ELEKTRİK DEVRELERİ - II

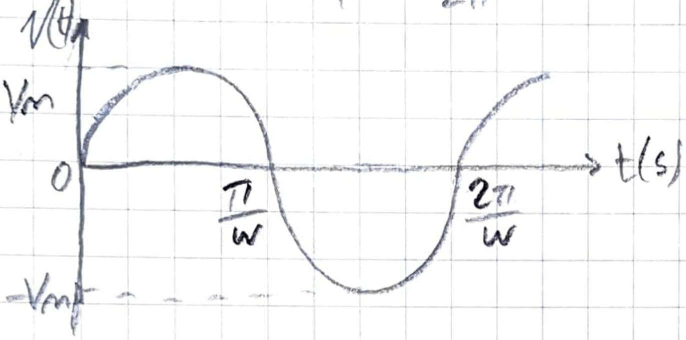
Sinüsoidal Kaynaklar ve Fazörler

$v(t) = V_m \sin wt$

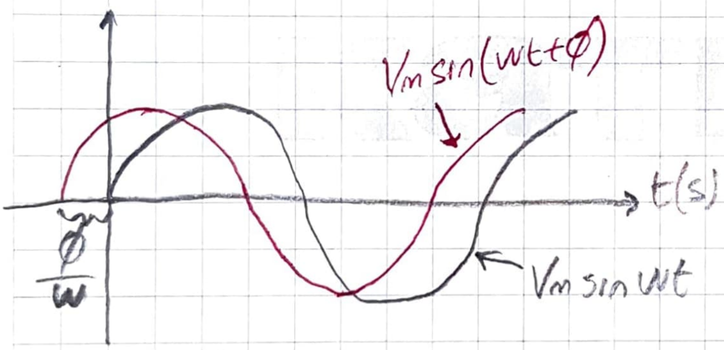
V_m: maximum (tepe) değer
w: açısal frekans (rad/s)

$v(t+T) = v(t)$ ise periyodiktir.

$T = \frac{2\pi}{w}$, $f = \frac{1}{T} = \frac{w}{2\pi}$, $w = 2\pi f$



En genel durumda; $v(t) = V_m \sin(wt + \phi)$



[t sıfırdaki değeri pozitif yada negatif olarak belirlenir.]

- $\cos\left(wt - \frac{\pi}{2}\right) = \sin wt$

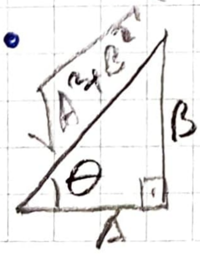
  $\sin\left(wt + \frac{\pi}{2}\right) = \cos wt$

  $\sin(wt \mp \pi) = -\sin wt$

  $\cos(wt \mp \pi) = -\cos wt$

  $\sin(wt \mp 2\pi) = \sin wt$

  $\cos(wt \mp 2\pi) = \cos wt$

- 

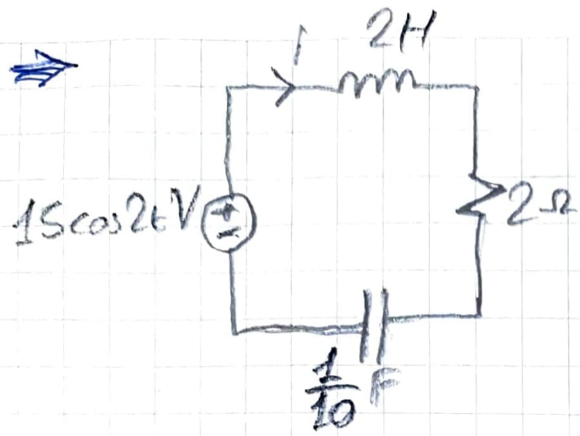
$$A\cos wt + B\sin wt = \sqrt{A^2+B^2}\left(\frac{A}{\sqrt{A^2+B^2}}\cos wt + \frac{B}{\sqrt{A^2+B^2}}\sin wt\right)$$

$$A\cos wt + B\sin wt = \sqrt{A^2+B^2}\left(\cos wt \cos\theta + \sin wt \sin\theta\right)$$

$$A\cos wt + B\sin wt = \sqrt{A^2+B^2}\cos(wt - \theta)$$

$$\theta = \tan^{-1}\frac{B}{A}$$

⇛



KVL

$$2\frac{di}{dt} + 2i + 10\int_{0+}^{t} i(\tau)\,d\tau + V_c(0+) = 15\cos 2t$$

Integralden kurtulmak için türev al

$$\frac{d^2i}{dt^2} + \frac{di}{dt} + 5i = -15\sin 2t$$

Yerine koy

$$i_f = A\cos 2t + B\sin 2t$$

$$(-4A\cos 2t - 4B\sin 2t) + (-2A\sin 2t + 2B\cos 2t)$$
$$+5(A\cos 2t + B\sin 2t) = -15\sin 2t$$

Katsayı eşitle

$A + 2B = 0$ $\qquad A = 6$

$-2A + B = -15$ $\qquad B = -3$

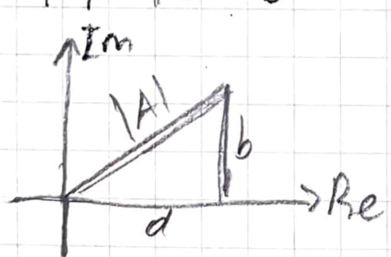
$$i_f = 6\cos 2t - 3\sin 2t \text{ A}$$
$$i_f = 3\sqrt{5}\cos(2t + 26{,}6^\circ)\text{ A}$$

Komplex Kaynaklar

$A = a + jb$ $\qquad i = j$

$A = |A|\angle\alpha$

$|A| = \sqrt{a^2+b^2}$, $\alpha = \tan^{-1}\frac{b}{a}$



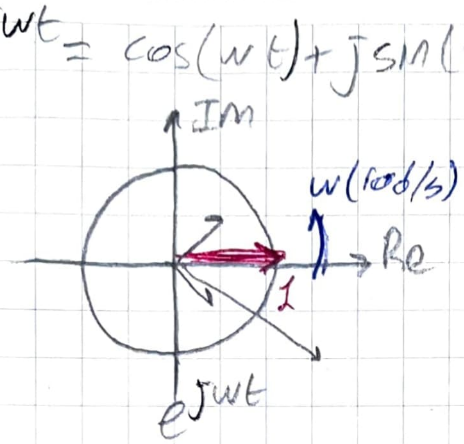
$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{j\theta} = 1\angle\theta$

$V_m e^{j(wt+\phi)}$ kompleks sayısının;

$$Re\left[V_m e^{j(wt+\phi)}\right] = V_m\cos(wt+\phi)$$
$$Im\left[V_m e^{j(wt+\phi)}\right] = V_m\sin(wt+\phi)$$

• $e^{jwt} = \cos(wt) + j\sin(wt)$



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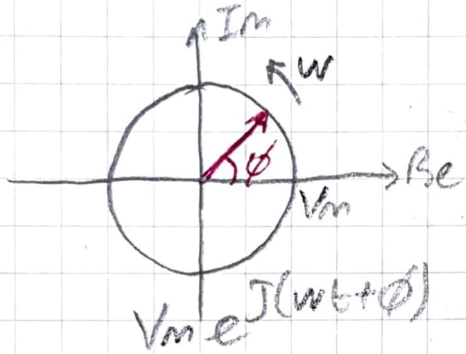


Fazörler

$V_g(t) = V_m . \cos(wt+\phi)$ > karşılaştır

$V_{g1}(t) = V_m . e^{j(wt+\phi)}$

$V_{g1}(t) = V_m e^{j\phi} . e^{jwt} = \bar{V} e^{jwt}$

$\bar{V}$: fazör değer

$\bar{V} = V_m e^{j\phi}$

$i(t) = \bar{I} e^{jwt}$

$v(t) = \bar{V} e^{jwt}$

Kaynak	Kompleks Kaynak	Fazör Kaynak
$A\cos(wt+\phi)$	$Ae^{j(wt+\phi)}$	$A\angle\phi$
$A\sin(wt+\phi)$	$Ae^{j(wt+\phi-90°)}$	$A\angle\phi-90°$

$\downarrow$

$A\cos(wt+\phi-90°)$

[Kafa karışmaması adına biz sadece cosinüs'ü fazöre dönüştürüp geri çapıracağız]

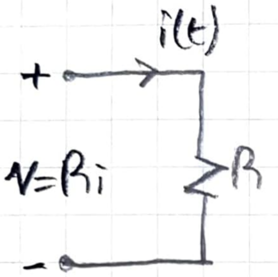
Fazörler İçin Akım ve Gerilim Kuralları

$$v(t) = \bar{V} e^{j\omega t}$$
$$i(t) = \bar{I} e^{j\omega t}$$

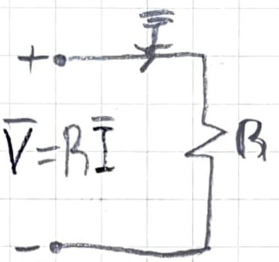
$$\bar{V} = |\bar{V}| \angle \phi_v$$
$$\bar{I} = |\bar{I}| \angle \phi_i$$

Ohm Kanunu; $\bar{V} e^{j\omega t} = R \bar{I} e^{j\omega t}$

$$\boxed{\bar{V} = R\bar{I}}$$



zaman domeni

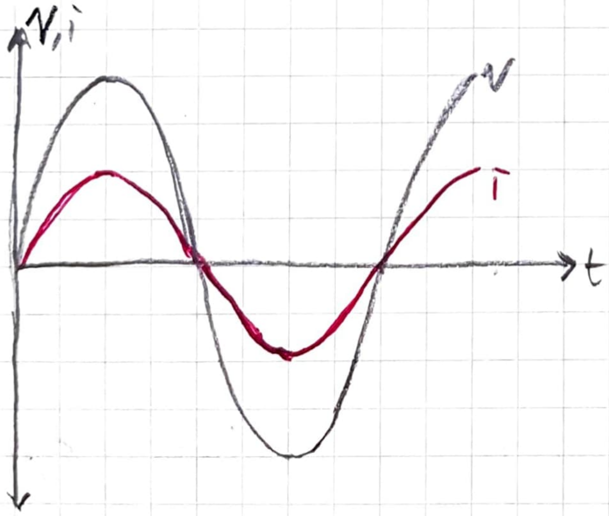


frekans domeni
(jw domeni)

- $v(t) = 12\cos(100t + 30°)\,V$, $R = 3\,\Omega$ ise

$$\bar{I} = \frac{\bar{V}}{R} = \frac{12\angle 30°}{3} = 4\angle 30°\,A$$

$$i(t) = 4\cos(100t + 30°)\,A$$



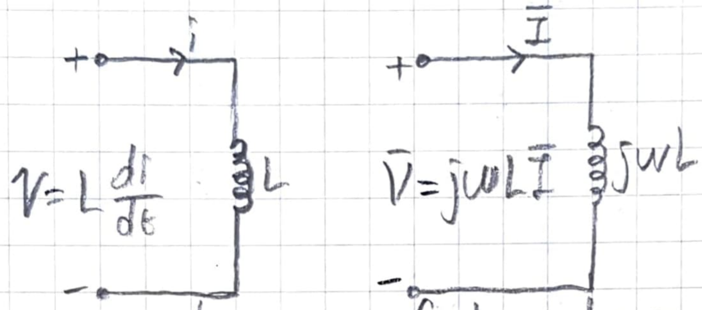
Faz farkı yok her ikisi de aynı açıda,
sadece 3'te 1'lik genlik farkı var.

Bobin Olursa; $v = L\dfrac{di}{dt}$

$$V_m e^{j(wt+\phi)} = L\frac{d}{dt}\left[I_m e^{j(wt+\phi_i)}\right]$$

$$= jwL\left[I_m e^{j(wt+\phi_i)}\right]$$

$$\boxed{\bar{V} = jwL\bar{I}}$$



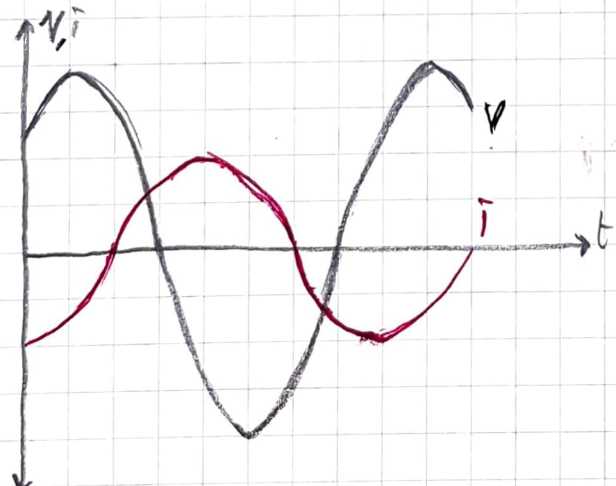
zaman domeni   frekans domeni

- $i = I_m\cos(wt+\phi_i)$ ise

$\bar{V} = jwL\bar{I}$

$\bar{V} = jwLI_m\angle\phi_i$

$\bar{V} = wLI_m\underline{\angle\phi_i+90°}$



$j = 1\underline{\angle 90°}$ oldugundan $90°$ ileride.

ilk olarak gerilim goruluyor sonra akım goruluyor (faz açısı)

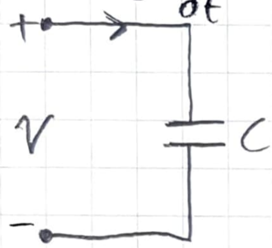
Kondansatör Olursa; $i = C\frac{dV}{dt}$

$$I_m e^{j(wt+\phi_i)} = C\frac{d}{dt}\left[V_m e^{j(wt+\phi_v)}\right]$$
$$= jwCV_m e^{j(wt+\phi_v)}$$

$$\bar{I} = jwC\bar{V}$$

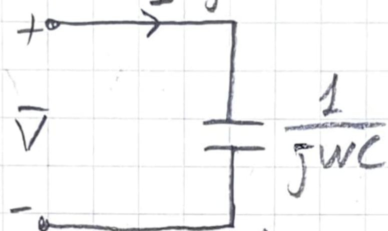
$$\boxed{\bar{V} = \frac{\bar{I}}{jwC}}$$

$i = C\frac{dV}{dt}$



Zaman domeni

$\bar{I} = jwC\bar{V}$



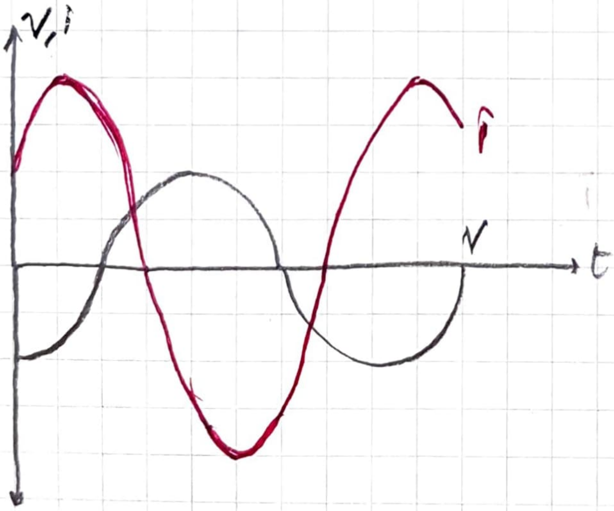
frekans domeni

- $v = V_m \cos(wt+\phi_v)$

$\bar{I} = (jwC)(V_m \angle \phi_v)$

$\bar{I} = wCV_m \angle \phi_v + 90°$

$i(t) = wCV_m \cos(wt + \phi_v + 90°)$



ilk olarak akım gorulur sonra gerilim gorulur

Empedans ve Admitans;

$$v(t) = V_m \cos(\omega t + \phi_v)$$
$$i(t) = I_m \cos(\omega t + \phi_i)$$

$$\bar{V} = V_m \angle \phi_v$$
$$\bar{I} = I_m \angle \phi_i$$

$$\boxed{\bar{V} = Z \bar{I}}$$ Z: Empedans

$$Z = |Z| \angle \phi_Z = \frac{V_m}{I_m} \angle \phi_v - \phi_i$$

$$|Z| = \frac{V_m}{I_m} \longrightarrow \phi_Z = \phi_v - \phi_i$$

$$\boxed{Z = R + jX}$$ X: Reaktans

$$R = Re(Z)$$
$$X = Im(Z)$$

$$|Z| = \sqrt{R^2 + X^2}, \quad \phi_Z = \tan^{-1} \frac{X}{R}$$

- $Z_R = R$
- $Z_L = j\omega L = \omega L \angle 90°$
- $Z_C = \dfrac{1}{j\omega C} = -j \dfrac{1}{\omega C} = \dfrac{1}{\omega C} \angle -90°$

Endoktif Reaktans; $X_L = \omega L$

$$Z_L = jX_L$$

Kapasitif Reaktans; $X_C = -\dfrac{1}{\omega C}$

$$Z_C = jX_C$$

Admitans (Y);

$$Y = \frac{1}{Z} = G + jB$$

$G = Re Y$ kondoktans

$B = Im Y$ suseptans

$$Y = G + jB = \frac{1}{Z} = \frac{1}{R + jX}$$

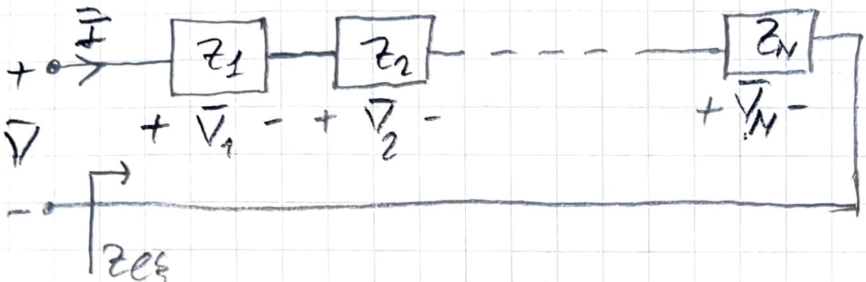
$$G \neq \frac{1}{R}$$
$$B \neq \frac{1}{X}$$

Kirchoff Kanunları ve Empedans Eşdeğeri

$$V_1 e^{j(\omega t+\theta_1)} + V_2 e^{j(\omega t+\theta_2)} + \dots + V_N e^{j(\omega t+\theta_N)} = 0$$

$$\bar{V}_1 + \bar{V}_2 + \dots + \bar{V}_N = 0 \quad , \quad \bar{V}_n = V_n \angle \theta_n \quad n = 1,2 \dots N$$

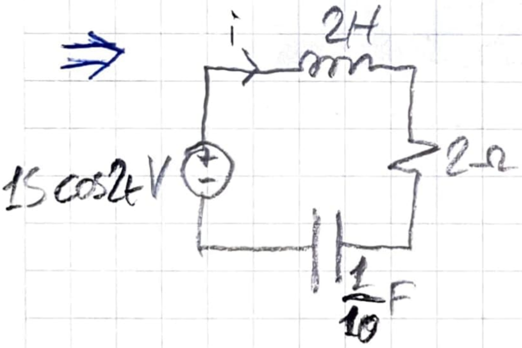
$$\bar{I}_1 + \bar{I}_2 + \dots + \bar{I}_N = 0 \quad , \quad \bar{I}_n = I_n \angle \theta_n \quad n = 1,2,\dots,N$$



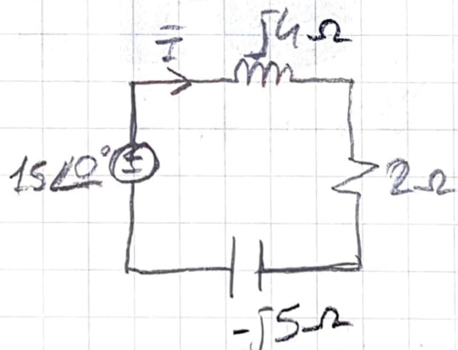
$$\bar{V}_1 = Z_1 \bar{I}$$
$$\bar{V}_2 = Z_2 \bar{I}$$
$$\bar{V}_3 = Z_3 \bar{I}$$

$$\bar{V} = \bar{V}_1 + \bar{V}_2 + \dots + \bar{V}_N$$
$$\bar{V} = (Z_1 + Z_2 + \dots + Z_N)\bar{I}$$
$$\bar{V} = Z_{eş} \bar{I}$$
$$Z_{eş} = Z_1 + Z_2 + Z_3 + \dots + Z_N$$
$$Z = R + jX$$

⇒



zaman domenindeki devre



fazörel devre

[kaynak fazör eleman empedans]

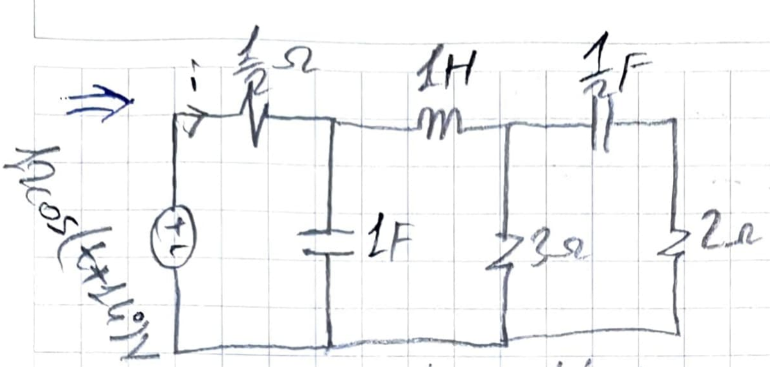
$$Z_L \bar{I} + Z_R \bar{I} + Z_C \bar{I} = 15\angle 0°$$

$$(j4 + 2 - j5)\bar{I} = 15\angle 0°$$

$$\bar{I} = \frac{15\angle 0°}{2-j} = \frac{15\angle 0°}{\sqrt{5}\angle -26{,}6°}$$

