Eckert Peters: Example Problem

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Abstract

This note provides a brief example of the kind of mathematical problems that feature heavily in Eckert & Peters 2018.

There are $i = 1, \ldots, N$ regions. Together these regions are inhabited by a mass $L = 1$ of agents. Across regions local stocks of agents hence sum to 1: $\sum_i L_i = 1$. Each region produces a good and a buys the goods from all other regions. The local production technology is linear in labor:

$$y_i = A_i l_i$$

where l_i is the amount of labor used and A_i is a region specific productivity shifter. In equilibrium we need $l_i = L_i$, ie the number of workers living i need to equal the number of workers employed in i. Workers are paid a wage w_i in region i, which is a second (after L_i) variable that we need to solve for.

It can be shown that the fraction of spending in region i that goes to the good produced in region j is:

$$\pi_{ij} = \frac{(\frac{w_j}{A_j})^{1-\sigma}}{\sum_k (\frac{w_k}{A_k})^{1-\sigma}}$$

One can show that this system results in the following set of two equations. First a labor market clearing condition, which postulates that the total labor income earned in i must be equal to the total money region i and all other regions' consumers pay for region i's good:

$$w_i L_i = \sum_j \pi_{ji} w_j L_j$$

And secondly a spatial equilibrium condition which states that workers are indifferent across regions:

$$L_i = \frac{\exp\left(\frac{w_i}{(\sum_k (\frac{w_k}{A_k})^{1-\sigma})^{\frac{1}{1-\sigma}}}\right)^{\eta}}{\sum_j \exp\left(\frac{w_j}{(\sum_k (\frac{w_j}{A_j})^{1-\sigma})^{\frac{1}{1-\sigma}}}\right)^{\eta}}$$

In addition we need one more equation. We define the numeraire of the system: $\sum_i w_i = 1$.

Assume that you are given parameters A_i such that $\sum_i A_i = 1$ and $A_i > 0$. Also $\sigma > 1$ and $\eta > 1$. With these parameters in hand write a code that solves for the two endogenous variables, the vector of w_i and L_i.

The code works best if you first guess a distribution of people L_i across space and then use

$$w_i = \frac{1}{L_i} \sum_j \pi_{ji} w_j L_j$$

to guess a wage, construct π_{ij}, and then use the resulting left hand side wage as a new guess for the wage. Be careful to enforce the normalization $\sum_i w_i = 1$ at each iteration. Also note that you may not want to update the wage fully, but take a convex combination of the updated wage and the old guess as the new guess.

Once this converged you updated the distribution of people by using the second equation:

$$L_i = \frac{\exp\left(\frac{w_i}{(\sum_k (\frac{w_k}{A_k})^{1-\sigma})^{\frac{1}{1-\sigma}}}\right)^{\eta}}{\sum_j \exp\left(\frac{w_j}{(\sum_k (\frac{w_j}{A_j})^{1-\sigma})^{\frac{1}{1-\sigma}}}\right)^{\eta}}$$

Together with the new, converged wage. Given this new distribution of L_r you need to re-compute the wage. Its a nested loop with the wage loop inside and the worker loop outside. You are finished when the wages have converged and plugging them into second equation gives you the same distribution of people as yoiu previously guessed.