Contest Design with Interim Types

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Federal Trade Commission

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Introduction

What if principal cannot discriminate

Contestants often differ in ability

- Heterogeneity reduces competitiveness and total effort
- Discrimination in favor of weaker player can correct for heterogeneity
- This requires information about player types

What if principal has this information *but cannot discriminate*

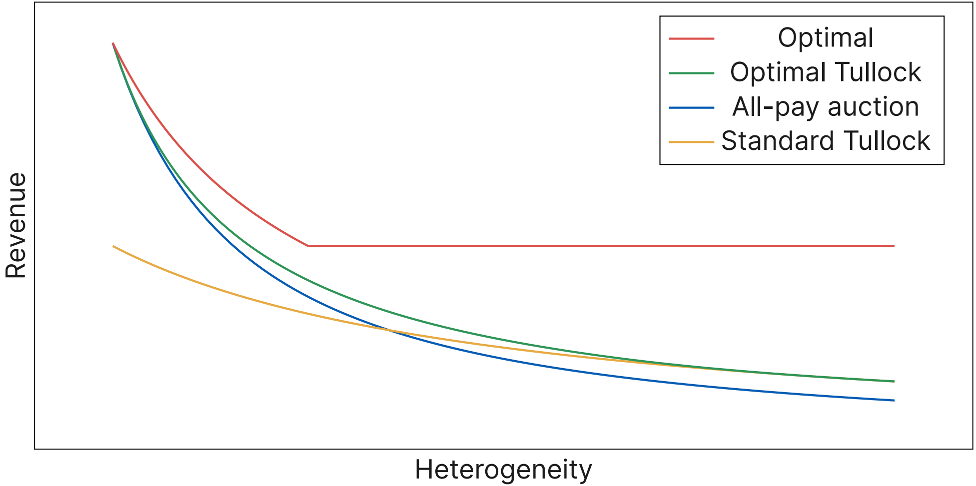
Known types without discrimination is design with interim types

All-knowing designer under anonymity still has interim type distribution

- Knowledge of interim type distribution is *powerful*
- Boring full-surplus extracting revelation mechanism:
    - Principal asks for types
    - Reported types do not match interim distribution $\implies$ collective punishment
    - Extract all surplus
- Argument assumes unlimited liability

Design with interim types and efficiency (type of limited liability)

Revenue from two-player contests



Related literature

"Structural" contest design[1]

- Ewerhart (2017), Franke, Leininger, et al. (2018), and Nti (2004)

Revenue dominance in anonymous, efficient contests

- Epstein et al. (2013), Fang (2002), and Franke, Kanzow, et al. (2014)

[1] This is a large literature. See Mealem and Nitzan (2016) for a review.

Model

Model (1): Setup

- Complete information, two-player[2] contest with unit prize
- Each player submits score $s_i \geq 0$ at linear cost $k_i > 0$ s.t. $k_2 > k_1$
- Principal chooses contest success functions (CSFs) to max expected revenue

$$p_i(s_i, s_{-i}) \in [0, 1]$$

- Solution concept is revenue-maximizing Nash equilibrium

Normalize $k_1 = 1$ and $k_2 = k > 1$ and call k heterogeneity

[2] Extend to n players later

Model (2): Timing

Timing of game is:

1. Types (k_1, k_2) are common knowledge[3]
2. Principal chooses CSFs and announces them to the players
3. Players submit scores (s_1, s_2) simultaneously
4. Player i receives payoff:

$$u_i(s_i; s_{-i}) = p_i(s_i, s_{-i}) - k_i s_i$$

[3] We restrict principal's use of information so knowledge of distribution is sufficient

Model (3): Restrictions

Two restrictions on principal's CSF:

Definition (Anonymous)
$p_1(x, y) = p_2(x, y)$ for all $x, y \geq 0$.

Definition (Efficient)
$p_1(x, y) + p_2(y, x) = 1$ for all $x, y \geq 0$.