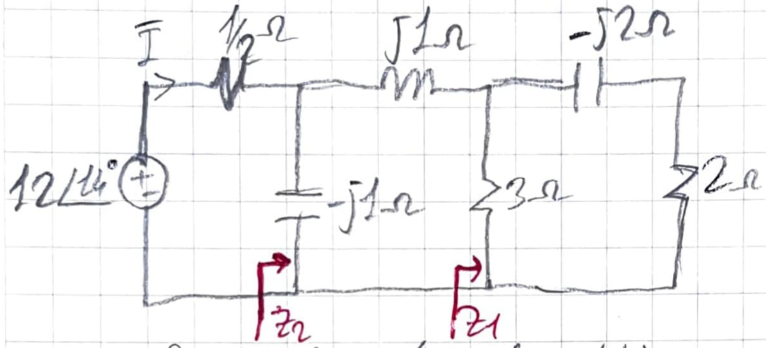
$$\bar{I} = 3\sqrt{5}\angle 26{,}6° \, A$$

$$i(t) = 3\sqrt{5}\cos(2t + 26{,}6°) A$$



i=?

zaman domenindeki devre



fazör devre (w=1 rad/s)

$$Z_1=\frac{3(2-j2)}{3+(2-j2)}=1{,}45-j0{,}621\ \Omega$$

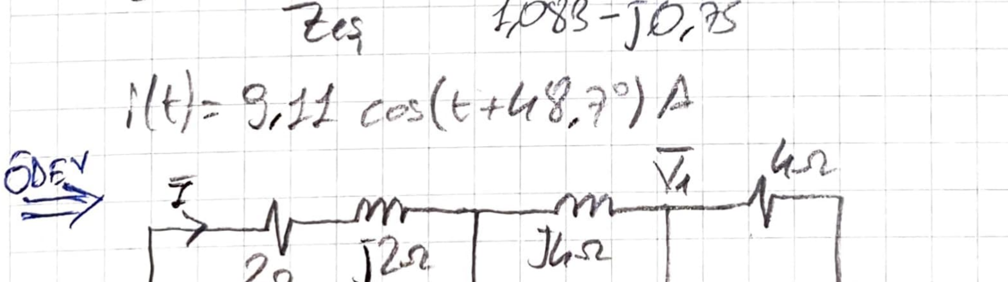
$$Z_2=\frac{(-j1)(Z_1+j1)}{-j1+(Z_1+j1)}=0{,}583-j0{,}75\ \Omega$$

$$Z_{eq}=\frac{1}{2}+Z_2=1{,}083-j0{,}75\ \Omega$$

$$\bar{I}=\frac{12\angle 14^\circ}{Z_{eq}}=\frac{12\angle 14^\circ}{1{,}083-j0{,}75}=9{,}11\angle 48{,}7^\circ\ A$$

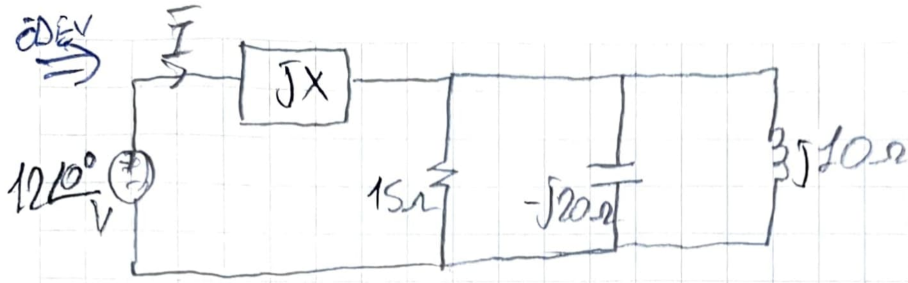
$$i(t)=9{,}11\cos(t+48{,}7^\circ)\ A$$

ÖDEV



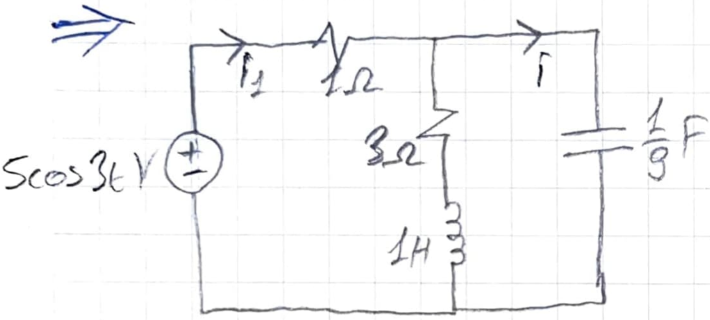
w=6 rad/s olduğuna göre i akımının ve V_1 geriliminin zaman domenindeki ifadesini bulunuz.

ÖDEV ⇒

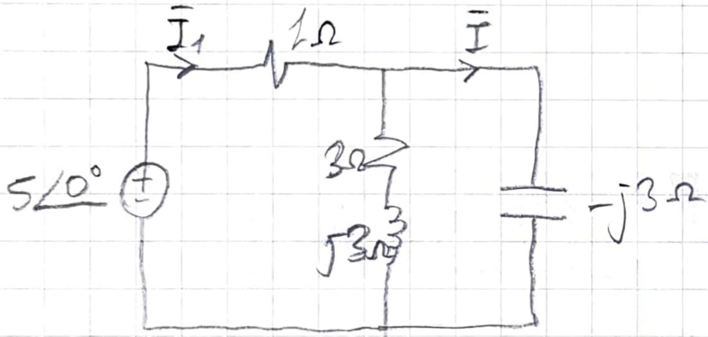


Kaynak tarafından devreye bakıldığında görülen empedansın reel olması için X reaktansı ne olmalıdır? Bu durumda kaynak akımının zaman domenindeki ifadesi ne olur? ($w=3$ rad/sn)

⇒



i = ?



$$\bar{Z} = 1 + \frac{(3+j3)(-j3)}{3+j3-j3} = 4 - j3\ \Omega$$

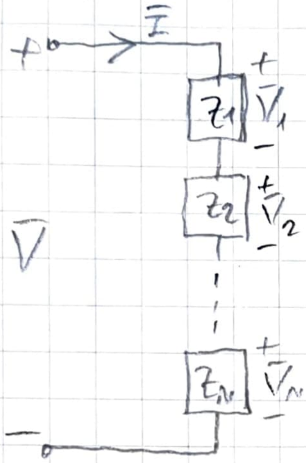
$$\bar{I}_1 = \frac{5\angle 0^\circ}{4-j3} = \frac{5\angle 0^\circ}{5\angle -36{,}9^\circ} = 1\angle 36{,}9^\circ\ A$$

$$\bar{I} = \frac{3+j3}{3+j3-j3}\,\bar{I}_1 = 1{,}41\angle 81{,}9^\circ\ A$$

$$i(t) = 1{,}41\cos(3t + 81{,}9^\circ)\ A$$

AC Sürekli Hal Analizi

- En genel durum; $\bar{V} = Z\bar{I}$

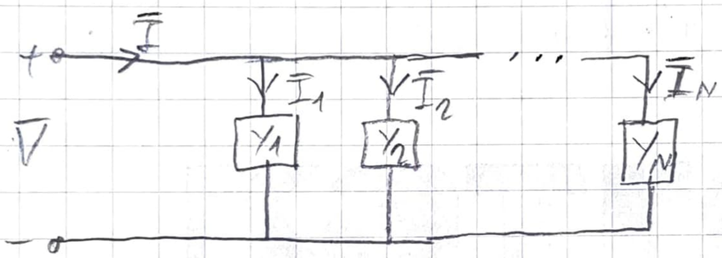


$$Z = Z_1 + Z_2 + \dots + Z_N$$

$$\bar{V}_i = \frac{\bar{V}}{Z_1 + Z_2 + \dots + Z_N} Z_i$$

$$\frac{\bar{V}_i}{\bar{V}_j} = \frac{Z_i}{Z_j}$$

[Akımları aynı en büyük empedans en büyük gerilim]

- 

[Akım → Admitans
Gerilim → Empedans]

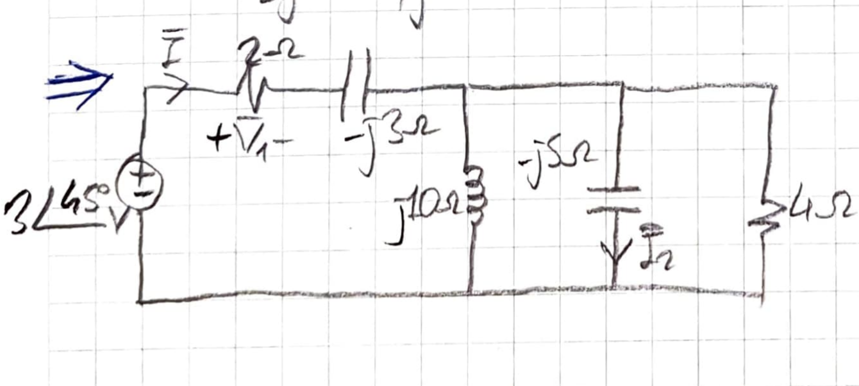
$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \dots + \bar{I}_N$$

$$Y = Y_1 + Y_2 + \dots + Y_N$$

$$\bar{I}_i = \frac{Y_i}{Y_1 + Y_2 + \dots + Y_N} \bar{I}$$

[Admitans hangisi büyükse akım büyüktür.]

$$\frac{\bar{I}_i}{\bar{I}_j} = \frac{Y_i}{Y_j}$$

⇒ 

V_1 gerilimi ve I_2 akımını bulunuz?

[w bilinmediğinden fazörde bulunacaktır]

$$Y = -j\frac{1}{10} + j\frac{1}{5} + \frac{1}{4} = \frac{1}{4} + j\frac{1}{10}\ S$$

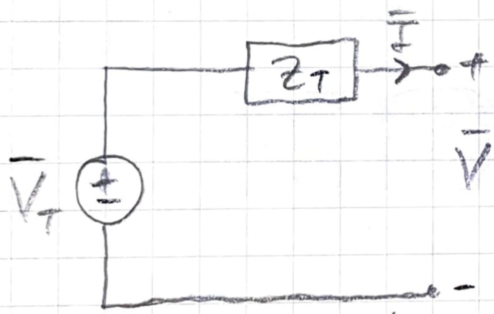
$$Z = \frac{1}{Y} = \frac{1}{\frac{1}{4} + j\frac{1}{10}} = 3{,}45 - j1{,}38\ \Omega$$

$$\bar{V}_1 = \frac{3\angle 45^\circ}{2 - j3 + (3{,}45 - j1{,}38)} \cdot 2 = 0{,}858\angle 83{,}8^\circ\ V$$

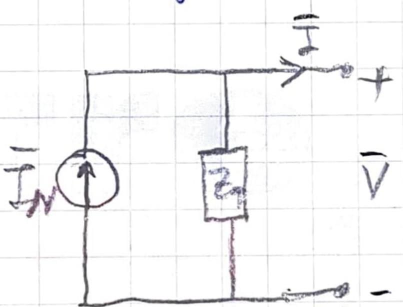
$$\bar{I}_2 = \frac{j\frac{1}{5}}{\frac{1}{4} + j\frac{1}{10}}\ \bar{I} = (0{,}743\angle 68{,}2^\circ)(0{,}429\angle 83{,}8^\circ)$$

$$\bar{I}_2 = 0{,}319\angle 152^\circ\ A$$

Thevenin ve Norton Eşdeğer Devreler



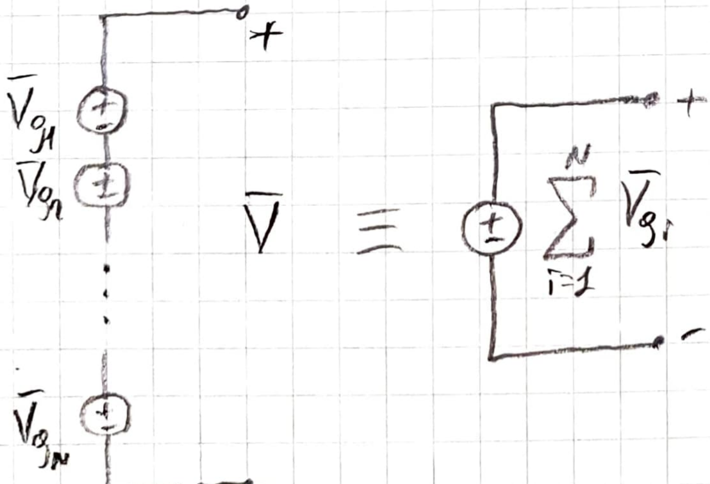
Thevenin eşdeğer devresi



Norton eşdeğer devresi

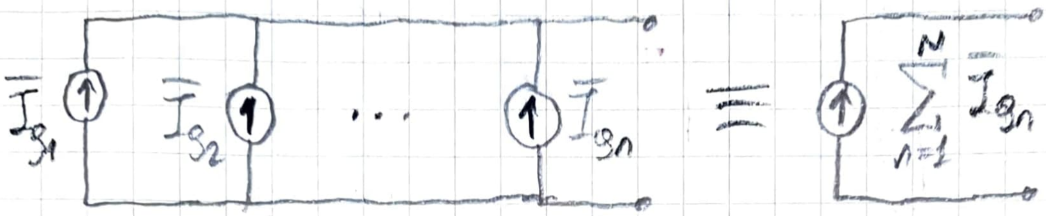
$$\boxed{Z_T = Z_N} \qquad \boxed{\bar{V}_T = Z_N \bar{I}_N}$$

Gerilim kaynakları seri bağlanırsa;

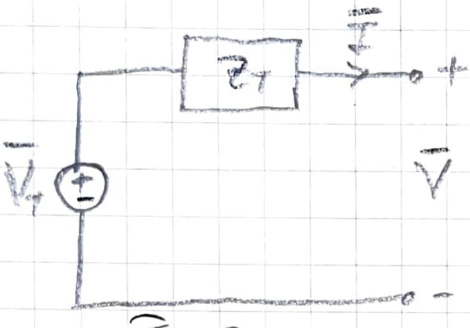


"Bütün kaynakların frekansları aynı"

Akım kaynakları paralel bağlanırsa;



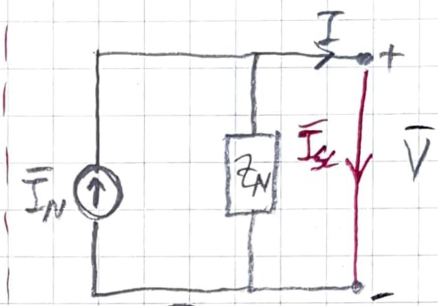
Açık devre durumunda;



$\bar{I} = 0$

$\bar{V} = \bar{V}_T = \bar{V}_{oc}$

Kısa devre Durumunda;

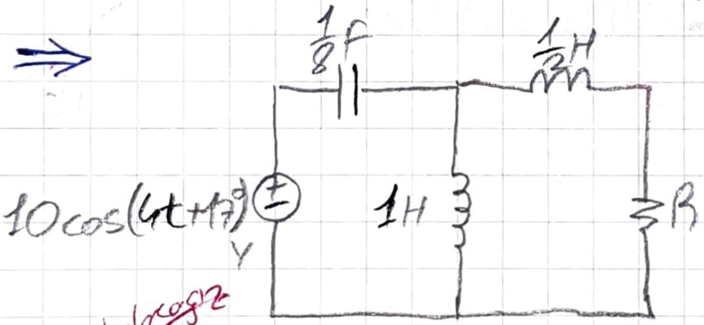


$\bar{V} = 0$

$\bar{I} = \bar{I}_{sc} = \bar{I}_N$

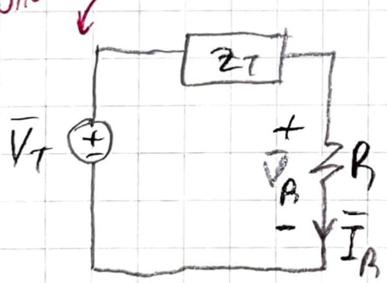
$$Z_T = Z_N = \frac{\bar{V}_{oc}}{\bar{I}_{sc}}$$

⇒



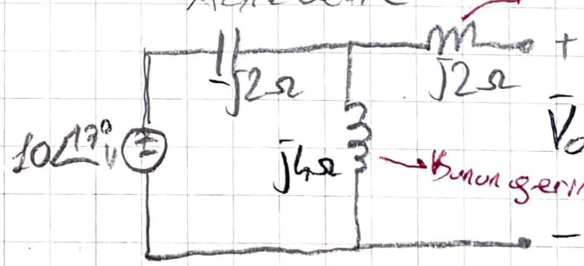
R direncinden akan akımın genliğinin 1A olması için R direnci ne olmalıdır.

Bunu bulacağız



1. adım

Açık devre

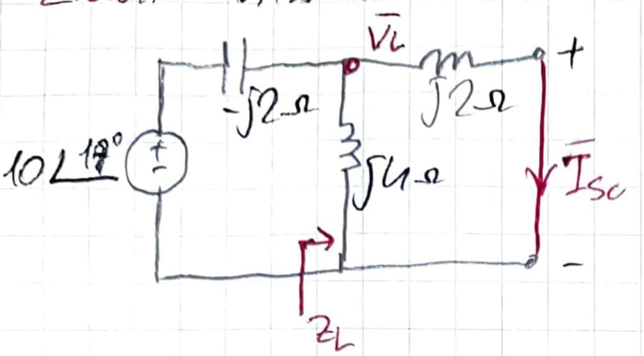


Bu bobinden akım akmaz

Bunun gerilimi

$$\bar{V}_{oc} = \frac{j4(10\angle 17^\circ)}{j4 - j2} = 20\angle 17^\circ \text{ V}$$

2. adım Kısa devre



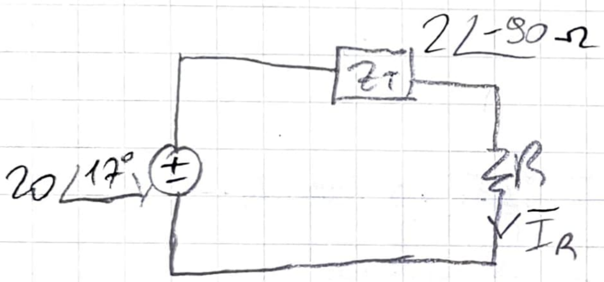
$$Z_L = \frac{j2 \cdot j4}{j2 + j4} = j\frac{4}{3}\ \Omega$$

$$\bar V_L = \frac{j\frac{4}{3}\,(10\angle 17^\circ)}{j\frac{4}{3} - j2} = 20\angle -163^\circ\ \text{V}$$

$$\bar I_{sc} = \frac{\bar V_L}{j2} = \frac{20\angle -163^\circ}{2\angle 90^\circ} = 10\angle 107^\circ\ \text{A}$$

$$Z_T = \frac{\bar V_{oc}}{\bar I_{sc}} = \frac{20\angle 17^\circ}{10\angle 107^\circ}$$

$$Z_T = 2\angle -90^\circ\ \Omega$$ (Devre kondansatör gibi davranır)



$$\bar I_R = \frac{\bar V_T}{Z_T + R} = \frac{20\angle 17^\circ}{-j2 + R}$$

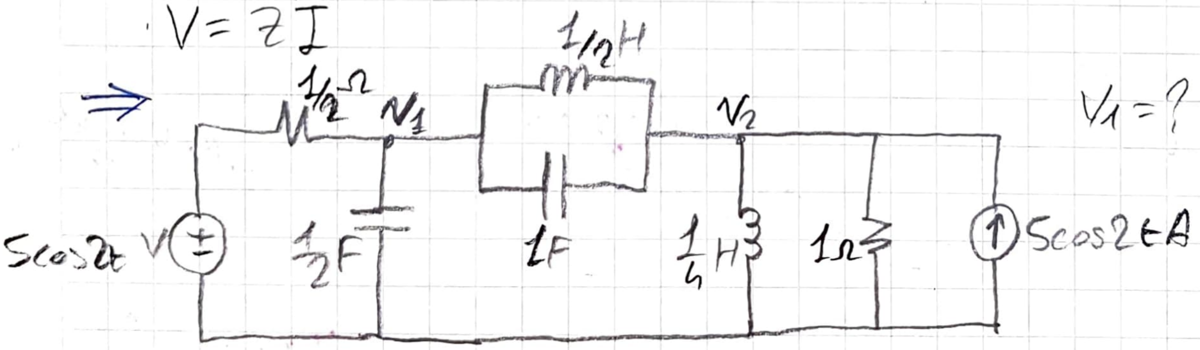
$$|\bar I_R| = \frac{20}{\sqrt{R^2 + 4}} = 1$$ Soruda verilmiş

