FİZİK-I

Sakarya Üniversitesi

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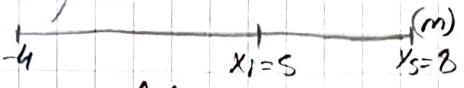
MUSAB UĞUR
150100037

TEK BOYUTTA HAREKET

Yerdeğiştirme (Vektörel) $\Delta\vec{X} = \vec{X}_s - \vec{X}_i$



$\Delta X = -9m$

Ortalama hız (Vektörel) $\vec{v} = \frac{\Delta\vec{X}}{\Delta t}$ (m/s)

Ortalama Sürat (skaler) = $\frac{\text{Toplam alınan yol}}{\text{zaman}}$

Anlık Hız $\Rightarrow \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{X}}{\Delta t} = \frac{d\vec{x}}{dt}$ → sıfıra giderken zamanda sapma olmadığı için anlık

Ortalama İvme $\Rightarrow \vec{a} = \frac{v_s - v_i}{t_s - t_i} = \frac{\Delta v}{\Delta t}$ (m/s^2)

Anlık İvme $\Rightarrow \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$ $\vec{a} = \frac{d\vec{v}}{dt}$ $\vec{a} = \frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right)$ $\vec{a} = \frac{d^2\vec{x}}{dt^2}$

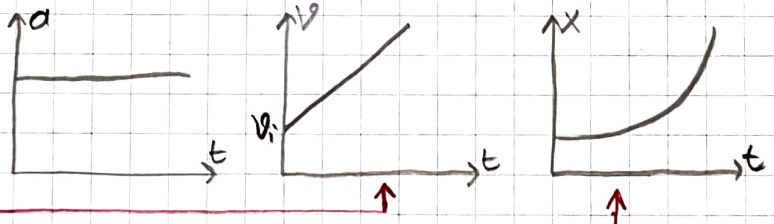
Sabit İvmeli Hareket

$t = 0 \Rightarrow v_i$

$t \Rightarrow v_s$

$a = \frac{v_s - v_i}{t}$

$v_s = v_i + at$



$v = \frac{v_s + v_i}{2}$ $\vec{v} = \frac{x_s - x_i}{t - 0}$

$x_s = x_i + v.t$

$x_s = x_i + \left(\frac{v_s + v_i}{2}\right).t$

$x_s = x_i + \left(\frac{v_i + a.t + v_i}{2}\right).t$

$x_s = x_i + v_i.t + \frac{1}{2}at^2$

$a = \frac{dv}{dt}$

$v = \int_{t_i=0}^{t_s=t} a\,dt$

$v = at\Big|_0^t$

$v = v_i + at$ $v = \frac{dx}{dt}$

$x = \int v\,dt$ $x = \int_0^t (v_i + at)\,dt$

$x = x_i + v_i t + \frac{1}{2}at^2$

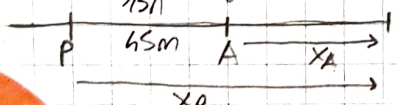
$\Delta = b^2 - 4ac = 900 + 120 = 1020$

$t_{1,2} = \frac{30 \mp \sqrt{1020}}{2}$

$t \simeq \frac{30 + 32}{2} \simeq 31s$

⇒ A → 45m/s, P

Araç geçtikten 1sn sonra polis 3m/s² ivmeyle hareket ediyor. Kaç sn sonra yakalar.



$x_A = 45.t$

$x_p = 0 + 0 + \frac{1}{2}.3.t^2$

$x_p = x_A + 45m$

$\frac{3}{2}t^2 = 45 + 45t$

$t^2 - 30t - 30 = 0$

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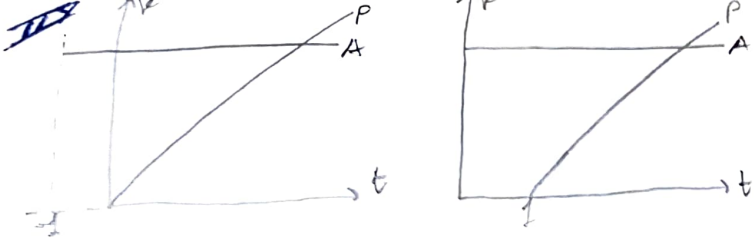
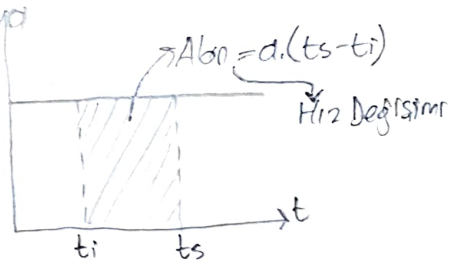
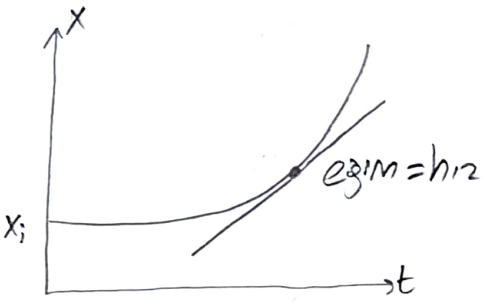


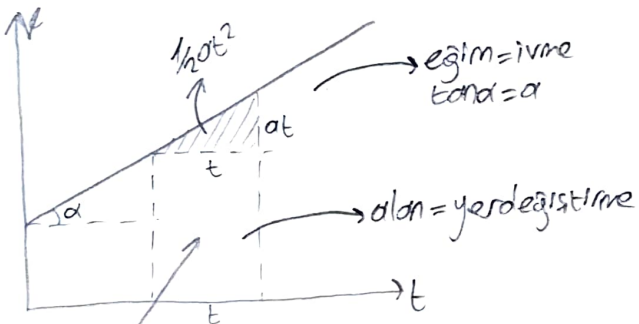
$V_s = V_i + at$





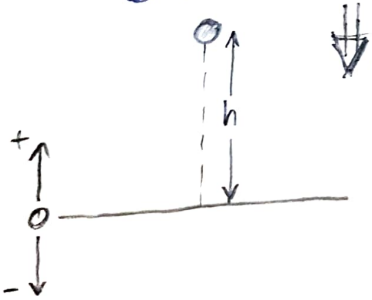
$\vec{v} = \frac{d\vec{x}}{dt}$

$\vec{a} = \frac{d\vec{v}}{dt}$



$x_s = x_i + v_i t + \frac{1}{2}at^2$

SERBEST DÜŞME

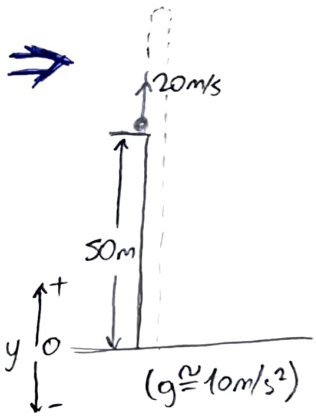


g (yerçekimi ivmesi)

$\vec{a} = \vec{g}$

$V_s = -gt$

$y_s = h - \frac{1}{2}gt^2$



d) Taşın max yüksekliğe ulaşma süresi:

$V_s = V_i + at$

$0 = 20 - g.t$

$t = 2s$

b) Max yükseklik:

$y_s = y_i + V_i.t^{max} + \frac{1}{2}at^2$

$h_{max} = 50 + 20m/s.2s - \frac{1}{2}10m/s^2.4s^2$

$= 70m$

c) Atıldığı yükseklikteki geri dönüş hızı: $V_s = -20m/s$

d) 5. saniyedeki hızı ve konumu:

$t = 5sn$ $V_s = 20m/s - 10m/s^2.5s$

$= -30\ m/s$

$y = 50m + 20m/s.5s - \frac{1}{2}10m/s^2.25s^2$

$= 25m$

e) Yere düştüğü andaki hızı:

Yere düşme süresi = t_o

$0 = 50m + 20m/s.t_o - \frac{1}{2}10m/s^2.t_o^2$

$t_o^2 - 4t_o - 10 = 0$

$t_o = \frac{4 + \sqrt{56}}{2}$

$t_o = 5,7s$

$V_s = 20 - 10m/s^2.5,7s$

$V_s = -37m/s$

Sabit İvmeli Hareket

$\vec{v} = \frac{x_s - x_i}{t}$

$x_s - x_i = \vec{v} \cdot t$

ortalama hız → $\vec{v} = \frac{v_s + v_i}{2}$

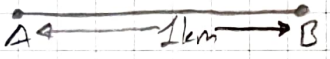
$t = \frac{v_s - v_i}{a}$

$x_s - x_i = \frac{v_s + v_i}{2} \cdot \frac{v_s - v_i}{a}$

$2a(x_s - x_i) = v_s^2 - v_i^2$

$v_s^2 = v_i^2 + 2a\Delta x$ → Zamansız hız formülü

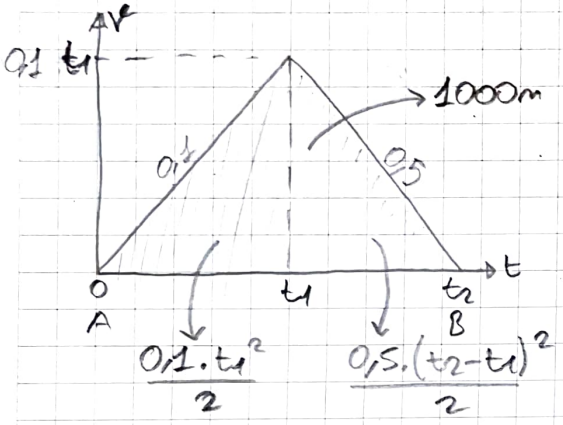
⇒



$a_{hızlanma} = 0{,}1 m/s^2$

$a_{yavaşlama} = -0{,}5 m/s^2$

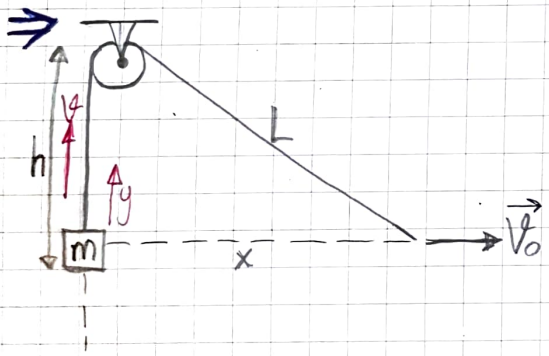
A'dan B'ye giden bir trenin B'ye en kısa kaç saniyede varır.



$0{,}1 \cdot t_1 = 0{,}5 \cdot (t_2 - t_1)$

$\frac{0{,}1 \cdot t_1^2}{2} + \frac{0{,}5 \cdot (t_2 - t_1)^2}{2} = 1000m$

⇒



m'in düşey hızını bulunuz?

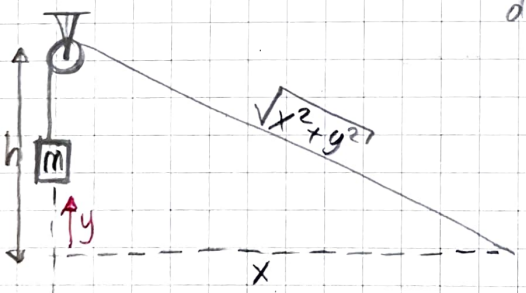
$L^2 = x^2 + y^2$

$V_o = \frac{dx}{dt}$

"zamana göre türev al"

$\frac{d}{dt} / L = \sqrt{x^2 + h^2} + (h - y)$

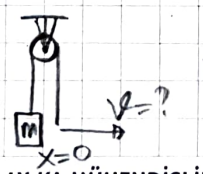
$0 = \frac{2x \cdot \frac{dx}{dy}}{2\sqrt{x^2 + h^2}} - \frac{dy}{dt}$



$V_y = \frac{dy}{dt} = x \cdot (x^2 + h^2)^{-1/2} \cdot V_o$

$a_y = \frac{dV_y}{dt}$ ... türev al

NOT:



$v = 0$

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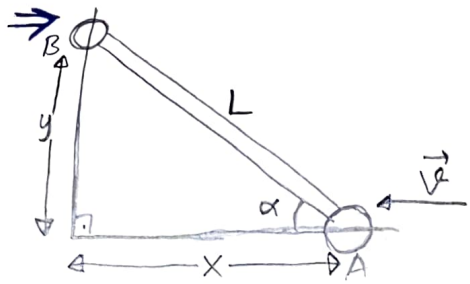


α, 60° olduğunda B'nin hızının büyüklüğü nedir?

$$x^2+y^2=L^2$$

$$\frac{dx}{dt}=v \qquad \frac{dy}{dt}=v_B$$

$$\frac{d}{dt}(x^2+y^2)=2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

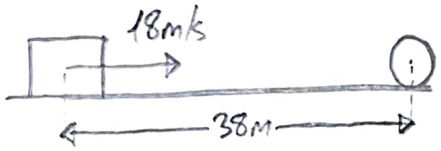
($\frac{dx}{dt}=v$, $\frac{dy}{dt}=v_B$)

$$\frac{x}{y}=\cot 60°$$

$$0=2xv+2yv_B \Rightarrow v_B=\frac{-x}{y}v$$

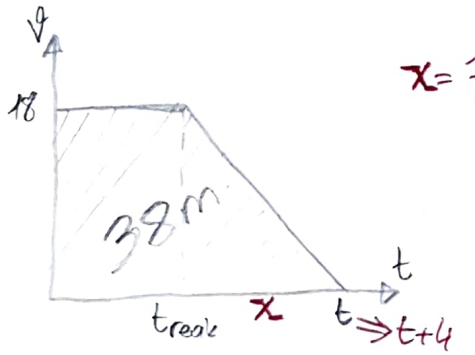
$$v_B=-\cot\alpha\, v,\ \alpha=60° \Rightarrow v_B=-\frac{\sqrt{3}}{3}v$$

⇒



$a_{max\ yavaşlama}=-4{,}5\ m/s^2$

Karenin topa çarpmaması için maximum reaksiyon süresi nedir?

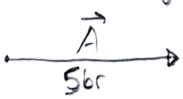


$$x=\frac{18}{4{,}5}=4$$

$$38m=18.t_r+18.(t-t_r)-\tfrac{1}{2}.4{,}5.(t-t_r)^2$$

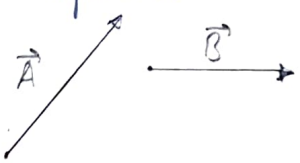
VEKTÖRLER

Yönü ve büyüklüğü vardır.

