Math 211.1

Solve $ax = b$.

Case 1: $a \neq 0 \Rightarrow x = \frac{b}{a}$ (Unique sol^n.)

Case 2: $a = 0$ and $b = 0$

$\Rightarrow 0 \cdot x = 0$ so all real numbers x work. (Infinitely many sol^{n}s.)

Case 3: $a = 0$ and $b \neq 0$

So no value of x works. (Inconsistent.)

Solve R_1: $x + 2y = 1$

R_2: $3x + 4y = 0$

$R_2 \leftarrow R_2 - 3R_1$

$x + 2y = 1$

$0x - 2y = -3$

$R_1 \leftarrow R_1 + R_2$

$x + 0y = -2$

$0x - 2y = -3$

So $x = -2$ $y = 3/2$

Augmented matrix:

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 4 & 0 \end{array}\right)$$

$R_2 \leftarrow R_2 - 3R_1$

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -2 & -3 \end{array}\right)$$

$R_1 \leftarrow R_1 + R_2$

$$\left(\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & -2 & -3 \end{array}\right)$$

Solve $x + 4y = 6$

$0x + 0y = 0.$

let $y = s \rightsquigarrow$ a parameter.

Then $x = 6 - 4s.$

Solve $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{array}\right)$

$R_2 \leftarrow R_2 - 5R_1$

$R_3 \leftarrow R_3 - 9R_1$

$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24 \end{array}\right)$

$R_3 \leftarrow R_3 - 2R_2$

$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$R_2 \leftarrow \frac{-1}{4} R_2$

$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$R_1 \leftarrow R_1 - 2R_2$

$\left(\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$

let $z = s$ a parameter.

So $x = -2 + s$

(because $x + 0y - s = -2$.)

and $y = 3 - 2s.$