26

MART
MARCH

PAZARTESİ
MONDAY

MART							2012
H	P	S	Ç	P	C	Ct	Pz
9				1	2	3	4
10	5	6	7	8	9	10	11
11	12	13	14	15	16	17	18
12	19	20	21	22	23	24	25
13	26	27	28	29	30	31	

MAT II

FAZ #3

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Belirsiz İntegral

Tanım: $f(x)$ tanımlı olduğunda $F'(x)=f(x)$ bağıntısını sağlayan bir F fonksiyonu varsa bu F fonksiyonu f'in ilkfonksiyonudur

Tanım: $f(x)$ herhangi fonksiyon türevi olsun. f'nin tüm ilkfonksiyonlarının sınıfına f fonksiyonunun x değişkenine göre belirsiz integrali denir

$$\boxed{F(x)=\int f(x)\,dx}$$

Temel İntegral Formülleri

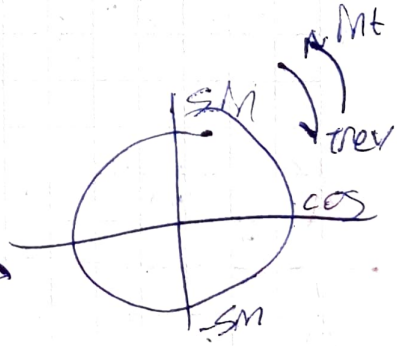
① $\int x^m dx = \frac{x^{m+1}}{m+1}+c \qquad (m\neq -1)$

② $\int x^{-1}dx = \int \frac{1}{x}dx = \ln|x|+c$

③ $\int a^x dx = \frac{a^x}{\ln a}+c$

④ $\int e^x dx = e^x+c$

⑤ $\int \sin x\,dx = -\cos x+c$



⑦ $\int \operatorname{cosec}^2 x\,dx = \int \frac{1}{\sin^2 x}dx = -\cot x+c = \int (1+\cot^2 x)dx$

⑧ $\int \sec^2 x\,dx = \int \frac{1}{\cos^2 x}dx = \tan x+c = \int (1+\tan^2 x)dx$

⑨ $\int \frac{1}{1+x^2}dx = \arctan x+c$

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(10) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + c$

(11) $\int \sinh x\,dx = \cosh x + c$

(12) $\int \cosh x\,dx = \sinh x + c$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\sinh x = \frac{e^x - e^{-x}}{2}$

(13) $\int \frac{dx}{\sqrt{1+x^2}} = \ln\left[x + \sqrt{1+x^2}\right] + c$

(14) $\int \frac{dx}{\sqrt{x^2-1}} = \ln\left[x + \sqrt{x^2-1}\right] + c$

İntegralin Özellikleri

(1) $\int \alpha f(x)\,dx = \alpha \int f(x)\,dx$

(2) $\int [f(x) \mp g(x)]\,dx = \int f(x)\,dx \mp \int g(x)\,dx$

$\star \int (6x^2 - 4x + 3)\,dx = 2x^3 - 2x^2 + 3x + c$

$\star \int \left(\sqrt[3]{x} + \frac{1}{2x^4}\right) dx = \int \left(x^{1/3} + \frac{1}{2}x^{-4}\right) dx$

$= \frac{x^{4/3}}{4/3} + \frac{1}{2}\frac{x^{-3}}{-3} + c$

$= \frac{3}{4}\sqrt[3]{x^4} - \frac{1}{6x^3} + c$

$$\int \left(\frac{5}{x} + \frac{4}{\cos^2 x} + 3e^x\right) dx = 5\ln|x| + 4\tan x + 3e^x + C$$

$$\int \frac{5x^2 - 6x + 3}{\sqrt{x}}\, dx = \int \left(\frac{5x^2}{\sqrt{x}}\right) dx - \int \frac{6x}{\sqrt{x}}\, dx + \int \frac{3}{\sqrt{x}}\, dx$$

$$= \int 5x^{3/2}\, dx - \int 6x^{1/2}\, dx + \int 3x^{-1/2}\, dx$$

$$= \frac{5x^{5/2}}{5/2} - \frac{6x^{3/2}}{3/2} + \frac{3x^{1/2}}{1/2}$$

$$= 2x^{5/2} - 4x^{3/2} + 6x^{1/2}$$

$$= 2\sqrt{x^5} - 4\sqrt{x^3} + 6\sqrt{x} + C$$

$$\int \left(3\sinh x + 4\cosh x + \frac{5}{\sin^2 x}\right) dx$$

$$= 3\cosh x + 4\sinh x - 5\cot x + C$$

$$\int \left(x^3 + 3^x + \frac{3}{\sqrt{x^2 - 1}}\right) dx = ?$$

$$= \frac{x^4}{4} + \frac{3^x}{\ln 3} + 3\ln\left(x + \sqrt{x^2 - 1}\right) + C$$

Ödev Türev formülleri

$(\cot x)' = [-1 + \cot^2 x]$

$= -\operatorname{cosec}^2 x$

$= -\frac{1}{\sin^2 x}$

$(\tan x)' = 1 + \tan^2 x$

$= \sec^2 x$

$= \frac{1}{\cos^2 x}$

hiperbolik
↓
+ − kuralı yok

sin cos

İntegral Alma Kuralları

① Değişken değiştirme

Tanım: ϕ sürekli türevlere sahip bir fonksiyon olmak üzere $x=\phi(t)$ dönüşümü yapıldığında x'in diferansiyeli $\frac{dx}{dt}=\phi'(t)$, $dx=\phi'(t)dt$ olacağından.

$$\int f(x)\,dx = \int f(\phi(t))\,\phi'(t)\,dt$$

Bundan integral çözülür daha sonra $t=\phi^{-1}(x)$ yazılarak sonuç bulunur.

★ $\int (1-4x)^6\,dx = ?$

$1-4x=t$
$-4dx=dt$

$$\int t^6 \cdot \frac{dt}{-4} = -\frac{1}{28}t^7 + C$$

$$= \frac{-(1-4x)^7}{28} + C$$

$\int \sin(7x)\,dx = ?$ $\quad 7x = \alpha$, $7dx = d\alpha$

$\int \sin\alpha \frac{d\alpha}{7} = \frac{-1}{7}\cdot\cos\alpha + C$

$= \frac{-\cos(7x)}{7} + C$

$\int \frac{(4+\ln x)^5}{x}\,dx = ?$ $\quad 4+\ln x = t$, $\frac{1}{x}dx = dt$

$\int t^5\,dt = \frac{t^6}{6} + C$

$= \frac{(4+\ln x)^6}{6} + C$

$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = ?$ $\quad \sqrt{x} = t$, $x = t^2$, $dx = 2t\,dt$

$\int \frac{e^t}{t} 2t\,dt = 2e^t + C$

$= 2e^{\sqrt{x}} + C$

$\int \tan x\,dx = ?$ $\quad \cos x = t$, $-\sin x\,dx = dt$

$\int \frac{\sin x}{\cos x}\,dx = \int \frac{-dt}{t}$

$= -\ln|t| + C$

$= -\ln|\cos x| + C$

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$\int e^{5x^2} dx$

sin / cos

$\int \cot x \, dx = ?$

$\int \frac{\cos x}{\sin x} dx = \int \frac{dt}{t}$

$\sin x = t$

$\cos x \, dx = dt$

$= \ln|t| + c$

$= \ln|\sin x| + c$

4 10 | 2
2 5 | 2
2 5 | 5

$\int \frac{x^4}{1+x^{10}} dx = ?$

~~$1+x^{10} = t$~~

~~$10x^9 dx = dt$~~

$x^5 = t$

$5x^4 dx = dt$

$\int \frac{dt}{5(1+t^2)} = \frac{1}{5}\int \frac{dt}{1+t^2}$

$= \frac{1}{5} \arctan(t) + c$

$= \frac{1}{5} \arctan x^5 + c$

$\int e^{5x} dx = ?$

$5x = t$

$5dx = dt$

$\int e^t \cdot \frac{dt}{5} = \frac{1}{5}\int e^t dt$

$= \frac{1}{5} e^t + c$

$= \frac{1}{5} e^{5x} + c$

ödev

$e^{ax} = a e^{ax}$

$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$ İntegral

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Türev arc cos ve arc cot sen bul !!!

$$\int e^{\frac{x}{5}}\,dx = 5e^{\frac{x}{5}} + C$$

formül

$$\int \frac{dx}{x^2+a^2} = ?$$

$x = at$

$dx = a\,dt$

$$\int \frac{a\,dt}{a^2t^2+a^2} = \int \frac{a\,dt}{a^2[t^2+1]}$$

$$= \frac{1}{a}\int \frac{dt}{t^2+1} = \frac{1}{a}\arctan t + C.$$

$$= \frac{1}{a}\arctan\frac{x}{a} + C$$

$$\int \frac{dx}{x^2+5} = \frac{1}{\sqrt{5}}\arctan\frac{x}{\sqrt{5}} + C$$

$a = \sqrt{5}$

formül

$$\int \frac{dx}{\sqrt{a^2-x^2}}$$

$x = at$

$dx = a\,dt$

$$\int \frac{a\,dt}{\sqrt{a^2-a^2t^2}} = \int \frac{a\,dt}{\sqrt{a^2[1-t^2]}} = \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \arcsin t + C$$

$$= \arcsin\frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$$

(1) $\cos^2 x = 1 - \sin^2 x$

$a^2 - a^2 \sin^2 x$

★ $\int \frac{dx}{\sqrt{3-2x-x^2}} = ?$

$3 - 2x - x^2 \overset{?}{=} a^2 - t^2$

$= -[x^2 + 2x - 3]$

$= -[(x+1)^2 - 4]$

$= 4 - (x+1)^2$

$= 2^2 - t^2$

$x+1 = t$

$dx = dt$

$\int \frac{dt}{\sqrt{2^2 - t^2}} = \arcsin \frac{t}{2} + c$

$= \arcsin\left(\frac{x+1}{2}\right) + c$

★ $\int x^2 \sqrt{x-2}\, dx = ?$

$x - 2 = t^2$

$dx = 2t\, dt$

$\int (t^2+2)^2 \, t \cdot 2t\, dt$

$(t^4 + 4t^2 + 4) 2t^2$

$2t^6 + 8t^4 + 8t^2$

$2\int t^2 [t^4 + 4t^2 + 4]\, dt$

$2\int (t^6 + 4t^4 + 4t^2)\, dt$

$2\left[\frac{t^7}{7} + \frac{4t^5}{5} + \frac{4t^3}{3}\right] + c = 2\left[\frac{(x-2)^{7/2}}{7} + 4\frac{(x-2)^{5/2}}{5} + 4\frac{(x-2)^{3/2}}{3}\right] + c$

(1) İntegralin herhangi bir yerinde $\sqrt{a^2 - x^2}$ böyle bir ifade varsa $x = a\sin t$ dönüşümü yapılır

★ $\int \frac{x+8}{\sqrt{9-x^2}}\, dx$

$\begin{bmatrix} x = 3\sin t \\ dx = 3\cos t\, dt \end{bmatrix} \rightarrow$

$\int \frac{3\sin t + 8}{\sqrt{9 - 9\sin^2 t}} 3\cos t\, dt = \int (3\sin t + 8)\, dt$

$= -3\cos t + 8t + c$

$3\cos t$

06 NİSAN APRIL CUMA FRIDAY

$\cos^2 x = 1 - \sin^2 x$ ($a^2 - x^2$)

$1 + \tan^2 x = \sec^2 x = \frac{1}{\cos^2 x}$

$\tan^2 x = \sec^2 x - 1$

Türev formülleri !!!!

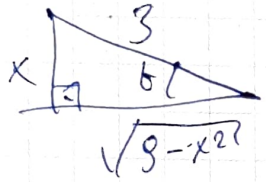
NİSAN 2012

H	P	S	Ç	P	C	Ct	Pz
13							1
14	2	3	4	5	6	7	8
15	9	10	11	12	13	14	15
16	16	17	18	19	20	21	22
17	23	24	25	26	27	28	29
18	30						

$\left[x = 3\sin t \rightarrow \frac{x}{3} = \sin t \rightarrow t = \arcsin \frac{x}{3} \right]$

$= -3\cos t + 8t + C$

$= -3\cos(\arcsin \frac{x}{3}) + 8 \arcsin \frac{x}{3} + C$



$= -\sqrt{9-x^2} + 8 \arcsin \frac{x}{3} + C$

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(2) İntegralin herhangi bir yerde $\sqrt{x^2 - a^2}$ varsa $\boxed{x = a\sec t}$ dönüşümü yapılır.

$\int \frac{dx}{x\sqrt{x^2-9}} = ?$

$x = 3\sec t$

$dx = 3\sec t \cdot \tan t \, dt$

$\int \frac{3\sec t \cdot \tan t \cdot dt}{3\sec t \sqrt{9\sec^2 t - 9}}$

$\int \frac{\tan t \, dt}{3\sqrt{\sec^2 t - 1}} = \int \frac{1}{3} dt = \frac{t}{3} + C$

$= \frac{1}{3} \operatorname{arcsec} \frac{x}{3} + C$

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$$\int \frac{dx}{\sqrt{1+x^2}} = \operatorname{arccosh} x$$

(3) İntegralin her hangi bir yerde $\sqrt{a^2+x^2}$ varsa $x = a\tan t$ dönüşümü yapılır.

$$\int \frac{dx}{x^2\sqrt{x^2+4}}$$

$$\begin{bmatrix} x = 2\tan t \\ dx = 2(1+\tan^2 t)\,dt \\ dx = 2\sec^2 x\,dt \end{bmatrix}$$

$$\int \frac{2\sec^2 t}{4\tan^2 t\sqrt{4\tan^2 t+4}}\,dt$$

$2\sec t$

$$\int \frac{1}{2}\,\frac{\sec^2 t}{\tan^2 t \cdot 2\sec t}\,dt$$

$$\frac{1}{4}\int \frac{\sec t}{\tan^2 t}\,dt$$

$$\frac{1}{4}\int \frac{\cos t}{\sin^2 t}\,dt \qquad \begin{bmatrix} \sin t = u \\ \cos t\,dt = du \end{bmatrix}$$

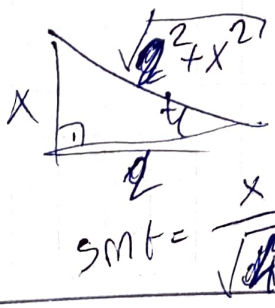
$$\frac{1}{4}\int \frac{du}{u^2} = \frac{1}{4}\int u^{-2}\,du$$

$$= \frac{1}{4}\,\frac{u^{-1}}{-1} + c$$

$$= -\frac{1}{4u} + c$$

$$= \frac{-1}{4\sin t} + c$$

$$= -\frac{\sqrt{4+x^2}}{4x} + c$$



$$\sin t = \frac{x}{\sqrt{4+x^2}}$$

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$\sqrt{\frac{4}{4}(4x^2-1)}$

$2x\sqrt{x^2-\frac{1}{4}}$

$$\int \frac{dx}{x\sqrt{4x^2-1}} = ?$$

$x = \frac{1}{2}\sec t$

$dx = \frac{1}{2}\sec t \tan t\, dt$

$$\int \frac{\frac{1}{2}\sec t \tan t\, dt}{2\cdot\frac{1}{2}\sec t \sqrt{\frac{1}{4}\sec^2 t - \frac{1}{4}}}$$

$\frac{1}{2}\tan t$

$t + c$

$\operatorname{arcsec} 2x + c$

(4) İntegralin herhangi bir yerinde $\sqrt[n]{ax+b}$ varsa ve $\text{ekok}(n_i) = p$ olsun; $ax+b = t^p$ değişken dönüşümü uygulanır. (Kök varsa aynı olabilir)

$$\int \frac{\sqrt[4]{x+1}^3 + 2}{\sqrt[6]{x+1}}\, dx = ?$$

4	6	2
2	2	3
1	1	1

$x+1 = t^{12}$

$dx = 12t^{11}dt$

$$\int \frac{t^3+2}{t^2} 12t^{11}dt$$

$$\int \left(t + \frac{2}{t^2}\right) 12t^{11}dt = 12\int \left(t^{12} + 2t^9\right) dt$$

$$= 12\left[\frac{t^{13}}{13} + \frac{2t^{10}}{10}\right] + c$$

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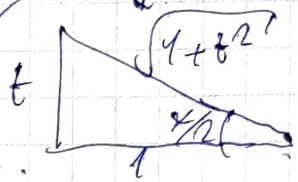
(5) İntegralin içinde trigonometrik ifadeler var ise yarım açı yöntemi denilen $\tan\frac{x}{2}=t$ değişken dönüşümü ile çözülür

$$\arctan t = \frac{x}{2}$$

$$2\arctan t = x$$

$$2\,\frac{1}{1+t^2}\,dt = dx$$



$$\sin\frac{x}{2} = \frac{t}{\sqrt{1+t^2}}$$

$$\cos\frac{x}{2} = \frac{1}{\sqrt{1+t^2}}$$

$$\sin 2x = 2\sin x\cos x$$

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$\boxed{\sin x = \frac{2t}{1+t^2}}$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos x = 2\cos^2\left(\frac{x}{2}\right) - 1$$

$$\cos x = \frac{2}{1+t^2} - 1$$

$$\boxed{\cos x = \frac{1-t^2}{1+t^2}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{2t}{1-t^2}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{1-t^2}{2t}$$

$1+t^2+t-t^2 \quad 2 \quad 4t$

$1+t^2+2t$

$2+2t^2+4t$

$t^2+2t+1 \quad (t+1)^2 \quad 2$

$\int \frac{1+\sin x}{(1+\cos x)\sin x}\,dx = ?$

$\tan\frac{x}{2} = t$

$\frac{2}{1+t^2}\,dt = dx$

$\sin x = \frac{2t}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$

$\int \frac{\left[1+\frac{2t}{1+t^2}\right]\frac{2}{1+t^2}}{\left[1+\frac{1-t^2}{1+t^2}\right]\frac{2t}{1+t^2}}\,dt$

$\int \frac{t^2+2t+1}{2t}\,dt$

$\frac{1}{2}\int t\,dt + \int 1\,dt + \frac{1}{2}\int \frac{1}{t}\,dt$

$\frac{t^2}{4} + t + \frac{1}{2}\ln|t| + c$

$\int \frac{\sin x - \cos x}{\cos x + \sin x}\,dx$

$\cos x + \sin x = t$

$(-\sin x + \cos x)\,dx = dt$

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$\int \cos 3x\,dx = ?$

$3x = t$

$3dx = dt$

$\int \cos ax\,dx = \frac{1}{a}\sin ax + c$

$\int \sin 5x\,dx = ?$

$5x = t$

$5dx = dt$

$\int \sin ax\,dx = \frac{-1}{a}\cos ax + c$

$\int \frac{dx}{5+3\cos x} = ?$

$\tan\frac{x}{2} = t$

$dx = \frac{2dt}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$

$= \int \frac{\frac{2dt}{1+t^2}}{5+3\frac{1-t^2}{1+t^2}}$

$= \int \frac{dt}{2^2+t^2} = \frac{1}{2}\arctan\frac{t}{2} + c$

$= \frac{1}{2}\arctan\left(\frac{\tan\frac{x}{2}}{2}\right) + c$

2 Nisan Integrasyon Örnek

Tanım: u ve v, x'e bağlı

$$\int \underbrace{f(x)}_{u}\,\underbrace{dx}_{dv} = u \cdot v - \int v\,du$$

LAPTUS
- Ln, log
- arc
- Pol (x^2, x^3)
- Trigo
- Üs

★ $\int x e^x dx = ?$

$u = x$, $dv = e^x dx$

$dx = du$, $\int e^x dx = \int dv$

$e^x = v$

$= x e^x - \int e^x dx$

$= x e^x - e^x + c$

★ $\int x e^{3x} dx = ?$

$u = x$, $dv = e^{3x} dx$

$dx = du$, $\frac{e^{3x}}{3} = v$

$= x \cdot \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} dx$

$= \frac{x e^{3x}}{3} - \frac{1}{3}\int e^{3x} dx$

$= \frac{x e^{3x}}{3} - \frac{1}{3} e^{3x} + c$

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$\int x \sin 2x\,dx = ?$

$x = u$
$dx = du$

$\sin 2x\,dx = dv$
$-\frac{1}{2}\cos 2x = v$

$= x\left(-\frac{1}{2}\cos 2x\right) - \int -\frac{1}{2}\cos 2x\,dx$

$= \frac{-x\cos 2x}{2} + \frac{1}{4}\sin 2x + c$

$\int \frac{x}{\sin^2 x}\,dx = ?$

$x = u$
$dx = du$

$\frac{dx}{\sin^2 x} = dv$
$-\cot x = v$

$= -x\cot x - \int -\cot x\,dx$

$= -x\cot x + \ln|\sin x| + c$

$\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$

$\sin x = t$
$\cos x\,dx = dt$

$= \int \frac{dt}{t}$
$= \ln|\sin x|$

$\int \frac{x}{\cos^2 x}\,dx = ?$

$x = u$
$dx = du$

$\frac{dx}{\cos^2 x} = dv$
$\tan x = v$

$= x\tan x - \int \tan x\,dx$

$= x\tan x + \ln|\cos x| + c$

$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

$\cos x = t$
$-\sin x\,dx = dt$

$= -\ln|\cos x| + c$

$\int x^4 \ln x \, dx$

$\ln x = u$ $\quad x^4 dx = dv$

$\frac{1}{x} dx = du$ $\quad \frac{x^5}{5} = v$

$= (\ln x)\frac{x^5}{5} - \int \frac{x^5}{5} \cdot \frac{1}{x} dx$

$= (\ln x)\frac{x^5}{5} - \frac{1}{5}\int x^4 dx$

$= (\ln x)\frac{x^5}{5} - \frac{x^5}{25} + C$

$\int \arctan x \, dx$

$\arctan x = u$ $\quad dx = dv$

$\frac{dx}{1+x^2} = du$ $\quad x = v$

$= \arctan x \cdot x - \int x \frac{1}{1+x^2} dx$

~~$= \arctan x \cdot x -$~~

$= \arctan x - \frac{1}{2}\ln|1+x^2| + C$

$\int \frac{x\,dx}{1+x^2}$

$1+x^2 = t$

$2x\,dx = dt$

$\int \frac{1}{t}\frac{dt}{2}$

$= \frac{1}{2}\ln|1+x^2| + C$

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Ters Laplace

$$\int e^{ax}\cos bx\,dx = ?$$

u $\quad dv$

$u = e^{ax}$
$du = a e^{ax} dx$

$\cos bx\,dx = dv$
$\frac{1}{b}\sin bx = v$

$$= e^{ax}\frac{1}{b}\sin bx - \int \frac{1}{b}\sin bx \cdot a e^{ax} dx$$

$$= \frac{1}{b}e^{ax}\sin bx - \frac{a}{b}\int e^{ax}\sin bx\,dx$$

u $\quad dv$

$u = e^{ax}$
$du = a e^{ax} dx$

$\sin bx\,dx = dv$
$-\frac{1}{b}\cos bx = v$

$$= e^{ax}\left(-\frac{1}{b}\cos bx\right) - \int \left(-\frac{1}{b}\cos bx\right) a e^{ax} dx$$

$$= -\frac{1}{b}e^{ax}\cos bx + \frac{a}{b}\int \cos bx\, e^{ax} dx$$

$$= \frac{1}{b}e^{ax}\sin bx + \frac{a}{b^2}e^{ax}\cos bx - \frac{a^2}{b^2}\int e^{ax}\cos bx\,dx$$

I

$$I = \frac{1}{b}e^{ax}\sin bx + \frac{a}{b^2}e^{ax}\cos bx - \frac{a^2}{b^2}I$$

$$\frac{a^2+b^2}{b^2}I = \frac{1}{b}e^{ax}\sin bx + \frac{a}{b^2}e^{ax}\cos bx$$

$$I = \frac{b^2}{a^2+b^2}\left[\frac{1}{b}e^{ax}\sin bx + \frac{a}{b^2}e^{ax}\cos bx\right] + c$$

27 NİSAN APRIL CUMA FRIDAY

$(x^2+5)^{1/2}$

NİSAN 2012

H	P	S	Ç	P	C	Ct	Pz
13							1
14	2	3	4	5	6	7	8
15	9	10	11	12	13	14	15
16	16	17	18	19	20	21	22
17	23	24	25	26	27	28	29
18	30						

$2x$.

$$\int \frac{xe^x}{(x+1)^2}\,dx$$

$u = xe^x$

$du = (e^x + xe^x)\,dx$

$\frac{dx}{(x+1)^2} = dv$

$\int \frac{dx}{(x+1)^2} = \int dv$

$x+1 = t$

$dx = dt$

$\int \frac{dt}{t^2} = v$

$\int t^{-2}\,dt = -t^{-1}$

$= \frac{-1}{t}$

$= \frac{-1}{x+1}$

$v = \frac{-1}{x+1}$

$= xe^x \frac{-1}{x+1} - \int \frac{-1}{x+1} e^x (x+1)\,dx$

$= \frac{-xe^x}{x+1} + \int e^x\,dx$

$= \frac{-xe^x}{x+1} + e^x + c$

$$\int \sqrt{x^2+5}\,dx$$

$\sqrt{x^2+5} = u$

$\frac{1}{2}(x^2+5)^{-1/2} \cdot 2x\,dx = du$

$\frac{x}{\sqrt{x^2+5}}\,dx = du$

$dx = dv$

$x = v$

$= \sqrt{x^2+5} \cdot x - \int x \frac{x}{\sqrt{x^2+5}}\,dx$

$\int \frac{x^2+5-5}{\sqrt{x^2+5}}\,dx = \int \frac{(x^2+5)-5}{\sqrt{x^2+5}}\,dx$

$= \int \sqrt{x^2+5}\,dx - 5\int \frac{1}{\sqrt{x^2+5}}\,dx$

I

Formül 13

$x = \sqrt{5}t$

$dx = \sqrt{5}\,dt$

$\int \frac{\sqrt{5}\,dt}{\sqrt{5}\sqrt{t^2+1}}$

$x\sqrt{x^2+5} + 5\int \frac{1}{\sqrt{x^2+5}}\,dx - I$

$x\sqrt{x^2+5} + 5\ln\left[x + \sqrt{5+x^2}\right] - I$

$$I = \frac{x\sqrt{x^2+5} + 5\ln\left[x+\sqrt{5+x^2}\right]}{2}$$

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H	P	S	Ç	P	C	Ct	Pz
18		1	2	3	4	5	6
19	7	8	9	10	11	12	13
20	14	15	16	17	18	19	20
21	21	22	23	24	25	26	27
22	28	29	30	31			

MAYIS 2012

LAPTO

MAYIS
MAY
CUMARTESİ
SATURDAY
05

$\int x^2 e^{-x} dx = ?$

$e^{-x} = u \qquad x^2 dx = dv$

$-e^{-x} dx = du \qquad \frac{x^3}{3} = v$

$= -e^{-x} \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot (-e^{-x}) dx$

$= -e^{-x} \frac{x^3}{3} + \frac{1}{3} \int x^3 e^{-x} dx$

$x^2 = u \qquad e^{-x} dx = dv$

$2x dx = du \qquad -e^{-x} = v$

$= x^2 \cdot (-e^{-x}) - \int (-e^{-x}) \cdot 2x dx$

$= -x^2 e^{-x} + \int 2x e^{-x} dx$

$x = u \qquad e^{-x} dx = dv$

$dx = du \qquad -e^{-x} = v$

$= -x^2 e^{-x} + 2[-x e^{-x} - e^{-x}] + c$

$-x e^{-x} + \int e^{-x} dx$

$-x e^{-x} - e^{-x} + c$

MAYIS
MAY
PAZAR
SUNDAY
06

NOT:

kendisi / Türev / Türev

$\int x^{10} e^x dx = e^x [x^{10} + 10x^9 + \dots] + c$

$\int x^3 e^x dx = e^x [x^3 + 3x^2 + 6x + 6] + c$

Sonsuz'ların 30 adımda sıfırlayan

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Bezier Curve

(3) İNDİRGEME KAĞANTILARI

$$\textcircled{1} \int \cos^n x\,dx = \int \underbrace{\cos^{n-1} x}_{u} \underbrace{\cos x\,dx}_{dv}$$

$u = \cos^{n-1} x$

$du = (n-1)\cos^{n-2} x \cdot (-\sin x)\,dx$

$dv = \cos x\,dx$

$v = \sin x$

$$= (\cos^{n-1} x)(\sin x) + (n-1)\int \sin^2 x \cos^{n-2} x\,dx$$

$\sin^2 x = (1-\cos^2 x)$

$$= \cos^{n-1} x \sin x + (n-1)\left[\int \cos^{n-2} x\,dx - \int \cos^n x\,dx\right]$$

$\int \cos^n x\,dx = I$

$$I = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x\,dx - (n-1) I$$

$$I = \frac{\cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x\,dx}{n}$$

$\int \sin^n x\,dx$ sen bul(!!

$$\int \cos^4 x\,dx = \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\int \cos^2 x\,dx$$

$$\int \cos^2 x\,dx = \frac{1}{2}\int (\cos 2x + 1)\,dx$$

$$= \frac{1}{2}\left[\frac{1}{2}\sin 2x + x\right].$$

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{16}\sin 2x + \frac{3x}{8} + c$$

Sözlü ödev

$$\int \cos^4 x\,dx = \int \underbrace{\cos^3 x}_{u}\underbrace{\cos x\,dx}_{dv}$$

$$\int \sin^5 x\,dx = ?$$

(2) $$\int \operatorname{cosec}^n x\,dx = \int \frac{1}{\sin^n x}\,dx$$

$$= \int \underbrace{\frac{1}{\sin^{n-2} x}}_{u} \cdot \underbrace{\frac{1}{\sin^2 x}\,dx}_{dv}$$

$u = \sin^{2-n} x$

$du = (2-n)\sin^{1-n} x\,(\cos x)\,dx$

$\frac{dx}{\sin^2 x} = dv$

$-\cot x = v \Rightarrow -\frac{\cos x}{\sin x} = v$

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$$= \sin^{2-n}x\left(\frac{-\cos x}{\sin x}\right) - \int \frac{-\cos x}{\sin x}(2-n)\sin^{1-n}x\cos x\,dx$$

$$= \frac{-\cos x}{\sin^{n-1}x} + (2-n)\int \frac{\cos^2 x}{\sin^n x}\,dx$$

$$\frac{1-\sin^2 x}{\sin^n x}$$

$$= \frac{-\cos x}{\sin^{n-1}x} + (2-n)\int \frac{1}{\sin^n x}\,dx - (2-n)\int \frac{1}{\sin^{n-2}x}\,dx$$

$$I = \frac{-\cos x}{\sin^{n-1}x} + (n-2)\int \frac{1}{\sin^{n-2}x}\,dx + (2-n)\,I$$

$$I = \frac{1}{(n-1)}\cdot\left[\frac{-\cos x}{\sin^{n-1}x} + (n-2)\int \frac{1}{\sin^{n-2}x}\,dx\right]$$

$$\int \operatorname{cosec}^6 x\,dx = \int \frac{1}{\sin^6 x}\,dx.$$

$$= \frac{-\cos x}{5\sin^5 x} + \frac{4}{5}\int \frac{1}{\sin^4 x}\,dx$$

$$\int \frac{1}{\sin^4 x}\,dx = \frac{-\cos x}{3\sin^3 x} + \frac{2}{3}\int \frac{1}{\sin^2 x}\,dx$$

$$= \frac{-\cos x}{3\sin^3 x} - \frac{2}{3}\cot x$$

$\int \sec^6 x\,dx$ bu!!!

