Math 211 2018-10-11

Parametric form: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}.$

$$\left.\begin{aligned} x_1 &= 1 + 2s \\ x_2 &= -1 + s \\ x_3 &= 2 + 3s \end{aligned}\right\} \quad \left.\begin{aligned} \frac{x_1 - 1}{2} &= s \\ x_2 + 1 &= s \\ \frac{x_3 - 2}{3} &= s \end{aligned}\right\}$$

Symmetric form:

$$\frac{x_1 - 1}{2} = x_2 + 1 = \frac{x_3 - 2}{3}$$

System of equations:

$$\left.\begin{aligned} \frac{x_1 - 1}{2} &= x_2 + 1 \\ x_2 + 1 &= \frac{x_3 - 2}{3} \end{aligned}\right\} \quad \left.\begin{aligned} x_1 - 1 &= 2x_2 + 2 \\ 3x_2 + 3 &= x_3 - 2 \end{aligned}\right\} \quad \begin{aligned} x_1 - 2x_2 + 0x_3 &= 3 \\ 0x_1 + 3x_2 - x_3 &= -5. \end{aligned}$$

What is the line through $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ parallel to $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}.$$

Find the intersection of

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$\begin{aligned} x_1 &= 3+t \\ x_2 &= 1-2t \\ x_3 &= 3+3t \end{aligned} \qquad \left.\begin{aligned} x_1 &= 4+2s \\ x_2 &= 6+3s \\ x_3 &= 1+s. \end{aligned}\right\} \quad \begin{aligned} 3+t &= 4+2s \\ 1-2t &= 6+3s \\ 3+3t &= 1+s. \end{aligned}$$

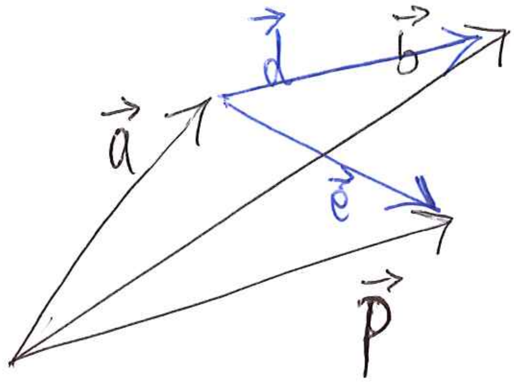
$$\Rightarrow \begin{aligned} t-2s &= 1 \\ -2t-3s &= 5 \\ 3t-s &= -2 \end{aligned} \Rightarrow \left(\begin{array}{cc|c} 1 & -2 & 1 \\ -2 & -3 & 5 \\ 3 & -1 & -2 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & -7 & 7 \\ 0 & 5 & -5 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right)$$

So $t=-1$ and $s=-1$.

point of intersection: $\begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$

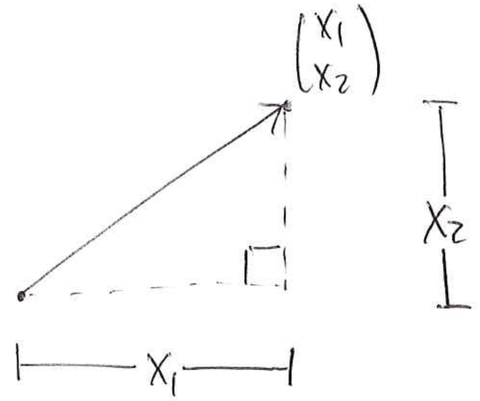
Find the plane passing through $\vec{a}=\begin{pmatrix}1\\0\\1\end{pmatrix}$, $\vec{b}=\begin{pmatrix}1\\2\\0\end{pmatrix}$ and $\vec{p}=\begin{pmatrix}0\\0\\3\end{pmatrix}$.



$$\vec{x} = \vec{a} + s\vec{d} + t\vec{e} = \begin{pmatrix}1\\0\\1\end{pmatrix} + s(\vec{b}-\vec{a}) + t(\vec{p}-\vec{a})$$

$$= \begin{pmatrix}1\\0\\1\end{pmatrix} + s\left[\begin{pmatrix}1\\2\\0\end{pmatrix} - \begin{pmatrix}1\\0\\1\end{pmatrix}\right] + t\left[\begin{pmatrix}0\\0\\3\end{pmatrix} - \begin{pmatrix}1\\0\\1\end{pmatrix}\right]$$

$$= \begin{pmatrix}1\\0\\1\end{pmatrix} + s\begin{pmatrix}0\\2\\-1\end{pmatrix} + t\begin{pmatrix}-1\\0\\2\end{pmatrix}.$$



length of $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ is $\sqrt{x_1^2 + x_2^2}$.

$$d\left(\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}\right) = \sqrt{(1-3)^2 + (-1-1)^2 + (3-0)^2}$$

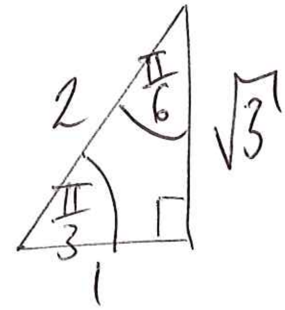
$$= \sqrt{4+4+9} = \sqrt{17}.$$

$$\left\|\begin{pmatrix} -3 \\ 4 \end{pmatrix}\right\| = \sqrt{9+16} = 5$$

$$\left\|\begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}\right\| = \sqrt{9+1+4} = \sqrt{14}.$$

$$\begin{pmatrix}1\\2\\0\\-1\end{pmatrix}\cdot\begin{pmatrix}0\\1\\2\\3\end{pmatrix} = 1\times 0 + 2\times 1 + 0\times 2 + (-1)\times 3 = -1$$

Find the angle between $\begin{pmatrix}1\\0\\-1\end{pmatrix}$ and $\begin{pmatrix}0\\1\\-1\end{pmatrix}$



$$\left.\begin{aligned} u\cdot v &= |u||v|\cos\theta \\ u\cdot v &= 1 \\ |u| &= \sqrt{2} = |v| \end{aligned}\right\} \quad \begin{aligned} \cos\theta &= \frac{1}{2} \\ \Rightarrow \theta &= \frac{\pi}{3}. \end{aligned}$$