HW6 : Design Optimization of a Ten-Bar Truss under Uncertainty

Optimization in Engineering
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A ten-bar truss as shown in Fig.1 is considered. An external force is applied to node 2 with $F = 10^4 kN$. All members have circular cross sections. Members $\{1, 2, 3, 4, 5, 6\}$ are identical with length $l = 914$ cm and members $\{7, 8, 9, 10\}$ are also identical.

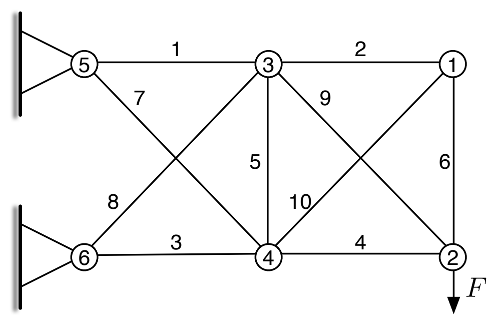

Figure 1: Ten-bar Truss

The deterministic design problem is to find the optimal radii of all bar members to

1. minimize the overall weight of the truss system
2. minimize the displacement at node 2 (δ_2)
3. no yielding and buckling occurring.

The problem is summarized in Eq.(1), where r_1 and r_2 are the radii of the cross-section areas for bars $1 \sim 6$ and $7 \sim 10$, respectively. Yield strength E, moment of inertia I, resulting force F, and tensile stress σ all have subscript i indicating the number of the bar element. δ_2 is the node 2 displacement. Structural steel (ASTM-A36) with density $\rho = 7860\text{kg/m}^3$, modulus of elasticity $E = 200\text{GPa}$, and yield strength $Y = 250$ MPa is used for this truss system. Finite element analysis is used to calculate the resulting forces and displacements on each member.

$$
\begin{aligned}
& \min_{r_1, r_2} (6 \cdot \pi r_1^2 l + 4 \cdot \pi r_2^2 \sqrt{2} l) \\
\text{s. t.} & \\
& F_i \le P^c = \tfrac{\pi^2 EI}{l^2} \\
& \sigma_i \le Y \\
& \delta_2 \le 2
\end{aligned}
\tag{1}
$$

Consider the existence of uncertainty resulting in variations in r_1, r_2, ρ, and F. Let all uncertainties are Gaussian with coefficient of variation (σ/μ) being 1/10. The design variables becomes the mean values of r_1 and r_2. Constraints have to be satisfied 99% of the time.

1. (15%) Please rewrite the mathematical formulation of the design problem with uncertainty

2. (25%) Please use Monte Carlo simulation with 100 samples to solve the problem. Rerun twice, are the results different? Did you face convergence difficulties? Why?

3. (25%) Please use Monte Carlo simulation with 1 million samples to solve the problem. Rerun twice, are the results different? Did you face convergence difficulties? Why?

4. (35%) Please use FOSM to solve the problem. Use Monte Carlo to verify the failure probability at the optimal. Did you get 99% results? Why not?