$$R = 19.9\ \Omega$$

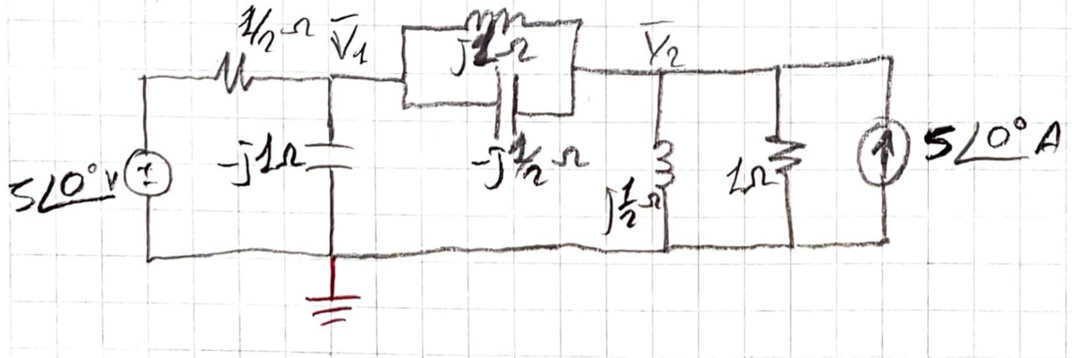
Dönüşüm Analizi

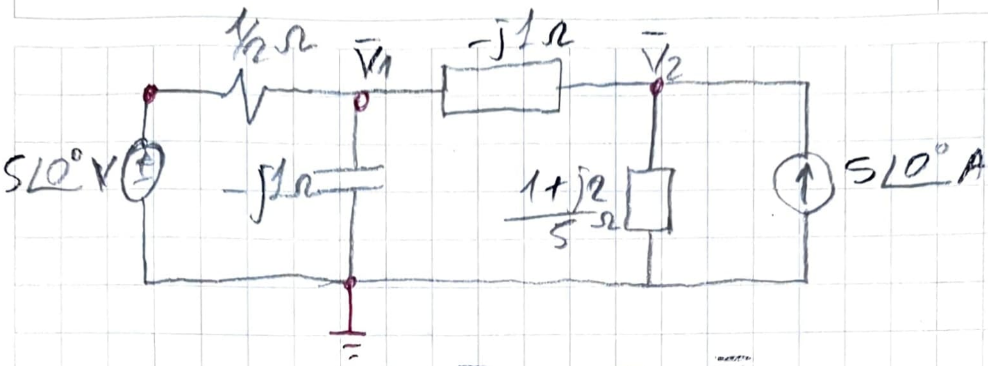
$$\bar V = Z \bar I$$

⇒



$V_1 = ?$ $V_2 = ?$

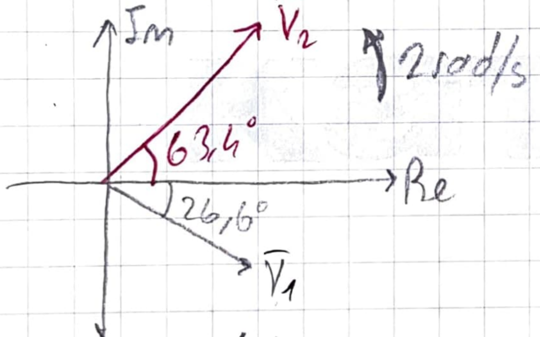




$$2(\bar{V}_1 - 5\angle 0^\circ) + \frac{\bar{V}_1}{j1} + \frac{\bar{V}_1 - \bar{V}_2}{-j1} = 0$$

$$\frac{\bar{V}_2 - \bar{V}_1}{-j1} + \frac{\bar{V}_2}{\frac{1+j2}{5}} = 5\angle 0^\circ$$

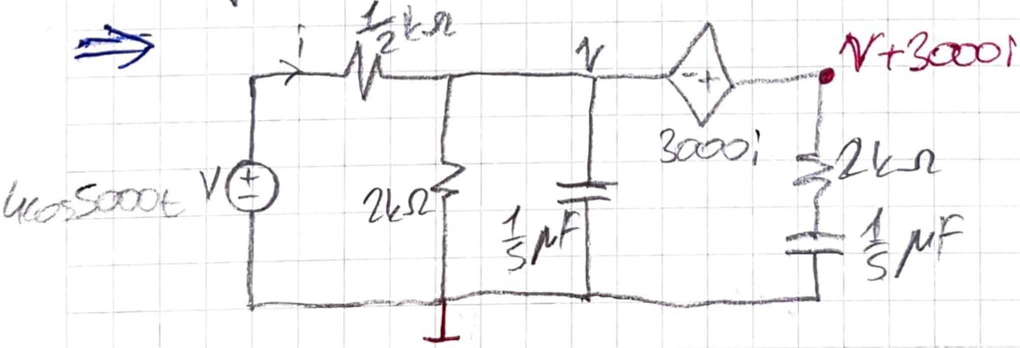
$$\left.\begin{array}{l}(2+j2)\bar{V}_1 - j1\bar{V}_2 = 10 \\ -j1\bar{V}_1 + (1-j1)\bar{V}_2 = 5\end{array}\right\} \quad \begin{array}{l}\bar{V}_1 = \sqrt{5}\angle -26{,}6^\circ V \\ \bar{V}_2 = 2\sqrt{5}\angle 63{,}4^\circ V\end{array}$$

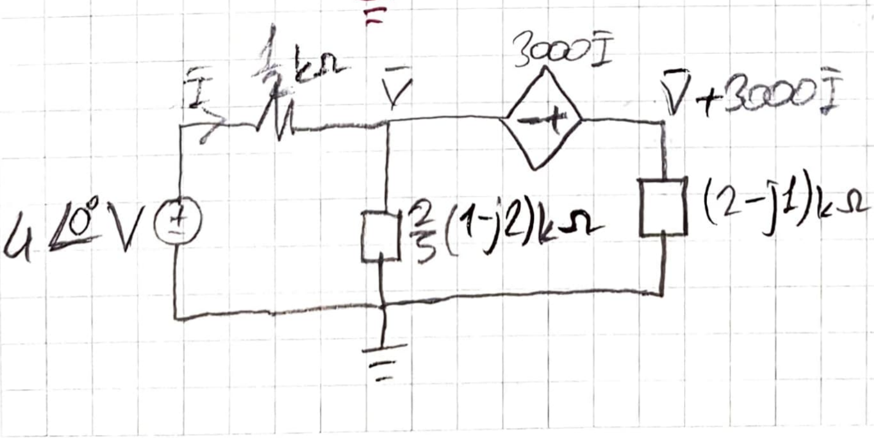


$v_1(t) = \sqrt{5}\cos(2t - 26{,}6^\circ) V$

$v_2(t) = 2\sqrt{5}\cos(2t + 63{,}4^\circ) V$

⇒



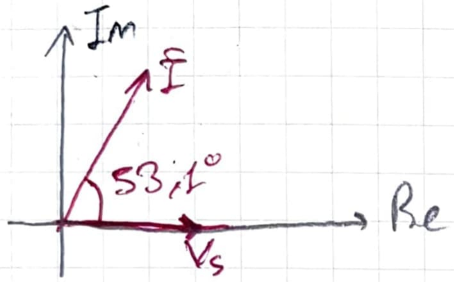


$$\frac{\bar{V}-4}{\frac{1}{2}\cdot 10^3}+\frac{\bar{V}}{\frac{2}{5}(1-j2)10^3}+\frac{\bar{V}+3000\bar{I}}{(2-j1)10^3}=0$$

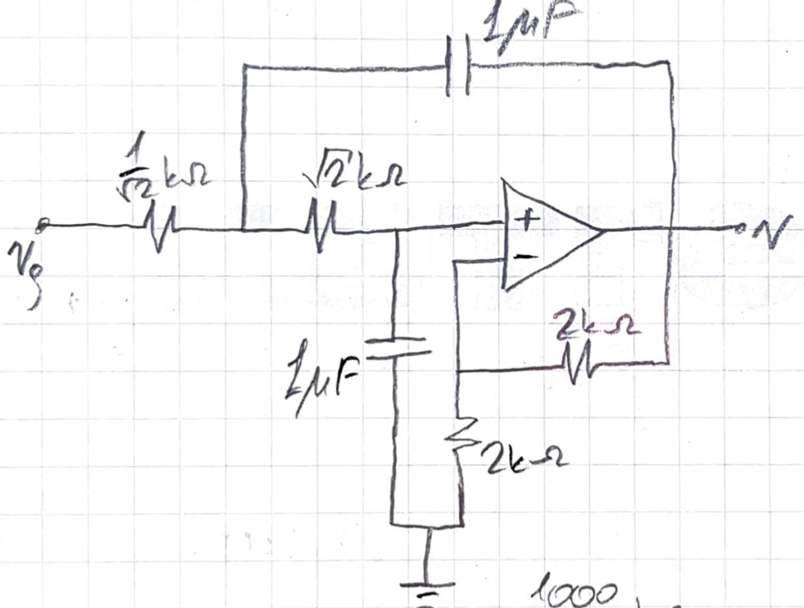
$$\bar{I}=\frac{4-\bar{V}}{\frac{1}{2}10^3}$$

$$\bar{I}=24\cdot 10^{-3}\angle 53{,}1^\circ A = 24\angle 53{,}1^\circ \, mA$$

$$i(t)=24\cos(5000t+53{,}1^\circ)\, mA$$

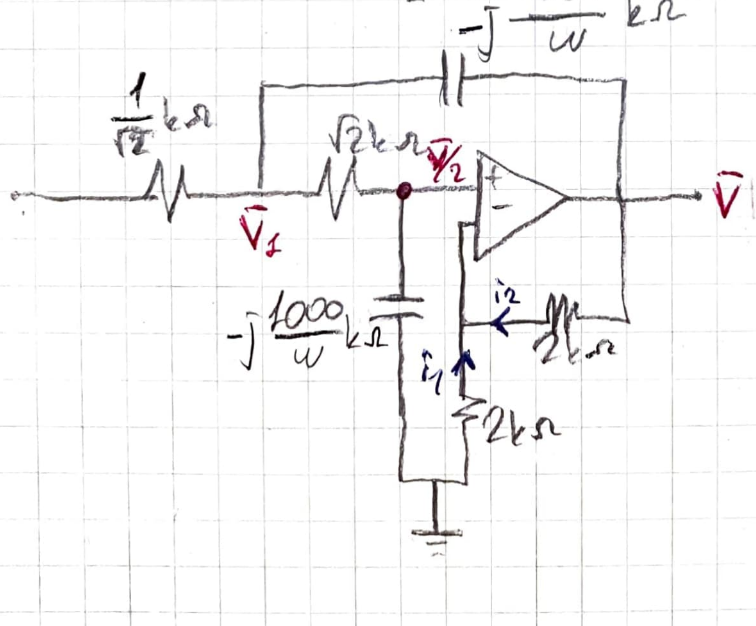


$\Rightarrow$



$V_g = V_m \cos \omega t \; V$

$V = ?$



$$i_1 + i_2 = 0$$

$$\frac{0-\bar{V}_n}{2k\Omega}=\frac{\bar{V}-\bar{V}_n}{2k\Omega}$$

$$\bar{V}_n = \bar{V}/2$$

$$\frac{\bar{V}_1 - V_m\angle 0^\circ}{\frac{1}{\sqrt{2}}10^3} + \frac{\bar{V}_1 - \left(\frac{\bar{V}}{2}\right)}{\sqrt{2}\,10^3} + \frac{\bar{V}_1 - \bar{V}}{\frac{-j10^6}{w}} = 0$$

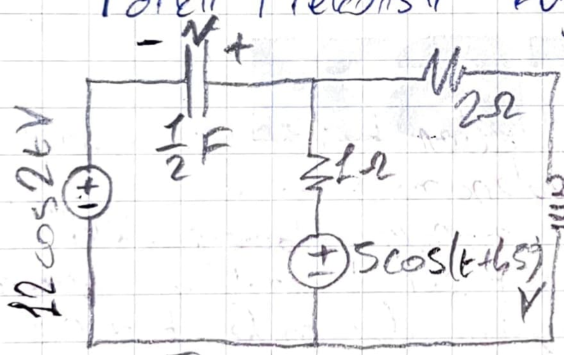
$$\frac{\frac{\bar{V}}{2} - \bar{V}_1}{\sqrt{2}\,10^3} + \frac{\frac{\bar{V}}{2}}{\frac{-j10^6}{w}} = 0$$

$$\theta = \tan^{-1}\frac{\frac{\sqrt{2}\,w}{1000}}{1-\left(\frac{w}{1000}\right)^2}$$

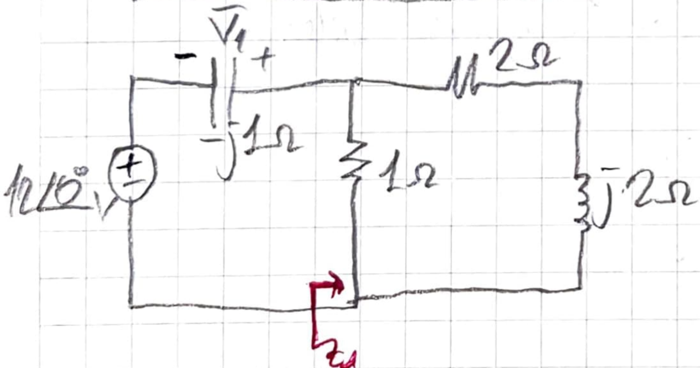
$$\bar{V} = \frac{2V_m\angle\theta}{\sqrt{1+\left(\frac{w}{1000}\right)^4}}\cos(wt+\theta)$$

[Düşük frekansları geçiren filtre devresi
$w \to \infty$ sıfır olur]

Farklı Frekanslı Kaynaklar



$V = ?$ (süperpozisyon kullanılıyor)



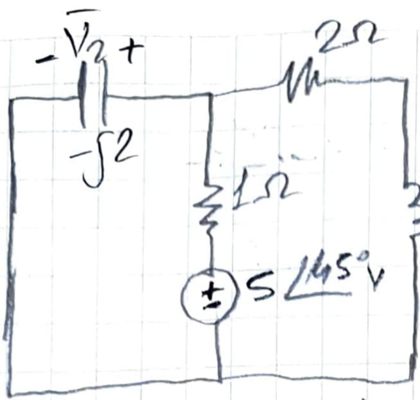
$w = 2$ rad/s için fazör devre

$$Z_d = \frac{1(2+j2)}{3+j2}$$

$$Z_d = 0{,}769 + j0{,}154\ \Omega$$

$$\bar{V}_1 = \frac{-j1}{-j1+Z_d}\,12\angle 0^\circ$$

$$\bar{V}_1 = 10{,}5\angle 138^\circ\ V$$



$\omega = 1$ rad/s için fazör devre

$$Z_b = \frac{(-j2)(2+j1)}{2-j1} = 1,6 - j1,2\ \Omega$$

$$\bar V_2 = \frac{Z_b}{Z_b+1}\, 5\angle 45^\circ$$

$$\bar V_2 = 3,5\angle 32,9^\circ \text{ V}$$

ω değerleri farklı olduğunda fazör toplanmaz

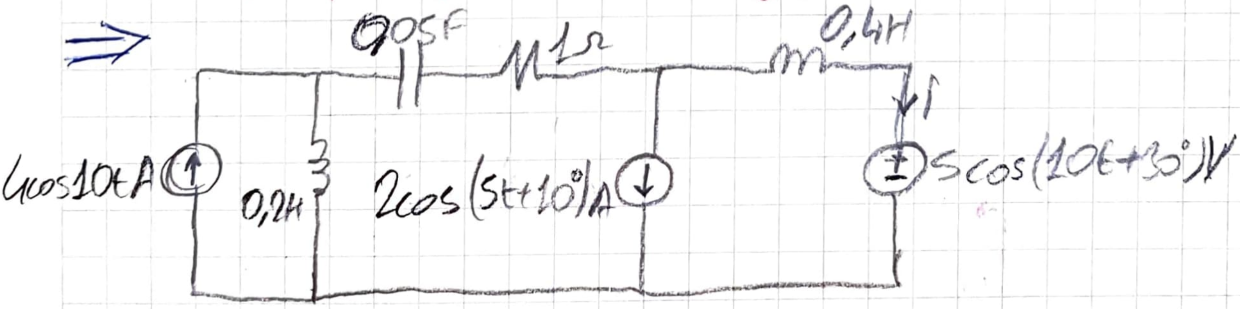
$v_1(t) = 10,5\cos(2t+138^\circ)$ V

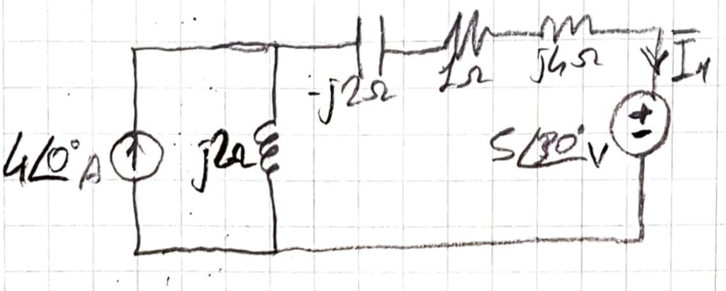
$v_2(t) = 3,5\cos(t+32,9^\circ)$ V

$v = v_1(t) + v_2(t) = 10,5\cos(2t+138^\circ) + 3,5\cos(t+32,9^\circ)$ V

NOT: Farklı frekansları temsil eden fazörlere süper pozisyon teoremi uygulanamaz. Süper pozisyon teoremi ancak onların zaman domenindeki ifadelerine uygulanabilir. Eğer kaynakların tamamı aynı frekansta olursa, bu durumda fazörlere süper pozisyon teoremi uygulanabilir.

$\Rightarrow$



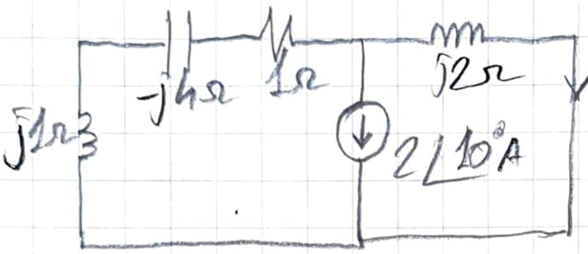


$\omega = 10$ rad/s için fazör devre

$$\bar{I}_1(-j2+1+j4)+j2(\bar{I}_1-4\angle 0^\circ)=5\angle 30^\circ$$

$$\bar{I}_1=\frac{5\angle 30^\circ + j8}{1+j4} = 2,76\angle -8,4^\circ \text{ A}$$

$$i_1(t)=2,76\cos(10t-8,4^\circ)\text{ A}$$



$w = 5$ rad/s için fazör devre

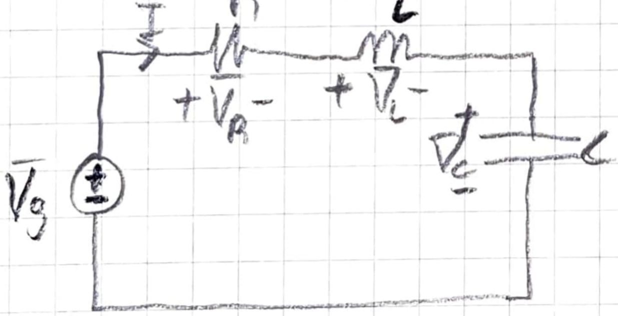
$$\bar{I}_2=\frac{1-j3}{1-j1}(-2\angle 10^\circ)$$

$$\bar{I}_2 = 4,47\angle 163^\circ \text{ A}$$

$$i_2(t) = 4,47\cos(5t+163^\circ)\text{ A}$$

$$i = i_1(t)+i_2(t) = 2,76\cos(10t-8,4^\circ)+4,47\cos(5t+163^\circ)\text{ A}$$

Fazör Diyagramı



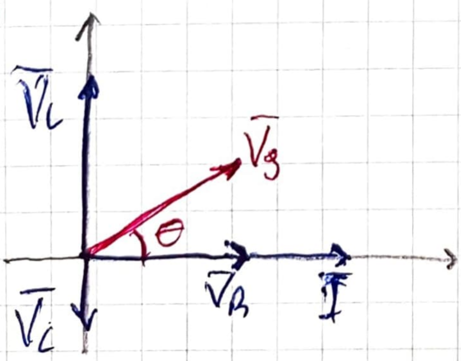
$$\bar{V}_R = R\bar{I} = R|\bar{I}|$$

$$\bar{V}_L = jwL\bar{I} = wL|\bar{I}|\angle 90^\circ$$

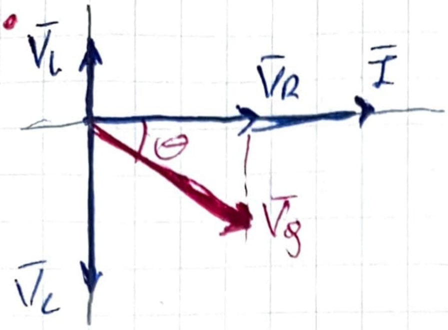
$$\bar{V}_C = -j\frac{1}{wC}\bar{I} = \frac{1}{wC}|\bar{I}|\angle -90^\circ$$

$$\bar{V}_g = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

Bu durum söz konusu

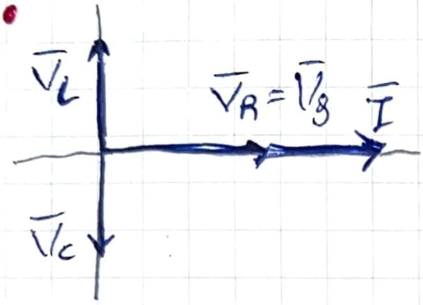


$|\bar{V}_L| > |\bar{V}_C|$ "endüktif"

• 

Kaposit

$|\bar{V}_L| < |\bar{V}_C|$

• 

$|\bar{V}_L| = |\bar{V}_C|$

Rezonans durumu

$$wL - \frac{1}{wC} = 0$$

$$w = \frac{1}{\sqrt{LC}}$$

Eğer devrede direnç olmazsa I sonsuza gider. Empedans sıfıra gider.

