BÖLÜM 2
VEKTÖR ANALİZİ

2.1. GİRİŞ

Bu bölümde 3 ana konu incelenmektedir:

1. Vektör Cebiri;
　　　　Vektörlerin toplanması, çıkarılması, çarpılması

2. Ortogonal Koordinat Sistemleri;
　　　　Kartezyen Koordinatlar
　　　　Silindirik Koordinatlar
　　　　Küresel Koordinatlar

3. Vektör Hesabı;
　　　　Vektörlerin Türevi ve Integrasyonu
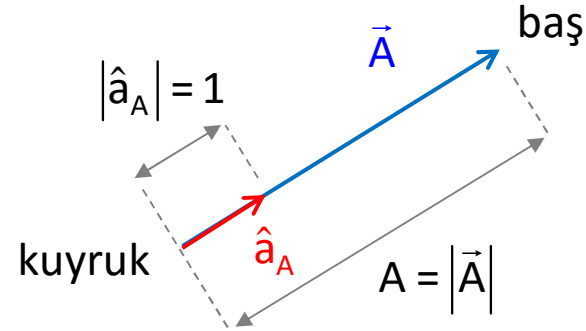　　　　Gradyant
　　　　Diverjans (Iraksama)
　　　　Rotasyonel (Dönel, Curl)

2.2. VEKTÖRLERİN TOPLANMASI VE ÇIKARILMASI

✓ Vektör

Bir $\vec{A}$ vektörü aşağıdaki gibi yazılır.

$$\vec{A} = \hat{a}_A A$$

$|\hat{a}_A| = 1$

$\vec{A}$ baş

$\hat{a}_A$

kuyruk

$A = |\vec{A}|$

$\hat{a}_A$;

    Birim vektördür

    Birim büyüklüğe sahiptir $|\hat{a}_A| = 1$

    Boyutsuzdur

    $\vec{A}$'nın yönünü belirtir

    $\hat{a}_A = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{\vec{A}}{A}$

A;

    $\vec{A}$'nın büyüklüğüdür
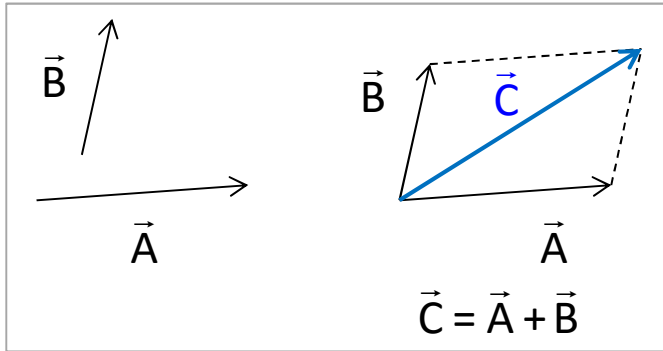
    $\vec{A}$ ile aynı birime sahiptir

    $A = |\vec{A}|$

✓ İki vektör uzayda farklı yerlerde de olsa eğer aynı büyüklüğe ve aynı yöne sahipse eşittir.
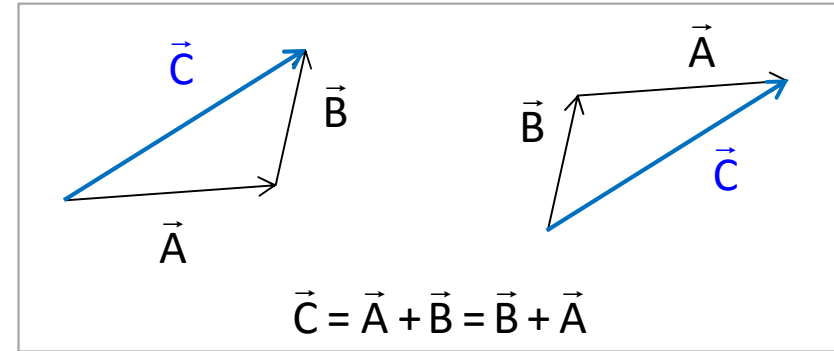
✓ Vektörlerin toplanması

Aynı veya zıt yönde olmayan iki $\vec{A}$ ve $\vec{B}$ vektörü bir düzlem belirler. Bunların toplamı aynı düzlemde yer alan bir $\vec{C}$ vektörüdür. $\vec{C} = \vec{A} + \vec{B}$ grafik olarak iki yolla elde edilir :

1. Paralelkenar kuralı

$$\vec{C} = \vec{A} + \vec{B}$$

2. Baş-Kuyruk kuralı

$$\vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

✓ Vektörlerin çıkarılması

1. Paralelkenar kuralı

$$\vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

2. Baş-Kuyruk kuralı

$$\vec{D} = -\vec{B} + \vec{A} = \vec{A} - \vec{B}$$

2.3. VEKTÖRLERİN ÇARPIMI

<u>vektörlerin çarpımı</u>

<u>skalar-vektör çarpımı</u>

$$k\vec{A} = \hat{a}_A(kA)$$

$$\frac{1}{k}\vec{A} = \hat{a}_A\left(\frac{1}{k}A\right)$$

k pozitifse vektörün yönü değişmez, büyüklüğü k veya 1/k kat değişir.

