MATH211: Linear Methods I

Matthew Burke

Thursday 6th December, 2018

Lecture on Thursday 6$^{\text{th}}$ December, 2018

Small examples

Examples

Example

Construct an example of a 2×2 matrix with one eigenvalue equal to 3 that is not diagonal, is not invertible but is diagonalisable.

Example

Find a matrix that is not the identity or the zero matrix such that

$$A^2 = A$$

Long examples

Google PageRank theory

Imagine modelling the internet following way:-

- Each page consists of links to other pages.
- We travel from one page to another by clicking a link at random.
- Therefore the probability of going from page j to page i is

$$\frac{L_{ji}}{Out(j)}$$

  where L_{ji} is the number of links from page j to page i and $Out(j)$ is the total number of links from page j.
- A steady state vector for this Markov chain (if it exists) gives the PageRank of the pages.

Google PageRank Example

Example (Simplified PageRank)

Suppose that the internet consists of three(!) pages P_1, P_2 and P_3. Suppose further that:-

- there are two links from P_1 to P_2
- there is one link from P_1 to P_3
- there is one link from P_2 to P_1
- there is one link from P_2 to P_3
- there is one link from P_3 to P_2

Then calculate the (relative) PageRanks for P_1, P_2 and P_3.

Examples

Example

Find the distance d between the lines

$$L_1 : \vec{x} = \begin{bmatrix} -6 \\ -7 \\ 7 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \text{ and } L_2 : \vec{x} = \begin{bmatrix} -7 \\ -14 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

and find a points A on L_1 and B on L_2 such that $d(A, B) = d$.

Examples

Example

Suppose that the sequence x_0, x_1, x_2 ... is defined by $x_0 = 2$, $x_1 = 1 + i$ and $x_{k+2} = (1 + i)x_{k+1} - ix_k$. Find a formula for x_k.

Examples

Example

A draconian government organises its education system as follows:-

- At age 11 all children take an exam.
- The children who pass attend a school of type A and those who fail attend a school of type B.
- Both schools administer weekly tests.
- Consider the fate of a certain individual Adam:-
  - Adam has probability $\frac{2}{3}$ of passing the first exam at age 11.
  - At school type A Adam has a $\frac{2}{5}$ chance of passing every weekly exam.
  - At school type B Adam has a $\frac{4}{5}$ chance of passing every weekly exam.

Model Adam's progress as a Markov chain and find all steady state vectors.

Examples

Example

Find equations for the lines through

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

that meet the line

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

at the two points at distance 3 from

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Examples

Example

Find the scalar equation for the plane passing through the point

$$\begin{bmatrix} 5 \\ -5 \\ 1 \end{bmatrix}$$

and containing the line

$$\vec{x} = \begin{bmatrix} 9 \\ -6 \\ -1 \end{bmatrix} + t \begin{bmatrix} -2 \\ -4 \\ 5 \end{bmatrix}$$

Example

Example

If possible diagonalise

$$\begin{bmatrix} 3 & -4 & 2 \\ 1 & -2 & 2 \\ 1 & -5 & 5 \end{bmatrix}$$

More small examples

Examples

Example

Compute the orthogonal projection of u onto v where

$$u = \begin{bmatrix} -2 \\ -10 \\ -3 \end{bmatrix} \text{ and } u = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$

Examples

Determine the volume of the parallelepiped with one vertex at the origin and with the three adjacent vertices given by:

$$\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} -4 \\ -4 \\ 1 \end{bmatrix}$$