

T.C.
SAKARYA ÜNİVERSİTESİ

MATEMATİK I

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"ÖDEV SORULARI VE CEVAPLARI"

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MATEMATİK-I ÖDEV SORULARI VE CEVAPLARI

SORU-1 $\lim_{x \rightarrow 3} \arctan\left(\frac{1}{x-3}\right) = ?$

CEVAP-1 $\lim_{x \rightarrow 3^+} \arctan\left(\frac{1}{x-3}\right) = -\frac{\pi}{2}$ Limit Yok
 $\lim_{x \rightarrow 3^-} \arctan\left(\frac{1}{x-3}\right) = +\frac{\pi}{2}$

SORU-2 $\lim_{x \rightarrow 0} \frac{x}{\sqrt[3]{x+1} - 1} = ?$

CEVAP-2 $\lim_{x \rightarrow 0} \frac{x}{\sqrt[3]{x+1} - 1} \cdot \frac{(\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + \sqrt[3]{1})}{(\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + \sqrt[3]{1})}$
 $\lim_{x \rightarrow 0} \frac{x(\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + \sqrt[3]{1})}{x} = 3$

SORU-3 $\lim_{x \rightarrow -\infty} \frac{3^x - 2^{x+1} + 5^{x-1}}{3^{x+1} + 2^{2x-1} + 5^x} = ?$ $\frac{0}{0}$ Belirsizlik Hali

CEVAP-3 $\lim_{x \rightarrow -\infty} \frac{2^x \cdot \left[\left(\frac{3}{2}\right)^x - 2 + \frac{1}{5}\left(\frac{5}{2}\right)^x\right]}{2^x \cdot \left[3\left(\frac{1}{2}\right)^x + \frac{1}{2} \cdot 2^x + \left(\frac{5}{2}\right)^x\right]} = \infty$

SORU-4 $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x^2} - 2\sqrt{x} + 1}{(x-1)^2} = ?$ $\frac{0}{0}$ Belirsizlik Hali

CEVAP-4 $\lim_{x \rightarrow 1} \frac{(\sqrt[3]{x} - 1)^2}{(x-1)^2} \cdot \frac{[(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1]^2}{[(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1]^2}$
 $\lim_{x \rightarrow 1} \frac{(x-1)^2}{(x-1)^2 \cdot [(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1]^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

SORU-5 $\lim_{x \rightarrow \infty} \frac{x+1 - \sqrt[3]{x^3+1}}{x} = ?$ $\frac{\infty}{\infty}$ Belirsizlik Hali

CEVAP-5 $\lim_{x \rightarrow \infty} \frac{(x+1) - \sqrt[3]{x^3+1}}{x}$
 $\lim_{x \rightarrow \infty} \frac{x + x \cdot \frac{1}{x} - \sqrt[3]{\left(1 + \frac{1}{x^3}\right) \cdot x^3}}{x}$
 $\lim_{x \rightarrow \infty} \frac{x\left(1 + \frac{1}{x} - \sqrt[3]{1 + \frac{1}{x^3}}\right)}{x} = 0$

SORU-6 $\lim_{x \rightarrow \infty} \sqrt{\frac{2x + \sqrt{3x}}{5x}} = ?$ $\frac{\infty}{\infty}$ Belirsizlik Hali

CEVAP-6 $\lim_{x \rightarrow \infty} \frac{\sqrt{x \cdot \left(2 + \sqrt{\frac{3}{x}}\right)}}{\sqrt{5x}} = \sqrt{\frac{2}{5}}$

SORU-7 $\lim_{x \rightarrow \infty} (\sqrt{x \cdot (x+a)} - x) = ?$ $\infty - \infty$ Belirsizlik Hali

CEVAP-7 $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+xa} - x}{\sqrt{x^2+xa} + x} \cdot (\sqrt{x^2+xa} + x)$
 $\lim_{x \rightarrow \infty} \frac{x^2+xa - x^2}{\sqrt{x^2\left(1 + \frac{a}{x}\right)} + x}$