<u>vektör-vektör çarpımı</u>

<u>skalar (nokta) çarpım</u>

$$\vec{A}\bullet\vec{B}$$

<u>vektör (çapraz) çarpım</u>

$$\vec{A}\times\vec{B}$$

"iki vektörün çarpımı" ifadesi yeterli değildir.
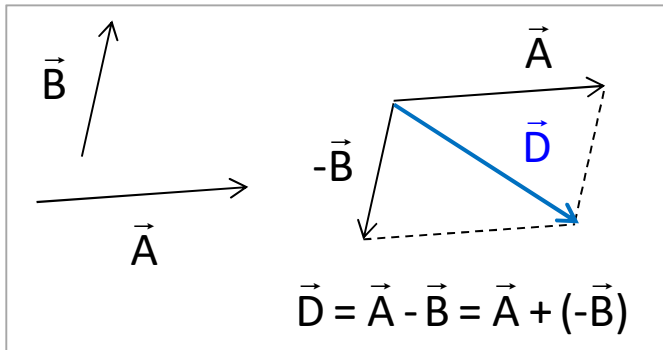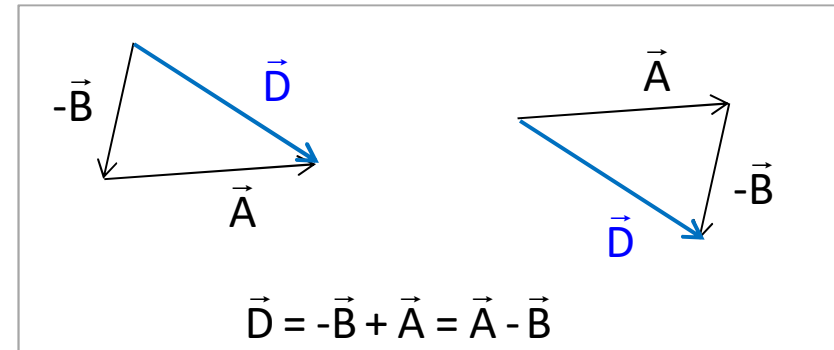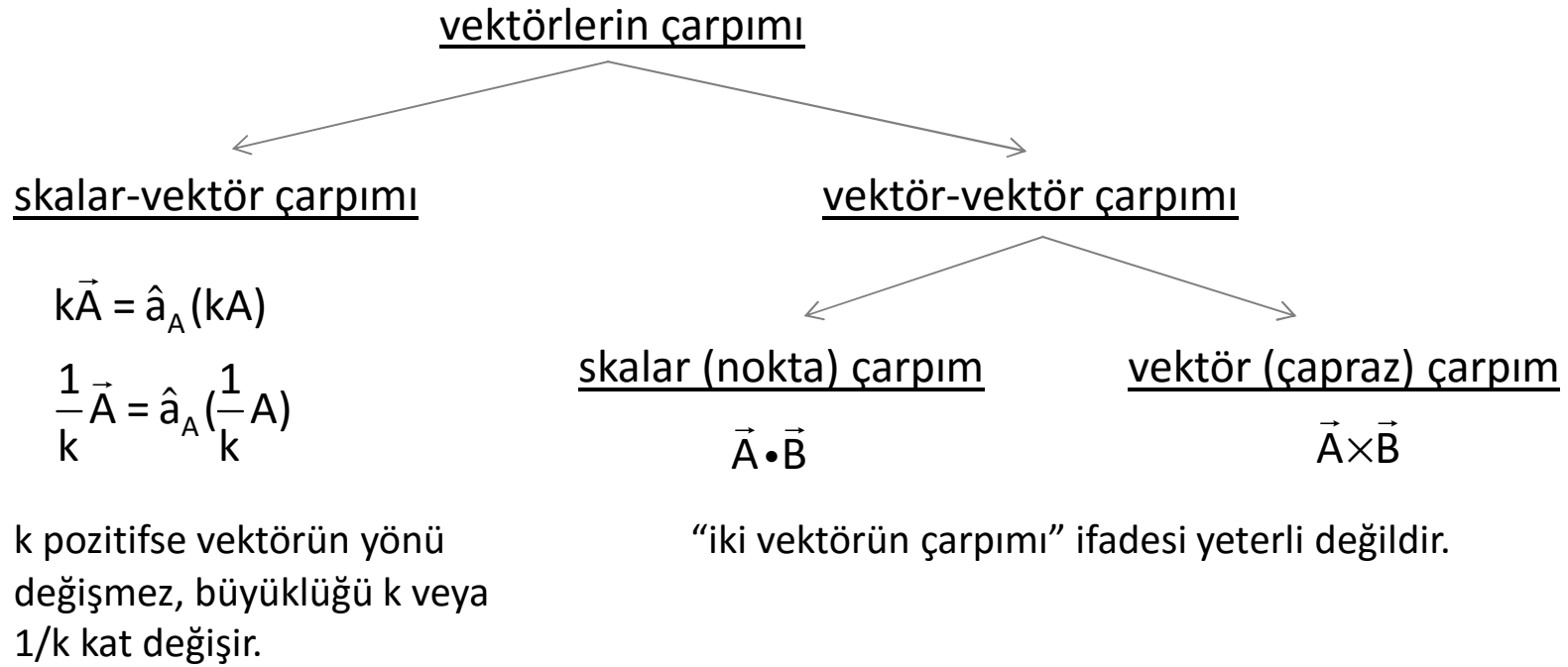
2.3.1. Skalar (Nokta) Çarpım

