Variance of Injection Velocity in a Cyclotron Accelerator: Effects on Particle Trajectories and Beam Width

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1 Abstract

For my Python project, I examined the effects on the trajectories of protons inside a cyclotron resulting from a slight variance in the initial injection velocity of the particles. The primary objective was to quantify the effects of such variance on the width of the extracted proton beam, although some other general effects on the orbits are briefly described as well. The investigation was carried out using a numerical computer model that employs the Boris integration method (reference: "Particle Push in Magnetic Field (Boris Method)", Particle in Cell Consulting LLC, https://www.particleincell.com/2011/vxb-rotation/). In accordance with expectation, I found that the final width of the beam increases in tandem with the injection velocity variance, although a few counter-intuitive phenomena were observed. In particular, the ability to extract a well-structured beam in the presence of such variance depends greatly on the position of the extractor, as the trajectories of particles across the velocity spectrum are parallel only in very specific locations along the final orbit. These effects are described in more detail in section four.

2 Introduction and Background

A cyclotron is a type of particle accelerator that uses electric and magnetic fields to accelerate charged particles outwards along a spiral trajectory. The type of simple cyclotron modeled here consists of a circular, disk-shaped chamber containing a uniform magnetic field. The ions are injected near the center, and are accelerated by an electric field that exists across the central region as shown in figure 1. The ions are bent into a semicircular path by the B-field, and are made to cross the E-field region repeatedly. The period of the revolution caused by the B-field can be derived from the Lorentz force law, and has the form $T = \frac{2\pi m}{qB}$. The E-field is varied according to a square-wave input of frequency $f = \frac{1}{T} = \frac{qB}{2\pi m}$ to ensure that each time the particles cross the central region, the E-field is aligned with their velocity, accelerating them. Once the particles reach a desired energy, the beam is extracted. A sample cyclotron trajectory is shown in figure 2. (Reference for general cyclotron basics: Hyperphysics by Dr. Rod Nave, Georgia State University: http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/cyclot.html)

Figure 1: Cyclotron

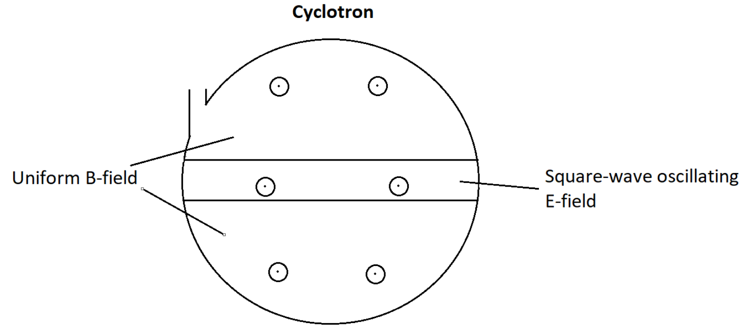
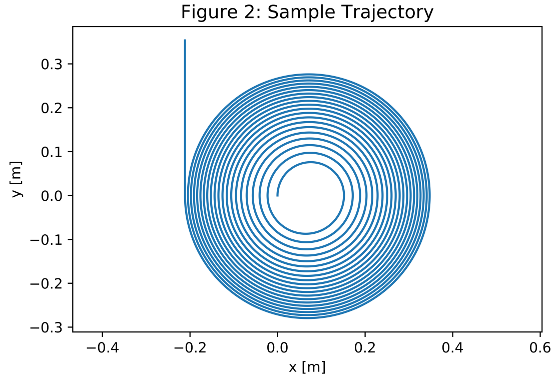


Figure 2: Sample Trajectory



The radius of a particular orbit depends on the proton's current velocity (and by extension, energy) given by $R = \frac{mv}{qB}$. The protons gain energy linearly with time, as each period of rotation brings the beam across the potential difference in the central region twice. The initial expectation was that if the injection velocity (and thus injection energy) is allowed to vary, there would be a corresponding variance in the radius of the final orbit, manifesting ultimately as a nonzero width in the extracted beam.

