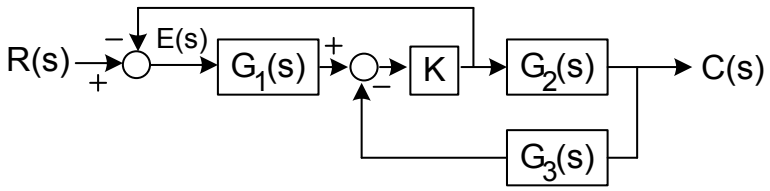
Otomatik Kontrol Yıl İçi Sınavı **14/Nisan/2011**

S_1

Aşağıda verilen Kontrol blok diyagramında, $G_1(s)=\frac{1}{s+1}, G_2(s)=10$ verilmektedir.

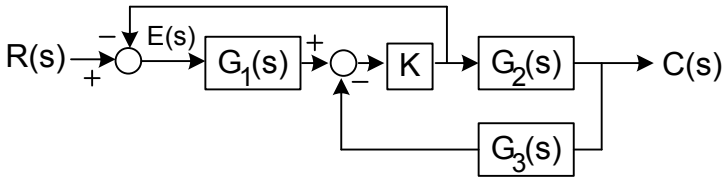


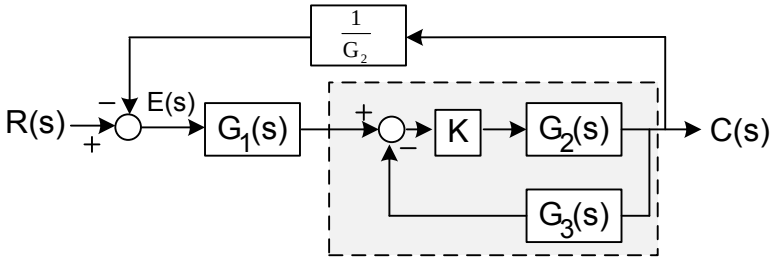
Kapalı-çevrim transfer fonksiyonu elde ediniz.

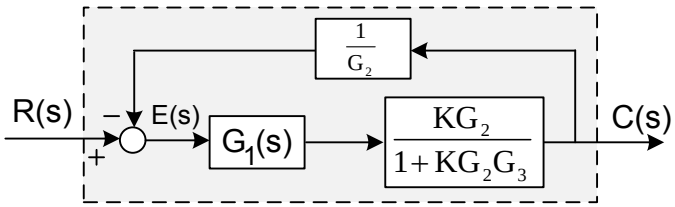
$\frac{C(s)}{R(s)}=?$

b) $r(t)=u(t)$ için $c(\infty)=1$ olması için $K=?$ hesap ediniz.

C1-a







$$R(s) \rightarrow \frac{G_1\frac{KG_2}{1+KG_2G_3}}{1+G_1\frac{KG_2}{1+KG_2G_3}\frac{1}{G_2}} \rightarrow C(s) \rightarrow$$

$$\frac{C(s)}{R(s)}=\frac{G_1\dfrac{KG_2}{1+KG_2G_3}}{1+G_1\dfrac{K\not{G}_2}{1+KG_2G_3}\dfrac{1}{\not{G}_2}}=\frac{G_1\dfrac{KG_2}{1+KG_2G_3}}{\dfrac{1+KG_2G_3+G_1K}{1+KG_2G_3}}=\frac{G_1KG_2}{1+KG_2G_3+G_1K}=\frac{\dfrac{1}{s+1}K*10}{1+K*10*\dfrac{s}{s+5}+\dfrac{1}{s+1}K}$$

$$\frac{C(s)}{R(s)}=\frac{\dfrac{1}{s+1}K*10}{1+K*10*\dfrac{s}{s+5}+\dfrac{1}{s+1}K}=\frac{\dfrac{1}{s+1}K*10}{\dfrac{(s+5)(s+1)+K*10*s(s+1)+(s+5)K}{(s+5)(s+1)}}=\frac{K*10(s+5)}{(s+5)(s+1)+K*10*s(s+1)+(s+5)K}$$