$$\vec{A}\bullet\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta = AB\cos\theta$$

$\vec{B}$

θ

$\vec{A}$
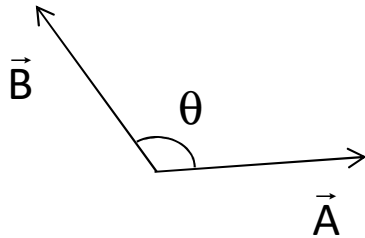
- ✓ Sonuç skalardır.
- ✓ θ, $\vec{A}$ ve $\vec{B}$ arasındaki küçük açıdır
- ✓ Nokta çarpım değişimlidir : $\vec{A}\bullet\vec{B} = \vec{B}\bullet\vec{A}$
- ✓ $\vec{A}\bullet\vec{A} = A^2$

$$A = \left|\vec{A}\right| = \sqrt{\vec{A}\bullet\vec{A}}$$

2.3.2. Vektör (Çapraz) Çarpım

$$\vec{A} \times \vec{B} = \hat{a}_N \left|\vec{A}\right|\left|\vec{B}\right| \sin\theta = \hat{a}_N AB\sin\theta$$

✓ Sonuç bir vektördür
✓ θ, $\vec{A}$ ve $\vec{B}$ arasındaki küçük açıdır
✓ Vektör çarpım değişimli değildir : $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
✓ $\hat{a}_N$, $\vec{A}$ ve $\vec{B}$'nin bulunduğu düzleme normal (dik) birim vektördür.

$\hat{a}_N$ veya $\vec{A} \times \vec{B}$'nin yönü sağ - el kuralı ile belirlenir.

