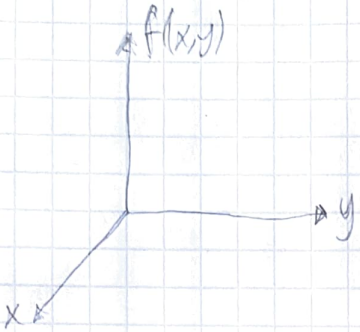
A.V. Oppenheim
Signals and Systems

İŞARET VE SİSTEMLER

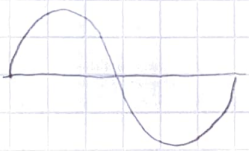


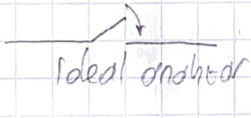
x, y → bağımsız değişken

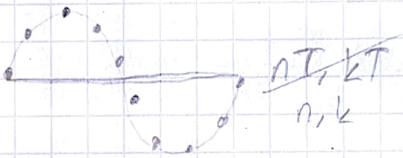
Sürekli - Zaman	Ayrık - Zaman
$x(t) = \sin(\omega t)$	$x[n] = \sin \omega n$
$-\infty < t < \infty$	$n = 0, \pm 1, \dots \pm \infty$
	$n = 1, 2, \dots -2/7$
$x(t)$	$x[n]$

i) Kendi doğasında ayrık
ii) Sürekli zaman işaretlerden elde edilir. (örnekleme)

NOT: Ayrık zamanda her zaman tam sayı olmak zorundadır.







Enerji ve Güç

$$E = \int_{t_1}^{t_2} |x(t)|^2 dt \qquad E = \sum_{n_1}^{n_2} |x[n]|^2$$

$$P = \frac{1}{t_2 - t_1} E \qquad P = \frac{1}{n_2 - n_1 + 1} \sum_{n_1}^{n_2} |x[n]|^2$$

$$E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt \qquad E_\infty = \lim_{N \to \infty} \sum_{-N}^{N} |x[n]|^2$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \qquad P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2$$

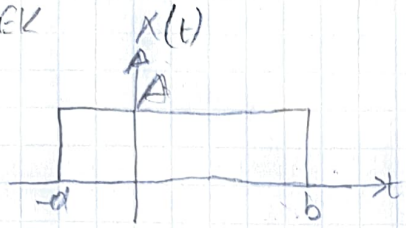
Enerji ve Güç işaretleri

I) $E_\infty < \infty \rightarrow P_\infty = 0$ enerji işareti

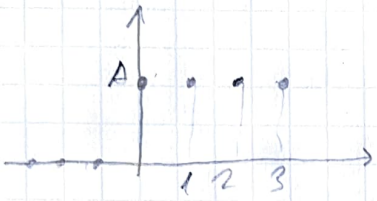
II) $E_\infty \rightarrow \infty$, $P_\infty < \infty$ güç işareti

III) $E_\infty \rightarrow \infty$, $P_\infty \rightarrow \infty$ ne enerji ne güç

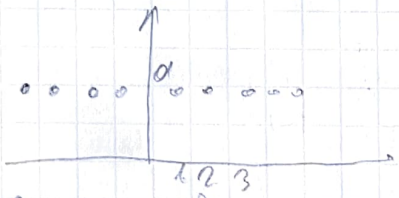
ÖRNEK



$E_\infty < \infty$ enerji dir.



Enerji dir. Sonlu



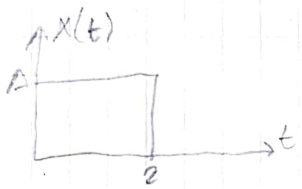
Enerji sonsuz, ama σ^2, ortalama

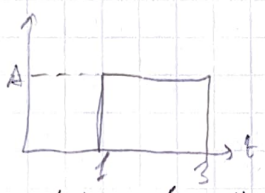
Bağımsız Değişkenler Üzerindeki İşlemler (Dönüşümler)

