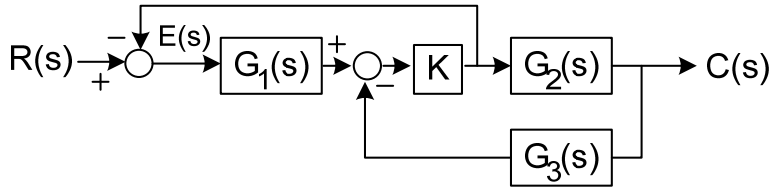


**S\_1**

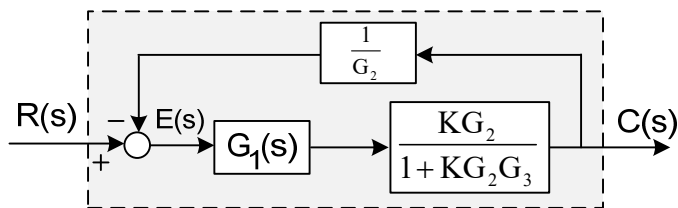
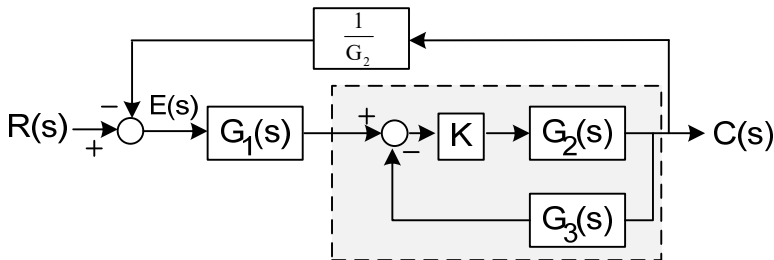
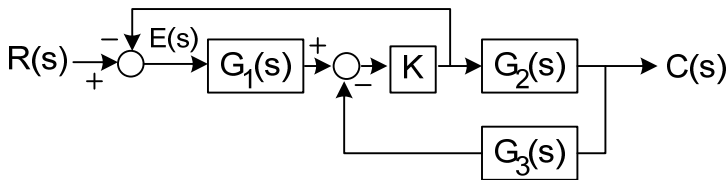
Aşağıda verilen Kontrol blok diyagramında,  $G_1(s) = \frac{1}{s+1}$ ,  $G_2(s) = 10$  verilmektedir.



Kapalı-çevrim transfer fonksiyonu elde ediniz.

$$\frac{C(s)}{R(s)} = ?$$

b)  $r(t) = u(t)$  için  $c(\infty) = 1$  olması için  $K = ?$  hesap ediniz.

**C1-a**

$$R(s) \rightarrow \frac{G_1 \frac{KG_2}{1 + KG_2G_3}}{1 + G_1 \frac{KG_2}{1 + KG_2G_3} \frac{1}{G_2}} C(s) \rightarrow$$

$$\frac{C(s)}{R(s)} = \frac{G_1 \frac{KG_2}{1+KG_2G_3}}{1+G_1 \frac{K \cancel{G_2}}{1+KG_2G_3} \frac{1}{\cancel{G_2}}} = \frac{G_1 \frac{KG_2}{1+KG_2G_3}}{\frac{1+KG_2G_3+G_1K}{1+KG_2G_3}} = \frac{G_1KG_2}{1+KG_2G_3+G_1K} = \frac{\frac{1}{s+1}K*10}{1+K*10*\frac{s}{s+5}+\frac{1}{s+1}K}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s+1}K*10}{1+K*10*\frac{s}{s+5}+\frac{1}{s+1}K} = \frac{\frac{1}{s+1}K*10}{\frac{(s+5)(s+1)+K*10*s(s+1)+(s+5)K}{(s+5)(s+1)}} = \frac{K*10(s+5)}{(s+5)(s+1)+K*10*s(s+1)+(s+5)K}$$

