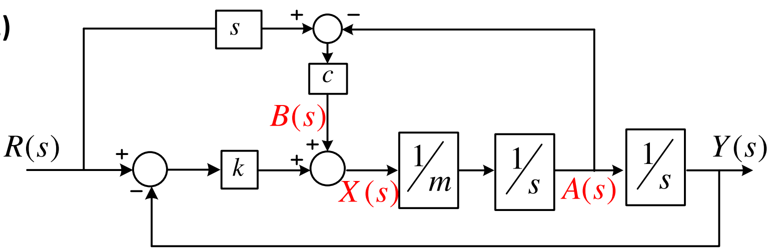
**Otomatik Kontrol Vize Sınavı** **11.04.2013**

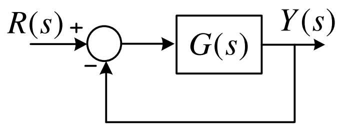
**S-1)**




Yanda blok diyagramı verilen sistem için;

a) $\frac{Y(s)}{R(s)}=?$

b) sisteme ait blok diyagram




şeklinde düzenlenmek istendiğindeG(s)=?

**a)**

1.yol, Doğrudan blok diyagramı üzerinden

1-) $B(s)=c(sR(s)-A(s))$

2-) $X(s)=B(s)+k(R(s)-Y(s))$

3-) $A(s)=\dfrac{X(s)}{sm}\rightarrow X(s)=smA(s)$

4-) $Y(s)=\dfrac{A(s)}{s}\rightarrow A(s)=sY(s)$

5-) 4 numaralı denklem 1'de yazılır ise $B(s)=c\left(sR(s)-sY(s)\right)$

6-) 4 ve 5 numaralı denklemler 2'de yazılır ise

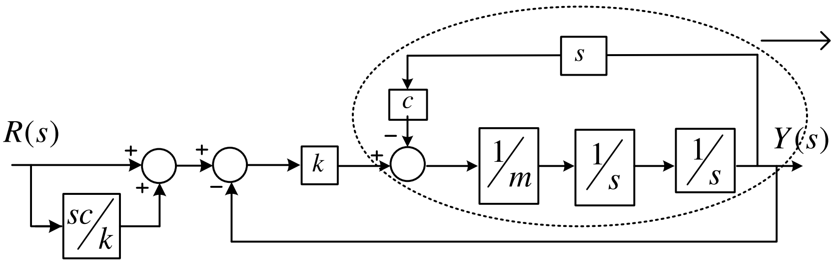
$$sm\left(sY(s)\right)=c\left(sR(s)-sY(s)\right)+k\left(R(s)-Y(s)\right)$$

$$s^2mY(s)=scR(s)-scY(s)+kR(s)-kY(s)$$

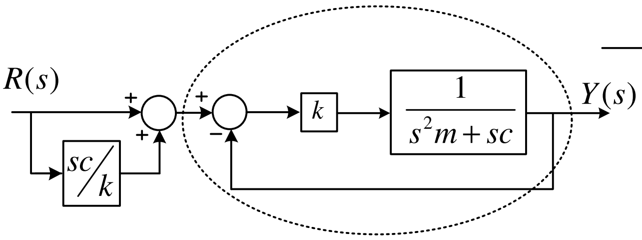
$$Y(s)\left(s^2m+sc+k\right)=R(s)\left(sc+k\right)$$