(3) $\int \tan^n x\,dx = \int \tan^{n-2} x \tan^2 x\,dx$

$= \int \tan^{n-2} x \left(\tan^2 x + 1 - 1\right) dx$

$= \int \tan^{n-2} x \left(\tan^2 x + 1\right) dx - \int \tan^{n-2} x\,dx$

$\int \tan^{n-2} x \left(\tan^2 x + 1\right) dx$ $\quad \tan x = t$, $\left(1+\tan^2 x\right) dx = dt$

$\int t^{n-2}\,dt = \frac{t^{n-1}}{n-1} + c$

$= \frac{\tan^{n-1} x}{n-1} + c$

$= \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$

$\int \tan^4 x\,dx = ?$

$= \frac{\tan^3 x}{3} - \int \tan^2 x\,dx$

$\int \tan^2 x\,dx = \int \left(\tan^2 x + 1\right) - (1)\,dx = \tan x - x$

$= \frac{\tan^3 x}{3} - \tan x + x + c$

$\int \cot^4 x\,dx$ bulun.

(4) $\int \frac{dx}{(a^2+x^2)^n} = ?$

Bir dışta derece azalır. Üzerken derece kendisinden yükselir.

$\int \frac{1}{(a^2+x^2)^{n-1}} dx$

$u \quad dv$

$u = (a^2+x^2)^{1-n}$

$du = (1-n)(a^2+x^2)^{-n}(2x)dx$

$dx = dv$

$x = v$

$= \frac{1}{(a^2+x^2)^{n-1}} \cdot x - \int x \cdot (1-n)(a^2+x^2)^{-n}(2x)dx$

$\int \frac{1}{(a^2+x^2)^{n-1}} dx = \frac{x}{(a^2+x^2)^{n-1}} - 2(1-n)\int \frac{x^2}{(a^2+x^2)^n} dx$

$x^2 + a^2 - a^2$

$= \frac{x}{(a^2+x^2)^{n-1}} - 2(1-n)\int \frac{1}{(a^2+x^2)^{n-1}} dx + 2(1-n)a^2 \int \frac{1}{(a^2+x^2)^n} dx$

$2(1-n)a^2 \int \frac{1}{(a^2+x^2)^n} dx = \frac{x}{(a^2+x^2)^{n-1}} + (1-2n)\int \frac{1}{(a^2+x^2)^{n-1}} dx$

$= \frac{x}{(a^2}$

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$$\int \frac{dx}{(a^2+x^2)^n} = \frac{1}{2(1-n)a^2}\left[\frac{x}{(a^2+x^2)^{n-1}} + (1-2n)\int \frac{1}{(a^2+x^2)^{n-1}}dx\right]$$

$$\int \frac{dx}{(4x^2+4x+10)^2} = \int \frac{dx}{((2x+1)^2+3^2)^2}$$

$$\int \frac{dt/2}{(t^2+3^2)^2} = \frac{1}{2}\int \frac{dt}{(3^2+t^2)^2}$$

$2x+1 = t$
$2dx = dt$

$a = 3$
$n = 2$

$$= \frac{1}{2}\left[\frac{-1}{18}\left(\frac{t}{(9+t^2)} + 3\int \frac{1}{(9+t^2)}dt\right)\right]$$

$$= \frac{1}{36}\left[\frac{t}{9+t^2} - \arctan\frac{t}{3}\right] + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}$$

$$= \frac{-1}{36}\left[\frac{2x+1}{4x^2+4x+10} - \arctan\frac{2x+1}{3}\right] + C$$

$$\int \frac{dx}{(1+x^2)^2} = ?$$

$x = \tan t$
$dx = (1+\tan^2 t)dt$

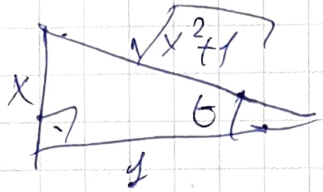
$$\int \frac{dx}{(a^2+x^2)^n}$$
$x = a\tan t$

$$\int \frac{(1+\tan^2 t)dt}{(1+\tan^2 t)^2}$$

$$\int \frac{dt}{1+\tan^2 t} = \int \frac{dt}{\frac{1}{\cos^2 t}} = \int \cos^2 t\, dt$$

$$= \int \frac{1}{2}(\cos 2t + 1)\,dt = \frac{1}{2}\left[\frac{1}{2}\sin 2t + t\right] + C$$

$x = \tan t$



$$\sin 2t = 2\sin t \cos t$$

$$= 2\,\frac{x}{\sqrt{x^2+1}} \cdot \frac{1}{\sqrt{x^2+1}}$$

$$= \frac{1}{4} \cdot \frac{2x}{(x^2+1)} + \frac{1}{2}\arctan x + C$$

BASIT KESIRLERE AYIRMA YÖNTEMI

$$\frac{P(x)}{Q(x)} = K(x) + \frac{R(x)}{Q(x)}$$

$$\int \frac{P(x)}{Q(x)}\,dx = \int K(x)\,dx + \int \frac{R(x)}{Q(x)}\,dx$$

?

$$\frac{R(x)}{Q(x)} \equiv \frac{M}{px+q} + \frac{N}{(px+q)^n} + \frac{S}{ax^2+bx+c} + \frac{T}{(ax^2+bx+c)^n}$$

$$\int \frac{3x^2+2x+3}{3x+2}dx = \int \left(x + \frac{3}{3x+2}\right)dx$$

$$= \int x\,dx + 3\int \frac{1}{3x+2}dx$$

$$= \frac{x^2}{2} + \ln|3x+2| + c$$

$3x+2=t$

$3dx=dt$

$$\int \frac{dx}{3x+2} = \int \frac{dt/3}{t}$$

$$= \frac{1}{3}\ln|t| + c$$

$$= \frac{1}{3}\ln|3x+2|$$

$$\int \frac{dx}{2-3x} = \int \frac{-dt/3}{t} = -\frac{1}{3}\ln|t| + c$$

$2-3x=t$

$-3dx=dt$

$$= -\frac{1}{3}\ln|2-3x| + c$$

Formül:

$$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b| + c$$

- $\int \frac{1}{2-\sqrt{2}x}dx = -\frac{1}{\sqrt{2}}\ln|2-\sqrt{2}x| + c$
- $\int \frac{dx}{x+1} = \ln|x+1| + c$

$$\int \frac{1}{(5x+2)^3}\,dx = \int \frac{1}{t^3}\,\frac{dt}{5} = \frac{1}{5}\int t^{-3}\,dt$$

$5x+2=t$

$5dx=dt$

$$= \frac{1}{5}\,\frac{t^{-2}}{-2} = \frac{-1}{10t^2} + C$$

$$= \frac{-1}{10(5x+2)^2} + C$$

$$\int \frac{2x+4+1-1}{x^2+5x+13}\,dx = \underbrace{\int \frac{2x+5}{x^2+5x+13}\,dx}_{a} + \underbrace{\int \frac{-1}{x^2+5x+13}\,dx}_{b}$$

$x^2+5x+13=t$

$(2x+5)dx=dt$

(a) $$\int \frac{dt}{t} = \ln|t| = \ln|x^2+5x+13|$$

(b) $$\int \frac{-1}{x^2+5x+13}\,dx = \int \frac{-1}{\left(x+\frac{5}{2}\right)^2+\frac{27}{4}}\,dx$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}$$

$x+5/2=t$

$dx=dt$

$$= \int \frac{-dt}{t^2+\left(\frac{3\sqrt{3}}{2}\right)^2} = \frac{-1}{\frac{3\sqrt{3}}{2}}\arctan\frac{t}{\frac{3\sqrt{3}}{2}}$$

$2x+5=2t$

$$= \frac{-2}{3\sqrt{3}}\arctan\left(\frac{2x+5}{3\sqrt{3}}\right)$$

Sonuç

$$I = \ln|x^2+5x+13| - \frac{2}{3\sqrt{3}}\arctan\left(\frac{2x+5}{3\sqrt{3}}\right) + C$$

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$$\int \frac{dx}{(4+x^2)^2} = I$$ indirgeme belirtisi

$$\int \underbrace{\frac{1}{(4+x^2)}}_{u} \underbrace{dx}_{dv}$$

$(4+x^2)^{-1} = u$	$dx = dv$
$\frac{-2x}{(4+x^2)^2} = du$	$x = v$

$$= \frac{1}{4+x^2}\,x - \int x \frac{-2x}{(4+x^2)^2}\,dx$$

$$= \frac{x}{4+x^2} + 2\int \frac{x^2+4-4}{(4+x^2)^2}\,dx$$

$$= \frac{x}{4+x^2} + 2\int \underbrace{\frac{x^2+4}{(4+x^2)^2}}_{\frac{1}{x^2+4}}\,dx - 8\underbrace{\int \frac{dx}{(4+x^2)^2}}_{I}$$

$$8\int \frac{1}{(4+x^2)^2}\,dx = \frac{x}{4+x^2} + \int \frac{dx}{x^2+4}$$

$$= \frac{x}{8(4+x^2)} + \frac{1}{16}\arctan\frac{x}{2} + C$$

II. Yol $x = 2\tan t$

18 MAYIS / MAY / CUMA / FRIDAY

Pada Gorpolara ayrılıyorsa ağır

MAYIS 2012

H	P	S	Ç	P	C	Ct	Pz
18		1	2	3	4	5	6
19	7	8	9	10	11	12	13
20	14	15	16	17	18	19	20
21	21	22	23	24	25	26	27
22	28	29	30	31			

$$\int \frac{4}{x^2-4}\,dx \Rightarrow \frac{A}{x-2} + \frac{B}{x+2} = \frac{4}{x^2-4}$$

$$Ax+2A+Bx-2B=4$$

$$A+B=0$$

$$2A-2B=4 \qquad A=1,\ B=-1$$

$$\int \frac{dx}{x-2} + \int \frac{-dx}{x+2}$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b|$$

$$\ln|x-2| + \ln|x+2| + c = \ln\left|\frac{x-2}{x+2}\right| + c$$

Tolga

$$\frac{2}{(x+1)(x-1)^3(3x^2+4)(x^2-4)(x^2+5x+13)^2(x^3+13)}$$

$$= \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3} + \frac{Ex+F}{3x^2+4} + \frac{H}{x-2} + \frac{I}{x+2}$$

$$+ \frac{Jx+K}{x^2+5x+13} + \frac{Mx+N}{(x^2+5x+13)^2} + \frac{2x^2+Yx+5}{x^3+13}$$

$$\int \frac{2x+1}{(x-1)^2}\,dx \qquad \frac{2x+1}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$Ax-A+B = 2x+1$$

$$A=2 \qquad B=3$$

$$\int \frac{2}{x-1}\,dx + \int \frac{3}{(x-1)^2}\,dx$$

$$2\ln|x-1| - \frac{3}{(x-1)} + c$$

$x-1=t$

$dx=dt$

$$= \int \frac{3}{t^2}\,dt$$

$$= 3t^{-1}$$

$$= \frac{-3}{(x-1)}$$

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x^2-2x+1

$(x-1)x^2+1$ $\quad x^3+x-x^2-1$

$$\int \frac{-2x+4}{(1+x^2)(x-1)^2}\,dx$$

$$\frac{-2x+4}{(x^2+1)(x-1)^2} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$(x-1)^2$ $\quad x-1$ $\quad x^2+1$

$$Ax^3-Ax^2+Ax+Bx^2-2Bx+B$$
$$Cx^3-Cx^2+Cx-C$$
$$Dx^2+D$$

$A+C=0$ $\quad C=-2$

$-A+B-C+D=0$ $\quad D=1$

$A-2B+C=-2$ $\quad A=2$

$B-C+D=4$ $\quad B=1$

$$\int \frac{-2}{x-1}\,dx = -2\ln|x-1|$$

$$\int \frac{1}{(x-1)^2}\,dx = \int \frac{1}{t^2}\,dt = \int t^{-2}\,dt = \frac{t^{-1}}{-1} = \frac{-1}{t} = \frac{-1}{x-1}$$

$x-1=t$
$dx=dt$

$$\int \frac{2x+1}{1+x^2}\,dx = \int \frac{2x}{1+x^2}\,dx + \int \frac{1}{1+x^2}\,dx$$

$1+x^2=t$
$2x\,dx=dt$

$$= \int \frac{dt}{t} + \arctan x$$
$$= \ln|1+x^2| + \arctan x$$

$$= -2\ln|x-1| - \frac{1}{x-1} + \ln|1+x^2| + \arctan x + c$$

$2x^4-6x^3+7x^2-2x-2 \;\big|\; x^3-3x^2+3x-1$

$2x^3-6x^2+6x^2-2x$

$2x$

x^2-2x+2

$x^2=2$

$$\int \frac{2x^4-6x^3+7x^2-2x-2}{x^3-3x^2+3x-1}\,dx$$

Derece eşit veya üst büyük ise tam kesir bul

$$= \int \left[2x + \frac{x^2-2}{(x-1)^3}\right] dx$$

$$= \underbrace{\int 2x\,dx}_{x^2} + \underbrace{\int \frac{x^2-2}{(x-1)^3}\,dx}$$

$$\frac{x^2-2}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

$$Ax^2-2Ax+A+Bx-B+C = x^2-2$$

$A=1$

$B=2$

$C=-1$

$$\int \frac{dx}{x-1} + \int \frac{2\,dx}{(x-1)^2} - \int \frac{dx}{(x-1)^3}$$

$x-1=t$

$dx=dt$

~~$\ln|x-1| - 2\ln|x-1|$~~

$$\ln|x-1| - \frac{2}{x-1} + \frac{1}{2(x-1)^2}$$

$$= x^2 + \ln|x-1| - \frac{2}{x-1} + \frac{1}{2(x-1)^2} + C$$

$$\int \frac{2}{x^2-1}\,dx \qquad \frac{2}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$Ax + A + Bx - B = 2$$

$$A+B=0$$
$$A-B=2$$
$$A=1 \quad B=-1$$

$$\int \frac{dx}{x-1} - \int \frac{dx}{x+1}$$

$$\ln|x-1| - \ln|x+1| + c$$

$$\int \frac{e^y\,dy}{e^{2y}+3e^y+2}$$

$$\frac{e^y}{(e^y+2)(e^y+1)} = \frac{A}{e^y+2} + \frac{B}{e^y+1}$$

$$A+B=1$$
$$2A+B=0$$
$$A=-1$$
$$B=1$$

$$\int \frac{-dy}{e^y+2} + 2\int \frac{dy}{e^y+1}$$

$$e^y = t$$
$$e^y\,dy = dt$$

$$\int \frac{1}{v^2+3v+2}\,dv$$

$$v = e^y$$
$$dv = e^y\,dy$$

$$= \ln|v+1| - \ln|v+2| + c$$

$$= \ln\left|\frac{e^y+1}{e^y+2}\right| + c$$

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$$\int \frac{\sin\theta\, d\theta}{\cos^2\theta + \cos\theta - 2}$$

$\cos\theta = u$

$-\sin\theta\, d\theta = du$

$$= \int \frac{-du}{u^2+u-2}$$

$+2$

-1

$$\frac{-1}{u^2+u-2} = \frac{A}{u+2} + \frac{B}{u-1}$$

$Au - A + Bu + 2B = -1$

$A + B = 0$

$-A + 2B = -1$

$B = -1/3$

$A = 1/3$

$$= \frac{1}{3}\int \frac{du}{u+2} - \frac{1}{3}\int \frac{du}{u-1}$$

$$= \frac{1}{3}\ln|\cos\theta+2| - \frac{1}{3}\ln|\cos\theta - 1| + C$$

$$= \frac{1}{3}\ln\left|\frac{\cos\theta+2}{\cos\theta-1}\right| + C$$

TRİGONOMETRİK İNTEGRALLER

$$\sin ax \sin bx = -\frac{1}{2}\left[\cos(a+b)x - \cos(a-b)x\right]$$

$$\sin ax \cos bx = \frac{1}{2}\left[\sin(a+b)x + \sin(a-b)x\right]$$

$$\cos ax \cos bx = \frac{1}{2}\left[\cos(a+b)x + \cos(a-b)x\right]$$

(1)(3) sm a+b → (2)

- $\cos(a+b)x = \cos(ax+bx)$

$= \cos(ax)\cos(bx) - sm(ax)\,sm(bx)$

- $\cos(a-b)x = \cos(ax-bx)$

$= \cos(ax)\cos(bx) + sm(ax)\,sm(bx)$

- $\cos(a-b)x - \cos(a+b)x = 2\,sm(ax)\,sm(bx)$

$\frac{1}{2}\left[\cos(a-b)x - \cos(a+b)x\right] = sm(ax)\,sm(bx)$

★ $\int sm\,7x\,sm\,3x\,dx = \int -\frac{1}{2}\left[\cos 10x - \cos 4x\right]dx$

$= -\frac{1}{2}\left[\frac{1}{10}\,sm\,10x - \frac{1}{4}\,sm\,4x\right] + c$

$\int \cos(ax)dx = \frac{1}{a}\,sm\,ax + c$

★ $\int sm\,5x\cos 3x\,dx = \int \frac{1}{2}\left[sm\,8x + sm\,2x\right]dx$

$= \frac{1}{2}\left[-\frac{1}{8}\cos 8x + \frac{-1}{2}\cos 2x\right] + c$

$\int sm\,ax\,dx = \frac{-1}{a}\cos ax + c$

★ $\int \cos 4x\cos 3x\,dx = \int \frac{1}{2}\left[\cos 7x + \cos x\right]dx$

$= \frac{1}{2}\left[\frac{1}{7}\,sm\,7x + sm\,x\right] + c$

sin → a
cos → b

$$\int \sin 3x \cos 5x\,dx = \frac{1}{2}\int [\sin 8x + \sin(-2x)]\,dx$$

$$= \int \frac{1}{2}[\sin 8x - \sin 2x]\,dx$$

$$= \frac{1}{2}\left[-\frac{1}{8}\cos 8x + \frac{1}{2}\cos 2x\right] + C$$

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① $\int \sin^m x \cos^n x\,dx$

Ikisinde tek

ⓐ m → tek (n çift / n tek) → $\cos x = t$

ⓑ n → tek (m çift / m tek) → $\sin x = t$

$m > n$ $\sin x = t$

$n > m$ $\cos x = t$

ⓒ m çift ve n çift

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x + 1 = 2\cos^2 x$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

$\int \sin^5 x \cos^2 x\,dx$ $\quad m=5,\ n=2$

$\cos x = t$

$-\sin x\,dx = dt$

$= \int \sin^4 x \cdot \sin x \cos^2 x\,dx$

$= \int (1-\cos^2 x)^2 \cos^2 x \sin x\,dx$

$= \int (1-t^2)^2 t^2 (-dt)$

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$$= -\int (1 - 2t^2 + t^4)\, t^2\, dt$$

$$= -\int (t^2 - 2t^4 + t^6)\, dt$$

$$= -\left[\frac{t^3}{3} - 2\frac{t^5}{5} + \frac{t^7}{7} \right] + C$$

$$= -\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C$$

$\int \sin^4 x \cos^3 x\, dx$ $\quad m = 4 \quad n = 3$

tek olan hariçtekine t de

$t = \sin x$

$dt = \cos x\, dx$

$$= \int \sin^4 x \cos^2 x \cos x\, dx$$

$$= \int \sin^4 x (1 - \sin^2 x) \cos x\, dx$$

$$= \int t^4 (1 - t^2)\, dt$$

$$= \int (t^4 - t^6)\, dt = \frac{t^5}{5} - \frac{t^7}{7} + C = \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$$

$\int \sin^9 x \cos^3 x\, dx$ $\quad m = 9 \quad n = 3 \quad m > n$

tek olana t de

$\sin x = t$

$\cos x\, dx = dt$

$$= \int \sin^9 x \cos^2 x \cos x\, dx$$

$$= \int t^9 (1 - t^2)\, dt$$

$$= \int (t^9 - t^{11})\, dt$$

$$= \frac{t^{10}}{10} - \frac{t^{12}}{12} + C$$

$$= \frac{\sin^{10} x}{10} - \frac{\sin^{12} x}{12} + C$$

$(\cos 5x)^7$ / $(3x^2+5x+4)^3$

$1-2\cos 2x+\cos^2 2x$

$\cos 2x - 2\cos^2 2x + \cos^3 2x$

28

MAYIS
MAY

PAZARTESİ
MONDAY

MAYIS 2012

H	P	S	Ç	P	C	Ct	Pz
18		1	2	3	4	5	6
19	7	8	9	10	11	12	13
20	14	15	16	17	18	19	20
21	21	22	23	24	25	26	27
22	28	29	30	31			

$$\int \sin^4 x \cos^2 x \, dx$$

$$= \int \left[\frac{1}{2}(1-\cos 2x)\right]^2 \times \frac{1}{2}\left[1+\cos 2x\right] dx$$

$$= \frac{1}{8}\int (1-2\cos 2x+\cos^2 2x)(1+\cos 2x)\, dx$$

$$= \frac{1}{8}\int (\underbrace{\cos^3 2x}_{①} - \underbrace{\cos^2 2x}_{②} - \underbrace{\cos 2x}_{③} + \underbrace{1}_{④})\, dx$$

④ $\int 1\,dx = x$

③ $\int -\cos 2x\, dx = -\frac{1}{2}\sin 2x$

② $\int -\cos 2x\, dx = \int -\frac{1}{2}\left[1+\cos 4x\right] dx$

$\cos 2x = 2\cos^2 x - 1$

$\cos 4x = 2\cos^2 2x - 1$

$$= -\frac{1}{2}\left[x + \frac{1}{4}\sin 4x\right]$$

$$= -\frac{x}{2} - \frac{1}{8}\sin 4x$$

① $\int \cos^3 2x\, dx = \int \cos^3 2x \cdot \sin^0 2x\, dx$

$\sin 2x = t$

$2\cos 2x\, dx = dt$

$$= \int \cos^2 2x \cos 2x\, dx$$

$$= \int (1-\sin^2 2x)\cos 2x\, dx$$

$$= \int (1-t^2)\, dt/2$$

$$= \frac{1}{2}\left[t - \frac{t^3}{3}\right] + C = \frac{1}{2}\left[\sin 2x - \frac{\sin^3 2x}{3}\right]$$

TMMOB MAKİNA MÜHENDİSLERİ ODASI 1954

$$= \frac{1}{8}\left[\frac{x}{2} - \frac{1}{8}\sin 4x - \frac{\sin^3 2x}{6}\right] + C$$

(2) $\int \tan^m x \sec^n x \, dx$

(a) n çift ise $\tan x = t$

(b) m tek ise $\sec x = t$

(c) m çift n tek ise $\tan^2 x = \sec^2 x - 1$ dek. kullanarak çöz

$\int \tan^6 x \sec^4 x \, dx$

$\tan x = t$

$\sec^2 x \, dx = dt$

$(\tan x)' = 1 + \tan^2 x = \sec^2 x = \frac{1}{\cos^2 x}$

$= \int \tan^6 x \sec^2 x \sec^2 x \, dx$

$= \int \tan^6 x (1 + \tan^2 x) \sec^2 x \, dx$

$= \int t^6 (1 + t^2) \, dt$

$= \int (t^6 + t^8) \, dt = \frac{t^7}{7} + \frac{t^9}{9} + C$

$= \frac{\tan^7 x}{7} + \frac{\tan^9 x}{9} + C$

$t^4 - 2t^2 + 1$

30

MAYIS
MAY

ÇARŞAMBA
WEDNESDAY

$\tan^2 x = \sec^2 x - 1$

MAYIS 2012

H	P	S	Ç	P	C	Ct	Pz
18		1	2	3	4	5	6
19	7	8	9	10	11	12	13
20	14	15	16	17	18	19	20
21	21	22	23	24	25	26	27
22	28	29	30	31			

★ $\int \tan^5 x \sec^3 x \, dx$

$\sec x = t$

$\sec x \tan x \, dx = dt$

$= \int \tan^4 x \sec^2 x \sec x \tan x \, dx$

$= \int (\sec^2 x - 1)^2 \sec^2 x \sec x \tan x \, dx$

$= \int (t^2 - 1)^2 t^2 \, dt$

$= \int (t^6 - 2t^4 + t^2) \, dt$

$= \frac{t^7}{7} - 2\frac{t^5}{5} + \frac{t^3}{3} + C$

$= \frac{\sec^7 x}{7} - 2\frac{\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$

★ $\int \tan^2 x \sec x \, dx$ $\qquad \tan^2 x = \sec^2 x - 1$

$= \int (\sec^2 x - 1) \sec x \, dx$

$= \int (\sec^3 x - \sec x) \, dx$

$= \underbrace{\int \sec^3 x \, dx}_{b} - \underbrace{\int \sec x \, dx}_{a}$

sec'in tek kuvvetlerinin integrali değişik yollar ile yapılır

(a) $\int \sec x \, dx = \int \frac{(\sec^2 x + \sec x \tan x)}{\sec x + \tan x} dx$

$(\sec x + \tan x)$

$\int \operatorname{cosec} x \, dx$

$(\operatorname{cosec} x + \cot x)$

(4'e 3' dene) !!!

$$= \int \frac{(\sec^2 x + \sec x \tan x)\,dx}{\sec x + \tan x}$$

$\sec x + \tan x = t$

$(\sec^2 x + \sec x \tan x)\,dx$

$$= \int \frac{dt}{t} = \ln|t| + c = \ln|\sec x + \tan x|$$

(b) $\int \sec^3 x\,dx = I$ (2'inci kuvvet gırılır.)

