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A Two Level Feedback System Design to Provide Regulation Reserve*

Bowen Zhang[1] and John Baillieul[2]

Abstract— **Demand side management has gained increasing importance as the penetration of renewable energy grows. Based on a Markov jump process model of a group of thermostatic loads, this paper proposes a two level feedback system to govern the interactions between an independent system operator (ISO) and a number of regulation reserve providers such that two objectives are achieved: 1) the ISO can optimally dispatch regulation signals to multiple providers in real time in order to reduce the requirement for expensive spinning reserves, and 2) each reserve provider can control its thermostatic loads to respond to the ISO signal. It is shown that the amount of regulation reserve that can be provided is implicitly restricted by a few fundamental parameters of the provider itself, such as the allowable set point choice and its thermal constant. An interesting finding is that there is a trade-off between an appliance's capacity for providing a large sustained reserve and its capacity of rapid reserve ramping. Simulations are presented to verify theoretical results.**

I. Introduction

Alternative sources like wind and solar make the electric energy grid harder to control. As a consequence, building level demand side management (DSM) has become a crucial resource for controlling the grid through either a price-based signalling control [1], [2] or a non-priced direct load control (DLC). Compared with pricing mechanisms that have been adopted in the Power Matching City in Netherlands [3] as well as the Olympic Peninsula project in western Washington [4], the operators in DLC will have a better understanding of the loads' response and thereby be able to provide more accurate high resolution reserve [5], such as the four seconds reserve operated by PJM.

DLC approaches for thermostatic loads have been studied extensively during the past decade for different purposes. An early investigation of load shifting has been studied with a state queuing model to illustrate the effect of set-point change [6], [7]. The notion of packetized DLC was proposed in [8], [9] to smooth electricity consumption by employing the idea of energy quantization. Regulation reserve load following can be found in [10], [11], [12], [13], and [14] with different state space models representing the dynamics of the evolution of the aggregated appliances and filtering techniques to estimate the building parameters. Results show that a single building with aggregated appliances is a promising resource for providing moderate amounts of reserves by acting as both a positive and negative consumption source from its average consumption level.

This paper reports research in two areas that are central to ensuring that reliable and accurate high resolution reserves can be provided by controlling aggregated loads in a building. First, there is need for an *analytical* expression of the reserve limitation that can be provided by a smart building having a group of aggregated appliances based on the appliances' thermal model parameters [15]. Second, there is need for a two way communication framework to identify the necessary information exchange between the ISO and *multiple* smart buildings to enhance the reliability of providing high resolution reserve, given the information of individual building's limitation. Without these pieces, a pilot program has shown that the energy reserve provision by commercial buildings can perform only half of the contracted amount [16].

The contribution of this paper is to provide the above missing pieces by developing a two level feedback system. We focus primarily on controlling thermostatic appliances in smart buildings where consumers in the building provide their preferred allowable set point range and are assumed to authorize the building operator to control their appliances within their preferred settings. The building operator has the authority to shift the set point of the appliances within the allowable range to modulate aggregated consumption and to track the regulation signal. At the lower level, the building operator can design a controller to track the regulation signal within a certain limit characterized by a few parameters of the building. At the higher level, information feedback from the smart buildings to the ISO characterize the buildings' limitation to respond and will be used by the ISO for optimal signal dispatch. We derive analytically the regulation reserve limitation of smart buildings based on its the capacity to provide long term sustained reserve and short term ramping reserve. It is further shown that two fundamental thermal parameters determining the limitation are the appliances' thermal constant τ and the appliances' temperature gain T_{g}. Large τ/T_g enables appliances to provide larger long term, but smaller short term regulation reserve. Small τ/T_g behaves oppositely. The optimal dispatch of reserve signals with knowledge of individual limitations reduces the amount of spinning reserves required from high-cost generators.

The paper proceeds as follows. Sec.II develops the state space model for which the feedback system is designed. Sec.III gives the overall two level feedback system design in which we introduce the lower level controller with feedback linearization, followed by a derivation of regulation reserve limitation. We then consider higher level information feed-

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[1]Corresponding author. Division of Systems Eng., Boston University, 15 St. Marys St., Brookline, MA 02446, email:bowenz@bu.edu

[2]Dept. of Electrical and Computer Eng., Dept. of Mechanical Eng., and Division of Systems Eng., Boston University, 110 Cummington St., Boston, MA 02215, email:johnb@bu.edu

back enabling the optimal dispatch of reserve signals. Sec.IV designs an observer. Sec.V illustrates the operation of our feedback control by simulations. Sec.VI concludes.