$\lim_{x \rightarrow \infty} \frac{xa}{x\left(\sqrt{1 + \frac{a}{x}} + 1\right)} = \frac{a}{2}$

SORU-8 $\lim_{x \rightarrow \infty} \left(x + \sqrt[3]{1-x^3}\right) = ?$

CEVAP-8 $\lim_{x \rightarrow \infty} \left(x + \sqrt[3]{1-x^3}\right) \cdot \frac{[x^2 - x\sqrt[3]{1-x^3} + (\sqrt[3]{1-x^3})^3]}{[x^2 - x\sqrt[3]{1-x^3} + (\sqrt[3]{1-x^3})^3]}$

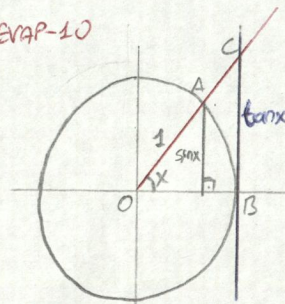
$\lim_{x \rightarrow \infty} \frac{x^3 + 1 - x^3}{x^2 - x\sqrt[3]{1-x^3} + (\sqrt[3]{1-x^3})^3} = \frac{1}{\infty} = 0$

SORU-9 $\lim_{x \rightarrow \infty} [x \cdot \sqrt{x^2+1} - x^2] = ?$ $\infty - \infty$ Belirsizlik Hali

CEVAP-9 $\lim_{x \rightarrow \infty} \frac{(x\sqrt{x^2+1} - x^2)}{x\sqrt{x^2+1} + x^2} \cdot (x\sqrt{x^2+1} + x^2)$
 $\lim_{x \rightarrow \infty} \frac{x^4 + x^2 - x^4}{x^2 \cdot \left(\sqrt{1 + \frac{1}{x^2}} + 1\right)} = \frac{1}{2}$

SORU-10 $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ olduğunu ispatlayınız?

CEVAP-10



$A(\widehat{AOB}) = \frac{1}{2} \cdot 1 \cdot \sin x$

$A(\widehat{AOB}) = \frac{x}{2\pi} \cdot \pi \cdot 1^2 = \frac{x}{2}$

$A(\widehat{COB}) = \frac{1}{2} \cdot 1 \cdot \tan x$

$A(\widehat{AOB}) < A(\widehat{AOB}) < A(\widehat{COB})$

$\frac{1}{2} |\sin x| < \frac{|x|}{2} < \frac{1}{2} |\tan x|$

$|\sin x| < |x| < |\tan x|$

$\frac{|\sin x|}{|\sin x|} < \frac{|x|}{|\sin x|} < \frac{|\tan x|}{|\sin x|}$

$1 > \frac{|\sin x|}{|x|} > |\cos x|$

I. Bölgede $\sin x(+)$, $x(+)$ dönme
 IV. Bölgede $\sin x(+)$, $x(+)$ dönme

$\lim_{x \rightarrow 0} 1 = 1$

$\lim_{x \rightarrow 0} \cos x = 1$

$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

SORU-11 $\lim_{x \rightarrow 1} \frac{x^x - 1}{x \ln x} = ?$

CEVAP-11 $\lim_{x \rightarrow 1} \frac{x^x - 1}{x \ln x} = \lim_{x \rightarrow 1} \frac{x^x - 1}{\ln(x^x - 1)} = 1$

SORU-12 $\lim_{x \rightarrow \pi/2} (1 + \cos x)^{\sec x} = ?$ 1^∞ Belirsizlik Hali

CEVAP-12 $\lim_{x \rightarrow \pi/2} (1 + \cos x)^{\sec x}$

$\lim_{x \rightarrow \pi/2} \left(\cos x \cdot \frac{3}{\cos x} \right) = e^3$

SORU-13 $\lim_{x \rightarrow 0} (1 + 3 \tan^2 x)^{\cot^2 x} = ?$ 1^∞ Belirsizlik Hali

CEVAP-13 $\lim_{x \rightarrow 0} (3 \tan^2 x)^{\cot^2 x}$

$\lim_{x \rightarrow 0} (3 \tan^2 x \cdot \cot^2 x) = e^3$

SORU-14 $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$ olduğunu ispatlayınız?