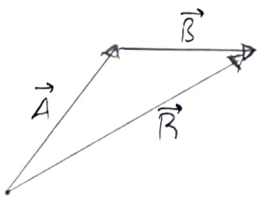


$|\vec{A}|=A=5$

Toplama:

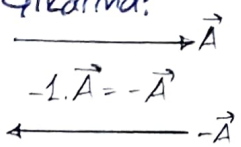


$\vec{A}+\vec{B}=$

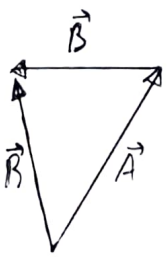


$=\vec{R}$

Çıkarma:

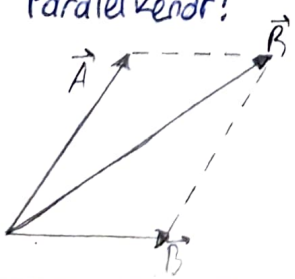


$\vec{A}-\vec{B}=$

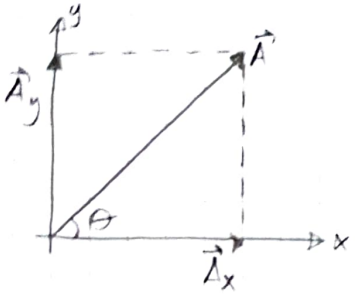


$=\vec{R}$

Paralelkenar:

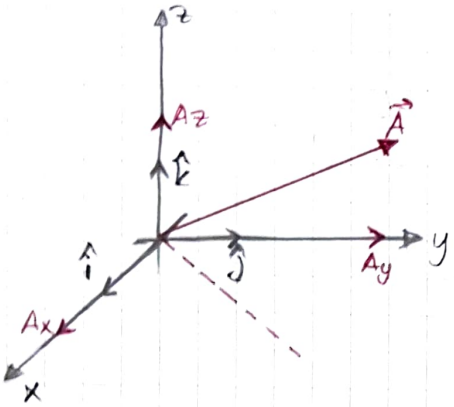


Dik Bileşenlerine Ayırma:



$$A_x = A \cdot \cos\theta$$
$$A_y = A \cdot \sin\theta$$
$$\vec{A} = \vec{A}_x + \vec{A}_y$$

Birim Vektör:



$$|\hat{k}| = |\hat{j}| = |\hat{i}| = 1$$

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$
$$= A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Skaler Çarpım:

$$\vec{A} \cdot \vec{B} = A \cdot B \cdot \cos\theta$$

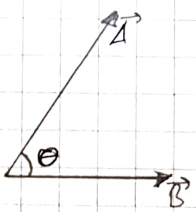
↓ Vektörler arası açı

$$\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z$$

Vektörel Çarpım:

$$|\vec{A} \times \vec{B}| = A \cdot B \cdot \sin\theta$$

↓ Vektörler arası açı

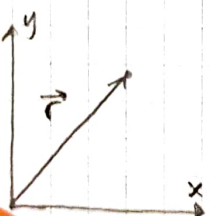


$$\vec{A} \times \vec{B} = \vec{R} \ (\otimes)$$
$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

(sağel kural: ilk vektör boş, diğer vek. kapar)

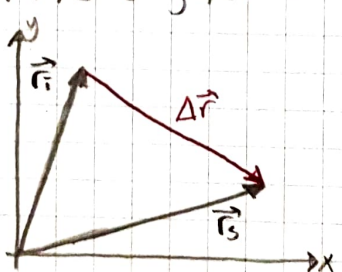
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

İKİ BOYUTTA HAREKET



$\vec{r}$: konum vektörü

$\Delta\vec{r}$: Yer değiştirme



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

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$\vec{r_s} = x_s\hat{i} + y_s\hat{j}$

$\vec{r_i} = x_i\hat{i} + y_i\hat{j}$

$\Delta\vec{r} = \vec{r_s} - \vec{r_i}$

$= (x_s - x_i)\hat{i} + (y_s - y_i)\hat{j}$

$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt}$

$= v_x\hat{i} + v_y\hat{j}$

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

- $\vec{r_s} = \vec{r_i} + \vec{v_i}.t + \frac{1}{2}\vec{a}.t^2$
  - $x_s = x_i + v_{x_i}.t + \frac{1}{2}.a_x.t^2$
  - $y_s = y_i + v_{y_i}.t + \frac{1}{2}a_y.t^2$
- $\vec{v_s} = \vec{v_i} + \vec{a}.t$

$\vec{v_s}^2 = \vec{v_i}^2 + 2a\Delta r$

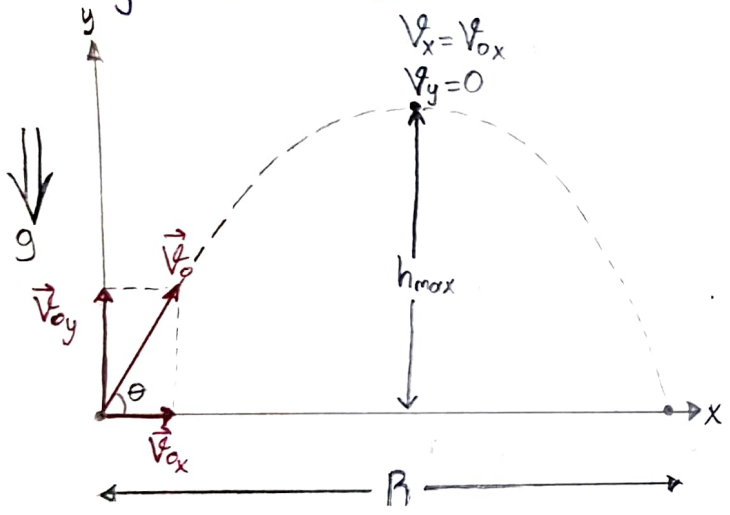
$\Downarrow$

$\vec{v_s}.\vec{v_s} = \vec{v_i}.\vec{v_i} + 2.\vec{a}.\Delta\vec{r}$

$(v_s)^2 = (v_{x_s} + v_{y_s})^2$

$v_{x_s}^2 + 2v_{x_s}.v_{y_s} + v_{y_s}^2$

Eğik Atış Hareketi



$v_{oy} = v_o.\sin\theta$

$v_{ox} = v_o.\cos\theta$

- $0 = v_{oy} - g.t_{max}$ → g ve v_{oy} ters yönde

$t_m = \frac{v_{oy}}{g}$

$t_u = \frac{2v_{oy}}{g}$

~~$-v_{oy} = v_{oy} - g.t_u$~~

$t_u = \frac{2v_{oy}}{g}$

- $R = v_{ox}.t_u$

$R = \frac{v_{ox}.2v_{oy}}{g} = \frac{2v_{ox}.v_{oy}}{g}$

- $h_{max} = v_{oy}.t_{max} - \frac{1}{2}.g.t_{max}^2$

$h_{max} = \frac{v_{oy}^2}{g} - \frac{v_{oy}^2}{2g}$

$h_{max} = \frac{v_{oy}^2}{2g} = \frac{v_o^2.\sin^2\theta}{2g}$

- $R = \frac{2v_o.\cos\theta.v_o.\sin\theta}{g} = \frac{v_o^2.\sin 2\theta}{g}$

- Parabolik yörüngenin ispatı;

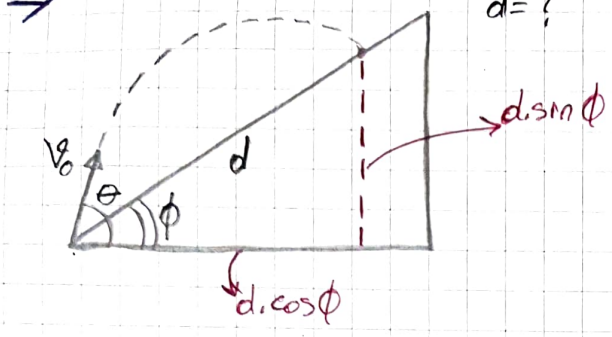
$x = v_o.\cos\theta.t$

$y = v_o.\sin\theta.t - \frac{1}{2}.g.t^2$

$t = \frac{x}{v_o.\cos\theta}$

$y = v_o.\sin\theta\frac{x}{v_o.\cos\theta} - \frac{1}{2}g.\left(\frac{x}{v_o.\cos\theta}\right)^2$

$y = \tan\theta.x - \frac{g}{2v_o^2.\cos^2\theta}.x^2$



$$d.\sin\phi = \tan\theta . d.\cos\phi - \frac{g}{2v_0^2.\cos^2\theta} . d^2.\cos^2\phi$$

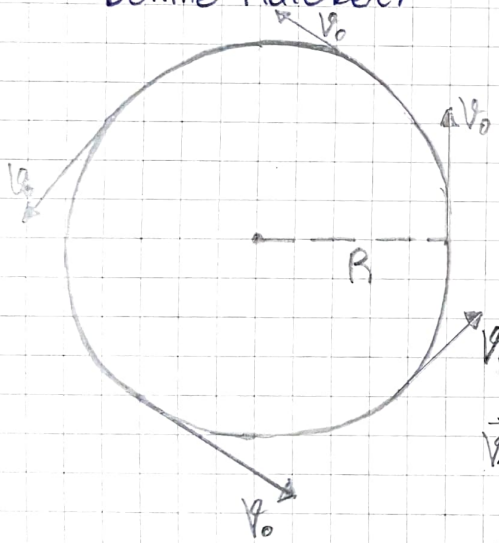
$$d\cos^2\phi = \frac{2v_0^2.\cos^2\theta}{g} . [\tan\theta.\cos\phi - \sin\phi]$$

$$d = \frac{2v_0^2}{g} . \left[\frac{\cos^2\theta.\tan\theta.\cos\phi - \sin\phi.\cos^2\theta}{\cos^2\phi}\right]$$

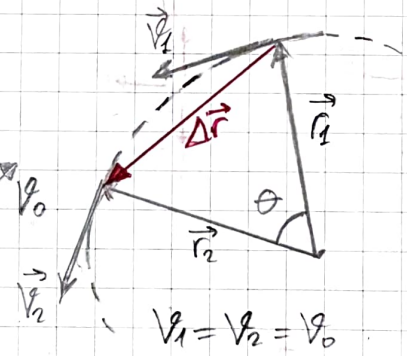
$$d = \frac{2v_0^2.\cos\theta}{\cos^2\phi} . [\sin\theta.\cos\phi - \cos\theta.\sin\phi]$$

$$d = \frac{2.v_0^2.\cos\theta.\sin(\theta-\phi)}{\cos^2\phi}$$

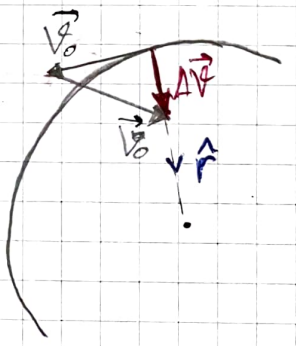
Dönme Hareketi



$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{v_s - v_i}{\Delta t}$$



$$v_1 = v_2 = v_0$$
$$r_1 = r_2 = R$$



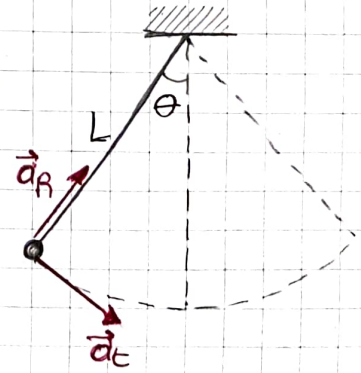
Benzerlik $\left\{ \frac{\Delta\vec{v}}{v_0} = \frac{\Delta\vec{r}}{R} \right.$

$$\Delta\vec{v} = \frac{\Delta\vec{r}.v_0}{R}$$

$$\vec{a}_R = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta\vec{r}}{\Delta t} . \frac{v_0}{R} \qquad \vec{a}_R = \frac{v_0^2}{R} . \hat{r}$$

($\frac{\Delta\vec{r}}{\Delta t} = v_0$)

NOT



BAL-AY-KA MÜHENDİSLİK ISITMA - SOĞUTMA SİSTEMLERİ İNŞAAT - GIDA - TURİZM SANAYİ TİCARET LTD. ŞTİ.

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Merkez } Akıncılar Mahallesi Gazi Bulvarı No. : 33/A Karesi/BALIKESİR

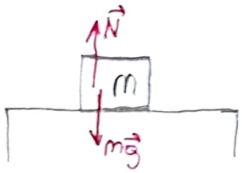
Depo-imalat } Yeni Sanayi Sitesi 33. Sokak No. : 12/A Karesi/BALIKESİR

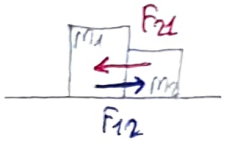
NEWTON'UN HAREKET YASALARI

1- Eylemsizlik: Bir sisteme dışarıdan bir kuvvet etki etmiyorsa, sabit hareket ediyorsa eylemsizlik vardır.

2- $\Sigma \vec{F} = m.\vec{a}$

3- Etki - Tepki

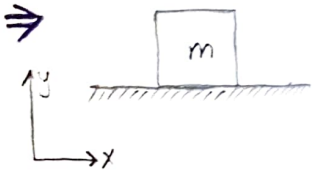




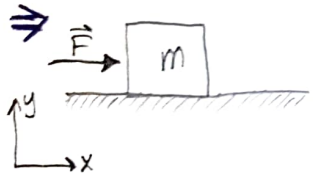
$F_{12} = F_{21}$

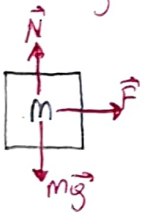
$\vec{F}_{12} = -\vec{F}_{21}$

Serbest Cisim Diyagramı



$\underbrace{N - mg}_{\Sigma F} = m.0$

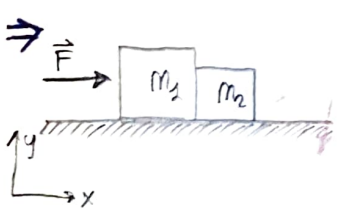


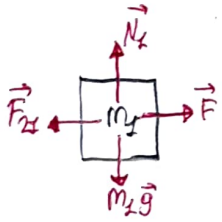


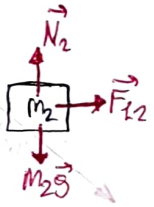
y: $N - mg = 0$

x: $F = m.a$

$a = \frac{F}{m}$







$F_{12} = F_{21} = F_{ET}$

$a_1 = a_2 = a$

y: $N_1 - m_1 g = 0$

$N_2 - m_2 g = 0$

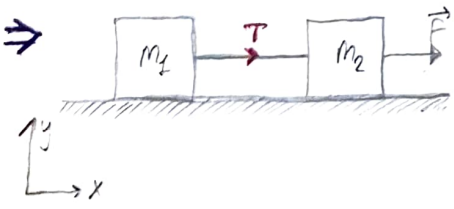
x: $F - F_{ET} = m_1.a_1$

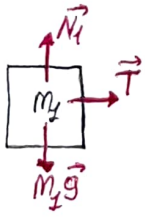
$F_{ET} = m_2.a_2$

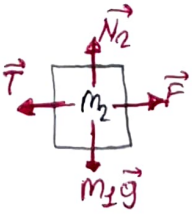
$F - F_{ET} = m_1.a$

$+ \quad F_{ET} = m_2.a$

$a = \frac{F}{m_1 + m_2}$







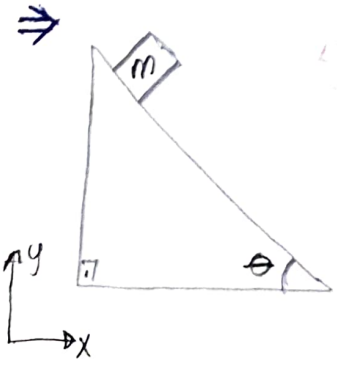
y: $N_1 - m_1 \vec{g} = 0$

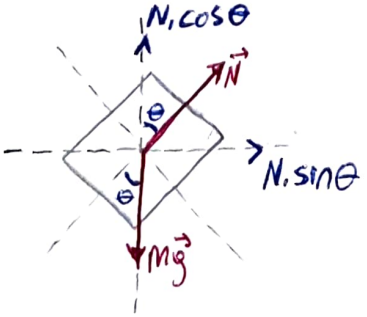
$N_2 - m_2 \vec{g} = 0$

x: $T = m_1 a$

$F - T = m_2 a$

$a = \frac{F}{m_1 + m_2}$





y: $N.\cos\theta - Mg = -M a_y$

x: $N.\sin\theta = M.a_x$

$a_x = \frac{N.\sin\theta}{M}$

$\frac{N\cos\theta - Mg}{M} = a_y$

$a = \sqrt{a_x^2 + a_y^2}$

$$\frac{N^2.\sin^2\theta}{M^2} + \frac{N^2.\cos^2\theta - 2NMg\cos\theta + M^2g^2}{M^2}$$

$$\frac{N^2 + M^2.g^2 - 2NMg.\cos\theta}{M^2}$$

$$\tan\theta = \frac{N.\cos\theta - Mg}{M} \Big/ \frac{N.\sin\theta}{M}$$

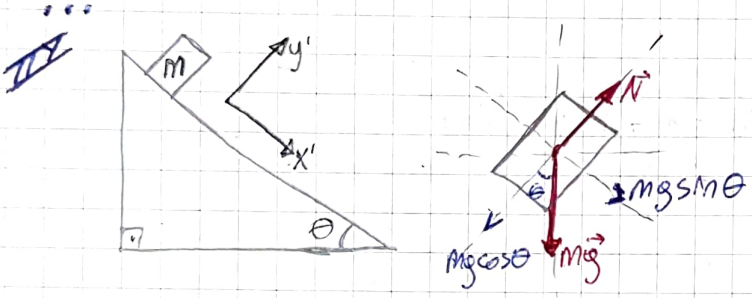
$$\tan\theta = \frac{N.\cos\theta - Mg}{N.\sin\theta}$$

Yerine yaz

$$N\cos\theta - N\sin\theta.\tan\theta = Mg$$

$$N = \frac{Mg}{\cos\theta - \sin\theta.\tan\theta}$$

...

