Math 211.3

E.g.

$$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 5 \\ 6 \end{pmatrix} = 5\begin{pmatrix} 1 \\ 4 \end{pmatrix} + 6\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5+12 \\ 20+18 \end{pmatrix} = \begin{pmatrix} 17 \\ 38 \end{pmatrix}$$

E.g.

$$\begin{pmatrix} 1 & 2 & 8 \\ 4 & 3 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix} = 5\begin{pmatrix} 1 \\ 4 \end{pmatrix} + 6\begin{pmatrix} 2 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 8 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 5+12+16 \\ 20+18-2 \end{pmatrix} = \begin{pmatrix} 33 \\ 36 \end{pmatrix}$$

Relationship between matrix equations and linear systems:

$$\begin{aligned} x + 3y &= 9 \\ 2x + 4y &= 3 \end{aligned} \quad \longleftrightarrow \quad \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$$

E.g. Find x_1, x_2, x_3, x_4 s.t.

$$x_1\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + x_2\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + x_3\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + x_4\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

i.e. Solve

$$\begin{pmatrix} 1 & 0 & 2 & -1 \\ 2 & -1 & 0 & 1 \\ 3 & 1 & 3 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

i.e. Solve the augmented matrix

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ 2 & -1 & 0 & 1 & 1 \\ 3 & 1 & 3 & 1 & 1 \end{array}\right).$$

$R_2 \leftarrow R_2 - 2R_1$; $R_3 \leftarrow R_3 - 3R_1$

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ 0 & -1 & -4 & 3 & -1 \\ 0 & 1 & -3 & 4 & -2 \end{array}\right)$$

$R_2 \leftarrow -R_2$

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ 0 & 1 & 4 & -3 & 1 \\ 0 & 1 & -3 & 4 & -2 \end{array}\right)$$

$R_3 \leftarrow R_3 - R_2$

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ 0 & 1 & 4 & -3 & 1 \\ 0 & 0 & -7 & 7 & -3 \end{array}\right)$$

$R_3 \leftarrow \frac{-1}{7} R_3$

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ 0 & 1 & 4 & -3 & 1 \\ 0 & 0 & 1 & -1 & 3/7 \end{array}\right)$$

$R_1 \leftarrow R_1 - 2R_3$; $R_2 \leftarrow R_2 - 4R_3$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1-6/7 \\ 0 & 1 & 0 & 1 & 1-12/7 \\ 0 & 0 & 1 & -1 & 3/7 \end{array}\right) \begin{array}{l} \to 1/7 \\ \to -5/7 \\ \end{array}$$

$x_4 = s$

$x_1 = 1/7 - s$

$x_2 = -5/7 - s$

$x_3 = 3/7 + s.$

(To answer the question we can choose s as any no. so set $s=0$.)

$x_1 = 1/7,\ x_2 = -5/7,\ x_3 = 3/7,\ x_4 = 0.$

$$\mathbb{R}^n \xrightarrow{A} \mathbb{R}^m \xrightarrow{B} \mathbb{R}^p.$$

$$\mathbb{R}^q \xrightarrow{C} \mathbb{R}^p.$$

Matrix product example:

$$\begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 2 \\ 0 & -2 & 4 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 4 & -1 & -2 \\ -1 & 4 & 0 \end{pmatrix}$$

First col.: $\begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = -\begin{pmatrix} -1 \\ 2 \end{pmatrix} + 0\begin{pmatrix} 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} 1+3 \\ -2+1 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

Second col.: $\begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 0 \\ -1 \end{pmatrix} + 0\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} -1 \\ 2+2 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}.$$

Third col: $\begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} = 2\begin{pmatrix} -1 \\ 2 \end{pmatrix} + 4\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4-4 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

So $BA = \begin{pmatrix} 4 & -1 & -2 \\ -1 & 4 & 0 \end{pmatrix}$.

Order of composition:

$$B(Ax) = (BA)x$$

$$A = \begin{pmatrix} 1 & 2 \\ -3 & 0 \\ 1 & -4 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -1 & 2 & 0 \\ 3 & -2 & 1 & -3 \end{pmatrix}$$

So AB makes sense as rows(B) = cols(A).

but BA does not make sense as cols$(B) \neq$ rows(A)

<u>Why?</u> $\left.\begin{array}{l} \mathbb{R}^4 \xrightarrow{B} \mathbb{R}^2 \\ \mathbb{R}^2 \xrightarrow{A} \mathbb{R}^3 \end{array}\right\}$ so $\mathbb{R}^4 \xrightarrow{B} \mathbb{R}^2 \xrightarrow{A} \mathbb{R}^3$ makes sense

but $\mathbb{R}^2 \xrightarrow{A} \mathbb{R}^3$

$\mathbb{R}^4 \xrightarrow{B} \mathbb{R}^2$ doesn't compose.