HW5 : Design under Uncertainty

Optimization in Engineering,
Department of Mechanical Engineering,
National Taiwan University

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Consider the problem below that has been widely used in the literature.

$$
\begin{aligned}
\min_{\boldsymbol{\mu}_\mathbf{X}} \; & f(\boldsymbol{\mu}_\mathbf{X}) = \mu_{X_1} + \mu_{X_2} \\
\text{s.t.} \quad & \Pr[g_1(\mathbf{X}) = 20 - X_1^2 X_2 > 0] \le 0.0013 \\
& \Pr[g_2(\mathbf{X}) = 1 - (X_1 + X_2 - 5)^2/30 \\
& \qquad -(X_1 - X_2 - 12)^2/120 > 0] \le 0.0013 \\
& \Pr[g_3(\mathbf{X}) = X_1^2 + 8X_2 - 75 > 0] \le 0.0013
\end{aligned} \tag{1}
$$

where $\begin{cases} X_1 \sim N(\mu_{X_1}, 0.3^2) \\ X_2 \sim N(\mu_{X_2}, 0.3^2) \end{cases}$

1. (25%) Please use Monte Carlo simulation with 100 samples to solve the problem. Rerun twice, are the results different? Did you face convergence difficulties? Why?

2. (25%) Please use Monte Carlo simulation with 1 million samples to solve the problem. Rerun twice, are the results different? Did you face convergence difficulties? Why?

3. (50%) Please use FOSM to solve the problem. Use Monte Carlo to verify the failure probability at the optimal. Did you get 99% results? Why not?

$$g_1 = 20 - X_1^2 X_2 \le 0$$

$$g_2 = 1 - \frac{(X_1 + X_2 - 5)^2}{30} - \frac{(X_1 - X_2 - 12)^2}{120} \le 0$$

$$g_3 = X_1^2 + 8X_2 - 75 \le 0$$

MATLAB : Element Manipulation