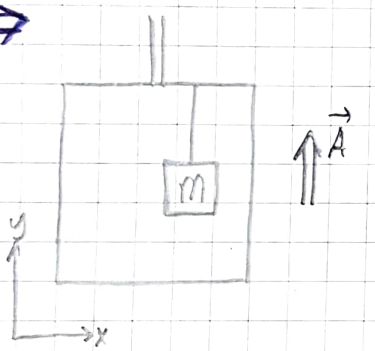


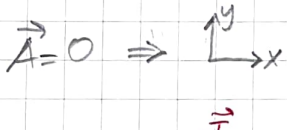
y': $N - Mg\cos\theta = 0$

x': $Mg.\sin\theta = M.a$

$a = g.\sin\theta$

⇒



$\vec{A} = 0 \Rightarrow$ 

$T - mg = 0$

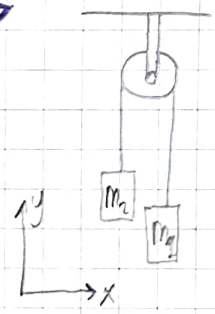
$\vec{A}\uparrow\uparrow \Rightarrow$ 

$T - mg = mA$

$T = m.(g + A)$

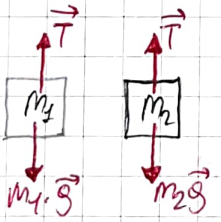
} Dışarıdan bakan gözlemci

⇒



$m_1 > m_2$

$a_1 = a_2 = a$



$T - m_1g = -m_1a$

$T - m_2g = m_2a$

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$$

Sürtünme Kuvveti

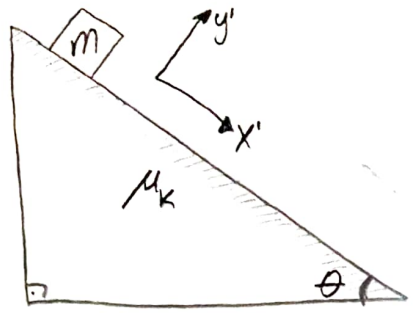
Statik (Durgun) Sürtünme Kuvveti

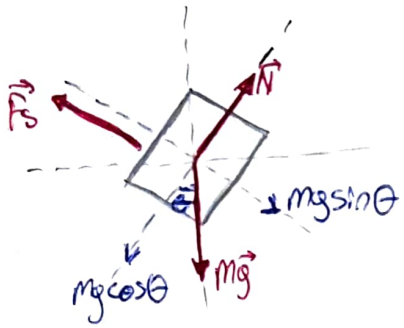
$\vec{F_s} \leq \mu_s.\vec{N}$ (statik sürtünme katsayısı)

Kinetik (Hareketli) Sürtünme Kuvveti

$\vec{F_s} = \mu_k.\vec{N}$ Genellikle → $\mu_s > \mu_k$

⇒



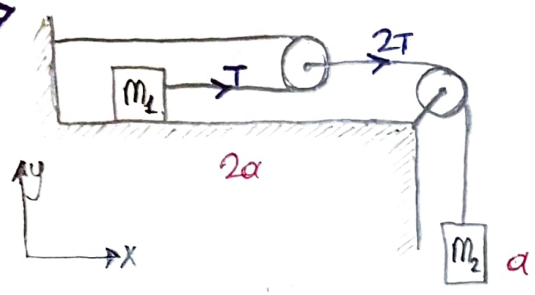


y': $N - mg\cos\theta = 0$

x': $mg\sin\theta - F_s = ma$

$F_s = \mu_k . N$

⇒

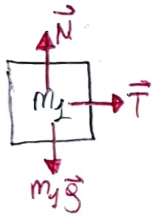


x	2x
a	2a

$x(t) = x_i + v_i . t + \frac{1}{2} a t^2$

$X = \frac{1}{2} a_1 t^2$

$2X = \frac{1}{2} a_2 . t_2$

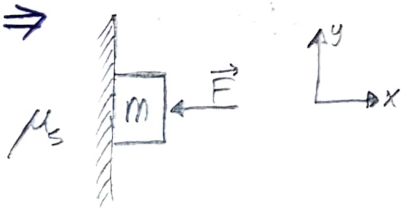


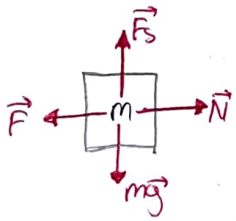
y: $N - m_1 g = 0$

$2T - m_2 g = -m_2 a$

x: $T = m_1 . 2a$

⇒





y: $F_s - mg = 0$

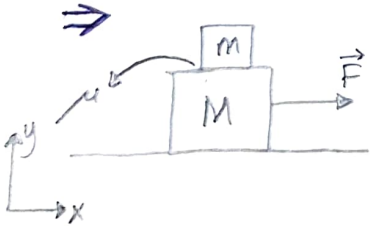
x: $N - F = 0$

$F_s = \mu_s N = \mu_s F$

$\mu_s F = m.g$

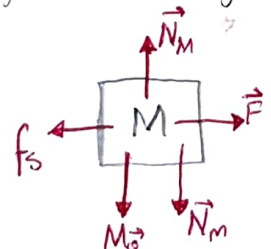
$F = \frac{m.g}{\mu_s}$

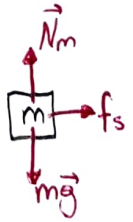
⇒



⇒a

m'in kaymadan ilerleye bilmesi için $\vec{F}_{max}$ = ?





$f_s = \mu . N_m$

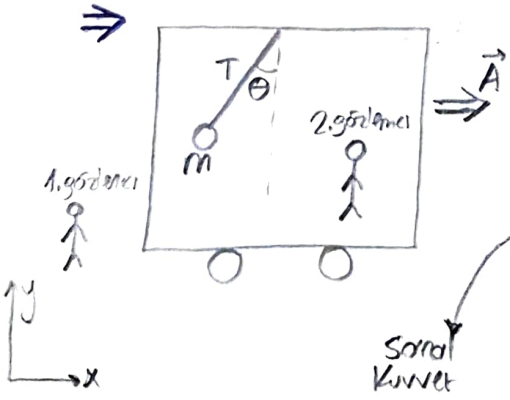
y: $\vec{N_M} - M\vec{g} - N_m = 0$

x: $F - f_s = Ma$

$N_m - mg = 0$

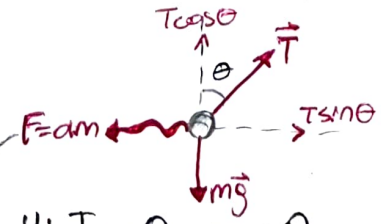
$f_s = ma$

⇒



$\vec{A}$

2. Gözlemci için;

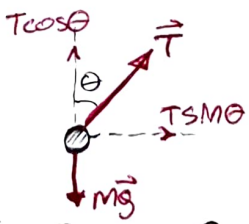


Sanal Kuvvet

y: $T\cos\theta - mg = 0$

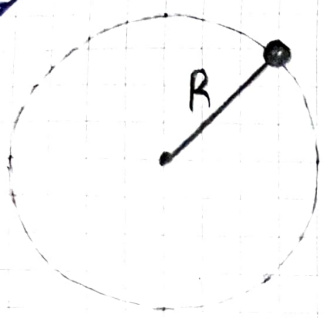
x: $T\sin\theta - mA = 0$

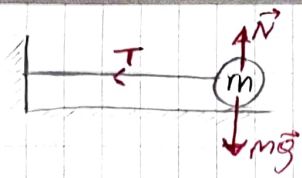
1. Gözlemci için;

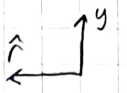


y: $T\cos\theta - mg = 0$

x: $T\sin\theta = mA$







silindirik referans

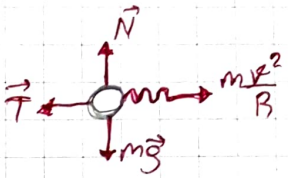
Durgun gözlemci için; $y: N - mg = 0$

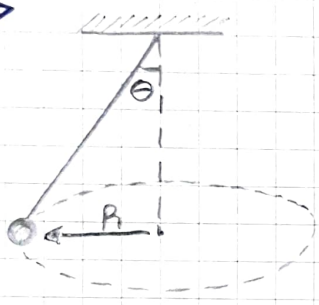
$r: T = \frac{mv^2}{R}$

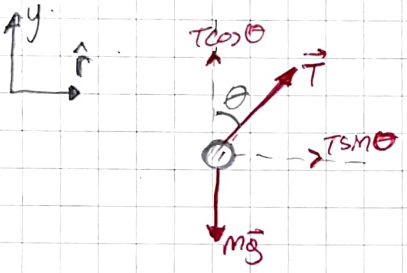
Cisimle birlikte hareket eden gözlemci için;

$y: N - mg = 0$

$r: T - m\frac{v^2}{R} = 0$

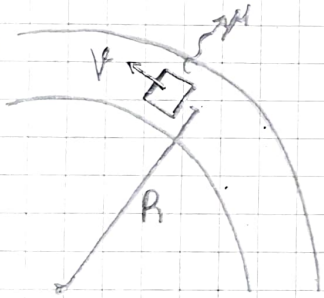


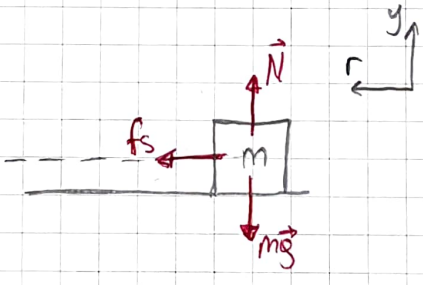




$y: T\cos\theta - mg = 0$

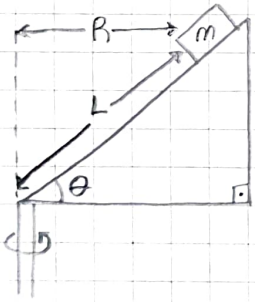
$x: T\sin\theta = m\frac{v^2}{R}$

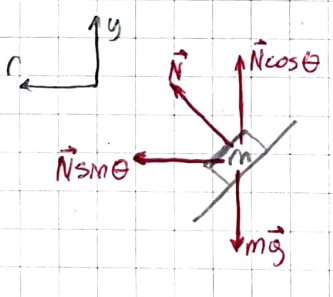




$y: N - mg = 0$

$x: f_s = m\frac{v^2}{R}$





$N\cos\theta - mg = 0$

$N\sin\theta = \frac{mv^2}{R}$

$R = L.\cos\theta$

$N = \frac{mg}{\cos\theta}$

$\Rightarrow \frac{mg}{\cos\theta} . \sin\theta = \frac{mv^2}{L\cos\theta}$

$v = (gL\sin\theta)^{\frac{1}{2}}$

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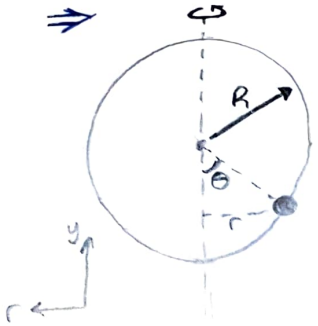
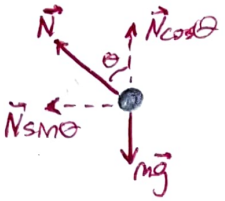


Topun dengede kalabilmesi için maximum θ açısı kaç derecedir? ($T = 0{,}45\,s$)



$N = \frac{mg}{\cos\theta}$ ← $N\cos\theta - mg = 0$ ← y

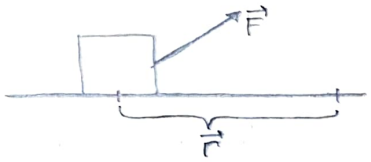
→ $N\sin\theta = \frac{mv^2}{r}$ ← x

$r = R.\sin\theta$ ↑ $v = \frac{2\pi r}{T}$ ↑

$$\frac{mg}{\cos\theta}.\sin\theta = \frac{m4\pi^2.r^2}{r.T^2}$$

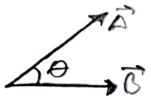
$$\tan\theta = \frac{4\pi^2.r}{g.T^2}$$

İŞ VE KİNETİK ENERJİ



$W = \vec{F}.\vec{r}$ $(N.m = \text{joule})$

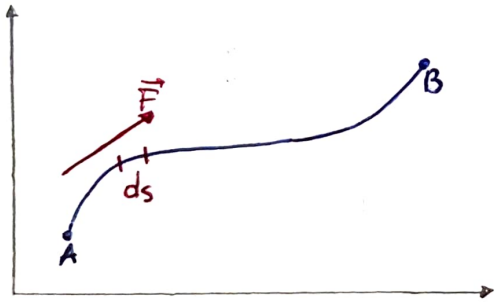
NOT: Skaler Çarpım



$\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|.\cos\theta$

$= A_x B_x + A_y B_y + A_z B_z$

- Değişken Kuvvet İçin;

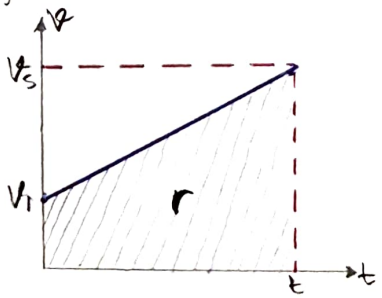


$W = \int_A^B \vec{F}.d\vec{s}$

- İş-Kinetik Enerji Teoremi

$W = F.r$

$W = m.a.r$



$r = \frac{1}{2}(v_i + v_s).t$

$a = \frac{v_s - v_i}{t}$

$W = m.\frac{v_s - v_i}{t}.\frac{1}{2}.(v_i + v_s).t$

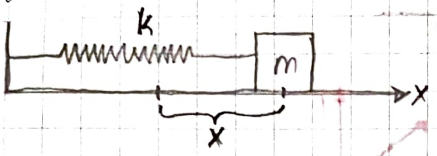
$W = \frac{1}{2}m v_s^2 - \frac{1}{2}m v_i^2$

Kinetik Enerji: $K = \frac{1}{2}mv^2$

$W = K_s - K_i$

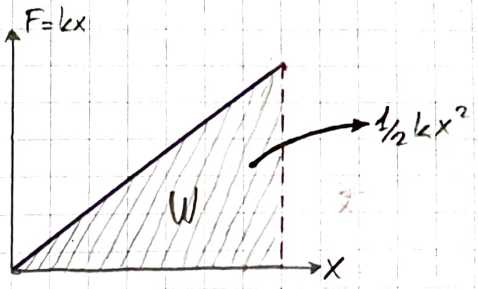
$W = \Delta K$ ⟶ iş kinetik enerji teoremi

- Yayın yaptığı iş



$$F = -k.x$$

$$W = \int \vec{F}.d\vec{s} = \int_0^x F.dx = \int_0^x kx.dx = \tfrac{1}{2}kx^2$$

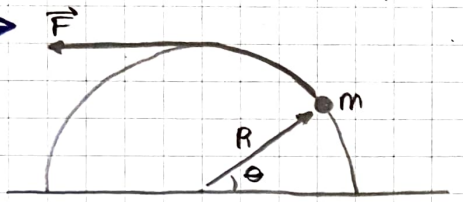


- Güç

$$\text{Güç} = P = \frac{dW}{dt} \quad (\text{Joule/s} = \text{watt}) \qquad P = \frac{d}{dt}(Fx)$$

$$\text{sabit kuvvet} \Rightarrow P = F\frac{dx}{dt} = F.v$$

$\Rightarrow$

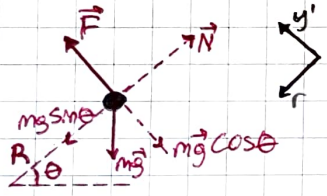


m kütleli cisim ip yardımıyla sabit hızla çekiliyor.

a) $F = mg\cos\theta$ olduğunu gösteriniz?