CEVAP-14 $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$ olduğunu ispatlayınız?

SORU-15 $\lim_{x \rightarrow 0} \frac{(1+x)^a - 1}{x} = a$ olduğunu gösteriniz?

CEVAP-15 $\lim_{x \rightarrow 0} \frac{(1+x)^a - 1}{x} = a$ olduğunu gösteriniz?

SORU-16 $\lim_{x \rightarrow 0} \frac{1}{x} \cdot \left[\frac{1}{x+3} - \frac{1}{3} \right] = ?$ $0 \cdot \infty$ Belirsizlik Hali

CEVAP-16 $\lim_{x \rightarrow 0} \frac{1}{x} \cdot \left[\frac{-x}{3x+3} \right] = -1/3$

SORU-17 $\lim_{x \rightarrow 0} \frac{1}{x} \ln \left(\frac{2+x}{2-x} \right) = ?$

CEVAP-17 $\lim_{x \rightarrow 0} \ln \left(\frac{2+x}{2-x} \right)^{1/x} = \ln e = 1$

SORU-18 $\lim_{x \rightarrow 0} \frac{\ln(3 - 2 \cos 3x)}{\ln^2(\sin 5x + 1)} = ?$

CEVAP-18 $\lim_{x \rightarrow 0} \frac{\ln(1 + 2 - 2 \cos 3x)}{\ln^2(\sin 5x + 1)} = \frac{(\sin 5x)^2 \cdot (2 - 2 \cos 3x)}{(2 - 2 \cos 3x) \cdot (\sin 5x)^2}$

$= \lim_{x \rightarrow 0} \frac{2 \cdot (1 - \cos 3x)}{\sin^2 5x} \cdot \frac{5x^2}{25x^2} = \frac{2 \cdot 1/2 \cdot 9}{25} = \frac{9}{25}$

SORU-19 $\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\cos \frac{\pi}{2} \cdot (\cos \frac{\pi}{4} - \sin \frac{\pi}{4})} = ?$ $0/0$ Belirsizlik Hali

CEVAP-19 $\lim_{x \rightarrow \pi} \frac{1 - \sin(\frac{\pi}{2} + \frac{u}{2})}{\cos \frac{\pi}{2} \cdot [\cos(\frac{\pi}{4} + \frac{u}{4}) - \sin(\frac{\pi}{4} + \frac{u}{4})]}$

$A = [\cos \frac{u}{4} \cdot \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \cdot \sin \frac{u}{4}] - [\sin \frac{u}{4} \cdot \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cdot \cos \frac{u}{4}]$

$A = \frac{-2}{\sqrt{2}} \sin \frac{u}{4} \Rightarrow \lim_{u \rightarrow 0} \frac{1 - \cos \frac{u}{2}}{\cos \frac{\pi}{2} \cdot \frac{\sqrt{2}}{2} \cdot \sin \frac{u}{4}}$

SORU-20 $\lim_{x \rightarrow \pi/2} \frac{\sin x + 2 \cot x}{(\pi - x) \cdot \sin \frac{x}{2}} = ?$

CEVAP-20 $\lim_{x \rightarrow \pi/2} \frac{\sin \pi/2 + 2 \cot \pi/2}{\pi/2 \cdot \sin \pi/4} = \frac{1}{\pi/2 \cdot \sqrt{2}/2} = \frac{2\sqrt{2}}{\pi}$

SORU-21 $\lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - \sqrt[3]{\cos x}}{\sin^2 x} = ?$

CEVAP-21 $\lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - \sqrt[3]{\cos x}}{\sin^2 x} \cdot \frac{x^2}{x^2}$

$\lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - 1}{x^2} \cdot \frac{\sqrt{\cos x} + 1}{\sqrt{\cos x} + 1} = \lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - 1}{x^2} \cdot \frac{\sqrt{\cos x} + 1}{\sqrt{\cos x} + 1}$