✓ Paralelkenarın alanı $= \left|\vec{A} \times \vec{B}\right| = AB\sin\theta$
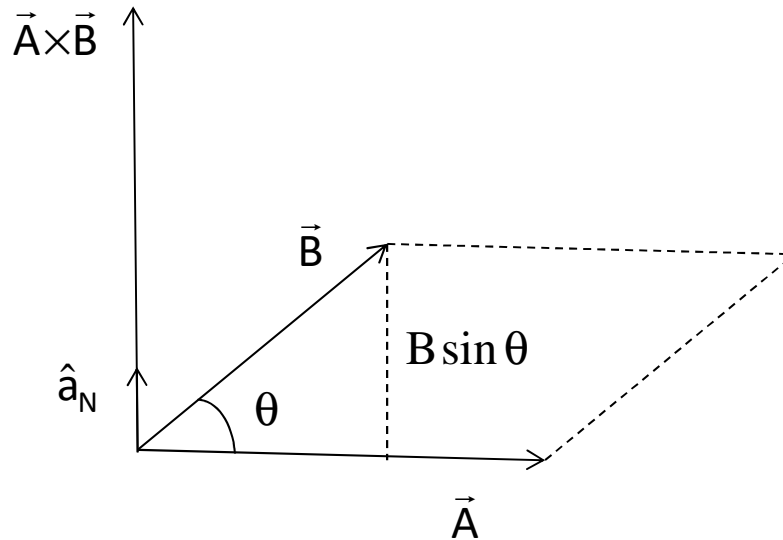