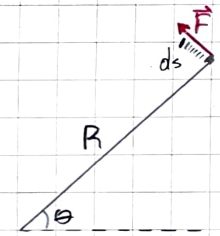
b) $W = \int \vec{F}\,d\vec{s}$ formülü ile yerden silindirin üst noktasına gelene kadarki iş nedir?

a)

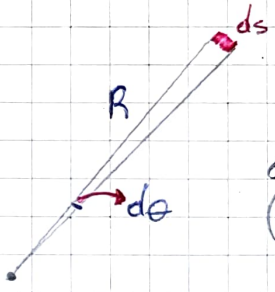


$$N - mg\sin\theta = 0$$
$$F - mg\cos\theta = 0$$

b)



$$W = \int \vec{F}\,d\vec{s} = \int F\,ds$$
$$= \int mg.\cos\theta\,ds$$
$$= mg\int \cos\theta\,ds$$
$$= mg\int_0^{\pi/2} \cos\theta\, R\,d\theta$$
$$= mgR\sin\theta\Big|_0^{\pi/2}$$
$$= mgR$$



$ds = R\,d\theta$ (θ radyan)

$\Rightarrow F = -kx + \beta x^3$
$k = 10\,N/m$
$\beta = 100\,N/m^3$
$\Delta x = 0,100\,m$
$\Big\}$ $W = ?$

$$W = \int \vec{F}.d\vec{x}$$
$$= \int_0^{0,100} -kx + \beta x^3\,dx$$
$$= -\frac{1}{2}kx^2 + \frac{\beta x^4}{4}\Big|_0^{0,1}$$

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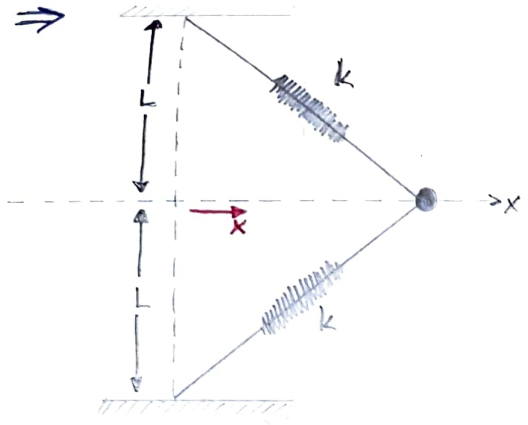
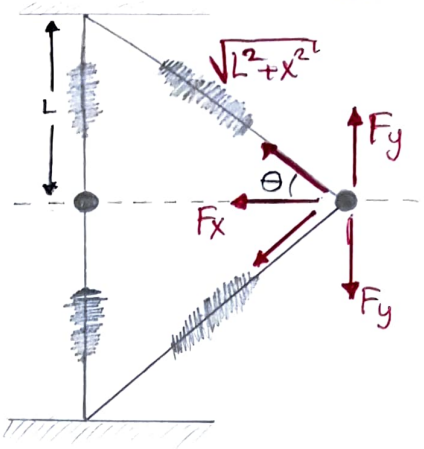




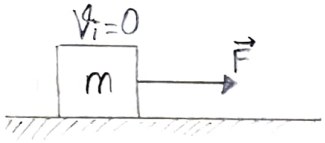
$$\vec{F} = -2k.x.\left(1-\frac{L}{\sqrt{x^2+L^2}}\right).\hat{j}$$

$$\sqrt{L^2+x^2}-L=\Delta X$$

$$F=-2F_y.\cos\theta.\hat{i}$$

$$F=-2.\left(k.\left(L-\sqrt{L^2+x^2}\right)\right)\frac{x}{\sqrt{L^2+x^2}}.\hat{i}$$

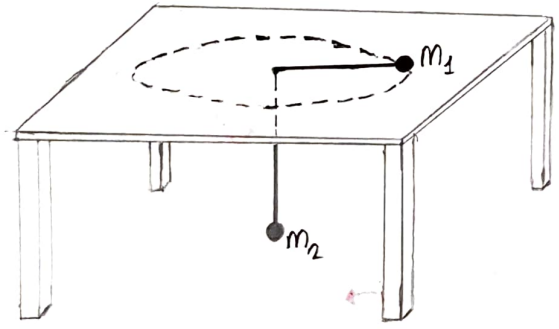
⇒

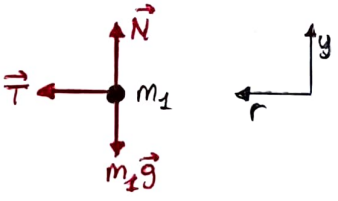


$$P=\left(\frac{F^2}{m}\right)$$

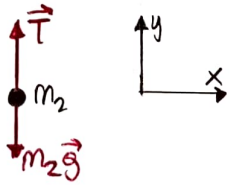
$$P=F.v$$
$$=F.a.t$$
$$=F.\frac{F}{m}.t$$

⇒



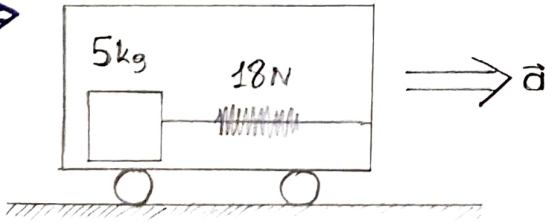


$$N-m_1g=0$$
$$T=\frac{m_1v^2}{R}$$



$$T-m_2g=0$$
$$T=m_2g$$

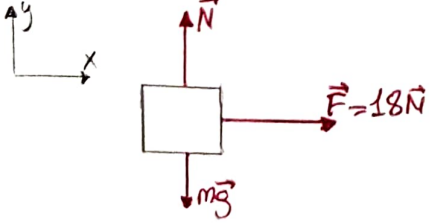
⇒



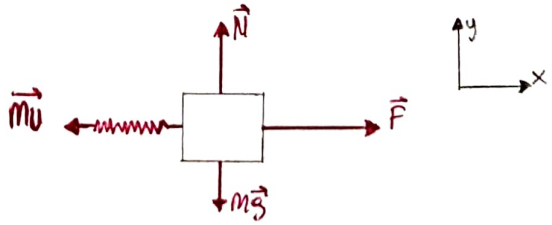
İvme: $18N=5kg.a$

$a=3,6\ m/s^2$

İçeriden bakan gözlemci;

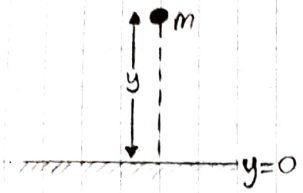


Dışarıdan bakan gözlemci;

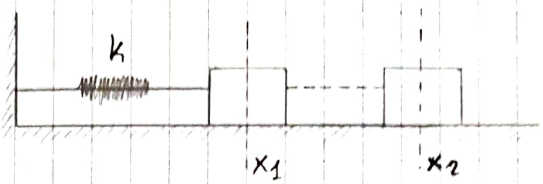


• Arada sabit ivmeli olsaydı yayın değeri sıfır (0) olurdu.

Potansiyel Enerji

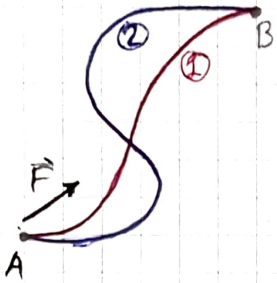


$$U_g = mgy$$



$$U_y = \frac{1}{2} k . \Delta x^2$$

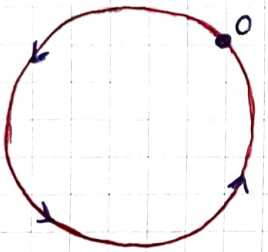
Korunumlu Kuvvet



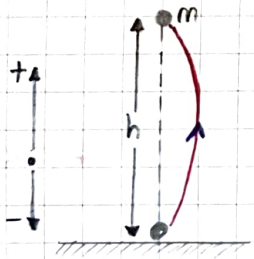
Korunumlu kuvvetin yaptığı iş iki nokta arasında seçilen yoldan bağımsızdır.

$$W_1 = W_2$$

Sürtünme kuvveti korunumlu değildir fakat yerçekimi ve yay kuvveti korunumludur.



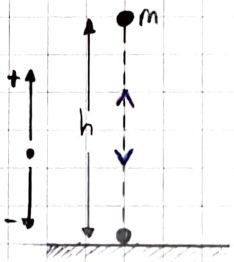
Kapalı bir eğri üzerinde hareket üzerinde korunumlu kuvvetin yaptığı iş sıfırdır.



$$W = -m\vec{g} . \vec{y}$$

$$(\cos 180° = -1)$$

$$W = mgy$$



$$W = 0$$

$$W = mgy_s - mgy_i$$

Mekanik Enerjinin Korunumu

$$E \equiv K + U$$

Sadece korunumlu kuvvetlerin etkisindeki yalıtılmış sistemlerde toplam mekanik enerji korunur.

$$E_{ilk} = E_{son}$$

$$K_i + U_i = K_s + U_s$$

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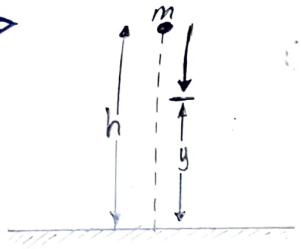
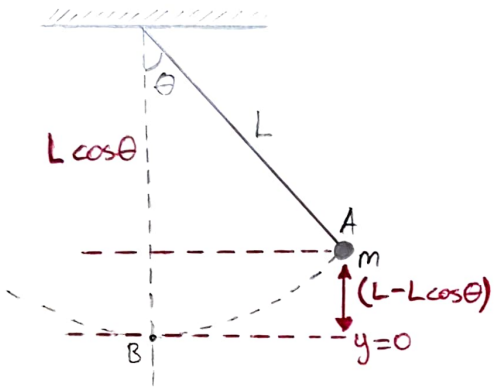


m kütleli cisim yerden h kadar yükseklikten serbest bırakılıyor. Cismin yerde y kadarki yükseklikteki hızı (V_y) kaçtır?

$$mgh = \frac{1}{2} m V_y^2 + mgy$$

$$V_y = \sqrt{2g(h-y)}$$

⇒



a) cismin B noktasındaki hızı (V_B) nedir?

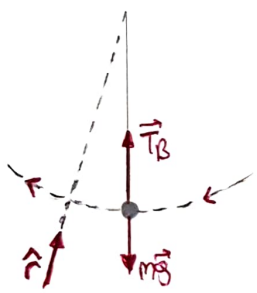
b) B noktasındaki ip gerilmesi (T_B) kaçtır?

a) $K_i + U_i = K_s + U_s$ (0 ve 0)

$$mgL(1-\cos\theta) = \frac{1}{2} m V_B^2$$

$$V_B = \sqrt{2gL(1-\cos\theta)}$$

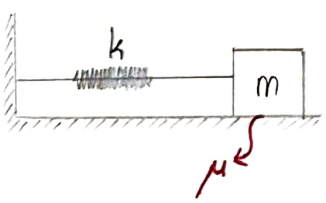
b)



$$T_B - mg = \frac{m V_B^2}{L}$$

$$T_B = mg + 2mg(1-\cos\theta)$$

★



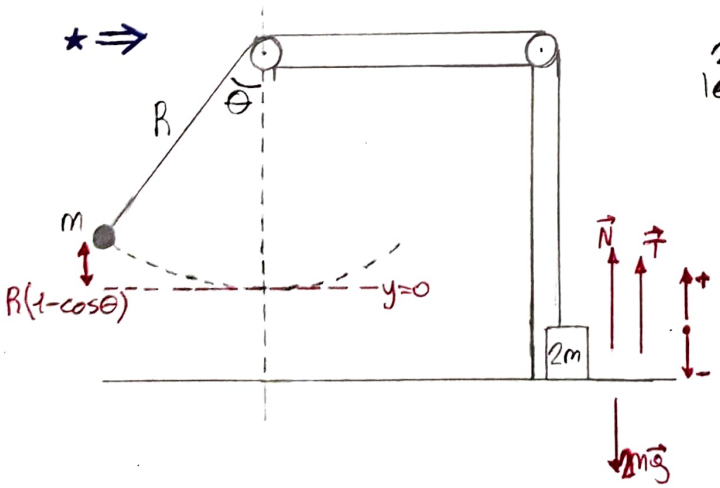
kuvvet	sistem
$W_{korunumlu} = -\Delta U$	
$W = \Delta K$	
$W_k + W_{ks} = \Delta K$	
$-\Delta U$	$W_{ks} = \Delta K + \Delta U$

Sistemin enerjisindeki değişim $-\frac{1}{2}kx^2$

Kuvvetin yaptığı iş $+\frac{1}{2}kx^2$

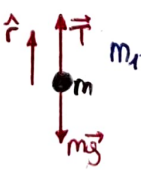
(Yay enerji kaybetti, m kütlesi enerji kazandı.)

★⇒



2m kütleli cisim yerden kalkmayacak şekilde m kütlesinin yapacağı maximum θ açısı kaçtır?

$$0 + mgR(1-\cos\theta) = \frac{1}{2} m V^2 + 0$$



$m_1 \rightarrow T - mg = \frac{mV^2}{R}$

$$T = mg + \frac{2mgR(1-\cos\theta)}{R}$$

$$T = mg + 2mg(1-\cos\theta)$$

$m_2 \rightarrow N + T - 2mg = 0$ (max θ için $N = 0$'dır)

$$2mg = mg + 2mg(1-\cos\theta)$$

$$\cos\theta = \frac{1}{2} \quad \theta = 60°$$

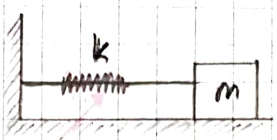
Korunumlu Kuvvet-Potansiyel Enerji İlişkisi

$F_x \cdot \Delta X = -\Delta U$

$F_x = -\frac{\Delta U}{\Delta X} \quad (\Delta X \to 0)$

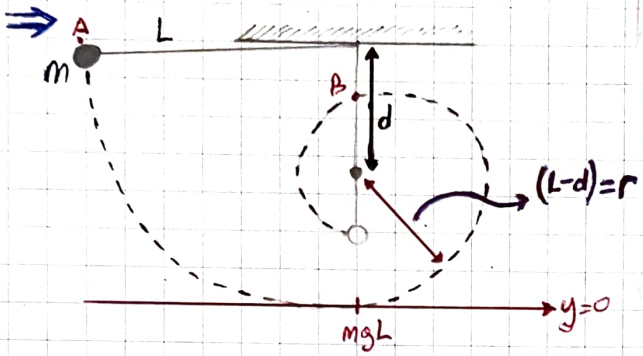
$F_x = -\frac{dU}{dX}$

NOT:



$U_y = \frac{1}{2} k x^2$

$\frac{dU}{dx} = kx \qquad F_x = -kx$



Sarkaçtaki top serbest bırakılıyor. m kütleli top duvardaki çiviye takılıyor. Topun tam bir çember çizebilmesi için (min) $d = 3L/2$ olduğunu gösterin.

$U_A = mgL$

$U_B = 2mg(L-d) + \frac{1}{2} m v^2$

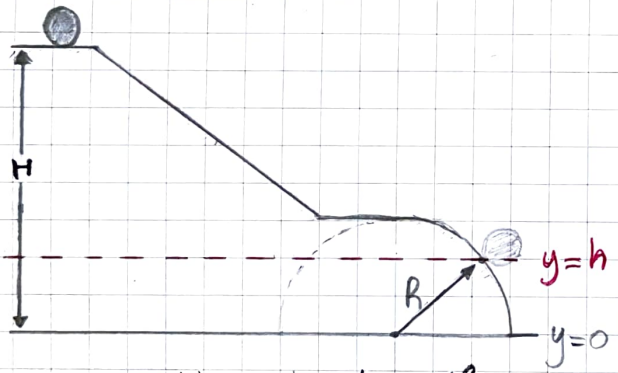
$mg = \frac{mv^2}{r}$

$gr = v^2$

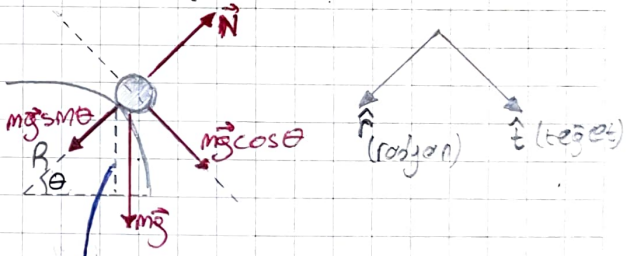
$g(L-d) = v^2$

$mgL = 2mg(L-d) + \frac{1}{2} mg(L-d)$

$L = 2(L-d) + \frac{1}{2}(L-d)$



Top kaydırakten kayarken hangi noktada yolla temasını kaybeder.



