MATH211: Linear Methods I

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Lecture on Thursday 27th September, 2018

Determinants

Examples

Traditional determinants

Examples

Last time

- Elementary matrices
- Elementary matrices as theoretical tool
- Matrix equations

Determinants

Two dimensional determinants

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix} \text{ and } \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix}$$

Then picture in two dimensions. The signed area of the parallelogram. Calculate.

Effect of scalar multiplication of row

Picture.

Determinants of diagonal matrix and scalar multiple

Determinant of diagonal. Determinant of scalar multiple of a matrix.

Effect of adding a multiple of one row to another

Determinant of triangular matrix

We can reduce a triangular matrix to a diagonal one.

Effect of swapping two rows

This essentially defines signed area in dimensions higher than three.

Questions?

Examples

Examples

Example

Find

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{vmatrix}$$

Example

Find

$$\begin{vmatrix} -3 & 5 & -6 \\ 1 & -1 & 3 \\ 2 & -4 & 1 \end{vmatrix}$$

Examples

Example

Find

$$\begin{vmatrix} 3 & 1 & 2 & 4 \\ -1 & -3 & 8 & 0 \\ 1 & -1 & 5 & 5 \\ 1 & 1 & 2 & -1 \end{vmatrix}$$

Example

If

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 4 \text{ find } \begin{vmatrix} -b_1 & -b_2 & -b_3 \\ a_1+2b_1 & a_2+2b_2 & a_3+2b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix}$$

Examples

Example

Find

$$\begin{vmatrix} 2 & 3 & 5 \\ 3 & 5 & 9 \\ 1 & 2 & 4 \end{vmatrix}$$

Questions?

Traditional determinants

Minors and cofactors

Definition

Let $A = [a_{ij}]$ be an $n \times n$ matrix. The ij^{th} minor of A, denoted as $minor\,(A)_{ij}$, is the determinant of the $n-1 \times n-1$ matrix which results from deleting the i^{th} row and the j^{th} column of A.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{bmatrix}$$

Recursive definition of determinant

Definition

The ij^{th} *cofactor* of A is

$$\mathrm{cof}(A)_{ij} = (-1)^{i+j} \mathit{minor}\,(A)_{ij}$$

It fact we can calculate the determinant using these matrices.

Definition

The *cofactor expansion* along row i is:

$$\begin{aligned}\det A &= \sum_{k=1}^{n} a_{ik}\mathrm{cof}(A)_{ik} \\ &= a_{11}\mathrm{cof}(A)_{11} + a_{12}\mathrm{cof}(A)_{12} + a_{13}\mathrm{cof}(A)_{13} + \cdots + a_{1n}\mathrm{cof}(A)_{1n}\end{aligned}$$

Examples

Examples

Example

Find the (1,2)-minor of

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 4 & 1 \\ 5 & 2 & 6 \end{bmatrix}$$

Example

Find

$$\begin{vmatrix} 1 & 1 & 3 \\ 2 & 4 & 1 \\ 5 & 2 & 6 \end{vmatrix}$$

Examples

Example

Find

$$\begin{vmatrix} 0 & 1 & -2 & 1 \\ 5 & 0 & 0 & 7 \\ 0 & 1 & -1 & 0 \\ 3 & 0 & 0 & 2 \end{vmatrix}$$

Example

Find

$$\begin{vmatrix} -8 & 1 & 0 & -4 \\ 5 & 7 & 0 & -7 \\ 12 & -3 & 0 & 8 \\ -3 & 11 & 0 & 2 \end{vmatrix}$$

Questions?