$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2 \cdot (\sqrt{\cos x} + 1)} = \lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2 \cdot (\sqrt{\cos x} + 1)}$

$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2} = -1/2$

SORU-22 $y = \ln^x (\cos^5 3^{7x^2})$ $y' = ?$

CEVAP-22 $y' = \ln(\cos^5 3^{7x^2})^x \cdot \ln[\ln(\cos^5 3^{7x^2})^x] \cdot 1$

$5 \cdot [\ln(\cos^5 3^{7x^2})]^4 \cdot \frac{\sin 3^{7x^2}}{\cos^5 3^{7x^2}} \cdot 3^{7x^2} \cdot \ln 3 \cdot 14x$

SORU-23 $y = \sec^x [\cos^x (\ln^x (x)^{x^3})]$ $y' = ?$

CEVAP-23 $y' = [\sec[\cos(\ln(x)^{x^3})^x]^x \cdot \ln(\sec[\cos(\ln(x)^{x^3})^x]^x) \cdot 1$

SORU-24 $f^{-1}(y_0) = \frac{1}{f'(x_0)}$ olduğunu ispatlayınız?

CEVAP-24 $f(x)=y$
 $f^{-1}(y)=x \rightarrow$ Her iterasyonun türevini al.

$$\frac{df^{-1}(y)}{dy} \cdot \frac{dy}{dx} = 1 \quad \frac{df^{-1}(y)}{dy} = \frac{1}{\frac{dx}{dy}} = \frac{1}{f'(x)} = \frac{1}{f'[f^{-1}(y)]}$$

SORU-25 $\tan(\arctan 3/4 + \arccot 5/8)$ değeri nedir?

CEVAP-25 $\arctan 3/4 = x$ $\arccot 5/8 = y$
 $\tan x = 3/4$ $\cot y = 5/8$
 $\tan y = 8/5$

$$\tan(x+y) = \frac{3/4 + 8/5}{1 - 3/4 \cdot 8/5} = \frac{-47}{4}$$

SORU-26 $g(x) = \arccot\left(\frac{x-1}{2}\right)$ $\left\{ \begin{array}{l} \text{fog}^{-1}(\pi/4) = ? \\ f(x) = \arcsin \frac{x}{2} \end{array} \right.$

CEVAP-26 $g^{-1}(x) = 2\cot x + 1$
 $\text{fog}^{-1}(x) = \arcsin\left(\frac{2\cot x + 1}{2}\right)$
 $\text{fog}^{-1}(\pi/4) = \arcsin(3/2)$

SORU-27 $y = \frac{x^3 + 5x^2 - 4x + 1}{x^2 - 4} \Rightarrow y^{(n)} = ?$

CEVAP-27 $y = (x+5) + \frac{21}{x^2-4} \Rightarrow (x+5) + 21 \cdot \left(\frac{1}{x-2} - \frac{1}{x+2}\right)$

$$y^{(n)} = 1 + (-1)^n \cdot (x-2)^{-n-1} + (-1)^{n-1} \cdot (x+2)^{-n-1} \cdot -1(x+1)^{-2} \dots$$

SORU-28 $y = \ln(x+1) \cdot \sin 5x \Rightarrow y^{(n)} = ?$

CEVAP-28 $(-1)^{n+1} \cdot (n-1)! \cdot (x+1)^{-n} \rightarrow f$
 $5^n \cdot \sin\left[n \cdot \frac{\pi}{2} + 5x\right] \rightarrow g$

$$y^{(n)} = \binom{n}{0} \cdot (-1)^{n+1} \cdot (n-1)! \cdot (x+1)^{-n} \cdot \sin 5x + (-1)^n \cdot (n-1)! \cdot (x+1)^{-n+1} \cdot n \cdot 5 \cdot \sin\left[n \cdot \frac{\pi}{2} + 5x\right] \dots$$