1) Ötelene 2) Ölçeklendirme 3) Terslene

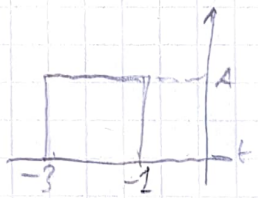
$x(t) \xrightarrow[\text{ötelenmiş hali}]{\text{to kadar}} x(t-t_0)$

$x[t] \xrightarrow[\text{ötelenmiş hali}]{\text{to kadar}} x[t-t_0]$

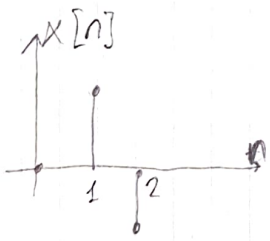


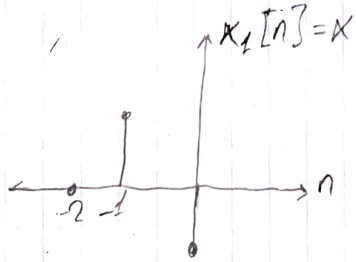


$x_1(t) = x(t-1)$



$x_2(t) = x(t+3)$



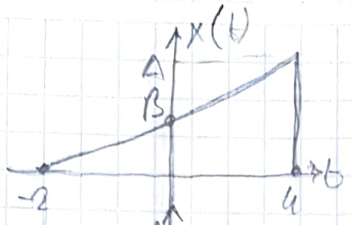


Ölçeklendirme

$x(t) \longrightarrow x_1(t) = x(\alpha t)$ $\qquad x[n] \longrightarrow x_1[n] = x[\alpha n]$

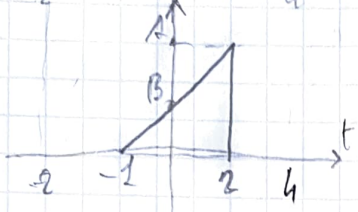
i) $0 < \alpha < 1$ işaret zaman domeninde genişliyor

ii) $\alpha > 1$ Zaman domeninde küçülüyor



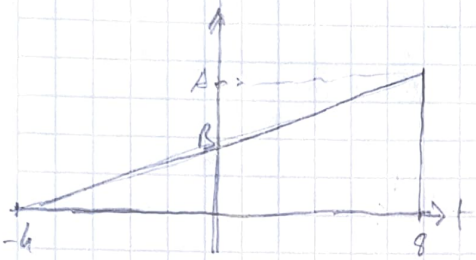
$x(t) = x_1(2t)$

$x_1(-2) = x(-4) = 0$



$x(t) = x_1(2t)$

İşaret daraldı $\alpha > 0$



$x(t) = x_2(t/2)$

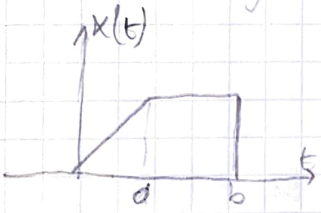
İşaret genişledi $0 < a < 1$

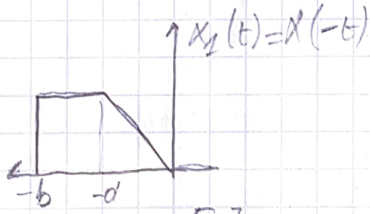
Terslenme

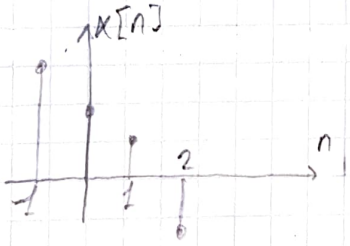
$x(t) \longrightarrow x_1(t) = x(-t)$

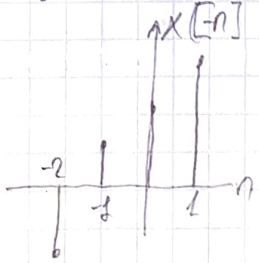
$x[n] \longrightarrow x_1[n] = x[-n]$

İşaretin dikey eksen etrafında 180° döndürülmesi









- Eğer $x(t)$ ve ya $x[n]$ işaretine bağımsız değişken üzerinde 3 işlemde gerçekleştirilecekse bu durumda işaret.
  1- Ötelenir
  2- Ölçeklendirilir
  3- Terslenir.