$\int \underbrace{\sec x}_{u} \underbrace{\sec^2 x\,dx}_{dv}$

$\sec x = u$	$\sec^2 x\,dx = dv$
$\sec x \tan x\,dx = du$	$\tan x = v$

$= \sec x \tan x - \int \tan^2 x \sec x\,dx$

$\tan^2 x = \sec^2 x - 1$

$= \sec x \tan x - \int (\sec^2 x - 1)\sec x\,dx$

$= \sec x \tan x - \int (\sec^3 x - \sec x)\,dx$

$I = \sec x \tan x - I + \int \sec x\,dx$

$I = \frac{1}{2}\left[\sec x \tan x + \ln|\sec x + \tan x|\right]$

Cevap

$= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + c$

(3) $\int \cot^m x \operatorname{cosec}^n x \, dx$

(a) n çift ise $\cot x = t$

(b) m tek ise $\operatorname{cosec} x = t$

(c) m çift, n tek ise $\cot^2 x = \operatorname{cosec}^2 x - 1$

$\int \cot^2 x \operatorname{cosec}^4 x \, dx$ $\quad \cot x = t$, $-\operatorname{cosec}^2 x \, dx = dt$

$= \int \cot^2 x \left(\cot^2 x + 1\right) \operatorname{cosec}^2 x \, dx$

$= \int t^2 \left(t^2 + 1\right) (-dt)$

$= -\int \left(t^4 + t^2\right) dt = -\left[\frac{t^5}{5} + \frac{t^3}{3}\right] + C$ $\quad \cot^2 x = \operatorname{cosec}^2 x - 1$

$= -\left[\frac{\cot^5 x}{5} + \frac{\cot^3 x}{3}\right] + C$

$(\cot x)' = -\left(1 + \cot^2 x\right) = -\operatorname{cosec}^2 x = -\frac{1}{\sin^2 x}$

$\int \cot^5 x \operatorname{cosec}^5 x \, dx$ $\quad \operatorname{cosec} x = t$, $-\operatorname{cosec} x \cot x \, dx = dt$

$= \int \cot^4 x \operatorname{cosec}^4 x \cot x \operatorname{cosec} x \, dx$

$= \int \left(\operatorname{cosec}^2 x - 1\right)^2 \operatorname{cosec}^4 x \cot x \operatorname{cosec} x \, dx$

$= \int \left(t^2 - 1\right)^2 \cdot t^4 \, dt$

$= \int \left(t^8 + 2t^6 - t^4\right) dt$

$\frac{x^2-1}{x^4-2x^2+1}$

$\int \cot^4 x \operatorname{cosec} x \, dx$

$\cot^2 x = \operatorname{cosec}^2 x - 1$

$\int (\operatorname{cosec}^2 x - 1)^2 \operatorname{cosec} x \, dx$

$\int (\underbrace{\operatorname{cosec}^5 x}_{c} - \underbrace{2\operatorname{cosec}^3 x}_{b} + \underbrace{\operatorname{cosec} x}_{a}) \, dx$

(a) $\int \operatorname{cosec} x \, dx = \int \frac{(\operatorname{cosec}^2 x + \operatorname{cosec} x \cot x) \, dx}{(\operatorname{cosec} x + \cot x)}$

$(\operatorname{cosec} x + \cot x) = t$

$(-\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x) \, dx = dt$

$= \int -\frac{dt}{t} = -\ln|t|$

$= -\ln|\operatorname{cosec} x + \cot x|$

(b) $\int \underbrace{\operatorname{cosec} x}_{u} \underbrace{\operatorname{cosec}^2 x \, dx}_{dv}$

$\operatorname{cosec} x = u$

$(-\operatorname{cosec} x \cot x) \, dx = du$

$\operatorname{cosec}^2 x \, dx = dv$

$-\cot x = v$

$= -\operatorname{cosec} x \cot x + \int \cot^2 x \operatorname{cosec} x \, dx$

$= -\operatorname{cosec} x \cot x - \int (\operatorname{cosec}^2 x - 1) \operatorname{cosec} x \, dx$

$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x + \int \operatorname{cosec} x \, dx$

$I = \frac{1}{2} \left[-\operatorname{cosec} x \cot x - \ln|\operatorname{cosec} x + \cot x| \right]$

$2\int \operatorname{cosec}^2 x \, dx = -\operatorname{cosec} x \cot x - \ln|\operatorname{cosec} x + \cot x|$

(c) $\int \operatorname{cosec}^5 x\,dx = \int \underbrace{\operatorname{cosec}^3 x}_{u} \underbrace{\operatorname{cosec}^2 x\,dx}_{dv}$

$-\operatorname{cosec}^3 x \cot x - \int (-\cot x)(-3\operatorname{cosec}^3 x \cot x)\,dx$

$-\operatorname{cosec}^3 x \cot x - 3\int \cot^2 x \operatorname{cosec}^3 x\,dx$

$\cot^2 x = \operatorname{cosec}^2 x - 1$

$\operatorname{cosec}^3 x = u$

$3\operatorname{cosec}^2 x(-\operatorname{cosec} x \cot x)\,dx = du$

$\operatorname{cosec}^2 x\,dx = dv$

$-\cot x = v$

$-\operatorname{cosec}^3 x \cot x - 3\int (\operatorname{cosec}^5 x - \operatorname{cosec}^3 x)\,dx$

$-\operatorname{cosec}^3 x \cot x - 3\int \operatorname{cosec}^5 x\,dx + 3\int \operatorname{cosec}^3 x\,dx$

$\int \operatorname{cosec}^5 x\,dx = \frac{1}{4}\left[-\operatorname{cosec}^3 x \cot x + \frac{3}{2}\left[-\operatorname{cosec} x \cot x - \ln|\operatorname{cosec} x + \cot x|\right]\right]$

$= \frac{-1}{4}\operatorname{cosec}^3 x \cot x - \frac{3}{8}\operatorname{cosec} x \cot x - \frac{3}{8}\ln|\operatorname{cosec} x + \cot x|$

Cevap

$= -\frac{1}{4}\operatorname{cosec}^3 x \cot x - \frac{3}{8}\operatorname{cosec} x \cot x - \frac{3}{8}\ln|\operatorname{cosec} x + \cot x|$

$+ \operatorname{cosec} x \cot x + \ln|\operatorname{cosec} x + \cot x| - \ln|\operatorname{cosec} x + \cot x|$

$= -\frac{1}{4}\operatorname{cosec}^3 x \cot x + \frac{5}{8}\operatorname{cosec} x \cot x - \frac{3}{8}\ln|\operatorname{cosec} x + \cot x| + c$

(4) $\int R(\sin x, \cos x)\,dx$

(a) $R(\sin x(-\cos x)) = -R(\sin x, \cos x)$

$\sin x = t$

(b) $R(-\sin x, \cos x) = -R(\sin x, \cos x)$

$\cos x = t$

(c) $R((-\sin x), (-\cos x)) = R(\sin x, \cos x)$

$\tan x = t$

$$\int \frac{\sin^3 x}{\cos^2 x}\,dx \qquad \cos x = t$$
$$-\sin x\,dx = dt$$

$$= \int \frac{\sin^2 x \sin x\,dx}{\cos^2 x}$$

$$= \int \frac{(1-\cos^2 x)\sin x\,dx}{\cos^2 x} = \int \frac{(1-t^2).(-dt)}{t^2} = \int \left(1 - \frac{1}{t^2}\right)dt$$

$$= \int dt - \int t^{-2}\,dt$$

$$= t - \frac{t^{-1}}{-1} + C$$

$$= t + \frac{1}{t} + C$$

$$= \cos x + \frac{1}{\cos x} + C$$

$$\int \sin^3 x \cot^5 x\,dx = \int \sin^3 x \frac{\cos^5 x}{\sin^5 x}\,dx$$

$$= \int \frac{\cos^5 x}{\sin^2 x}\,dx$$

$\sin x = t$

$\cos x\,dx = dt$

$$= \int \frac{(1-\sin^2 x)^2 \cos x\,dx}{\sin^2 x}$$

$$= \int \frac{(1-t^2)^2\,dt}{t^2} = \int \frac{(1-2t^2+t^4)\,dt}{t^2}$$

$$= \int \frac{1}{t^2}\,dt - 2\int dt + \int t^2\,dt$$

$$= -\frac{1}{t} - 2t + \frac{t^3}{3} + C$$

$$= \frac{-1}{\sin x} - 2\sin x + \frac{\sin^3 x}{3} + C$$

$$\int \frac{1}{\sin^2 x \cos^2 x}\,dx$$

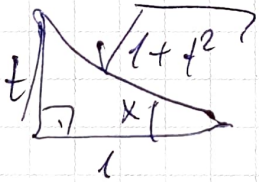
$\tan x = t$

$\frac{1}{\cos^2 x}\,dx = dt$

$(1+\tan^2 x)\,dx = dt$

$$dx = \frac{dt}{1+\tan^2 x}$$

$$dx = \frac{dt}{1+t^2}$$



$$\cos x = \frac{1}{\sqrt{1+t^2}}$$

$$\sin x = \frac{t}{\sqrt{1+t^2}}$$

$$= \int \frac{1}{\frac{t^2}{1+t^2} \cdot \frac{1}{(1+t^2)^2}} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{1+2t^2+t^4}{t^2}\, dt$$

$$= \int \left(t^{-2} + 2 + t^2\right) dt$$

$$= \frac{-1}{t} + 2t + \frac{t^3}{3} + c$$

$$\int \operatorname{sm}\frac{7x}{2} \operatorname{sm}\frac{x}{2}\, dx$$

$$= \int -\frac{1}{2}\left[\cos 4x - \cos 3x\right] dx$$

$$= -\frac{1}{2}\int \cos 4x\, dx + \frac{1}{2}\int \cos 3x\, dx$$

$$= -\frac{1}{8}\operatorname{sm}4x - \frac{1}{6}\operatorname{sm}3x + c$$

$$\int \operatorname{sm}^3 x \cos^2 x\, dx$$

$\cos x = t$
$-\operatorname{sm} x\, dx = dt$

$$= \int \operatorname{sm}^2 x \cos^2 x \operatorname{sm} x\, dx$$

$$= \int \left(1-\cos^2 x\right)\cos^2 x \operatorname{sm} x\, dx$$

$$= \int \left(1-t^2\right) t^2 (-dt)$$

$\int \tan^3 x \sec^4 x \, dx$

$\sec x = t$

$\sec x \tan x \, dx = dt$

$= \int \tan^2 x \sec^3 x \tan x \sec x \, dx$

$= \int (\sec^2 x - 1) \sec^3 x \tan x \sec x \, dx$

$= \int (t^2 - 1) t^3 \, dt$

$= \int (t^5 - t^3) \, dt$

$= \frac{t^6}{6} - \frac{t^4}{4} + C = \frac{\sec^6 x}{6} - \frac{\sec^4 x}{4} + C$

$\int \cot^4 x \operatorname{cosec}^6 x \, dx$

$\cot x = t$

$-\operatorname{cosec}^2 x \, dx = dt$

$= \int \cot^4 x \operatorname{cosec}^4 x \operatorname{cosec}^2 x \, dx$

$= \int \cot^4 x (\cot^2 x + 1)^2 \operatorname{cosec}^2 x \, dx$

$= \int t^4 (t^2 + 1)^2 (-dt)$

$= -\int (t^8 + 2t^6 + t^4) \, dt$

$= -\left[\frac{t^9}{9} + 2\frac{t^7}{7} + \frac{t^5}{5} \right] + C$

$= -\frac{1}{9} \cot^9 x - \frac{2}{7} \cot^7 x - \frac{1}{5} \cot^5 x + C$

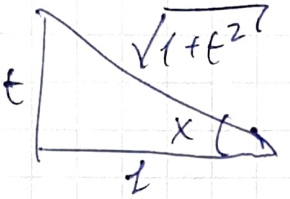
$$\int \frac{\sin^2 x}{4+\cos^2 x}\,dx$$

$\tan x = t$

$(1+\tan^2 x)\,dx = dt$

$dx = \frac{dt}{1+\tan^2 x}$

$dx = \frac{dt}{1+t^2}$



$\sin x = \frac{t}{\sqrt{1+t^2}}$

$\cos x = \frac{1}{\sqrt{1+t^2}}$

$$= \int \frac{\frac{t^4}{(1+t^2)^2}}{4+\frac{1}{1+t^2}} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{t^4}{(1+t^2)^3} \cdot \frac{1+t^2}{5+4t^2}\,dt$$

$$= \int \frac{t^4}{(1+t^2)^2\,(5+4t^2)}\,dt$$

$$= \int \frac{t^4}{4t^6+13t^4+14t^2+5}\,dt$$

$$\sin^4 x = (1-\cos^2 x)^2 = 1 - 2\cos^2 x + \cos^4 x$$

$$1 - 2\cos^2 x + \cos^4 x \;\Big|\; \frac{4+\cos^2 x}{\cos^2 x - 6}$$

$$25$$

$$\int \left((\cos^2 x - 6) + \frac{25}{4+\cos^2 x} \right) dx$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\underbrace{\int \frac{1}{2}[\cos 2x + 1]\,dx}_{1} \underbrace{- \int 6\,dx}_{2} + \underbrace{\int \frac{25}{4+\cos^2 x}\,dx}_{3}$$

(1) $\frac{1}{2}\left[\frac{1}{2}\sin 2x + x\right]$

(2) $-6x$

(3) $\int \frac{25}{4 + \frac{1}{1+t^2}} \cdot \frac{dt}{1+t^2} = \int \frac{25}{5+4t^2}\,dt = \int \frac{\frac{25}{2}\,dv}{5+v^2}$

$\begin{bmatrix} 2t = v \\ 2\,dt = dv \end{bmatrix}$

$$= \frac{25}{2}\left[\frac{1}{\sqrt{5}} \arctan \frac{v}{\sqrt{5}}\right] = \frac{25}{2}\left[\frac{1}{\sqrt{5}} \arctan \frac{2t}{\sqrt{5}}\right]$$

cikmis

$$= \frac{1}{4}\sin 2x + \frac{x}{2} - 6x + \frac{25}{2\sqrt{2}} \arctan \frac{2(\tan x)}{\sqrt{5}} + C$$

İRRASYONEL FONKSİYONLARIN İNTEGRALİ

(1) $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

(a) $ax^2+bx+c = k^2-u^2$ $\quad k=sbt$ $\quad u-x$'li fonk

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \int \frac{dx}{\sqrt{k^2-u^2}} = \arcsin\frac{u}{k}+c$$

(b) $ax^2+bx+c = u^2 \mp p$ $\quad s=sbt$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \int \frac{du}{\sqrt{u^2 \mp p}} = \ln\left|u+\sqrt{u^2 \mp p}\right|+c$$

$$\int \frac{dx}{\sqrt{-x^2+2x+3}}$$

$$-x^2+2x+3 = -\left[x^2-2x-3\right] = -\left[x^2-2x+1-4\right]$$

$$= -\left[(x-1)^2-4\right] = 4-(x-1)^2$$

$$= 2^2-(x-1)^2$$

$$= \int \frac{dx}{\sqrt{2^2-(x-1)^2}}$$

$$\boxed{\begin{aligned} x-1 &= u \\ dx &= du \end{aligned}}$$

$$= \int \frac{du}{\sqrt{2^2-u^2}}$$

$$= \arcsin\frac{u}{2}+c$$

$$= \arcsin\left(\frac{x-1}{2}\right)+c$$

$$\int \frac{dx}{\sqrt{4x^2+4x+3}} \qquad 4x^2+4x+3 = (2x+1)^2+2$$

$$= \int \frac{dx}{\sqrt{(2x+1)^2+2}} \qquad \boxed{\begin{array}{l} 2x+1=u \\ 2dx=du \end{array}}$$

$$= \int \frac{du/2}{\sqrt{u^2+2}} = \frac{1}{2}\ln\left|u+\sqrt{u^2+2}\right|+c$$

$$= \frac{1}{2}\ln\left|2x+1+\sqrt{(2x+1)^2+2}\right|+c$$

$$\int \frac{dx}{\sqrt{x^2-2x+3}} \qquad x^2-2x+3 = (x-1)^2+2$$

$$= \int \frac{dx}{\sqrt{(x-1)^2+2}} \qquad \boxed{\begin{array}{l} x-1=u \\ dx=du \end{array}}$$

$$= \int \frac{du}{\sqrt{u^2+2}} = \ln\left|u+\sqrt{u^2+2}\right|+c$$

$$= \ln\left|x-1+\sqrt{x^2-2x+3}\right|+c$$

$$\int \frac{dx}{\sqrt{21-8x-4x^2}}$$

$$-\left[4x^2+8x-21\right] = (2x+2)^2 - 25$$

$$= \int \frac{dx}{\sqrt{5^2-(2x+2)^2}}$$

$$\boxed{\begin{array}{l} 2x+2=u \\ 2dx=du \end{array}}$$

$$= \int \frac{du/2}{\sqrt{5^2-u^2}} = \frac{1}{2}\arcsin\left(\frac{u}{5}\right)+c$$

$$= \frac{1}{2}\arcsin\left(\frac{2x+2}{5}\right)+c$$

$$\int \frac{dx}{\sqrt{x^2-4x+5}}$$

$$x^2-4x+5 = (x-2)^2+1$$

$$= \int \frac{dx}{\sqrt{(x-2)^2+1}}$$

$$\boxed{\begin{array}{l} x-2=u \\ dx=du \end{array}}$$

$$= \int \frac{du}{\sqrt{u^2+1}} = \ln\left|u+\sqrt{u^2+1}\right|+c$$

$$= \ln\left|x-2+\sqrt{x^2-4x+5}\right|+c$$

$$\int \frac{dx}{\sqrt{4x^2-4x+5}}$$

$$4x^2-4x+5 = (2x-1)^2+4$$

$$= \int \frac{dx}{\sqrt{(2x-1)^2+4}}$$

$$\boxed{\begin{array}{l} 2x-1=u \\ 2dx=du \end{array}}$$

$$= \int \frac{du/2}{\sqrt{u^2+4}} = \frac{1}{2}\ln\left|u+\sqrt{u^2+4}\right|+c$$

$$= \frac{1}{2}\ln\left|2x-1+\sqrt{4x^2-4x+5}\right|+c$$

(2) $\int \frac{mx+n}{\sqrt{ax^2+bx+c}}\,dx$

$ax^2+bx+c=t$

$(2ax+b)\,dx=dt$

ile

$(mx+n)\,dx$ orantı bağlantı

$\int \frac{3x+2}{\sqrt{x^2+4x+1}}\,dx$

$x^2+4x+1=t$

$(2x+4)\,dx=dt$

$3x+2 = 3\left[x+\frac{2}{3}\right]$

$= \frac{3}{2}\left[2x+\frac{4}{3}\right]$

$= \frac{3}{2}\left[2x+4+\frac{4}{3}-4\right]$

$= \frac{3}{2}\left[2x+4-\frac{8}{3}\right]$

$= \frac{3}{2}\left[2x+4\right]-4$

$= \int \frac{\frac{3}{2}(2x+4)-4}{\sqrt{x^2+4x+1}}\,dx = \frac{3}{2}\int \frac{2x+4}{\sqrt{x^2+4x+1}}\,dx - 4\int \frac{1}{\sqrt{x^2+4x+1}}\,dx$

1 2

(1) $\frac{3}{2}\int \frac{2x+4}{\sqrt{x^2+4x+1}}\,dx = \frac{3}{2}\int \frac{dt}{\sqrt{t}} = \frac{3}{2}\,\frac{t^{1/2}}{1/2}$

$= 3\sqrt{t} = 3\sqrt{x^2+4x+1}$

(2) $-4\int \frac{dx}{\sqrt{x^2+4x+1}}$

$x^2+4x+1 = (x+2)^2-3$

$-4\int \frac{1}{\sqrt{(x+2)^2-3}}\,dx$

$x+2=u$
$dx=du$

$-4\int \frac{du}{\sqrt{u^2-3}} = -4\ln\left[u+\sqrt{u^2-3}\right] = -4\ln\left[x+2+\sqrt{x^2+4x+1}\right]$

cevap $= 3\sqrt{x^2+4x+1} - 4\ln\left[x+2+\sqrt{x^2+4x+1}\right] + c$

$\int \frac{x+3}{\sqrt{x^2+2x+2}}\,dx$

$x^2+2x+2 = t$

$(2x+2)\,dx = dt \quad = \quad 2(x+1)\,dx = dt$

$x+3 = \frac{1}{2}[2x+6]$

$= \frac{1}{2}[2x+2+6-2]$

$x+3 = x+1+2$

$= \int \frac{(x+1)+2}{\sqrt{x^2+2x+2}}\,dx = \int \frac{(x+1)\,dx}{\sqrt{x^2+2x+2}} + 2\int \frac{dx}{\sqrt{x^2+2x+2}}$

(1) (2)

(1) $\int \frac{dt/2}{\sqrt{t}} = \frac{1}{2}\int t^{-1/2}\,dt = \frac{1}{2}\,\frac{t^{1/2}}{1/2} = \sqrt{t} = \sqrt{x^2+2x+2}$

(2) $2\int \frac{dx}{\sqrt{x^2+2x+2}} = 2\int \frac{dx}{\sqrt{(x+1)^2+1}}$

$x+1=v$
$dx=dv$

$= 2\int \frac{dv}{\sqrt{v^2+1}} = 2\left(\ln\left[v+\sqrt{v^2+1}\right]\right) = 2\left(\ln\left[x+1+\sqrt{x^2+2x+1}\right]\right)$

Cevap: $\sqrt{x^2+2x+1} + 2\ln\left[x+1+\sqrt{x^2+2x+1}\right]+c$

(3) $\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$

$\frac{1}{px+q} = t$

~~$\int \frac{dx}{(x-1)\sqrt{x^2+3}}$~~

$\frac{1}{x-1} = t$

$\frac{1}{t} = x-1$

$\frac{t}{t}+1 = x$

$\frac{t+1}{t} = x$

$x^2 = \frac{t^2+2t+1}{t^2}$

$x^2+3 = \frac{4t^2+2t+1}{t^2}$

$\frac{-1}{t^2}dt = dx$

$= \int \frac{t\,\frac{-1}{t^2}\,dt}{\sqrt{\frac{4t^2+2t+1}{t^2}}} = \int \frac{\frac{-dt}{t^2}}{\frac{\sqrt{4t^2+2x+1}}{t}}$

$= \int \frac{-dt}{\sqrt{4t^2+2t+1}}$

$2t^2-2t+2t^2$

HAZİRAN						2012	
H	P	S	Ç	P	C	Ct	Pz
22					1	2	3
23	4	5	6	7	8	9	10
24	11	12	13	14	15	16	17
25	18	19	20	21	22	23	24
26	25	26	27	28	29	30	

HAZİRAN *JUNE*

SALI *TUESDAY*

19

$$= \int \frac{-dt}{\sqrt{\left(2t+\frac{1}{2}\right)^2+\frac{3}{4}}} = \int \frac{-dv/2}{\sqrt{v^2+3/4}} = -\frac{1}{2}\ln\left[v+\sqrt{v^2+\frac{3}{4}}\right]+c$$

$2t+\frac{1}{2}=v$

$2dt=dv$

$$= -\frac{1}{2}\ln\left[2t+\frac{1}{2}+\sqrt{4t^2+2t+1}\right]+c$$

$$= -\frac{1}{2}\ln\left[\frac{2}{x-1}+\frac{1}{2}+\sqrt{4\left(\frac{1}{x-1}\right)^2+\frac{2}{x-1}+1}\right]+c$$

$$\int \frac{dx}{(x+1)\sqrt{x^2-2x+2}}$$

$\frac{1}{x+1}=t$

$\frac{1}{t}=x+1$

$\frac{1}{t}-1=x$

$\frac{1-t}{t}=x$

$\frac{-1}{t^2}dt=dx$

$x^2=\frac{t^2+2t+1}{t^2}=\frac{1-2t+t^2}{t^2}$

$-2x=\frac{2t-2}{t}$

$+2$

$$= \int \frac{t\ \frac{-dt}{t^2}}{\sqrt{\frac{t^2+2t+1}{t^2}+\frac{2t-2}{t}+2}} = \int \frac{-dt}{\sqrt{5t^2-4t+1}}$$

$$= \int \frac{-dt}{\sqrt{\left(\sqrt{5}t-\frac{2}{\sqrt{5}}\right)^2+\frac{1}{5}}} = \int \frac{-dv/\sqrt{5}}{\sqrt{v^2+1/5}}$$

$\sqrt{5}t-\frac{2}{\sqrt{5}}=v$

$\sqrt{5}dt=dv$

$$= \frac{-1}{\sqrt{5}}\ln\left[v+\sqrt{v^2+1/5}\right]+c$$

$$= -\frac{1}{\sqrt{5}}\ln\left[\sqrt{5}t-\frac{2}{\sqrt{5}}+\sqrt{5t^2-4t+1}\right]$$

$t=\frac{1}{x+1}$

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$x^2, x^3 \ldots$ — Bir derece düşüşü — Ax^2+Bx+C … Payda

(4) $$\int \frac{P_n(x)}{\sqrt{ax^2+bx+c}}\,dx = Q_{n-1}(x)\sqrt{ax^2+bx+c} + \lambda \int \frac{1}{\sqrt{ax^2+bx+c}}\,dx$$

Bu format yazılır, sonra her iki tarafın türevi alınarak bilinmeyenler bulunur. Bilinmeyenler sağ tarafta yerine yazılır.

$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx = (Ax+B)\sqrt{1-x^2} + \lambda \int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\frac{x^2}{\sqrt{1-x^2}} = A\sqrt{1-x^2} + (Ax+B)\frac{-x}{\sqrt{1-x^2}} + \frac{\lambda}{\sqrt{1-x^2}}$$

$\left(\sqrt{1-x^2}\right)$

$$x^2 = A(1-x^2) - x(Ax+B) + \lambda$$

$$x^2 = A - Ax^2 - Ax^2 + Bx + \lambda$$

$$x^2 = x^2[-2A] + x[-B] + [A+\lambda]$$

$$A = -1/2 \quad B = 0 \quad \lambda = \frac{1}{2}$$

$$= \left(-\frac{1}{2}x\right)\sqrt{1-x^2} + \frac{1}{2}\int \frac{dx}{\sqrt{1-x^2}}$$

$$= \left(-\frac{1}{2}x\right)\sqrt{1-x^2} + \frac{1}{2}\arcsin x + c$$

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$(x^2+4)^{1/2} = \frac{1}{2}(x^2+4)^{-1/2}\cdot(2x)$

A $\int \frac{x^4+4x^2}{\sqrt{x^2+4}}\,dx = (Ax^3+Bx^2+Cx+D)\sqrt{x^2+4} + \lambda\int \frac{1}{\sqrt{x^2+4}}\,dx$

$\frac{x^4+4x^2}{\sqrt{x^2+4}} = (3Ax^2+2Bx+C)\sqrt{x^2+4} + (Ax^3+Bx^2+Cx+D)\frac{x}{\sqrt{x^2+4}} + \lambda\frac{1}{\sqrt{x^2+4}}$

$x^4+4x^2 = (3Ax^2+2Bx+C)(x^2+4) + (Ax^3+Bx^2+Cx+D)x + \lambda$

$x^4+4x^2 = 3Ax^4+2Bx^3+Cx^2+12Ax^2+8Bx+4C$
$+Ax^4+Bx^3+Cx^2+Dx+\lambda$

$x^4+4x^2 = x^4[4A] + x^3[3B] + x^2[12A+2C] + x[8B+D] + 4C+\lambda$

$A=1/4 \quad B=0 \quad C=1/2 \quad D=0 \quad \lambda=-2$

$= \left(\frac{x^3}{4}+\frac{x}{2}\right)\sqrt{x^2+4} - 2\int \frac{1}{\sqrt{x^2+4}}\,dx$

$= \left(\frac{x^3}{4}+\frac{x}{2}\right)\sqrt{x^2+4} - 2\ln\left[x+\sqrt{x^2+4}\right] + C$

(5) $\int \frac{dx}{(x-p)^n \sqrt{ax^2+bx+c}}$

$\boxed{\frac{1}{(x-p)} = t}$

$\int \frac{dx}{(x-1)^2 \sqrt{x^2+2x-2}}$

$\frac{1}{x-1} = t$

x^2+2x-2

$\frac{1}{t} = x-1$

$\frac{t^2+2t+1}{t^2} + \frac{2t+2}{t} - 2 = \frac{t^2+4t+1}{t^2}$

$\frac{1}{t} + 1 = x$

$\frac{t+1}{t} = x$

$\boxed{\frac{-1}{t^2} dt = dx}$

$= \int \frac{t^2 \cdot \frac{-dt}{t^2}}{\sqrt{\frac{t^2+4t+1}{t^2}}}$

$= \int \frac{-t\,dt}{\sqrt{t^2+4t+1}}$

2. yöntem

$t^2+4t+1 = v$

$(2t+4)dt = dv$

$-t = -\frac{1}{2}[2t]$

$= -\frac{1}{2}[2t+4-4]$

$= -\frac{1}{2}[2t+4] + 2$

$= \int \frac{-\frac{1}{2}[2t+4]+2}{\sqrt{t^2+4t+1}} dt$

$= -\frac{1}{2} \int \frac{2t+4}{\sqrt{t^2+4t+1}} dt + 2 \int \frac{dt}{\sqrt{t^2+4t+1}}$

① ②

① $-\frac{1}{2}\int \frac{2t+4}{\sqrt{t^2+4t+1}}\,dt = -\frac{1}{2}\int \frac{dv}{\sqrt{v}} = -\frac{1}{2}\int v^{-1/2}\,dv = -\frac{1}{2}\frac{v^{1/2}}{1/2} = -\sqrt{v}$

$= -\sqrt{t^2+4t+1}$

② $2\int \frac{dt}{\sqrt{t^2+4t+1}}$ 1. yöntem $= 2\int \frac{dt}{\sqrt{(t+2)^2-3}}$

$= 2\ln\left[t+2+\sqrt{t^2+4t+4}\right]$

genel

$= -\sqrt{t^2+4t+1} + 2\ln\left[t+2+\sqrt{t^2+4t+4}\right] + C$

$= -\sqrt{\frac{1}{(x-1)^2}+\frac{4}{x-1}+1} + 2\ln\left[\frac{1}{x-1}+2+\sqrt{\frac{1}{(x-1)^2}+\frac{4}{x-1}}\right] + C$

$\int \frac{dx}{(x+1)^2\sqrt{x^2-2x}}$

$\frac{1}{x+1} = t$

$\frac{1}{t} = x+1$

$\frac{1}{t} - 1 = x$

$\frac{1-t}{t} = x$

$-\frac{1}{t^2}\,dt = dx$

x^2-2x

$\frac{1-2t+t^2}{t^2} - 2\frac{1-t}{t}$

$= \frac{3t^2-4t+1}{t^2}$

$= \int \frac{t^2\,\frac{-1}{t^2}\,dt}{\sqrt{\frac{3t^2-4t+1}{t^2}}}$

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$$= \int \frac{t\,dt}{\sqrt{3t^2-4t+1}}$$

$3t^2-4t+1 = u$

$(6t-4)dt = du$

$-t = -\frac{1}{6}[6t]$

$= -\frac{1}{6}[6t-4+4]$

$= -\frac{1}{6}(6t-4) - \frac{2}{3}$

$$= \int \frac{-\frac{1}{6}(6t-4) - \frac{2}{3}}{\sqrt{3t^2-4t+1}}\,dt$$

$$= -\frac{1}{6}\int \frac{6t-4}{\sqrt{3t^2-4t+1}}\,dt - \frac{2}{3}\int \frac{1}{\sqrt{3t^2-4t+1}}\,dt$$

