16)(b^2+1)$$

So $b = \pm 4$ and $a = \mp 1$. So $a+bi = \pm(1-4i)$.

So $z_{1,2} = \frac{3-2i \pm (1-4i)}{2}$; $z_1 = \frac{3-2i+1-4i}{2} = \frac{4-6i}{2} = 2-3i$

$$z_2 = \frac{3-2i-1+4i}{2} = \frac{2+2i}{2} = 1+i.$$

If $A = \begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix}$ then $A - \lambda I = \begin{pmatrix} 4-\lambda & -2 \\ -1 & 3-\lambda \end{pmatrix}$

$$\chi(\lambda) = \begin{vmatrix} 4-\lambda & -2 \\ -1 & 3-\lambda \end{vmatrix} = (4-\lambda)(3-\lambda) - 2 = 12 - 4\lambda - 3\lambda + \lambda^2 - 2$$

$$= \lambda^2 - 7\lambda + 10 = (\lambda - 5)(\lambda - 2)$$

The eigenvalues are:

$\lambda = 5$ with alg. mult. $= 1$

$\lambda = 2$ with alg. mult. $= 1$

If $A = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & -2 \\ 0 & -1 & 2 \end{pmatrix}$ then $A - \lambda I = \begin{pmatrix} 4-\lambda & 1 & 2 \\ 0 & 3-\lambda & -2 \\ 0 & -1 & 2-\lambda \end{pmatrix}$

$$\begin{vmatrix} 4-\lambda & 1 & 2 \\ 0 & 3-\lambda & -2 \\ 0 & -1 & 2-\lambda \end{vmatrix} = \begin{vmatrix} 4-\lambda & 1 & 2 \\ 0 & 1 & \lambda-2 \\ 0 & 3-\lambda & -2 \end{vmatrix}$$

$$= \begin{vmatrix} 4-\lambda & 1 & 2 \\ 0 & 1 & \lambda-2 \\ 0 & 0 & -2-(3-\lambda)(\lambda-2) \end{vmatrix} = (4-\lambda)\left[-2-(3-\lambda)(\lambda-2)\right]$$

$$-2-(3-\lambda)(\lambda-2)$$

$$= (4-\lambda)\left[-2-(3\lambda-6-\lambda^2+2\lambda)\right]$$

$$= (4-\lambda)(\lambda^2-5\lambda+4) = -(\lambda-4)(\lambda-4)(\lambda-1)$$

$$= -(\lambda-4)^2(\lambda-1)$$

Therefore the eigenvalues are:

$\lambda = 4$ has alg. mult. $=2$

$\lambda = 1$ has alg. mult. $=1$.

① We know $\begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix}$ has eigenvalues 5 and 2.

<u>Find E_5:</u> $A - \lambda I = \begin{pmatrix} 4-\lambda & -2 \\ -1 & 3-\lambda \end{pmatrix}$

i.e. solve $\left(\begin{array}{cc|c} -1 & -2 & 0 \\ -1 & -2 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $y = s$, $x = -2s$

So $E_5 = s\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

<u>Find E_2:</u> $\cancel{\begin{pmatrix} 3 & -2 \\ -1 & \end{pmatrix}}$ $\left(\begin{array}{cc|c} 2 & -2 & 0 \\ -1 & 1 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$y = s$ $x = s$ so $E_2 = s\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

②

If $A=\begin{pmatrix}4 & 1 & 2\\ 0 & 3 & -2\\ 0 & -1 & 2\end{pmatrix}$ $\lambda=1,4.$

Find E_1: $\left(\begin{array}{ccc|c}3 & 1 & 2 & 0\\ 0 & 2 & -2 & 0\\ 0 & -1 & 1 & 0\end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c}3 & 1 & 2 & 0\\ 0 & 1 & -1 & 0\end{array}\right)$

$\rightsquigarrow \left(\begin{array}{ccc|c}3 & 0 & 3 & 0\\ 0 & 1 & -1 & 0\end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c}1 & 0 & 1 & 0\\ 0 & 1 & -1 & 0\end{array}\right).$ $z=s$, $y=s$, $x=-s$

So $E_1 = s\begin{pmatrix}-1\\1\\1\end{pmatrix} = \left\langle \begin{pmatrix}-1\\1\\1\end{pmatrix} \right\rangle$

Find E_4: $\left(\begin{array}{ccc|c}0 & 1 & 2 & 0\\ 0 & -1 & -2 & 0\\ 0 & -1 & -2 & 0\end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c}0 & 1 & 2 & 0\end{array}\right).$

$x=s$, $z=t$

$y=-2t$

$E_4 = \begin{pmatrix}s\\-2t\\t\end{pmatrix} = s\begin{pmatrix}1\\0\\0\end{pmatrix} + t\begin{pmatrix}0\\-2\\1\end{pmatrix}$

Basic eigenvectors.