SORU-29 $y = \sin^2 bx \Rightarrow y^{(n)} = ?$

CEVAP-29 $y = (\sin bx)^2$
 $\sin^2 bx = 1/2 \cdot (1 - \cos 2bx)$

$$y^{(n)} = \frac{1}{2} \cdot (2b)^n \cdot \sin\left[n \cdot \frac{\pi}{2} + 2bx\right]$$

SORU-30 $y = (x^2 + 5x) \cdot 3^{-7x} \Rightarrow y^{(n)} = ?$

CEVAP-30 $g = x^2 + 5x$ $f = 3^{-7x}$
 $g^{(0)}(x) = 2x + 5$ $f^{(n)}(x) = (-7)^n \cdot (\ln 3)^n \cdot 3^{-7x}$
 $g^{(1)}(x) = 2$

$$y^{(n)} = 1 \cdot (-7)^n \cdot (\ln 3)^n \cdot (3^{-7x}) \cdot (2x+5) + n \cdot (-7)^{n-1} \cdot (\ln 3)^{n-1} \cdot 3^{-7x} \cdot 2$$

SORU-31 $y = \frac{x^4+1}{x^3-x} \Rightarrow y^{(n)} = ?$

CEVAP-31 $y = \frac{x^4+1}{x \cdot (x^2-1)} = \frac{x^4+1}{x} \cdot \frac{1}{x^2-1}$

$$g^{(0)} = (-1) \cdot (x^2-1)^{-2} \quad f^{(0)} = \frac{(x^4+1)}{x} \cdot (x)^{-1}$$

$$g^{(1)} = (-1) \cdot (-2) \cdot (x^2-1)^{-3} \quad g^{(2)} = (-1) \cdot (-2) \cdot (-3) \cdot (x^2-1)^{-4}$$

$$\vdots$$

$$g^{(n)} = (-1)^n \cdot n! \cdot (x^2-1)^{-(n+2)}$$

$$u' = 4x^3 \quad u'' = 12x^2 \quad u''' = 24x \quad u^{(n)} = 0$$

$$v' = -1(x)^{-2} \quad v'' = (-1) \cdot (-2) \cdot (x)^{-3} \quad v^{(n)} = (-1)^n \cdot n! \cdot x^{-(n+2)}$$

$$f^{(n)} = 1 \cdot (-1)^n \cdot n! \cdot x^{-(n+1)} \cdot (x^4+1) + \binom{n}{2} \cdot (-1)^{n-2} \cdot (n-2)! \cdot x^{-(n-2)} \cdot 12x^2 + \binom{n}{3} \cdot (-1)^{n-3} \cdot (n-3)! \cdot x^{-(n-3)} \cdot 24x + \binom{n}{4} \cdot (-1)^{n-4} \cdot (n-4)! \cdot x^{-(n-4)} \cdot 24 + 0$$

$$y^{(n)} = \binom{n}{0} \cdot f^{(n)} \cdot \frac{1}{x^2-1} + \binom{n}{1} \cdot f^{(n-1)} \cdot (-1) \cdot (x^2-1)^{-2} + \binom{n}{2} \cdot f^{(n-2)} \cdot (-1)^2 \cdot (x^2-1)^{-3} + \dots + \binom{n}{n} \cdot f^{(0)} \cdot g^{(n)}$$

SORU-32 $\lim_{x \rightarrow \pi/2} (x - \pi/2)^{\cos x}$ limitini türev kullanarak hesaplayınız?

CEVAP-32 0^0 Belirsizlik Hali

$\ln y = \lim_{x \rightarrow \pi/2} \cos x \cdot \ln(x - \pi/2)$ $0 \cdot \infty$ Belirsizlik Hali

$\ln y = \lim_{x \rightarrow \pi/2} \frac{\ln(x - \pi/2)}{\sec x}$ $\frac{\infty}{\infty}$ Belirsizlik Hali

$\ln y = \lim_{x \rightarrow \pi/2} \frac{\frac{1}{x - \pi/2}}{\sec x \cdot \tan x}$

$\ln y = \lim_{x \rightarrow \pi/2} \frac{\cos^2 x}{\sin x (x - \pi/2)}$

$y = e^0$
 $y = 1$