Sayfa 18 örnek

- $x(-t/2 - 1) = x_1(t)$

1) öteleme $x_{1a}(t) = x(t-1)$

2) ölçekleme $x_{1b}(t) = x_{1a}(t/2)$ → Bu kısma gidecek

3) Tersleme $x_{1c}(t) = x_{1b}(-t)$

- $x_1(t) = x(-t/2 - 1)$

- 

$x_1[n] =$

Periyodik İşaretler

$x(t) = x(t+T)$

$x(t)$ işaret "T" değeri ile periyodiktir.

$x(t) = x(t+T) = x(t+2T) = \ldots = x(t+kT)$ $k = 0, 1, 2, \ldots$

$x[n] = x[n+N] = \ldots = x[n+kN]$ $k = 0, 1, 2 \ldots$

NOT: $x_1(t) = x_1(t+T_1)$

$x_2(t) = x_2(t+T_2)$ verilmiş olsun.

$x_3(t) = x_1(t) + x_2(t)$ işaretinin periyodikliği için ne söylenir.

$\frac{T_1}{T_2} = \frac{r}{q}$, r ve q tamsayı ise $x_3(t)$, $T_3 = qT_1 = rT_2$ ile periyodiktir.

Çift ve Tek İşaret

$x(t) = x(-t)$ çift cos

$x(t) = -x(-t)$ tek sin

Bir işaret ne çift ne tek olabilir mi?

$x(t) = u(t)$

$Ey = \frac{x(t) + x(-t)}{2}$ çift bileşen

$Od = \frac{x(t) - x(-t)}{2}$ tek bileşen (ters simetrik)

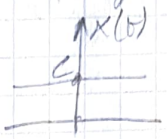
Sürekli zaman Üstel ve Sinüzoidal İşaretler

$x(t) = Ce^{at}$

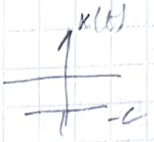
C ve a karmaşık sayı / $x+jy$, $|z|\angle\phi$

1. özel durum, C ve a reel sayı! (İmajiner bileşenler sıfır)

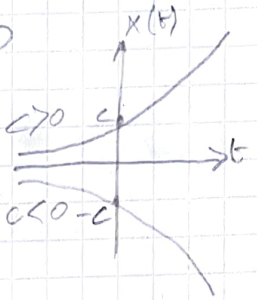
$x(t) = Ce^{at}$ i) $a=0$



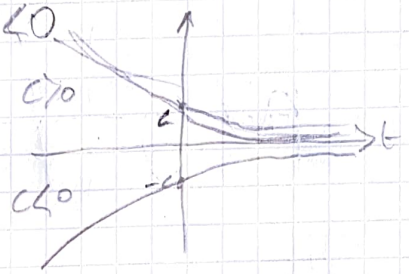
veya



ii) $a>0$



iii) $a<0$



2. özel durum ($C=1$, $a=j\omega_0$, reel bileşen sıfır)

$x(t) = Ce^{at} = e^{j\omega_0 t}$

$x(t) = x(t+T)$

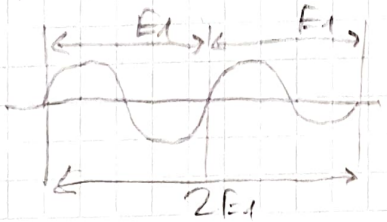
$e^{j\omega_0 t} = e^{j\omega_0(t+T)} = e^{j\omega_0 t} e^{j\omega_0 T}$

$e^{j\omega_0 T} = 1 \quad T = \frac{2\pi}{\omega_0}$

$e^{jx} = \cos x + j \sin x$

- $e^{j\omega_0 t}$ işareti tüm "ω_0" lar için periyodiktir
- $x(t) = A\sin(\omega_0 t + \phi)$
- Periyodik, karmaşık üstel ve sinüzoidal işaretlerin güç işareti olduğu gösterilebilir.