Results

Full surplus extraction with anonymity or efficiency alone

Note: full surplus is one which requires $s_1 = 1$ and $s_2 = 0$

If not efficient,

- Principal sets reserve score of 1

If not anonymous,

- Principal allocates to Player 2 unless $s_1 \geq 1$

No full surplus extraction with anonymity and efficiency

No anonymous, efficient CSF can extract full surplus

- Both players must have payoff zero and $s_1 = 1$, $s_2 = 0$
- Player 1 has profitable deviation because $p(0, 0) = 0.5$

Yet to demonstrate one cannot get arbitrarily close to full surplus extraction[4]

[4] In fact, with $n > 2$ players and $m < n - 1$ prizes, principal can get arbitrarily close

When heterogenity low, optimal is APA with bid caps

If $k \leq 2$, optimal anonymous, efficient contest

- Implementable using all-pay auction with bid cap at $\frac{1}{2k}$

$$p(x, y) = \begin{cases} 1 & \text{if } \frac{1}{2k} \geq x > y \text{ or } y > \frac{1}{2k} \\ \frac{1}{2} & \text{if } x = y \\ 0 & \text{if } \frac{1}{2k} \geq y > x \text{ or } x > \frac{1}{2k} \end{cases}$$

- Both players score $\frac{1}{2k}$ and split prize

Optimal to extract effort from both players because heterogeneity is low

When heterogenity high, optimal is difference form

If $k \geq 2$, optimal anonymous, efficient contest

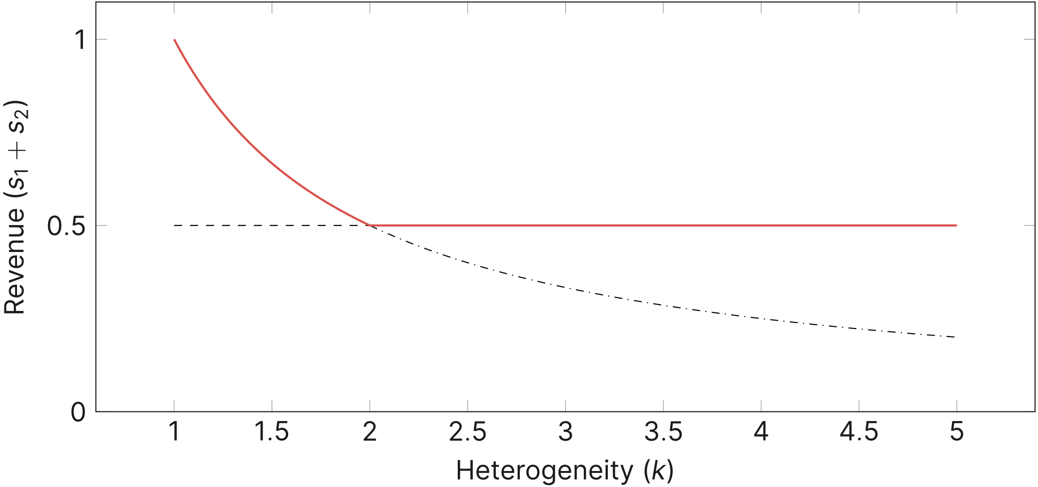
- Implementable using difference-form contest

$$p(x,y) = \begin{cases} 1 & \text{if } x - y > \frac{1}{2} \\ \frac{1}{2} + x - y & \text{if } x - y \in \left[-\frac{1}{2}, \frac{1}{2}\right] \\ 0 & \text{if } x - y < -\frac{1}{2}. \end{cases}$$

- Player 1 scores $\frac{1}{2}$ and Player 2 scores zero

Not worth extracting effort from Player 2 because heterogeneity is high

Two Contests that Maximize Revenue



More players

Only interesting with one fewer prizes than players

If $m < n - 1$ prizes:

- Request $\frac{1-\epsilon}{k_i}$ effort from players 1 to m for $1 - \epsilon$ of prize
- Request $\frac{m\epsilon}{k_{m+1}}$ from Player $m + 1$ for $m\epsilon$ of prize
- At least one player has no prize
- **If player imitates another, give both prizes to players with unique scores**