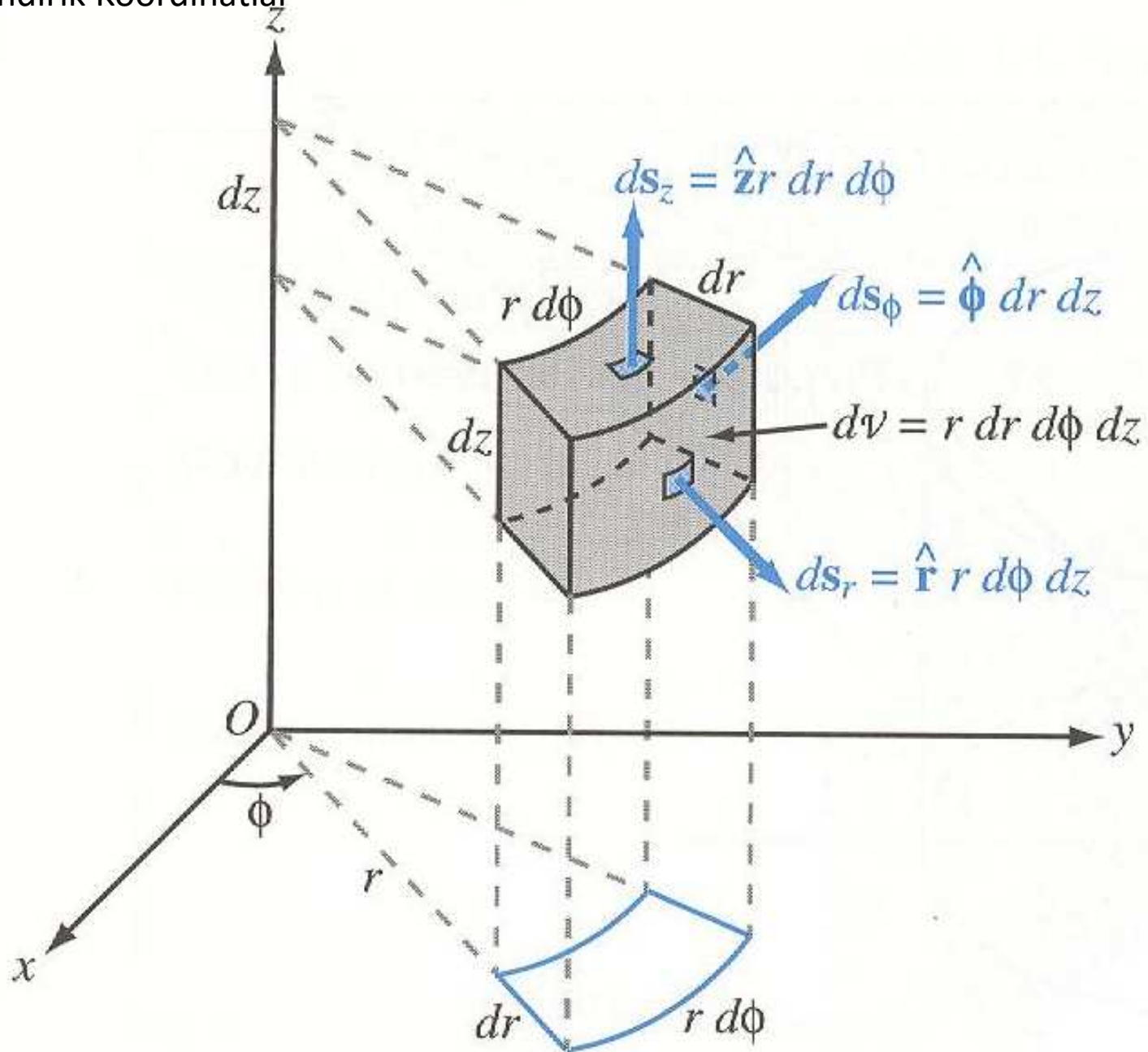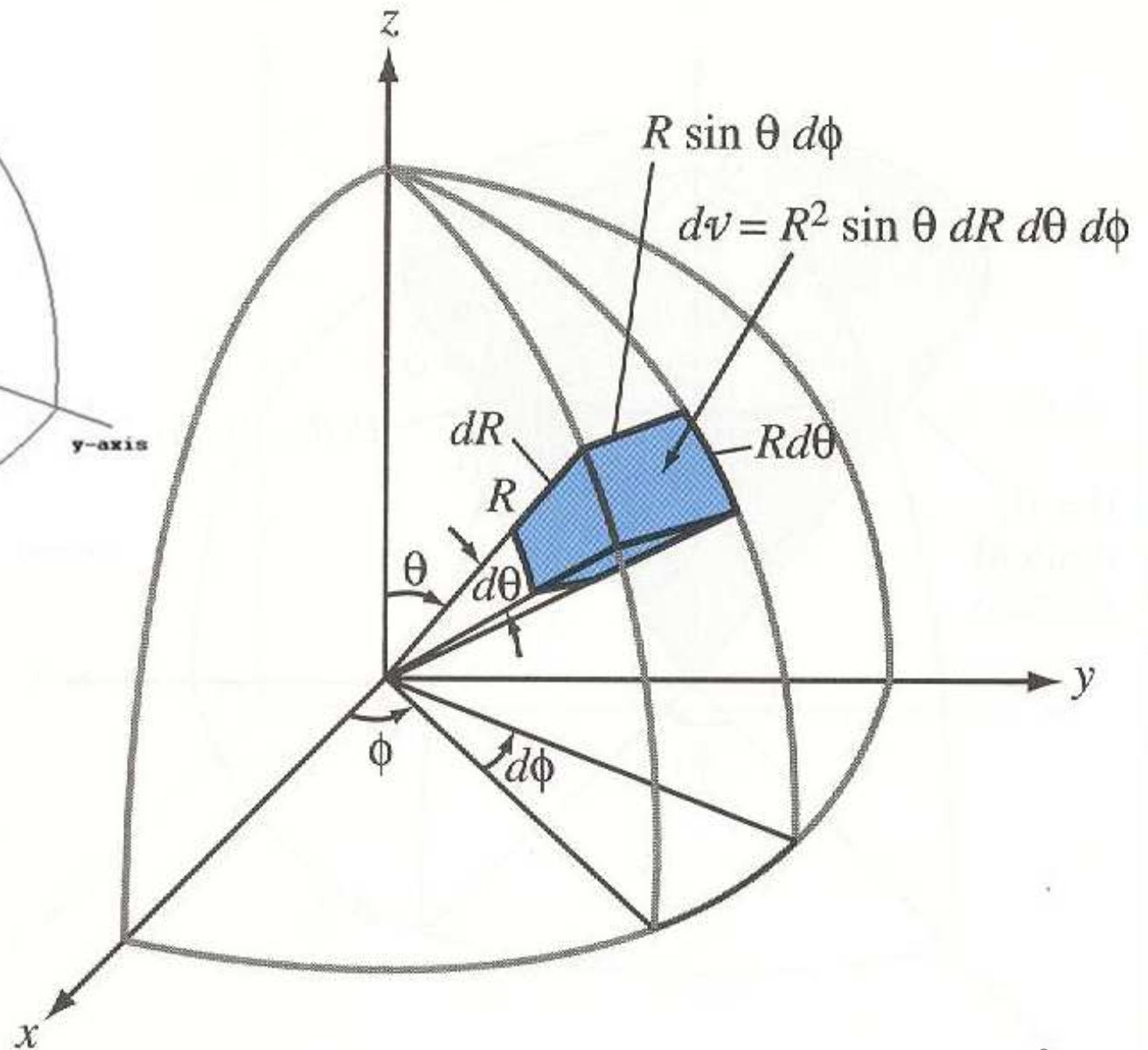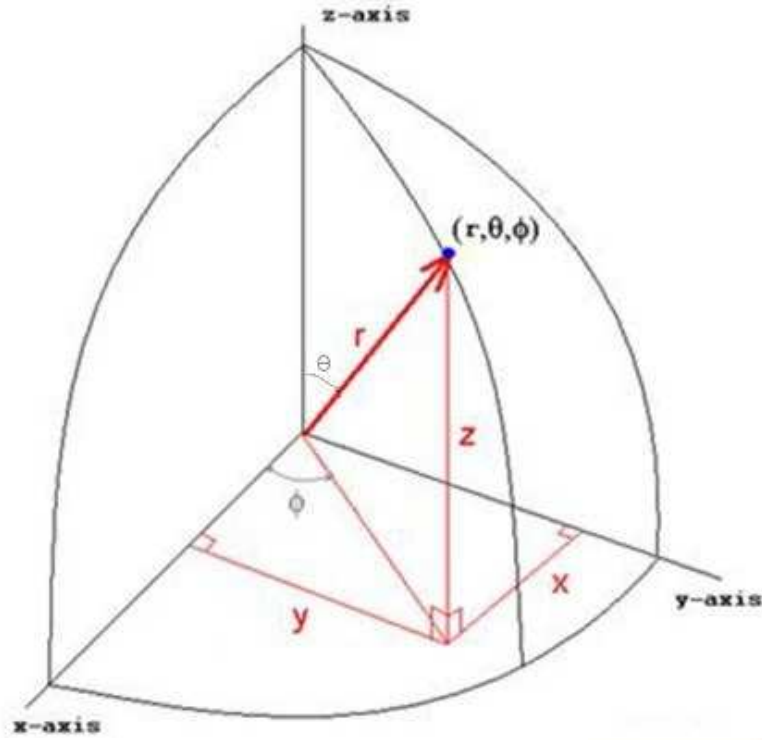## 