$$\frac{C(s)}{R(s)} = T(s) = \frac{10K(s+5)}{(s+5)(s+1)+10Ks(s+1)+(s+5)K}$$

$$\frac{C(s)}{R(s)} = T(s) = \frac{10K(s+5)}{s^2+s+5s+5+10Ks^2+10Ks+Ks+5K}$$

$$\frac{C(s)}{R(s)} = T(s) = \frac{10K(s+5)}{(10K+1)s^2+(11K+6)s+5+5K}$$

### C1-b

$$r(t) = u(t) \xrightarrow{s} R(s) = \frac{1}{s} \text{ bulunur.}$$

$$c(\infty) = \lim_{t \rightarrow \infty} c(t) \xrightarrow{s} c(\infty) = \lim_{s \rightarrow 0} sC(s) = \lim_{s \rightarrow 0} sT(s)R(s) \text{ dir.}$$

İstenen

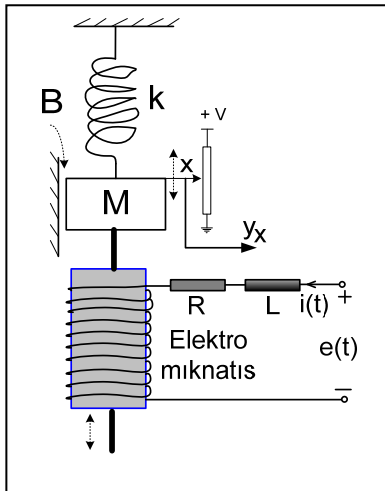
$$c(\infty) = \lim_{s \rightarrow 0} sT(s)R(s) = 1 \rightarrow$$

$$c(\infty) = \lim_{s \rightarrow 0} s \frac{10K(s+5)}{(10K+1)s^2+(11K+6)s+5+5K} \frac{1}{s} = 1 \rightarrow$$

$$\lim_{s \rightarrow 0} \cancel{s} \frac{10K(\cancel{0}+5)}{(10K+1)\cancel{0}^2+(11K+6)\cancel{0}+5+5K} \frac{1}{\cancel{s}} = 1$$

$$\frac{K*50}{5+5K} = 1 \rightarrow K = \frac{1}{9}$$

## S\_2



Yanda verilen kütle, yay ve elektromıknatıs sisteminde, elektro mıknatıs kuvveti  $F(t) = k_m \cdot i(t)$  olarak verilmektedir.

- Sisteme ait dinamik denklemleri yazınız. Kontrol blok diyagramını elde ediniz. ( $y_x = x(t)$  ölçülen konum)
- $y_r$  referans konum girişi olmak üzere, ayrık-zaman sayısal kapalı çevrim kontrol blok diyagramını çiziniz.

Not: Kütle yay dengede

## C2-a

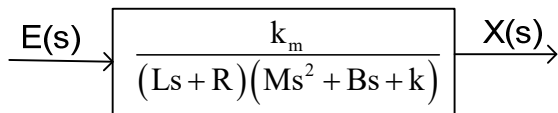
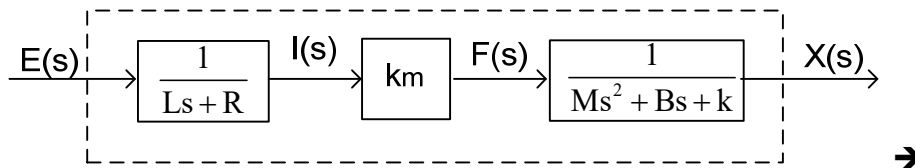
$$1) \quad e(t) = Ri(t) + L \frac{d}{dt} i(t) \xrightarrow{s} E(s) = RI(s) + LsI(s) \rightarrow I(s) = \frac{1}{Ls + R} E(s)$$

$$2) \quad f(t) = k_m i(t) \xrightarrow{s} F(s) = k_m I(s) \rightarrow F(s) = k_m I(s)$$

