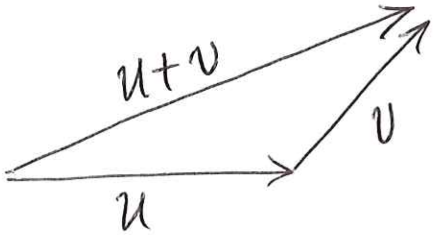
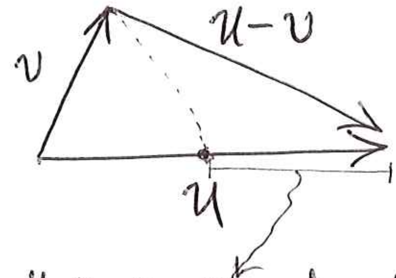
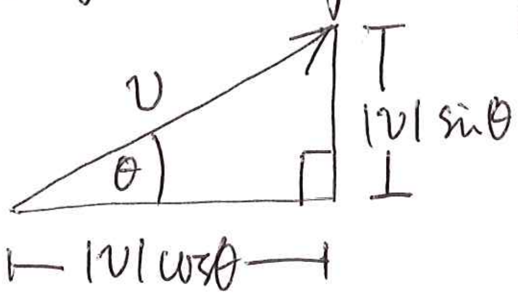
Math211 2018-10-25





$$\|u+v\| \leq \|u\| + \|v\| \qquad\qquad \big|\,\|u\| - \|v\|\,\big| \leq \|u-v\|$$

Lagrange Identity:



Recall $\sin^2\theta + \cos^2\theta = 1$

Pythagorean Identity.

$$\|u\cdot v\|^2 + \|u\times v\|^2 = \left(\|u\|\|v\|\cos\theta\right)^2 + \left(\|u\|\|v\|\sin\theta\right)^2$$

$$= \|u\|^2\|v\|^2\left(\cos^2\theta + \sin^2\theta\right) = \|u\|^2\|v\|^2.$$

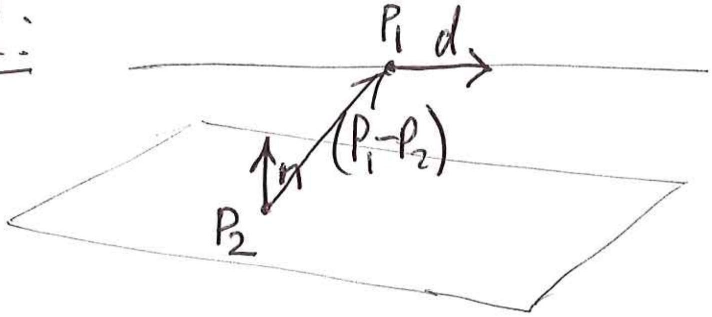
Lagrange: $\|u\cdot v\|^2 + \|u\times v\|^2 = \|u\|^2\|v\|^2$.

Shortest distance between X_1: $x - y + 2z = 2$

$$L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}}_{P_1} + t \underbrace{\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}}_{d}.$$

Case 1: X_1 intersects $L_1 \Rightarrow$ dist $= 0$.

Case 2:



i.e. L_1 is parallel to X_1.

To determine whether Case 1 or Case 2 we need $n \cdot d$.

$$X_1: \; x - y + 2z = 2; \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} - 2 = 0; \quad \to (1 \; -1 \; 2 \,|\, 2)$$

Aim: $n \cdot (x - P_2) = 0$

$$\underbrace{\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}}_{= n} \cdot \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{= P_2} \right) = 0$$

So: $n \cdot d = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 1 + 1 - 2 = 0.$

$$\text{So dist} = \left| \frac{n}{\|n\|} \cdot (P_1 - P_2) \right| = \frac{1}{\sqrt{4+1+1}} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \left[\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]$$

$$= \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \frac{1}{\sqrt{6}} (1 - 1 + 4) = \frac{4}{\sqrt{6}} = \frac{4}{6}\sqrt{6}$$

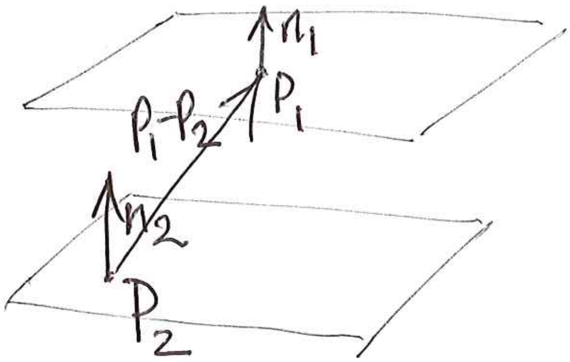
$$= \frac{2}{3}\sqrt{6}.$$

Shortest distance:

$X_1: x - y + 2z = 5 \qquad X_2: 2x - 2y + 4z = 4$

Case 1: X_1 intersects $X_2 \Rightarrow$ dist $= 0$.

Case 2:



I.e. X_1 parallel to X_2

$X_1: x - y + 2z = 5$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} - 5 = 0$$

$$\underbrace{\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}}_{n_1} \cdot \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \underbrace{\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}}_{P_1} \right) = 0$$

$X_2: 2x - 2y + 4z = 4$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} - 4 = 0$$

$$\underbrace{\begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}}_{n_2} \cdot \left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{P_2} \right] = 0$$

$$\text{dist} = \left| \frac{n_2}{\|n_2\|} \cdot (P_1 - P_2) \right| = \frac{1}{\sqrt{4+4+16}} \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} \cdot \left[\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right].$$

$$= \frac{1}{\sqrt{24}} \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{2\sqrt{6}} (2 + 0 + 4) = \frac{3}{\sqrt{6}} = \frac{3}{6}\sqrt{6}$$

$$= \tfrac{1}{2}\sqrt{6}.$$

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ y \\ -x+2y \end{pmatrix} \; ; \quad S\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xy \\ x+y \end{pmatrix}.$$

$$T\left(\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}\right) = T\begin{pmatrix} x+a \\ y+b \end{pmatrix} = \begin{pmatrix} 2(x+a) \\ y+b \\ -(x+a)+2(y+b) \end{pmatrix}.$$

$$T\begin{pmatrix} x \\ y \end{pmatrix} + T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2x \\ y \\ -x+2y \end{pmatrix} + \begin{pmatrix} 2a \\ b \\ -a+2b \end{pmatrix} = \begin{pmatrix} 2(x+a) \\ y+b \\ -(x+a)+2(y+b) \end{pmatrix}$$

so satisfies 1st axiom...