Math 211. 2018-09-25

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ -10 & -12 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 \leftarrow -2R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

First col:

$$1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 5 \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -10 \end{pmatrix}$$

Second col:

$$2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 4 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 6 \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -12 \end{pmatrix}.$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left(A + 3\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{pmatrix}\right)^T = \begin{pmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{pmatrix}$$

$$A + 3\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 5 & 8 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 5 & 8 \end{pmatrix} - \begin{pmatrix} 3 & -3 & 0 \\ 3 & 6 & 12 \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 3 & 3 \\ -2 & -1 & -4 \end{pmatrix}.$$

If $A=\begin{pmatrix}4&1\\1&3\end{pmatrix}$ and $C=\begin{pmatrix}1&3\\2&-5\end{pmatrix}$ then find elementary matrices s.t. $C=FEA$.

$$\begin{pmatrix}4&1\\1&3\end{pmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{pmatrix}1&3\\4&1\end{pmatrix}$$

$R_2 \leftarrow R_2-2R_1$

$$\begin{pmatrix}1&3\\2&-5\end{pmatrix}$$

So: $C=\begin{pmatrix}1&0\\-2&1\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}4&1\\1&3\end{pmatrix}=\begin{pmatrix}1&0\\-2&1\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}A.$

Find a matrix $\mathcal{U}$ such that $\mathcal{U}\begin{pmatrix}3&0&1\\2&-1&0\end{pmatrix}$ is in reduced row echelon form.

$$\left(\begin{array}{ccc|cc}3&0&1&1&0\\2&-1&0&0&1\end{array}\right)$$

$R_1 \leftarrow \frac{1}{3}R_1$

$$\left(\begin{array}{ccc|cc}1&0&1/3&1/3&0\\2&-1&0&0&1\end{array}\right)$$

$R_2 \leftarrow R_2-2R_1$

$$\left(\begin{array}{ccc|cc}1&0&1/3&1/3&0\\0&-1&-2/3&-2/3&1\end{array}\right)$$

$R_2 \leftarrow -R_2$

$$\left(\begin{array}{ccc|cc}1&0&1/3&1/3&0\\0&1&2/3&2/3&-1\end{array}\right)$$

So $\mathcal{U}=\begin{pmatrix}1/3&0\\2/3&-1\end{pmatrix}$.

Find A such that

$$(3I - A^T)^{-1} = 2\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

$$3I - A^T = \frac{1}{2}\frac{1}{3-2}\begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$$

$$-A^T = \begin{pmatrix} 3/2 & -1/2 \\ -1 & 1/2 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} -3/2 & -1/2 \\ -1 & -5/2 \end{pmatrix}$$

$$A = \begin{pmatrix} 3/2 & 1 \\ 1/2 & 5/2 \end{pmatrix}$$

Prove that if $A^3 = 4I$ then A is invertible.

$$\left.\begin{aligned} A(A^2) &= 4I \\ A(1/4A^2) &= I \end{aligned}\right\} \text{ so } A^{-1} = \frac{1}{4}A^2$$