Math 211 2018-10-09

Interpolate $(0,1), (1,2), (2,5), (3,10)$.

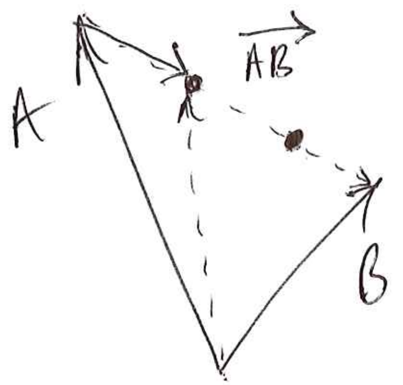
Try $a + bx + cx^2 + dx^3 = f(x)$.

$$\left.\begin{aligned}
\underline{(0,1):}\quad & a + 0b + 0c + 0d = 1\\
\underline{(1,2):}\quad & a + b + c + d = 2\\
\underline{(2,5):}\quad & a + 2b + 4c + 8d = 5\\
\underline{(3,10):}\quad & a + 3b + 9c + 27d = 10
\end{aligned}\right\}$$

Solve
$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1\\ 1 & 1 & 1 & 1 & 2\\ 1 & 2 & 4 & 8 & 5\\ 1 & 3 & 9 & 27 & 10\end{array}\right)$$

Midpoint of $\begin{pmatrix}-1\\-4\\3\end{pmatrix}$ and $\begin{pmatrix}5\\0\\-3\end{pmatrix}$ is $\frac{1}{2}\left[\begin{pmatrix}-1\\-4\\3\end{pmatrix}+\begin{pmatrix}5\\0\\-3\end{pmatrix}\right]=\frac{1}{2}\begin{pmatrix}4\\-4\\0\end{pmatrix}=\begin{pmatrix}2\\-2\\0\end{pmatrix}$.

Trisecting the between $A=\begin{pmatrix}2\\3\\5\end{pmatrix}$ and $B=\begin{pmatrix}8\\-6\\2\end{pmatrix}$.



First point: $\overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$

$$= (A-0) + \frac{1}{3}(B-A)$$

$$= \frac{1}{3}(2A+B).$$

$$= \frac{1}{3}\left(2\begin{pmatrix}2\\3\\5\end{pmatrix} + \begin{pmatrix}8\\-6\\2\end{pmatrix}\right) = \frac{1}{3}\begin{pmatrix}12\\0\\12\end{pmatrix} = \begin{pmatrix}4\\0\\4\end{pmatrix}.$$

Second point: $\overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB}$

$$= (A-0) + \frac{2}{3}(B-A) = \frac{1}{3}(A+2B).$$

$$= \frac{1}{3}\left[\begin{pmatrix}2\\3\\5\end{pmatrix} + 2\begin{pmatrix}8\\-6\\2\end{pmatrix}\right] = \frac{1}{3}\begin{pmatrix}18\\-9\\9\end{pmatrix} = \begin{pmatrix}6\\-3\\3\end{pmatrix}.$$

$$P=\begin{pmatrix}1\\-2\\1\end{pmatrix},\quad Q=\begin{pmatrix}-3\\0\\5\end{pmatrix},\quad X=\begin{pmatrix}2\\-1\\5\end{pmatrix},\quad Y=\begin{pmatrix}4\\-2\\3\end{pmatrix}$$

- Is $\overrightarrow{PQ}$ parallel to $\overrightarrow{XY}$?

