- that the diagonals of a parallelogram bisect each other.

If

$$\begin{tikzcd} & B \arrow[r] & D \\ A \arrow[ur] \arrow[r] & C \arrow[ur] & \end{tikzcd}$$

is a parallelogram

we know by definition that

$$\overrightarrow{AB} = \overrightarrow{CD} \quad \text{and} \quad \overrightarrow{BD} = \overrightarrow{AC}$$

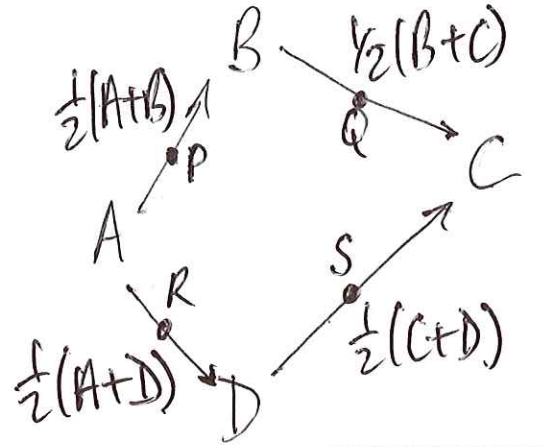
From the first: $B - A = D - C$

$$\Rightarrow C + B = D + A.$$

Therefore

$$\text{midpoint}(B, C) = \tfrac{1}{2}(B+C) = \tfrac{1}{2}(A+D) = \text{midpoint}(A, D).$$

Prove that if $ABCD$ is an arbitrary quadrilateral then the midpoints of the sides form a parallelogram.



We need to show that

(1) $\overrightarrow{PQ} = \overrightarrow{RS}$

(2) $\overrightarrow{RP} = \overrightarrow{SQ}$

And indeed:

(1)
$$\begin{aligned}\overrightarrow{PQ} = Q - P &= \tfrac{1}{2}(B+C) - \tfrac{1}{2}(A+B) \\ &= \tfrac{1}{2}(C-A) \\ &= \tfrac{1}{2}(C+D) - \tfrac{1}{2}(A+D) \\ &= S - R = \overrightarrow{RS}\end{aligned}$$

(2)
$$\begin{aligned}\overrightarrow{RP} = P - R &= \tfrac{1}{2}(A+B) - \tfrac{1}{2}(A+D) \\ &= \tfrac{1}{2}(B-D) \\ &= \tfrac{1}{2}(B+C) - \tfrac{1}{2}(C+D) \\ &= Q - S = \overrightarrow{SQ}.\end{aligned}$$