II. Markov Jump Thermal Process Modelling

We model the system as a continuous time, discrete state Markov jump process where a state i is defined as a pair of temperature values and binary appliance operating states $\{T_i, \text{on(off)}\}$. Previous literature has considered similar definitions [6], [12], [17]. The departure of our model from previous ones is that we consider the state in a Markov setting and establish the implicit relations between thermal and the Markov jump processes. Denote the comfort band as $[T_{\text{min}}, T_{\text{max}}]$ and discretize the temperature in the band into bins of width δ, with the number of temperature bins being $N = \Delta_{\text{band}}/\delta$, where Δ_{band} is the width of the comfort band. We say that an appliance is in state i for $i = 1, \ldots, N$ if T_i, the ambient temperature that the appliance regulates, satisfies $T_i \in [T_{\text{min}} + (i-1)\delta, T_{\text{min}} + i\delta]$ with status off, and i for $i = N+1, \ldots, 2N$ if $T_i \in [T_{\text{max}} - (i-N-2)\delta, T_{\text{max}} - (i-N-1)\delta]$ with status on.

First we model the Markov transition of the uncontrolled thermal process with $u = 0$. Denote by α the transition rate when the thermostat is off, and β the rate when the thermostat is on; see Fig.1(a). In the duty off process, the probability that an appliances will be in state i at time $t+1$ given that it is in state j at time t is,

$$p_{i,j} = \begin{cases} \alpha\Delta t + o(\Delta t) & \text{if } i = j+1 \\ 1 - \alpha\Delta t + o(\Delta t) & \text{if } i = j \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where $o(\Delta t)$ means $\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$. We eliminate the probability of Markov jump between non-adjacent states since the decision interval $\Delta t = 4$s is small in high resolution reserve provision. The above equation can be interpreted that the transition probability between adjacent states linearly increases with small Δt.

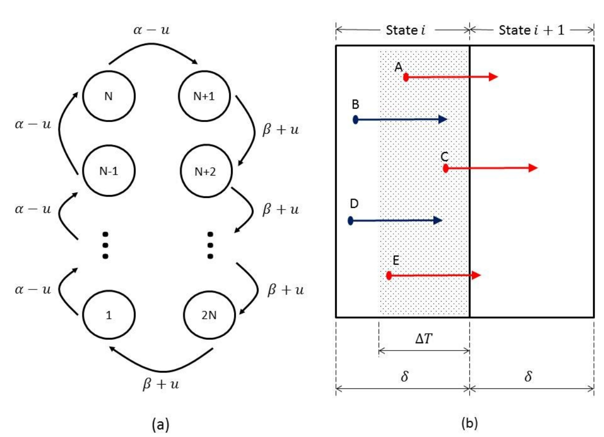

Fig. 1. Markov jump process model. (a) Markov chain transition rate diagram. (b) Transition from state i to $i+1$. With temperature rise ΔT (length of arrow), appliances whose temperatures are inside the dotted area (states A, C, E) change state and those that are outside (states B, D) remain in the same state.

The temperature change ΔT within a small time Δt is proportional to its warming rate r_{off} due to inside-outside temperature difference: $\Delta T = r_{\text{off}}\Delta t$. Assuming that an appliance's actual temperature is uniformly distributed among its bins, then the probability that the Markov jump happens is equal to the probability that the appliance temperature is in the dotted area in Fig.1(b). The probability of being in the dotted area is,

$$p_{\text{dot}} = \frac{r_{\text{off}}\Delta t}{\delta} = \frac{\Delta_{\text{band}}\Delta t}{t_{\text{off}}\delta} = \frac{N\Delta t}{t_{\text{off}}}, \quad (2)$$

where the second equality follows from the relation between warming rate and duty cycle. Since $p_{i,i+1} = p_{\text{dot}}$, comparing (1) with (2) we have $\alpha = N/t_{\text{off}}$. Similarly, $\beta = N/t_{\text{on}}$ for the duty on process. These two equalities are the implicit relation between the duty cycle and the Markov model.