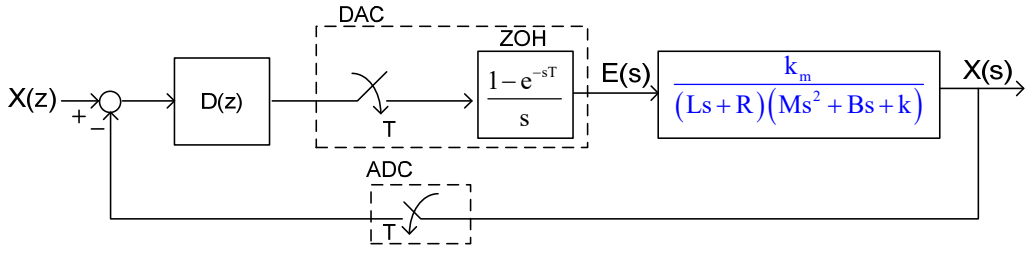
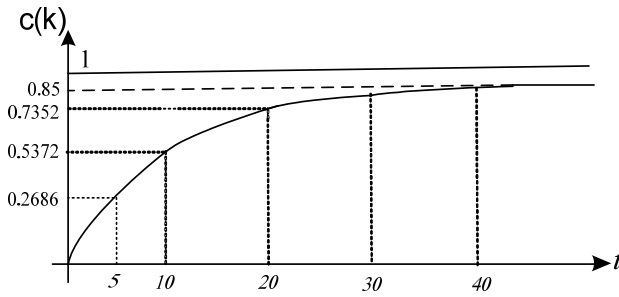
$$3) \quad f(t) = kx(t) + B \frac{d}{dt} x(t) + M \frac{d^2}{dt^2} x(t) \xrightarrow{s} F(s) = kX(s) + BsX(s) + Ms^2X(s) \rightarrow$$

$$X(s) = \frac{1}{Ms^2 + Bs + k} F(s)$$

blok diyagram



$$\frac{X(s)}{E(s)} = \frac{k_m}{(Ls + R)(Ms^2 + Bs + k)}$$

**C2-b****S\_3**

Yanda I. Dereceden sisteme ait açık-çevrim birim basamak cevabı verilmiştir.

a)  $G(s)$  transfer fonksiyonunu bulunuz.

b) örnekleme zamanı  $T = \frac{\tau}{5}$  olmak üzere

$\tau$  : zaman sabiti

Ayrık zaman açık-çevrim transfer fonksiyonu  $G(z)$  elde ediniz (ZOH' lu).

c) Birim basamak giriş için  $c(k)$  'yı elde ediniz ve  $k=5$  için  $c(k)$  'yı hesaplayınız.

**C3-a**

$\frac{K}{\tau s + 1}$  1. Dereceden modelin parametreleri deneysel yol ile elde edilecektir.

**Açık çevrim kazanç:**  $K$  ise  $K = \frac{v_{\text{çıkış}}(\infty)}{v_{\text{giriş}}(\infty)} = \frac{0.85}{1} = 0.85$  elde edilir.

**Açık çevrim zaman sabiti:**  $\tau$  ise  $0.632 * v_{\text{çıkış}}(\infty) = 0.5372$  'ye denk gelen zaman değeri olup birim basamak cevap eğrisinden  $\tau = 10sn$  olarak okunur.

1. Dereceden transfer fonksiyonunda parametreler yerlerine konursa ,

$$G(s) = \frac{0.85}{10s + 1} \text{ bulunur.}$$

**C3-b**

$$z \square e^{sT}, T = \frac{\tau}{5} = \frac{10}{5} = 2sn$$

$$X(z) = \sum_{i=1}^n \left\{ \frac{1}{(m-1)!} \frac{d^{m-1}}{ds^{m-1}} \left[ (s-s_i)^m X(s) \frac{z}{z-e^{sT}} \right] \right\}_{s=s_i}$$

$$Z\{T(s)G_{ZOH}(s)\} = TG_{ZOH}(z) = Z\left\{\frac{0.85}{10s+1} \frac{1-e^{-Ts}}{s}\right\} = \frac{0.85}{10}(1-z^{-1})Z\left\{\frac{1}{s(s+0.1)}\right\} \rightarrow$$