$$\frac{C(s)}{R(s)}=T(s)=\frac{10K(s+5)}{(s+5)(s+1)+10Ks(s+1)+(s+5)K}$$

$$\frac{C(s)}{R(s)}=T(s)=\frac{10K(s+5)}{s^2+s+5s+5+10Ks^2+10Ks+Ks+5K}$$

$$T(s)=\frac{C(s)}{R(s)}=\frac{10K(s+5)}{(10K+1)s^2+(11K+6)s+5+5K}$$

C1-b

$r(t)=u(t)\xrightarrow{S}R(s)=\frac{1}{s}$ bulunur.

$c(\infty)=\lim_{t\to\infty}c(t)\xrightarrow{S}c(\infty)=\lim_{s\to0}sC(s)=\lim_{s\to0}sT(s)R(s)$ dir.

İstenen

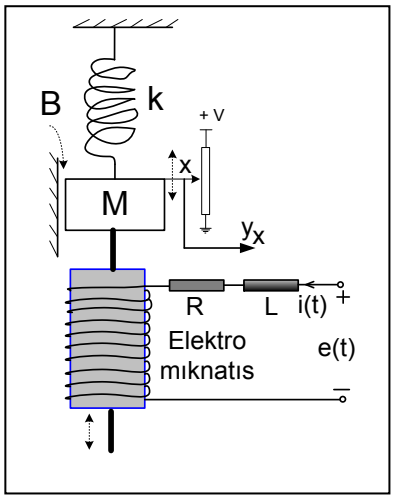
$c(\infty)=\lim_{s\to0}sT(s)R(s)=1$ ➔

$$c(\infty)=\lim_{s\to0}s\frac{10K(s+5)}{(10K+1)s^2+(11K+6)s+5+5K}\frac{1}{s}=1$$ ➔

$$\lim_{s\to0}\not{s}\frac{10K(0+5)}{(10K+1)0^2+(11K+6)0+5+5K}\frac{1}{\not{s}}=1$$

$$\frac{K*50}{5+5K}=1 \rightarrow K=\frac{1}{9}$$

S_2



Yanda verilen kütle, yay ve elektromıknatıs sisteminde, elektro mıknatıs kuvveti $F(t)=k_m * i(t)$ olarak verilmektedir.

a- Sisteme ait dinamik denklemleri yazınız. Kontrol blok diyagramını elde ediniz. ($y_x = x(t)$ ölçülen konum)

b- y_r referans konum girişi olmak üzere, ayrık-zaman sayısal kapalı çevrim kontrol blok diyagramını çiziniz.

Not: Kütle yay dengede

C2-a

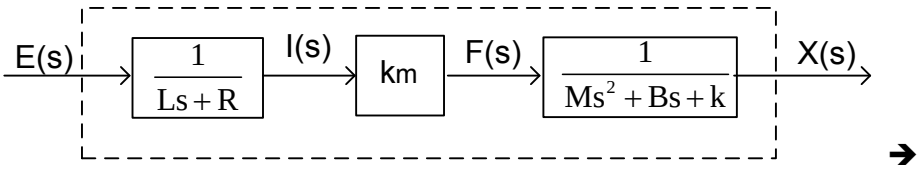
1) $e(t)=Ri(t)+L\frac{d}{dt}i(t) \xrightarrow{S} E(s)=RI(s)+LsI(s)$ ➔ $I(s)=\frac{1}{Ls+R}E(s)$