$x(t) = \sin\omega t$, $\cos\omega t$, $A\cos(\omega t + \phi)$ $\qquad P = \frac{E}{\text{periyot}}$



güçtür $P_\infty < \infty$

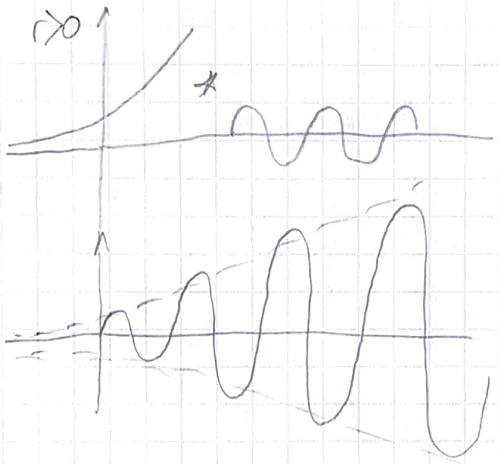
- $e^{j\omega_0 t}$ işareti $T = \frac{2\pi}{\omega_0}$ ile periyodiktir ve $e^{j\omega_0 t}$ nin harmonik ilişkili işaret kümesi $\phi_k(t) = e^{jk\omega_0 t}$ $k = 0, \pm 1, \pm 2, \ldots, \infty$ tane işaret bulunur.
Genel olarak "k. harmonik" nın açısal frekansı $k\omega_0$, periyodu ise $T/k \Rightarrow \frac{2\pi}{k\omega_0}$ olarak verilir.

3. Durum $x(t) = Ce^{at}$ (C ve a kompleks sayı)

$C = |C|e^{j\theta}$, $a = r + j\omega_0$

$x(t) = |C|e^{j\theta} \cdot e^{(r+j\omega_0)t} = |C|e^{rt} \cdot e^{j(\omega_0 t + \theta)}$

genliği belirleyecek (işaretin zarfı)



Sönümsüz işaret

Ayrık zaman üssel ve sinüzoidal işaretler

Sayfa 32 (2. Hafta)

- $C = |C|e^{j\theta}$, $\alpha = |\alpha|e^{j\omega_0}$

$x[n] = |C||\alpha|^n e^{j(\omega_0 n + \theta)}$

- $x[n] = e^{j\omega_0 n}$ işareti 2π ile periyodiktir.

Sürekli zamanda $x(t) = e^{j\omega_0 t} \rightarrow 2\pi$

$x(t) = x(t + T)$

$x[n] = e^{j(\omega_0 + 2\pi)n} = e^{j\omega_0 n}$

$x(t) = e^{j\omega_0 t} \neq e^{j(\omega_0 + 2\pi)t}$

Sayfa 60

$x[n] = \cos(\omega_0 n) = Re\{e^{j\omega_0 n}\}$

0 ve 2π salınım değerleri
π ve π/n salınım değerleri

$x[n]$ $\quad$ $x[n+N]$

$e^{j w_0 n} = e^{j w_0 (n+N)} = e^{j w_0 n} \cdot \underbrace{e^{j w_0 N}}_{1}$

$e^{j w_0 N} = 1 \Rightarrow w_0 N = 2\pi \cdot m \quad , m = 1, 2, 3, \ldots$

$$\boxed{N = \frac{2\pi}{w_0} \cdot m}$$

↓ Tamsayı

$\cos(2n)$ periyodik değil $\longrightarrow N = \frac{2\pi}{w_0 = 2} \cdot m = \pi m$

$\cos(\frac{3\pi}{5} n)$ periyodik $\longrightarrow N = \frac{2\pi}{\frac{3\pi}{5}} \cdot m = \frac{10}{3} \cdot m, \; m = 3$

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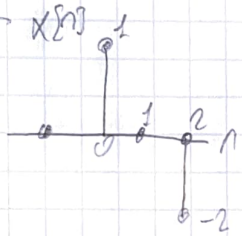
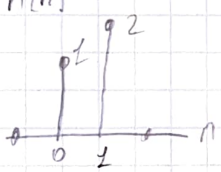
3. Hafta





$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] = ?$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$= \ldots + 0 + x[0]h[n] + 0 + x[2]h[n-2] + 0$