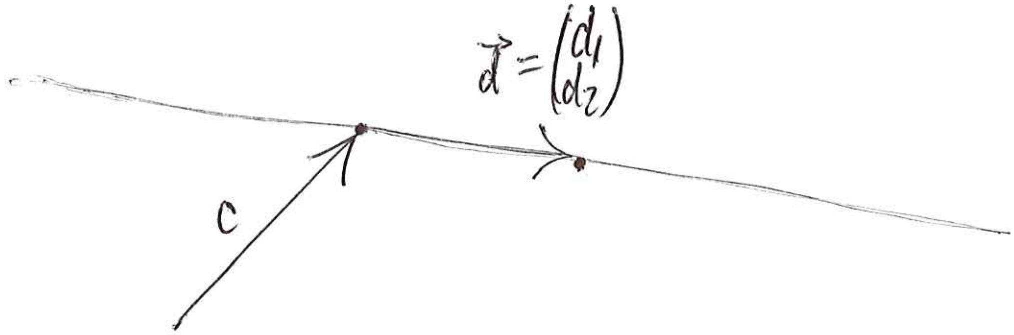
$$\overrightarrow{PQ}=\underset{\substack{\shortparallel\\ Q}}{\begin{pmatrix}-3\\0\\5\end{pmatrix}}-\underset{\substack{\shortparallel\\ P}}{\begin{pmatrix}1\\-2\\1\end{pmatrix}}=\begin{pmatrix}-4\\2\\4\end{pmatrix}.\quad \overrightarrow{XY}=\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\-1\\5\end{pmatrix}=\begin{pmatrix}2\\-1\\-2\end{pmatrix}$$

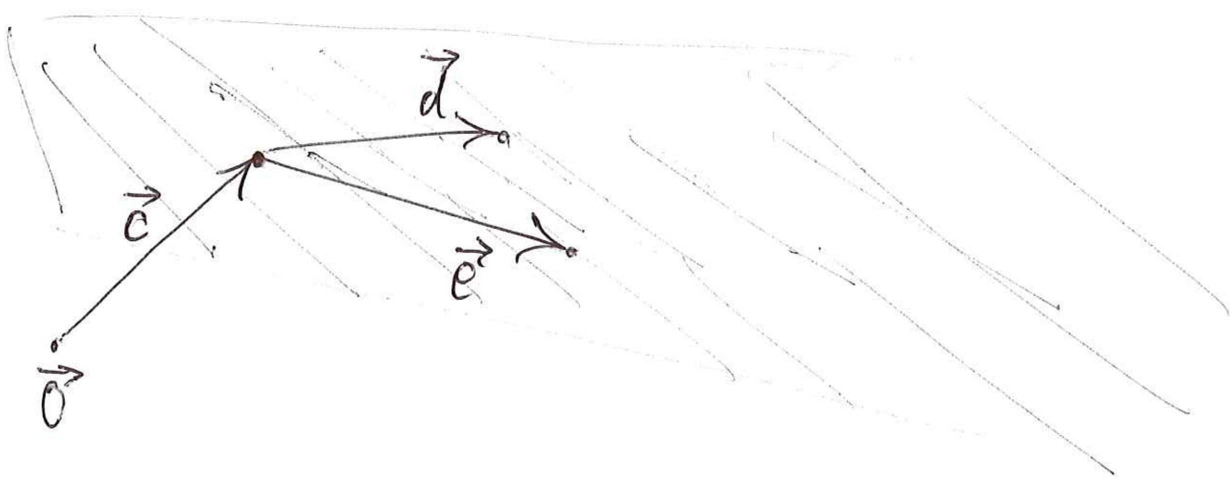
So $\overrightarrow{PQ}=-2\overrightarrow{XY}$ so PQ and XY are parallel.

- Is $\overrightarrow{PX}$ parallel to $\overrightarrow{QY}$?

$$\overrightarrow{PX}=\begin{pmatrix}2\\-1\\5\end{pmatrix}-\begin{pmatrix}1\\-2\\1\end{pmatrix}=\begin{pmatrix}1\\1\\4\end{pmatrix},\quad \overrightarrow{QY}=\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}-3\\0\\5\end{pmatrix}=\begin{pmatrix}7\\-2\\-2\end{pmatrix}.$$

So $\overrightarrow{PX}$ and $\overrightarrow{QY}$ are <u>not</u> parallel.





Convert the following into parametric form:

$$\frac{x-2}{3} = \frac{y-1}{2} = z+3.$$

So
$$\left.\begin{aligned} x-2 &= 3s \\ y-1 &= 2s \\ z+3 &= s \end{aligned}\right\} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

If $P = \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix}$, $Q = \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$.

then the line passing through P & Q is:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix} + s\,\overrightarrow{PQ} = \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix} + s\left[\begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix}\right]$$

$$= \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix} + s\begin{pmatrix} -5 \\ 5 \\ -2 \end{pmatrix}.$$