(1) $-\frac{1}{6}\int \frac{6t-4}{\sqrt{3t^2-4t+1}}\,dt = -\frac{1}{6}\int \frac{du}{\sqrt{u}} = -\frac{1}{6}\int u^{-1/2}\,du = -\frac{1}{6}\frac{u^{+1/2}}{1/2}$

$= -\frac{1}{3}\sqrt{u}$

$= -\frac{1}{3}\sqrt{3t^2-4t+1}$

(2) $-\frac{2}{3}\int \frac{dt}{\sqrt{3t^2-4t+1}} = -\frac{2}{3}\int \frac{1}{\sqrt{\left(\sqrt{3}t-\frac{2}{\sqrt{3}}\right)^2-\frac{1}{3}}}\,dt$

$\sqrt{3}t - \frac{2}{\sqrt{3}} = u$

$\sqrt{3}\,dt = du$

$= -\frac{2}{3}\int \frac{du/\sqrt{3}}{\sqrt{u^2-\frac{1}{3}}}$

$$= \frac{-2}{3\sqrt{3}} \ln\left[u + \sqrt{u^2 - \frac{1}{3}}\right]$$

$$= \frac{-2}{3\sqrt{3}} \ln\left[\sqrt{3}t - \frac{2}{\sqrt{3}} + \sqrt{3t^2 - 4t + 1}\right]$$

cevap

$$= -\frac{1}{3}\sqrt{3t^2 - 4t + 1} - \frac{2}{3\sqrt{3}} \ln\left[\sqrt{3}t - \frac{2}{\sqrt{3}} + \sqrt{3t^2 - 4t + 1}\right] + c$$

$$t = \frac{1}{x+1}$$

⑥ BİNOM INT.

$$\int x^r (a + bx^p)^q \, dx$$

ⓐ q = tam sayı ise ekok(paydа, paydа) = k olsun

$$x = t^k$$

ⓑ $q \neq$ tam sayı ise q'nun paydası n ise $\frac{r+1}{p}$ = Tam sayı ise

$$(a + bx^p) = t^n$$

ⓒ $q \neq$ tamsayı, $\frac{r+1}{p} \neq$ Tamsayı, $\frac{r+1}{p} + q$ = tamsayı ve q'nun paydası n ise

$$a x^{-p} + b = t^n$$

$$\int \sqrt[3]{x}\left(1+2\sqrt{x}\right)^2 dx = \int x^{1/3}\left(1+2x^{1/2}\right)^2 dx$$

$r=1/3 \quad p=1/2 \quad q=2$

Üçlü ile çözülür (q = tamsayı)

$\text{ekok}(2,3)=6=k$

$$x = t^k = t^6$$

$$dx = 6t^5 dt$$

$$= \int t^2\left(1+2t^3\right)^2 6t^5 dt$$

$$= 6\int t^7\left(1+2t^3\right)^2 dt$$

$$= 6\int t^7\left(1+4t^3+4t^6\right) dt$$

$$= 6\int\left(t^7+4t^{10}+4t^{13}\right) dt$$

$$= 6\left[\frac{t^8}{8}+\frac{4t^{11}}{11}+\frac{4t^{14}}{14}\right]+C$$

$$= 6\left[\frac{x^{8/6}}{8}+\frac{4x^{11/6}}{11}+\frac{4x^{14/6}}{14}\right]+C$$

$$\int \frac{\sqrt{1+\sqrt[3]{x}}}{\sqrt[3]{x^2}}\,dx = \int x^{-2/3}\left[1+x^{1/3}\right]^{1/2}dx$$

$r=-\frac{2}{3} \quad p=\frac{1}{3} \quad q=\frac{1}{2}$

$q \neq$ Tamsayı $\quad \frac{r+1}{p}=\frac{1/3}{1/3}=1 =$ tamsayı

b şıkkı ile çözülür

$n=2 \rightarrow$ q'nun paydası

paydası kare t^n'deki n

$1+x^{1/3}=t^2$

$\frac{1}{3}x^{-2/3}dx=2t\,dt$

$t=\sqrt{1+\sqrt[3]{x}}$

$$= \int 6t\,dt\,(t^2)^{1/2} = 6\int t^2\,dt = 6\,\frac{t^3}{3}+C = 2t^3+C$$

$$= 2\left(\sqrt{1+\sqrt[3]{x}}\right)^3+C$$

$$\int \sqrt{\frac{x}{1-x^3}}\,dx = \int \frac{\sqrt{x}}{\sqrt{1-x^3}}\,dx = \int x^{1/2}\left(1-x^3\right)^{-1/2}dx$$

$r=1/2 \quad p=3 \quad q=-1/2$

$\frac{r+1}{p}=\frac{3/2}{3}=1/2$

$\frac{r+1}{p}+q=0 \rightarrow$ Tamsayı

$n=2$

c şıkkına göre çözülür

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$x^{-3} - 1 = t^2 \longrightarrow x^{-3} = t^2 + 1$

$-3x^{-4}\,dx = 2t\,dt$

$dx = -\frac{2}{3} x^4 t\,dt$

$\frac{1}{x^3} = t^2 + 1 \rightarrow \frac{1}{x^3} - 1 = t^2$

$\frac{1}{t^2+1} = x^3 \quad | \quad t = \sqrt{\frac{1-x^3}{x^3}}$

$dx = -\frac{2}{3}\left(\frac{1}{t^2+1}\right)^{4/3} t\,dt \qquad \left(\frac{1}{t^2+1}\right)^{1/3} = x$

$$= \int \frac{1}{(t^2+1)^{1/6}} \left[1 - \frac{1}{t^2+1}\right]^{-1/2} \cdot \left(\frac{-2}{3}\right) \frac{t}{(t^2+1)^{4/3}}\,dt$$

$$= -\frac{2}{3} \int \frac{1}{(t^2+1)^{3/2}} \frac{t^{-1} \cdot t}{(t^2+1)^{-1/2}}\,dt$$

$$= -\frac{2}{3} \int \frac{1}{t^2+1}\,dt = -\frac{2}{3} \arctan t + C$$

$$= -\frac{2}{3} \arctan \sqrt{\frac{1-x^3}{x^3}} + C$$

TEMMUZ							2012
H	P	S	Ç	P	C	Ct	Pz
26							1
27	2	3	4	5	6	7	8
28	9	10	11	12	13	14	15
29	16	17	18	19	20	21	22
30	23	24	25	26	27	28	29
31	30	31					

4- P - %5-kuş
5- S - %35-ve
6- P - %5-kuş
7- P - %5-10 yıl
8 ? - %50-[illegible]

Listende

TEMMUZ
JULY
CUMARTESİ
SATURDAY
07

$$\int \frac{2x^4 - 3x^3 + 5}{3x^2}\,dx$$

$2x^4 - 3x^2 + 5 \mid 3x^2$

$\frac{2}{3}x^2$

$$\int \left(\frac{2}{3}x^2 + \frac{5}{3}\frac{1}{x^2}\right)dx = \frac{2}{3}\int x^3 + \frac{5}{3}\frac{x^{-1}}{-1} - \frac{x^2}{2}$$

$-x$

$$\frac{2}{3}x^3 - \frac{5}{3}x^{-1} - \frac{x^2}{2} + C$$

$$\int x^5 \ln x\,dx = \frac{x^6}{36}(A\ln x + B) + C \qquad A, B, C = ?$$

çarpım

$\ln x = u$

$\frac{1}{x}dx = du$

$x^3 dx = dv$

TEMMUZ
JULY
PAZAR
SUNDAY
08

$$\left(\frac{x^6}{36}[A\ln x + B] + C\right)' = x^5 \ln x$$

$$\frac{6x^5}{36}(A\ln x + B) + \frac{x^6}{36}\frac{A}{x} + 0 = x^5 \ln x$$

$$\ln x = \frac{A\ln x + B}{6} + \frac{A}{36}$$

$$\ln x = \frac{A}{6}\ln x + \frac{B}{6} + \frac{A}{36}$$

$A = 6$

$B = -1$

$C \in \mathbb{R}$

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★ Türevi kendisinin N'i katının eşit olan pozitif bir f fonksiyonu için $f(0)=1$ dir. göre $f(x)=?$

$$f'(x)=2f(x)$$

$$\frac{f'(x)}{f(x)}=2$$

$$=\int \frac{f'(x)}{f(x)}dx=\int 2\,dx$$

$f(x)=u$

$f'(x)dx=du$

$$=\int \frac{du}{u}=2x+c$$

$$\ln|u|=2x+c$$

$$\ln|f(x)|=2x+c$$

$$\ln f(x)=2x+c \quad \rightarrow \text{pozitif}$$

$$f(x)=e^{2x+c}$$

$$f(0)=1=e^{c} \Rightarrow c=0$$

$$\boxed{f(x)=e^{2x}}$$

$$\int \frac{x^3}{1+x^8}\,dx$$

obek (3, 8)

$2^3 ; 3$

$1+x^8 = t^2$

$8x^7\,dx = 2t\,dt$

$x^4 = t$

$4x^3\,dx = dt$

$$= \frac{1}{4}\int \frac{dt}{1+t^2}$$

$$= \frac{1}{4}\arctan t + c$$

$$= \frac{1}{4}\arctan x^4 + c$$

$$\int \sin\left((2x+1)\pi\right)\,dx$$

toplus

$(2x+1)\pi = t$

$2\pi\,dx = dt$

$$= \int \sin t \cdot \frac{dt}{2\pi}$$

$$= -\frac{1}{2\pi}\cos t + c$$

$$= -\frac{1}{2\pi}\cos\left((2x+1)\pi\right) + c$$

$$\int \frac{x\,dx}{\sqrt{x+1}}$$

II. yol
+1 -1
ekle

$x+1 = t^2$

$dx = 2t\,dt$

$$= \int \frac{(t^2-1)\,2t\,dt}{t}$$

$$= 2\int (t^2-1)\,dt$$

$$= 2\,\frac{t^3}{3} - 2t + c$$

$$= \frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + c$$

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$e^x - e^x/\ln e$

$\sqrt{a^2-x^2} \to a\sin t$

$\sqrt{x^2-a^2} \to a\sec t$

$\sqrt{a^2+x^2} \to a\tan t$

11

TEMMUZ
JULY

ÇARŞAMBA
WEDNESDAY

TEMMUZ						2012	
H	P	S	Ç	P	C	Ct	Pz
26							1
27	2	3	4	5	6	7	8
28	9	10	11	12	13	14	15
29	16	17	18	19	20	21	22
30	23	24	25	26	27	28	29
31	30	31					

$$\int \frac{dx}{x^2\sqrt{16-x^2}}$$

$\frac{1}{x^2} = t \to x = \frac{1}{\sqrt{t}}$

$x^2 = t$

$-2x^{-3}\,dx = dt$

$-2\,dx = x^3\,dt$

$dx = \frac{x^3}{-2}\,dt$

$\left(\frac{1}{t}\right)^{3/2}$

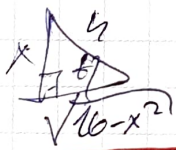
$x = 4\sin t$

$dx = 4\cos t\,dt$

$$I = \int \frac{4\cos t}{16\sin^2 t\sqrt{16-16\sin^2 t}}\,dt$$

$$= \frac{1}{16}\int \frac{4\cos t}{4\cos t}\,dt = \frac{1}{16}t + C$$

$$= -\frac{1}{16}\cot t + C = -\frac{1}{16}\cdot\frac{\sqrt{16-x^2}}{x} + C$$



$$\int \frac{2^x\,dx}{\sqrt{1-4^x}}$$

$2^x = t$

$2^x \ln 2\,dx = dt$

$$= \frac{1}{\ln 2}\int \frac{dt}{\sqrt{1-t^2}}$$

$$= \frac{1}{\ln 2}\arcsin t + C$$

$$= \frac{1}{\ln 2}\arcsin(2^x) + C$$

$$\int \frac{e^{2x}}{\sqrt{e^x+1}}\,dx \qquad 1+e^x = t^2$$

$$e^x dx = 2t\,dt$$

$$= \int \frac{(t^2-1)\,2t\,dt}{\sqrt{t^2}} = 2\int (t^2-1)\,dt \;\; 2\left(\frac{t^3}{3}-t\right)+c$$

$$= \frac{2}{3}\left(e^x+1\right)^{3/2} - \sqrt{e^x+1}+c$$

$$\int \frac{1}{1+\cos x}\,dx$$

$$\tan\frac{x}{2} = t$$

$$\frac{x}{2} = \arctan t$$

$$x = 2\arctan t$$

$$dx = 2\,\frac{1}{1+t^2}\,dt$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$= \int \frac{1}{1+\frac{1-t^2}{1+t^2}}\,\frac{2}{1+t^2}\,dt$$

$$= \int \frac{1+t^2}{2}\cdot\frac{2}{1+t^2}\,dt$$

$$= t+c = \tan\frac{x}{2}+c$$

$$1+t^2+1-t^2 = 2$$

$$\frac{2}{1+t^2}$$

$$\int \frac{1}{1+\left[2\cos^2\left(\frac{x}{2}\right)-1\right]}\,dx = \int \frac{dx}{2\cos^2\left(\frac{x}{2}\right)}$$

$$\frac{x}{2} = t$$

$$\frac{dx}{2} = dt$$

3.

$1+2t^2$

$$\int \frac{dx}{2+\sin x-\cos x}$$

$\tan\frac{x}{2}=t$

$$=\int \frac{\frac{2}{1+t^2}dt}{2+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}} = \int \frac{\frac{2}{1+t^2}}{\frac{3t^2+2t+1}{1+t^2}}dt$$

$$=\int \frac{2}{3t^2+2t+1}dt$$

$t^2+2t+3=u$

$2t+2$

$$\frac{2}{t^2+2t+3}=\frac{A}{t+1}+\frac{B}{t+2}$$

$$=\int\left(\frac{2}{t+1}+\frac{2}{t+2}\right)dt$$

$At+2A+Bt+B=2$

$A+B=0$

$2A+B=2$

$A=2 \quad B=-2$

$$=\ln|t+1|-\ln|t+2|+C$$

$$=\ln\left|\frac{\tan\frac{x}{2}+1}{\tan\frac{x}{2}+2}\right|+C$$

$$\int \frac{A\,dx}{a^2+x^2}=A\frac{1}{a}\arctan\frac{x}{a}+C$$

$$=2\int \frac{dt}{\left(\sqrt{3}t+\frac{1}{\sqrt{3}}\right)^2+\frac{2}{3}}$$

$\sqrt{3}t+\frac{1}{\sqrt{3}}=u$

$\sqrt{3}dt=du$

$$=2\int \frac{\frac{du}{\sqrt{3}}}{u^2+\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2} = \frac{2}{\sqrt{3}}\cdot\frac{1}{\frac{\sqrt{2}}{\sqrt{3}}}\arctan\frac{u}{\frac{\sqrt{2}}{\sqrt{3}}}+C$$

$$=\sqrt{2}\arctan\frac{\sqrt{3}\left(\sqrt{3}t+\frac{1}{\sqrt{3}}\right)}{\sqrt{2}}$$

$= \sqrt{2}\arctan\frac{3t+1}{\sqrt{2}} + C$

$= \sqrt{2}\arctan\frac{3(\tan\frac{x}{2})+1}{\sqrt{2}} + C$

$\int x^2\cos x\,dx$ — Laplace

$x^2 = u \quad | \quad \cos x\,dx = dv$
$2x\,dx = du \quad | \quad \sin x = v$

$= x^2\sin x - \int (\sin x)\cdot 2x\,dx$

$x = u \quad | \quad \sin x\,dx = dv$
$dx = du \quad | \quad -\cos x = v$

$= x^2\sin x - 2\left[-x\cos x + \int \cos x\,dx\right]$

$= x^2\sin x + 2x\cos x - 2\sin x + C$

$\int \underbrace{\cos(\ln x)}_{u}\underbrace{dx}_{dv}$

$\ln x = t$
$\frac{dx}{x} = dt$

$\cos(\ln x) = u$
$\frac{1}{x}(-\sin(\ln x))\,dx = du$
$dx = dv$
$x = v$

$= (\cos(\ln x))x - \int x\frac{1}{x}(-\sin(\ln x))\,dx$

$\sin(\ln x) = u$
$\frac{1}{x}(\cos(\ln x))\,dx = du$

$= x\cos(\ln x) + \int \sin(\ln x)\,dx$

$= x\cos(\ln x) + x\sin(\ln x) - \int x\frac{1}{x}\cos(\ln x)\,dx$

$I = \frac{1}{2}\left[x(\cos(\ln x) + \sin(\ln x))\right] + C$

$(\log_a u)' = \frac{u'}{u} \log_a e$

$\int \log_5 x \, dx$

$\log_5 x = u$ | $dx = dv$

$\left(\frac{1}{x} \log_5 e\right) dx = du$ | $x = v$

$= (\log_5 x) x - \int x \frac{1}{x} \log_5 e \, dx$

$= x \log_5 x - x \log_5 e + C$

$\int sm^n dx = \int \underbrace{sm^{n-1} x}_{u} \underbrace{sm x \, dx}_{dv}$

$sm^{n-1} x = u$ | $sm x \, dx = dv$

$(n-1) \cos^{n-2} x \, dx = du$ | $-\cos x = v$

$-sm^{n-1} x \cos x + \int \cos x \, (n-1) \cos^{n-2} x \, dx$

$$\int \frac{dx}{x^2+x-6} = ?$$

$$\frac{1}{x^2+x-6} = \frac{A}{x+3} + \frac{B}{x-2}$$

$$Ax - 2A + Bx + 3B = 1$$

$$A+B=0$$

$$-2A+3B=1$$

$$B = 1/5 \quad A = -1/5$$

$$-\frac{1}{5}\ln|x+3| + \frac{1}{5}\ln|x-2| + c$$

$$\int \frac{x^4}{x^4-1}\,dx$$

$$\frac{x^4}{x^4-1} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1} + \frac{D}{x-1}$$

$$(x^2+1)(x+1)(x-1)$$

$$(x-1)(x^2+1) \qquad (x+1)(x^2+1)$$

$$D = 1/4 \quad C = -1/4$$

$$A = 0 \quad B = 1/2$$

$$\int \left(1 + \frac{1}{x^4-1}\right)dx$$

$$= x + \frac{1}{4}\ln|x-1| + \frac{1}{4}\ln|x+1| + \frac{1}{2}\arctan x + c$$

$$\int \cos 2x \cos 3x\,dx$$

$$\cos ax \cos bx = \frac{1}{2}\left[\cos(a+b)x + \cos(a-b)x\right]$$

$$= \frac{1}{2}\int (\cos 5x + \cos x)\,dx$$

$$= \frac{1}{2}\left[\frac{1}{5}\sin 5x + \sin x\right] + c$$

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$\int \tan^3 x \sec^4 x \, dx$

$\tan x = t$
$\sec^2 x \, dx = dt$
$1 + \tan^2 x = \sec^2 x$

$= \int \tan^3 x \sec^2 x \sec^2 x \, dx$

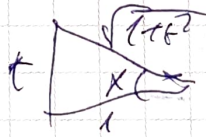
$= \int t^3 (1 + t^2) \, dt$

$= \int (t^3 + t^5) \, dt = \frac{t^4}{4} + \frac{t^6}{6} + c = \frac{\tan^4 x}{4} + \frac{\tan^6 x}{6} + c$

$\int \frac{dx}{(\sin x + \cos x)^2}$

($\tan x = t$ veya $\tan \frac{x}{2} = t$ denenebilir)

$\tan x = t$
$x = \arctan t$
$dx = \frac{1}{1+t^2} \, dt$



$= \int \frac{\frac{1}{1+t^2} \, dt}{\left(\frac{t}{\sqrt{1+t^2}} + \frac{1}{\sqrt{1+t^2}}\right)^2}$

$= \int \frac{1}{(t+1)^2} \, dt$

$t + 1 = v$
$dt = dv$

$= \int v^{-2} \, dv$

$= -v^{-1} + c = -\frac{1}{t+1} + c = -\frac{1}{\tan x + 1} + c$

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$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \int x^{-1/2}\left(1+x^{1/4}\right)^{1/3}dx$$

$$\left(1+x^{1/4}\right) = t^3$$

$$\frac{1}{4}x^{-3/4}dx = t^2\,dt$$

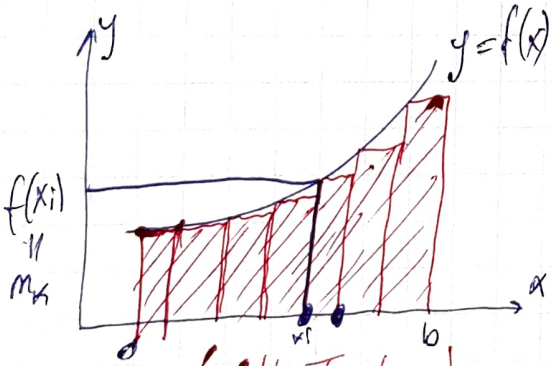
$$dx = 4t^2 x^{3/4}\,dt$$

$$dx = (t^3-1)^3\,4t^2\,dt$$

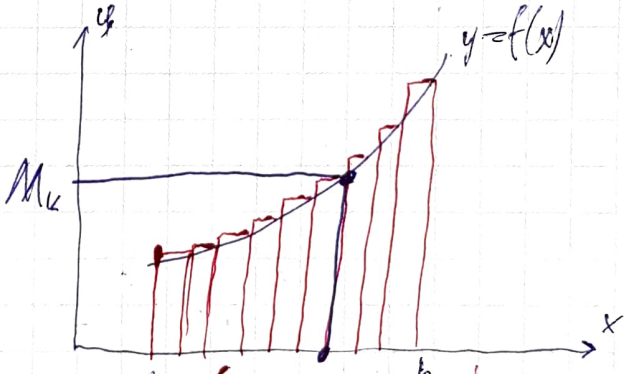
$$x^{1/4} = t^3-1$$

$$x = (t^3-1)^4 \quad , \quad x^{-1/2} = (t^3-1)^{-2}$$

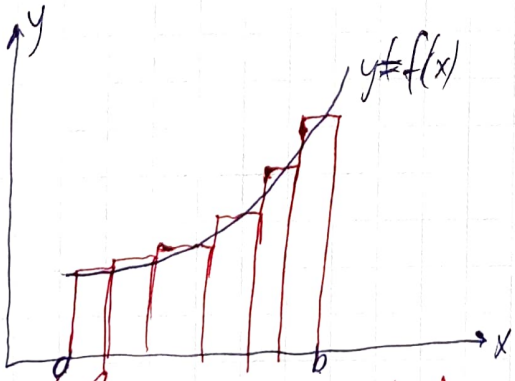
BELİRLİ İNTEGRAL



(Alt Toplam)



(Üst Toplam)



(Riemann Toplamı)

Tanım: $f:[a,b]\to\mathbb{R}$ fonksiyon sınırlı olsun $[a,b]$ aralığının $P=\{x_0,\ldots,x_n\}$ parçalanışı için

$M_k = \max\{f(x),\ x_{k-1}\le x\le x_k\}$

$m_k = \min\{f(x),\ x_{k-1}\le x\le x_k\}$

olsun →

$$A(f,P) = \sum_{i=1}^{n} \Delta x_i M_i$$

$$\Delta x_4 = x_4 - x_3$$

$$\bar{U}(f,P) = \sum_{i=1}^{n} \Delta x_i M_i$$

$$R(f,P) = \sum_{i=1}^{n} \Delta x_i f(x_i^*)$$

f fonksiyonunun P parçalanışa göre Reimann toplamı denir

Tanım: f, $[a,b]$ üzerinde tanımlı sınırlı bir fonksiyon olsun ve

$$x_k^* \in [x_{k-1}, x_k]$$

$$\lim_{\|P\| \to 0} \sum_{i=1}^{n} f(x_i^*) \Delta x_i = I$$

limiti varsa, bu limite a'dan b'ye kadar f fonksiyonunun integrali denir ve $\int_a^b f(x)dx$ ile gösterilir

Teorem: f $[a,b]$ aralığı integrallenebilir.

$\forall x \in (a,b)$ için $F'(x) = f(x)$ ise

$$\int_a^b f(x)\,dx = f(x)\Big|_a^b = f(b) - f(a)$$

(Newton-Leibnitz)

$$\int_1^2 2x\,dx = x^2\Big|_1^2 = 2^2 - 1^2 = 3$$

$$\int_0^{\pi/2} \cos x\,dx = \sin x\Big|_0^{\pi/2} = 1$$

Belirli İnt Özellikleri

1. f türevi sınırlı ise integrallenebilir.
2. Sürekli ise integrallenebilir.
3. Monoton ise integrallenebilir
4. $\int_a^a f(x)\,dx = 0$
5. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
6. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \dots = \int_a^b f(u)\,du$

(7) $c \in [a,b]$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$$\int_0^{\pi/2} \sin^{3/2}x \cos x \, dx$$

$\sin x = t$

$\cos x \, dx = dt$

$$= \int_0^{\pi/2} t^{3/2} dt$$

$$= \frac{t^{5/2}}{5/2}\Big|_0^{\pi/2} = \frac{2}{5} t^{5/2}\Big|_0^{\pi/2} = \frac{2}{5}\sin^{5/2}x\Big|_0^{\pi/2} = \frac{2}{5}$$

(8) $$\int_a^b f(x)dx = uv\Big|_a^b - \int_a^b v\,du$$

$$\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x} dx$$

$\sin x = t$

$\cos x \, dx = dt$

$$= \int_0^{\pi/2} \frac{dt}{1+t^2} = \arctan t\Big|_0^{\pi/2}$$

$x = 0 \to \sin 0 = 0$

$x = \pi/2 \to \sin \pi/2 = 1$

$$= \arctan t\Big|_0^1$$

$$= \arctan 1 - \arctan 0$$

$$= \pi/4 - 0 = \pi/4$$

Lagrange

$$\int_0^{\pi} x\cos x\,dx$$

$x = v$ $\quad \cos x\,dx = dv$

$dx = dv$ $\quad \sin x = v$

$$x\sin x\Big|_0^{\pi} - \int_0^{\pi} \sin x\,dx$$

$$(\pi \sin \pi) - [1 - 1] = -2$$

---

(9) $f(x) \geq 0 \rightarrow \int_a^b f(x)dx \geq 0$

(10) $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$

(11) ~~f fonk tek ise~~

$f(-x) = -f(x)$ tek

$f(-x) = +f(x)$ çift

$f(-x) = x^2$ çift

$f(-x) = -x^3$ tek

$f(x) = \frac{x^2}{1+x^4}$

$f(-x) = \frac{x^2}{1+x^4}$

çift

$f(x) = x^3 + 1$

$f(-x) = -x^3 + 1$

Ne Tek Ne çift

f tek ise $\int_{-a}^{a} f(x)dx = 0$

f çift ise $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

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$$\int_{-\pi/2}^{\pi/2} \cos^2 x\,dx$$

$f(x) = \cos x$

$f(-x) = \cos x$

$$= 2\int_{0}^{\pi/2} \cos^2 x\,dx$$

$$= 2\int_{0}^{\pi/2} \frac{1}{2}\left[\cos 2x + 1\right] dx$$

$$= \frac{1}{2}\sin 2x + x \Big|_0^{\pi/2} = \left(\frac{1}{2}\sin\pi + \frac{\pi}{2}\right) - \left(\frac{1}{2}\sin 0 + 0\right)$$

$$= \pi/2$$

$$\int_{-\pi}^{\pi} \frac{x^3 \cos x}{1+\sin^{10} x}\,dx = 0$$

13) İntegralin Toplamı

f, $[a,b]$ integrallenebilen fonk.

$$F(x) = \int_{v(x)}^{u(x)} f(t)\,dt$$

$$F'(x) = \frac{d}{dt}\int_{v(x)}^{u(x)} f(t)\,dt = f\left(u(x)\right)u'(x) - f\left(v(x)\right)v'(x)$$

(Leibniz)

$F(x) = \int_x^{x^2} \sin(t^2)\,dt$ $\quad F'(x) = ?$

$= \sin(x^4) \cdot 2x - \sin(x^2) \cdot 1$

$F(x) = \int_5^x \tan t\,dt$ $\quad F'(x) = ?$

$= \tan x \cdot 1 - \tan 5 \cdot 0$

$= \tan x$

$\lim_{x\to 0} \frac{1}{x^2} \int_0^x \frac{t^2}{t^4+1}\,dt = ?$

$= \frac{1}{0} \cdot \int_0^0 (\,)\,dt = \infty \cdot 0$ Belirsizlik!