ALTERNATİF AKIMDA GÜÇ

Ortalama (Aktif) Güç

$p(t) = v(t)\,i(t)$ anı güç (kuvvet hart)

Anı güç periyodiktir:

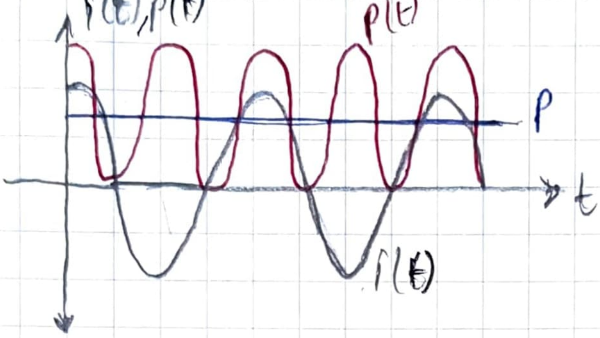
$$p(t+T) = v(t+T) \cdot i(t+T)$$
$$= v(t)\,i(t) = p(t)$$

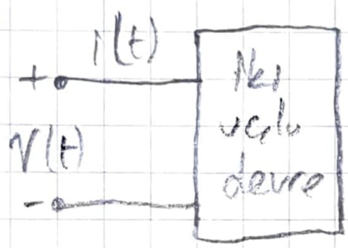
$$\boxed{P = \frac{1}{T}\int_{t_1}^{t_1+T} p(t)\,dt}$$

Anı gücün bir periyod boyunca ortalaması, ortalama gücü verir.

R direncinin akımı $i(t) = I_m \cos \omega t$ olsun

$$p(t) = R\,i(t)^2 = R\,I_m^2 \cos^2 \omega t = \frac{R I_m^2}{2}(1 + \cos 2\omega t)$$





$$v(t) = V_m \cos(\omega t + \phi_v)$$

$$i(t) = I_m \cos(\omega t + \phi_i)$$

$$T = \frac{2\pi}{\omega}$$

$$p(t) = v(t)\,i(t) = V_m \cos(\omega t + \phi_v)\, I_m \cos(\omega t + \phi_i)$$

$$P = \frac{\omega V_m I_m}{2\pi} \int_0^{2\pi/\omega} \cos(\omega t + \phi_v) \cos(\omega t + \phi_i)\, dt$$

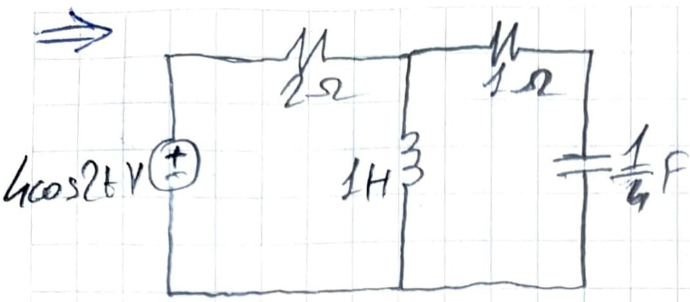
$$\boxed{P = \frac{V_m I_m}{2} \cos(\phi_v - \phi_i)}$$

V_m: gerilimin maximum değeri
I_m: akımın maximum değeri

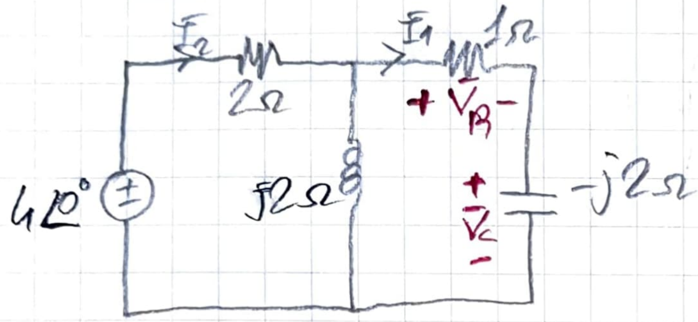
$$\bar{V} = V_m \angle \phi_v = |\bar{V}_m| \angle \phi_v$$

$$\bar{I} = I_m \angle \phi_i = |\bar{I}_m| \angle \phi_i$$

$$P = \frac{1}{2} |\bar{V}_m| |\bar{I}_m| \cos(\phi_v - \phi_i)$$



Elemanların çektiği ortalama güçleri bul?



$$Z = 2 + (j2)(1-j2) = 6 + j2\ \Omega$$

$$\bar{I} = \frac{4\angle 0^\circ}{6+j2} = 0{,}632\angle -18{,}4^\circ\ A$$

$$\bar{I}_1 = \frac{j2}{1}\bar{I} = 1{,}26\angle 71{,}6^\circ\ A$$

$$\bar{V}_R = 1 \cdot \bar{I}_1 = 1{,}26\angle 71{,}6^\circ\ V$$

$$P_R = \frac{1}{2}|\bar{V}_R||\bar{I}_R|\cos(71{,}6^\circ - 71{,}6^\circ)$$

$$P_R = 0{,}80\,W$$

$$\bar{V}_C = (-j2)\,\bar{I}_1 = 2{,}52\angle -18{,}4^\circ\ V$$

$$P_C = \frac{1}{2}|\bar{V}_C||\bar{I}_1|\cos(-18{,}4^\circ - 71{,}6^\circ) = 0$$

Kondansatör aktif güç çekmiyor.
Yani ısınmıyor.

Etkin (RMS) Değer

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i(t)^2 dt}$$

Sinosoidal durumda;

$$V_{rms} = \frac{V_m}{\sqrt{2}} \quad , \quad I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{P = V_{rms} I_{rms} \cos(\phi_v - \phi_i)}$$

Komplex Güç

$$\bar{V}_{rms} = |\bar{V}_{rms}| \angle \phi_v$$

$$\bar{I}_{rms} = |\bar{I}_{rms}| \angle \phi_i$$

$$\boxed{\bar{S} = \bar{V}_{rms} \bar{I}^{*}_{rms}}$$ Komplex Güç (* eşlenik)

$$|\bar{S}| = |\bar{V}_{rms}||\bar{I}^{*}_{rms}| = |\bar{V}_{rms}||\bar{I}_{rms}|$$

$$\angle\bar{S} = \angle\bar{V}_{rms} + \angle\bar{I}^{*}_{rms}$$

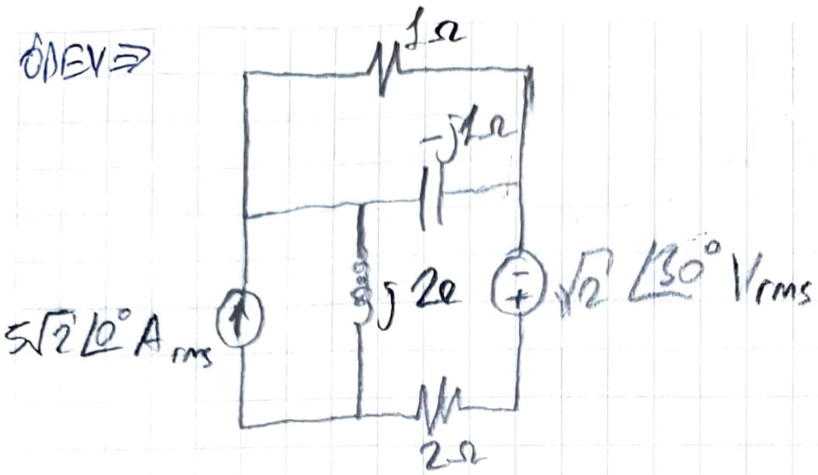
$$P = Re(\bar{S}) = |\bar{V}_{rms}||\bar{I}_{rms}| \cos(\phi_v - \phi_i)$$

Reaktif Güç

$$Q = |V_{rms}||I_{rms}| \sin(\phi_v - \phi_i)$$

$$\boxed{\bar{S} = \bar{V}_{rms} \bar{I}^{*}_{rms} = P + jQ}$$

ÖDEV ⇒



Kaynaklardan gelen komplex güçleri bulunuz?

- $\bar{V}_{rms} = Z \bar{I}_{rms}$ —— *

  $\bar{S} = \bar{V}_{rms} \bar{I}^{*}_{rms} = Z \bar{I}_{rms} \bar{I}^{*}_{rms}$

  $\bar{S} = Z |\bar{I}_{rms}|^2$

- $P = Re(Z) |\bar{I}_{rms}|^2$

  $P = \frac{Re(Z)}{|Z|^2} |\bar{V}_{rms}|^2$

  } gerilim ve empedans belli ise

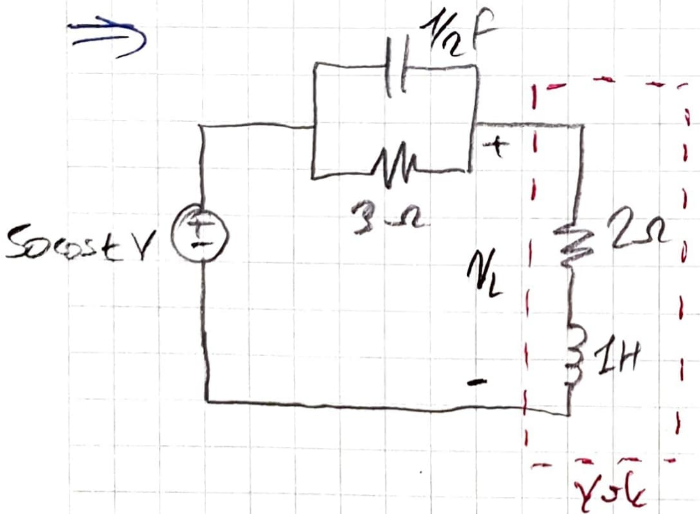
- $\bar{I}_{rms} = Y \bar{V}_{rms}$ —— **

  $\bar{S} = \bar{V}_{rms} \bar{I}^{*}_{rms} = Y^{*} |\bar{V}_{rms}|^2$

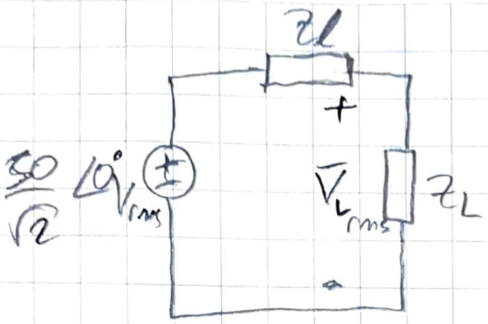
  $P = Re(Y^{*}) |\bar{V}_{rms}|^2 = Re(Y) |\bar{V}_{rms}|^2$

  $P = \frac{Re(Y)}{|Y|^2} |\bar{I}_{rms}|^2$

⇒



Yükün çektiği ortalama gücü bulunuz.



$$Z_\ell = \frac{3(-j2)}{3-j2} = 0{,}923 - j1{,}38\,\Omega$$

$$Z_L = 2 + j1\,\Omega$$

$$\overline{V}_{L_{rms}} = \frac{\frac{50}{\sqrt{2}}}{Z_\ell + Z_L} Z_L = 26{,}9\angle 34{,}1^\circ\ V_{rms}$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{2+j1} = 0{,}4 - j0{,}2\ S$$

$$P_L = Re(Y)\,|\overline{V}_{L_{rms}}|^2$$

$$P_L = 0{,}4(26{,}9)^2$$

$$P_L = 289\ W$$

SÜPERPOZİSYON VE GÜÇ

$$v = v_1 + v_2$$
$$i = i_1 + i_2$$

$$p(t) = v(t)\,i(t)$$
$$p(t) = (v_1+v_2)(i_1+i_2)$$
$$p(t) = v_1 i_1 + v_2 i_2 + v_1 i_2 + v_2 i_1$$
$$p(t) = p_1(t) + p_2(t) + v_1 i_2 + v_2 i_1$$

Anı güçte süperpozisyon teoreminin genel bir ifadesi yoktur.

$$\overline{V} = \overline{V}_1 + \overline{V}_2$$
$$\overline{I} = \overline{I}_1 + \overline{I}_2$$

$$\overline{S} = \overline{V}\,\overline{I}^* = (\overline{V}_1 + \overline{V}_2)(\overline{I}_1 + \overline{I}_2)^*$$
$$\overline{S} = \overline{V}_1 \overline{I}_1^* + \overline{V}_2 \overline{I}_2^* + (\overline{V}_1 \overline{I}_2^* + \overline{V}_2 \overline{I}_1^*)$$
$$\overline{S} = \overline{S}_1 + \overline{S}_2 + (\overline{V}_1 \overline{I}_2^* + \overline{V}_2 \overline{I}_1^*)$$

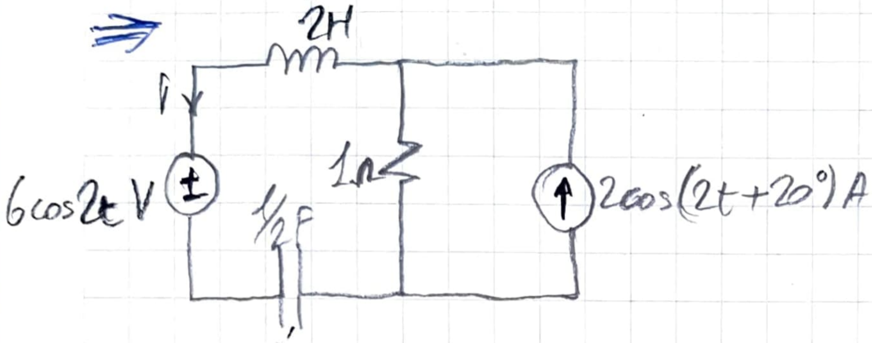
Aynı frekans için süperpozisyon teoremi kompleks güce uygulanamaz.

$$P = P_1 + P_2 + Re(\bar{V}_1 \bar{I}_2^* + \bar{V}_2 \bar{I}_1^*)$$

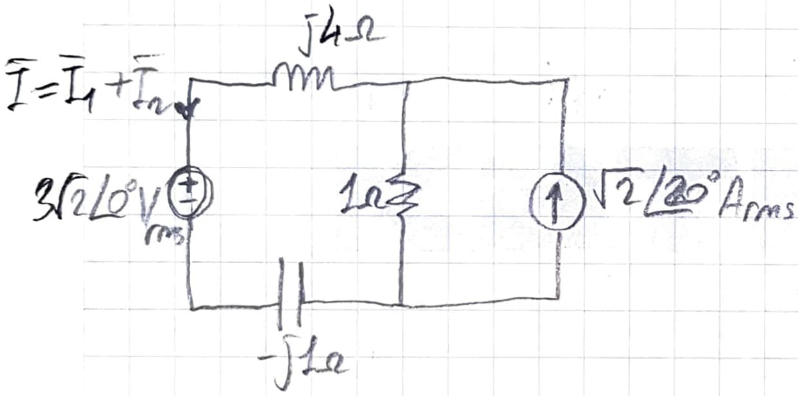
Aynı frekans varsa süperpozisyon teoremi ortalama (Aktif) güce uygulanamaz.

$$Q = Q_1 + Q_2 + Im(\bar{V}_1 \bar{I}_2^* + \bar{V}_2 \bar{I}_1^*)$$

Aynı frekans varsa süperpozisyon teoremi reaktif güce uygulanamaz.



Gerilim kaynağının gücünü bulunuz



$$\bar{I}_1 = -\frac{3\sqrt{2}\angle 0°}{1+j3} = -0{,}424 + j1{,}27\ A_{rms}$$

$$\bar{I}_2 = \frac{\sqrt{2}\angle 20°}{1+j3} = 0{,}278 - j0{,}35\ A_{rms}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = -0{,}146 + j0{,}92\ A_{rms}$$

$$\bar{S} = \bar{V}\bar{I}^* = (3\sqrt{2}\angle 0°)(-0{,}146 + j0{,}92)$$

$$\bar{S} = -0{,}62 - j3{,}9\ VA$$

Frekanslar Farklı ise; $(w_1 \neq w_2)$

$$p(t) = p_1 + p_2 + (v_1 i_2 + v_2 i_1)$$

$$v_1(t) = V_{m_1} \cos(w_1 t + \phi_1)$$

$$i_2(t) = I_{m_2} \cos(w_2 t + \phi_2)$$

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T (p_1 + p_2 + v_1 i_2 + v_2 i_1)\,dt$$

$$P = P_1 + P_2 + \frac{1}{T}\int_0^T (v_1 i_2 + v_2 i_1)\,dt$$

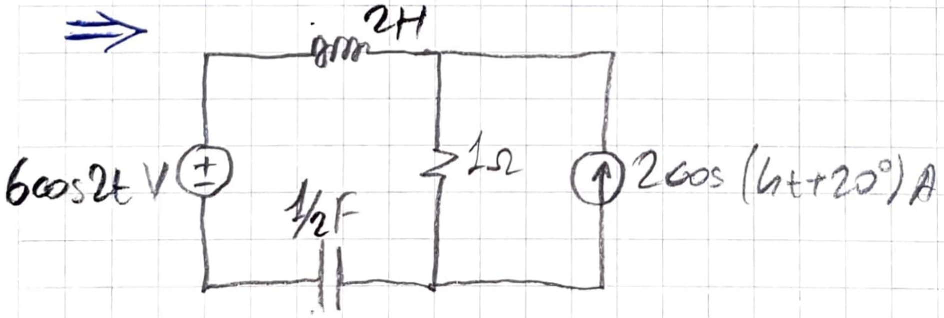
$w_1 \neq w_2$ için

$$\frac{1}{T}\int_0^T (v_1 i_2)\,dt = \frac{1}{T}\int_0^T V_{m_1}\cos(w_1 t + \phi_1)\, I_{m_2}\cos(w_2 t + \phi_2)\,dt = 0$$

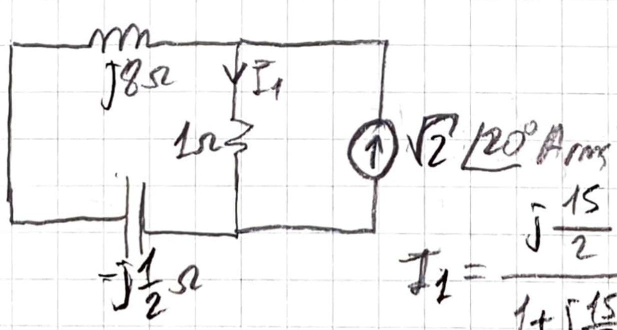
$$P = P_1 + P_2$$

Farklı frekanslı kaynaklardan oluşan ortalama güç için süperpozisyon teoremi uygulanabilir.

$\Rightarrow$

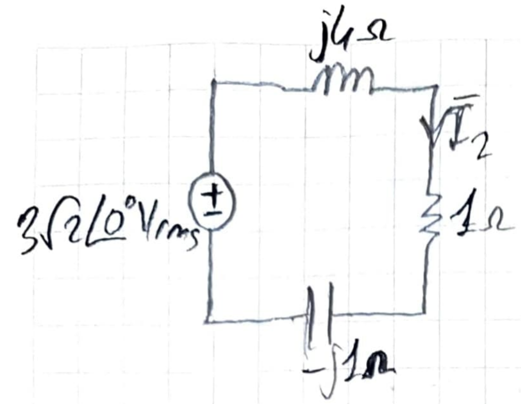


Devrenin harcadığı aktif gücü bul



$$I_1 = \frac{j\frac{15}{2}}{1 + j\frac{15}{2}} \sqrt{2}\angle 20° = 1{,}40\angle 27{,}6° \text{ Arms}$$

$$P_1 = \mathrm{Re}(Z)|I_1|^2 = R|I_1| = 1(1{,}40)^2 = 1{,}96\,W$$

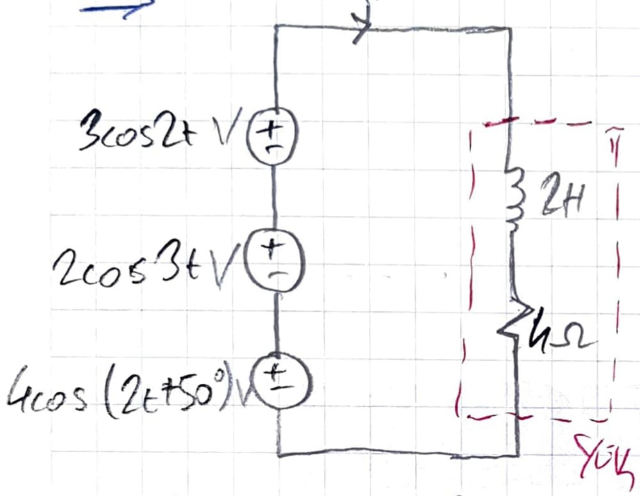


$$\bar{I}_2 = \frac{3\sqrt{2}\angle 0}{1+j3} = 1,34\angle -71,6^\circ \text{ Arms}$$

$$P_2 = R\,|\bar{I}_2|^2 = 1.(1,34)^2 = 1,80\text{W}$$

$$P = P_1 + P_2$$
$$P = 1,96 + 1,80 = 3,76\text{W}$$

$\Rightarrow$



Yükün çektiği komplex güç?

$$\bar{I}_1 = \frac{1,5\sqrt{2}\angle 0^\circ + 2\sqrt{2}\angle 50^\circ}{4+j4}$$

$$\bar{I}_1 = 0,795\angle -16,2 \text{ Arms}$$

$$\bar{S}_1 = Z_1\,|\bar{I}_1|^2 = (4+j4)\,0,795^2 = 2,53 + j2,53 \text{ VA}$$

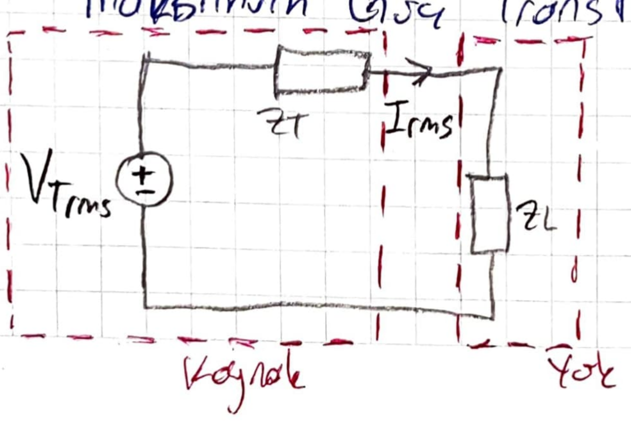
$$\bar{I}_2 = \frac{\sqrt{2}\angle 0^\circ}{4+j6} = 0,196\angle -56,3 \text{ Arms}$$

$$\bar{S}_2 = Z_2\,|\bar{I}_2|^2 = (4+j6)\,0,196^2 = 0,784 + j0,118 \text{ V}$$

$$\bar{S} = \bar{S}_1 + \bar{S}_2 = 3,314 + j2,648 \text{ VA}$$

NOT: Farklı frekansların oluşturduğu komplex güç için süperpozisyon teoremi uygulanabilir.