$$\frac{Y(s)}{R(s)}=\frac{sc+k}{s^2m+sc+k}$$

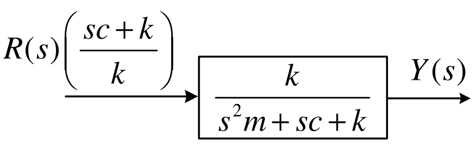
2.yol,




$$\longrightarrow \frac{1}{s^2m+sc}$$

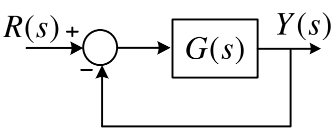



$$\longrightarrow \frac{k}{s^2m+sc+k}$$



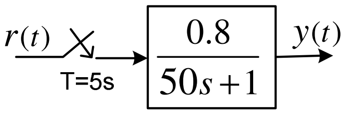

$$\rightarrow \frac{Y(s)}{R(s)}=\frac{sc+k}{s^2m+sc+k}$$

**b)**




$$\rightarrow \frac{Y(s)}{R(s)}=\frac{G(s)}{1+G(s)}=\frac{sc+k}{s^2m+sc+k}\rightarrow G(s)=\frac{sc+k}{s^2m}$$

**S-2)**




$r(t) = 5u(t)$ basamak giriş için;

i) $Y(z)$ ifadesini elde ediniz.

ii) $y(kT) = y(k) = ?$ elde ediniz.

ii) $y(kT)$'nin son değerini $y(\infty) = \lim_{k\to\infty} y(kT)$ ve $y(\infty) = \lim_{z\to 1}(z-1)Y(z)$ ifadelerinden ayrıayrı hesaplayınız.

**i)** $Y(s) = R^*(s)G(s) \rightarrow Y^*(s) = R^*(s)G^*(s) \rightarrow Y(z) = R(z)G(z)$

$$r(t) = 5u(t) \rightarrow R(z) = 5\frac{z}{z-1}$$

$$G(z) = Z\{G(s)\}_{T=5s} = Z\left\{\frac{0.8}{50s+1}\right\}_{T=5s} = Z\left\{\frac{0.8/50}{s+1/50}\right\}_{T=5s} = \frac{0.8}{50}Z\left\{\frac{1}{s+1/50}\right\}_{T=5s}$$

$$G(z) = \frac{0.8}{50}\left[\left. \left(s+1/50\right)\frac{1}{s+1/50}\frac{z}{z-e^{sT}}\right|_{s=-1/50}\right]_{T=5s} = \frac{0.016z}{z-0.905}$$

$$Y(z) = R(z)G(z) = 5\frac{z}{z-1}\frac{0.016z}{z-0.905} = \frac{0.08z^2}{(z-1)(z-0.905)}$$

**ii)** 1.yol rezidü yöntemi kullanılarak,

$$y(k) = (z-1)\left.\frac{0.08z^2}{(z-1)(z-0.905)}z^{k-1}\right|_{z=1} + (z-0.905)\left.\frac{0.08z^2}{(z-1)(z-0.905)}z^{k-1}\right|_{z=0.905}$$

$$y(k) = 0.842 - 0.762 * 0.905^k$$

2.yolbasit kesirlere ayırma yöntemi kullanılarak,

$$\frac{Y(z)}{z} = \frac{0.08z}{(z-1)(z-0.905)} = \frac{A}{(z-1)} + \frac{B}{(z-0.905)} \Rightarrow A = 0.842, B = -0.762$$

$$\frac{Y(z)}{z} = \frac{0.842}{z-1} - \frac{0.762}{z-0.905} = \frac{0.842z}{z-1} - \frac{0.762z}{z-0.905} \quad \rightarrow y(k) = 0.842 - 0.762 * 0.905^k$$

**iii)** $y(\infty) = \lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1}(z-1)\frac{0.08z^2}{(z-1)(z-0.905)} = \frac{0.08*1^2}{(1-0.905)} = 0.842$

$$y(\infty) = \lim_{k\to\infty} y(k) = \lim_{k\to\infty}\left(0.842 - 0.762 * 0.905^k\right) = 0.842$$

3.SORU BURDA

**S-3)**

**a) (10p)**

$$u_{ort}(\mathrm{t}) = K \cdot u(\mathrm{t})$$

$u_{ort}(\mathrm{t}) = Ri_c(\mathrm{t}) + \mathrm{V_c}(\mathrm{t}) \;\; ve \;\; i_c(\mathrm{t}) = C(\mathrm{t}) \cdot \dfrac{d\,\mathrm{V_c}(\mathrm{t})}{dt}$ olduğuna göre;