When control is applied to shift the set point at a thermal rate of r_{set} (the unit of r_{set} is the same as r_{on} or r_{off}, namely ${}^{o}F/t$), there is a transition between adjacent states within Δt. Specifically, when we change the set point, the absolute temperature in each individual room does not change instantly, but its relative position to the comfort band changes as the comfort band shifts along with the set point. When the set point rises, namely $r_{\text{set}} > 0$, we can show that the control incurred Markov transition rate between adjacent states is $u = r_{\text{set}}/\delta$. The rate is the same with $r_{\text{set}} < 0$. The combined transition rate by the thermal process and set point shifting process is the sum of these two individual processes as shown in Fig.1(a). Note that the allowable set of u is $L_{\text{u}} = \{u | -\beta \leq u \leq \alpha\}$ to maintain a non-negative Markov rate. Otherwise, the system fails to be a Markov chain.

Similar to (1), we can write for non-zero u,

$$p_{i,j} = \begin{cases} (\alpha - u)\Delta t + o(\Delta t) & \text{if } i = j+1 \\ 1 - (\alpha - u)\Delta t + o(\Delta t) & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$$

Let $x(t)$ be a vector whose i^{th} component is the probability that an appliance is found in the i^{th} state in the Markov chain. It can be shown [18] that the dynamics of $x(t)$ is ,

$$\dot{x}(t) = [A + Bu(t)]x(t), \quad (3)$$

where both A and B are composed of four N by N matrices

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \qquad B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}.$$

A_{11} (A_{22}) has value $-\alpha$ $(-\beta)$ on its main diagonals and value α (β) on its sub-diagonals, respectively. A_{12} (A_{21}) is a zero matrix except that the value of the $(1, N)^{\text{th}}$ entry is β (α), respectively. B_{11} (B_{22}) has value 1 (-1) on its main diagonals and value -1 (1) on its sub-diagonals, respectively. B_{12} (B_{21}) is a zero matrix except that the value of the $(1, N)^{\text{th}}$ entry is 1 (-1), respectively. The output of the system, namely the aggregated consumption, is

$$y(t) = Cx(t), \quad (4)$$

where $C = [\underbrace{0, \ldots, 0}_{\text{N}}, \underbrace{N_{\text{c}}, \ldots, N_{\text{c}}}_{\text{N}}]$, and N_{c} is the total number of appliances in the pool.

III. Two Level Feedback System Design

Based on the model developed in Sec.II, we are able to design our two level feedback system; see Fig.2. In the building level feedback, we design a state feedback law such that the building can track the required regulation signal within certain limits. In the higher level feedback, each building sends to the ISO its information to characterize the building's capability to respond. The ISO, after receiving all information from each provider, dispatches real time signals after solving an optimization problem. Both the higher and lower level controls are operated at a scale that adequately deals with the update frequency of the reserve signals. The next sub-section will briefly discuss the building level controller design based on feedback linearzation.

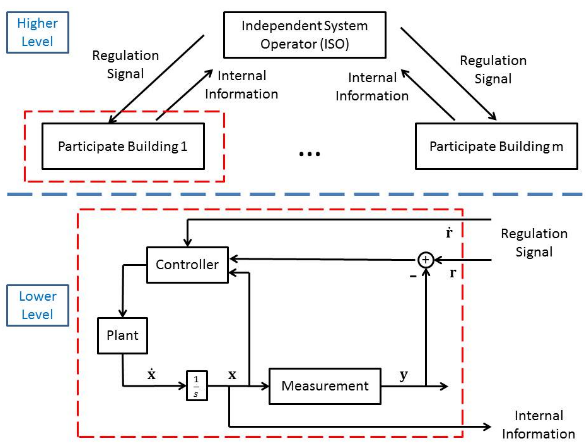

Fig. 2. Two level feedback system where the lower feedback contains individual tracking controller for a given regulation signal, and the higher feedback enables the communication between the ISO and the regulation providers such that information is exchanged to benefit real time operation.