Maksimum Güç Transferi



$$\bar{I}_{rms} = \frac{\bar{V}_{T_{rms}}}{Z_T + Z_L}$$

$$P_L = Re(Z_L)\,|\bar{I}_{rms}|^2 = \frac{R(Z_L)\,|\bar{V}_{T_{rms}}|^2}{|Z_T + Z_L|^2}$$

$$Z_L = R_L + jX_L$$
$$Z_T = R_T + jX_T$$

$$P_L = \frac{R_L}{(R_L + R_T)^2 + (X_L + X_T)^2}\,|\bar{V}_{T_{rms}}|^2$$

Maksimum güç için;

[Birinci türevini sıfıra eşitle]

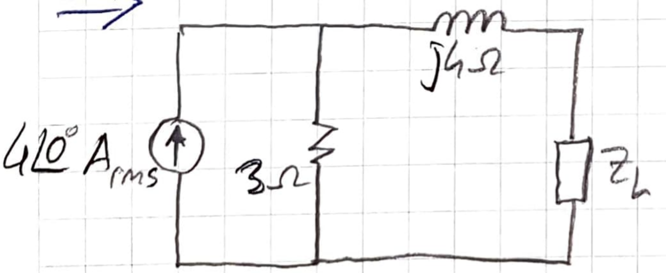
$\boxed{Z_L = Z_T^*}$ olmalı

$$R_L = R_T$$
$$X_L = -X_T$$

$$P_{max} = \frac{Re(Z_T)\,|\bar{V}_{T_{rms}}|^2}{|2Re(Z_T)|^2} = \frac{|\bar{V}_{T_{rms}}|^2}{4Re(Z_T)}$$

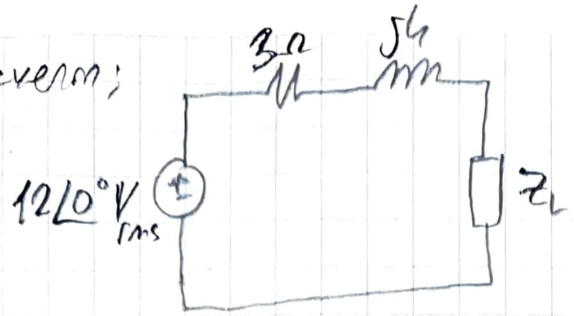
$$P_{max} = \frac{Re(Y_T)\,|\bar{V}_{T_{rms}}|^2}{4} = \frac{|\bar{I}_{T_{rms}}|^2}{4Re(Y_T)}$$

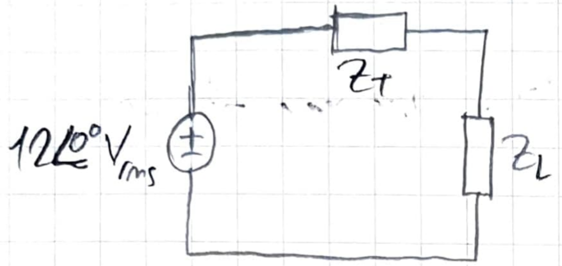
$\Rightarrow$



Yükün çektiği maximum gücü bul?

Thevenin;





$$Z_T = 3+j4\ \Omega$$

maximum güç için

$$Z_L = Z_T^* = 3-j4\ \Omega$$

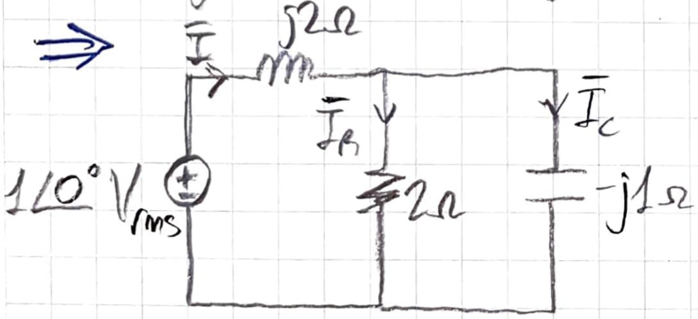
$$P_{max} = \frac{12^2}{4.3} = 12\,W$$

6. Güç Dönüşümü

$$\sum_{k,j} \bar{S}_{k,j} = 0$$

Herhangi bir devrede bütün elemanların çektiği kompleks güçlerin toplamı sıfırdır.

$\Rightarrow$



Devrenin kompleks güç bilançosunu açıklayınız?

$$Z = j2 + \frac{2(-j1)}{2-j1} = 0,4 + j1,2\ \Omega$$

$$\bar{I} = \frac{1\angle 0^\circ}{0,4+j1,2} = 0,791\angle -71,6^\circ\ A_{rms}$$

Devrenin çektiği güç;

$$\bar{S}_d = \bar{V}\bar{I}^* = 1\angle 0^\circ (0{,}791\angle 71{,}6^\circ) = 0{,}25 + j0{,}75 \text{ VA}$$

$$\bar{S}_L = \bar{Z}_L|\bar{I}|^2 = (j2)(0{,}791)^2 = 0 + j1{,}25 \text{ VA}$$

$$\bar{I}_R = \frac{0{,}791\angle -71{,}6^\circ (-j1)}{2-j1} = 0{,}354\angle -135^\circ \text{ A}_{rms}$$

$$\bar{S}_R = Z_R|\bar{I}_R|^2 = 2.(0{,}354)^2 = 0{,}25 + j0 \text{ VA}$$

$$\bar{I}_C = \bar{I} - \bar{I}_R = 0{,}707\angle -45^\circ \text{ A}_{rms}$$

$$\bar{S}_C = Z_C|\bar{I}_C|^2 = (-j1)(0{,}707)^2 = -j0{,}50 \text{ VA}$$

Kaynağın çektiği güç;

$$\bar{S} = -\bar{V}\bar{I}^* = -0{,}25 - j0{,}75 \text{ VA}$$

Reaktif Güç ve Güç Faktörü

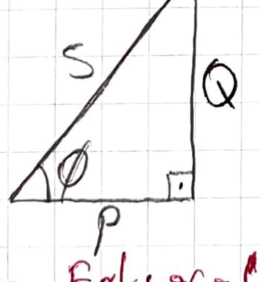
$$P = V_{rms} I_{rms} \cos\phi \ (\text{W})$$

$$Q = V_{rms} I_{rms} \sin\phi \ (\text{VA}_r)$$

$$\phi = \phi_v - \phi_i$$

Görünen Güç (S);

$$S = V_{rms} I_{rms} \ (\text{VA})$$



Kompleks gücün genliği görünen güce eşittir.

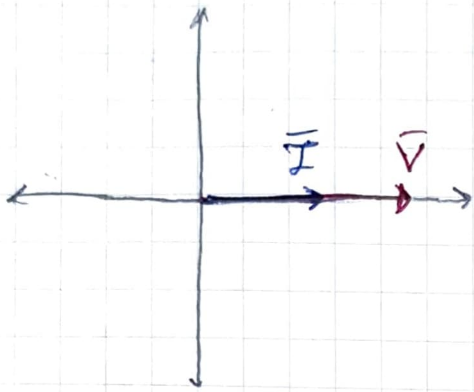
Güç Faktörü;

$$pf = \frac{P}{S} = \frac{P}{V_{rms} I_{rms}}$$

Sinüsoidal durumda;

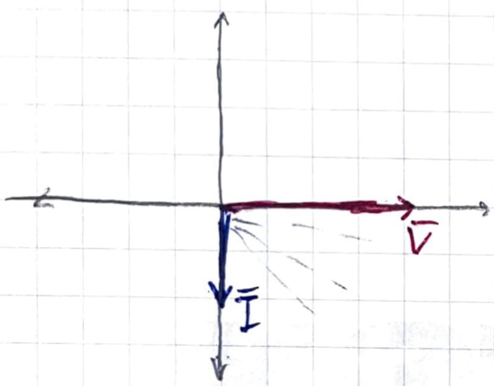
$pf = \cos\phi$

[$\cos\phi$ → tek frekans için
Güç fak. → çok " "]

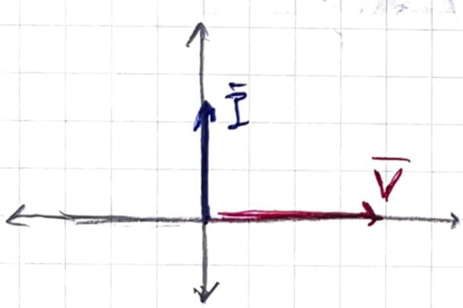


saf omik

(Akım gerilim aynı faz)



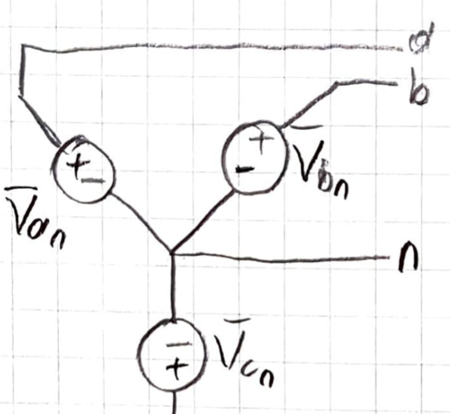
saf endüktif (geri)

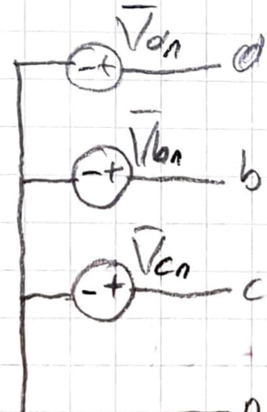


saf kapasitif (ileri)

Üç Fazlı Devreler

Üç fazlı Yıldız-Yıldız (Y/Y) sistemler





$\bar{V}_{an} = V_p \angle 0°$

$\bar{V}_{bn} = V_p \angle -120°$

$\bar{V}_{cn} = V_p \angle 120°$

Pozitif (abc) sıralı

$\bar{V}_{an} = V_p \angle 0°$

$\bar{V}_{bn} = V_p \angle 120°$

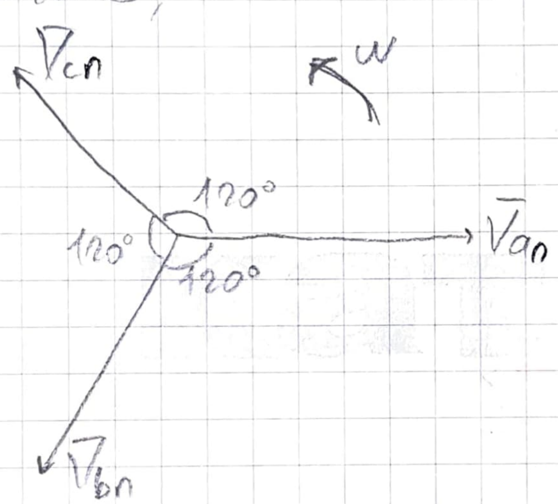
$\bar{V}_{cn} = V_p \angle -120°$

negatif (acb) sıralı

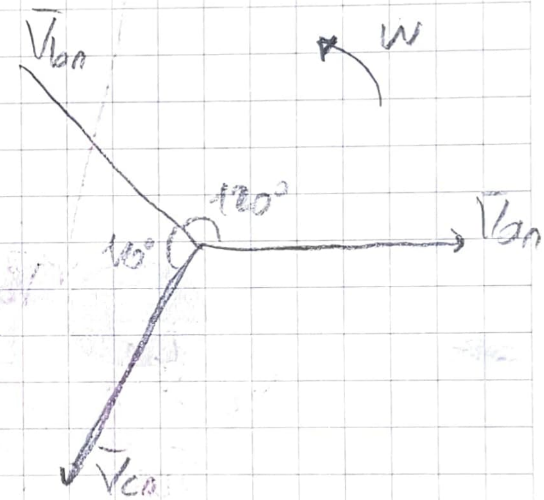
Dengeli sistemlerde;

$$\bar{V}_{an} + \bar{V}_{bn} + \bar{V}_{cn} = 0$$

Fazörler;



pozitif sıralı



negatif sıralı

$$\bar{V}_{bn} = \bar{V}_{an} \angle -120°$$

$$\bar{V}_{cn} = \bar{V}_{an} \angle 120°$$

Faz arası veya hat gerilimi;

$$\bar{V}_{ab} = \bar{V}_{an} - \bar{V}_{bn} = \bar{V}_{an} + \bar{V}_{nb}$$

$$= V_p \angle 0° + V_p \angle 60°$$

$$= V_p + V_p \left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3}\, V_p \left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)$$

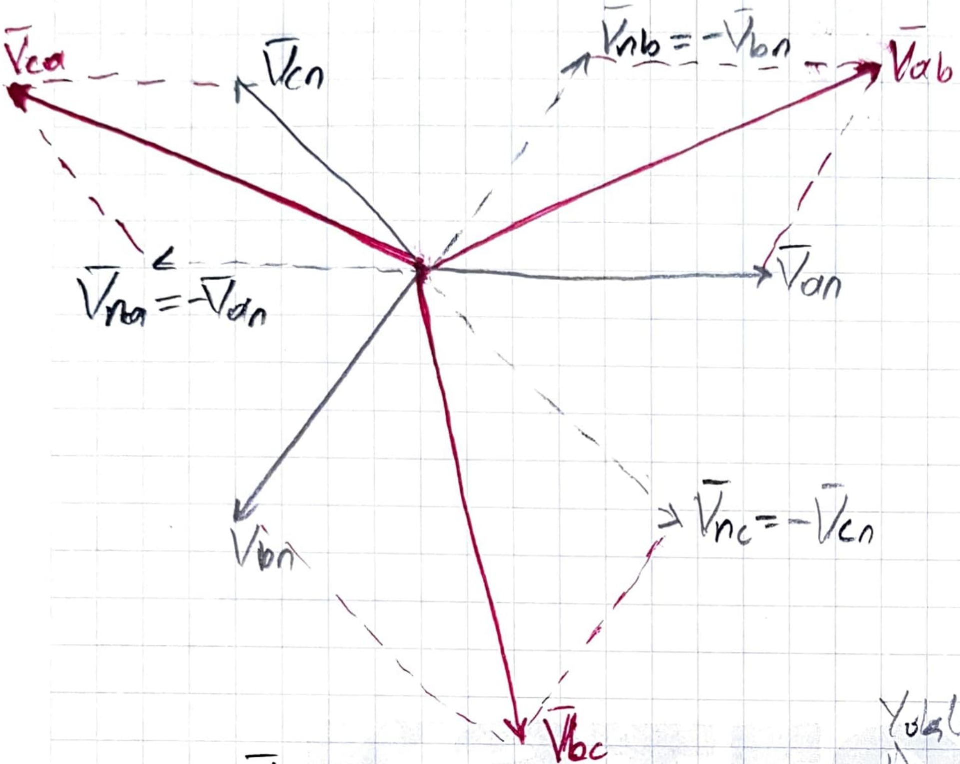
$$= \sqrt{3}\, V_p \angle 30°$$

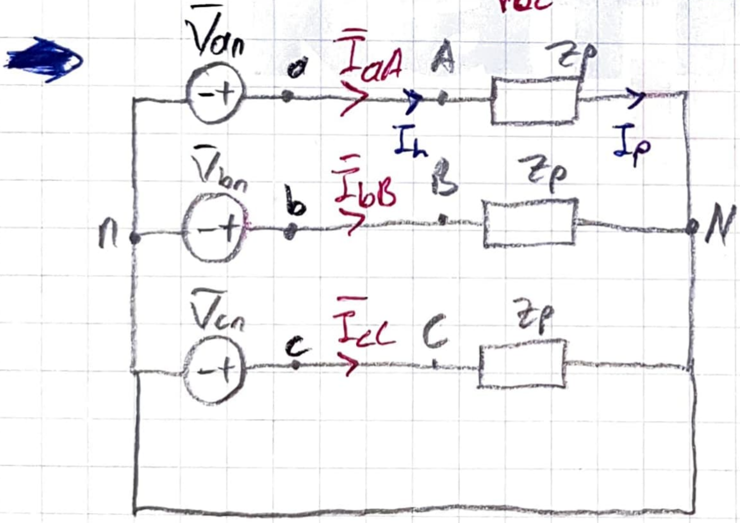
$$\bar{V}_{bc} = \sqrt{3}\, V_p \angle -90$$

$$\bar{V}_{ca} = \sqrt{3}\, V_p \angle -210°$$

$$\boxed{V_L = \sqrt{3}\, V_p}$$

$$\bar{V}_{ab} = V_L \angle 30^\circ$$
$$\bar{V}_{bc} = V_L \angle -90^\circ$$
$$\bar{V}_{ca} = V_L \angle -210^\circ$$





Yollar eşitse $V_N = 0$
Dengeli olduğundan
$V_N = V_n = 0$

$$\bar{I}_{aA} = \frac{\bar{V}_{an}}{Z_p}$$

$$\bar{I}_{bB} = \frac{\bar{V}_{bn}}{Z_p} = \frac{\bar{V}_{an} \angle -120^\circ}{Z_p}$$

$$\bar{I}_{bB} = \bar{I}_{aA} \angle -120^\circ$$

$$\bar{I}_{cC} = \frac{\bar{V}_{cn}}{Z_p} = \frac{\bar{V}_{aA} \angle 120^\circ}{Z_p}$$

$$\bar{I}_{cC} = \bar{I}_{aA} \angle 120^\circ$$

$$-\bar{I}_{nN} = \bar{I}_{aA} + \bar{I}_{bB} + \bar{I}_{cC} = 0 \text{ (sistem Dengeli)}$$

// En Basit Yıldız - Yıldız Sistemi

$\bar{I}_{aA} = I_L \angle \theta = I_p \angle -\theta$ θ: Yük açısı (Empedansın açısı)

$\bar{I}_{bB} = I_L \angle -\theta - 120° = I_p \angle -\theta - 120°$

$\bar{I}_{cC} = I_L \angle -\theta + 120° = I_p \angle -\theta + 120°$

$$Z_p = R_p + jX_p$$

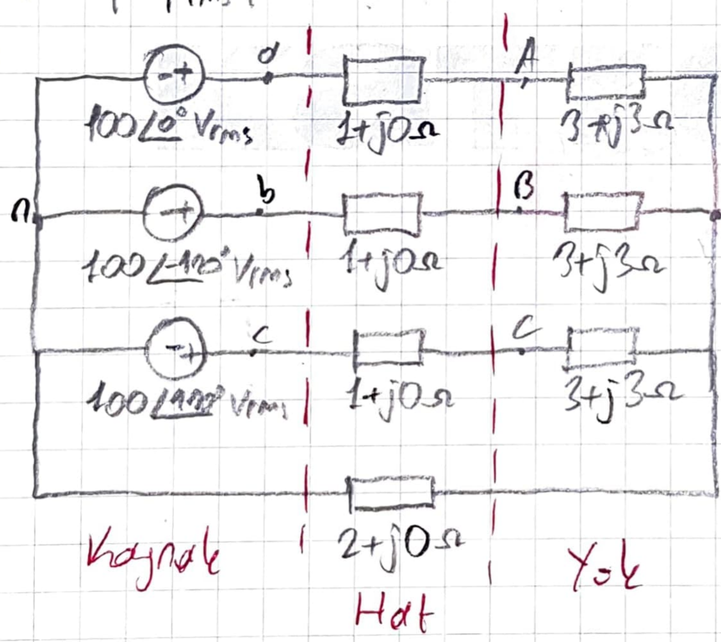
$$\theta = \tan^{-1} \frac{X_p}{R_p}$$

Her bir faz için; $P_p = V_{p_{rms}} \cdot I_{p_{rms}} \cdot \cos\theta$

Toplam güç; $P = 3P_p$

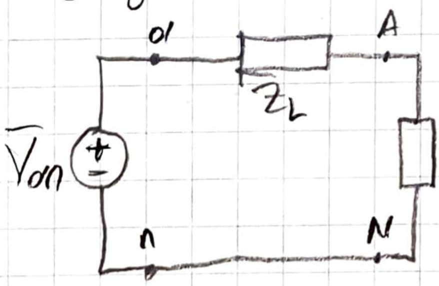
$$P_p = |I_{p_{rms}}|^2 \, Re(Z_p)$$ // Aktif güç $I^2 R$

$\Rightarrow$



[Yükler eşitse V_N sıfırdır]

Dengeli sistemler için;



Bir fazlı eşdeğer devre

$$Z = Z_L + Z_p = 4 + j3 \, \Omega$$

$$\bar{I}_{aA} = \frac{100\angle 0°}{5\angle 36{,}9°} = 20\angle -36{,}9° \text{ Arms}$$

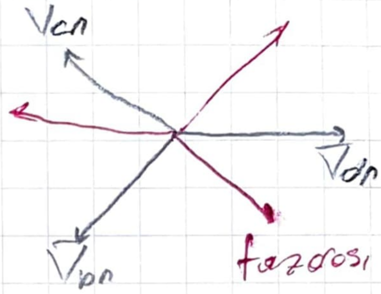
$$\bar{I}_{bB} = 20\angle -156{,}9° \text{ Arms}$$

$$\bar{I}_{cC} = 20\angle -276{,}9° \text{ Arms}$$

⇒ Hat (Faz arası) gerilimi 380 Vrms olan dengeli kaynaktan beslenen yıldız bağlı dengeli bir yük $P = 1{,}71\,kW$, $\cos\phi = 0{,}9$ geri (endüktif) bir güç çekmektedir. Hat akımını ve yük empedansını bulunuz.