$\frac{0}{0}$ veya $\frac{\infty}{\infty}$ çıkar

L'Hospital ile çöz

$= \lim_{x=0} \frac{\int_0^x \frac{t^2}{t^4+1}\,dt}{x^2} = \frac{0}{0}$ (Türev al / Türev al)

$= \lim_{x\to 0} \frac{\frac{x^2}{x^4+1}}{2x}$

$= \lim_{x\to 0} \frac{x}{2(x^4+1)} = \frac{0}{2} = 0$

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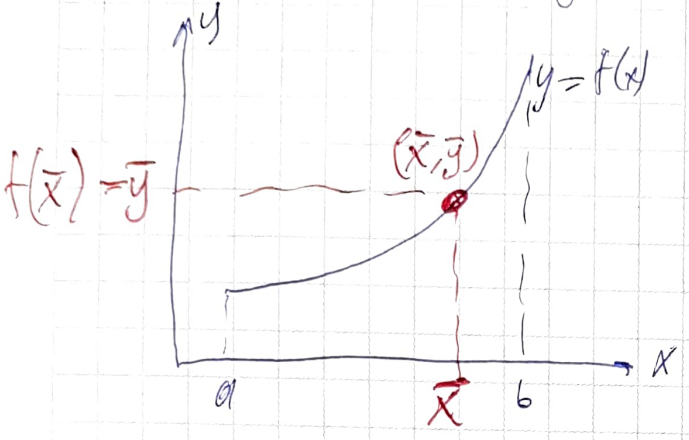
Tanım: (Ortalama Değer) = Aritmetik Mekez

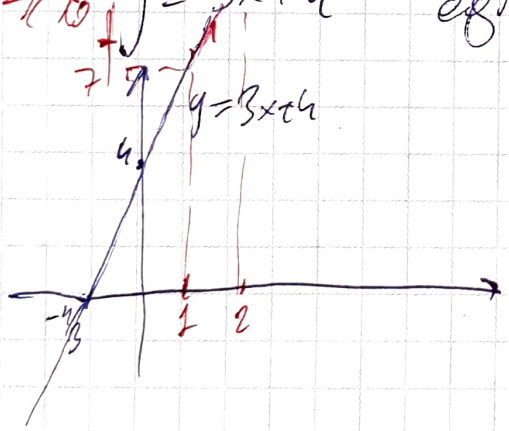


f, [a,b] aralığında integrallenebilir bir fonksiyon olsun. y = f(x) fonksiyonunun [a,b] aralığındaki ortalama değeri

$$\bar{y} = \frac{1}{b-a}\int_a^b f(x)\,dx \quad , \quad f(\bar{x}) = \bar{y}$$

→ Bu eşitliğe ortalama değer teoremi denir.

Ör: $y = 3x + 4$ eğrisinin [1,2] aralığındaki ort. değeri



$$\bar{y} = \frac{1}{2-1}\int_1^2 (3x+4)\,dx$$

$$\bar{y} = 3\frac{x^2}{2} + 4x \Big|_1^2$$

$$\bar{y} = (6+8) - \left(\frac{3}{2} + 4\right)$$

$$\bar{y} = 17/2 \cong 8{,}5$$

$$\bar{y} = 3\bar{x} + 4$$

$$17/2 = 3\bar{x} + 4 \qquad \bar{x} = 3/2$$

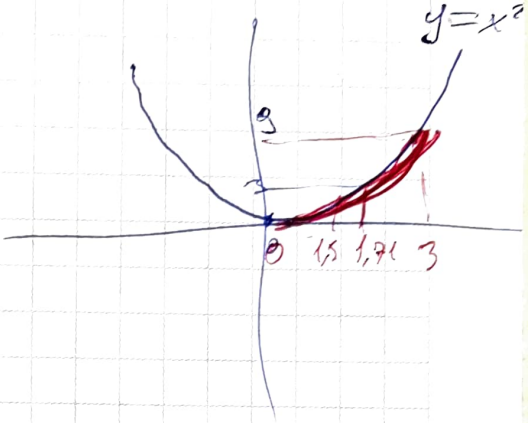
$y = x^2$, $[0, 3]$ eğrisi ort

$$\bar{y} = \frac{1}{3-0}\int_0^3 x^2\,dx$$

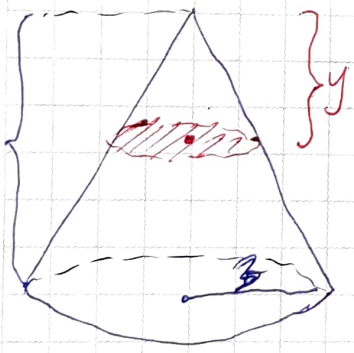
$$\bar{y} = \frac{1}{3}\left(\frac{x^3}{3}\right)\Big|_0^3$$

$$\bar{y} = 3$$

$$\bar{x} = \sqrt{3} \cong 1{,}71$$



Taban yarıçapı = 3 yüksekliği = 12 dik dairesel koni. Tepesinden paralel y birim uzaklık kesiliyor. Dairesel kesitin alanı = ?



$$\bar{y} = \frac{1}{12}\int_0^{12} \pi r^2\,dy$$

$$\bar{y} = \frac{\pi}{12}\left(\frac{r^3}{3}\right)\Big|_0^{12}$$

$$\bar{y} = 48\pi$$

$$\pi r^2 = 48\pi$$

$$r = 48$$

$A = \pi r^2$

$\frac{r}{3} = \frac{y}{12}$

$r = \frac{y}{4}$

$A = \pi \frac{y^2}{16}$

$$\bar{y} = \frac{1}{12}\int_0^{12} \frac{1}{16}\pi y^2\,dy$$

$$= \frac{1}{12}\frac{1}{16}\pi\left[\frac{y^3}{3}\right]\Big|_0^{12}$$

$$= 3\pi$$

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$$\int_{-1}^{2} |x|\,dx = \int_{0}^{2} |x|\,dx + \int_{-1}^{0} |x|\,dx$$

$$|x| = \begin{cases} -x, & x<0 \\ x, & x\geq 0 \end{cases}$$

$$= \int_{-1}^{0} -x\,dx + \int_{0}^{2} x\,dx$$

$$= \frac{-x^2}{2}\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_{0}^{2}$$

$$= (0) - \left(-\frac{1}{2}\right) + (2) - (0)$$

$$= 5/2$$

$$\int_{0}^{3} [\![x]\!]\,dx$$

Kendinden büyük olmayan en büyük sayı (Tam değer)

$[\![3,9]\!] = 3$

$[\![3,2]\!] = 3$

$[\![3]\!] = 3$

$[\![-2,4]\!] = -3$

$[\![-2,7]\!] = -3$

$[\![4,2]\!] = 4$

$$= \int_{0}^{1} 0\,dx + \int_{1}^{2} 1\,dx + \int_{2}^{3} 2\,dx$$

$$= x\Big|_{1}^{2} + 2x\Big|_{2}^{3}$$

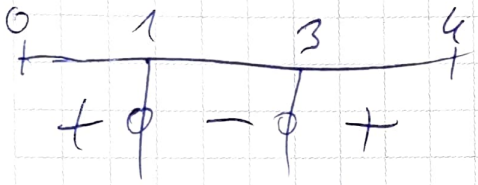
$$= 1 + 2 = 3$$

$$\int_0^2 x[x]\,dx$$

$$= \int_0^1 x \cdot 0\,dx + \int_1^2 x \cdot 1\,dx = 3/2$$

$$\int_0^4 \operatorname{sgn}(x^2-4x+3)\,dx$$

$$\operatorname{sgn}(f) = \begin{cases} 1, & f>0 \\ 0, & f=0 \\ -1, & f<0 \end{cases}$$



$$= \int_0^1 1\,dx + \int_1^3 -1\,dx + \int_3^4 1\,dx$$

$f(x) = 2|x|$, $[-1, 1]$ aralığı ort bul.

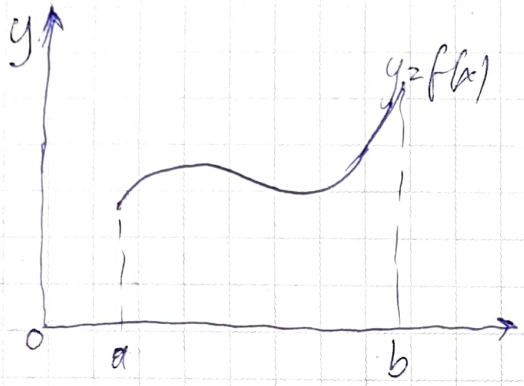
$$\bar{y} = \frac{1}{1-(-1)} \int_{-1}^{1} 2|x|\,dx$$

$$\bar{y} = \int_{-1}^{1} |x|\,dx$$

$$\bar{y} = \int_{-1}^{0} -x\,dx + \int_0^1 x\,dx$$

$$\bar{y} = 1$$

ALAN HESABI



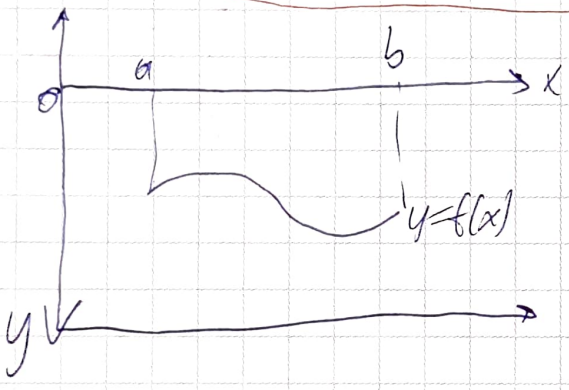
$y = f(x) \geq 0$

$x = a$

$x = b$

Ox - eksen $(y = 0)$

$$A = \int_a^b f(x)\,dx$$



$y = f(x) \leq 0$

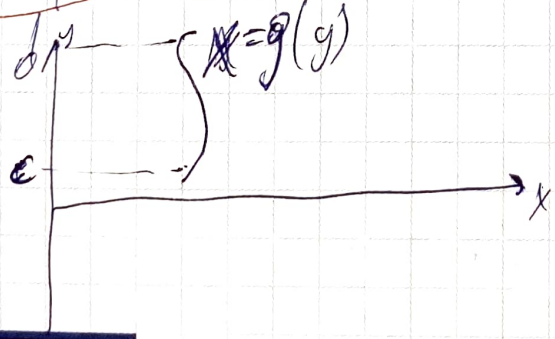
$x = a$

$x = b$

Ox - eksen $(y = 0)$

$$A = -\int_a^b f(x)\,dx$$

$$A = \int_a^b f(x)\,dx - \int_b^c f(x)\,dx$$



$x = g(y) \geq 0$

$y = c$

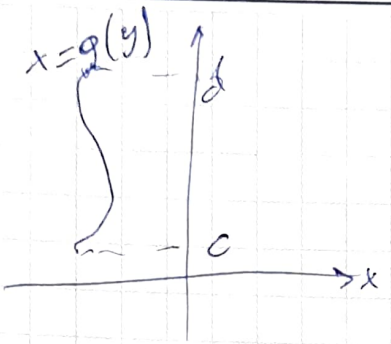
$y = d$

Oy - eksen $(x = 0)$

$$A = \int_c^d g(y)\,dy$$

$2\frac{6^3}{3} - \frac{3^2}{3}$

$9(9-1)2.3.2.2.3$



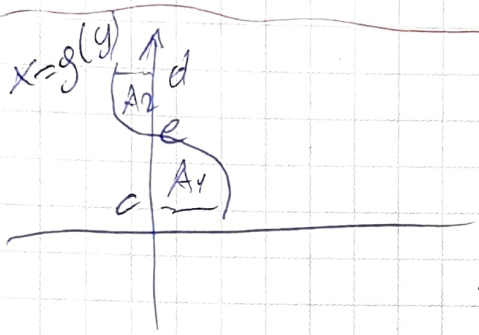
$x = g(y) \leq 0$

$y = c$

$y = d$

Oy - ekseni

$$A = -\int_c^d g(y)\,dy$$

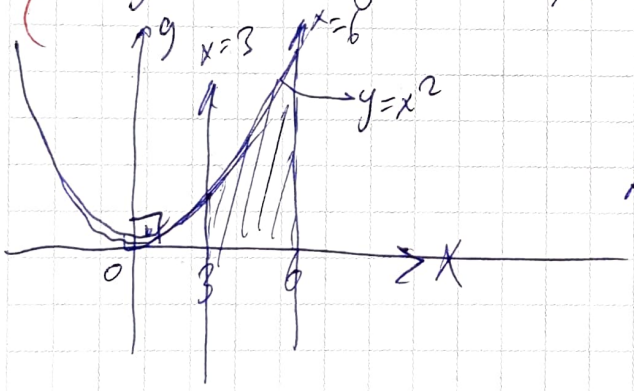


$A = A_1 + A_2$

$$A = \int_c^e g(y)\,dy - \int_e^d g(y)\,dy$$

$y = x^2$ eğrisi

$x = 3$, $x = 6$, Ox - ekseni



$$A = \int_3^6 x^2\,dx$$

$$= \frac{x^3}{3}\Big|_3^6$$

$$= 63 \text{ br}^2$$

$y = x^3 - 3x$ Ox - ek

$y = 0$

$0 = x(x^2 - 3)$

$x_1 = 0$

$x_2 = \sqrt{3}$

$x_3 = -\sqrt{3}$

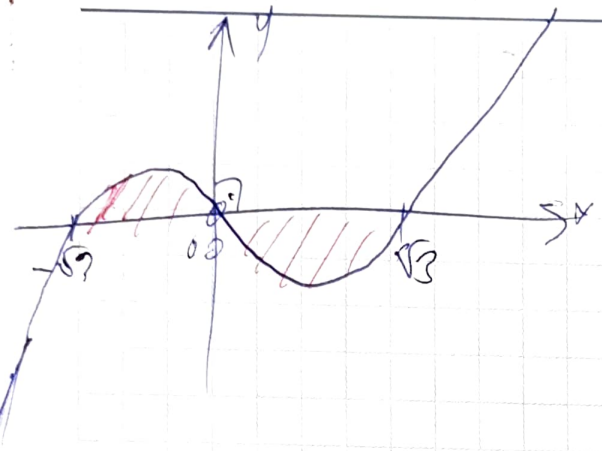
03

AGUSTOS
AUGUST

CUMA
FRIDAY

AGUSTOS 2012

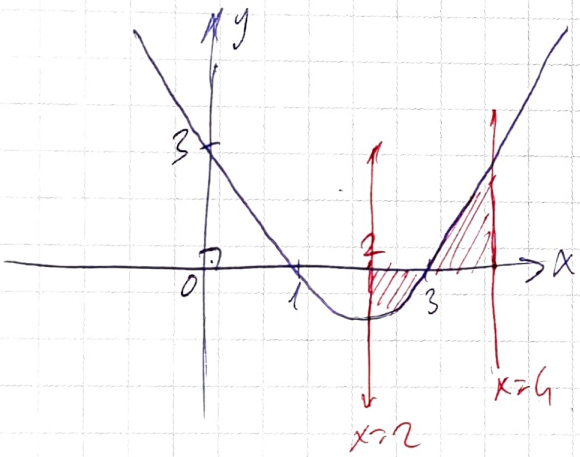
H	P	S	Ç	P	C	Ct	Pz
31			1	2	3	4	5
32	6	7	8	9	10	11	12
33	13	14	15	16	17	18	19
34	20	21	22	23	24	25	26
35	27	28	29	30	31		



$$A = \int_{-\sqrt{3}}^{\sqrt{3}} (x^3 - 3x)\,dx$$

$$A = 2\int_{0}^{\sqrt{3}} (x^3 - 3x)\,dx$$

$y = x^2 - 4x + 3 \qquad x = 2 \qquad x = 4 \qquad 0x - e\ell$



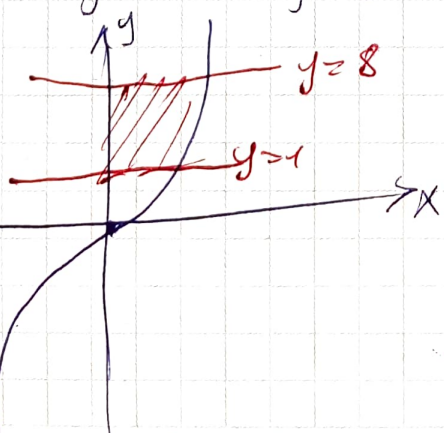
$$A = -\int_{2}^{3} (x^2 - 4x + 3)\,dx + \int_{3}^{4} (x^2 - 4x + 3)\,dx$$

$$A = -\left(\frac{x^3}{3} - \frac{4x^2}{2} + 3x\right)\Bigg|_{2}^{3} + \left(\frac{x^3}{3} - \frac{4x^2}{2} + 3x\right)\Bigg|_{3}^{4}$$

$$A = \frac{2}{3} + \left(\frac{4}{3} - 0\right)$$

$$A = 2\,br^2$$

$y = x^3 \qquad y = 1 \qquad y = 8 \qquad 0y - e\ell$



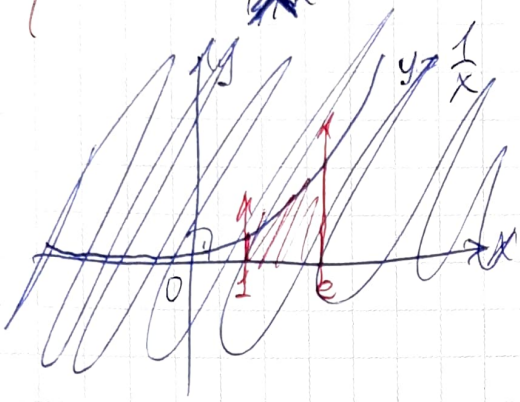
$$A = \int_{1}^{8} \sqrt[3]{y}\,dy$$

$$\sqrt[3]{y} = y^{1/3}$$

$$\frac{3}{4} y^{4/3}\Big|_{1}^{8} \qquad \frac{4\ell}{3} \cdot \frac{3}{4} = \frac{12}{4}\,br^2$$

$8^{4/3} = 2^4$

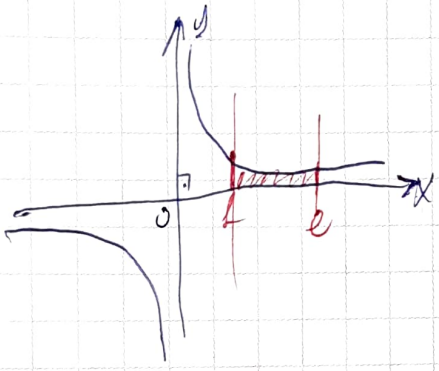
$y = \frac{1}{x}$ $x = 1$ $x = e$ Ox-eksen

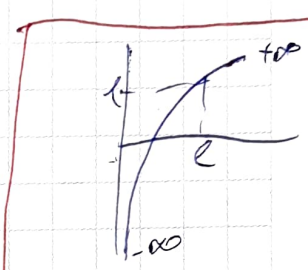


$$A = \int_1^e \frac{1}{x}\,dx$$

$$= \ln x \Big|_1^e$$

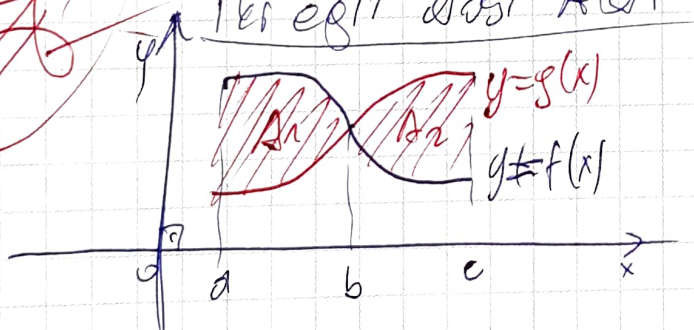
$$= 1 \text{ birim}^2$$





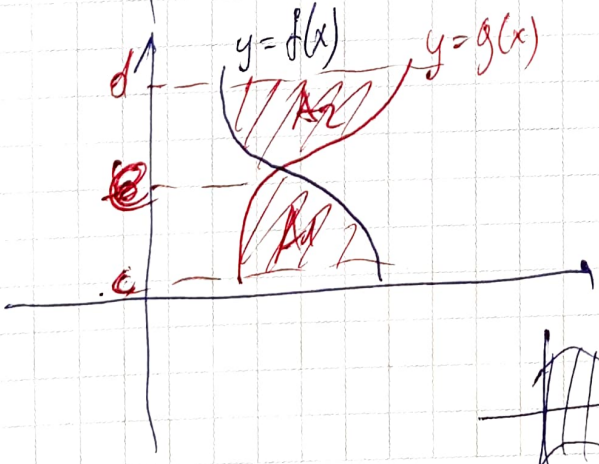
$\ln 0^+ = -\infty$
$\ln 1 = 0$
$\ln e = 1$
$\ln \infty = +\infty$

İki eğri arası Alan

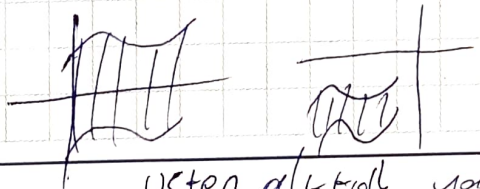


$A = A_1 + A_2$

$$A = \int_a^b (f(x) - g(x)) + \int_b^c (g(x) - f(x))$$



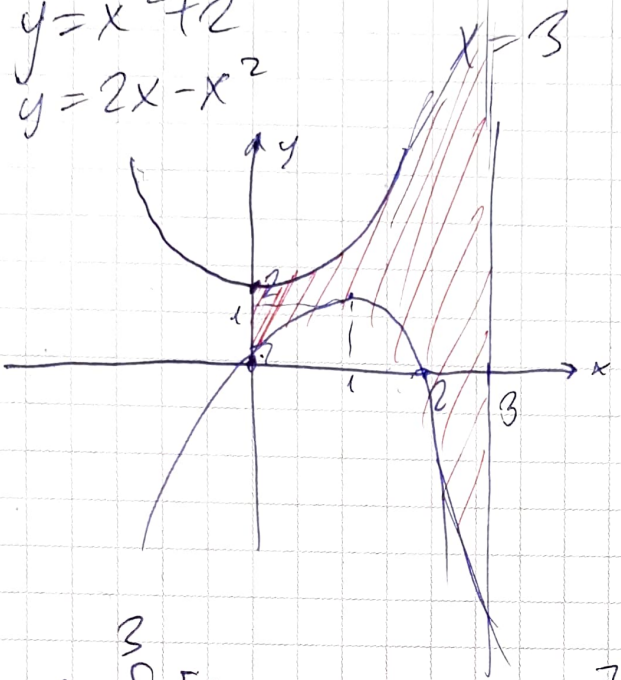
$$A = \int_c^e (f(x) - g(x)) + \int_e^d (g(x) - f(x))$$



Üsten alttaki yada sağdan soldaki gibi.

$y = x^2 + 2$

$y = 2x - x^2$

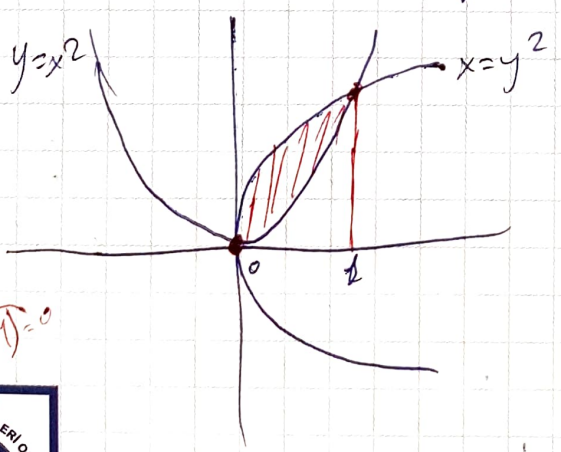


$$A = \int_0^3 \left[(x^2+2) - (2x - x^2) \right] dx$$

$$A = \int_0^3 (2x^2 - 2x + 2)\, dx$$

$$A = 2\frac{x^3}{3} - \frac{2x^2}{2} + 2x \Big|_0^3 = 15 \text{ br}^2$$

$y = x^2$ ile $x = y^2$ arası alan



$$A = \int_0^1 (\sqrt{x} - x^2)\, dx$$

$1/3 \text{ br}^2$

$x^2 = \sqrt{x}$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$y^2 = \frac{y+4}{3}$

$3y^2 - y - 4 = 0$

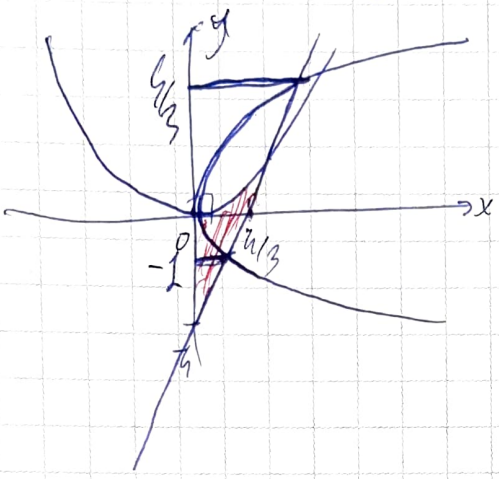
$3y \quad +1$

-4

$y+1 \quad 3y-4 \quad \frac{4}{3} \quad -1$

$x = y^2$

A , $y = 3x - 4$



$$A = \int_{-1}^{4/3} \left(\frac{y+4}{3} - y^2 \right) dy$$

$$A = \int_{-1}^{4/3} \left(\frac{1}{3}y + \frac{4}{3} - y^2 \right) dy$$

$$= \frac{1}{3}\frac{y^2}{2} + \frac{4}{3}y - \frac{y^3}{3} \Big|_{-1}^{4/3}$$

$$\left(\frac{1}{3} \cdot \frac{16}{9} \cdot \frac{1}{2} + \frac{4}{3}\frac{4}{3} - \frac{1}{3}\frac{4^3}{3^3} \right) - \left(\frac{1}{3}\frac{1}{2} + \frac{4}{3}(-1) + \frac{1}{2} \right)$$

B $y = 3x - x^3$, $y = x$

$x(3 - x^2)$

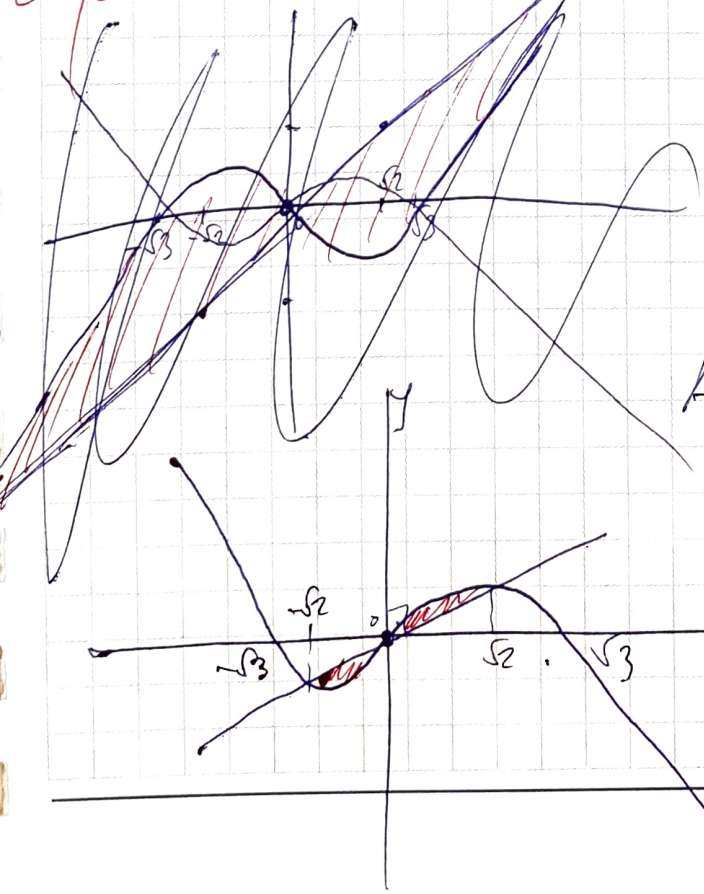
$-x^3 + 2x$

$x^3 - 2x = 0$

$-(x^3 - 2x)$

$x(x^2 - 2) = 0$

$0 \quad \sqrt{2} \quad -\sqrt{2}$



$$A = 2\int_0^{\sqrt{2}} (3x - x^3 - x)\, dx$$

$$= 2\left(-\frac{x^4}{4} + 3x^2 \right) \Big|_0^{\sqrt{2}}$$

$$= -2 + 12$$

$$= 10 br^2$$

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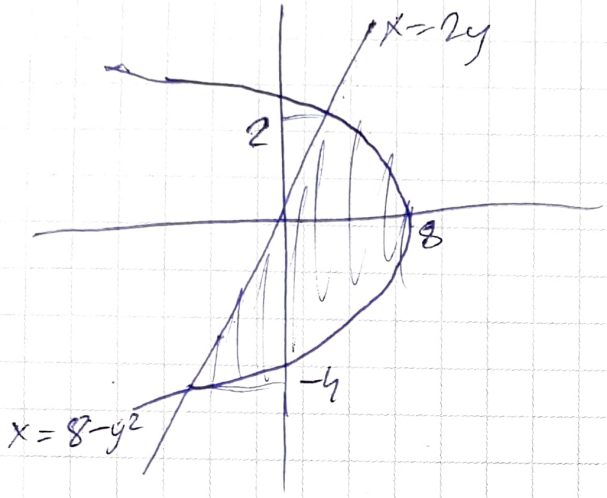
$8 - y^2 = 2y$

$y^2 + 2y - 8 = 0$

$y_1 = -4 \quad y_2 = 2$

$x = 2y \qquad x = 8 - y^2$



$$A = \int_{-4}^{2} (8 - y^2 - 2y)\,dy$$

$$\left(8y - \frac{y^3}{3} - y^2\right)\Big|_{-4}^{2}$$

Vize Sonrası

Parametrik D.V.E.B.