$u_{ort}(\mathrm{t}) = R \cdot C(\mathrm{t}) \cdot \dfrac{d\,\mathrm{V_c}(\mathrm{t})}{dt} + \mathrm{V_c}(\mathrm{t})$ olarak elde edilir. Buradan;

$\dfrac{d\,\mathrm{V_c}(\mathrm{t})}{dt} = \dfrac{u_{ort}(\mathrm{t}) - \mathrm{V_c}(\mathrm{t})}{RC(\mathrm{t})}$ burada $C(\mathrm{t}) = \dfrac{k_c}{V_c(\mathrm{t})}$ soruda belirtildiği gibi denklemde yerine koyulduğunda;

$\dfrac{d\,\mathrm{V_c}(\mathrm{t})}{dt} = \dfrac{u_{ort}(\mathrm{t}) - \mathrm{V_c}(\mathrm{t})}{R\dfrac{k_c}{V_c(\mathrm{t})}} = u_{ort}(\mathrm{t})\dfrac{\mathrm{V_c}(\mathrm{t})}{Rk_c} - \dfrac{\mathrm{V_c}(\mathrm{t})^2}{Rk_c}$ olarak elde edilir.

$f_1 = \dfrac{d\,\mathrm{V_c}(\mathrm{t})}{dt} = u_{ort}(\mathrm{t})\dfrac{\mathrm{V_c}(\mathrm{t})}{Rk_c} - \dfrac{\mathrm{V_c}(\mathrm{t})^2}{Rk_c}$ lineer olmayan diferansiyel denklem.

**b) (5p)**

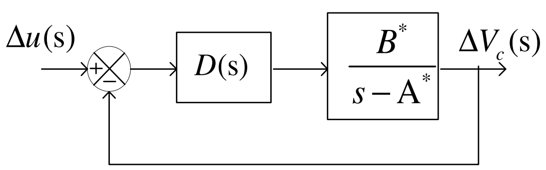
$\mathrm{V_c}(\mathrm{t}) = \mathrm{V_0}$ **olmak üzere;**

$$A^* = \left[\frac{\partial f_1}{\partial V_c(\mathrm{t})}\right]_{V_0, u_0} = \left[\frac{\partial\left(u_{ort}(\mathrm{t})\dfrac{\mathrm{V_c}(\mathrm{t})}{Rk_c} - \dfrac{\mathrm{V_c}(\mathrm{t})^2}{Rk_c}\right)}{\partial V_c(\mathrm{t})}\right] = \frac{u_0}{Rk_c} - \frac{2V_0}{Rk_c} \quad \boxed{A^* = \frac{u_0 - 2V_0}{Rk_c}}$$

$$B^* = \left[\frac{\partial f_1}{\partial u(\mathrm{t})}\right]_{V_0, u_0} = \left[\frac{\partial\left(u_{ort}(\mathrm{t})\dfrac{\mathrm{V_c}(\mathrm{t})}{Rk_c} - \dfrac{\mathrm{V_c}(\mathrm{t})^2}{Rk_c}\right)}{\partial u(\mathrm{t})}\right] = \frac{V_0}{Rk_c} \quad \boxed{B^* = \frac{V_0}{Rk_c}}$$

**c) (10p)**

$s\Delta V_c(\mathrm{s}) = \mathrm{A}^* \, \Delta V_c(\mathrm{s}) + B^* \Delta u(\mathrm{s}) \quad \boxed{\dfrac{\Delta V_c(\mathrm{s})}{\Delta u(\mathrm{s})} = \dfrac{B^*}{s - \mathrm{A}^*}}$ $D(\mathrm{s})$ kontrolör olmak üzere sürekli-zaman kapalı çevrim kontrol blok diyagramını çiziniz.




