

# MATH211: Linear Methods I

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# Lecture on Thursday 6<sup>th</sup> December, 2018

Small examples

Long examples

More small examples

## Small examples

# Examples

## Example

Construct an example of a  $2 \times 2$  matrix with one eigenvalue equal to 3 that is not diagonal, is not invertible but is diagonalisable.

## Example

Find a matrix that is not the identity or the zero matrix such that

$$A^2 = A$$

## Long examples

# Google PageRank theory

Imagine modelling the internet following way:-

- ▶ Each page consists of links to other pages.
- ▶ We travel from one page to another by clicking a link at random.
- ▶ Therefore the probability of going from page  $j$  to page  $i$  is

$$\frac{L_{ji}}{Out(j)}$$

where  $L_{ji}$  is the number of links from page  $j$  to page  $i$  and  $Out(j)$  is the total number of links from page  $j$ .

- ▶ A steady state vector for this Markov chain (if it exists) gives the PageRank of the pages.

# Google PageRank Example

## Example (Simplified PageRank)

Suppose that the internet consists of three(!) pages  $P_1$ ,  $P_2$  and  $P_3$ . Suppose further that:-

- ▶ there are two links from  $P_1$  to  $P_2$
- ▶ there is one link from  $P_1$  to  $P_3$
- ▶ there is one link from  $P_2$  to  $P_1$
- ▶ there is one link from  $P_2$  to  $P_3$
- ▶ there is one link from  $P_3$  to  $P_2$

Then calculate the (relative) PageRanks for  $P_1$ ,  $P_2$  and  $P_3$ .

# Examples

## Example

Find the distance  $d$  between the lines

$$L_1 : \vec{x} = \begin{bmatrix} -6 \\ -7 \\ 7 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \quad \text{and} \quad L_2 : \vec{x} = \begin{bmatrix} -7 \\ -14 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

and find a points  $A$  on  $L_1$  and  $B$  on  $L_2$  such that  $d(A, B) = d$ .



# Examples

## Example

Suppose that the sequence  $x_0, x_1, x_2 \dots$  is defined by  $x_0 = 2$ ,  $x_1 = 1 + i$  and  $x_{k+2} = (1 + i)x_{k+1} - ix_k$ . Find a formula for  $x_k$ .

# Examples

## Example

A draconian government organises its education system as follows:-

- ▶ At age 11 all children take an exam.
- ▶ The children who pass attend a school of type  $A$  and those who fail attend a school of type  $B$ .
- ▶ Both schools administer weekly tests.
- ▶ Consider the fate of a certain individual Adam:-
  - ▶ Adam has probability  $\frac{2}{3}$  of passing the first exam at age 11.
  - ▶ At school type  $A$  Adam has a  $\frac{2}{5}$  chance of passing every weekly exam.
  - ▶ At school type  $B$  Adam has a  $\frac{4}{5}$  chance of passing every weekly exam.

Model Adam's progress as a Markov chain and find all steady state vectors.

# Examples

## Example

Find equations for the lines through

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

that meet the line

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

at the two points at distance 3 from

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

# Examples

## Example

Find the scalar equation for the plane passing through the point

$$\begin{bmatrix} 5 \\ -5 \\ 1 \end{bmatrix}$$

and containing the line

$$\vec{x} = \begin{bmatrix} 9 \\ -6 \\ -1 \end{bmatrix} + t \begin{bmatrix} -2 \\ -4 \\ 5 \end{bmatrix}$$

# Example

## Example

If possible diagonalise

$$\begin{bmatrix} 3 & -4 & 2 \\ 1 & -2 & 2 \\ 1 & -5 & 5 \end{bmatrix}$$

More small examples

# Examples

## Example

Compute the orthogonal projection of  $u$  onto  $v$  where

$$u = \begin{bmatrix} -2 \\ -10 \\ -3 \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$