Math211.2

$$x_1 + 0x_2 + 5x_3 + 0x_4 + x_5 = 1$$

$$x_1 = 1 - 5x_3 - x_5 = 1 - 5s - t.$$

Solve:

$$\left(\begin{array}{ccccc|c} 0 & 0 & 0 & -2 & -8 & 4 \\ -3 & 6 & -4 & -9 & 3 & -1 \\ -1 & 2 & -2 & -4 & -3 & 3 \\ 1 & -2 & 1 & 3 & -1 & 1 \end{array}\right)$$

$R_1 \leftrightarrow R_4$

$$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ -3 & 6 & -4 & -9 & 3 & -1 \\ -1 & 2 & -2 & -4 & -3 & 3 \\ 0 & 0 & 0 & -2 & -8 & 4 \end{array}\right)$$

$R_2 \leftarrow R_2 + 3R_1$; $R_3 \leftarrow R_3 + R_1$

$$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & -1 & 0 & 0 & 2 \\ 0 & 0 & -1 & -1 & -4 & 4 \\ 0 & 0 & 0 & -2 & -8 & 4 \end{array}\right)$$

$R_2 \leftarrow -R_2$

$$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & -1 & -1 & -4 & 4 \\ 0 & 0 & 0 & -2 & -8 & 4 \end{array}\right)$$

$R_3 \leftarrow R_3 + R_2$

$$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & -1 & -4 & 2 \\ 0 & 0 & 0 & -2 & -8 & 4 \end{array}\right)$$

$R_3 \leftarrow -R_3$

$$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & -2 & -8 & 4 \end{array}\right)$$

$R_4 \leftarrow R_4 + 2R_3$

$$\left(\begin{array}{ccccc|c} 1 & -2 & 1 & 3 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Row echelon form.

$R_1 \leftarrow R_1 - R_2$

$$\left(\begin{array}{ccccc|c} 1 & -2 & 0 & 3 & -1 & 3 \\ 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$R_1 \leftarrow R_1 - 3R_3$

$$\left(\begin{array}{ccccc|c} \textcircled{1} & -2 & 0 & 0 & -13 & 9 \\ 0 & 0 & \textcircled{1} & 0 & 0 & -2 \\ 0 & 0 & 0 & \textcircled{1} & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Leading variables: x_1, x_3, x_4.
Non-leading: x_2, x_5.
So set $x_2 = s \rightsquigarrow$ parameter.
$x_5 = t$

$$x_1 = 2s + 13t + 9$$
$$x_3 = -2$$
$$x_4 = -4t - 2.$$

Solve $\left(\begin{array}{cccc|c} \textcircled{1} & 0 & 0 & -4 & 0 \\ 0 & \textcircled{1} & 0 & 7 & 2 \\ 0 & 0 & \textcircled{1} & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

Leading vars: x_1, x_2, x_3

So $x_4 = s$
$x_1 = 4s$
$x_2 = -7s + 2$
$x_3 = 2s - 1.$

If homog. then there is at least one solution:

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = 0$$

$$x_1 = 0$$

$$x_2 = 0$$

$$\vdots$$

$$x_n = 0$$