2) $f(t)=k_m i(t) \xrightarrow{S} F(s)=k_m I(s)$ ➔ $F(s)=k_m I(s)$

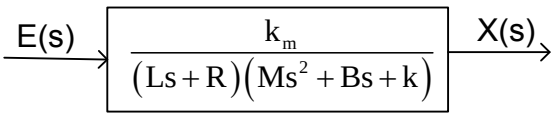
3) $f(t)=kx(t)+B\frac{d}{dt}x(t)+M\frac{d^2}{dt^2}x(t) \xrightarrow{S} F(s)=kX(s)+BsX(s)+Ms^2X(s)$ ➔

$$X(s)=\frac{1}{Ms^2+Bs+k}F(s)$$

blok diyagram

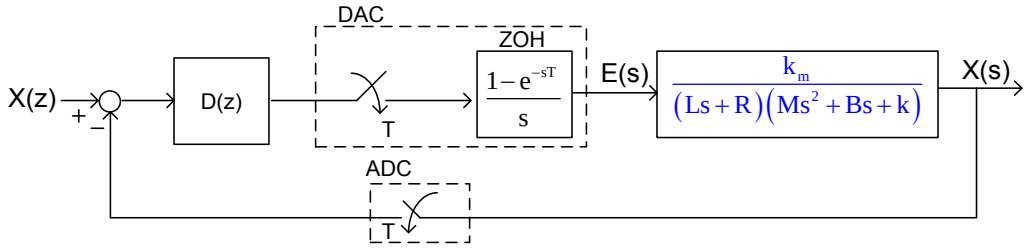


➔

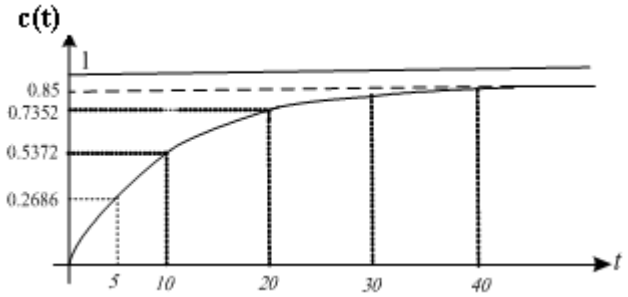


$$\frac{X(s)}{E(s)}=\frac{k_m}{(Ls+R)(Ms^2+Bs+k)}$$

C2-b



S_3



Yanda I. Dereceden sisteme ait açık-çevrim birim basamak cevabı verilmiştir.

a) $G(s)$ transfer fonksiyonunu bulunuz.

b) örnekleme zamanı $T=\frac{\tau}{5}$ olmak üzere

τ : zaman sabiti

Ayrık zaman açık-çevrim transfer fonksiyonu $G(z)$ elde ediniz (ZOH' lu).

c) Birim basamak giriş için $c(k)$' yı elde ediniz ve k=5 için $c(k)$ yı hesaplayınız.

C3-a

$\frac{K}{\tau s+1}$ 1. Dereceden modelin parametreleri deneysel yol ile elde edilecektir.

Açık çevrim kazanç: K ise $K=\frac{v_{çıkış}(\infty)}{v_{giriş}(\infty)}=\frac{0.85}{1}=0.85$ elde edilir.

Açık çevrim zaman sabiti: τ ise $0.632*v_{çıkış}(\infty)=0.5372$'ye denk gelen zaman değeri olup birim basmak cevap eğrisinden $\tau=10sn$ olarak okunur.

1. Dereceden transfer fonksiyonunda parametreler yerlerine konursa ,

$G(s)=\frac{0.85}{10s+1}$ bulunur.

C3-b

$z \triangleq e^{sT}$, $T = \frac{\tau}{5} = \frac{10}{5} = 2sn$

$$X(z) = \sum_{i=1}^{n} \left\{ \frac{1}{(m-1)!} \frac{d^{m-1}}{ds^{m-1}} \left[(s-s_i)^m X(s) \frac{z}{z-e^{sT}} \right] \Big|_{s=s_i} \right\}$$

$$Z\{T(s)G_{ZOH}(s)\} = TG_{ZOH}(z) = Z\left\{ \frac{0.85}{10s+1} \frac{1-e^{-Ts}}{s} \right\} = \frac{0.85}{10}(1-z^{-1}) Z\left\{ \frac{1}{s(s+0.1)} \right\} \rightarrow$$