A. Building Level Feedback Linearization Design

We use feedback linearization [19] to solve the building level controller for the non-linear system described in (3) and (4). The system has relative degree 1 since

$$\dot{y} = C\dot{x}(t) = C[A + Bu(t)]x(t),$$

depends on the control u if

$$\begin{aligned} x(t) \quad &\in \{x(t) \in \mathbb{R}^{2\mathrm{N}} | CBx(t) \neq 0\} \\ &= \{x(t) \in \mathbb{R}^{2\mathrm{N}} | x_{\mathrm{N}}(t) + x_{2\mathrm{N}}(t) > 0\}, \end{aligned} \tag{5}$$

which can be guaranteed as long as the Markov chain is irreducible and ergodic which will be the case for proper choice of $u \in (-\beta, \alpha)$ as detailed below. (5) means the number of controllable appliances near the comfort band is positive, and the set point shift can change the aggregated consumption.

To design a tracking controller for the relative degree 1 system where $R(t)$ is the regulation signal given by the ISO, we need $R(t), \dot{R}(t)$ to be available and bounded for all $t > 0$. This will be assured since the ISO regulation signal is bounded: $R(t) \in [-R_{\mathrm{r}}, R_{\mathrm{r}}]$, where R_{r} is amount of reserve sold to the market, and $R(t)$ is updated every $\Delta t = 4\mathrm{s}$ prior to consumption for the building to obtain $\dot{R}(t)$ [20],

$$\dot{R}(t) = \frac{R(t + \Delta t) - R(t)}{\Delta t}.$$

The time derivative of the tracking error,

$$\dot{e}(t) = \dot{y}(t) - \dot{R}(t) = C[A + Bu(t)]x(t) - \dot{R}(t), \tag{6}$$

will asymptotically approach 0 if $u(t)$ is chosen such that

$$\dot{e}(t) = Ke(t), K < 0. \tag{7}$$

The controller

$$u(t) = -\frac{CAx(t)}{CBx(t)} + \frac{1}{CBx(t)}[-K(\xi(t) - R(t)) + \dot{R}(t)] \tag{8}$$

satisfies the requirement because substituting (8) into (6) yields (7).

B. Long and Short Term Reserve Limitation

In this section we address the first issue identified in Sec.I – what is the regulation reserve limitation that a building can provide given its model parameters and customers' specified settings. This question has two parts: 1) in the long term, we need the accumulated shift of set point to be within the allowable range given by the customers to provide sustain level of reserve, and 2) in the short term, we need the signalling change to be within certain limits such that the controller can track it. The first proposition will discuss the long term limitation.

Proposition 1 For a given allowable set point range $[T_{\mathrm{set}} - \frac{1}{2}\Delta_{\mathrm{set}}, T_{\mathrm{set}} + \frac{1}{2}\Delta_{\mathrm{set}}]$ from customers, the accumulated amount of reserve that a building can provide up to time t, $S(t) = \sum_{i=0}^{t} R(i)$, is bounded by

$$S(t) \leq \frac{N_c \tau \Delta_{\mathrm{set}}}{2T_{\mathrm{g}}}. \tag{9}$$

Proof. Denote the base consumption level as R_{b}, the total consumption up to time t for a given sequence of $R(i)$ is,

$$P_{\mathrm{cool}} = tR_{\mathrm{b}} + \sum_{i=0}^{t} R(i) = tR_{\mathrm{b}} + S(t),$$

with an average consumption level of

$$P_{\mathrm{ave}} = R_{\mathrm{b}} + \frac{S(t)}{t}. \tag{10}$$

Providing the above amount of regulation reserve is equivalent to maintaining at a consumption level of P_{ave} to provide sustained response up to time t. The relation between the average consumption and non-zero control is,

$$P_{\mathrm{ave}} = N_{\mathrm{c}} \frac{t_{\mathrm{on}}}{t_{\mathrm{on}} + t_{\mathrm{off}}} = N_{\mathrm{c}} \frac{\frac{N}{\beta+u}}{\frac{N}{\alpha-u} + \frac{N}{\beta+u}} = N_{\mathrm{c}} \frac{\alpha - u}{\alpha + \beta}. \tag{11}$$