$mgH = mgh + \frac{1}{2} mv^2$

$mv^2 = 2(mgH - mgh)$

$g\sin\theta = \frac{2g \cdot (H-h)}{R}$

$\frac{2gH - gR\sin\theta}{2g} = h$

$h = \frac{2H - R\sin\theta}{2}$

$h = R\sin\theta$

$mg\cos\theta = m \cdot a_t$

$mg\sin\theta - N = m \cdot a_r$

Temasın Kaybolması $\Rightarrow N = 0$

$mg\sin\theta = \frac{mv^2}{R}$

$\Rightarrow h = \frac{2H}{3}$

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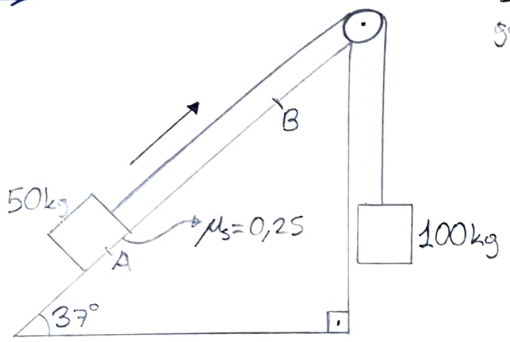
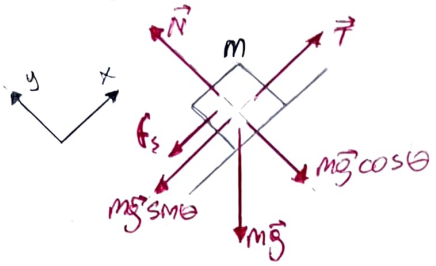
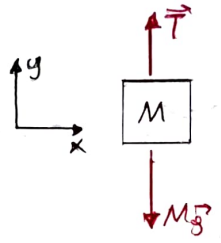
⇒

50kg'lık blok A'dan B'ye hareket ediyor. $|AB| = 20m$ olduğuna göre bloğun kinetik enerjisindeki değişim nedir?





$$N - mg\cos\theta = 0$$
$$T - mg\sin\theta - f_s = ma$$



$$T - Mg = -Ma$$
$$f_s = \mu_k \cdot N = \mu_k \cdot mg \cdot \cos\theta$$

$$Mg - Ma - mg\sin\theta - \mu_k \cdot mg \cdot \cos\theta = ma$$

$$a = \frac{g(M - m\sin\theta - \mu_k m\cos\theta)}{M+m}$$

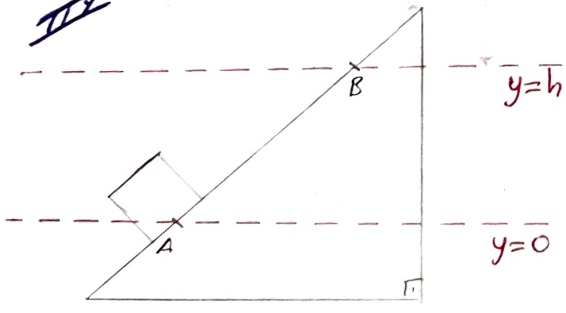
$$V_B^2 = 2a|AB| \qquad (V_A = 0 \text{ kabul edelim})$$

$$\Delta K = \tfrac{1}{2} m V_B^2 = \frac{mg(M - m\sin\theta - \mu_k \cdot m \cdot \cos\theta)}{M+m} \cdot |AB|$$

$$= \frac{50 \cdot 10 \cdot (100 - 50 \cdot \sin 37° - 0,25 \cdot 50 \cdot \cos 37°)}{100+50} \cdot 20$$

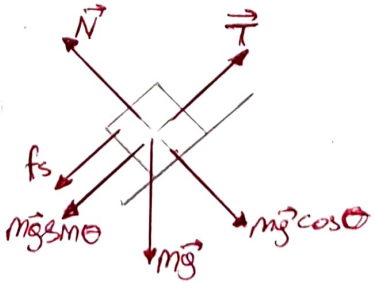
$$= 4000 j$$

III



$U_A = 0$ $\qquad K_A = 0$

$U_B = mgh$ $\qquad K_B = \tfrac{1}{2} m V_B^2$



$$W - U = K$$
$$(T - f_s - mg\sin\theta) \cdot |AB| - U_B = K_B$$

LİNEER MOMENTUM

$\vec{P} \equiv m.\vec{V}$ $\quad P_x = mV_x$, $P_y = mV_y$, $P_z = mV_z$

$\vec{P} = P_x\,\hat{i} + P_y\,\hat{j} + P_z\,\hat{k}$

$\frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{V}) = m\frac{d\vec{V}}{dt} = m\vec{a}$

- $\sum\vec{F} = \frac{d\vec{P}}{dt}$ sisteme etki eden net dış kuvvet sıfır ise

$\sum\vec{F} = 0$ ve $\frac{d\vec{P}}{dt} = 0$ dır.

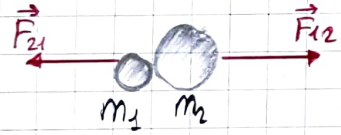
- Momentum (P) korunur $\vec{P}_{ilk} = \vec{P}_{son}$ → Lineer momentumun korunumu

İTME

$\vec{F} = \frac{d\vec{P}}{dt}$ $\quad d\vec{P} = \vec{F}.dt$ $\quad \Delta\vec{P} = \vec{P}_{son} - \vec{P}_{ilk} = \int_{t_i}^{t_s}\vec{F}\,dt$

$I = \int_{t_i}^{t_s}\vec{F}\,dt$

ÇARPIŞMALAR



$\Delta\vec{P}_1 = \int_{t_i}^{t_s}\vec{F}_{21}\,dt$

$\Delta\vec{P}_2 = \int_{t_i}^{t_s}\vec{F}_{12}\,dt$

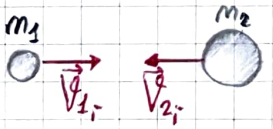
$\Delta\vec{P}_1 + \Delta\vec{P}_2 = 0$

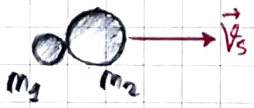
$\vec{P}_{1s} - \vec{P}_{1i} + \vec{P}_{2s} - \vec{P}_{2i} = 0$

$\boxed{\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1s} + \vec{P}_{2s}}$ sistemin toplam momentumu değişmiyor.

Tamamen Esnek Olmayan Çarpışma

Momentum (P) korunur, Kinetik Enerji (K) korunmaz.





$m_1\vec{V}_{1i} + m_2\vec{V}_{2i} = (m_1 + m_2).\vec{V}_s$

$\vec{V}_s = \frac{m_1.\vec{V}_{1i} + m_2.\vec{V}_{2i}}{(m_1 + m_2)}$

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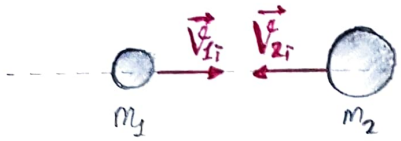
Elastik Çarpışma

Momentum (P) ve Kinetik Enerji (K) korunur.



$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1s} + m_2\vec{v}_{2s}$$

$$\tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2 = m_1v_{1s}^2 + m_2v_{2s}^2$$

$$v_{1s} = \left(\frac{m_1-m_2}{m_1+m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1+m_2}\right)v_{2i}$$

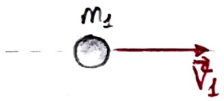
$$v_{2s} = \left(\frac{2m_1}{m_1+m_2}\right)v_{1i} + \left(\frac{m_2-m_1}{m_1+m_2}\right)v_{2i}$$

İki Boyuttaki Çarpışmalar

$\vec{P}_{ilk} = \vec{P}_{son}$

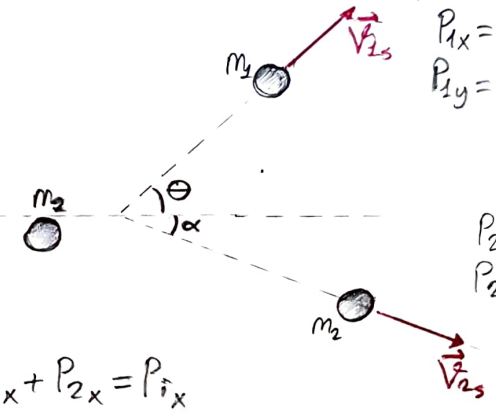
$\vec{P}_{ix} = \vec{P}_{sx}$

$\vec{P}_{iy} = \vec{P}_{sy}$



$P_{1x} = m_1 . v_i$

$P_{2y} = 0$



$P_{1x} = m_1 v_{1s} . \cos\theta$

$P_{1y} = m_1 v_{1s} . \sin\theta$

$P_{2x} = m_2 v_{2s} . \cos\theta$

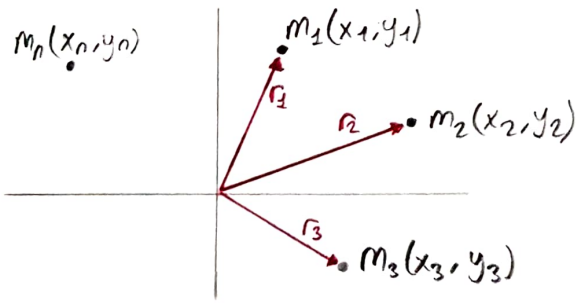
$P_{2y} = m_2 v_{2s} . \sin\theta$

$P_{1x} + P_{2x} = P_{ix}$

$P_{1y} - P_{2y} = 0$

KÜTLE MERKEZİ

Parçacık için:



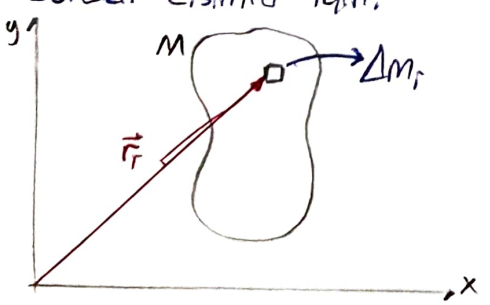
$$X_{km} = \frac{m_1.x_1 + m_2.x_2 + m_3.x_3 + \ldots + m_n.x_n}{m_1 + m_2 + m_3 + \ldots + m_n}$$

$$Y_{km} = \frac{m_1.y_1 + m_2.y_2 + m_3.y_3 + \ldots + m_n.y_n}{m_1 + m_2 + m_3 + \ldots + m_n}$$

$$\vec{r}_{km} = \frac{1}{M}\sum_i m_i.\vec{r}_i$$

(Toplam Kütle ← M)

Sürekli cisimler için:



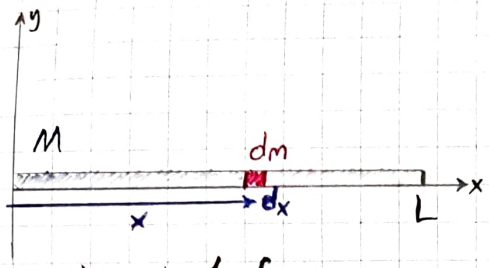
$$\vec{r}_{km} = \frac{1}{M}\sum_i \Delta m_i.\vec{r}_i$$

$$\lim_{\Delta m_i \to 0} \vec{r}_{km} = \frac{1}{M}\int \vec{r}\,dm$$

$$X_{km} = \frac{1}{M}\int x\,dm$$

$$Y_{km} = \frac{1}{M}\int y\,dm$$

⇒



$$X_{km} = \frac{1}{M}\int x\,dm$$

$\lambda = \frac{M}{L}$ → Homojen çubuk olduğu için birim uzunluktaki kütleye bakılır.

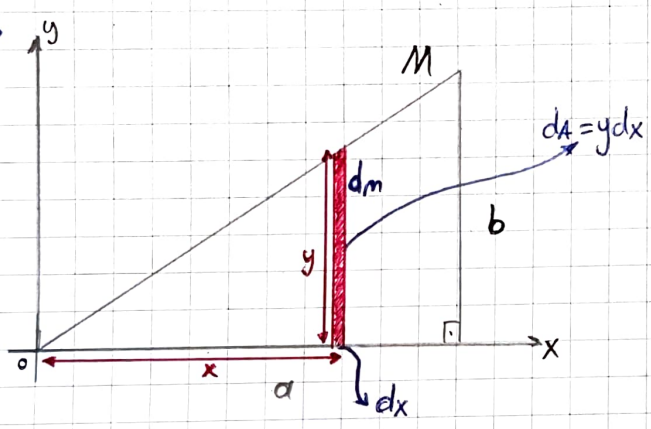
$dm = \lambda\,dx$

$$X_{km} = \frac{1}{M}\int_0^L x\lambda\,dx$$

$$= \frac{\lambda}{M}\cdot\frac{x^2}{2}\Big|_0^L$$

$$X_{km} = \frac{\lambda L^2}{2m} \qquad X_{km} = \frac{L}{2}$$

⇒



$$X_{km} = \frac{1}{M}\int x\,dm$$

$\sigma = \frac{M}{\frac{1}{2}ab} \qquad dm = \sigma . y\,dx$

$$X_{km} = \frac{1}{M}\sigma\int xy\,dx \qquad \frac{b}{a} = \frac{y}{x} \quad y = \frac{xb}{a}$$

$$= \frac{1}{M}\sigma\frac{b}{a}\int_0^a x^2\,dx$$

$$= \frac{1}{M}\sigma\frac{b}{a}\frac{x^3}{3}\Big|_0^a$$

$$= \frac{1}{M}\cdot\sigma\cdot\frac{b}{a}\cdot\frac{a^3}{3}$$

$$= \frac{1}{M}\cdot\frac{2M}{ab}\cdot\frac{b}{a}\cdot\frac{a^3}{3} = \frac{2a}{3}$$

$$Y_{km} = \frac{1}{M}\int y\,dm$$

$\sigma = \frac{M}{ab} \qquad dm = \sigma . x\,dy$

$\frac{a}{b} = \frac{x}{y} \quad x = \frac{ay}{b}$

$$Y_{km} = \frac{1}{M}\sigma\int yx\,dy$$

$$= \frac{1}{M}\sigma\frac{a}{b}\int_0^b y^2\,dy$$

$$= \frac{1}{M}\sigma\frac{a}{b}\cdot\frac{y^3}{3}\Big|_0^b$$

$$= \frac{1}{M}\sigma\frac{a}{b}\cdot\frac{b^3}{3}$$

$$= \frac{1}{M}\cdot\frac{M}{ab}\cdot\frac{b}{a}\cdot\frac{a^3}{3} = \frac{b}{3}$$

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Parçacık Sistemlerinin Hareketi

- $\vec{v}_{km} = \frac{d\vec{r}_{km}}{dt}$

$$= \frac{1}{M}\sum_i m_i \frac{d\vec{r}_i}{dt} = \frac{\sum_i m_i \vec{v}_i}{M}$$

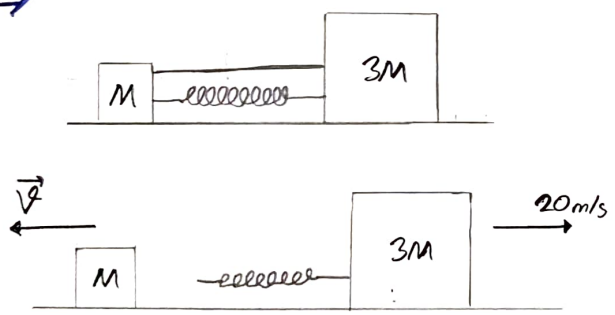
$$M\vec{v}_{km} = \sum_i m_i \vec{v}_i = \sum_i \vec{P}_i = \vec{P}_{toplam}$$

- $\vec{a}_{km} = \frac{d\vec{v}_{km}}{dt}$

$$= \frac{1}{M}\sum_i m_i \vec{a}_i$$

$$M\vec{a}_{km} = \sum_i m_i \vec{a}_i = \sum_i \vec{F}_i = \frac{d\vec{P}_{toplam}}{dt}$$

⇒



Sürtünmesiz düzlemde, ip kesiliyor. $M = 0{,}35\,kg$ olduğuna göre v_M ve U_{yay} kaçtır?

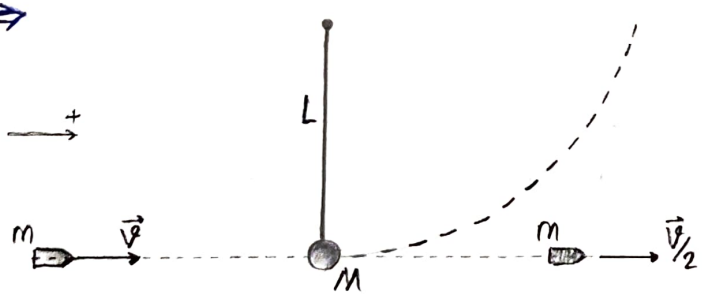
$$P_{ilk} = 0 = 3M \cdot 20\,m/s - M \cdot v_M$$

$$v_M = 6\,m/s$$

$$U_{yay} = \frac{1}{2}M \cdot 6^2 + \frac{1}{2} \cdot 3M \cdot 2^2$$

$$U_{yay} = 8{,}4\,kgm^2/s^2$$

⇒



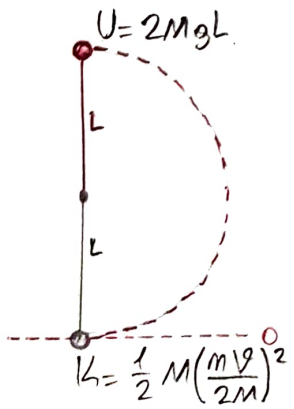
Çubuğa takılmış M kütleli cisme mermi gömülüyor deliyor geçiyor. Cismin tam dönüş yapabilmesi için $\vec{v}$'nin minimum hızı kaçtır?

Momentum Korunumu

$$\vec{P}_{ilk} = \vec{P}_{son}$$

$$m\vec{v} = \frac{m\vec{v}}{2} + M \cdot \vec{v}_x$$

$$\vec{v}_x = \frac{mv}{2M}$$



-Çubuk olduğu için; $K = U$

Eğer ip olsaydı, merkezcil kuvvet ağırlığına eşit olurdu.