2.4. ORTOGONAL KOORDİNAT SİSTEMLERİ
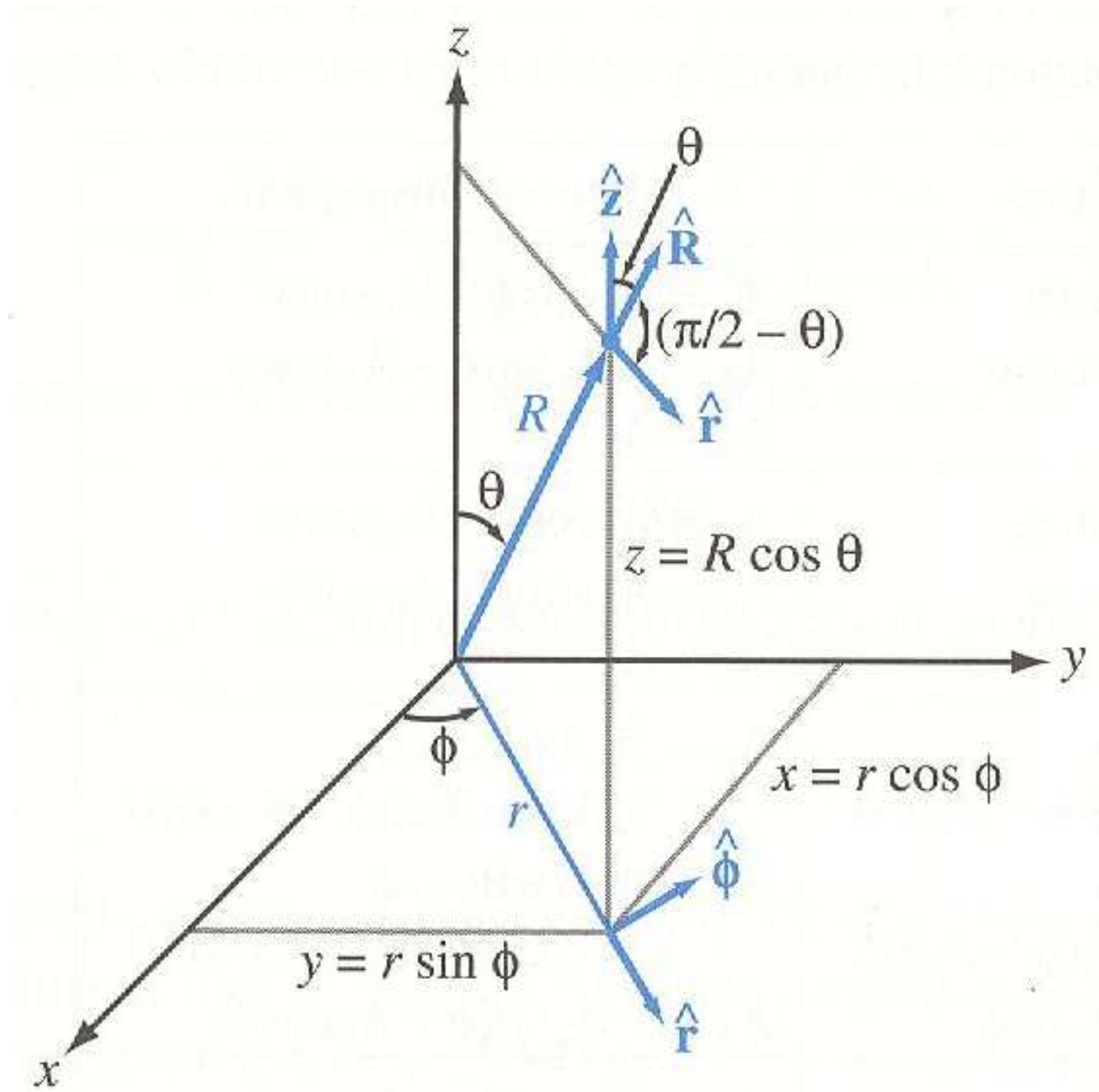
2.4.1. Kartezyen Koordinatlar