• 380 → faz arası
Bize faz nötr lazım

• Endüktif olduğundan
$\cos\phi$ pozitiftir



$$P = 3P_p$$

$$P_p = V_{p\,rms}\, I_{p\,rms} \cos\phi$$

$$P_1 = \frac{1710}{3} = 570\,W$$

$$570 = \frac{380}{\sqrt{3}}\, I_p\, 0{,}9$$

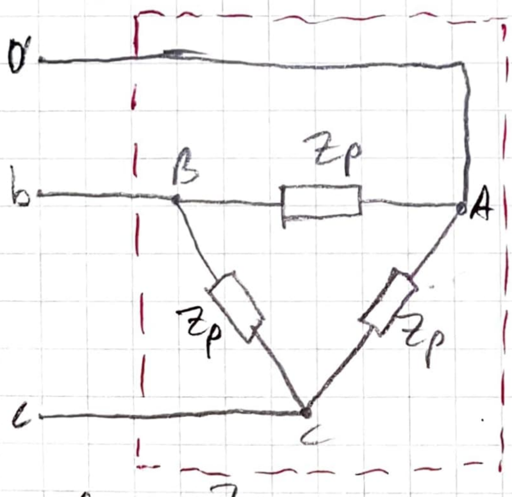
$$I_p = I_L = 2{,}89\,A\,rms$$

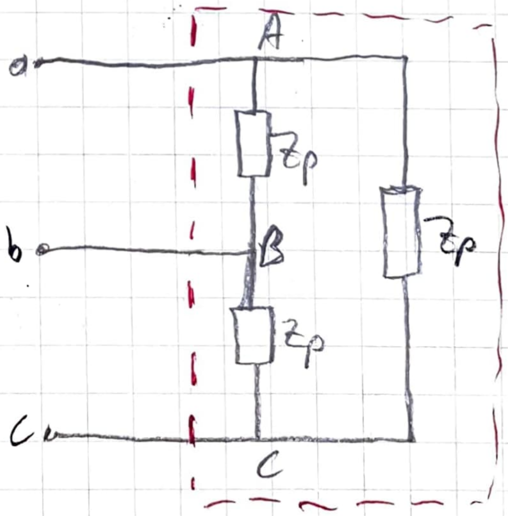
$$|Z_p| = \frac{|V_p|}{|I_p|} = \frac{\frac{380}{\sqrt{3}}}{2{,}89} = 76{,}12\,\Omega$$

$$\phi = \cos^{-1} 0{,}9 = 25{,}84^\circ$$

$$Z_p = 76{,}12 \angle 25{,}84^\circ\ \Omega$$

Üçgen Yükler





Yıldız → faz nötr
Üçgen → faz arası

→ $\bar{V}_{AB} = V_L \angle 30^\circ$

$\bar{V}_{BC} = V_L \angle -90^\circ$

$\bar{V}_{CA} = V_L \angle 150^\circ$

V_L : Faz arası (hat) gerilim

$V_L = V_p$ üçgen bağlı olduğundan

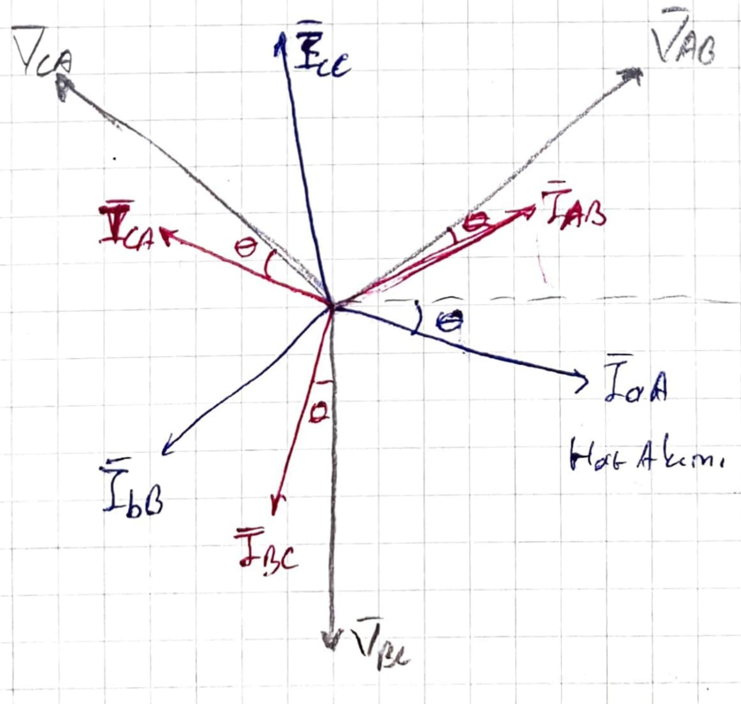
→ $Z_p = |Z_p| \angle \theta$

$$\bar{I}_{AB} = \frac{\bar{V}_{AB}}{Z_p} = I_p \angle 30^\circ - \theta$$

$$\bar{I}_{BC} = \frac{\bar{V}_{BC}}{Z_p} = I_p \angle -90^\circ - \theta$$

$$\bar{I}_{CA} = \frac{\bar{V}_{CA}}{Z_p} = I_p \angle 150^\circ - \theta$$

$$I_p = \frac{V_L}{|Z_p|}$$



$\xrightarrow{KCL}$ $\bar{I}_{aA} = \bar{I}_{AB} - \bar{I}_{CA}$

$\bar{I}_{aA} = \sqrt{3}\, I_p \angle -\theta$

$\bar{I}_{bB} = \sqrt{3}\, I_p \angle -120° - \theta$

$\bar{I}_{cC} = \sqrt{3}\, I_p \angle -240° - \theta$

$$\boxed{I_L = \sqrt{3}\, I_p}$$

$\rightarrow$ $\bar{I}_{aA} = I_L \angle -\theta$

$\bar{I}_{bB} = I_L \angle -120° - \theta$

$\bar{I}_{cC} = I_L \angle -240° - \theta$

$\Rightarrow$ Üçgen bağlı bir yük $\cos\phi = 0{,}8$ endüktif olarak 1,5 kW'lık bir güç çekmektedir. Hat gerilimi 250 Vrms olduğuna göre hat akımını bulunuz?

$P = 3P_p$

$P_p = V_{L_{rms}} I_{P_{rms}} \cos\phi$

$P_p = \frac{1500}{3} = 500W$

$500 = 250\, I_p \cdot 0{,}8$

$I_p = 2{,}5\ A_{rms}$

$I_L = \sqrt{3}\, I_p = 4{,}33\ A_{rms}$

En Genel Durumda

Yıldız Sistemde

$P = 3P_p = 3V_p I_p \cos\phi$

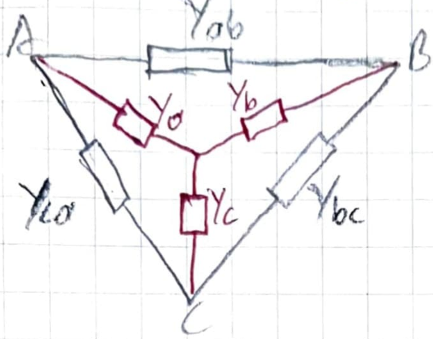
V_p: faz (faz-nötr) gerilimi
I_p: faz akımı

Üçgen bağlı sistemde

$P = \sqrt{3}\, V_L I_L \cos\phi$

V_L: Hat (faz arası) gerilim
I_L: Hat akımı

Yıldız - Üçgen Dönüşümü



$$Y_{ab} = \frac{Y_a Y_b}{Y_a + Y_b + Y_c}$$

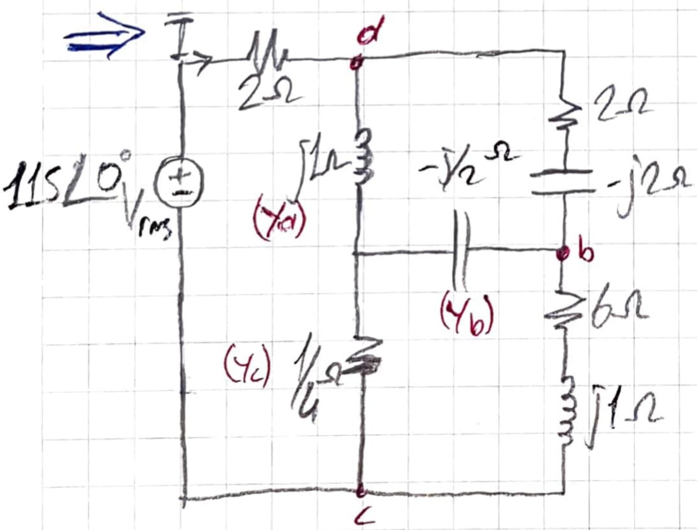
$$Y_{bc} = \frac{Y_b Y_c}{Y_a + Y_b + Y_c}$$

$$Y_{ca} = \frac{Y_c Y_a}{Y_a + Y_b + Y_c}$$

$$Z_a = \frac{Z_{ab} \cdot Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

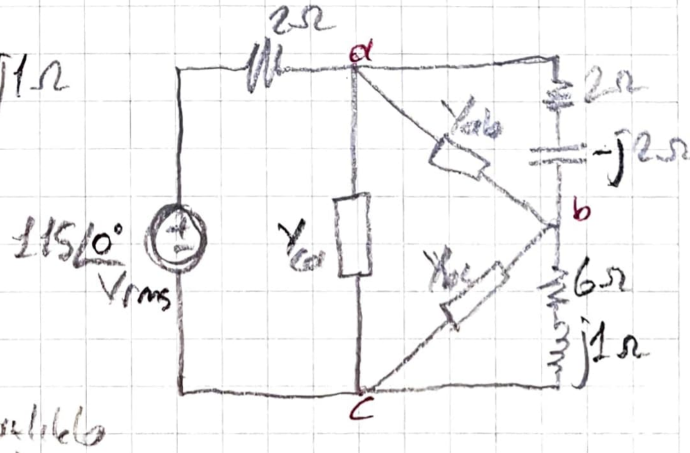
$$Z_b = \frac{Z_{bc} Z_{ab}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

$$Z_c = \frac{Z_{ca} Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}}$$



I akımını bulunuz?

$$\left(Y_b = \frac{1}{Z_b} \right)$$



Seri paralel
devresi rahatlıkla
alına bilecek
Yani hepsi yıldız veya üçgen olacak

$Y_a = -j1 \ S$

$Y_b = j2 \ S$

$Y_c = 4 \ S$

$Y_a + Y_b + Y_c = 4 + j \ S$

$$Y_{ab} = \frac{(-j1)(j2)}{4+j} = \frac{2}{4+j}$$

$$Y_{bc} = \frac{j8}{4+j}$$

$$Y_{ca} = \frac{-4j}{4+j}$$

$$\frac{2}{4+j} + \frac{1}{2-j2} = \frac{8-3j}{10-6j}$$ paralel olduğu için topla

$$\frac{j8}{4+j} + \frac{1}{6+j} = \frac{-4+48j}{23+10j}$$ (Y_{bc} paralel)

$$Z_1 = \frac{10-6j}{8-3j} + \frac{23+10j}{-4+48j}$$

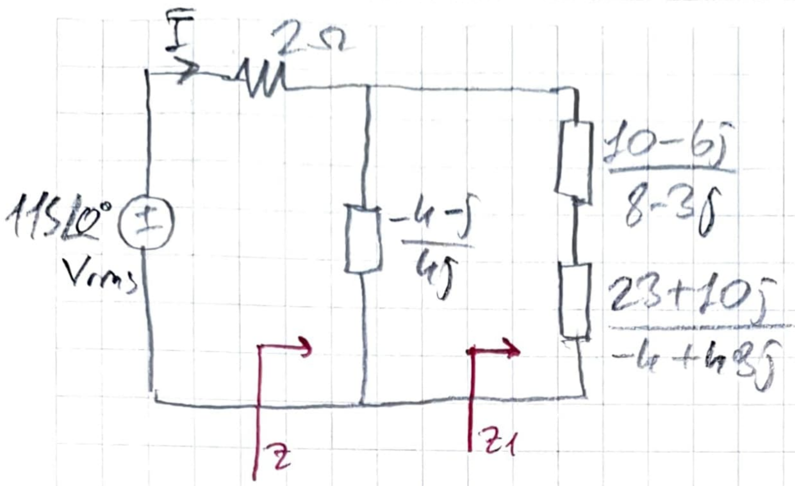
$$Z_1 = 1{,}51 - j0{,}73 \ \Omega$$

$$Z = 2 + \frac{[(-4-j)/4j] Z_1}{[(-4-j)/4j] + Z_1}$$

$$Z = 2{,}54 + j1{,}23 \ \Omega$$

$$\bar{I} = \frac{115\angle 0^\circ}{Z} = \frac{115\angle 0}{2{,}54 + j1{,}23}$$

$$\bar{I} = 40{,}7 \angle -25{,}8 \ A_{rms}$$



Laplace Dönüşümü (Geçici hal)

$$F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

Örnek: $f(t) = e^{-at}\, u(t)$

$$F(s) = \int_{0}^{\infty} e^{-at} e^{-st} dt = \int_{0}^{\infty} e^{-(s+a)t} dt = \frac{1}{s+a}$$

Ters Laplace Dönüşümü

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s)\, e^{st}\, ds$$

Laplace Dönüşümü Özellikleri

	$f(t)$		$F(s)$
1- Doğrusallık	$c_1 f_1(t) + c_2 f_2(t)$	$\longrightarrow$	$c_1 F_1(s) + c_2 F_2(s)$
2- Türev alma	$\frac{d}{dt} f(t)$	$\longrightarrow$	$sF(s) - f(0-)$

3- n. Dereceden Türev

$$\frac{d^n}{dt^n} f(t) \longrightarrow s^n F(s) - s^{n-1} f(0-) - s^{n-2} f'(0-) \ldots - f^{(n-1)}(0-)$$

4- İntegral alma

$$\int_{0^-}^{t} f(\tau)\,d\tau \longrightarrow \frac{F(s)}{s}$$

5- Zaman domeninde öteleme

$$f(t-t_0)\,u(t-t_0),\ t>0 \longrightarrow e^{-st_0}F(s)$$

6- Frekans domeninde öteleme

$$e^{-s_0 t}f(t) \longrightarrow F(s+s_0)$$

7- Zaman - frekans ölçeklemesi

$$f(ct),\ c>0 \longrightarrow \frac{1}{c}F\left(\frac{s}{c}\right)$$

8- t ile çarpma

$$tf(t) \longrightarrow -\frac{d}{ds}F(s)$$

9- n. dereceden t ile çarpma

$$t^n f(t) \longrightarrow (-1)^n \frac{d^n}{ds^n}F(s)$$

$\Rightarrow$ $F(s) = \dfrac{2(s+10)}{(s+1)(s+4)}$

$$F(s) = \frac{2(s+10)}{(s+1)(s+4)} = \frac{A}{s+1} + \frac{B}{s+4}$$

$$(s+1)F(s) = \frac{2(s+10)}{s+4} = A + \frac{B(s+1)}{s+4}$$

$$(s+4)F(s) = \frac{2(s+10)}{s+1} = \frac{A(s+4)}{s+1} + B$$

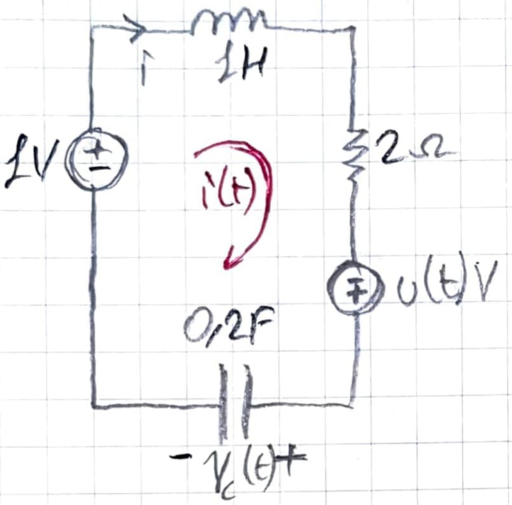
$$A = (s+1)F(s)\Big|_{s=-1} = \frac{2.9}{3} = 6$$

$$B = (s+4)F(s)\Big|_{s=-4} = \frac{2.6}{-3} = -4$$

$$F(s) = \frac{6}{s+1} - \frac{4}{s+4}$$

$$f(t) = 6e^{-t} - 4e^{-4t}$$

$\Rightarrow$



$t>0$ için $i=?$

$i(0-)=0$

$V_c(0-)=1V$

$$\frac{di}{dt} + 2i + \left[5\int_{0-}^{t} i(\tau)\,d\tau + 1\right] = 1 + u(t)$$

$$[sI(s) - 0] + 2I(s) + \frac{5I(s)}{s} + \frac{1}{s} = \frac{1}{s} + \frac{1}{s}$$

$$I(s) = \frac{1}{s^2+2s+5} = \frac{A}{s+1-j2} + \frac{A^*}{s+1+j2}$$

$$i(t) = \frac{1}{2}e^{-t}\cos(2t-90^\circ) = \frac{1}{2}e^{-t}\sin 2t \ A$$

S Domeninde Devre Analizi

$$V_1(t) + V_2(t) + V_3(t) + V_4(t) = 0$$

$$V_1(s) + V_2(s) + V_3(s) + V_4(s) = 0$$

$$i_1(t) + i_2(t) + i_3(t) + i_4(t) = 0$$

$$I_1(s) + I_2(s) + I_3(s) + I_4(s) = 0$$

$$v_R(t) = R\, i_R(t) \Longrightarrow V_R(s) = R\, I_R(s)$$

$$v_L(t) = L \frac{d}{dt} i_L(t) \Longrightarrow V_L(s) = sL\, I_L(s) - L\, i_L(0^-)$$

$$i_c(t) = C \frac{d}{dt} v_c(t) \Longrightarrow I_c(s) = sC\, V_c(s) - C\, v_c(0^-)$$

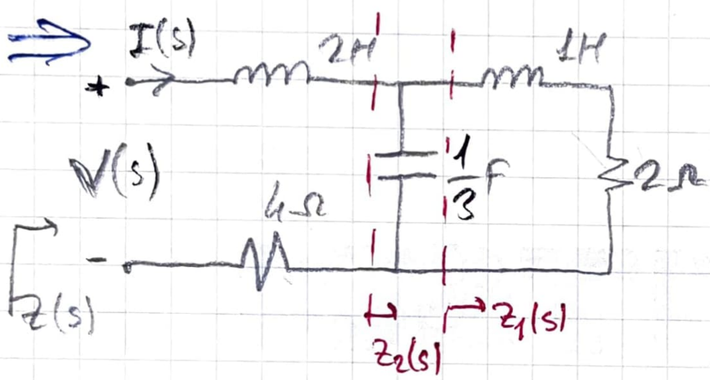
$$V_c(s) = \frac{1}{sC} I_c(s) + \frac{1}{s} v_c(0^-)$$