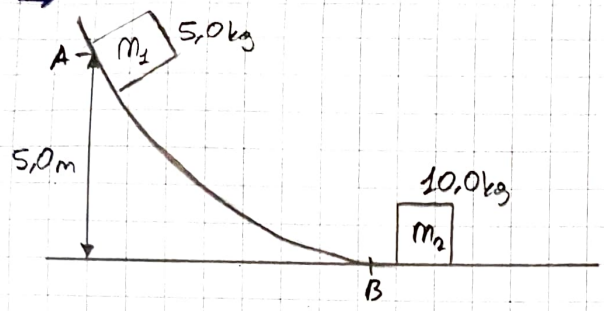
$$K = U$$

$$\frac{1}{2}M\left(\frac{mv}{2M}\right)^2 = 2MgL$$

$$v = \frac{4M\sqrt{gL}}{m}$$

$$\vec{v}_x = \frac{m \cdot \frac{4M\sqrt{gL}}{m}}{2M}$$

$$= 2\sqrt{gL}$$



Cisimler sürtünmesiz düzlemde elastik çarpışma yapıyor. Çarpışmadan sonra m_1'in çıkabileceği maximum yükseklik kaçtır?

A: $U = 5{,}0\,kg \cdot 5{,}0\,m \cdot 10\,m/s^2$
$= 250\,J$

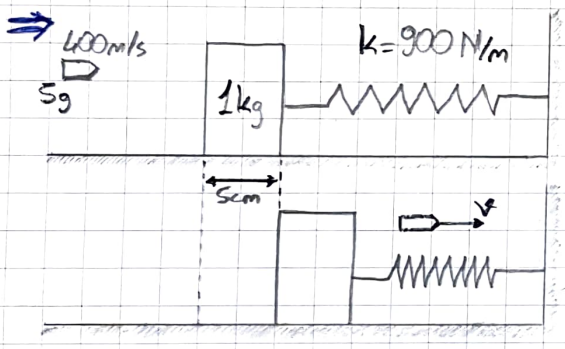
B: $\frac{1}{2}\,5{,}0\,kg\,v^2 = 250\,J$
$v = 10{,}0\,m/s$

$v_{1i} = 10{,}0\,m/s$, $v_{2i} = 0$

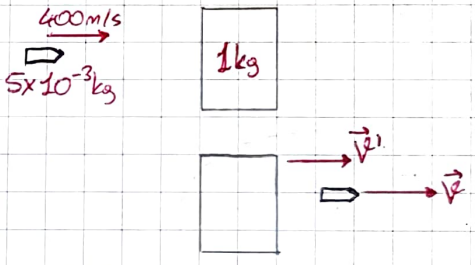
$$v_{1s} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$
$$v_{2s} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

$\rightarrow$ $v_{1s} = -\frac{10}{3}\,m/s$, $v_{2s} = \frac{20}{3}\,m/s$

$K = U \Rightarrow \frac{1}{2}\,5{,}0\,kg \cdot \left(\frac{10}{3}\,m/s\right)^2 = 5{,}0\,kg \cdot 10\,m/s \cdot h \Rightarrow h = 5/9\,m$



Mermi'nin bloktan çıktıktan sonraki hızı ve çarpışma sırasında kaybedilen enerji nedir? (Kütle kaybı ihmal ediliyor)



- $P_{ilk} = 5\times10^{-3}\,kg \times 400\,m/s = 2\,kg\,m/s$
  $P_{son} = 5\times10^{-3}\,kg \cdot v + 1\,kg \cdot v'$

$U_{yay} = \frac{1}{2}\,900\,N/m \cdot 25\times10^{-4}\,m^2$
$= K = \frac{1}{2}\,1\,kg \cdot v'^2$
$v' = 1{,}5\,m/s$

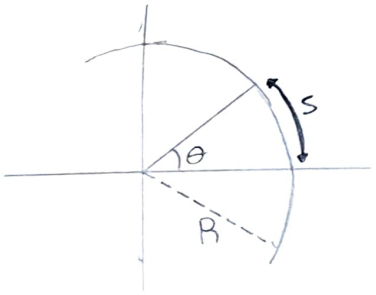
$P_{ilk} = P_{son}$
$2\,kg\,m/s = 5\times10^{-3}\,kg \cdot v + 1\,kg \cdot 1{,}5\,m/s$
$0{,}5\,kg\,m/s = 5\times10^{-3}\,kg\,v$
$v = 100\,m/s$

- $E_{ilk} = \frac{1}{2} \cdot 5\times10^{-3}\,kg\,(4\cdot10^2)^2$
$= 400\,J$

$E_{son} = \frac{1}{2}\,5\times10^{-3}\,kg\,(1\times10^2)^2 + \frac{1}{2}\,900\,N/m\,(5\times10^{-3})^2$
$\cong 26$

Enerji kaybı: $E_{son} - E_{ilk} = -374\,J$ kayıp

Radyan:



$\theta = \frac{s}{R}$ (rad)

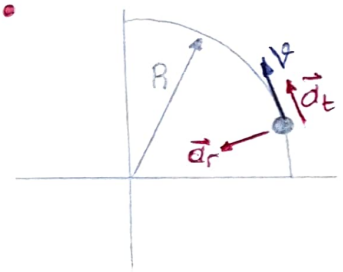
360°de 2π'ye karşılık gelir.

Açısal Hız: $\omega = \frac{d\theta}{dt}$ (rad/s)

Açısal İvme: $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ (rad/s²)

Hareket Denklemlerine Benziyor

$$\begin{cases} \theta_s = \theta_i + \omega_i \cdot t + \frac{1}{2}\alpha t^2 \\ \omega_s = \omega_i + \alpha t \\ \omega_s^2 = \omega_i^2 + 2\alpha\Delta\theta \end{cases}$$



$v = \frac{ds}{dt}$

$= \frac{d}{dt}(R\theta)$

$= R\frac{d\theta}{dt}$

$v = R\omega$

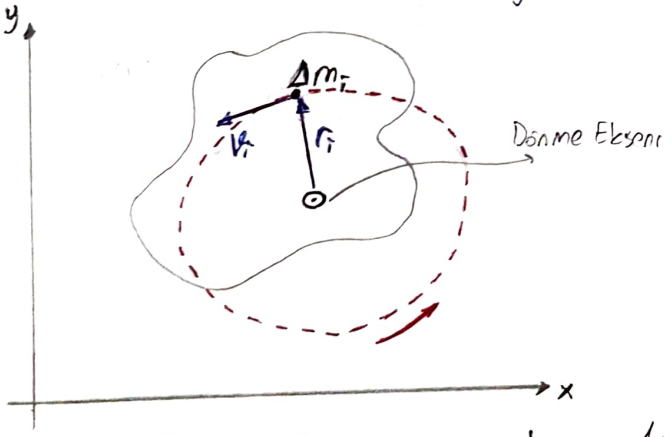
$a_t = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = R\frac{d\omega}{dt}$

$a_t = R\alpha$

$a_r = \frac{v^2}{R} = \frac{R^2\omega^2}{R}$

$a_r = R\cdot\omega^2$

Dönen Bir Cismin Kinetik Enerjisi:



$\Delta K_i = \frac{1}{2}\Delta m_i v_i^2$

$\sum K_i = \frac{1}{2}\sum_i \Delta m_i v_i^2$

$v_i = r_i\omega$

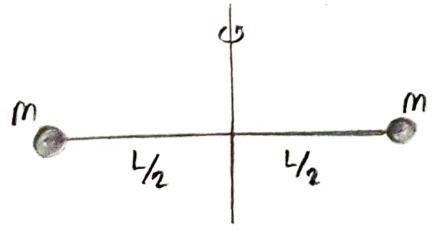
$\sum K_i = \frac{1}{2}\sum_i \Delta m_i r_i^2 \omega^2$

Dönme Kinetik Enerjisi: $K_D = \lim_{\Delta m_i \to 0} \frac{1}{2}\underbrace{\sum_i \Delta m_i r_i^2}_{I:\text{Eylemsizlik}}\omega^2$

Parçacıklardan Oluşan Sistem İçin: $I = \sum_i m_i r_i^2$

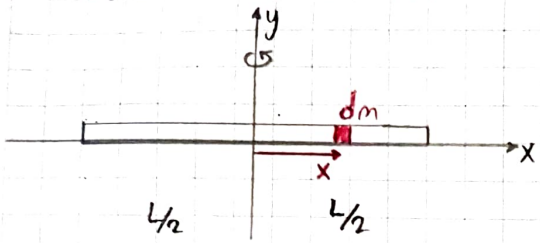
Sürekli Cisimler İçin: $I = \int r^2 dm$ $\quad K_D = \frac{1}{2} I \omega^2$

- (Parçacık)



$I = \frac{mL^2}{4} + \frac{mL^2}{4} = \frac{mL^2}{2}$

• (Kütle Merkezine Göre)



Uzunluklu, M kütleli

$$I = \int r^2 dm \qquad \lambda = \frac{M}{L} \qquad dm = \lambda dx$$

$$I = \int_{-L/2}^{L/2} x^2 \lambda dx$$

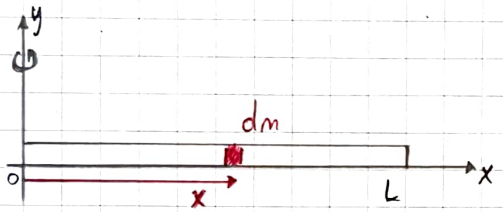
$$= \lambda \frac{x^3}{3} \Big|_{-L/2}^{L/2}$$

$$= \lambda \left[\frac{L^3}{24} + \frac{L^3}{24} \right]$$

$$= \frac{M}{L} \cdot \frac{L^3}{12}$$

$$I_{KM} = \frac{1}{12} M L^2$$

•



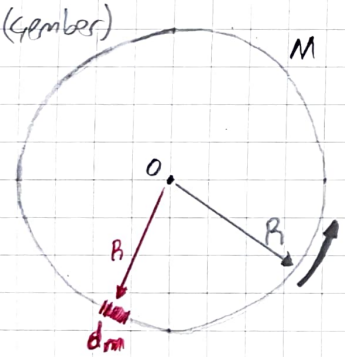
$$\lambda = \frac{M}{L}$$

$$dm = \lambda dx$$

$$I = \int_0^L x^2 \lambda dx$$

$$= \lambda \frac{x^3}{3} \Big|_0^L = \frac{1}{3} M L^2$$

• (Gemberi)



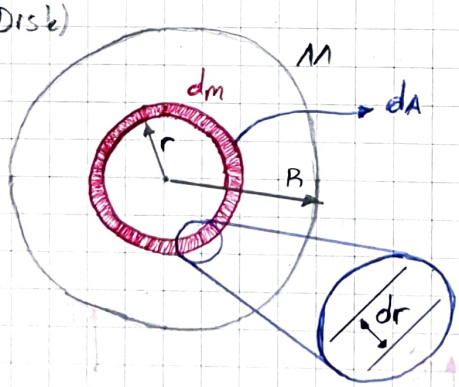
Kütle merkezinden dik geçen eksen tarafından döndürülüyor. Gemberin eylemsizlik momenti nedir?

$$I_{KM} = \int R^2 dm$$

$$= R^2 \int dm$$

$$= M R^2$$

• (Disk)



$$\sigma = \frac{M}{\pi R^2}$$

$$dA = 2\pi r dr$$

$$dm = \sigma dA$$

$$= \sigma 2\pi r dr$$

$$I = \int r^2 dm$$

$$= \int_0^R r^2 \sigma 2\pi r dr$$

$$= 2\pi\sigma \frac{r^4}{4} \Big|_0^R$$

$$= 2\pi \frac{M}{\pi R^2} \cdot \frac{R^4}{4}$$

$$= \frac{1}{2} M R^2$$

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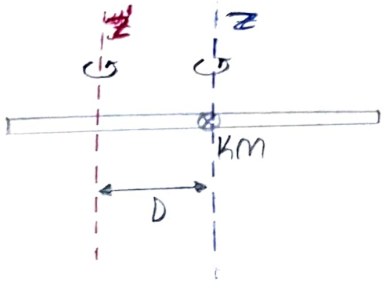
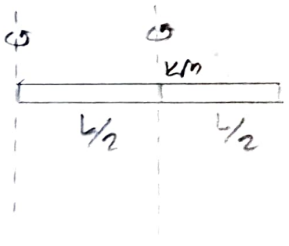
Paralel Eksenler Teoremi



$$I' = I_{KM} + MD^2$$

$$I_{KM} = \frac{1}{12} ML^2$$

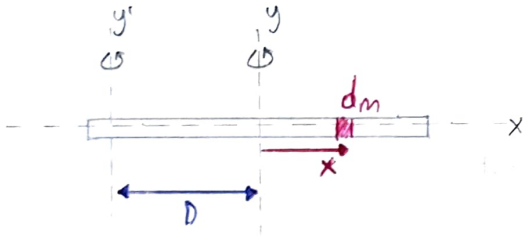


$$I' = \frac{1}{12} ML^2 + \frac{ML^2}{4}$$

$$= \frac{1}{3} ML^2$$

(Kütle merkezini hesapla sonra diğer eksene geç.)

- (İspat)



$$I_{KM} = \int x^2 dm$$

$$I' = \int (x+D)^2 dm$$

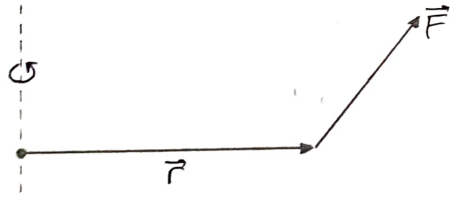
$$I' = \int (x^2 + 2xD + D^2)\, dm$$

$$= \underbrace{\int x^2 dm}_{I_{KM}} + \underbrace{\int 2xD\, dm}_{0} + \underbrace{\int D^2 dm}_{MD^2}$$

$$\hookrightarrow 2D \int x\, dm$$

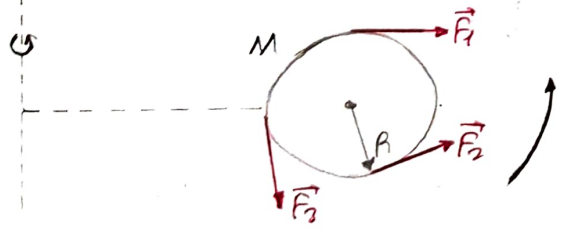
$$I' = I_{KM} + MD^2$$

TORK



$$\vec{\tau} = \vec{r} \times \vec{F}$$

- (makara)



$$\sum \tau = RF$$

$$= R.Ma$$

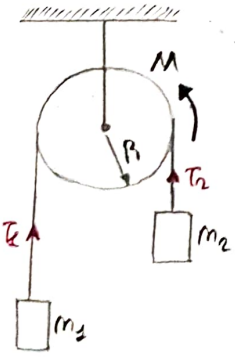
$$= RMR\alpha$$

$$= (MR^2)\alpha$$

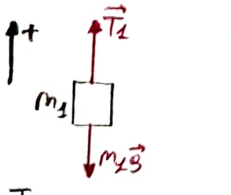
$$= I\alpha$$

$$\sum \tau = I\alpha$$

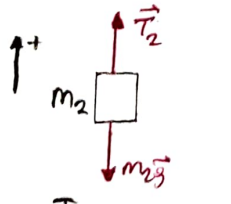
$\Rightarrow$



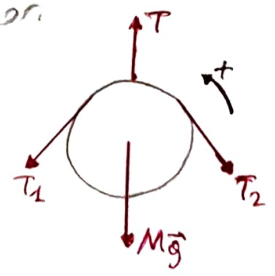
$m_1 > m_2$, ip makaradan kaymadan dönüyor.



$$T_1 - m_1 g = -m_1 a$$



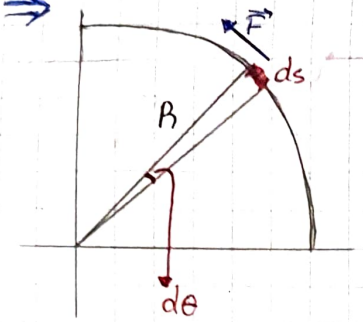
$$T_2 - m_2 g = m_2 a$$



$$T - Mg - T_1 - T_2 = 0$$

$$T_1 R - T_2 R = I\alpha$$

$$a = R\alpha$$



$dW = \vec{F} \cdot d\vec{s}$

$ds = R\,d\theta$

$dW = F \cdot R\,d\theta$

$dW = \tau\, d\theta$

$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta$

$\tau = I\alpha$

$= I \frac{d\omega}{dt}$

$= I \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt}$

$W = \int I \frac{d\theta}{dt} \cdot \frac{d\omega}{d\theta} \cdot d\theta$ ($\frac{d\theta}{dt} = \omega$)

