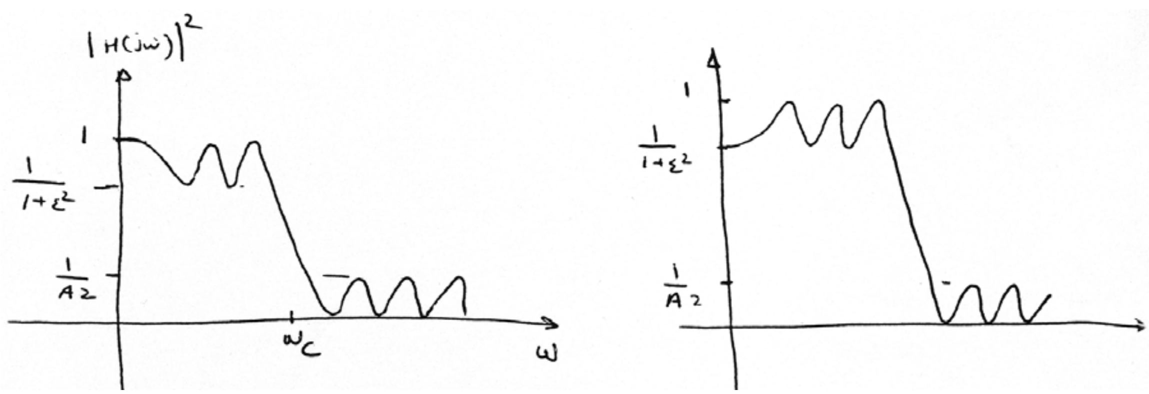
Elliptic Filters

The magnitude squared frequency response of the normalized low-pass elliptic filter of order n is defined by

$$\left|H_n(j\omega)\right|^2 = \frac{1}{1+\varepsilon^2 R_n^2(\omega)}$$

where $R_n(\omega)$ is a Chebyshev rational function of ω determined from the specified ripple characteristics

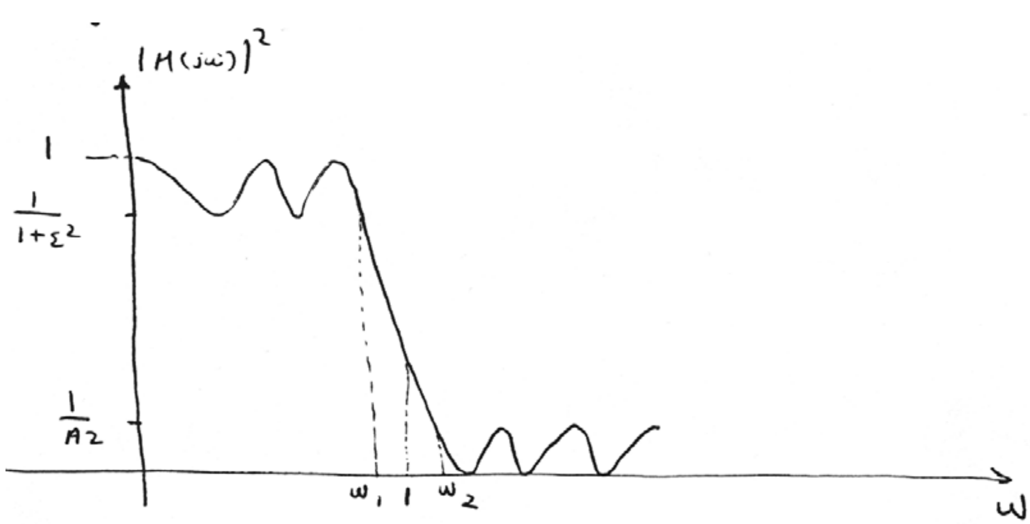


n odd **n even**

Figure 1: Magnitude squared frequency response of elliptic LP filters of odd and even orders.

Unlike the Butterworth and Chebyshev filters, $\omega = 1$ is calculated using

$$\sqrt{\omega_1 \omega_2} = 1$$



A parameter, ω_r representing the sharpness of the transition region is defined as

$$\omega_r = \frac{\omega_2}{\omega_1}$$

Thus a large value of ω_r indicates a large transition band, while a small value of ω_r indicates a small transition band.