Bütün başlangıç koşulları sıfır ise;

$$\boxed{V(s) = Z(s)\, I(s)}$$

$$Z_R(s) = R, \quad Z_L(s) = sL, \quad Z_C(s) = \frac{1}{sC}$$

$\Longrightarrow$



$Z(s) = ?$

$V(s) = \frac{1}{s^2}$ olması durumunda $I_c(s)$ akımını bulunuz? (Başlangıç koşulları sıfır)

$$Z_1(s) = s + 2$$

$$Z_2(s) = \frac{1}{\frac{1}{s+2} + \frac{s}{3}} = \frac{3(s+2)}{s^2+2s+3}$$

$$Z(s) = Z_2(s) + 2s + 4$$

$$= \frac{3(s+2) + (2s+4)(s^2+2s+3)}{s^2+2s+3}$$

$$= \frac{2s^3 + 8s^2 + 17s + 18}{s^2+2s+3}$$

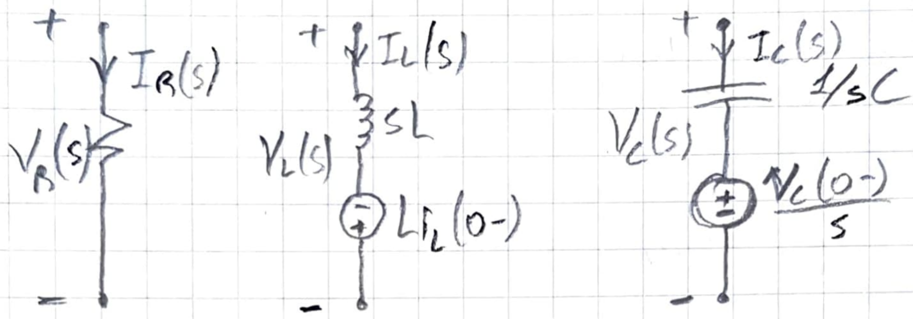
$$I(s) = Y(s)\, V(s) = \frac{s^2+2s+3}{2s^3+8s^2+17s+18} \cdot \frac{1}{s^2}$$

$$I_c(s) = \frac{\frac{s}{3}}{\frac{s}{3} + \frac{1}{s+2}} I(s) = \frac{s+2}{s(2s^3+8s^2+17s+18)}$$

- $V_R(s) = R\, I_R(s)$

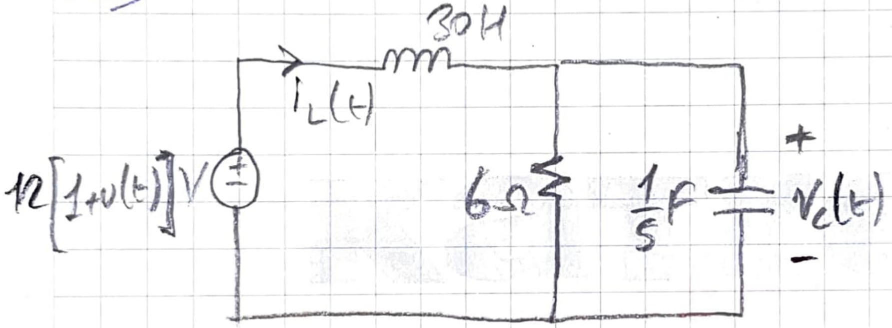
$V_L(s) = sL\, I_L(s) - L i_L(0-)$

$V_C(s) = \frac{1}{sC} I_C(s) + \frac{1}{s} V_C(0-)$



[Başlangıç koşulları kaynak olarak bağlanır]

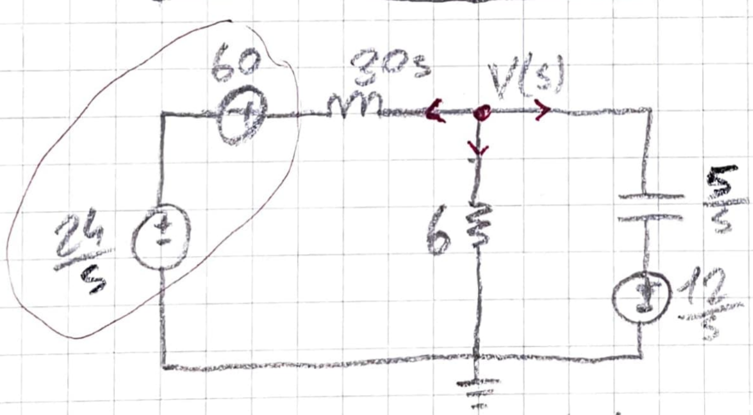
$\Longrightarrow$



$t>0$ için $V_C(t) = ?$

$i_L(0-) = 2A$

$V_C(0-) = 12V$



KCL $\quad V(s) = \dfrac{12s^2 + 10s + 4}{s\left(s^2 + \frac{5}{6}s + \frac{1}{6}\right)}$

$$V(s) = \frac{24}{s} + \frac{24}{s+\frac{1}{2}} - \frac{36}{s+\frac{1}{3}}$$

$$V_C(t) = 12\left(2 + 2e^{-t/2} - 3e^{-t/3}\right) V \quad , \quad t>0$$

Aktif güç → direnç üzerinde

Vize Tekrar

$S = 12\ kVA$

$\bar{S} = 6 + j6\ kVA$

- $P = V_{rms} I_{rms} \cos\phi$ (W) Aktif güç

  $Q = V_{rms} I_{rms} \sin\phi$ (VAr) Reaktif güç

  $S = V_{rms} I_{rms} = |\bar{S}|$ (VA) Görünür güç

  $\bar{S} = \bar{V}_{rms} \bar{I}^{*}_{rms} = P + jQ$ (VA) Kompleks güç

- $v(t) = V_m \sin(wt + \phi)$

  $v(t) = 5\cos(314t + 30^\circ)\ V$

  $w = 2\pi f$ w: rad/sn

  $P = \frac{1}{2} V_m I_m \cos\phi$

  $Q = \frac{1}{2} V_m I_m \sin\phi$

  sinüsoidal Durumda

  $V_{rms} = \frac{V_m}{\sqrt{2}}$

  $I_{rms} = \frac{I_m}{\sqrt{2}}$

- 

  $\bar{S} = \bar{V} \bar{I}^{*}$

$$\boxed{\begin{aligned} Q &= 3|I|^2 \, Im(Z_y) \\ \bar{V}_y &= \bar{I} \cdot Z_y \end{aligned}}$$

Frekans Cevabı

$V_o(s) = H(s)\,V_i(s)$ Başlangıç koşulu sıfır

$H(s)$: transfer fonksiyonu

$s \rightarrow jw$

$$\boxed{\bar{V}_o = \bar{H}(jw)\,\bar{V}_i}$$

⇒ Transfer fonksiyonu $H(s) = \dfrac{2s}{s^2+4s+1}$ olan bir devrenin girişine $v_i(t) = 12\cos(2t-30°)$ V uygulanırsa sürekli halde çıkış ne olur?

$\bar{V}_o = \bar{H}(jw)\,\bar{V}_i$

$\bar{V}_i = 12\angle -30$ V

$\bar{H}(jw) = \dfrac{2(jw)}{(jw)^2+4(jw)+1}$ $w=2$

$\bar{H}(j2) = \dfrac{4j}{(2j)^2+4(2j)+1}$

$\bar{V}_o = \bar{H}(j2)\,12\angle -30°$

$\bar{V}_o = 5{,}62\angle -50{,}6°$ V

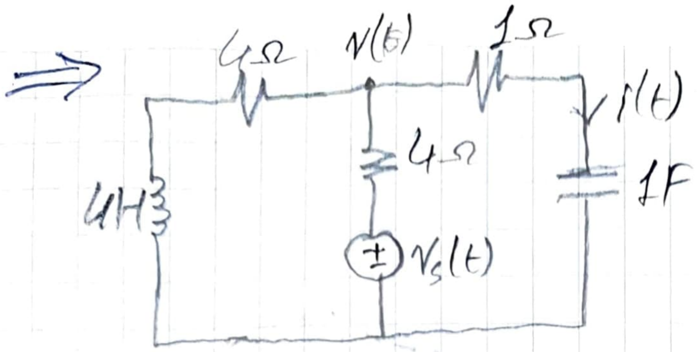
$v_o(t) = 5{,}62\cos(2t-50{,}6°)$ V

- $v_i(t) = 10\cos(100t+20°)$ olursa;

$\bar{H}(j100) = \dfrac{2(100)j}{(100j)^2+4(100j)+1} = 0{,}019\angle -87°$

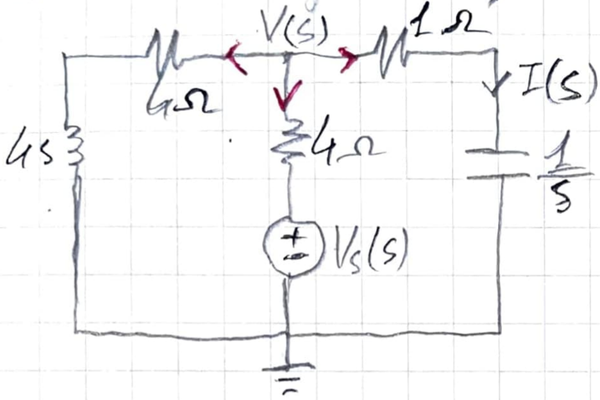
$\bar{V}_o = 0{,}019\angle -87° \cdot 10\angle 20° = 0{,}19\angle -67°$

$v_o(t) = 0{,}19\cos(100t-67°)$ V



$v_s(t) = 6\cos t$ V olması durumunda $i(t)$ akımının sürekli haldeki değerini bulunuz.

Transfer fonksiyonu ile;



$$\frac{V(s)}{4+4s} + \frac{V(s)-V_s(s)}{4} + \frac{V(s)}{1+\frac{1}{s}} = 0$$

$$\frac{1}{4+4s} V(s) + \frac{1}{4}\left[V(s)-V_s(s)\right] + \frac{s}{s+1} V(s) = 0$$

$$V(s) = \frac{s+1}{5s+2} V_s(s)$$

$$I(s) = \frac{V(s)}{1+\frac{1}{s}} = \frac{s}{s+1} V(s) = \frac{s}{5s+2} V_s(s)$$

$$H(s) = \frac{I(s)}{V_s(s)} = \frac{s}{5s+2}$$

$$s \to j\omega$$

$$\bar{H}(j\omega) = \frac{j\omega}{5(j\omega)+2}$$

$$\bar{I} = \bar{H}(j1)\,\bar{V}_s$$

$$\bar{I} = \frac{j1}{5(j1)+2} 6\angle 0°$$

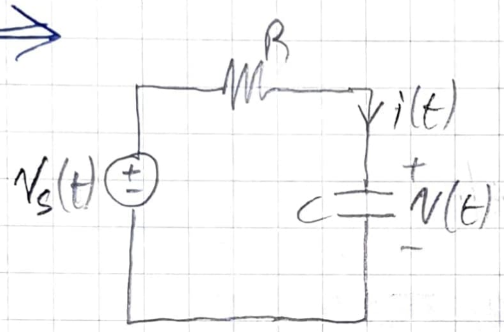
$$\bar{I} = (0{,}186\angle 21{,}8°)(6\angle 0°) = 1{,}12\angle 21{,}8° \text{ A}$$

$$i(t) = 1{,}12\cos(t + 21{,}8°) \text{ A}$$

- $|\bar{V}_o| = |\bar{H}(jw)|\,|\bar{V}_i|$

$$\angle \bar{V}_o = \angle \bar{H}(jw) + \angle \bar{V}_i$$

⇒



v(t) geriliminin genlik ve faz açısının frekansa göre değişimini çiziniz?

$$V(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_s(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} V_s(s)$$

$$H(s) = \frac{V(s)}{V_s(s)} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

[Her zaman en büyük s in katsayısı 1 olacak şekilde transfer fonk. düzenlenir]

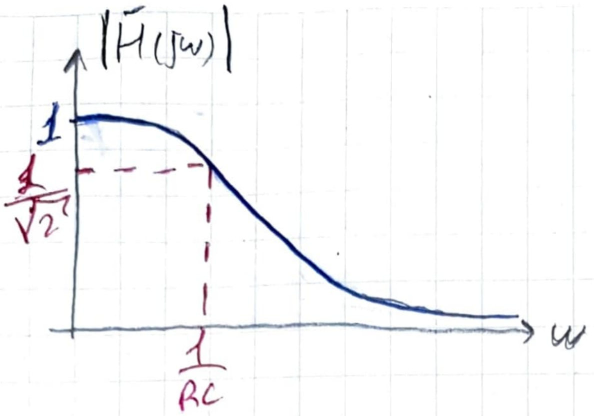
$$\bar{H}(jw) = \frac{\frac{1}{RC}}{jw + \frac{1}{RC}}$$

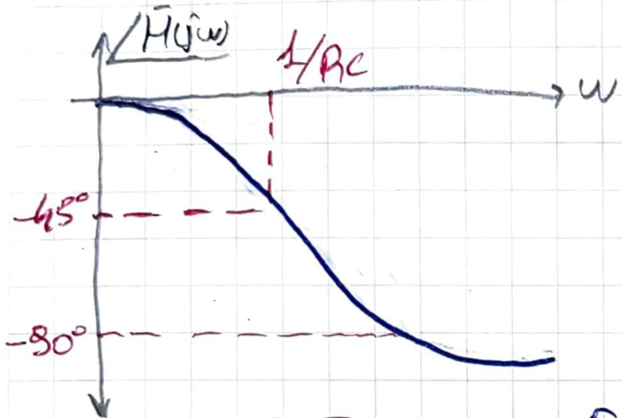
$$|\bar{H}(jw)| = \frac{\left|\frac{1}{RC}\right|}{\left|jw + \frac{1}{RC}\right|} = \frac{\frac{1}{RC}}{\sqrt{w^2 + \left(\frac{1}{RC}\right)^2}}$$ ← frekans sonsuza giderken sıfır

$$\angle H(jw) = -\tan^{-1} wRC$$

SDEN → Filtre ve İki kapılı
Mat analiz → eleman değerleri → prob üç eğri

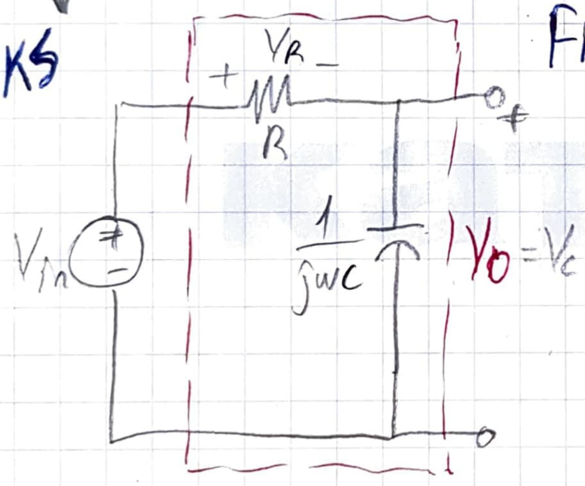
Düşük frekansı geçiriyor Yüksek frekansı geçirmiyor





NKS

Filtreler



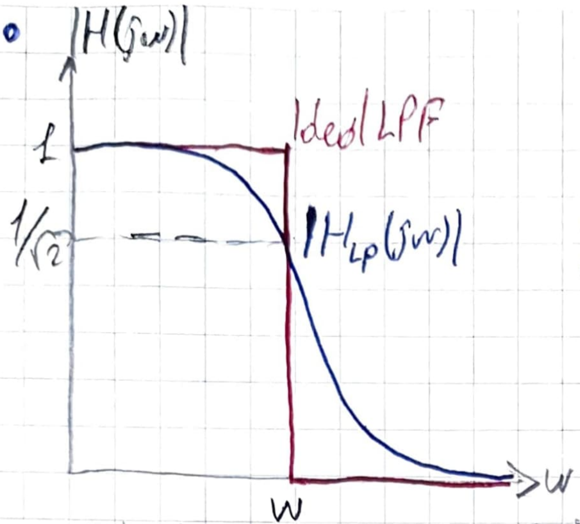
RC yüksek geçiren veya alçak geçiren filtre

$$H_{LP}(j\omega) = \frac{V_c}{V_{in}} = \frac{1/(j\omega C)}{R + [1/(j\omega C)]} = \frac{1}{j\omega RC + 1}$$

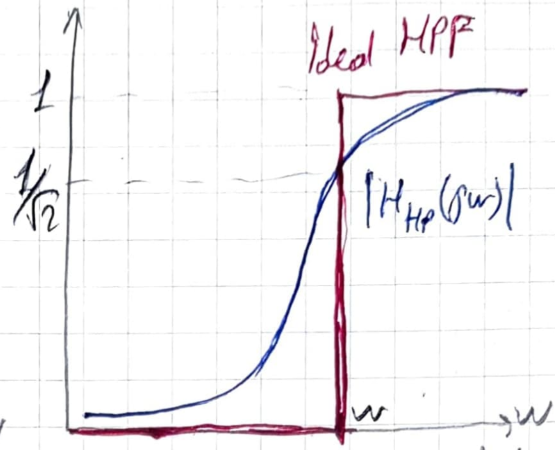
I. Dereceden alçak geçiren pasif filtre devresi
Transfer fonksiyonunun genliği kazancı verir.

$$|H_{LP}(j\omega)| = \frac{1}{\sqrt{R^2C^2w^2+1}}$$

çıkışlar kondansatör → Alçak
" " direnç → yüksek



Alçak geçiren filtre (LPF)



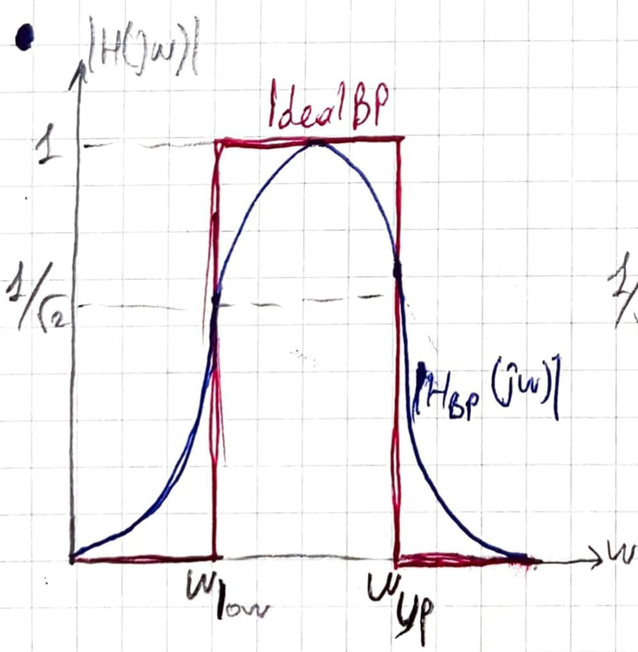
Yüksek geçiren filtre (HPF)

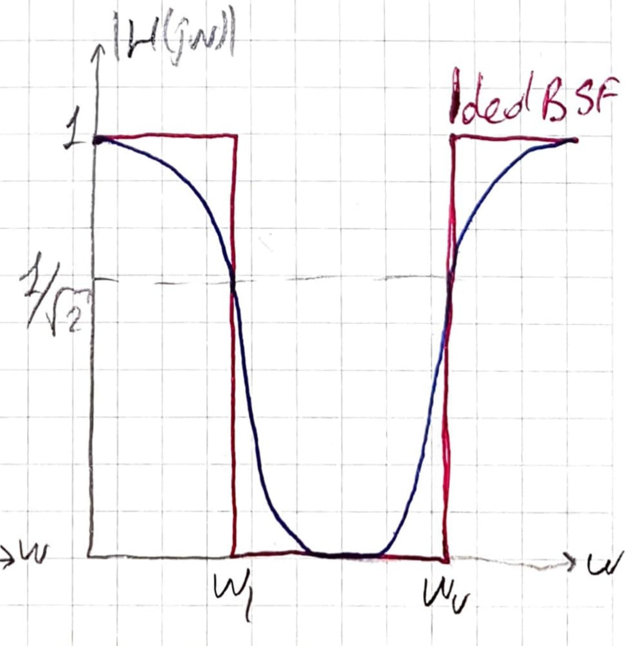
$$W_c = \frac{1}{RC}$$

$$H_{HP}(jw) = \frac{V_R}{V_{in}}$$

$$= \frac{R}{R + [1/jwC]} = \frac{jwRC}{jwRC + 1}$$

$$|H_{HP}(jw)| = \frac{RCw}{\sqrt{R^2C^2w^2 + 1}}$$



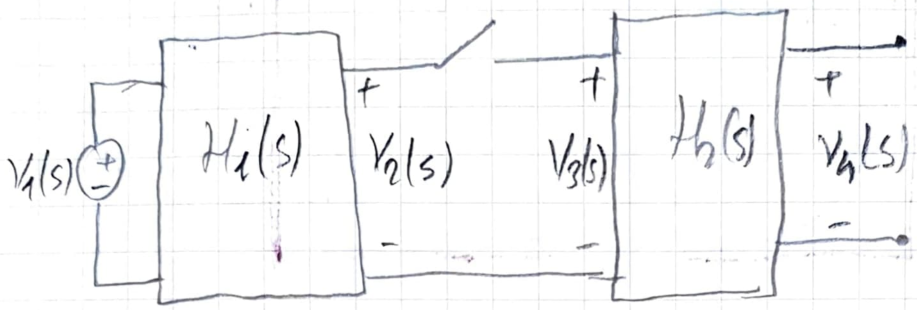


Band geçiren filtre

• Alçak geçiren ile yüksek geçiren filtreyi çarparak band geçiren filtre

$$H_{BP}(j\omega) = H_{LP}(j\omega)\, H_{HP}(j\omega)$$

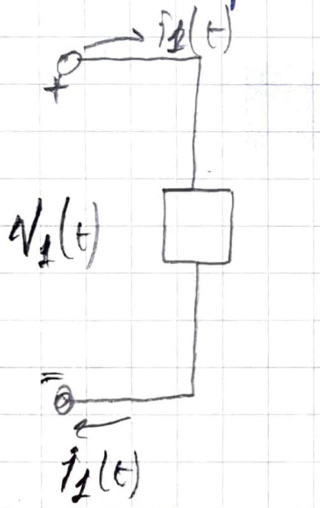
• Kaskad (Cascade)



