Math211 2018-10-02

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

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$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{vmatrix} = 1\begin{vmatrix} 5 & 6 \\ 0 & 9 \end{vmatrix} - 2\begin{vmatrix} 0 & 6 \\ 0 & 9 \end{vmatrix} + 3\begin{vmatrix} 0 & 5 \\ 0 & 0 \end{vmatrix}$$

$$= 1\cdot(45-0) - 2(0-0) + 3(0-0)$$

$$= 45$$

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$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}.$$

$$= (45-48) - 2(36-42) + 3(32-35)$$

$$= -3 - 2(-6) + 3(-3)$$

$$= 0.$$

$$\begin{vmatrix} 0 & 1 & -2 & 1 \\ 5 & 0 & 0 & 7 \\ 0 & 1 & -1 & 0 \\ 3 & 0 & 0 & 2 \end{vmatrix} =$$

$$0\begin{vmatrix} 0 & 0 & 7 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{vmatrix} - 1\begin{vmatrix} 5 & 0 & 7 \\ 0 & -1 & 0 \\ 3 & 0 & 2 \end{vmatrix} + (-2)\begin{vmatrix} 5 & 0 & 7 \\ 0 & 1 & 0 \\ 3 & 0 & 2 \end{vmatrix} - \begin{vmatrix} 5 & 0 & 0 \\ 0 & 1 & -1 \\ 3 & 0 & 0 \end{vmatrix}$$

(not enough time in lecture to expand out these 3x3 determinants)

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$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{pmatrix}, \begin{pmatrix} 0 & 1 & -2 & 1 \\ 5 & 0 & 0 & 7 \\ 0 & 1 & -1 & 0 \\ 3 & 0 & 0 & 2 \end{pmatrix}$$

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Around 1st col:

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{vmatrix} = 1\cdot\begin{vmatrix} 5 & 6 \\ 0 & 9 \end{vmatrix} - 0\begin{vmatrix} 2 & 3 \\ 0 & 9 \end{vmatrix} + 0\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}.$$

$$= 1\cdot(45-0) = 45.$$

Along 4th row:

$$\begin{vmatrix} 0 & 1 & -2 & 1 \\ 5 & 0 & 0 & 7 \\ 0 & 1 & -1 & 0 \\ 3 & 0 & 0 & 2 \end{vmatrix} =$$

$$-3\begin{vmatrix} 1 & -2 & 1 \\ 0 & 0 & 7 \\ 1 & -1 & 0 \end{vmatrix} + 0\begin{vmatrix} 0 & -2 & 1 \\ 5 & 0 & 7 \\ 0 & -1 & 0 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 & 1 \\ 5 & 0 & 7 \\ 0 & 1 & 0 \end{vmatrix} + 2\begin{vmatrix} 0 & 1 & -2 \\ 5 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

because $(-1)^{4+1} = -1$.

$$= -3\begin{vmatrix} 1 & -2 & 1 \\ 0 & 0 & 7 \\ 1 & -1 & 0 \end{vmatrix} + 2\begin{vmatrix} 0 & 1 & -2 \\ 5 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$(-1)^{2+3} = -1$

$$= -3\left[-0\begin{vmatrix} -2 & 1 \\ -1 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - 7\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}\right]$$

$$+2\left[0\begin{vmatrix} 0 & 0 \\ 1 & -1 \end{vmatrix} - 5\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} + 0\begin{vmatrix} 1 & -2 \\ 0 & 0 \end{vmatrix}\right]$$

$$= 21 \begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} - 10 \begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}$$

$$= 21(-1+2) - 10(-1+2)$$

$$= 11$$