(1) $x = g(t)$
$y = h(t)$ $\longrightarrow$ $t_1 \le t \le t_2$

$x = a$
$x = b$ ve Ox eksen

$$A = \int_{t_1}^{t_2} |h(t)|\, g'(t)\,dt$$

$|y|$ $\quad$ x'

(2) $x = g(t)$
$y = h(t)$ $\quad t_3 \le t \le t_4$

$y = c$
$y = d$ ve Oy ekseni

$$A = \int_{t_3}^{t_4} |g(t)|\, h'(t)\, dt$$

$|x| \quad y'$

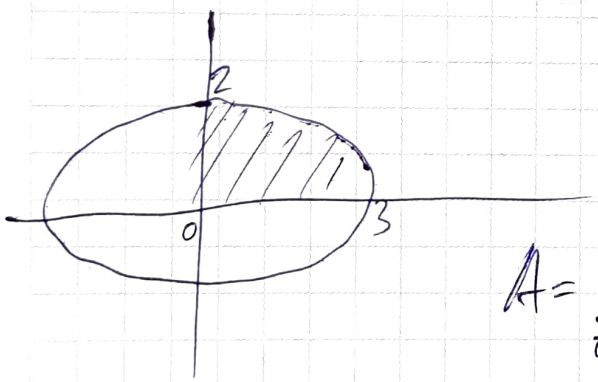
$x = 3\cos t$ $\quad x=0 \quad t_1 = ?$
$y = 2\sin t$ $\quad x=3 \quad t_2 = ?$

$x = 3\cos t \Rightarrow 0 = 3\cos t$

$t_1 = \frac{\pi}{2}$

$\Rightarrow 3 = 3\cos t$

$t_2 = 0$



$$A = \int_{\pi/2}^{0} |2\sin t| \cdot (-3\sin t)\, dt$$

$$= 6 \int_{0}^{\pi/2} \sin^2 t\, dt$$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1}{2}[1 - \cos 2x]$

$$= 6 \int_{0}^{\pi/2} \frac{1}{2}[1 - \cos 2t]\, dt$$

$$= 3\left[t - \frac{\sin 2t}{2}\right]_0^{\pi/2}$$

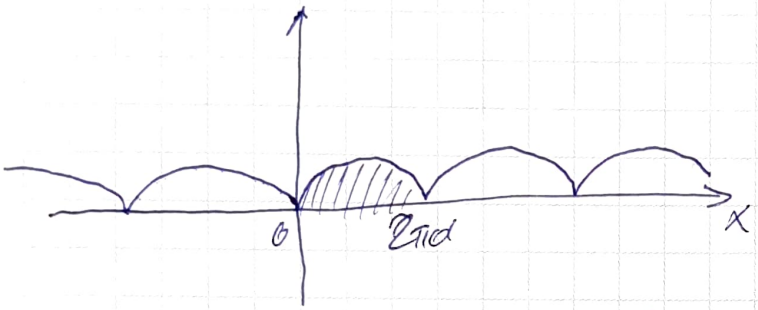
$$= 3\left(\left(\frac{\pi}{2} - 0\right) - (0 - 0)\right)$$

$$= \frac{3\pi}{2} \text{ br}^2$$

Cevap $4A = 6\pi$ br^2

A $x = a(t - \sin t)$
$y = a(1 - \cos t)$

cycloid eğrisinin bir eğrisinin bir yayının Ox ekseni ile yaptığı alanı hesaplayınız



$x = 0 \Rightarrow 0 = a(t - \sin t) \Rightarrow t_1 = 0$
$x = 2\pi a \Rightarrow 2\pi a = a(t - \sin t) \Rightarrow t_2 = 2\pi$

$$A = \int_0^{2\pi} |a(1-\cos t)| \underbrace{a(1-\cos t)}_{x'} dt$$

$|a| \geq 0$

-1 ile 1 arasında (pozitif sıfır)

$$= a^2 \int_0^{2\pi} (1-\cos t)^2 dt$$

$$= a^2 \int_0^{2\pi} [1 - 2\cos t + \cos^2 t] dt$$

(1) $\int_0^{2\pi} 1\,dt = t\Big|_0^{2\pi} = 2\pi$

(2) $\int_0^{2\pi} 2\cos t\,dt = 2\sin t\Big|_0^{2\pi} = 0$

(3)

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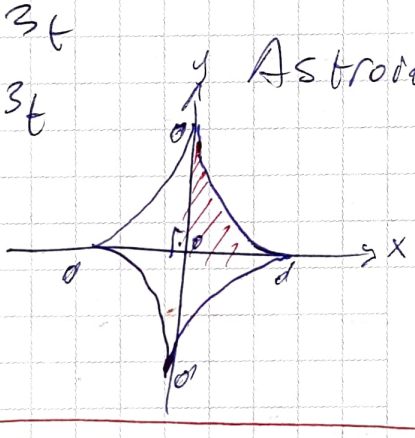
$$\int_0^{2\pi} \cos^2 t\,dt = \int_0^{2\pi} \frac{1}{2}[\cos 2t + 1]\,dt$$

$$= \frac{1}{2}\left[\frac{\sin 2t}{2} + t\right]_0^{2\pi} = \frac{1}{2}[(0+2\pi) - (0+0)]$$

$$= \pi$$

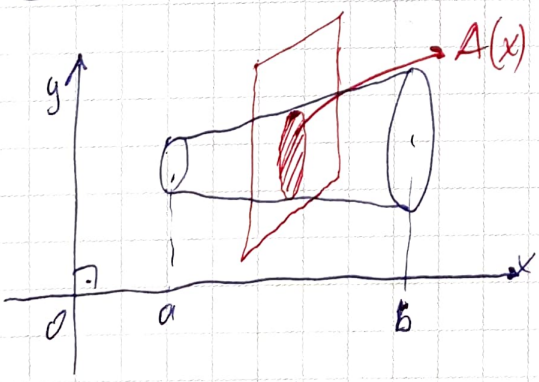
Cevap $A = a^2[2\pi - 0 + \pi] = 3\pi a^2 br^2$

$x = a\cos^3 t$
$y = a\sin^3 t$

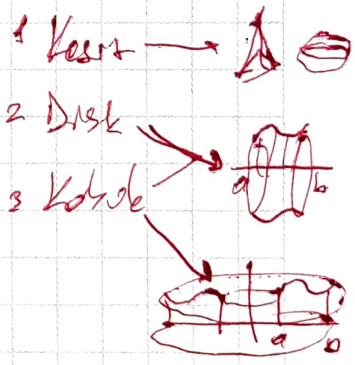


Hacim Hesabı

(1) Kesit Yöntemi



[a,b] aralığına yerleştirilen cismin Ox-eksenine dik kesilerek elde edilen kesitlerin alanı A(x) olsun



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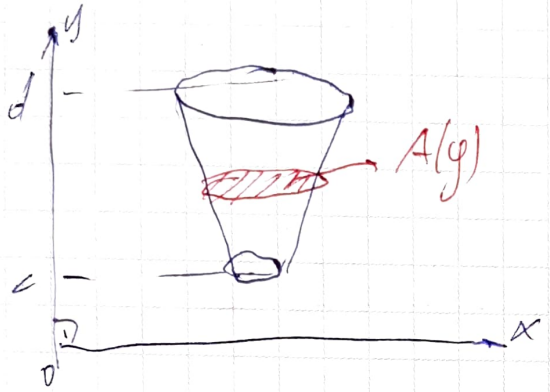
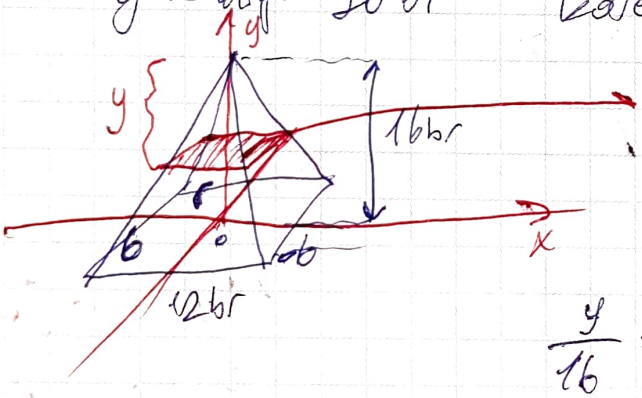
$\frac{8^3}{16} \cdot \frac{16^{4/2}}{3}$

$$H = \int_a^b A(x)\,dx$$



$$H = \int_c^d A(y)\,dy$$

Tabanının bir kenarı 12 br olan kare yüksekliği 16 br kare prizmik hacmin



$$A(y) = r^2 = \frac{9}{16}y^2$$

$$\frac{y}{16} = \frac{r}{12}$$

$$r = 3/4\ y$$

$$H = \int_0^{16} \frac{9}{16}y^2\,dy$$

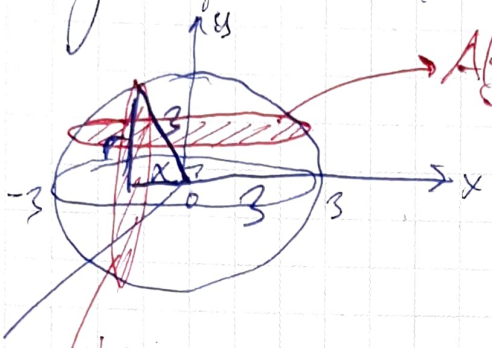
$$= \frac{9}{16}\left(\frac{y^3}{3}\right)\Big|_0^{16}$$

$$= 768\ br^3$$

Parent → $\frac{1}{3}b^2h$

$\frac{1}{3}.16^2.16$

yarıçapı = 3 br kare hacmi



$A(y) = \pi r^2$

$r = \sqrt{9 - x^2}$

$A(x) = \pi r^2$

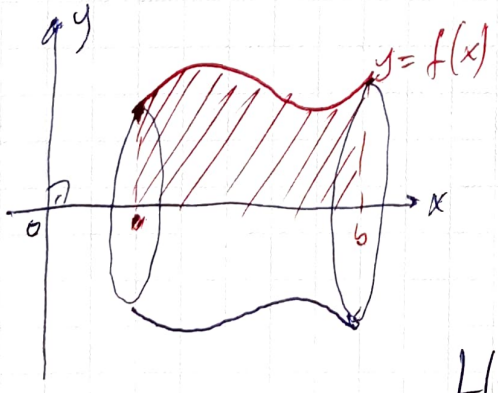
$$H = \int_{-3}^{3} A(x)\,dx$$

$$= \int_{-3}^{3} \pi (9 - x^2)\,dx$$

$$= 2\pi \left(9x - \frac{x^3}{3}\right)\Big|_0^3$$

$$= 36\pi \text{ br}^3$$

② Disk Yöntemi



$y = f(x)$
$x = a$
$x = b$
Ox - eksen

→ Ox ekseni etrafında dönsün

$$H = \pi \int_0^b [f(x)]^2\,dx$$

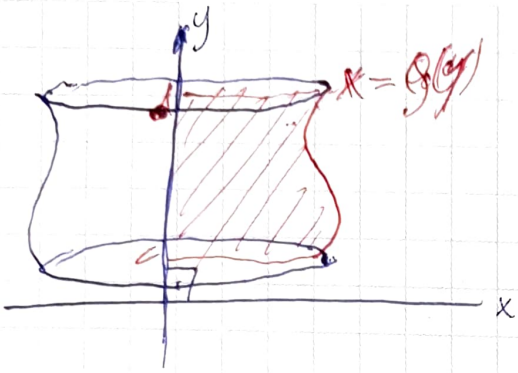
Lagrange Eğrilerine Bak !!!

15

AĞUSTOS
AUGUST

ÇARŞAMBA
WEDNESDAY

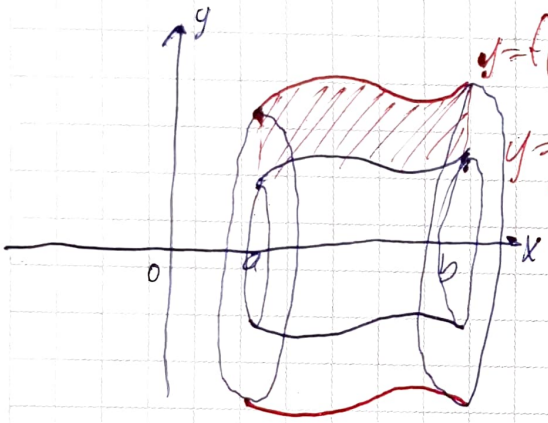
AĞUSTOS						2012	
H	P	S	Ç	P	C	Ct	Pz
31			1	2	3	4	5
32	6	7	8	9	10	11	12
33	13	14	15	16	17	18	19
34	20	21	22	23	24	25	26
35	27	28	29	30	31		



$x = g(y)$
$y = c$
$y = d$
Oy-ekseni

→ Oy-ek etrafı döndür

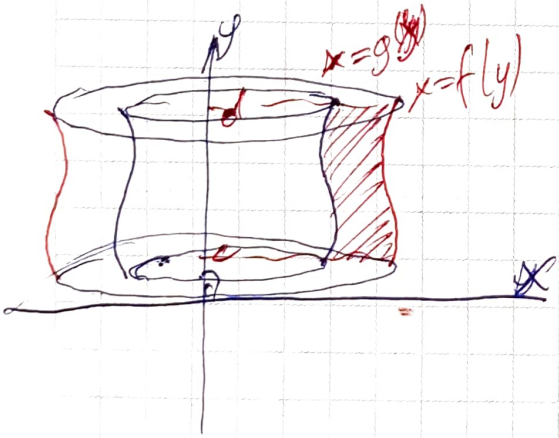
$$H = \pi \int_c^d [g(y)]^2 \, dy$$



$y = f(x)$
$y = g(x)$
$x = a$
$x = b$

→ Ox-ek etrafı döner

$$H = \pi \int_a^b \left([f(x)]^2 - [g(x)]^2 \right) dx$$

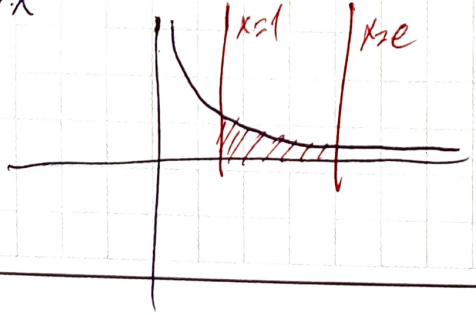


$x = f(y)$
$x = g(y)$
$y = a$
$y = d$

→ Oy-ek etrafı döner

$$H = \pi \int_c^d \left([f(y)]^2 - [g(y)]^2 \right) dy$$

$y = \frac{1}{\sqrt{x}}$, $x = 1$, $x = e$, ox-ek



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$x^4 = 4x$
$x^3 - 4 = 0$

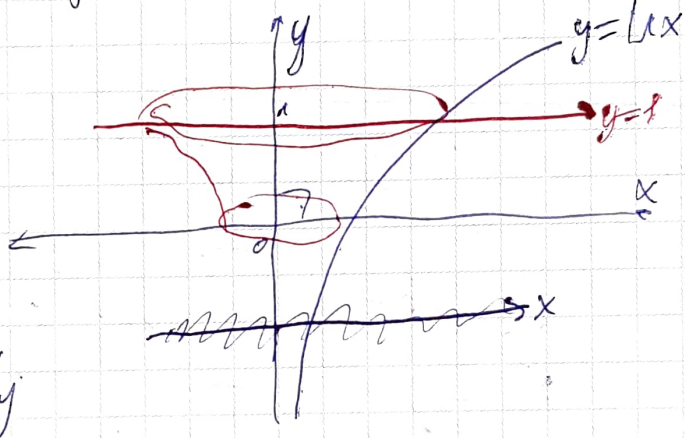
$$H = \pi \int_1^e \left(\frac{1}{\sqrt{x}}\right)^2 dx$$

$$= \pi \left(\ln x\right) \Big|_1^e$$

$$= \pi \text{ br}^3$$

$y = \ln x$
Ox - ek
Oy - ek
$y = 1$

Oy - ekseni dan $H = ?$



$\ln 0^+ = -\infty$
$\ln 1 = 0$
$\ln e = 1$
$\ln \infty = \infty$

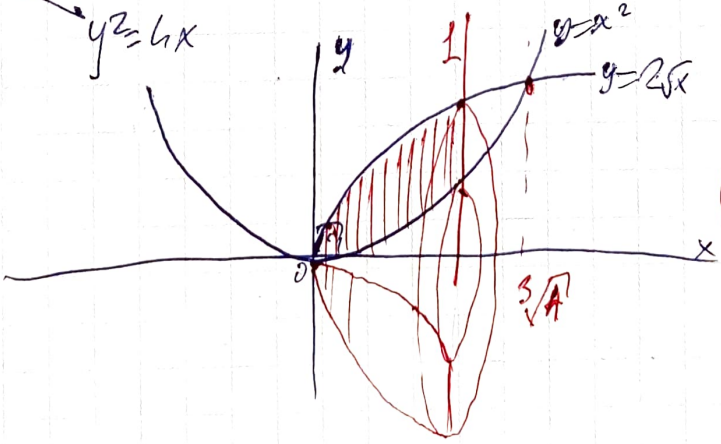
$$H = \pi \int_0^1 (e^y)^2 dy$$

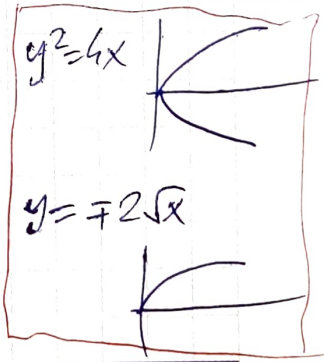
$$= \pi \frac{e^{2y}}{2} \Big|_0^1$$

$$= \frac{\pi}{2} \left[e^2 - 1\right] \text{ br}^3$$

$y = 2\sqrt{x}$ → $y^2 = 4x$
$y = x^2$
$x = 1$

Ox - ekseninde dönder





$$H = \pi \int_0^4 \left((2\sqrt{x})^2 - (x^2)^2 \right) dx$$

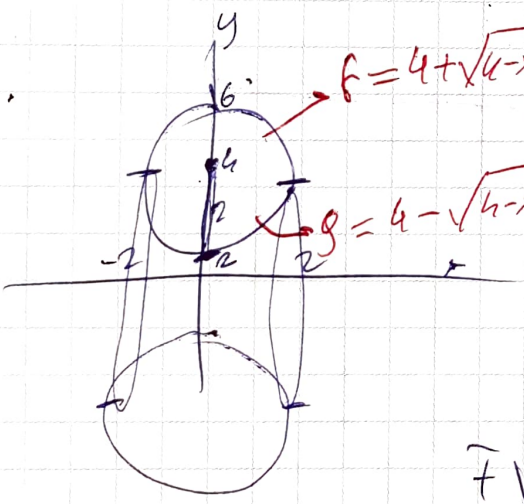
$$= \pi \int_0^4 (4x - x^4)\, dx$$

$$= \pi \left(4\frac{x^2}{2} - \frac{x^5}{5} \right) \Big|_0^1$$

$$= \frac{9\pi}{5} br^3$$

---

merkez $(0,4)$, $r=2$ çember Ox ekseni etrafında dönsün $H=?$



$$(x-0)^2 + (y-4)^2 = 4$$

$$x^2 + (y-4)^2 = 4$$

$$4 - x^2 = (y-4)^2$$

$$\mp\sqrt{4-x^2} = y-4$$

$$4 \mp \sqrt{4-x^2} = y$$

$$H = \pi \int_{-2}^{2} \left[\left(4+\sqrt{4-x^2}\right)^2 - \left(4-\sqrt{4-x^2}\right)^2 \right] dx$$

$$= 2\pi \int_0^2 \left(16\sqrt{4-x^2}\right) dx$$

$\int \sqrt{4-x^2}\,dx$

a^2-x^2
x^2-a^2
x^2+a^2

$x = 2\sin t$

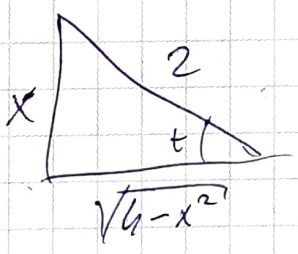
$dx = 2\cos t\,dt$

$= \int \sqrt{4-4\sin^2 t}\; 2\cos t\,dt$

$= \int 4\cos^2 t\,dt$

$= \int 4\,\frac{1}{2}\left[\cos 2t + 1\right] dt$

$= 2\left[\frac{1}{2}\sin 2t + t\right] = \sin 2t + t$



$\sin 2t = 2\sin t\cos t$

$= 2\,\frac{x}{2}\,\frac{\sqrt{4-x^2}}{2}$

$= \frac{x\sqrt{4-x^2}}{2}$

$H = 32\pi\left[\frac{x\sqrt{4-x^2}}{2} + 2\arcsin\frac{x}{2}\right]\Big|_0^2$

$= 32\pi\left[\pi - 0\right]$

$= 32\pi^2\ br^3$

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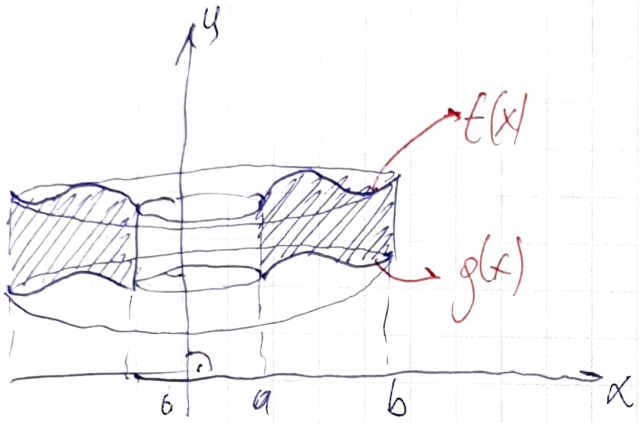
Dişe → y sınır ise y bulunur ve sağ dan sol ya bakılır

4 + 8 → 3

4

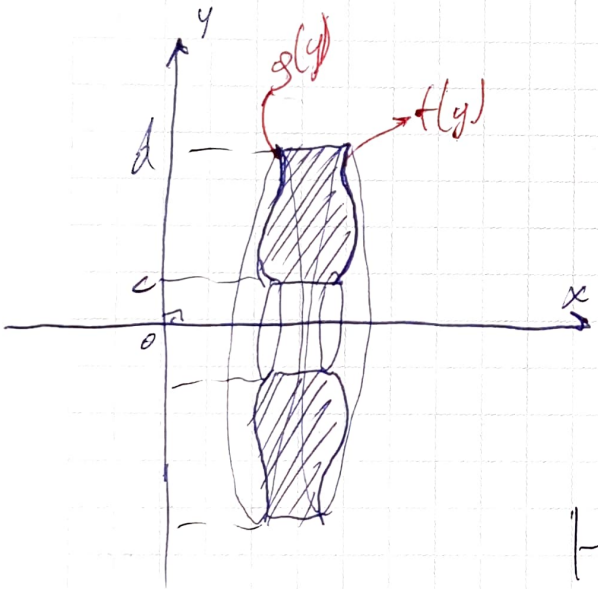
(3) KABUK YÖNTEMİ



$y = f(x)$
$y = g(x)$
$x = a$
$x = b$

→ oy etrafında dön

$$H = 2\pi \int_a^b |x[f(x) - g(x)]|\,dx$$



$x = f(y)$
$x = g(y)$
$y = c$
$y = d$

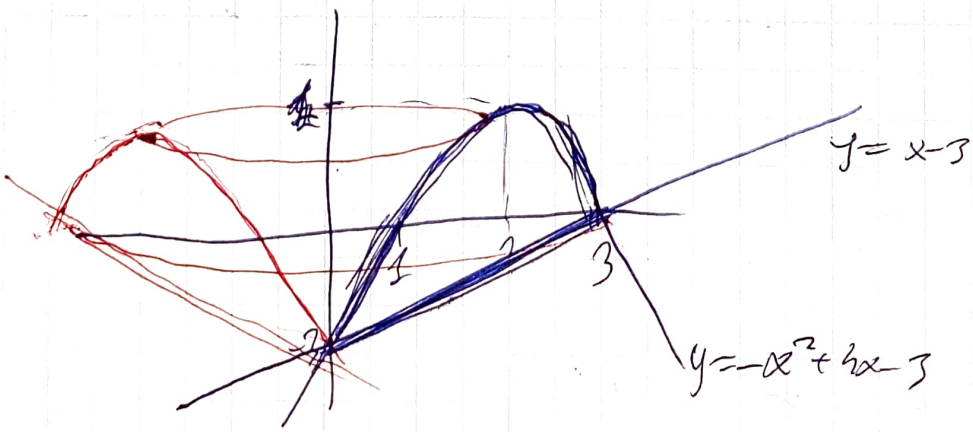
→ Ox etrafında dön

$$H = 2\pi \int_c^d |y[f(y) - g(y)]|\,dy$$

A $y = -x^2 + 4x - 3$
$y = x - 3$

Oy - etrafında dön

$x^2 - 4x + 3$
-3
-1



$$H = 2\pi \int_0^3 \left| x\left[-x^2+4x-3-x+3\right] \right| dx$$

$$= 2\pi \int_0^3 \left| x\left[-x^2+3x\right] \right| dx$$

$$= 2\pi \int_0^3 \left| -x^3+3x^2 \right| dx$$

$$= 2\pi \int_0^3 \left| x^2(3-x) \right| dx$$

x^2: pozitif; $(3-x)$: pozitif (0'dan 3'e)

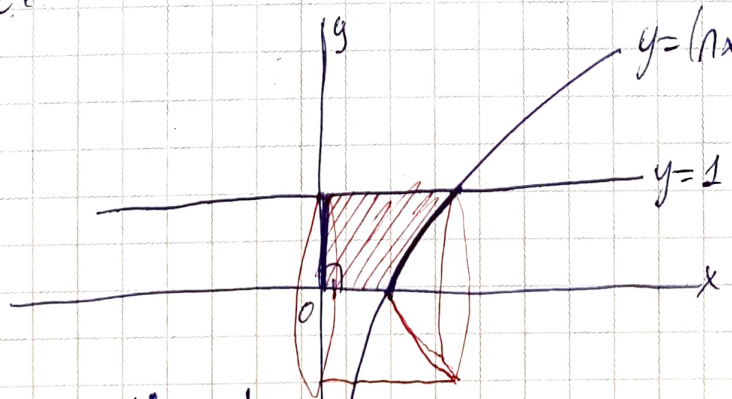
$$= 2\pi \int_0^3 (3x^2-x^3)\, dx$$

$$= 2\pi \left(x^3 - \frac{x^4}{4}\right)\Big|_0^3 =$$

$y = \ln x$
Ox-ek
Oy-ek
$y = 1$

Ox-ek dön

$H = ?$



$$H = 2\pi \int_0^1 \left| y\left[e^y - 0\right] \right| dy$$

$$H = 2\pi \int_0^1 |ye^y| dy$$

$\int ye^y dy$ LAPLUS

$y = v \quad e^y dy = dw$

$dy = dw \quad e^y = v$

$ye^y - \int e^y dy$

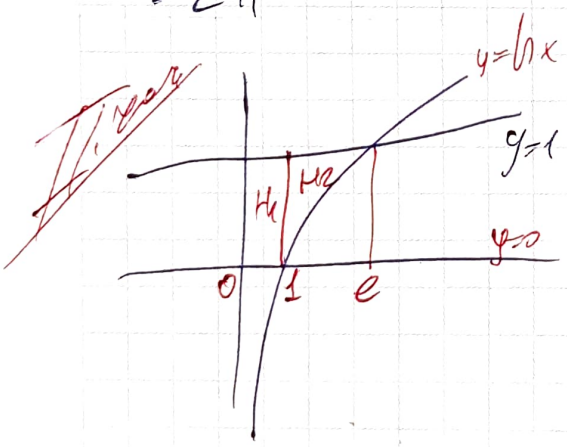
$ye^y - e^y$

$e^y[y-1]$

$$H = 2\pi \left(e^y[y-1] \right)\Big|_0^1$$

$= 2\pi \left(e^1(1-1) - e^0(0-1) \right)$

$= 2\pi$



$H = H_1 + H_2$ → sıkıntılı

$= \pi \int_0^1 (1^2 - 0^2) dx + \pi \int_1^e (1^2 - h_x^2) dx$

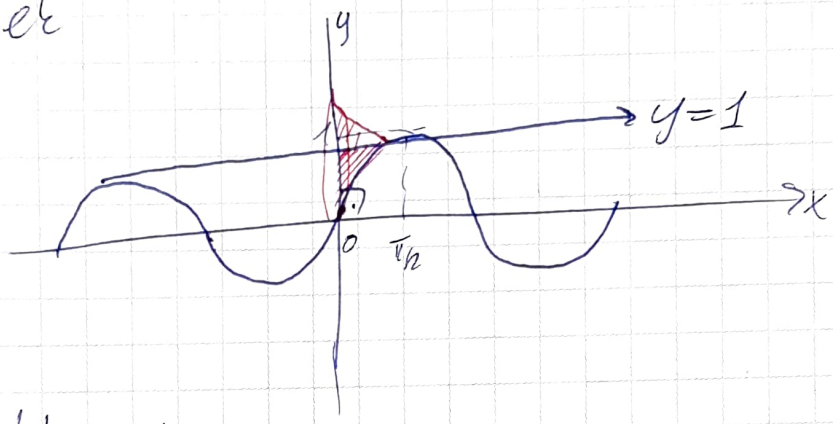
$= \pi + \pi$

$= 2\pi$

$H = \pi \int_a^b (\text{uzaklıkların})^2 dx$

$y = \sin x$
$y = 1$
Oy - ek

$y = 1$ etrafında dönsə $H = ?$
Dəstə çəkin



$f = \text{uzaqlıq} = 1 - \sin x$

$$H = \pi \int_0^{\pi/2} (1 - \sin x)^2 dx$$

$$= \pi \int_0^{\pi/2} (1 - 2\sin x + \sin^2 x) dx$$

$$= \pi \left(x + 2\cos x + \frac{x}{2} - \frac{\sin 2x}{4} \right) \Big|_0^{\pi/2}$$

$$= \frac{3}{4}\pi^2 - 2\pi \text{ br}^3$$

Parametrik Denklemleri Verilen Eğrilerin Alanları

$x = v(t)$ $\quad t_1 \le t \le t_2$
$y = \varphi(t)$

Sınırladığı alan Ox-ekseninden $H = ?$

$$H = \pi \int_{t_1}^{t_2} y^2 x' \, dt$$

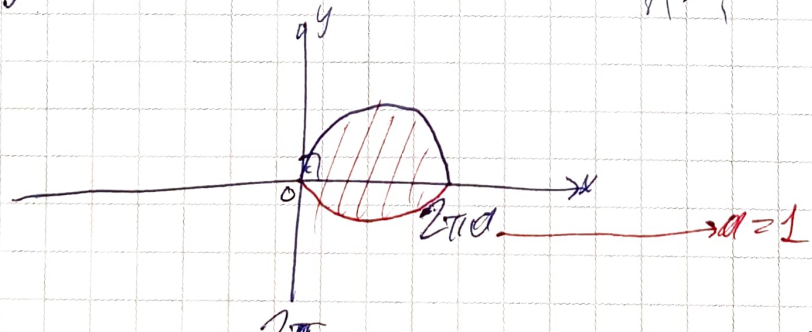
$x = v(t)$ $\quad t_3 \le t \le t_4$
$y = \varphi(t)$