$$ds_z = \hat{z}\, dx\, dy$$

$$ds_y = \hat{y}\, dx\, dz$$

$$ds_x = \hat{x}\, dy\, dz$$

$$dv = dx\, dy\, dz$$
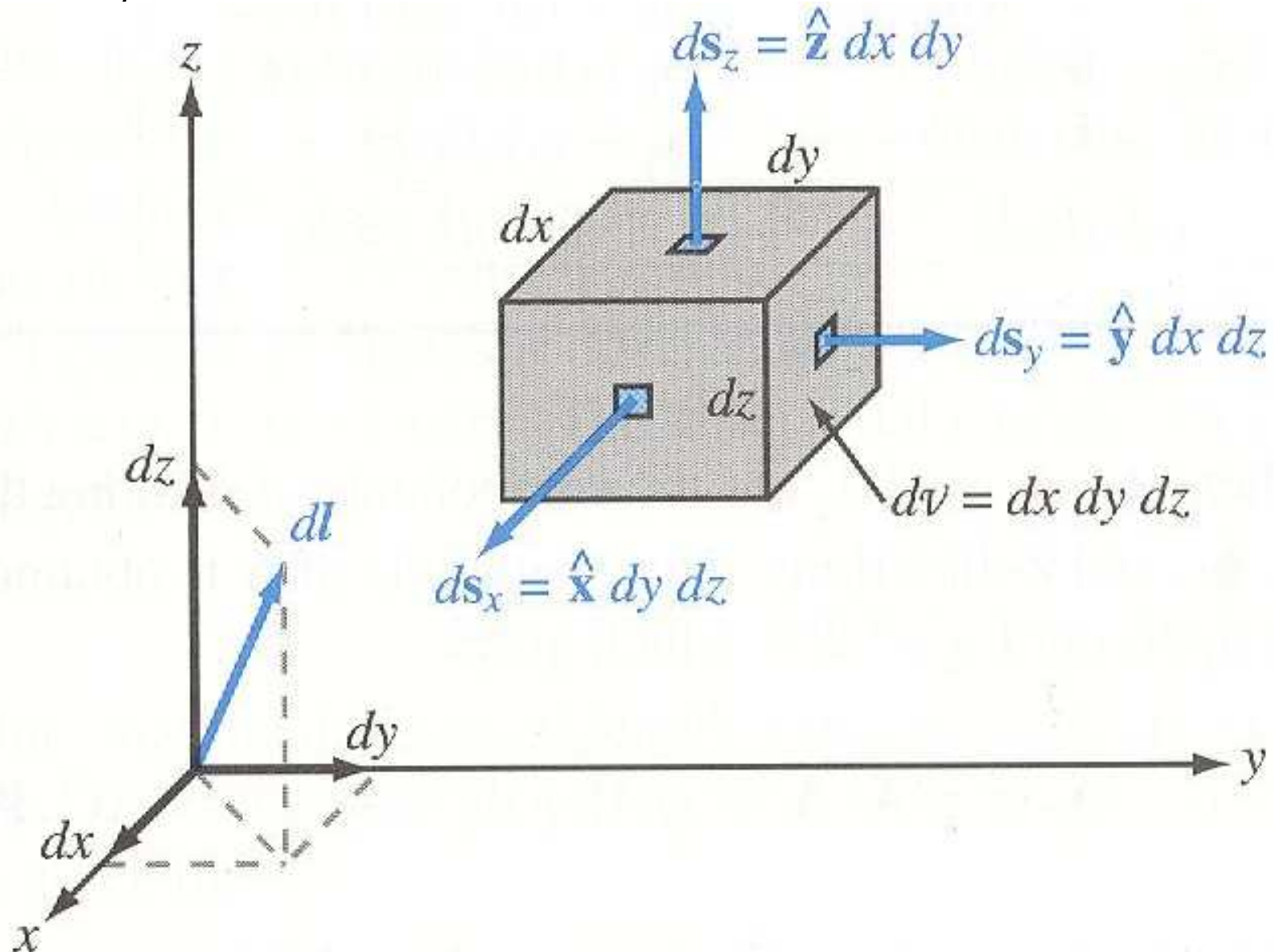
2.4.2. Silindirik Koordinatlar

2.4.3. Küresel Koordinatlar

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation, $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of $\mathbf{A}$, $\|A\| =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1,$ for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1,$ for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1,$ for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product, $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product, $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $dl =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume, $d\mathcal{V} =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Koordinat Sistemleri Arasındaki Ilişkiler