Arbitrarily close to full surplus extraction

Three players and two prizes has all interesting attributes of n players

Optimal anonymous, efficient mechanism obtains revenue

$$\begin{cases} \frac{1}{2k_1} + \frac{1}{2k_2} & \text{if } \frac{k_3}{k_2} \geq 3 \\ \frac{1}{2k_1} + \frac{3}{2k_3} & \text{if } \frac{k_3}{k_2} \leq 3 \leq \frac{k_3}{k_1} \\ \frac{3}{k_3} & \text{if } \frac{k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \geq 2 \\ \frac{1}{k_1} + \frac{3 - k_3/k_1}{2k_2} & \text{if } \frac{k_3}{k_1} \leq 3 \leq \frac{k_2+k_3}{k_1} \text{ and } \frac{k_3}{k_2} \leq 2 \\ \frac{6 - \frac{k_2+k_3}{k_1}}{2k_1} & \text{if } \frac{k_2+k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \leq 2 \end{cases}$$

Similar to the two player case, *no prize for Player 3*

Three players and two prizes has all interesting attributes of n players

Optimal anonymous, efficient mechanism obtains revenue

$$\begin{cases} \frac{1}{2k_1} + \frac{1}{2k_2} & \text{if } \frac{k_3}{k_2} \geq 3 \\ \frac{1}{2k_1} + \frac{3}{2k_3} & \text{if } \frac{k_3}{k_2} \leq 3 \leq \frac{k_3}{k_1} \\ \frac{3}{k_3} & \text{if } \frac{k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \geq 2 \\ \frac{1}{k_1} + \frac{3 - k_3/k_1}{2k_2} & \text{if } \frac{k_3}{k_1} \leq 3 \leq \frac{k_2+k_3}{k_1} \text{ and } \frac{k_3}{k_2} \leq 2 \\ \frac{6 - \frac{k_2+k_3}{k_1}}{2k_1} & \text{if } \frac{k_2+k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \leq 2 \end{cases}$$

Similar to the two player case, *split one prize between Player 2 and Player 3*

Three players and two prizes has all interesting attributes of n players

Optimal anonymous, efficient mechanism obtains revenue

$$\begin{cases} \frac{1}{2k_1} + \frac{1}{2k_2} & \text{if } \frac{k_3}{k_2} \geq 3 \\ \frac{1}{2k_1} + \frac{3}{2k_3} & \text{if } \frac{k_3}{k_2} \leq 3 \leq \frac{k_3}{k_1} \\ \frac{3}{k_3} & \text{if } \frac{k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \geq 2 \\ \frac{1}{k_1} + \frac{3 - k_3/k_1}{2k_2} & \text{if } \frac{k_3}{k_1} \leq 3 \leq \frac{k_2+k_3}{k_1} \text{ and } \frac{k_3}{k_2} \leq 2 \\ \frac{6 - \frac{k_2+k_3}{k_1}}{2k_1} & \text{if } \frac{k_2+k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \leq 2 \end{cases}$$

Give half of Player 1 and Player 2's prize to Player 3

Three players and two prizes has all interesting attributes of n players

Optimal anonymous, efficient mechanism obtains revenue

$$\begin{cases} \frac{1}{2k_1} + \frac{1}{2k_2} & \text{if } \frac{k_3}{k_2} \geq 3 \\ \frac{1}{2k_1} + \frac{3}{2k_3} & \text{if } \frac{k_3}{k_2} \leq 3 \leq \frac{k_3}{k_1} \\ \frac{3}{k_3} & \text{if } \frac{k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \geq 2 \\ \frac{1}{k_1} + \frac{3 - k_3/k_1}{2k_2} & \text{if } \frac{k_3}{k_1} \leq 3 \leq \frac{k_2+k_3}{k_1} \text{ and } \frac{k_3}{k_2} \leq 2 \\ \frac{6 - \frac{k_2+k_3}{k_1}}{2k_1} & \text{if } \frac{k_2+k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \leq 2 \end{cases}$$

IR binding for *Player 2*, transfer *half* Player 2's prize and some of Player 3's