Sınırladığı alan Oy-ekseninden $H = ?$

$$H = \pi \int_{t_3}^{t_4} x^2 y' \, dt$$

$x = (t - \sin t)$
$y = (1 - \cos t)$

Cycloid eğrisinin bir parçasının Ox ekseninden $H = ?$



$$H = \pi \int_0^{2\pi} (1 - \cos t)^2 (1 - \cos t) \, dt$$

$$= \pi \int_0^{2\pi} (1 - 3\cos t + 3\cos^2 t - \cos^3 t) \, dt$$

$\cos^2 t = \frac{1}{2}[1+\cos 2t]$

$$H = \pi\left[t - 3\sin t + \frac{3t}{2} + \frac{3\sin 2t}{4} + \sin t - \frac{\sin 3t}{3}\right]\Big|_0^{2\pi}$$

$$H = 5\pi^2 br^3$$

$\int \cos^3 t\,dt$

$\int \cos^2 t \cos t\,dt$

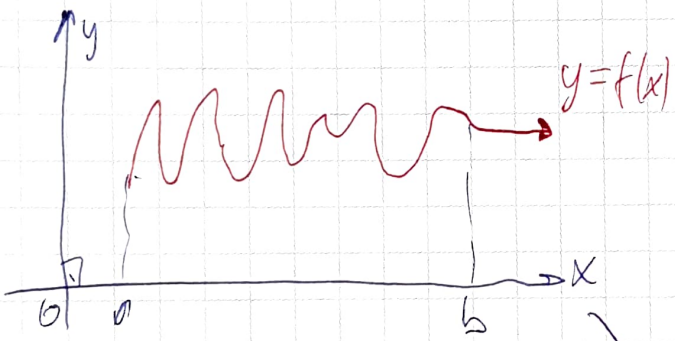
$\int (1-\sin^2 t)\cos t\,dt$

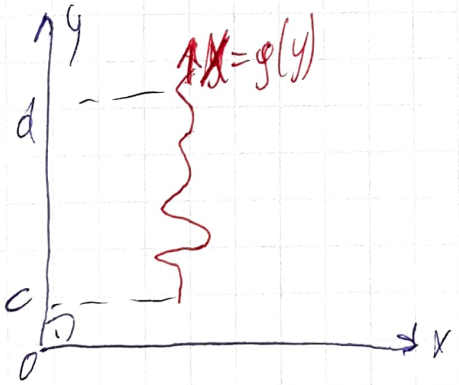
$\int (1-u^2)\,du$

$= u - \frac{u^3}{3}$

$= \sin t - \frac{\sin^3 t}{3}$

Eğra Uzunluğu





$y = f(x)\,,\ [a,b]$

$$\ell = \int_a^b \sqrt{1+(y')^2}\,dx$$

$x = g(y)\,,\ [c,d]$

$$\ell = \int_c^d \sqrt{1+(x')^2}\,dy$$

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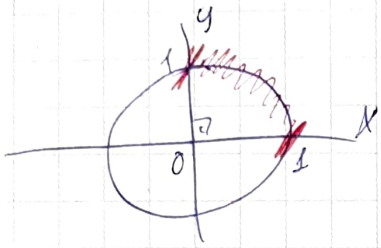
$r=1$ olan çemberin çevre uzunluğu



$x^2+y^2=1$

$y=\sqrt{1-x^2}$

$y=(1-x^2)^{1/2}$

$y'=\frac{1}{2}(1-x^2)\cdot(-2x)$

$y'=\frac{-x}{\sqrt{1-x^2}}$

$(y')^2=\frac{x^2}{1-x^2}$

$$l=4\int_0^1 \sqrt{1+\frac{x^2}{1-x^2}}\,dx$$

$$=4\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$$

$$=4\arcsin|_0^1$$

$$=4\cdot\frac{\pi}{2}$$

$$=2\pi$$

$\frac{1-8x^2+x^4}{16x^2}$

$y = \ln x - \frac{x^2}{8}$, $[2,4]$ aralığı $\ell = ?$

$y' = \frac{1}{x} - \frac{1}{4}x \Rightarrow (y')^2 = \frac{1}{x^2} - \frac{1}{2} + \frac{x^2}{16}$

$$\ell = \int_2^4 \sqrt{1 + \left(\frac{1}{x^2} - \frac{1}{2} + \frac{x^2}{16}\right)} \, dx$$

$\frac{x^4+8x^2+1}{16x^2}$

$(a-b)^2 = a^2 - 2ab + b^2$

$+4ab$

$= (a+b)^2$

$$1 + (y')^2 = 1 + \left(\frac{1}{x} - \frac{x}{4}\right)^2$$

$$= 1 + \left(\frac{1}{x}\right)^2 - \frac{1}{2} + \left(\frac{x}{4}\right)^2$$

$$= \left(\frac{1}{x}\right)^2 + \frac{1}{2} + \left(\frac{x}{4}\right)^2$$

$$= \left(\frac{1}{x} + \frac{x}{4}\right)^2$$

$$\ell = \int_2^4 \left(\frac{1}{x} + \frac{x}{4}\right) dx = \ln x + \frac{x^2}{8} \Big|_2^4$$

$= \left(\ln 2 + \frac{3}{2}\right)$ br

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Parametrik Eğrinin Eğri Uzunluğu

$x = \upsilon(t)$ $\quad t_1 \le t \le t_2$

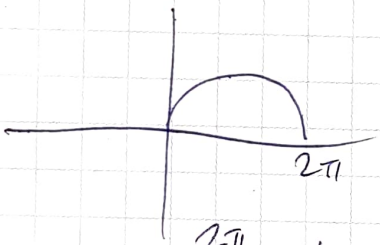
$y = \vartheta(t)$

$$\ell = \int_{t_1}^{t_2} \sqrt{(x')^2 + (y')^2}\, dt$$

$x = (t - \sin t)$

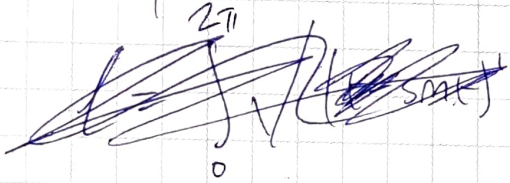
$y = (1 - \cos t)$ $\quad 0 \le t \le 2\pi$

Parametrik eğrinin bir yayının eğri uzunluğu



$x' = 1 - \cos t$

$y' = \sin t$



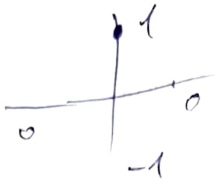
$$\ell = \int_0^{2\pi} \sqrt{(1-\cos t)^2 (\sin t)^2}\, dt$$

$$(x')^2 + (y')^2 = 1 - 2\cos t + \underbrace{\cos^2 t + \sin^2 t}_{1}$$

$$= 2(1 - \cos t)$$

$$= 2\left[1 - \left(1 - 2\sin^2 \frac{t}{2}\right)\right]$$

$$= 4 \sin^2 \frac{t}{2}$$



$1+\cos t$

$$\ell = \int_0^{2\pi} \sqrt{4\sin^2\frac{t}{2}}\,dx$$

$$= \int_0^{2\pi} 2\sin\frac{t}{2}\,dt$$

$$= -4\cos\frac{t}{2}\Big|_0^{2\pi}$$

$$= 8br$$

APT
Gelistirilmis
Sarac
Tekniti

$x = \cos t \qquad -\pi \le t \le \pi$

$y = t + \sin t$

eğrinin
düzgün olup olmadığını
araştırınız

$$(x')^2 + (y')^2 = \sin^2 t + 1 + 2\cos t + \cos^2 t$$

$$= 2(1+\cos t)$$

$$= 2\left(1 + (2\cos^2\frac{t}{2} - 1)\right)$$

$$= 4\cos^2\frac{t}{2}$$

$$\ell = \int_{-\pi}^{\pi} \sqrt{4\cos^2\frac{t}{2}}\,dt$$

$$= \int_{-\pi}^{\pi} 2\cos\frac{t}{2}\,dt = 8\int_0^{\pi/2} \left|\cos\frac{t}{2}\right| dt = 8\sqrt{2}$$

$$4\sin\frac{t}{2}\Big|_{-\pi}^{\pi} = 8br$$

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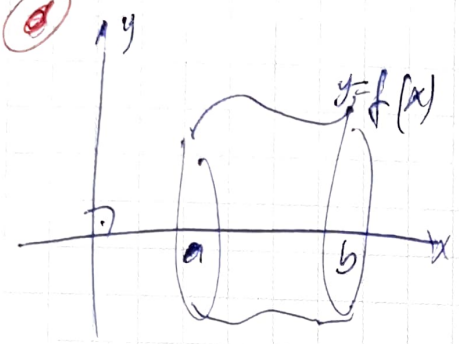
Dönel Yüzeylerin Alanı

(a)

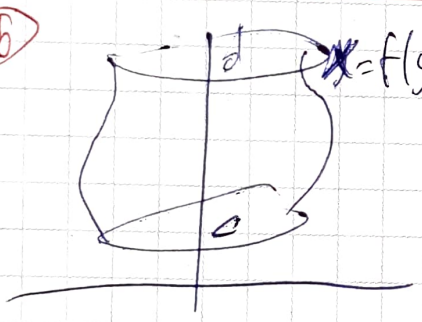


$y=f(x)$ fonk $[a,b]$
Ox ekseni etrafında dönel yüzeyin yüzey alanı

$$S = 2\pi \int_a^b |y| \sqrt{1+(y')^2}\,dx$$

$$= 2\pi \int_a^b |f(x)| \sqrt{1+[f'(x)]^2}\,dx$$

(b)

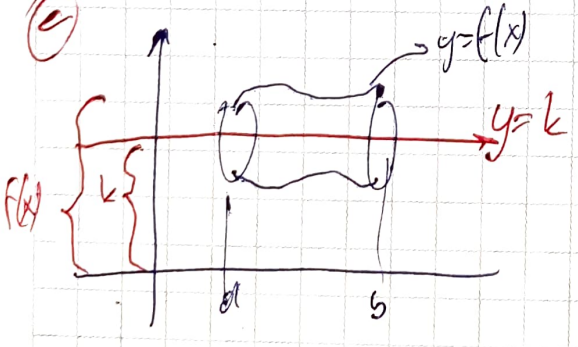


$x=f(y)$ fonk $[c,d]$
Oy ekseni etrafında dönel yüzey alanı

$$S = 2\pi \int_c^d |x| \sqrt{1+(x')^2}\,dy$$

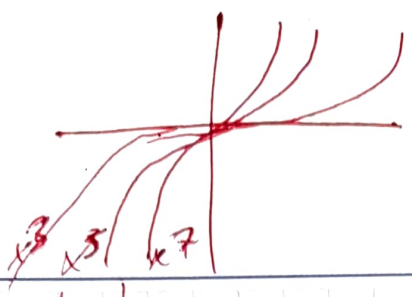
$$= 2\pi \int_c^d |f(y)| \sqrt{1+[f'(y)]^2}\,dy$$

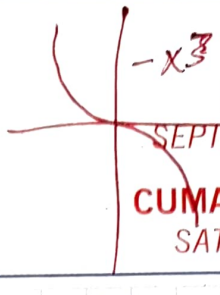
(c)



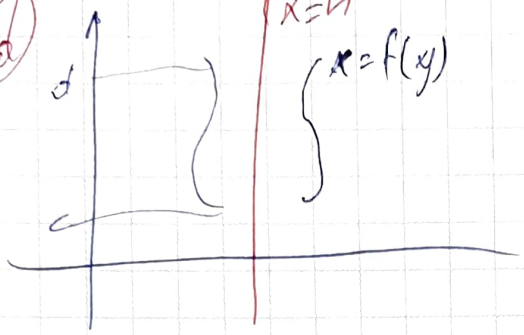
$y=f(x)$ fonk $[a,b]$
$y=k$ ekseni etrafında dönel yüzey alanı

$$S = 2\pi \int_a^b |f(x)-k| \sqrt{1+[f'(x)]^2}\,dx$$





(a)



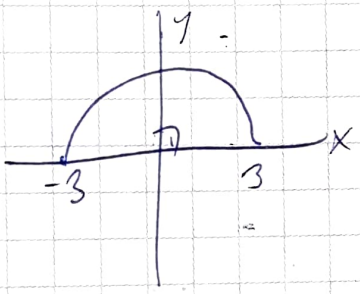
$x = f(y)$ fonk. $[c,d]$

$x = h$ et dön

yüzey alanı

$$S = 2\pi \int_c^d |f(y) - h| \sqrt{1 + [f'(y)]^2}\, dy$$

$r = 3$ küresel yüzey alanı



$x^2 + y^2 = 9$

$$y = \pm\sqrt{9 - x^2} \Rightarrow y' = \frac{-x}{\sqrt{9-x^2}}$$

EYLÜL SEPTEMBER PAZAR SUNDAY 02

$$S = 2\pi \int_{-3}^{3} |\sqrt{9-x^2}| \sqrt{1 + \frac{x^2}{9-x^2}}\, dx$$

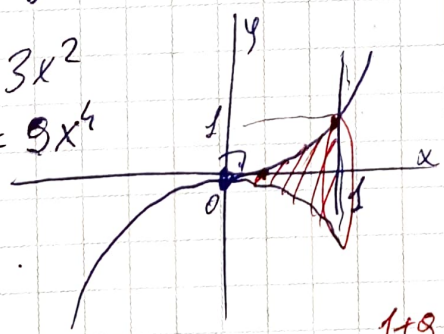
$$= 2\pi \int_0^3 \sqrt{9-x^2}\, \frac{3}{\sqrt{9-x^2}}\, dx$$

$$= 12\pi x \Big|_0^3 = 36\pi \text{ br}^2$$

$y = x^3$ $\quad 0 \le x \le 1$ $\quad$ Ox-etrafı $\quad S = ?$

$y' = 3x^2$

$(y')^2 = 9x^4$



$$S = 2\pi \int_0^1 |x^3| \sqrt{1 + 9x^4}\, dx$$

$$= 2\pi \int_0^1 x^3 \sqrt{\underbrace{1 + 9x^4}_{u}}\, dx$$

$1 + 9x^4 = u$

$36x^3 dx = du$

$$S = 2\pi \int_0^1 \sqrt{v}\, \frac{dv}{36}$$

$$= \frac{\pi}{18} \left. \frac{v^{3/2}}{3/2} \right|_0^1$$

$$= \frac{\pi}{27} \left. (1+8x^4)^{3/2} \right|_0^1$$

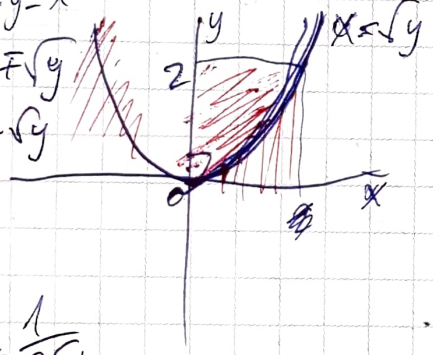
$$= \frac{\pi}{27} [40^{3/2} - 1] \text{ br}^2$$

$x = \sqrt{y}$, $0 \le y \le 2$ Oy- etrafında dönen yüzey alanı

tek çözüm $y = x^2$

$x = \mp\sqrt{y}$

$x = +\sqrt{y}$



$x' = \frac{1}{2\sqrt{y}}$

$(x')^2 = \frac{1}{4y}$

$$S = 2\pi \int_0^2 |\sqrt{y}| \sqrt{1 + \frac{1}{4y}}\, dy$$

$$S = 2\pi \int_0^2 \sqrt{y}\, \frac{\sqrt{4y+1}}{2\sqrt{y}}\, dy$$

$$= \pi \int_0^2 \sqrt{4y+1}\, dy$$

$4y+1 = v$

$4dy = dv$

$$= \pi \int_0^2 v^{1/2}\, \frac{dv}{4}$$

$$= \frac{\pi}{4} \left. \frac{v^{3/2}}{3/2} \right|_0^2$$

$$= \frac{\pi}{6} \left. (4y+1)^{3/2} \right|_0^2$$

$$= \frac{\pi}{6} (27 - 1) = \frac{13}{3} \text{ br}^2$$

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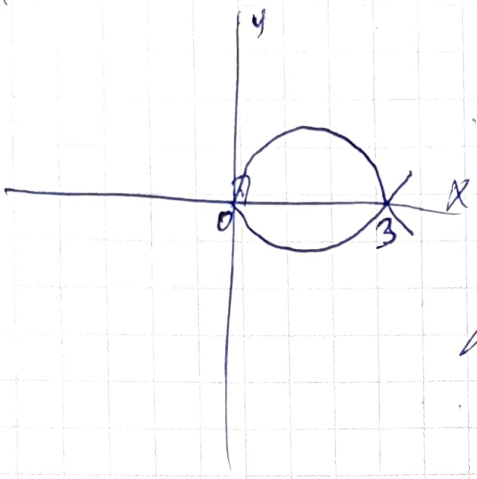
$9y^2 = x(3-x)^2$ ilmiğinin ox-ekseni etrafında dönmesiyle oluşan yüzey alanı $S = ?$



$$y = \sqrt{\frac{x(3-x)^2}{9}} = \frac{1}{3}\sqrt{x}(3-x)$$

$$S = 2\pi \int_0^3 y\sqrt{1+(y')^2}\,dx$$

$$y' = \frac{1}{3}\left[\frac{1}{2\sqrt{x}}(3-x) + \sqrt{x}(-1)\right]$$

$$= \frac{3-x}{6\sqrt{x}} - \frac{\sqrt{x}}{3}$$

$$(y')^2 = \frac{9-6x+x^2}{36x} - \frac{6\sqrt{x}^2 - 2x\sqrt{x}}{18\sqrt{x}} + \frac{x}{9}$$

$2\sqrt{x}$ $\quad 4x$

$$= \frac{9-6x+x^2-12x+4x^2+4x^2}{36x}$$

$$= \frac{9x^2-18x+9}{36x}$$

$$1+(y')^2 = \frac{9x^2+18x+9}{36x} = \frac{x^2+2x+1}{4x}$$

$$S = 2\pi \int_0^3 \left| \frac{1}{3}\sqrt{x}\,(3-x) \right| \sqrt{\frac{(x+1)^2}{4x}}\, dx$$

$$= 2\pi \int_0^3 \frac{1}{3}\sqrt{x}\,(3-x)\,\frac{(x+1)}{2\sqrt{x}}\, dx$$

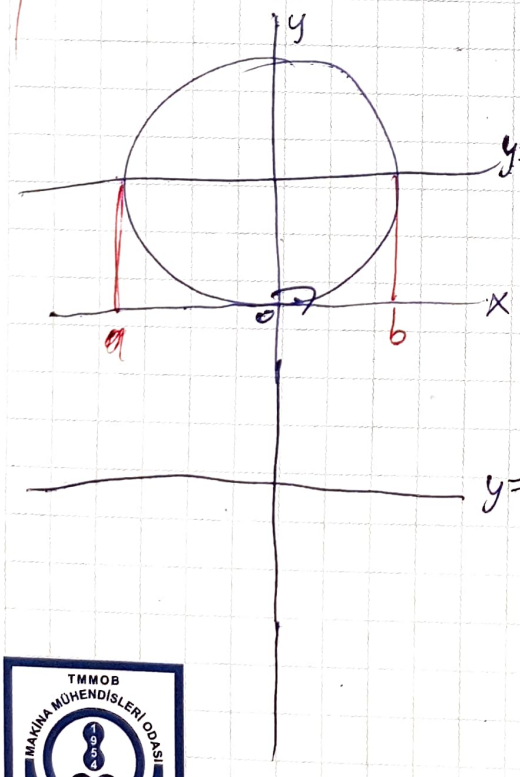
$$= \frac{\pi}{3}\int_0^3 (-x^2+2x+3)\,dx$$

$$= \frac{\pi}{3}\left[\frac{-x^3}{3} + x^2 + 3x \right]\Big|_0^3$$

$$= \frac{\pi}{3}\left[-9+9+3 \right] = 3\pi \text{ br}^2$$

A) $x^2+(y-1)^2=1$, $y=-1$ ve ox , $S=?$

$$y = 1 \mp \sqrt{1-x^2}$$



$$S = 2\pi \int_a^b |f(x)-k| \sqrt{1+[f'(x)]^2}\, dx$$

(a) $y = 1 + \sqrt{1-x^2}$ (üst kısım)

$$S_1 = 2\pi \int_{-1}^{1} \left| 1 + \sqrt{1-x^2} - (-1) \right| \sqrt{\frac{1}{1+x^2}}\, dx$$

$$y' = \frac{-x}{\sqrt{1-x^2}} \Rightarrow (y')^2 = \frac{x^2}{1-x^2} \Rightarrow 1+(y')^2 = \frac{1}{1-x^2}$$

(b) $y = 1 - \sqrt{1-x^2}$ (alt kısım)

$$1+(y')^2 = \frac{1}{1-x^2}$$

$$S_1 = 2\pi \int_{-1}^{1} \left| 1 + \sqrt{1-x^2} - (-1) \right| \sqrt{\frac{1}{1-x^2}}\, dx$$

$$S_2 = 2\pi \int_{-1}^{1} \left| 1 - \sqrt{1-x^2} - (-1) \right| \sqrt{\frac{1}{1-x^2}}\, dx$$

$$S_1 + S_2 = S = 2\pi \int_{-1}^{1} |4| \frac{1}{\sqrt{1-x^2}}\, dx$$

$$= 16\pi \int_{0}^{1} \frac{1}{\sqrt{1-x^2}}\, dx$$

$$= 16\pi \arcsin x \Big|_0^1$$

$$= 16\pi (\arcsin 1 - \arcsin 0)$$

$$= 16\pi \left(\pi/2 - 0 \right) = 8\pi^2 \text{ br}^2$$

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Parametrik Eğrilerin Oluşturduğu Yüzey Alanları

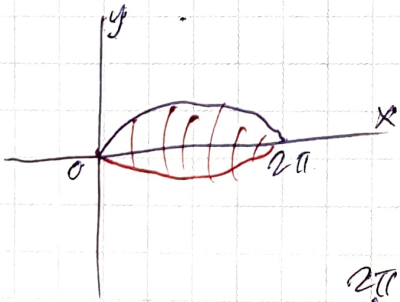
(a) $x = u(t)$, $y = v(t)$ $\quad t_1 \le t \le t_2$ $\quad$ Eğrinin Ox-e etrafında dönmesi ile oluşan eksenel yüzey alanı

$$S = 2\pi \int_{t_1}^{t_2} |y| \sqrt{(x')^2 + (y')^2}\, dt$$

(b) $x = u(t)$, $y = v(t)$ $\quad t_3 \le t \le t_4$ $\quad$ Par. eğrinin Oy-ek etrafında dönmesi ile oluşan eksenel yüzey alanı

$$S = 2\pi \int_{t_3}^{t_4} |x| \sqrt{(x')^2 + (y')^2}\, dt$$

Örnek: $x = t - \sin t$, $y = 1 - \cos t$ $\quad 0 \le t \le 2\pi$ $\quad$ Ox-ek etrafında $S = ?$



$x' + y' = 1 - \cos t + \sin t$

$(x')^2 + (y') = 1 - 2\cos t + \cos^2 t + \sin^2 t$

$= 2(1 - \cos t)$

$$S = 2\pi \int_0^{2\pi} |1 - \cos t| \sqrt{2 - 2\cos t}\, dt$$

$$= 2\sqrt{2}\,\pi \int_0^{2\pi} (1 - \cos t) \sqrt{1 - \cos t}\, dt$$

EYLÜL						2012	
H	P	S	Ç	P	C	Ct	Pz
35						1	2
36	3	4	5	6	7	8	9
37	10	11	12	13	14	15	16
38	17	18	19	20	21	22	23
39	24	25	26	27	28	29	30

$1-\cos t = v$

$\sin t\, dt = dv$

EYLÜL SEPTEMBER
CUMARTESİ SATURDAY
08

$$1-\cos t = 1-\left[1-2\sin^2\frac{t}{2}\right] = 2\sin^2\frac{t}{2}$$

$$S = 2\cdot 2\pi \int_0^{2\pi} (1-\cos t)\sqrt{2\sin^2\frac{t}{2}}\, dt$$

$$= 4\pi \int_0^{2\pi} 2\sin^2\frac{t}{2}\sin\frac{t}{2}\, dt$$

$$= 8\pi \int_0^{2\pi} \left(1-\cos^2\frac{t}{2}\right)\sin\frac{t}{2}\, dt$$

$\cos\frac{t}{2} = v$

$-\frac{1}{2}\sin\frac{t}{2}\, dt = dv$

$$= 8\pi \int_0^{2\pi} (1-v^2)(-2dv)$$

$$= -16\pi \left[v-\frac{v^3}{3}\right]\Big|_0^{2\pi}$$

$$= -16\pi \left[\cos\frac{t}{2} - \frac{\cos^3 t/2}{3}\right]\Big|_0^{2\pi}$$

EYLÜL SEPTEMBER
PAZAR SUNDAY
09

$$= -16\pi \left[\left(-1+\frac{1}{3}\right)-\left(1-\frac{1}{3}\right)\right]$$

$$= \frac{64}{3}\pi\, br^2$$

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Genelleştirilmiş (Hatalı Olmayan) İntegral

I. Tip Sınırlardan en az bir sonsuz olsun diğer sınır sayı ise yerine yazdığında sorun olmaması gerekir.

(a) $y=f(x)$ fonk $[a,t]$ integrallenebilir olsun

$$\int_a^{\infty} f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$$

(b) $y=f(x)$ fonk $[t,b]$ için

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{t\to-\infty} \int_t^b f(x)\,dx$$

(c) $y=f(x)$ fonk için

$$\int_{-\infty}^{+\infty} f(x)dx = \lim_{t\to-\infty} \int_t^c f(x)\,dx + \lim_{s\to\infty} \int_c^s f(x)\,dx$$

Bu integrallerin sonucu varsa "yakınsak" sayı yoksa sonuç sonsuz veya belirsiz ise "ıraksak" denir.

$\frac{e^x}{\ln a}$

$$\int_1^\infty e^{-x}\,dx$$

$$= \lim_{t\to\infty} \int_1^t e^{-x}\,dx$$

$$= \lim_{t\to\infty} \left(-e^{-x}\right)\Big|_1^t$$

$$= \lim_{t\to\infty} \left(-e^{-t} + e^{-1}\right)$$

$$= \lim_{t\to\infty} \left(-\frac{1}{e^t} + \frac{1}{e}\right)$$

$$= \frac{-1}{e^\infty} + \frac{1}{e}$$

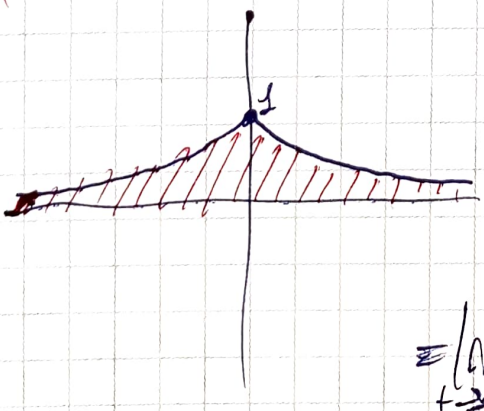
$$= \frac{-1}{\infty} + \frac{1}{e}$$

$$= 0 + \frac{1}{e} = \frac{1}{e}$$

sonuç çıktı yani yakınsak yakınsaklık oranı $(1/e)$'dir.