$$TG_{ZOH}(z) = 0.085 \frac{z-1}{z} \left\{ \cancel{\left(s - (-0.1)\right)} \frac{1}{s(s+0.1)} \frac{z}{z-e^{sT}} \Big|_{s=-0.1} + \cancel{\left(s - (0)\right)} \frac{1}{\cancel{s}(s+0.1)} \frac{z}{z-e^{sT}} \Big|_{s=0} \right\}$$

$$TG_{ZOH}(z) = 0.085 \frac{z-1}{z} \left\{ \frac{1}{-0.1} \frac{z}{z-e^{-0.1*2}} + \frac{1}{(0+0.1)} \frac{z}{z-e^{0*2}} \right\} = 0.085 \frac{z-1}{z} \left\{ -10 \frac{z}{z-0.8187} + 10 \frac{z}{z-1} \right\}$$

$$TG_{ZOH}(z) = 0.85 \frac{z-1}{\cancel{z}} \left\{ -\frac{\cancel{z}}{z-0.8187} + \frac{\cancel{z}}{z-1} \right\} = 0.85 \left( 1 - \frac{z-1}{z-0.8187} \right) = 0.85 \left( \frac{z-0.8187-z+1}{z-0.8187} \right) \rightarrow$$

$$TG_{ZOH}(z) = \frac{0.1541}{z-0.8187}$$

**C3-c**

Giriş işareti  $r(t) = u(t) \xrightarrow{s} R(s) = \frac{1}{s} \xrightarrow{z} R(z) = \frac{z}{z-1}$  bulunur.

$$\frac{C(z)}{R(z)} = TG_{ZOH}(z)$$

$$C(z) = TG_{ZOH}(z) R(z)$$

$$C(z) = TG_{ZOH}(z) R(z) = \frac{0.1541}{z-0.8187} \frac{z}{z-1}$$

$$C(z) = \frac{0.1541z}{(z-0.8187)(z-1)}$$

$$x(k) = \sum_{i=1}^n \left\{ \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \left[ (z-z_i)^m X(z) z^{k-1} \right] \right\}_{z=z_i}$$

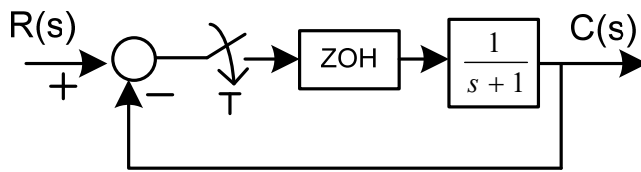
$$c(k) = Z\{C(z)\} = Z\left\{ \frac{0.1541z}{(z-0.8187)(z-1)} \right\}$$

$$c(k) = \left\{ \cancel{(z-(0.8187))} \frac{0.1541z}{(z-0.8187)(z-1)} z^{k-1} \right\}_{z=0.8187} + \left\{ \cancel{(z-1)} \frac{0.1541z}{(z-0.8187)(z-1)} z^{k-1} \right\}_{z=1}$$

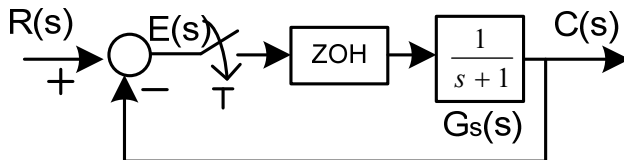
$$c(k) = \frac{0.1541 * 0.8187^k}{(0.8187-1)} + \frac{0.1541 * 1^k}{(1-0.8187)} = 0.85(1-0.8187^k)$$

$$c(k) = 0.85(1-0.8187^k) \Rightarrow c(5) = 0.85(1-0.8187^5) \Rightarrow c(5) = 0.5373$$