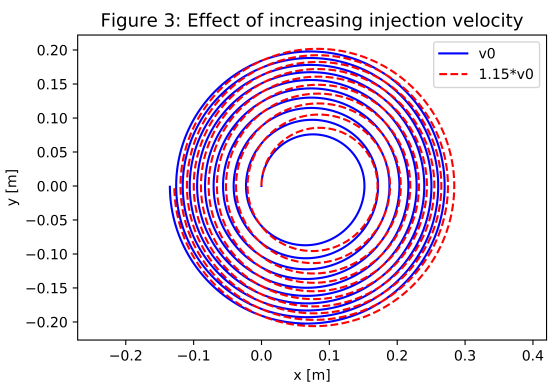
3 Model and Setup

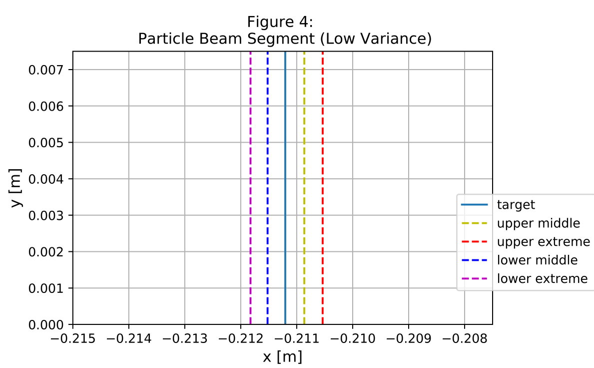
The cyclotron was modeled as a flat, circular disk of radius 0.285m centered approximately at the point (0.0681m, 0m), with the proton injector lying at the origin. The disk contains a uniform magnetic field of magnitude 0.75T directed toward positive z (out of the page). The E-field region is 0.05m across vertically, and the potential difference across it is 50kV. The E-field oscillates according to a square-wave input with a frequency of approximately 11.45MHz, and has a phase offset such that the electric field inverts at ever 1-quarter and 3-quarter period (ie, the electric field inverts each time the y-component of the velocity if the injected protons is 0). The extractor was placed on leftmost point of the disk, so that the extracted beam travels toward +y. The protons were injected with an initial target velocity of $v_0 = 5 * 10^6$m/s (roughly 0.13 MeV), and were extracted after 20 rotation periods at a final velocity of $2.01 * 10^7$m/s (roughly 2.1 MeV). The radius of the final orbit was approximately 0.279m for this idealized target case. Three separate proton beams were simulated in addition to this ideal case. In the first beam, the "low variance" case, the injection velocities varied within 1.25% of the target value. The 2nd beam was the "middle variance" case, where the injection velocities varied within 2.5% of the target value. Finally, a "high variance" case was simulated, with the injection velocities varying within 5% of the target value.

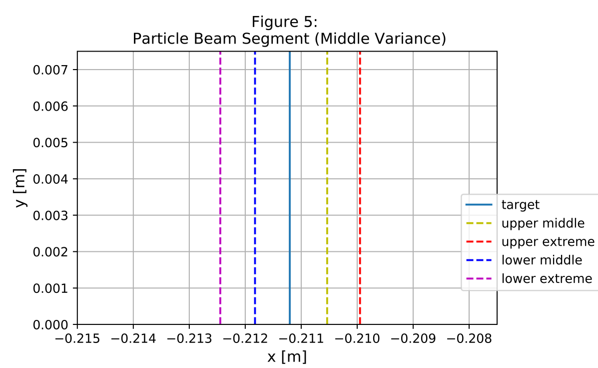
The numerical python model makes use of the Boris integration scheme to simulate the trajectories of particles. Particles in cyclotrons are known to emit radiation as they move, but this is not modeled here for simplicity (reference: "Cyclotron radiation", wikipedia www.en.wikipedia.org/wiki/Cyclotron_radiation). Relativistic effects are also ignored. The loop used to simulate and store the trajectories has a very similar structure to what was used in computer homework 7. The simulation loops for just over 20 rotation periods, with 20,000 iterations per period to ensure accuracy. For extracting the beam, real-world cyclotrons often use electrostatic deflectors or stripping foil (reference: "Injection and extraction for cyclotrons" by W. Kleeven, Ion Beam Applications: https://cds.cern.ch/record/1005057/files/p271.pdf). In this model, the magnetic field simply drops to zero in the region where both $x \leq -0.208m$ and $y \geq 0m$ (corresponding to the far left edge of the disk, the desired extractor position), allowing the beam to travel in a straight-line path for the rest of the simulation as shown in the sample trajectory. Four particle trajectories were simulated for each beam in order to properly characterize the three cases. Two of these particles were chosen to be at the upper and lower extremes of the injection velocity range, while the other two had injection velocities halfway between the target case and each extreme. For instance, in the high variance case, the trajectories simulated had injection velocities of $1.05v_0$, $0.95v_0$, $1.025v_0$, and $0.975v_0$, where v_0 is the target velocity. These trajectories were plotted along with the target trajectory, and this process was carried out for all three beam cases. Finally, I plotted the beam width as a function of the degree of velocity variance. Since the extracted beams were vertically oriented, it suffices to simply subtract the x-coordinate of the leftmost particle in the beam from that of the rightmost to obtain the beam width.