Sürekli-zaman kapalı çevrim kontrol blok diyagramını

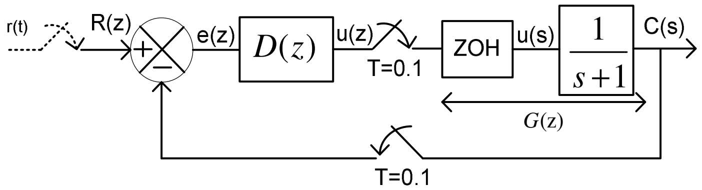
**S-4)**I.dereceden zaman sabiti 1 ve açık-çevrim kazancı 1 olan sistemin transfer fonksiyonuG(s) aşağıda belirtildiği gibi elde edilir;

$$\tau = 1; K = 1$$

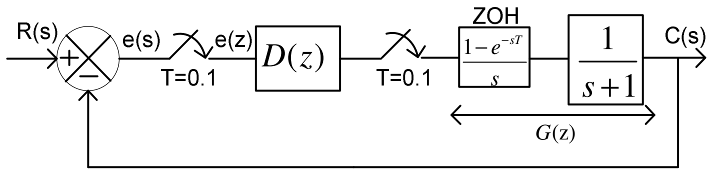
$$G(\mathrm{s}) = \frac{K}{\tau s + 1} = \frac{1}{s+1}$$

**a)(10p)**

Sistem, sayısal kontrolör "**D(z)**" ile kontrol edildiği için kapalı-çevrim kontrol blok diyagram; **<u>ayrık-zaman</u>** kapalı çevrim kontrol blok diyagram olarak çizilmelidir.




Veya




Şeklinde çizilebilir.

**b) (10p)**

<u>Ayrık-zaman açık-çevrim transfer fonksiyonu</u>: T=0.1

$$A\c{C}TF = D(\mathrm{z})\,\mathrm{G(z)}; D(\mathrm{z}) = K$$

$$G(\mathrm{z}) = \mathrm{Z}\left\{\frac{1-e^{-sT}}{s}\cdot\frac{1}{s+1}\right\} = (1-\mathrm{z}^{-1})\,\mathrm{Z}\left\{\frac{1}{s(s+1)}\right\}$$

$$G(\mathrm{z}) = \frac{z-1}{z}\left\{\frac{1}{s(\mathrm{s}+1)}\,s\,\frac{z}{z-e^{sT}}\Bigg|_{s=0} + \frac{1}{\mathrm{s}(s+1)}(s+1)\frac{z}{z-e^{sT}}\Bigg|_{s=-1}\right\} \text{ T=0.1}$$

$$G(\mathrm{z}) = \frac{z-1}{z}\left\{\frac{z}{z-1} - \frac{z}{z-e^{-0.1}}\right\} = 1 - \frac{z}{z-0.9048}$$

$$G(\mathrm{z}) = \frac{0.0952}{z-0.9048}$$

$$\boxed{A\c{C}TF = \mathrm{K}\frac{0.0952}{z-0.9048}}$$ olarak elde edilir.

**c)(5p)**

<u>Ayrık-zaman kapalı çevrim transfer fonksiyonu</u>;

$$T(\mathrm{z}) = \frac{C(\mathrm{z})}{R(\mathrm{z})} = \frac{D(\mathrm{z})\cdot\mathrm{G(z)}}{1 + D(\mathrm{z})\cdot\mathrm{G(z)}}$$ olarak hesap edilmelidir.

Buradan;

$$\mathrm{D(z)}\cdot\mathrm{G(z)} = \mathrm{K}\frac{0.0952}{z-0.9048}$$ olduğuna göre (bkz 'b' şıkkı);

$$T(\mathrm{z}) = \frac{\mathrm{K}\dfrac{0.0952}{z-0.9048}}{1+\mathrm{K}\dfrac{0.0952}{z-0.9048}} = \boxed{\frac{0.0952K}{z-0.9048+0.0952K}}$$ olarak elde edilir.