

# Lecture 1

Thursday 6<sup>th</sup> September, 2018

# Lecture 1

Admin

One/Two variables

Three variables

Reduced row echelon form

Admin

## Office hours and contact details

office MS 448

office hours 11:00-12:00 and 15:30-16:30 Tuesday and Thursday

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# Notes and textbook

d2l > Content > Additional Course Material

[Class notes L02/L03](#) Notes from these lectures

[General lecture notes](#) Slides describing the course in detail

[Additional practice problems](#) for the whole course

[External resources](#) A curated list of videos, books and Q&A forums

[Free online textbook](#) 'A first course in linear algebra' by Kuttler

# Course structure

d2l > Content > Course Information > Course Schedule

d2l > Content > Course Information > How to Access Lyryx

- ▶ 10 Lyryx assignments
  - ▶ Don't worry about 1-2 marks; use to increase marks in final
  - ▶ First due on 23 September 11:59 PM
  
- ▶ ONE midterm
  - ▶ Friday 26 October
  
- ▶ Final Examination
  - ▶ Not yet scheduled: between 10-20 December

## More support

d2l > Content > Course Information > (various)

- ▶ Labs
- ▶ Continuous tutorials
- ▶ Student success centre
- ▶ Engineering academic success centre

## General advice

- ▶ Figure out which resources work for you sooner rather than later.
  - ▶ Probably far too many to actually use
- ▶ My advice: emphasise practising typical questions. E.g.
  - ▶ attend labs
  - ▶ go to continuous tutorials with specific questions
  - ▶ use Lyryx effectively



# Linear equations in one variable

Find all real numbers  $x$  such that

$$ax = b$$

where  $a$  and  $b$  are real numbers.

# Linear equation in two variables

Find all real numbers  $x$  and  $y$  such that both

$$x + 2y = 1$$

$$3x + 4y = 0$$

hold.

Questions?

# Elementary row operations

But what operations are we allowed to do on the rows?

The following operations will not change the solutions:-

1. Swap two rows.
2. Add a multiple of one row to another row.
3. Multiply a row by a scalar.

See pictures on Jupyter notebook.

# Inconsistent equations

Find all real numbers  $x$  and  $y$  such that both

$$x + 2y = 1$$

$$5x + 10y = 2$$

hold.

# Infinitely many solutions

Find all real numbers  $x$  and  $y$  such that both

$$3x + 12y = 18$$

$$4x + 16y = 24$$

hold.

Questions?

Find all  $x$ ,  $y$  and  $z$  such that

$$x + 2y + 3z = 4$$

$$5x + 6y + 7z = 8$$

$$9x + 10y + 11z = 12$$

hold.



Find all  $x$ ,  $y$  and  $z$  such that

$$x + y + 2z = -1$$

$$2x + y + 3z = 0$$

$$0x + -2y + 1z = 2$$

hold.

Find all  $x$ ,  $y$  and  $z$  such that

$$x + 2y + 3z = 4$$

$$5x + 6y + 7z = 8$$

$$3x + 4y + 5z = 1$$

hold.

Find all  $x$ ,  $y$  and  $z$  such that

$$6x + 4y + 2z = 4$$

$$3x + 2y + 1z = 2$$

$$9x + 6y + 3z = 6$$

hold.

Questions?

## Row echelon form

A matrix is in *row echelon form* iff:-

- ▶ All rows consisting entirely of zeros are at the bottom.
- ▶ The first nonzero entry in each nonzero row is a 1 (called the **leading 1** for that row).
- ▶ Each leading 1 is to the right of all leading 1's in rows above it.

For instance:

$$\begin{bmatrix} 0 & 1 & * & * & * & * & * & * \\ 0 & 0 & 0 & 1 & * & * & * & * \\ 0 & 0 & 0 & 0 & 1 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where \* can be any number.

# Reduced row echelon form

A matrix is in *reduced row echelon form* iff:-

- ▶ It is a row-echelon matrix.
- ▶ Each leading 1 is the only nonzero entry in its column.

$$\begin{bmatrix} 0 & 1 & * & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 1 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 1 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where \* can be any number.

- ▶ Row echelon form  $\leftrightarrow$  ready for back-substitution.
- ▶ Reduced row echelon form  $\leftrightarrow$  read off solutions.

Another advantage of reduced row echelon form is that:

### Theorem

*Reduced row echelon form is unique.*

## Reading off solutions

- ▶ The augmented matrix

$$\left[ \begin{array}{ccccc|c} 1 & 0 & 5 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

is in reduced row echelon form.

- ▶ If there are any rows of the form  $(0 \ 0 \dots 0 | 1)$  then we know immediately that there are no solutions.



## Identify leading 1s and parameter blocks

- ▶ The leading 1s are shown in green

$$\left[ \begin{array}{ccccc|c} 1 & 0 & 5 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

and the other non-zero numbers are shown in blue and red.

- ▶ Variables associated to the leading 1s are the *leading variables*.
- ▶ In this case  $x_1$ ,  $x_2$  and  $x_4$  are leading and  $x_3$  and  $x_5$  are not.

The non-leading variables can take any value so we assign parameters to them.

In this case:

$$x_3 = s$$

$$x_5 = t$$

and then we solve for the leading variables in terms of these parameters:

$$x_1 = 1 - 5s - t$$

$$x_2 = -s - 2t$$

$$x_4 = -4t$$

Questions?