$$TG_{ZOH}(z) = 0.085 \frac{z-1}{z} \left\{ \cancel{(s-(-0.1))} \frac{1}{\cancel{s}(s+0.1)} \frac{z}{z-e^{sT}} \Big|_{s=-0.1} + \cancel{(s-(-0))} \frac{1}{\cancel{s}(s+0.1)} \frac{z}{z-e^{sT}} \Big|_{s=0} \right\}$$

$$TG_{ZOH}(z) = 0.085 \frac{z-1}{z} \left\{ \frac{1}{-0.1} \frac{z}{z-e^{-0.1*2}} + \frac{1}{(0+0.1)} \frac{z}{z-e^{0*2}} \right\} = 0.085 \frac{z-1}{z} \left\{ -10 \frac{z}{z-0.8187} + 10 \frac{z}{z-1} \right\}$$

$$TG_{ZOH}(z) = 0.85 \frac{z-1}{\cancel{z}} \left\{ -\frac{\cancel{z}}{z-0.8187} + \frac{\cancel{z}}{z-1} \right\} = 0.85 \left(1 - \frac{z-1}{z-0.8187} \right) = 0.85 \left(\frac{z-0.8187-z+1}{z-0.8187} \right) \rightarrow$$

$$TG_{ZOH}(z) = \frac{0.1541}{z-0.8187}$$

C3-c

Giriş işareti $r(t) = u(t) \xrightarrow{S} R(s) = \frac{1}{s} \xrightarrow{z} R(z) = \frac{z}{z-1}$ bulunur.

$$\frac{C(z)}{R(z)} = TG_{ZOH}(z)$$

$$C(z) = TG_{ZOH}(z) R(z)$$

$$C(z) = TG_{ZOH}(z) R(z) = \frac{0.1541}{z-0.8187} \frac{z}{z-1}$$

$$C(z) = \frac{0.1541z}{(z-0.8187)(z-1)}$$

$$x(k)=\sum_{i=1}^{n}\left\{\frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\left[(z-z_i)^m X(z)z^{k-1}\right]\Big|_{z=z_i}\right\}$$

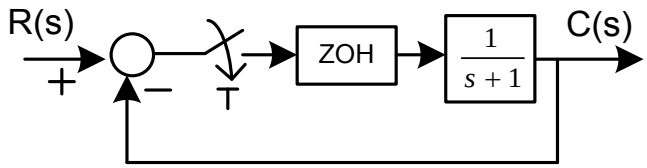
$$c(k)=Z\{C(z)\}=Z\left\{\frac{0.1541z}{(z-0.8187)(z-1)}\right\}$$

$$c(k)=\left\{\cancel{(z-(0.8187))}\frac{0.1541z}{\cancel{(z-0.8187)}(z-1)}z^{k-1}\Big|_{z=0.8187}+\cancel{(z-(1))}\frac{0.1541z}{(z-0.8187)\cancel{(z-1)}}z^{k-1}\Big|_{z=1}\right\}$$

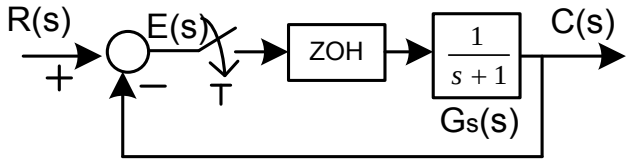
$$c(k)=\frac{0.1541*0.8187^k}{(0.8187-1)}+\frac{0.1541*1^k}{(1-0.8187)}=0.85(1-0.8187^k)$$

$c(k)=0.85(1-0.8187^k)$ ➔ $c(5)=0.85(1-0.8187^5)$ ➔ $c(5)=0.5373$

S_4 T= 0.1 sn için $r(t)=u(t)$ için $C(z)$'i elde ediniz. $C(\infty)$ değerini hesap ediniz.