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi$ $\quad + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi$ $\quad + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi$ $\quad + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi$ $\quad + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi$ $\quad + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi$ $\quad + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi$ $\quad + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi$ $\quad + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

KOORDİNAT SİSTEMLERİ DÖNÜŞÜMLERİ

Kartezyen $\leftrightarrow$ Silindirik Dönüşümü

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_r \\ A_\phi \\ A_z \end{pmatrix}, \quad x = r\cos\phi \quad y = r\sin\phi \quad z = z$$

$$\begin{pmatrix} A_r \\ A_\phi \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}, \quad r = \sqrt{x^2 + y^2} \quad \phi = \tan^{-1}\left(\frac{y}{x}\right) \quad z = z$$

Kartezyen $\leftrightarrow$ Küresel Dönüşümü

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} A_R \\ A_\theta \\ A_\phi \end{pmatrix}, \quad x = R\sin\theta\cos\phi \quad y = R\sin\theta\sin\phi \quad z = R\cos\theta$$

$$\begin{pmatrix} A_R \\ A_\theta \\ A_\phi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}, \quad R = \sqrt{x^2 + y^2 + z^2} \quad \theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right) \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

Silindirik $\leftrightarrow$ Küresel Dönüşümü

$$\begin{pmatrix} A_r \\ A_\phi \\ A_z \end{pmatrix} = \begin{pmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} A_R \\ A_\theta \\ A_\phi \end{pmatrix}, \quad r = R\sin\theta \quad \phi = \phi \quad z = R\cos\theta$$

$$\begin{pmatrix} A_R \\ A_\theta \\ A_\phi \end{pmatrix} = \begin{pmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} A_r \\ A_\phi \\ A_z \end{pmatrix}, \quad R = \sqrt{r^2 + z^2} \quad \theta = \tan^{-1}\left(\frac{r}{z}\right) \quad \phi = \phi$$

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \mathbf{a}_x \frac{\partial V}{\partial x} + \mathbf{a}_y \frac{\partial V}{\partial y} + \mathbf{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \mathbf{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \mathbf{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \mathbf{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, φ, z)

$$\nabla V = \hat{\mathbf{r}} \frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}} \frac{1}{r} \frac{\partial V}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r} \begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & r A_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}} \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\boldsymbol{\phi}} \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{\mathbf{z}} \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\phi) - \frac{\partial A_r}{\partial \phi} \right]$$

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, φ)

$$\nabla V = \hat{\mathbf{R}} \frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}} \frac{1}{R} \frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}} \frac{1}{R \sin \theta} \frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 A_R) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{R \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}}R & \hat{\boldsymbol{\phi}}R \sin \theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & R A_\theta & (R \sin \theta) A_\phi \end{vmatrix}$$

$$= \hat{\mathbf{R}} \frac{1}{R \sin \theta} \left[\frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right] + \hat{\boldsymbol{\theta}} \frac{1}{R} \left[\frac{1}{\sin \theta} \frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R} (R A_\phi) \right] + \hat{\boldsymbol{\phi}} \frac{1}{R} \left[\frac{\partial}{\partial R} (R A_\theta) - \frac{\partial A_R}{\partial \theta} \right]$$

$$\nabla^2 V = \frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\partial V}{\partial R} \right) + \frac{1}{R^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{R^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

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SOME USEFUL VECTOR IDENTITIES

$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta_{AB}$ Scalar (or dot) product

$\mathbf{A} \times \mathbf{B} = \hat{\mathbf{n}} AB \sin \theta_{AB}$ Vector (or cross) product, $\hat{\mathbf{n}}$ normal to plane containing $\mathbf{A}$ and $\mathbf{B}$

$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \times \mathbf{B})$

$\nabla(U + V) = \nabla U + \nabla V$

$\nabla(UV) = U\nabla V + V\nabla U$

$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B}$

$\nabla \cdot (U\mathbf{A}) = U\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla U$

$\nabla \times (U\mathbf{A}) = U\nabla \times \mathbf{A} + \nabla U \times \mathbf{A}$

$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B}$

$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

$\nabla \cdot (\nabla \times \mathbf{A}) = 0$

$\nabla \times \nabla V = 0$

$\nabla \cdot \nabla V = \nabla^2 V$

$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

$\int_v (\nabla \cdot \mathbf{A})\, dv = \oint_S \mathbf{A} \cdot d\mathbf{s}$ Divergence theorem (S encloses v)

$\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\mathbf{l}$ Stokes's theorem (S bounded by C)