MATH211: Linear Methods I

Matthew Burke

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Lecture on Thursday 20th September, 2018

Matrix inversion algorithm

Examples

Properties of inverses

Other matrix equations

Last time

- (i,j)-entry proofs
- Identity matrices
- Inverse matrices
- Small examples of inverses

Matrix inversion algorithm

Aim of matrix inversion

Given B find A such that $BA = I_n$:

$$\begin{bmatrix} B\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} & B\begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} & \dots B\begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

I.e. find linear combinations of the columns of B that give each of the standard basis vectors.

The algorithm

For the first standard basis vector we solve:

$$Bx = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftrightarrow \left[\begin{array}{cccc|c} b_{11} & b_{12} & \dots & b_{1m} & 1 \\ b_{21} & b_{22} & \dots & b_{2m} & 0 \\ \dots & \dots & \dots & \dots & 0 \\ b_{p1} & b_{p2} & \dots & b_{pm} & 0 \end{array}\right]$$

and we can group all together as:

$$\left[\begin{array}{cccc|cccc} b_{11} & b_{12} & \dots & b_{1m} & 1 & 0 & \dots & 0 \\ b_{21} & b_{22} & \dots & b_{2m} & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ b_{p1} & b_{p2} & \dots & b_{pm} & 0 & 0 & \dots & 1 \end{array}\right]$$

Reduction

Row reduce

$$\left[\begin{array}{cccc|cccc} b_{11} & b_{12} & \dots & b_{1m} & 1 & 0 & \dots & 0 \\ b_{21} & b_{22} & \dots & b_{2m} & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ b_{p1} & b_{p2} & \dots & b_{pm} & 0 & 0 & \dots & 1 \end{array}\right]$$

to

$$[R|A] = \left[\begin{array}{cccc|cccc} 1 & 0 & \dots & \star & a_{11} & a_{12} & \dots & a_{1n} \\ 0 & 1 & \dots & \star & a_{21} & a_{22} & \dots & a_{2n} \\ 0 & 0 & \dots & \star & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \star & a_{m1} & a_{m2} & \dots & a_{mn} \end{array}\right]$$

where R is in reduced row echelon form.

Finishing off

$$[R|A] = \left[\begin{array}{cccc|cccc} 1 & 0 & \dots & \star & a_{11} & a_{12} & \dots & a_{1n} \\ 0 & 1 & \dots & \star & a_{21} & a_{22} & \dots & a_{2n} \\ 0 & 0 & \dots & \star & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \star & a_{m1} & a_{m2} & \dots & a_{mn} \end{array}\right]$$

If R is:-

the identity matrix then B is invertible with inverse A

anything else then B is not invertible

Relationship to solutions of systems

Recall that a system of equations is equivalently a matrix equation:

$$Ax = b$$

so if A is invertible

$$x = A^{-1}Ax = A^{-1}b$$

is the *unique solution* to $Ax = b$.

This method is rarely the most efficient thing to do.

Questions?

Examples

Example

Questions?

Properties of inverses

Inverse of transpose

Inverses of products

Alternative characterisation of invertible matrices

For square matrices $BA = I$ is the same as $AB = I$...

Other matrix equations

Example

Questions?

Questions?