4 Results and Conclusions

In general, it was found that an increase in the injection velocity shifts the center of the spiral trajectory toward the +x direction. Similarly, a decrease in injection velocity shifts the center of the trajectory toward -x. To illustrate this effect, figure 3 shows two sample trajectories, one of which has an injection velocity 15% larger than the other. It is also apparent in this figure that the two trajectories become parallel only as the beam crosses the E-field. As a result of this misalignment of the trajectories, it appears here that any attempt at extracting the beam at a different location than the one described in section 3 could end in failure if significant variance in injection velocity is present, as the velocities of extracted particles may lie at substantially different angles. Additionally, particles may miss the extractor outright. If the extractor is in the position described in section 3, then the consequence of the horizontal shifting effect is a nonzero beam width. Figures 4, 5 and 6 display plots of a segment of the extracted beam in each of the three variance scenarios describes in section3. It can be seen that due to this horizontal shifting effect, the protons that have an injection higher than the target end up on the +x side of the beam, while the protons that had a lower injection velocity than the target were on the -x side. This remains true across all three beams. The largest contributers to width were the most extreme cases in all 3 beams. The low variance beam had a width of approximately 1.34mm, while the middle and high variance beams had widths of 2.48mm and 5.20mm respectively. The middle variance beam is roughly 1.85 times wider than the low-variance beam, and the high-variance beam is roughly 2.1 time wider than the middle-variance beam. The high variance beam is roughly 3.89 times wider than the low variance beam. According to these findings, the relationship between beam width and injection velocity variance is nonlinear, as the slope of the function from 2.5% to 5% is slightly steeper than from 1.25% to 2.5%.







These results indicate That that significant deviation from a target injection velocity is generally undesirable, as it restricts placement of the extractor, and additionally risks portions of the beam missing the extractor all together. Collisions with the walls of the cyclotron may also be a concern with greater velocity variance. The intensity of the beam is also reduced due to the spreading out of the constituent particles. In applications where a very low beam width is required, it should be ensured that the velocities of injected particles are very well-constrained to a specific target.

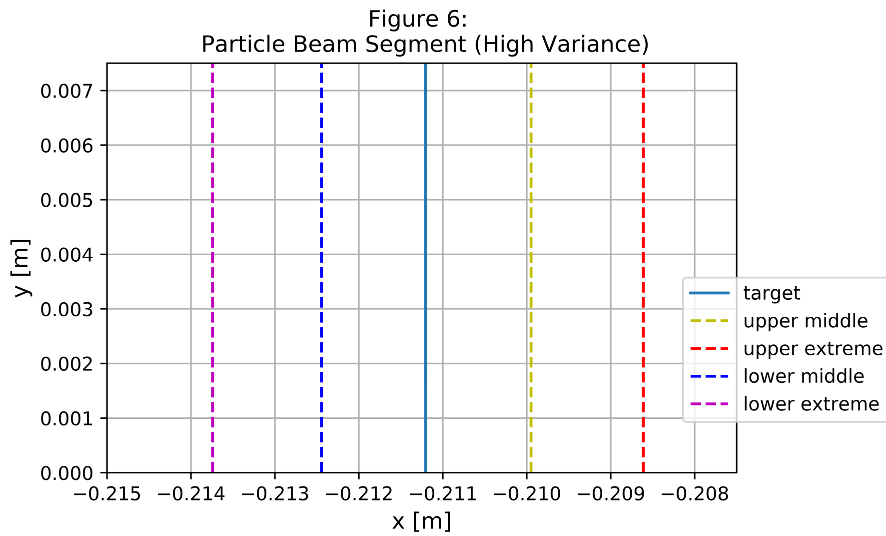


Figure 6:
Particle Beam Segment (High Variance)
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
y [m]
−0.215
−0.214
−0.213
−0.212
−0.211
−0.210
−0.209
−0.208
x [m]
target
upper middle
upper extreme
lower middle
lower extreme

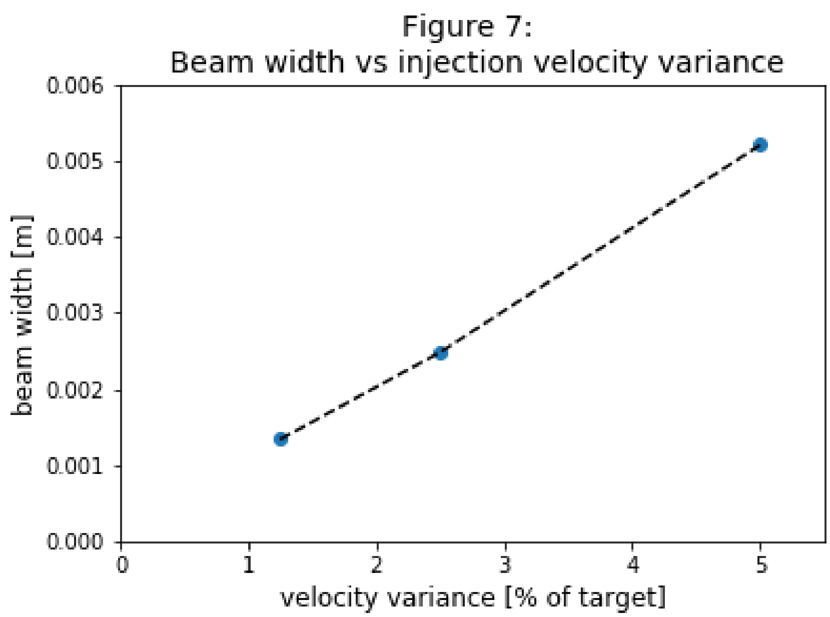


Figure 7:
Beam width vs injection velocity variance
0.006
0.005
0.004
0.003
0.002
0.001
0.000
beam width [m]
0
1
2
3
4
5
velocity variance [% of target]