The general transfer function $H_n(s)$, for the normalized low-pass elliptic filter is given from odd and even n by

$$H_n(s) = \frac{H_0}{(s+s_0)} \prod_{i=1}^{\frac{n-1}{2}} \frac{s^2 + A_{0i}}{s^2 + B_{1i}s + B_{0i}}, \text{ odd n}$$

$$H_n(s) = H_0 \prod_{i=1}^{\frac{n}{2}} \frac{s^2 + A_{0i}}{s^2 + B_{1i}s + B_{0i}}, \text{ even n}$$

To design a filter $n, H_0, S_0, A_{0i}, B_{1i}, B_{0i}$ have to be determined from the design specifications

(1) ε

(2) A

(3) ω_r

or equivalently G_1, G_2 and ω_r where

$$G_1 = 20\log\left[\frac{1}{\sqrt{(1+\varepsilon^2)}}\right] = 20\log\left|H_n(j\omega_1)\right|$$

$$G_2 = 20\log\left[\frac{1}{A^2}\right] = 20\log\left|H_n(j\omega_2)\right|$$

By finding the G_1 and G_2 in this fashion, the ω_r requirement will not be satisfied exactly; however, an ω_r can be selected that exceeds the requirements.

Determination of n for Normalized Elliptic Filters

The design of a low-pass normalized elliptic filter to satisfy the specifications G_1, G_2 and ω_r using Table 3.6 is straight forward.

(1) Find G_1 and G_2 in dB.
(2) Find the ω_r portion of the table for which a value less than the determined ω_r is found.
(3) Find the corresponding order n.
(4) The values of ω_1 and ω_2 are obtained by

$$\omega_1 = \sqrt{\frac{1}{\omega_r}} \qquad \text{and} \qquad \omega_2 = \sqrt{\omega_r}$$

Thus it can be clearly verified that the requirements have been met.

The design of an un-normalized elliptic low-pass filter satisfying a G_1 dB ripple, cutoff at ω_1' and a G_2 dB gain at ω_2' can be obtained by LP ->LP transformation of a mutable normalized elliptic filter

$$H(s) = H_{LP}(s)|_{s \to \frac{s}{\omega_0}}$$

For the elliptic filter ω_0 is selected to be the geometric mean of ω_1' and ω_2'

$$\omega_0 = \sqrt{\omega_1' \omega_2'}$$

The corresponding LP requirements, ω_1, ω_2 and ω_r (transition) are obtained as follows

$$\omega_1 = \frac{\omega_1'}{\omega_0}$$

$$\omega_2 = \frac{\omega_2'}{\omega_0}$$

$$\omega_r = \frac{\omega_2}{\omega_1} = \frac{\omega_2'}{\omega_1'}$$

Example #1

Find the transfer function for an elliptic low-pass filter with –2 dB cutoff value at 10,000 rad/s and a stop band attenuation of 40 dB for all ω past 14,400 rad/s.

Solution

$$\omega_1' = 10{,}000 rad/s \qquad G_1 = -2dB$$
$$\omega_2' = 14{,}400 rad/s \qquad G_2 = -40dB$$

Then

$$\omega_0 = \sqrt{\omega_2'\omega_1'} = \sqrt{(1*10^4)(1.44*10^4)} = 12{,}000$$
$$\omega_1 = \frac{\omega_1'}{\omega_0} = \frac{10{,}000}{12{,}000} = \frac{5}{6}$$
$$\omega_2 = \frac{\omega_2'}{\omega_0} = \frac{14{,}400}{12{,}000} = \frac{6}{5}$$
$$\omega_r = \frac{\omega_2}{\omega_1} = \frac{\frac{6}{5}}{\frac{5}{6}} = 1.44$$

From the –2 dB and –40 dB part of the table it is seen that for n = 4 given ω_r = 1.40542. Thus

$$H_{LP}(s) = \frac{0.01(s^2 + 7.25202)(s^2 + 1.57676)}{(s^2 + 0.467290s + 0.212344)(s^2 + 0.127954s + 0.677934)}$$

The required LP is obtained by

$$H(s) = H_{LP}(s)\Big|_{s \to \frac{s}{12{,}000}}$$

Example #2

Find the transfer function H(s) for a normalized elliptic filter that will satisfy the following conditions:

$$G_1 = -0.5dB$$
$$G_2 = -30dB$$
$$\omega_r = 1.21$$

From table 3.6a with pass-band ripple of –0.5 dB and stop-band gain of –30 dB, the smallest value of n that satisfies the ω_r is n = 5, giving ω_r = 1.12912.

$$H_5(s) = \frac{0.118807(s^2 + 2.14490)(s^2 + 1.18122)}{(s + 0.511701)(s^2 + 0.480774s + 0.648724)(s^2 + 0.088080s + 0.907216)}$$

Where

$$\omega_1 = \frac{1}{\sqrt{\omega_r}} = 0.941087$$
$$\omega_2 = \sqrt{\omega_r} = \sqrt{1.12912} = 1.06260$$