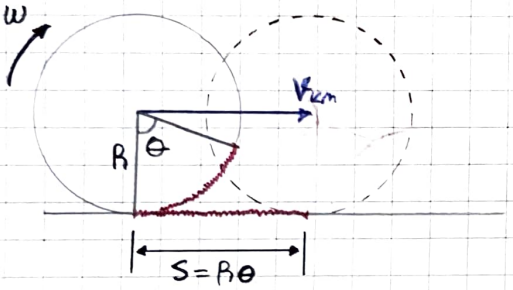
$W = \int_{\omega_1}^{\omega_2} I\,\omega\, d\omega$

$W = \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2$

Güç: $P = \frac{dW}{dt} = \tau \frac{d\theta}{dt}$

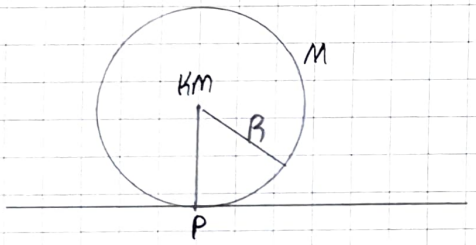
$= \tau\,\omega$

Kaymadan Yuvarlanma



$V_{km} = \frac{ds}{dt} = \frac{d(R\theta)}{dt} = R\omega$

$a_{km} = \frac{dV_{km}}{dt} = R\alpha$

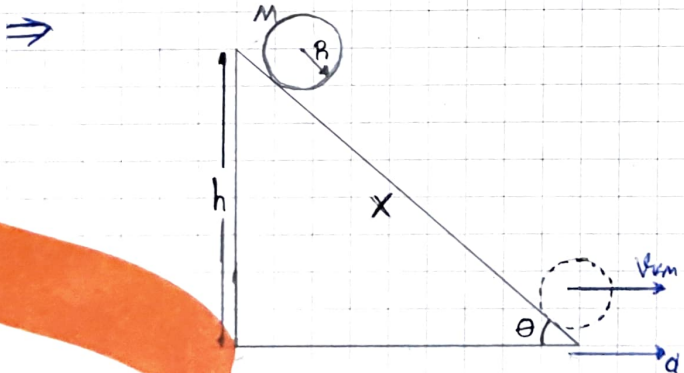


$K = \frac{1}{2} \cdot I_p \cdot \omega^2$

Paralel Eksenler Teoremi; $I_p = I_{km} + MR^2$

$K = \frac{1}{2} I_{km} \omega^2 + \frac{1}{2} MR^2 \omega^2$

$K = \frac{1}{2} I_{km} \omega^2 + \frac{1}{2} M V_{km}^2$ $\quad V_{km} = R \cdot \omega$



Cisim kaymadan yuvarlanıyor. Cismin kütle merkezinin hızı (V_{km}) ve cismin ivmesi nedir?
($I_{km} = \frac{2}{5} MR^2$)

Kinematik Denklemler ile Çözüm;

$E_{ilk} = E_{son}$

$Mgh = \frac{1}{2} (I_{km} \omega^2 + M V_{km}^2)$

$V_{km} = R \cdot \omega$

$Mgh = \frac{1}{2} \left(\frac{I_{km}}{R^2} + M \right) V_{km}^2$

$V_{km} = \left(\frac{2gh}{1 + I_{km}/MR^2} \right)^{1/2} = \left(\frac{10}{7} gh \right)^{1/2}$

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a → $v_s^2 = v_i^2 + 2a\Delta x$

$v_{km}^2 = 2ax$

$a = \frac{v_{km}^2}{2x} = \frac{10}{7}\cdot\frac{gh}{x} = \frac{10}{7} g\sin\theta$

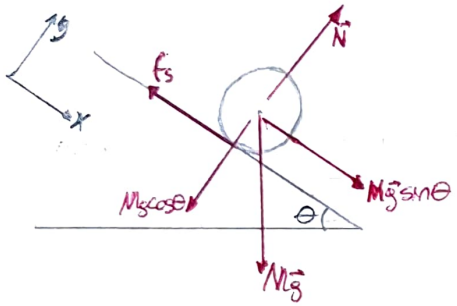
Sabit Gider

Olduğu Yerde Döner

Yuvarlanıyor (Sürtünme enerji kaybı değil Yuvarlanmasını sağlıyor)

NOT: $v_{km} = \sqrt{\frac{10}{7} gx\sin\theta}$ yuvarlanma olmasaydı $\sqrt{2gx\sin\theta}$ olurdu.

Dinamik Olarak Çözem;



$Mg\sin\theta - f_s = Ma_{km}$

$N - Mg\cos\theta = 0$

$\tau_{km} = f_s . R = I_{km} . \alpha$

$I_{km} = \frac{2}{5} MR^2$

$a_{km} = \alpha . R$

$f_s = \frac{I_{km}.\alpha}{R} = \frac{\frac{2}{5}MR^2 . a_{km}/R}{R} = \frac{2}{5} Ma_{km}$

$v_s^2 = v_i^2 + 2ax$

$v_s^2 = 2\cdot\frac{5g}{7} x\sin\theta$

$= \frac{10}{7} gx\sin\theta$

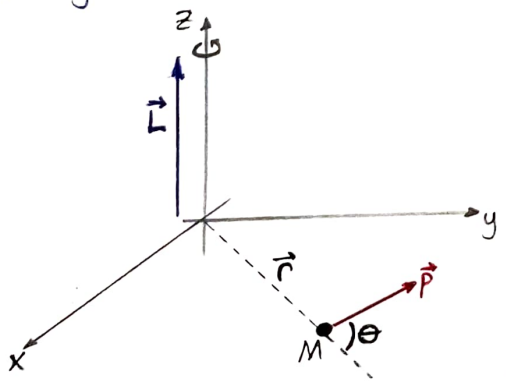
$Mg\sin\theta = Ma_{km} + \frac{2}{5} Ma_{km}$ (f_s)

$Mg\sin\theta = \frac{7}{5} Ma_{km}$

$a_{km} = \frac{5}{7} g\sin\theta$

Parçacığın Açısal Momentumu

$\vec{L} \equiv \vec{r} \times \vec{P}$



$L = rp\sin\theta$

$= mvr\sin\theta$

$\sum\tau = \vec{r} \times \sum\vec{F}$

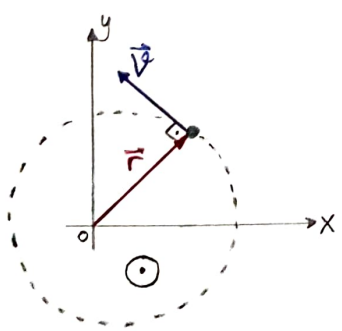
$= \vec{r} \times \frac{d\vec{P}}{dt}$

$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p})$

$= \vec{r} \times \frac{d\vec{P}}{dt} + \frac{d\vec{r}}{dt} \times \vec{P}$ → 0

$\sum\tau = \frac{d\vec{L}}{dt}$ sisteme dışarıdan tork etki etmiyorsa (sıfır ise) L zamanla değişmiyordur. Ve açısal momentum korunur.

⇒ O noktasına göre açısal momentumu bulunuz?



$L = rmv\sin 90°$

$= mvr$

$\vec{L} = mvr\hat{k}$

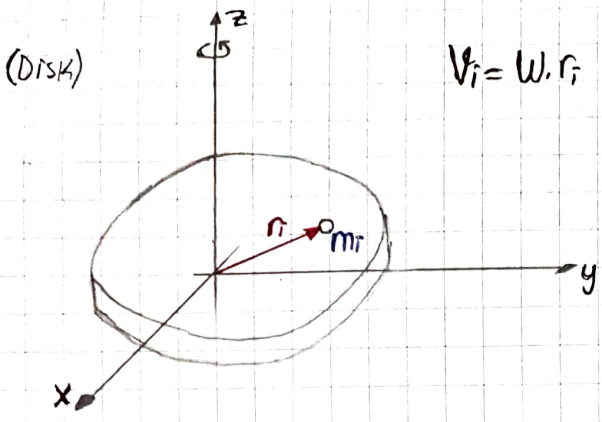
$v = \omega . r$

$\vec{L} = mr^2 . \omega\hat{k}$

$\vec{L} = I_o\omega\hat{k}$

Dönen Katı Cismin Açısal Momentumu

(DİSK)



$V_i = \omega . r_i$

$L_i = m_i . r_i^2 . \omega$

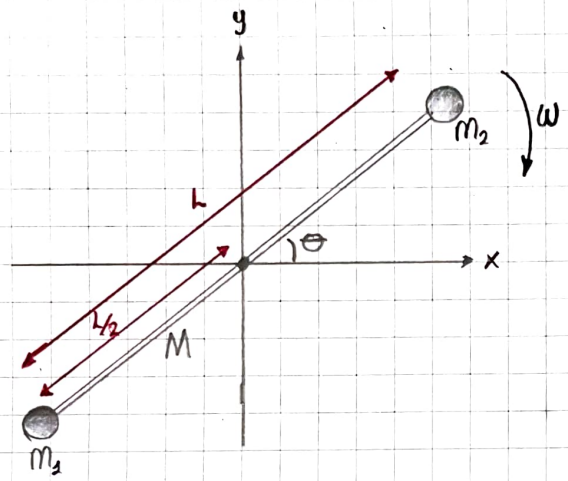
$\vec{L} = \sum_i L_i$

$= \left(\sum_i m_i r_i^2\right) \omega$

I

$L = I . \omega$

$\Rightarrow$



M kütleli çubuğa m_1 ve m_2 kütleli cisimler bağlanıyor. Çubuğun boyunun yarısı kadarlık noktadan ω açısal hızı ile sistem döndürülüyor. Sistemin açısal momentumu kaçtır?

Parçacık olarak kabul edildiğinde;

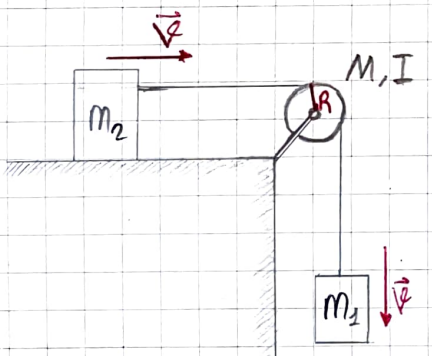
$I_1 = m_1 . \frac{L^2}{4}$ $\qquad L_1 = I_1 . \omega$

$I_2 = m_2 . \frac{L^2}{4}$ $\qquad L_2 = I_2 . \omega$

$I_{çubuk} = \frac{1}{12} M L^2$ $\qquad L_3 = I_{çubuk} . \omega$

$L = \sum_i L_i = L_1 + L_2 + L_3$

$\Rightarrow$



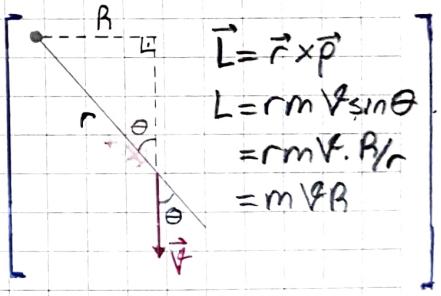
Sistem serbest bırakıldığında tork ve açısal momentum korunumlarını kullanarak sistemin ivmesini bulunuz?

$\sum \tau_{dış} = \frac{d\vec{L}}{dt}$

$L = m_1 V R + m_2 V R + I . \omega$

$\omega = \frac{V}{R}$

(Herhangi bir anda hızlarını V aldık) $\quad L = m_1 V R + m_2 V R + \frac{I V}{R}$



$\vec{L} = \vec{r} \times \vec{p}$

$L = r m V \sin\theta$

$= r m V . R/r$

$= m V R$

$m_1 g R = \frac{d}{dt}\left(m_1 V R + m_2 V R + I/R\, V\right)$

Simdeside oluşturan dış tork

$= \left(m_1 R + m_2 R + \frac{I}{R}\right) \frac{dV}{dt}$

a

$a = \frac{m_2 g R}{m_1 R + m_2 R + I/R}$

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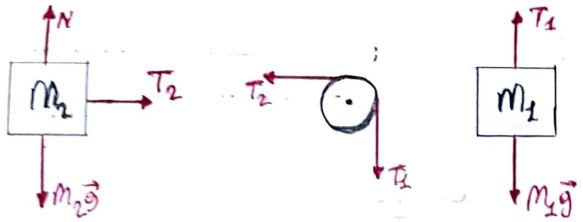
Dinamikteki



$T_2 = m_2 a$ $\quad$ $T_1 R - T_2 R = I\alpha$

$T_1 = m_1 g - m_1 a$ $\quad$ $m_1 g - T_1 = m_1 a$

$a = \alpha R$

$$(m_1 g - m_1 a)R - m_2 aR = I\frac{a}{R}$$

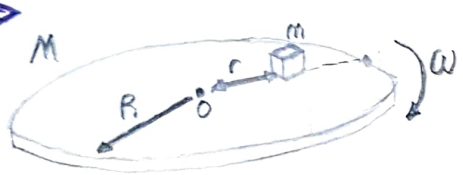
$$m_1 gR = (m_1 R + m_2 R + I/R)a$$

$$a = \frac{m_1 gR}{m_1 R + m_2 R + I/R}$$

Açısal Momentumun Korunumu

$$\sum \tau_{dış} = \frac{d\vec{L}}{dt} \qquad \sum \tau_{dış} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \qquad \vec{L}_{son} = \vec{L}_{ilk}$$

⇒



$M = 100kg$, $m = 60kg$, $R = 2,0m$, $W = 2\ rad/s$

Cisim en uç kenarda iken $W = 2rad/s$ ile dönen plakanın üzerinde merkeze doğru ilerliyor. Merkezden uzaklığı 0,5m olduğunda açısal ivme kaçtır? (ivmelenme yok)

$L_{ilk} = L_{son}$

$I_{ilk} \cdot W_{ilk} = I_{son} \cdot W_{son}$

$I_{ilk} = \frac{1}{2} MR^2 + mR^2$

$I_{son} = \frac{1}{2} MR^2 + mr^2$

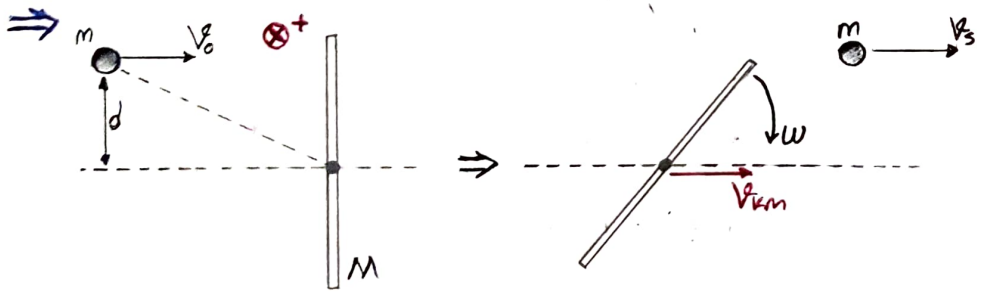
$$(\tfrac{1}{2}MR^2 + mR^2)W_{ilk} = (\tfrac{1}{2}MR^2 + mr^2)W_{son}$$

$$W_{son} = \frac{(\frac{1}{2}MR^2 + MR^2)W_{ilk}}{(\frac{1}{2}MR^2 + mr^2)}$$

$$W_{son} = \frac{\frac{1}{2}100kg \cdot 4m^2 + 60kg \cdot 4m^2}{\frac{1}{2}100kg \cdot 4m^2 + 60kg \cdot 0,25m^2} \cdot 2,0\ rad/sn$$

$$= \frac{440}{215} \cdot 2,0 \cong 4\ rad/s$$

⇒



$m = 2kg$, $M = 1kg$, $V_o = 3m/s$

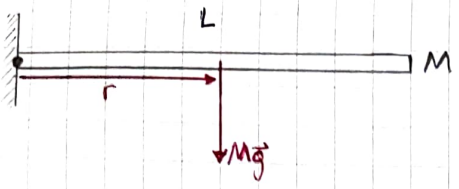
$I_{km} = 1,33\ kg\,m^2$, $d = 2m$

Top ile çubuk yatay düzlemde elastik çarpışma yapıyor. V_s ve W kaçtır?

