Math 211 2018-10-04

Find the inverse of $\begin{pmatrix} 4 & 0 & 3 \\ 1 & 9 & 7 \\ 0 & 6 & 4 \end{pmatrix} = A.$

$$\mathrm{cof}_{11}(A) = (-1)^{1+1}\begin{vmatrix} 9 & 7 \\ 6 & 4 \end{vmatrix} = 36-42 = -6$$

$$\mathrm{cof}_{12}(A) = (-1)^{1+2}\begin{vmatrix} 1 & 7 \\ 0 & 4 \end{vmatrix} = -(4) = -4$$

$$\mathrm{cof}_{13}(A) = (-1)^{1+3}\begin{vmatrix} 1 & 9 \\ 0 & 6 \end{vmatrix} = 6$$

$$\mathrm{cof}_{21}(A) = (-1)^{2+1}\begin{vmatrix} 0 & 3 \\ 6 & 4 \end{vmatrix} = -(-18) = 18.$$

$$\mathrm{cof}_{22}(A) = (-1)^{2+2}\begin{vmatrix} 4 & 3 \\ 0 & 4 \end{vmatrix} = 16.$$

$$\mathrm{cof}_{23}(A) = (-1)^{2+3}\begin{vmatrix} 4 & 0 \\ 0 & 6 \end{vmatrix} = -24$$

$$\mathrm{cof}_{31}(A) = (-1)^{3+1}\begin{vmatrix} 0 & 3 \\ 9 & 7 \end{vmatrix} = -27$$

$$\mathrm{cof}_{32}(A) = (-1)^{3+2}\begin{vmatrix} 4 & 3 \\ 1 & 7 \end{vmatrix} = -(28-3) = -25$$

$$\mathrm{cof}_{33}(A) = (-1)^{3+3}\begin{vmatrix} 4 & 0 \\ 1 & 9 \end{vmatrix} = 36$$

$$\mathrm{cof}(A) = \begin{pmatrix} -6 & -4 & 6 \\ 18 & 16 & -24 \\ -27 & -25 & 36 \end{pmatrix}, \quad \mathrm{adj}(A) = \begin{pmatrix} -6 & 18 & -27 \\ -4 & 16 & -25 \\ 6 & -24 & 36 \end{pmatrix}.$$

$$\det(A) = 4(-6) + 0 + 3\cdot 6 = -6.$$

$$A^{-1} = \frac{-1}{6}\begin{pmatrix} -6 & 18 & -27 \\ -4 & 16 & -25 \\ 6 & -24 & 36 \end{pmatrix}.$$

Find x_2 such that

$$\begin{pmatrix} 3 & 1 & -1 \\ 5 & 2 & 0 \\ 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}.$$

Use Cramer's rule: $x_2 = \dfrac{\det(A_2)}{\det(A)}.$

$$|A| = \begin{vmatrix} 3 & 1 & -1 \\ 5 & 2 & 0 \\ 1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 \\ 5 & 2 & 0 \\ 1 & 1 & -1 \end{vmatrix} = -4$$

$$R_1 \leftarrow R_1 - R_3$$

$$|A_2| = \begin{vmatrix} 3 & -1 & -1 \\ 5 & 2 & 0 \\ 1 & 1 & -1 \end{vmatrix} \overset{R_1 \leftarrow R_1 - R_3}{=} \begin{vmatrix} 2 & -2 & 0 \\ 5 & 2 & 0 \\ 1 & 1 & -1 \end{vmatrix} \overset{R_1 \leftarrow R_1 + R_2}{=} \begin{vmatrix} 7 & 0 & 0 \\ 5 & 2 & 0 \\ 1 & 1 & -1 \end{vmatrix} = -14$$

$$x_2 = \frac{-14}{-4} = \frac{7}{2}.$$

Suppose A is invertible. Find $\det(\operatorname{adj}(A))$.

We know $\quad A \operatorname{adj}(A) = \det(A) I$.

Recall:

$\det(kB) = k^n \det(B)$

So:

$$\det(A)\det(\operatorname{adj}(A)) = \det(\det(A) I).$$

$$= \det(A)^n \det(I)$$

$$= \det(A)^n$$

$$\Rightarrow \det(\operatorname{adj}(A)) = \det(A)^{n-1}$$

Find an interpolating polynomial for

$$(0,1),\ (1,2),\ (2,5),\ (3,10).$$

So we need to find a, b, c, d s.t.

$a + bx + cx^2 + dx^3$ passes through the points.

i.e.

$$a + 0b + 0c + 0d = 1$$

$$a + b + c + d = 2$$

$$a + 2b + 4c + 8d = 5$$

$$a + 3b + 9c + 27d = 10$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 4 & 8 & 5 \\ 1 & 3 & 9 & 27 & 10 \end{array}\right)$$

$$
\left(\begin{array}{cccc|c}
1 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 \\
0 & 2 & 4 & 8 & 4 \\
0 & 3 & 9 & 27 & 9
\end{array}\right)
\rightarrow
\left(\begin{array}{cccc|c}
1 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 \\
0 & 0 & 2 & 6 & 2 \\
0 & 0 & 6 & 24 & 6
\end{array}\right)
$$

$$
\rightarrow
\left(\begin{array}{cccc|c}
1 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 3 & 1 \\
0 & 0 & 6 & 24 & 6
\end{array}\right)
\rightarrow
\left(\begin{array}{cccc|c}
1 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 3 & 1 \\
0 & 0 & 0 & 6 & 0
\end{array}\right)
$$

$R_2 \leftarrow R_2 - R_4;\ R_3 \leftarrow R_3 - R_4.$

$$
\rightarrow
\left(\begin{array}{cccc|c}
1 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 3 & 1 \\
0 & 0 & 0 & 1 & 0
\end{array}\right)
\rightarrow
\left(\begin{array}{cccc|c}
1 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0
\end{array}\right)
$$

$$
\rightarrow
\left(\begin{array}{cccc|c}
1 & & & 0 & 1 \\
 & 1 & & & 0 \\
 & & 1 & & 1 \\
0 & & & 1 & 0
\end{array}\right)
\rightarrow
$$

$a = 1$

$c = 1.$

If A, B and C are 4×4 matrices with

$$\det(A) = -1, \quad \det(B) = 2, \quad \det(C) = 1$$

then

$$\det(2A^2B^{-1}(C^T)^3BA^{-1}) = 2^4(\det(A))^2\frac{1}{\det(B)}(\det(C))^3\det(B)\frac{1}{\det(A)}$$

$$= 2^4(-1)^2(1)^3\frac{1}{-1} = -2^4 = -16$$

If A is a 3×3 matrix and

$$\det(2A^{-1}) = -4 = \det(A^3(B^{-1})^T)$$

Find $\det(A)$ and $\det(B)$.

First $\left.\begin{aligned} -4 &= \det(2A^{-1}) \\ &= 2^3\det(A^{-1}) = 2^3\frac{1}{\det(A)} \end{aligned}\right\}$ $\det(A) = \frac{8}{-4} = -2$

Second $-4 = \det(A^3(B^{-1})^T) = \det(A)^3\det(B^{-1})$

$$= (-2)^3\frac{1}{\det(B)}$$

So $\det(B) = \frac{-8}{-4} = 2.$