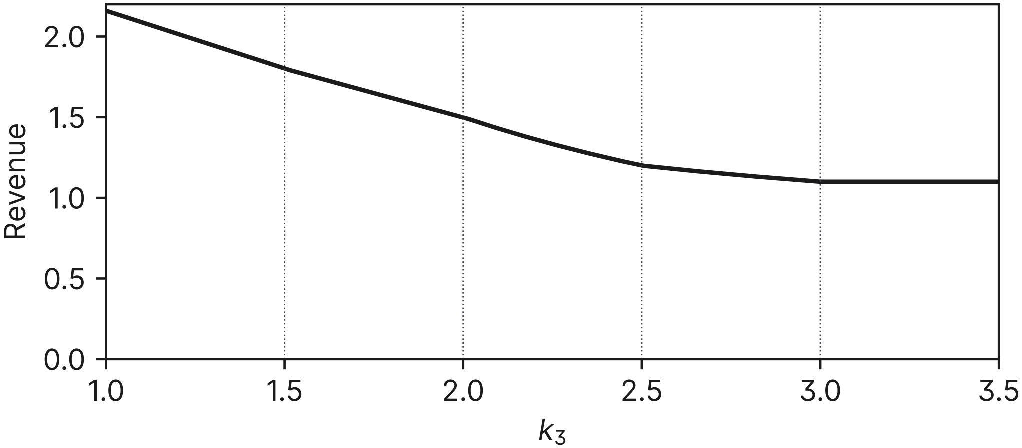
Three players and two prizes has all interesting attributes of n players

Optimal anonymous, efficient mechanism obtains revenue

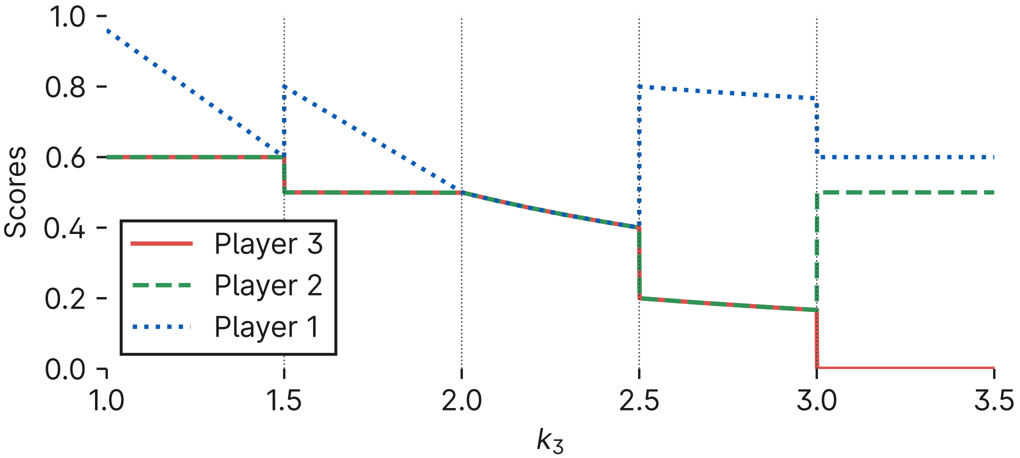
$$\begin{cases} \frac{1}{2k_1} + \frac{1}{2k_2} & \text{if } \frac{k_3}{k_2} \geq 3 \\ \frac{1}{2k_1} + \frac{3}{2k_3} & \text{if } \frac{k_3}{k_2} \leq 3 \leq \frac{k_3}{k_1} \\ \frac{3}{k_3} & \text{if } \frac{k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \geq 2 \\ \frac{1}{k_1} + \frac{3 - k_3/k_1}{2k_2} & \text{if } \frac{k_3}{k_1} \leq 3 \leq \frac{k_2+k_3}{k_1} \text{ and } \frac{k_3}{k_2} \leq 2 \\ \frac{6 - \frac{k_2+k_3}{k_1}}{2k_1} & \text{if } \frac{k_2+k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \leq 2 \end{cases}$$

IR binding for *everyone*, transfer *some* of players 1 and 2's prize to Player 3

Revenue from three-player contests ($k_1 = 5/6$ and $k_2 = 1$)



Scores from three-player contests ($k_1 = 5/6$ and $k_2 = 1$)



Thank You!

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Appendix