$y = \frac{1}{1+x^2}$ ile ox-ek arasından kalan alan

$$A = \int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$



$$= \lim_{t\to-\infty} \int_t^0 \frac{1}{1+x^2}\,dx + \lim_{s\to\infty} \int_0^s \frac{1}{1+x^2}\,dx$$

$$= \lim_{t\to-\infty} \left(\arctan x \Big|_t^0\right) + \lim_{s\to\infty} \left(\arctan x \Big|_0^s\right)$$

$= \lim_{t\to-\infty} (\arctan 0 - \arctan t) + \lim_{s\to\infty} (\arctan s - \arctan 0)$

$= -\arctan(-\infty) + \arctan(\infty)$

$= \frac{\pi}{2} + \frac{\pi}{2}$

$= \pi$ br² yakınsak

$\int_0^\infty x^2 dx$

$= \lim_{t\to\infty} \int_0^t x^2 dx$

$= \lim_{t\to\infty} \left(\frac{x^3}{3}\right)\Big|_0^t$

$= \lim_{t\to\infty} \left(\frac{t^3}{3}\right) = \infty$ ıraksak

II. TİP

(a) $y = f(x)$, $[a,b]$

$\int_a^b f(x)dx = \lim_{t\to b} \int_a^t f(x)dx$

Sınırsız Sayı

(b) $y = f(x)$, $[a,b]$

$\int_a^b f(x)dx = \lim_{t\to a} \int_t^b f(x)dx$

(c) $y = f(x)$, $[a,b]$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$= \lim_{t \to a} \int_t^c f(x)\,dx + \lim_{z \to b} \int_c^z f(x)\,dx$$

$$\int_0^1 \frac{dx}{\sqrt[4]{1-x}}$$

$$= \lim_{t \to 1} \int_0^t (1-x)^{-1/4}\,dx$$

$$= \lim_{t \to 1} \left[-\frac{4}{3}(1-x)^{3/4} \Big|_0^t \right]$$

$1-x = v$
$-dx = dv$

$$\int v^{-1/4}(-dv)$$

$$= -\frac{4}{3}(1-x)^{3/4}$$

$$= \lim_{t \to 1} \left[-\frac{4}{3}\left((1-t)^{3/4} - 1 \right) \right]$$

$$= 4/3$$

yakınsak

$$\int_{-1}^{1} \frac{1}{x^2}\,dx \to \int_{-1}^{0} \frac{1}{x^2}\,dx + \int_{0}^{1} \frac{1}{x^2}\,dx$$

II. Tip

$$\int_0^2 \frac{1}{x^2}\,dx$$

$$= \lim_{t\to 0} \int_t^2 \frac{1}{x^2}\,dx$$

$$= \lim_{t\to 0} \int_t^2 x^{-2}\,dx$$

$$= \lim_{t\to 0} \left(-x^{-1}\right)\Big|_t^2$$

$$= \lim_{t\to 0} \left(-\frac{1}{2} + \frac{1}{t}\right)$$

$$= \infty$$

İraksak

$$\int_{-1}^{1} \frac{1}{1-x^2}\,dx$$

$$= \int_{-1}^{0} \frac{1}{1-x^2}\,dx + \int_{0}^{1} \frac{1}{1-x^2}\,dx$$

$$= \lim_{t\to -1} \int_t^0 \frac{1}{1-x^2}\,dx + \lim_{s\to 1} \int_0^s \frac{1}{1-x^2}\,dx$$

$$= \lim_{t\to -1} \arcsin x\Big|_t^0 + \lim_{s\to 1} \arcsin x\Big|_0^s$$

$$= \lim_{t\to -1} (\arcsin 0 - \arcsin t) + \lim_{s\to 1} (\arcsin s - \arcsin 0)$$

$$\int \frac{1}{1-x^2}\,dx = \frac{A}{1-x} + \frac{B}{1+x} \qquad = \frac{1}{2} \ln\left|\frac{1+x}{1-x}\right|$$

$$= \lim_{t\to -1}\left[\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|\Big|_t^0\right] + \lim_{s\to 1}\left[\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|\Big|_0^s\right]$$

$$= \lim_{t\to -1}\frac{1}{2}\left[\ln 1 - \ln\left|\frac{1+t}{1-t}\right|\right] + \lim_{s\to 1}\left[\frac{1}{2}\left[\ln\left|\frac{1+s}{1-s}\right| - \ln 1\right]\right]$$

$$= \frac{\infty}{2} + \frac{\infty}{2} = \infty$$ ıraksak

III. Tür sayının bir sonsuz sayının diğer sıkıntı çıkarır.

$\int_a^{\infty} f(x)\,dx$ → sonsuz, → sıkıntı

III. Tip ⇒ II. tip ≠ I. tip

$\int_{-\infty}^{a} f(x)\,dx$ → sıkıntı, → sonsuz

$\int_{-\infty}^{\infty} f(x)\,dx$ arada a sayısı sıkıntı.

$$\int_{-\infty}^{\infty} = \int_{-\infty}^{-1} + \int_{-1}^{0} + \int_{0}^{1} + \int_{1}^{\infty}$$

$x^{-1} - x^{-2}$

$$\int_0^\infty \frac{1}{x^2}\,dx = \underbrace{\int_1^\infty \frac{1}{x^2}\,dx}_{\text{I. tip}} + \underbrace{\int_0^1 \frac{1}{x^2}\,dx}_{\text{II. tip}}$$

$$= \lim_{t\to\infty} \int_1^t \frac{1}{x^2}\,dx + \lim_{s\to 0} \int_s^1 \frac{1}{x^2}\,dx$$

$$\int_1^\infty \frac{dx}{x^3} = \lim_{t\to\infty} \int_1^t \frac{dx}{x^3}$$

$$= \lim_{t\to\infty} \frac{-1}{2x^2}\Big|_1^t$$

$$= \lim_{t\to\infty} \left(\frac{-1}{2t^2} + \frac{1}{2}\right)$$

$$= 0 + \frac{1}{2}$$

$$= 1/2 \quad \text{(yakınsak)}$$

$$\int_2^\infty \frac{dx}{x\sqrt{x^2-1}} = \lim_{t\to\infty} \int_2^t \frac{dx}{x\sqrt{x^2-1}}$$

$$\int \frac{dx}{x\sqrt{x^2-1}}$$

$\frac{1}{x} = t \Rightarrow \frac{1}{t} = x$

$\frac{-1}{x^2}dx = dt$

$\frac{-1}{t^2}dt = dx$

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$\lim \frac{2}{0} = \infty$

$\frac{2}{0}$ = tanımsız

$x^{-3/2}$ $-\frac{1}{2}$ $x^{3/2}$

$-2x^{-1/2}$

$$= \int \frac{t \cdot \frac{-1}{t^2}\,dt}{\sqrt{\frac{1}{t^2} - t}} = \int_b \frac{-dt}{\frac{\sqrt{1-t^2}}{t}} = -\int \frac{dt}{\sqrt{1-t^2}}$$

$= -\arcsin t$

$= -\arcsin\left(\frac{1}{x}\right)$

$$= \lim_{t \to \infty} \left(-\arcsin\left(\frac{1}{x}\right)\right)\Big|_2^t$$

$$= \lim_{t \to \infty} \left(-\arcsin\frac{1}{t} + \arcsin\frac{1}{2}\right) = 0 + \frac{\pi}{6} = \frac{\pi}{6}$$

$$\int_0^\infty x e^{-x}\,dx$$

ÖDEV

$$\int_0^1 \frac{dx}{x\sqrt{x}}$$

$$= \lim_{t \to 0} \int_t^1 \frac{dx}{x\sqrt{x}}$$

$$= \lim_{t \to 0} \left(-2x^{-1/2}\right)\Big|_t^1$$

$$= \lim_{t \to 0} \left(-2 + \frac{2}{\sqrt{t}}\right)$$

$= -2 + \infty$

$= \infty$

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~~arcsin x~~ =

$$\int_0^1 \frac{\arcsin x}{\sqrt{1-x^2}}\,dx = \lim_{t\to 1} \int_0^t \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$$

$\arcsin x = u$

$\frac{1}{\sqrt{1-x^2}}\,dx = du$

$$= \lim_{t\to 1} \left(\frac{(\arcsin x)^2}{2} \right) \Big|_0^t \qquad \left(\frac{u^2}{2}\right)$$

$$= \lim_{t\to 1} \left(\frac{\arcsin^2 t}{2} - \frac{\arcsin^2 0}{2} \right)$$

$$= \frac{\pi^2}{8} \quad \text{(yakınsak)}$$

$$\int_0^1 \frac{dx}{\sqrt{x-x^2}} = \underbrace{\int_0^{1/2} \frac{dx}{\sqrt{x-x^2}}}_{a} + \underbrace{\int_{1/2}^1 \frac{dx}{\sqrt{x-x^2}}}_{b}$$

(a) $$\int_0^{1/2} \frac{dx}{\sqrt{x-x^2}} = \lim_{t\to 0} \int_t^{1/2} \frac{dx}{\sqrt{x-x^2}}$$

$$\int \frac{dx}{\sqrt{x-x^2}} = \int \frac{dx}{\sqrt{x^2\left(\frac{1}{x}-1\right)}} = \int \frac{dx}{x\sqrt{\frac{1}{x}-1}}$$

$\frac{1}{x} = v$

$\frac{1}{x^2}dx = -dv$

$$= \int \frac{dv}{\sqrt{\left(\frac{1}{2}\right)^2 - v^2}}$$

$x - x^2 = -\left[x^2 - x\right]$

$= -\left[\left(x-\frac{1}{2}\right)^2 - \frac{1}{4}\right]$

$= \frac{1}{4} - \left(x-\frac{1}{2}\right)^2$

$= \left(\frac{1}{2}\right)^2 - v^2$

$x - \frac{1}{2} = v$

$dx = dv$

$= \arcsin \frac{v}{1/2}$

$= \arcsin 2v$

$= \arcsin(2x-1)$

$$= \lim_{t\to 0} \arcsin(2x-1)\Big|_t^{1/2}$$

$$= \lim_{t\to 0} \left(\arcsin 0 - \arcsin(2t-1)\right)$$

$= \arcsin 0 - \arcsin(-1)$

$= 0 - \left(-\frac{\pi}{2}\right) = \pi/2$

(b)

$$= \lim_{t\to 1} \arcsin(2x-1)\Big|_{1/2}^{t}$$

$$= \lim_{t\to 1} \left(\arcsin(2t-1) - \arcsin 0\right)$$

$= \arcsin 1 - \arcsin 0$

$= \pi/2$

(a)+(b) $\Rightarrow \frac{\pi}{2} + \frac{\pi}{2} = \pi$

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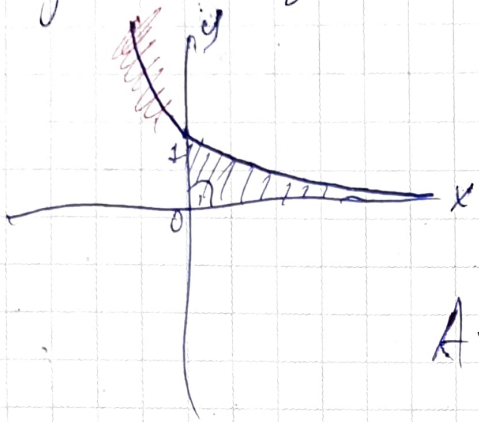
$y = e^{-x}$, Oy-ek, Ox-ek



$$A = \int_0^{\infty} e^{-x}\,dx$$

$$= \lim_{t\to\infty} \int_0^{t} e^{-x}\,dx$$

$$= \lim_{t\to\infty} \left(-e^{-x}\right)\Big|_0^{t}$$

$$= \lim_{t\to\infty} \left(-e^{-t} + 1\right)$$

$$= 1 \text{ br}^2$$

$y = e^{-x}$, Oy-ek, Ox-ek, Ox-etrafında dönme
H=?

$$H = \pi \int_a^b f^2\,dx$$

$$H = \pi \int_0^{\infty} e^{-2x}\,dx$$

$$= \pi \lim_{t\to\infty} \int_0^{t} e^{-2x}\,dx$$

Ox-et dr. $S = 2\pi \int_0^4 |y| \sqrt{1+(y')^2}\, dx$

Oy-et dr. $S = 2\pi \int_0^{\cdot} |x| \sqrt{1+(x')^2}\, dy$

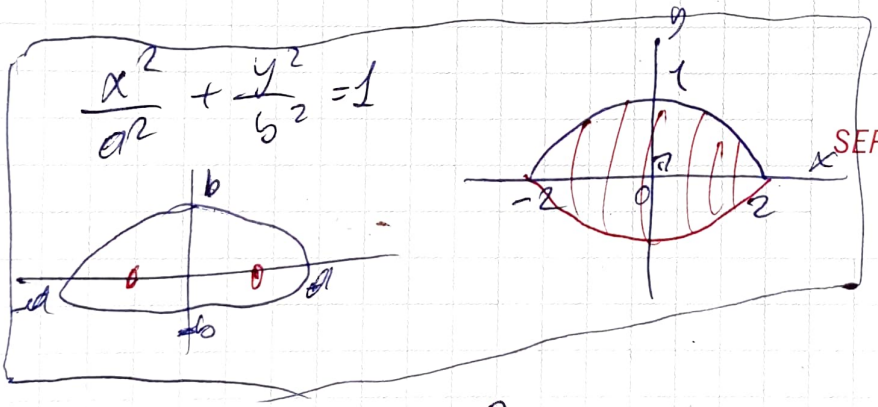
$$= \pi \lim_{t\to\infty} \left(-\frac{e^{-2t}}{2} + \frac{e^0}{2} \right)$$

$$= \pi/2 \quad br^3$$

$x^2 + 4y^2 = 4$ elipsi üst yarısı Ox-et dr. $S = ?$

$$y = \frac{\mp\sqrt{4-x^2}}{2}$$

$$y' = \frac{1}{2}(-2x)\frac{1}{2\sqrt{4-x^2}} = \frac{-x}{2\sqrt{4-x^2}}$$



$$(y')^2 = \frac{x^2}{4(4-x^2)}$$

$$S = 2\pi \int_{-2}^{2} \left| \frac{\sqrt{4-x^2}}{2} \right| \sqrt{1 + \frac{x^2}{4(4-x^2)}}\, dx$$

$$S = 2\pi \int_0^2 \sqrt{4-x^2}\, \frac{\sqrt{16-3x^2}}{2\sqrt{4-x^2}}\, dx$$

$$S = \pi \int_0^{\cdot} \sqrt{16-3x^2}\, dx$$

$$= \pi \int_0^{\sqrt{3}} \sqrt{16 - 3x^2}\, dx$$

$x = a \sin t$

$a^2 = x^2$

$x^2 - a^2$

$a^2 + x^2$

$$x = \frac{4}{\sqrt{3}} \sin t$$

$$dx = \frac{4}{\sqrt{3}} \cos t\, dt$$

$$16 - 3x^2$$

$$\left(\frac{4}{\sqrt{3}}\right)^2 - x^2$$

$$= \pi \int_0^{2} \sqrt{16 - 3\,\frac{16}{3}\sin^2 t}\cdot \frac{4}{\sqrt{3}} \cos t\, dt$$

$$= \frac{4}{\sqrt{3}} \pi \int_0^{2} 4 \cos t \cos t\, dt$$

$$= \frac{16}{\sqrt{3}} \pi \int_0^{2} \cos^2 t\, dt$$

$$\cos^2 t = \frac{1}{2}[1 + \cos 2t]$$

$$= \frac{16}{\sqrt{3}} \pi \int_0^{2} \frac{1}{2}[1 + \cos 2t]\, dt$$

$$= \frac{8}{\sqrt{3}} \pi \left(t + \frac{\sin 2t}{2} \right) \Bigg|_{0=0}^{2=\pi/3}$$

$x = 0 \Rightarrow t = 0$

$x = 2 \Rightarrow t = \pi/3$

$$= \frac{8}{\sqrt{3}} \pi \left(\frac{\pi}{3} + \frac{1}{4} \right) br^2$$

Ox-etrafı dön $\Rightarrow S = 2\pi \int_{t_1}^{t_2} |y| \sqrt{(x')^2+(y')^2}\, dt$

Oy-etrafı dön $\Rightarrow S = 2\pi \int_{t_1}^{t_2} |x| \sqrt{(x')^2+(y')^2}\, dt$

$x = e^t \sin t \qquad 0 \le t \le \pi/2$

$y = e^t \cos t$

Ox-etrafı dön yüzey alanı = ?

$x' = e^t \sin t + e^t \cos t$

$y' = e^t \cos t - e^t \sin t$

$(x')^2 = e^{2t}\sin^2 t + 2e^{2t}\sin t\cos t + e^{2t}\cos^2 t$

$(y')^2 = e^{2t}\cos^2 t - 2e^{2t}\sin t\cos t + e^{2t}\sin^2 t$

$(x')^2 + (y')^2 = 2e^{2t}$

$$S = 2\pi \int_0^{\pi/2} |e^t \cos t| \sqrt{2e^{2t}}\, dt$$

$$= 2\sqrt{2}\pi \int_0^{\pi/2} e^{2t}\cos t\, dt$$

LAPTO

$x = t^3$
$y = 2t + 3$
$-1 \le t \le 1$

Oy-et. dön
$S = ?$

$x' = 3t^2$
$y' = 2$
$(x')^2 + (y')^2 = 9t^4 + 4$

$$S = 2\pi \int_{-1}^{1} |t^3| \sqrt{9t^4 + 4}\, dt$$

$$= 4\pi \int_{0}^{1} t^3 \sqrt{9t^4 + 4}\, dt.$$

$9t^4 + 4 = u$
$36t^3 dt = du$

$$= 4\pi \int_{0}^{1} \sqrt{u}\, \frac{du}{36}$$

$$= \frac{1}{9}\pi \int_{0}^{1} \sqrt{u}\, du$$

$$= \frac{2\pi}{27}\left[13^{3/2} - 8\right] br^2$$

$x = 6\cos t$
$y = t + \sin t$
$-\pi \le t \le \pi$
eğri varlığı

$$\ell = \int_{t_1}^{t_2} \sqrt{(x')^2 + (y')^2}\, dt$$

$x' = -\sin t$
$y' = 1 + \cos t$
$(x')^2 + (y')^2 = \sin^2 t + 1 + 2\cos t + \cos^2 t$
$= 2 + 2\cos t$

$$\ell = \int_{-\pi}^{\pi} \sqrt{2}\sqrt{1+\cos t}\, dt$$

$\cos t = 2\cos^2 \frac{t}{2} - 1$

$$= 2\sqrt{2} \int_0^{\pi} \sqrt{2\cos^2 \tfrac{t}{2}}\, dt$$

$$= 4 \int_0^{\pi} \cos \tfrac{t}{2}\, dt$$

$$= 8 \sin \tfrac{t}{2} \Big|_0^{\pi}$$

$= 8$ br

$\frac{\sin 2x}{\cos}$

$\frac{1}{\cos 2x}$

$x = \ln(\sec y)$ eğrisi $y=0$, $y=\frac{\pi}{3}$ doğruları

x sınır $\longrightarrow$ $l = \int_a^b \sqrt{1+(y')^2}\,dx$

y sınır $\longrightarrow$ $l = \int_c^d \sqrt{1+(x')^2}\,dy$

$x' = \frac{1}{\sec y}\cdot(\sec y \tan y) = \tan y$

$$l = \int_0^{\pi/3} \sqrt{1+\tan^2 y}\,dy$$

$$= \int_0^{\pi/3} \sqrt{\sec^2 y}\,dy$$

$$= \int_0^{\pi/3} \sec y\,dy$$

$(\sec y + \tan y)$

$$= \int_0^{\pi/3} \frac{\sec^2 y + \sec y \tan y}{\sec y + \tan y}\,dy$$

$\sec y + \tan y = t$

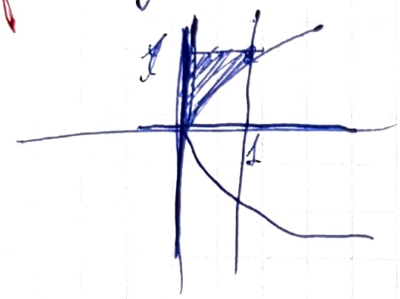
$(\sec^2 y + \sec y \tan y)\,dy = dt$

$$= \ln|\sec y + \tan y| \Big|_0^{\pi/3}$$

$$= \ln\left|\sec\tfrac{\pi}{3} + \tan\tfrac{\pi}{3}\right| - \ln|\sec 0 + \tan 0|$$

$$= \ln|2+\sqrt{3}| - \ln|1+0| = \ln|2+\sqrt{3}| \text{ br}$$

$x=y^2$, $x=0$, $y=1$ Ox. etr. dön $H=?$



$$H=\pi\int_0^1 (\sqrt{x})^2\,dx$$

$$=\pi\left[\frac{x^2}{2}\right]_0^1$$

$$=\pi/2\,br^3$$

Kabuk

$$H=2\pi\int_0^b |y[f(y)-g(y)]|\,dy$$

$$=2\pi\int_0^1 |y(y^2-0)|\,dy$$

$$=2\pi\int_0^1 y^3\,dy$$

$$=\pi/2\,br^3$$

Disk

$$H=\pi\int_0^1 (f(x))^2-(g(y))^2\,dx$$

$$=\pi\int_0^1 1^2-(\sqrt{x})^2\,dx$$

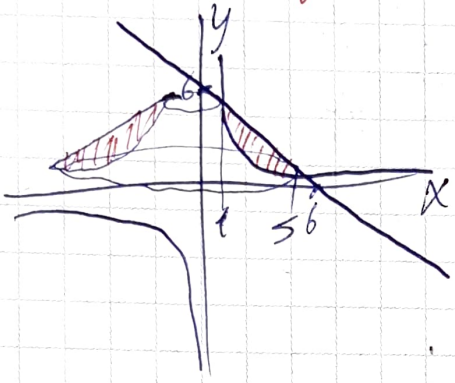
$$=\pi\int_0^1 (1-x)\,dx$$

$$=\pi\left[x-\frac{x^2}{2}\right]_0^1$$

$$=\pi/2\,br^3$$

$xy=5 \rightarrow y=\frac{5}{x}$ Oyeik $H=?$

$y+x=6 \rightarrow y=6-x$



$\frac{5}{x}=6-x$

$5=6x-x^2$

$x^2-6x+5=0$

$x \quad -5$

$x \quad -1$

$x=5$

$x=1$

$$H=2\pi\int_1^5 |x[f(x)-g(x)]|dx$$

$$=2\pi\int_1^5 |x[(6-x)-(\frac{5}{x})]|dx$$

$$=2\pi\int_1^5 |6x-x^2-5|dx$$

$$=2\pi\int_1^5 (6x-x^2-5)dx$$

$-(x^2-6x+5)$

1 5

$+ \quad \phi \quad - \quad \phi \quad +$

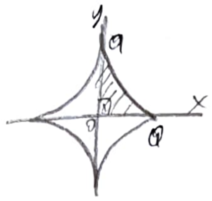
$- \quad \quad + \quad \quad -$

* Parametrik Denk. Verilen Eğrilerin Alanı

① $x = g(t)$, $y = h(t)$ $t_1 \leq t \leq t_2$ $x = a$, $x = b$ Ox - eksen $A = \int_{t_1}^{t_2} |y|\, x' \, dt$

② $x = g(t)$, $y = h(t)$ $t_3 \leq t \leq t_4$ $y = c$, $y = d$ Oy - ek $A = \int_{t_3}^{t_4} |x|\, y' \, dt$

✱ $x = a\cos^3 t$, $y = a\sin^3 t$



$x = 0$, $x = a$ $0 = a\cos^3 t$, $a = a\cos^3 t$ $t_1 = \pi/2$, $t_2 = 0$

$$A = \int_{\pi/2}^{0} |a\sin^3 t|\, 3a\cos^2 t\,(-\sin t)\, dt$$

$$= 3a^2 \int_0^{\pi/2} \sin^4 t \cos^2 t \, dt$$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$$= 3a^2 \int_0^{\pi/2} \left[\frac{1}{2}(1 - \cos 2t)\right]^2 \left[\frac{1}{2}(1 + \cos 2t)\right] dt$$

$$= \frac{3a^2}{8} \int_0^{\pi/2} (1 - 2\cos 2t + \cos^2 2t)(1 + \cos 2t)\, dt$$

$$= \frac{3a^2}{8} \int_0^{\pi/2} (\underbrace{\cos^3 2t}_{④} - \underbrace{\cos^2 2t}_{③} - \underbrace{\cos 2t}_{②} + \underbrace{1}_{①})\, dt$$

① $\int_0^{\pi/2} dt = \pi/2$

② $-\int_0^{\pi/2} \cos 2t \, dt = -\frac{1}{2}\sin 2t \Big|_0^{\pi/2} = -\frac{1}{2}(\sin\pi - \sin 0) = 0$

③ $-\int_0^{\pi/2} \cos^2 2t \, dt = -\int_0^{\pi/2} \frac{1}{2}(1 + \cos 4t)\, dt = -\frac{1}{2}\left(t + \frac{1}{4}\sin 4t\right)\Big|_0^{\pi/2} = -\frac{1}{2}\left[\left(\frac{\pi}{2} + \frac{1}{4}\sin 2\pi\right) - \left(0 + \frac{1}{4}\sin 0\right)\right] = -\frac{\pi}{4}$

④ $\int_0^{\pi/2} \cos^3 2x \, dx = \int_0^{\pi/2} \cos^3 2x \sin^0 2x \, dx = 0$ $\boxed{\begin{matrix}\sin 2t = u \\ 2\cos 2t\, dt = du\end{matrix}}$

$$4A = \frac{3a^2}{8} \cdot 4 \cdot \left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{3a^2}{4}$$

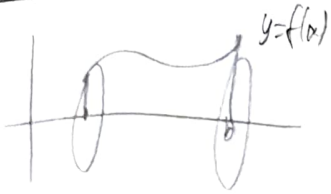
Bu konuda anlaşılmayan [illegible] bağdaştırma [illegible] Saygılarımla

HACİM HESABI

① Kesit Yöntemi (Düzgün cisimler)

$$H = \int_a^b A(x)\,dx \quad ; \quad H = \int_c^d A(y)\,dy$$

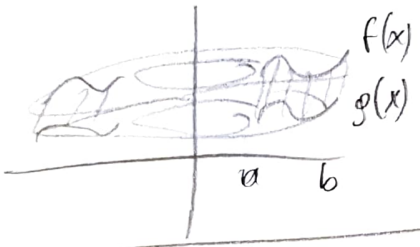
② Disk Yöntemi



$$H = \pi \int_a^b [f(x)]^2\,dx \qquad \approx \pi r^2$$

(sınırlar ile dönme yönü aynı)

③ Kabuk Yöntemi



$$H = 2\pi \int_a^b \left| x\,[f(x) - g(x)] \right| dx \qquad \approx (2\pi r)(r)$$

★ Parametrik Eğrilerin Hacimleri

① $x = u(t)$, $y = v(t)$ — $t_1 \le t \le t_2$ — Ox-etrafında dön

$$H = \pi \int_{t_1}^{t_2} y^2 x'\,dx$$

② $x = u(t)$, $y = v(t)$ — $t_3 \le t \le t_4$ — Oy-etrafında dön

$$H = \pi \int_{t_3}^{t_4} x^2 y'\,dy$$

EĞRİ UZUNLUĞU

① $y = f(x)$, $[a,b]$,

$$\ell = \int_a^b \sqrt{1 + (y')^2}\,dx$$

② $x = g(y)$, $[c,d]$,

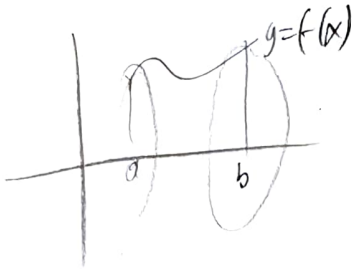
$$\ell = \int_c^d \sqrt{1 + (x')^2}\,dy$$

★ Parametrik Eğrilerin Eğri Uzunluğu

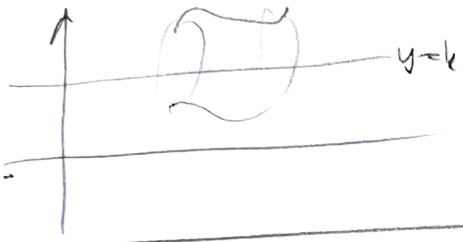
$x = u(t)$, $y = v(t)$ — $t_1 \le t \le t_2$

$$\ell = \int_{t_1}^{t_2} \sqrt{(x')^2 + (y')^2}\,dt$$

DÖNEL YÜZEYİN YÜZEY ALANI



$$S = 2\pi \int_a^b |f(x)| \sqrt{1 + (f'(x))^2}\, dx$$



$$S = 2\pi \int_a^b |f(x) - k| \sqrt{1 + (f'(x))^2}\, dx$$

* Parametrik Eğrilerin Oluşturduğu Yüzey Alanı

① $x = u(t)$, $y = v(t)$, $t_1 \le t \le t_2$, Ox-etrafı

$$S = 2\pi \int_{t_1}^{t_2} |y| \sqrt{(x')^2 + (y')^2}\, dt$$

② $x = u(t)$, $y = v(t)$, $t_3 \le t \le t_4$, Oy-etrafı

$$S = 2\pi \int_{t_3}^{t_4} |x| \sqrt{(x')^2 + (y')^2}\, dt$$

GENELLEŞTİRİLMİŞ İNTEGRAL

I. TİP Sınırların en az biri sonsuz, diğer sayı ise yerine koyunca sorun çıkmamalı

$$\int_a^{\infty} f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$$

$$\int_{-\infty}^{b} f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$$

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{t\to\infty} \int_t^c f(x)dx + \lim_{s\to\infty} \int_c^s f(x)dx$$

II. TİP

$$\int_a^{b} f(x)dx = \lim_{t\to b} \int_a^{t} f(x)dx$$

$$\int_a^{b} f(x)dx = \lim_{t\to a} \int_t^{b} f(x)dx$$

$$\int_a^{b} f(x)dx = \int_a^{c} f(x)dx + \int_c^{b} f(x)dx$$

$$= \lim_{t\to a} \int_t^{c} f(x)dx + \lim_{z\to b} \int_c^{z} f(x)dx$$

III. TİP

III. tip $\longrightarrow$ II. tip + I. tip

$$\int_a^{\infty} f(x)dx$$

$$\int_{-\infty}^{a} f(x)dx$$

$\int_{-\infty}^{\infty} f(x)dx \longrightarrow$ orada "a" sayısı sıfırdır

---

$$\int_0^{\infty} x e^{-x} dx$$