**S\_4** T= 0.1 sn için  $r(t) = u(t)$  için  $C(z)$ 'i elde ediniz.  $C(\infty)$  değerini hesap ediniz.



**C4**



$$E(s) = R(s) - C(s) \Rightarrow E^*(s) = R^*(s) - C^*(s)$$

$$C(s) = (R^*(s) - C^*(s)) G_{zoh}(s) G_s(s) = R^*(s) G_{zoh}(s) G_s(s) - C^*(s) G_{zoh}(s) G_s(s) \Rightarrow$$

$$C^*(s) = R^*(s) G_{zoh} G_s^*(s) - C^*(s) G_{zoh} G_s^*(s)$$

$$C^*(s) + C^*(s) G_{zoh} G_s^*(s) = R^*(s) G_{zoh} G_s^*(s)$$

$$\frac{C^*(s)}{R^*(s)} = \frac{G_{zoh} G_s^*(s)}{1 + G_{zoh} G_s^*(s)}$$

$$\frac{C(z)}{R(z)} = \frac{G_{zoh} G_s(z)}{1 + G_{zoh} G_s(z)} \Rightarrow \text{doğrudan yazılabilir. (Çıkarım bilgi amaçlı verilmiştir.)}$$

$z \neq e^{sT}$  olduğu göz önüne alınır ve  $T = 0.1$  örnekleme zamanı olmak üzere  $z$ - dönüşümü yapılır.

$$X(z) = \sum_{i=1}^n \left\{ \frac{1}{(m-1)!} \frac{d^{m-1}}{ds^{m-1}} \left[ (s-s_i)^m X(s) \frac{z}{z-e^{sT}} \right] \right\}_{s=s_i}$$

$$G_{zoh} G_s(z) = Z \{ G_{zoh}(s) G_s(s) \} = Z \left\{ \frac{1-e^{-sT}}{s} \frac{1}{s+1} \right\} = (1-z^{-1}) Z \left\{ \frac{1}{s(s+1)} \right\}$$

$$G_{zoh} G_s(z) = (1-z^{-1}) \left\{ \cancel{(s-(0))} \frac{1}{s(s+1)} \frac{z}{z-e^{sT}} \Big|_{s=0} + \cancel{(s-(-1))} \frac{1}{s(s+1)} \frac{z}{z-e^{sT}} \Big|_{s=-1} \right\}$$

$$G_{zoh} G_s(z) = (1-z^{-1}) \left\{ \frac{1}{(0+1)} \frac{z}{z-e^{0*0.1}} + \frac{1}{-1} \frac{z}{z-e^{(-1)*0.1}} \right\}$$

$$G_{zoh} G_s(z) = \left( \frac{z-1}{z} \right) \left\{ \frac{z}{z-1} - \frac{z}{z-0.9048} \right\} = 1 - \frac{z-1}{z-0.9048}$$

$$G_{zoh} G_s(z) = \frac{0.0952}{z-0.9048} \rightarrow \text{ileri yol transfer fonksiyonu elde edilir.}$$

$$\frac{C(z)}{R(z)} = \frac{\frac{0.0952}{z-0.9048}}{1 + \frac{0.0952}{z-0.9048}} = \frac{\frac{0.0952}{z-0.9048}}{\frac{z-0.9048+0.0952}{z-0.9048}}$$

$$\frac{C(z)}{R(z)} = \frac{0.0952}{z-0.8096} \quad \text{Kapalı çevrim transfer fonksiyonu elde edilir. Buradan}$$

$$C(z) = \frac{0.0952}{z-0.8096} R(z) \quad \text{cevap ifadesi yazılır.}$$

$$\text{Giriş işareti } r(t) = u(t) \xrightarrow{s} R(s) = \frac{1}{s} \xrightarrow{z} R(z) = \frac{z}{z-1} \text{ dir.}$$

$$C(z) = \frac{0.0952}{z-0.8096} \frac{z}{z-1}$$

$$C(z) = \frac{0.0952z}{(z-0.8096)(z-1)}$$

$$c(\infty) = \lim_{t \rightarrow \infty} c(t) = \lim_{s \rightarrow 0} sC(s) = \lim_{z \rightarrow 0} (z-1)C(z)$$

$$c(\infty) = \lim_{z \rightarrow 1} \cancel{(z-1)} \frac{0.0952z}{(z-0.8096)\cancel{(z-1)}} = \frac{0.0952*1}{(1-0.8096)\cancel{(z-1)}}$$

$$c(\infty) = 0.5$$