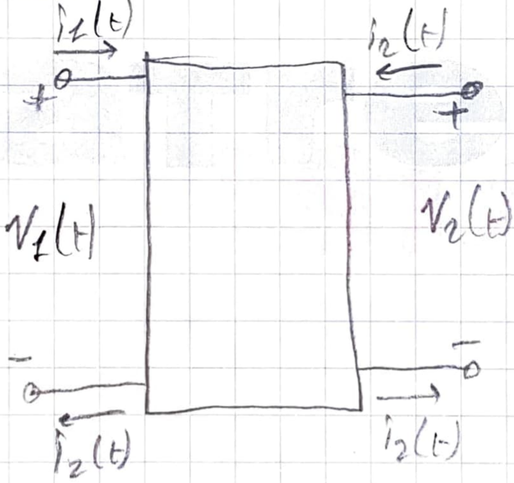
$$H(s) = H_1(s)\, H_2(s)$$

Birinin çıkışı diğerinin girişine bağlanması

İki Kapılı Devreler

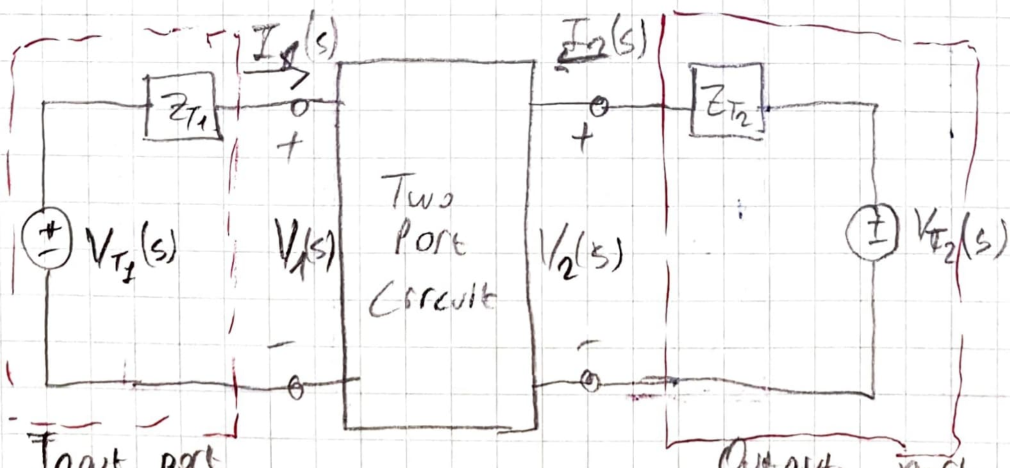


Tek kapılı devre



İki kapılı devre

İki tane terminali olan devreler



Input port termination circuit

Output port termination circuit

Çıkış kapısını $I_2(s)=0$ yapmak istiyoruz. Ve giriş kapısına I_1 akım kaynağı bağlıyoruz.

$$V_1\big|_{I_2=0} = z_{11} I_1 \qquad V_2\big|_{I_2=0} = z_{21} I_1$$

$I_2=0$ için çıkışı açık devre yapmalıyız

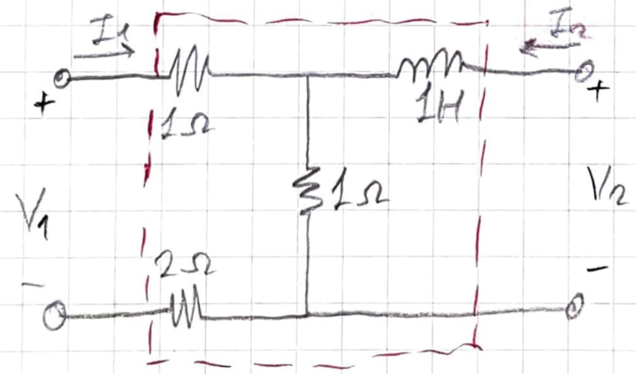
$$V_1\big|_{I_1=0} = z_{12} I_2 \qquad V_2\big|_{I_1=0} = z_{22} I_2$$

V_1 ve V_2 yi bulup topla

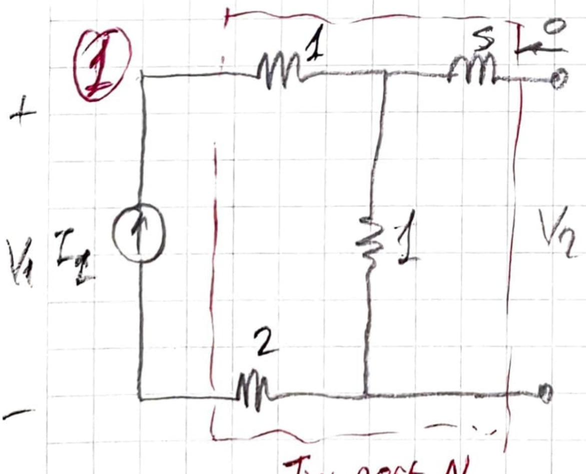
$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

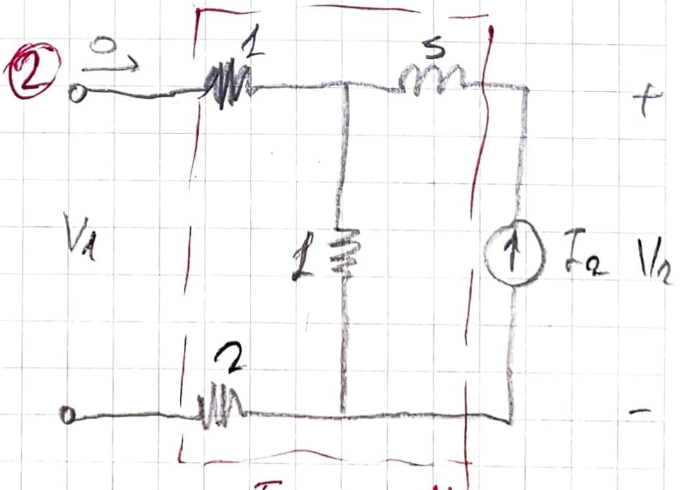
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}}_{\text{empedans parametreleri matrisi}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



Empedans parametreleri matrisini bulun?



Tw port N



Two port N

Thus

1a $I_2 = 0$

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \qquad z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

$V_1 = 4I_1$

$V_2 = I_1$ $\Rightarrow z_{11} = 4 \ , \ z_{21} = 1$

2a $I_1 = 0$

$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} \qquad z_{22} = \frac{V_2}{I_1}\bigg|_{I_1=0}$$

$V_1 = I_2$

$V_2 = (s+1)I_2 \Rightarrow z_{12} = 1 \ , \ z_{22} = s+1$

End

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & s+1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

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Özet

$v(t) = V_m \sin(wt + \phi)$

$V_m \to$ max değer

$w \to$ açısal frekans

Fazörler için Akım ve Gerilim Kuralları

$v(t) = \bar{V} e^{jwt}$

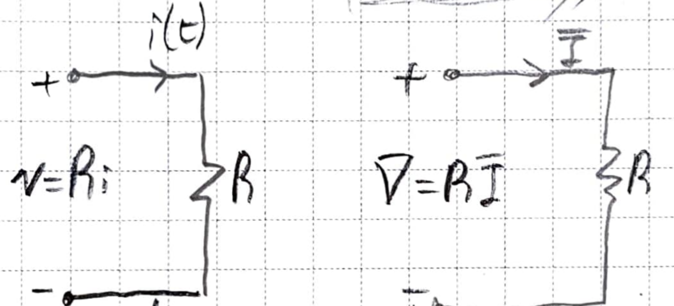
$i(t) = \bar{I} e^{jwt}$

$\bar{V} = |\bar{V}| \angle \phi_v$

$\bar{I} = |\bar{I}| \angle \phi_i$

Ohm kanunu: $\bar{V} e^{jwt} = R \bar{I} e^{jwt}$

$\boxed{\bar{V} = R \bar{I}}$

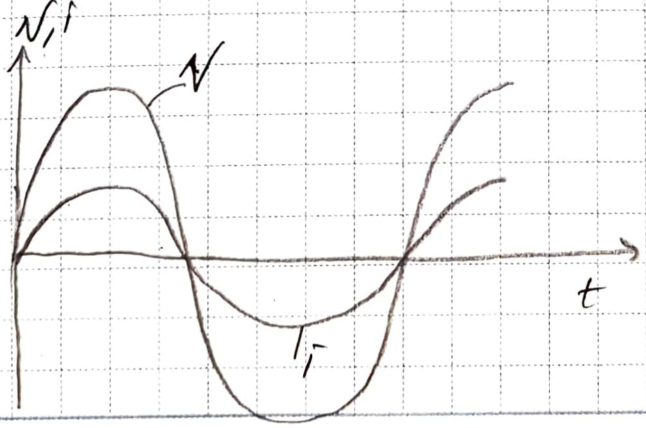


zaman domeni

frekans domeni (jw domeni)

- $v(t) = 12\cos(100t + 30^\circ)$ V
  $R = 3\,\Omega$ ise $\bar{I} = \dfrac{\bar{V}}{R} = \dfrac{12\angle 30^\circ}{3} = 4\angle 30^\circ$ A

  $i(t) = 4\cos(100t + 30^\circ)$ A



[faz farkı yok her ikisi aynı konumda]

3'te 1'i

TEKSAN
TARİH / DATE:
SAAT / TIME:
KONU / SUBJECT:

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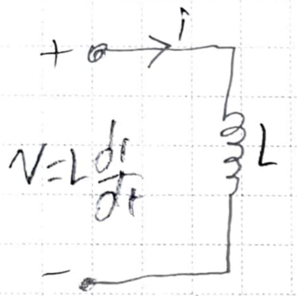
Bobin Akısı

$$V = L\frac{di}{dt}$$

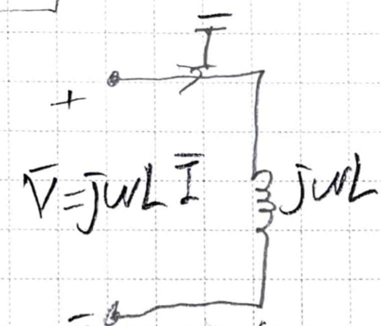
$$V_m e^{j(wt+\phi)} = L\frac{d}{dt}\left[I_m e^{j(wt+\phi_i)}\right]$$

$$= jwL\left[I_m e^{j(wt+\phi_i)}\right]$$

$$\boxed{\bar{V} = jwL\bar{I}}$$



Zaman domeni



Frekans domeni

$j = 1\angle 90°$ olduğundan 90° ileride

İlk olarak gerilim görülüyor sonra akım görülüyor (fazörsel)

- $i = I_m\cos(wt+\phi_i)$ ise

$\bar{V} = jwL\bar{I}$

$\bar{V} = jwL I_m\angle\phi_i$

$\bar{V} = wL I_m\angle\phi_i+90°$

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Kondansatör durumu;

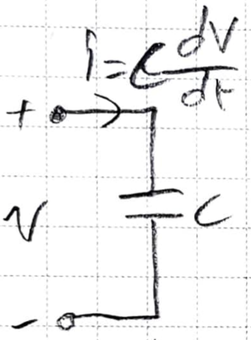
$$i = C\frac{dV}{dt}$$

$$I_m e^{j(wt+\phi_I)} = C\frac{d}{dt}\left[V_m e^{j(wt+\phi_V)}\right]$$

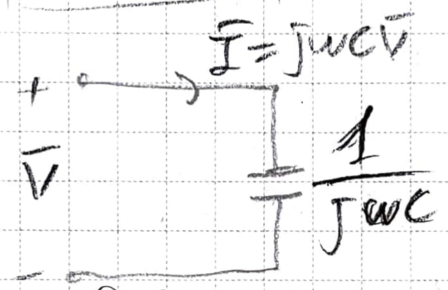
$$= jwCV_m e^{j(wt+\phi_V)}$$

$$\bar{I} = jwC\bar{V}$$

$$\boxed{\bar{V} = \frac{\bar{I}}{jwC}}$$



zaman domeni



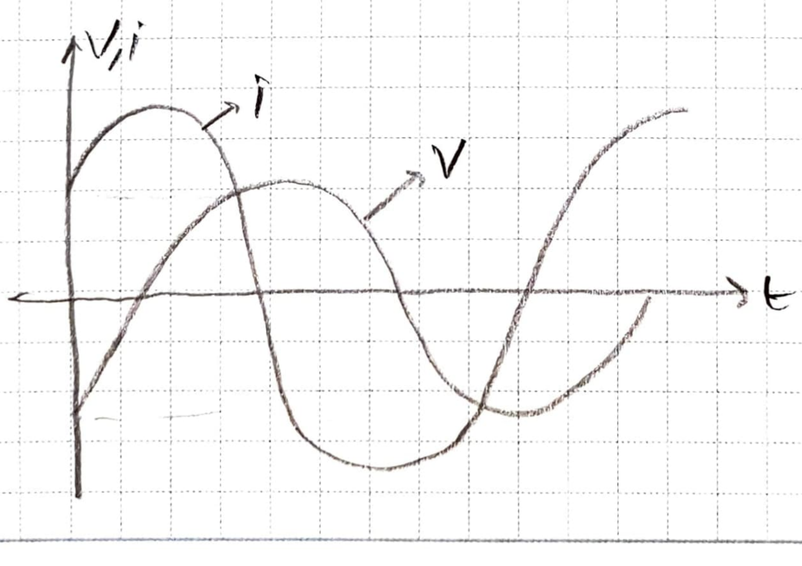
frekans domeni

$$v = V_m \cos(wt+\phi_v)$$

$$\bar{I} = (jwC)(V_m \angle \phi_v)$$

$$\bar{I} = wCV_m \angle \phi_v + 90°$$

$$i(t) = wCV_m \cos(wt+\phi_v+90°)$$



İlk olarak akım görülür sonra gerilim görülür

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Empedans ve Admitans;

$$v(t) = V_m \cos(wt + \phi_v)$$

$$i(t) = I_m \cos(wt + \phi_i)$$

$$\bar{V} = V_m \angle \phi_v$$

$$\bar{I} = I_m \angle \phi_i$$

$$\boxed{\bar{V} = Z\bar{I}}$$ Z : Empedans

$$Z = |Z| \angle \phi_Z = \frac{V_m}{I_m} \angle \phi_v - \phi_i$$

$$|Z| = \frac{V_m}{I_m} \rightarrow \phi_Z = \phi_v - \phi_i$$

$$\boxed{Z = R + jX}$$ X : Reaktans

$$R = Re(Z)$$

$$X = Im(Z)$$

$$|Z| = \sqrt{R^2 + X^2} \qquad \phi_Z = \tan^{-1}\frac{X}{R}$$

- $Z_R = R$
- $Z_L = jwL = wL \angle 90°$
- $Z_C = \frac{1}{jwC} = -j\frac{1}{wC} = \frac{1}{wC} \angle -90°$

Endüktif Reaktans:

$$X_L = wL$$

$$Z_L = jX_L$$

Kapasitif Reaktans

$$X_C = -\frac{1}{wC}$$

$$Z_C = jX_C$$

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Admitans (Y)

$$Y = \frac{1}{Z} = G + jB$$

$G = \text{Re}\,Y$ kondüktans

$B = \text{Im}\,Y$ süseptans

$$Y = G + jB = \frac{1}{Z} = \frac{1}{R + jX}$$

$$\boxed{\begin{array}{l} G \neq \frac{1}{R} \\ B \neq \frac{1}{X} \end{array}}$$

Komplex eşli
Komplex kopya
Sinaşardaki kopya
Ye kuntar

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Bobin $\rightarrow V = L\frac{dI}{dt}$

Kondansatör $\rightarrow I = C\frac{dV}{dt}$

$i_c = C\frac{dV}{dt}$ $\quad I_a + I_f + I_c = 0 \quad V_p = V_n = 0$

$$\frac{V_g - V_n}{R_a} + \frac{V_2 - V_n}{R_f} + C\frac{d}{dt}(V_2 - V_n) = 0$$

$$\frac{V_g}{R_a} + \frac{V_2}{R_f} + C\frac{dV_2}{dt} = 0$$

$t > 0$

$$\frac{V_2}{R_f} + C\frac{dV_2}{dt} = 0$$

$$\frac{1}{R_f C}V_2 + \frac{dV_2}{dt} = 0$$

$$V_2(t) = Ke^{+st}$$

$$s + \frac{1}{R_f C} = 0$$

I

Zorlanmış çözüm

Değer ver (A)

A'yı çöz

[sağ taraf kalsın]

Doğal Çözüm

denklem yerine "s" koy

s'i çöz

[sağ taraf sıfır]

Fedakârlık yapılmadan Zafer kazanılmaz

Dünyayı, Ben Kurtaracam

II

$$s^2 + a_1 s + a_0 = 0$$

$$s^2 + 2\xi\omega_0 s + \omega_0^2 = 0$$

$$\omega_0^2 = a_0 \Rightarrow \omega_0 = \sqrt{a_0}$$

$$2\xi\omega_0 = a_1$$

sönüm katsayısı $\xi = \frac{a_1}{2\sqrt{a_0}}$

$$s_{1,2} = \left[-\xi \mp \sqrt{(\xi^2 - 1)}\right]\omega_0$$

- Aşırı sönümlü Durum ($\xi > 1$)

$s_1 = \sigma_1$

$s_2 = \sigma_2$

$$x_n(t) = K_1 e^{\sigma_1 t} + K_2 e^{\sigma_2 t}$$

- Az sönümlü Durum ($\xi < 1$)

$s_1 = \sigma + j\omega$ $\quad \omega \neq 0$

$s_2 = \sigma - j\omega$

$$x_n(t) = \beta_1 e^{\sigma t}\cos\omega t + \beta_2 e^{\sigma t}\sin\omega t$$

- Kritik sönümlü Durum ($\xi = 1$)

$s_1 = s_2 = \sigma$

$$x_n(t) = K_1 e^{\sigma t} + K_2 t e^{\sigma t}$$

$y^{(4)} + y^{(2)} = 3x^2 + 4\sin x \cancel{2} \cos x$

$\alpha^4 + \alpha^2 = 0$

$\alpha^2(\alpha^2+1) = 0$

$\alpha_1 = 0 \quad \alpha_2 = 0 \quad \alpha_3 = i \quad \alpha_4 = -i$

$y_h = C_1 + C_2 x + C_3 \cos x + C_4 \sin x$

$(-2\cos x)$

$y'_{P_1} = -A\sin(x) + B\cos(x)$

$y''_{P_1} = -A\cos(x) - B\sin(x)$ *

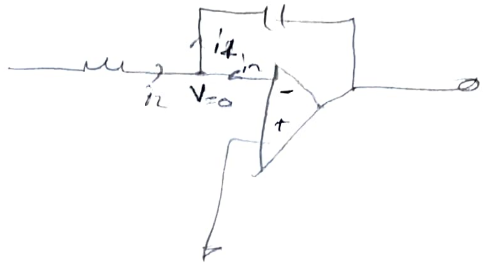
$y'''_{P_1} = A\sin(x) - B\cos(x)$

$y^{IV}_{P_1} = A\cos(x) + B\sin(x)$ *

$-A\cos(x) - B\sin(x) + A\cos(x) + B\sin(x) = -2\cos(x)$

$i = c\frac{dV}{dt}$

$V_i = 1V$



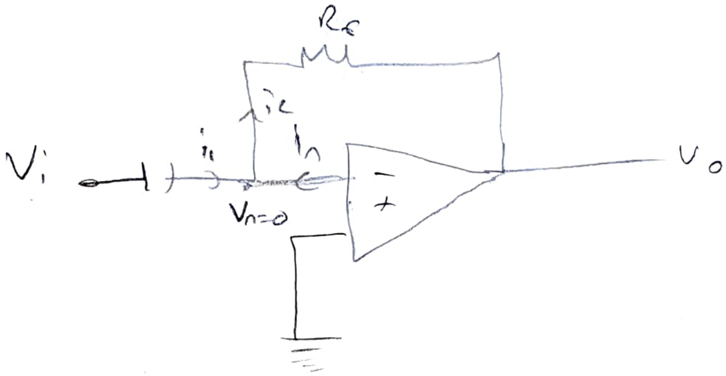
$i_2 - i_1 + i_n = 0$

$i_2 = i_1$

$$\frac{V_i - 0}{R_1} = 10^{-6}\frac{d(V_n - V_o)}{dt}$$

$$\frac{dt}{R_1}\cdot 10^{6} = d(V_n - V_o) \Rightarrow \frac{dt}{R_1}\cdot 10^{6} = -dV_o$$

$$\frac{t}{R_1}\cdot 10^{6} = -V_o$$



$V_n = i_n = 0$

$i_1 - i_2 + i_n = 0$

$i_1 = i_2$

$$c\frac{d(V_i - 0)}{dt} = \frac{V_n - V_o}{R_f}$$

$$d(V_i) = \frac{-V_o\cdot dt}{R_f C} \Rightarrow \boxed{V_i = \frac{-V_o\cdot t}{R_f C}}$$