C4



$E(s)=R(s)-C(s)$ ➔ $E^*(s)=R^*(s)-C^*(s)$

$C(s)=(R^*(s)-C^*(s))G_{zoh}(s)G_s(s)=R^*(s)G_{zoh}(s)G_s(s)-C^*(s)G_{zoh}(s)G_s(s)$ ➔

$C^*(s)=R^*(s)G_{zoh}G_s^*(s)-C^*(s)G_{zoh}G_s^*(s)$

$C^*(s)+C^*(s)G_{zoh}G_s^*(s)=R^*(s)G_{zoh}G_s^*(s)$

$$\frac{C^*(s)}{R^*(s)}=\frac{G_{zoh}G_s^*(s)}{1+G_{zoh}G_s^*(s)}$$

$$\frac{C(z)}{R(z)}=\frac{G_{zoh}G_s(z)}{1+G_{zoh}G_s(z)}$$ ➔ doğrudan yazılabilir. (Çıkarım bilgi amaçlı verilmiştir.)

$z \triangleq e^{sT}$ olduğu göz önüne alınır ve $T = 0.1$ örnekleme zamanı olmak üzere z- dönüşümü yapılır.

$$X(z) = \sum_{i=1}^{n} \left\{ \frac{1}{(m-1)!} \frac{d^{m-1}}{ds^{m-1}} \left[(s-s_i)^m X(s) \frac{z}{z-e^{sT}} \right] \Big|_{s=s_i} \right\}$$

$$G_{zoh}G_s(z) = Z\{G_{zoh}(s)G_s(s)\} = Z\left\{ \frac{1-e^{-sT}}{s} \frac{1}{s+1} \right\} = (1-z^{-1}) Z\left\{ \frac{1}{s(s+1)} \right\}$$

$$G_{zoh}G_s(z) = (1-z^{-1}) \left\{ \cancel{(s-(0))} \frac{1}{s(s+1)} \frac{z}{z-e^{sT}} \Big|_{s=0} + \cancel{(s-(-1))} \frac{1}{s\cancel{(s+1)}} \frac{z}{z-e^{sT}} \Big|_{s=-1} \right\}$$

$$G_{zoh}G_s(z) = (1-z^{-1}) \left\{ \frac{1}{(0+1)} \frac{z}{z-e^{0*0.1}} + \frac{1}{-1} \frac{z}{z-e^{(-1)0.1}} \right\}$$

$$G_{zoh}G_s(z) = \left(\frac{z-1}{z} \right) \left\{ \frac{z}{z-1} - \frac{z}{z-0.9048} \right\} = 1 - \frac{z-1}{z-0.9048}$$

$G_{zoh}G_s(z) = \dfrac{0.0952}{z-0.9048}$ ➔ ileri yol transfer fonksiyonu elde edilir.

$$\frac{C(z)}{R(z)} = \frac{\dfrac{0.0952}{z-0.9048}}{1+\dfrac{0.0952}{z-0.9048}} = \frac{\dfrac{0.0952}{z-0.9048}}{\dfrac{z-0.9048+0.0952}{z-0.9048}}$$

$\dfrac{C(z)}{R(z)} = \dfrac{0.0952}{z-0.8096}$ Kapalı çevrim transfer fonksiyonu elde edilir. Buradan

$C(z) = \dfrac{0.0952}{z-0.8096} R(z)$ cevap ifadesi yazılır.

Giriş işareti $r(t) = u(t) \xrightarrow{S} R(s) = \dfrac{1}{s} \xrightarrow{z} R(z) = \dfrac{z}{z-1}$ dir.

$$C(z) = \frac{0.0952}{z-0.8096} \frac{z}{z-1}$$

$$C(z) = \frac{0.0952z}{(z-0.8096)(z-1)}$$

$$c(\infty) = \lim_{t\to\infty} c(t) = \lim_{s\to 0} sC(s) = \lim_{z\to 0} (z-1)C(z)$$

$$c(\infty) = \lim_{z\to 1} \cancel{(z-1)} \frac{0.0952z}{(z-0.8096)\cancel{(z-1)}} = \frac{0.0952*1}{(1-0.8096)\cancel{(z-1)}}$$

$$c(\infty) = 0.5$$