$\vec{L}_I = \vec{L}_s$ $\qquad$ $m V_o d = m V_s d + I W$

$\vec{P}_I = \vec{P}_s$ $\qquad$ $m V_o = m V_s + M V_{km}$

$K_I = K_s$ $\qquad$ $\frac{1}{2} m V_o^2 = \frac{1}{2} m V_s^2 + \frac{1}{2} M V_{km}^2 + \frac{1}{2} I W^2$



Kütle merkezinin açısal ivmesi:

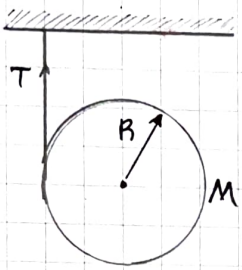
$\vec{\tau} = \vec{r} \times \vec{F}$ $\quad Mg\frac{L}{2} = I\alpha$ $\quad Mg\frac{L}{2} = \frac{1}{3}ML^2\alpha$

$\tau = Mg\frac{L}{2}$ $\quad I = \frac{1}{3}ML^2$ $\quad \alpha = \frac{3g}{2L}$

Kütle merkezinin lineer ivmesi:

$a_{km} = \alpha \cdot \frac{L}{2} \Rightarrow a_{km} = \frac{3g}{2L} \cdot \frac{L}{2} = \frac{3g}{4}$

$\Rightarrow$



Tavana sabitlenmiş bir yoyo serbest bırakılıyor. Buna göre;

a) $T = ?$

$I = \frac{1}{2}MR^2$

$Mg - T = Ma$

$TR = I\alpha$

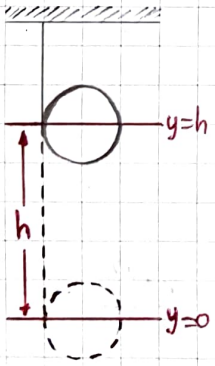
$a = \alpha \cdot R \Rightarrow \alpha = \frac{a}{R}$

$TR = I \cdot \frac{a}{R} \Rightarrow a = \frac{T \cdot R^2}{I}$

$Mg - T = \frac{M \cdot T \cdot R^2}{I} = \frac{M \cdot R^2 \cdot T}{\frac{1}{2}MR^2}$

$3T = Mg \Rightarrow T = \frac{Mg}{3}$

b) Disk h kadar düştüğünde kütle merkezinin çizgisel hızı nedir?



Enerjiden; $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$\omega = \frac{v}{R}$ $\quad I = \frac{1}{2}MR^2$

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2$

$v = \sqrt{4gh/3}$

Kinematikten; $a = \frac{2T}{M} = \frac{2Mg}{3M} = \frac{2}{3}g$

$v_s^2 = v_i^2 + 2aX$

$v_s^2 = 2 \cdot \frac{2g}{3}h$

$v_s = \sqrt{\frac{4gh}{3}}$

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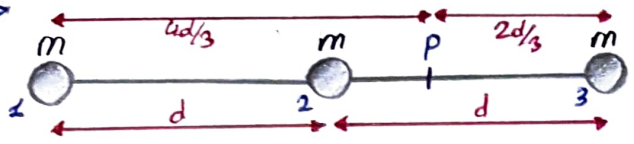


Tavana bağlı çubuk P noktasından asılıyor ve serbest bırakılıyor. Buna göre;

a) Sisteme etki eden tork = ?

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau_1 = mg\,4d/3 \quad \odot$$

$$\tau_2 = mg\,d/3 \quad \odot$$

$$\tau_3 = mg\,2d/3 \quad \otimes$$

$$\tau = \tau_1 + \tau_2 + \tau_3$$

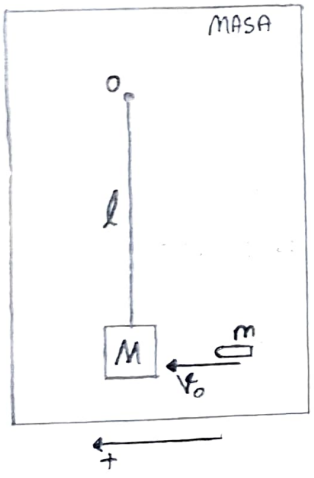
$$= mgd \quad \odot$$

b) Atıldığı andaki açısal ivmesi = ?

$$\sum \vec{\tau} = I.\alpha$$

$$I = m\frac{16d^2}{9} + m\frac{d^2}{9} + m\frac{4d^2}{9} = \frac{7}{3}md^2$$

$$mgd = \frac{7}{3}md^2.\alpha \qquad \alpha = \frac{3g}{7d}$$



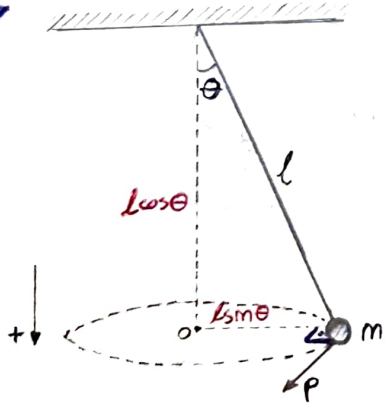
Yatay düzlemde masa üzerinde bir ucu sabitlenmiş çubuğa M kütleli cisim perçinleniyor. Mermi ile blok sisteminin açısal momentumu nedir? (Masada sürtünme yok)

$$P:\ mV_0 = (m+M).V$$

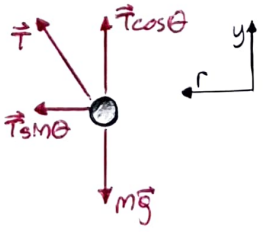
$$V = \frac{mV_0}{(m+M)}$$

$$\vec{L} = \vec{r} \times \vec{P}$$

$$= \ell m V_0 \quad \otimes$$



Topun döndüğü çemberin merkezine göre açısal momentumu nedir?



$$r = \ell \sin\theta$$

$$T\cos\theta - mg = 0$$

$$T\sin\theta = \frac{mV^2}{\ell\sin\theta}$$

$$T = \frac{mg}{\cos\theta}$$

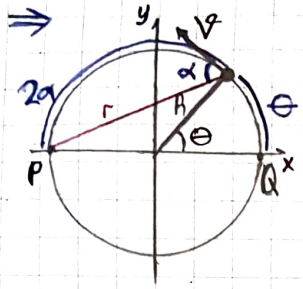
$$\frac{mg.\sin\theta}{\cos\theta} = \frac{mV^2}{\ell\sin\theta}$$

$$V^2 = g\ell.\frac{\sin^2\theta}{\cos\theta}$$

$$\vec{L} = \vec{r} \times \vec{P} \Rightarrow L = rP = rmV$$

$$L = \ell\sin\theta.m\left(g.\ell.\frac{\sin^2\theta}{\cos\theta}\right)^{1/2}$$

⇒



Hareket Q noktasından başlayan sabit hızlı parçacığın P noktasına göre açısal momentumunun zamana bağlı fonksiyonunu yazınız?

$\vec{L} = r P \sin\alpha \; \hat{k}$

$W = \frac{V}{R} \qquad Q = W.t \qquad Q = \frac{V}{R}.t$

$\theta + 2\alpha = \pi$

$\alpha = \frac{\pi}{2} - \frac{\theta}{2}$

$\vec{L} = r m V \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$

$= r m V \cos\left(\frac{\theta}{2}\right)$

$= r m V \cos\left(\frac{V}{2R} t\right)$

$r^2 = R^2 + R^2 - 2.R.R.\cos(180-\theta)$

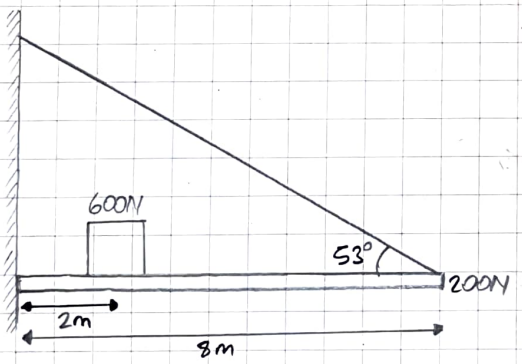
$r^2 = R^2 + R^2 + 2R^2.\cos\theta$

$r^2 = 2R^2.(1+\cos\theta)$

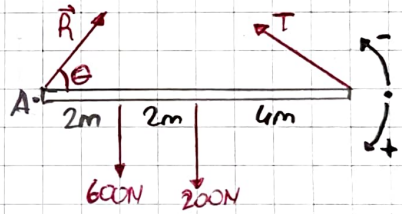
$r = R\sqrt{2(1+\cos\left(\frac{V}{R} t\right))}$

$L = R\sqrt{2\left[1+\cos\left(\frac{V}{R} t\right)\right]}\; m V \cos\left(\frac{V}{2R} t\right)$

⇒



Çubuğun duvara yapmış olduğu bileşke kuvvet ve bu kuvvetin çubuk ile arasındaki açı kaçtır? (Çubuk homojen)



$A.\sum\vec{\tau} = 0$

$600.2 + 200.4 - T\sin 53.8 = 0$

$T.6,4 = 2000N$

$T = 312,5N$

$\sum\vec{F} = 0$

$X: R\cos\theta - T\cos 53 = 0$

$y: R\sin\theta + T\sin 53 - 800N = 0$

$R\cos\theta = 187,5N$

$R\sin\theta = 232,5N$

$\tan\theta = \frac{232,5N}{187,5N} = 1,24$

$\theta = 51°$

$R = 297N$

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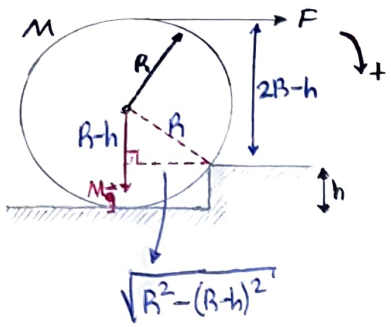
⇒

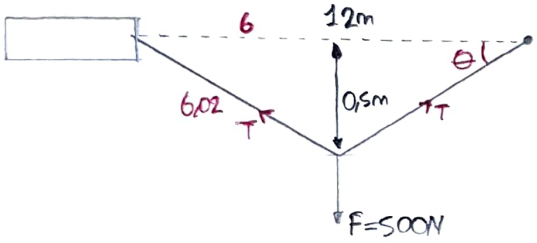


M kütleli topun basamaktan çıkması için gereken F kuvveti nedir?

$$F.(2R-h) - Mg\sqrt{R^2-(R-h)^2} = 0$$

$$F = \frac{Mg\sqrt{R^2-(R-h)^2}}{2R-h}$$

⇒



Esnek olmayan bir ipin tam ortasından kuvvet uygulanıyor. İpte oluşan gerilme kuvveti kaçtır?

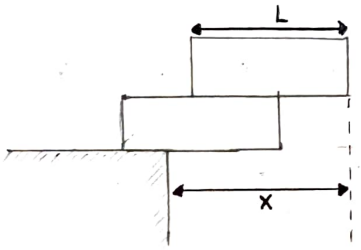
$$2T\sin\theta - 500N = 0$$

$$T\sin\theta = 250N$$

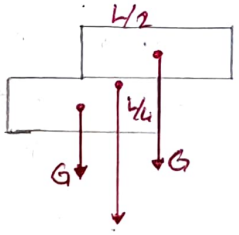
$$\sin\theta = \frac{0,5}{6,02} \qquad T = \frac{250.6,02}{0,5} = 3010N$$

$$T.\frac{6}{6,02} \Rightarrow 3010.\frac{6}{6,02} \simeq 3000$$

⇒

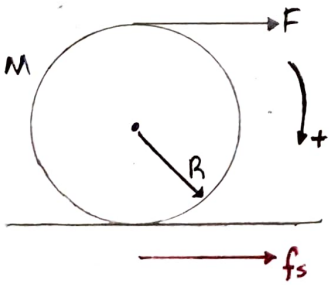


Özdeş cisimlerin düşmemesi için maximum x değeri nedir?



$$x = \frac{3L}{4}$$

⇒



F kuvveti ile tel bobin çekilince, kütlesi ve yarıçapı korunuyor. Kütle merkezinin ivmesi nedir?

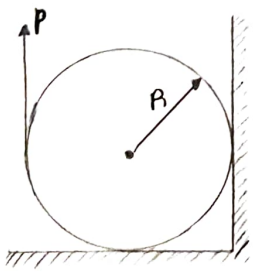
R/ $F + f_s = Ma$ $\qquad I = \frac{1}{2}MR^2$

$+ \quad FR - f_sR = I\alpha$

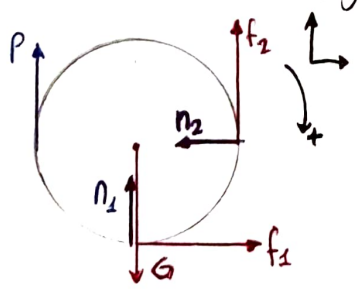
$$2FR = MaR + I\frac{a}{R}$$

$$a = \frac{2FR}{MR + \frac{1}{2}MR} = \frac{4F}{3M}$$

⇒



Büton yüzeyler arası statik sürtünme katsayısı 0,5 olduğuna göre P'nin kürenin ağırlığı olan G cinsinden değeri kaçtır?



$$\sum \vec{F} = 0$$

$$\sum \vec{\tau} = 0$$

$f_1 = n_1 = \mu_s \cdot n_1$

$P + f_2 + n_1 = G$

$PR - f_2 R - f_1 R = 0$

$P = f_1 + f_2$

$f_1 = \mu_s n_1$

$f_2 = \mu_s \cdot n_2$

$P = \mu_s (n_1 + n_2)$

$f_1 = \mu_s n_1$

$f_2 = \mu_s^2 n_1$

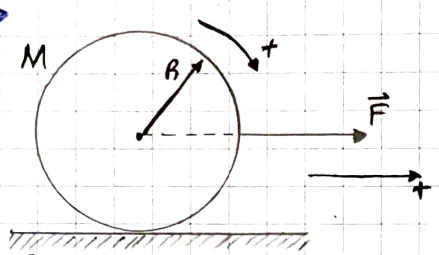
$\mu_s (n_1 + \mu_s n_1) + \mu_s^2 n_1 + n_1 = G$

$0{,}5(1{,}5 n_1) + 0{,}25 n_1 + n_1 = G$

$n_1 = G/2 \quad n_2 = G/4$

$P = 1/2 (G/2 + G/4) = 3G/8$

⇒



$(I_{km} = 1/2 MR^2)$

Silindirin merkezinden F kuvveti ile çekiliyor?
a) Kütle merkezinin ivmesi nedir?
b) Statik sürtünme katsayısı nedir?

a) $F - f_s = Ma$

$KM: f_s R = I\alpha$

$f_s = \mu Mg$

$\mu Mg R = I \frac{a}{R}$

$f_s = \frac{I\alpha}{R^2}$

$F - \frac{I\alpha}{R^2} = Ma$

$F = \left(\frac{I}{R^2} + M\right) a$

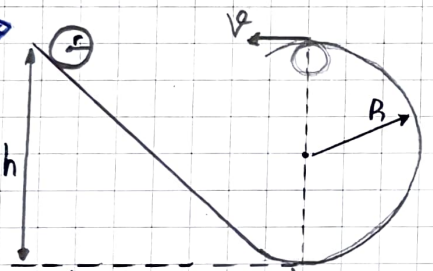
$F = 3/2 Ma$

$a_{km} = \frac{2F}{3M}$

b) $\mu M g R = 1/2 MR^2 \cdot \frac{2F}{3MR}$

$\mu = \frac{F}{3Mg}$

⇒



$(I_{km} = 2/5 MR^2)$

Küre kaymadan yuvarlanıyor. Büyük halkayı dönmek için h'ın R ve r cinsinden değeri nedir?

$mg(h+r) = mg(2R - r) + 1/2 mV^2 + 1/2 I\omega^2$

$V = \omega \cdot R$

- Minimum hızı merkezcil kuvvetin ağırlığına eşit olmalıdır.

$mg = \frac{mV^2}{(R-r)} \qquad mV^2 = mg(R-r)$

$mg(h+r) = mg(2R-r) + 1/2 mg(R-r) + 1/2 \cdot 2/5 mr^2 \frac{g(R-r)}{r^2}$

⇒

$m = 2$ konum grafiği $\vec{r} = 6\hat{i} + 5t\hat{j}$ olan cismin açısal momentum kaçtır?

$\vec{L}(t) = ?$

$\vec{L} = \vec{r} \times \vec{P}$

$\vec{P} = m\vec{V} = m\frac{d\vec{r}}{dt} = m5\hat{j}$ m/s

$\vec{L} = (6\hat{i} + 5t\hat{j}) \cdot (5m\hat{j})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 5t & 0 \\ 0 & 5m & 0 \end{vmatrix} = 30m\hat{k} = 60\hat{k}$

$m = 2$

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