For an uncontrolled process at average consumption,

$$R_{\mathrm{b}} = N_{\mathrm{c}} \frac{\alpha}{\alpha + \beta}. \tag{12}$$

From (10)–(12), the equivalent control to provide the required sustained reserve is

$$u = -\frac{S(t)(\alpha+\beta)}{tN_c}.$$

The set point shift after time t by the above control u is,

$$\begin{aligned} T(t) &= T(0) + tr_{set} = T(0) + tu\delta \\ &= T(0) - t\frac{S(t)(\alpha+\beta)}{tN_c}\frac{T_{\text{band}}}{N} \\ &= T(0) - \frac{S(t)}{N_c}(r_{\text{on}} + r_{\text{off}}) \\ &= T(0) - \frac{S(t)}{N_c}\frac{T_g}{\tau}. \end{aligned}$$

The third equation is based on the relation between the Markov rate and thermal rate. The last equation says that the sum of cooling and warming rate equals the fraction T_g/τ because $r_{\text{off}} = r_{\text{amb}}$ and $r_{\text{on}} = r_{\text{app}} - r_{\text{amb}}$, where r_{amb} is the warming rate caused by the ambient temperature that is higher than room temperature, and $r_{\text{app}} = T_g/\tau$ is the cooling rate caused by the operation of the air conditioner compressor [15]. To maintain the set point shift within the allowable band with initial condition $T(0) = T_{\text{set}}$, we need $T(t) \in [T_{\text{set}} - \frac{1}{2}\Delta_{\text{set}}, T_{\text{set}} + \frac{1}{2}\Delta_{\text{set}}]$ which yields (9). ■

Remark 1 The long term reserve limitation is proportional to three parameters: N_c, Δ_{set}, and τ/T_g. The intuition for the first two parameters is that a large appliance population and allowable set point shift authorized by customers enable the operator to provide sustained reserve. For the fraction τ/T_g, a large value of τ slows down, and T_g speeds up, the rate at which the room temperature moves to the boundary of comfort band so that appliances with large τ and small T_g is more capable to provide sustained response.

To consider the short term reserve limitation, the possible ramp rate of consumption is limited by the state $x(t)$ because we are shifting the set point rather than directly turning appliances on or off. Aside from $x(t)$, we are interested in finding thermal parameters that characterize the short term limitation similar to those found in the first proposition. The desired result is the following.

Proposition 2 For a given comfort band around the set point $[T_{\text{set}} - \frac{1}{2}\Delta_{\text{band}}, T_{\text{set}} + \frac{1}{2}\Delta_{\text{band}}]$, the amount of reserve that a building can provide for the short term is limited by,

$$-Nx_{2N}\frac{N_c T_g}{\tau\Delta_{\text{band}}} \le \Delta R \le Nx_N\frac{N_c T_g}{\tau\Delta_{\text{band}}}. \quad (13)$$

Proof. In the dynamic operation when the tracking error is 0, the feedback controller is given by,

$$u = \frac{1}{N_c(x_N + x_{2N})}[N_c(\alpha x_N - \beta x_{2N}) - \Delta R]. \quad (14)$$

Since the allowable control set is $u \in [-\beta, \alpha]$, then ΔR is restricted by

$$-N_c(\alpha+\beta)x_{2N} \le \Delta R \le N_c(\alpha+\beta)x_N.$$

Using the relation between transition rate and duty cycle developed in Sec.II and the fact

$$\Delta_{\text{band}} = t_{\text{off}} r_{\text{off}} = t_{\text{on}} r_{\text{on}},$$

the following inequality is seen to hold:

$$-Nx_{2N}\frac{N_c(r_{\text{on}} + r_{\text{off}})}{\Delta_{\text{band}}} \le \Delta R \le Nx_N\frac{N_c(r_{\text{on}} + r_{\text{off}})}{\Delta_{\text{band}}}.$$

Using the fact that $r_{\text{on}} + r_{\text{off}} = T_g/\tau$ yields (13). ■

Remark 2 The short term reserve capacity is proportional to N_c, and inversely proportional to Δ_{band} and τ/T_g. The proportionality to N_c shares the same explanation as in Remark 1. The two inverse proportionalities can be explained that small value of Δ_{band} makes the Markov transition faster, and large value of T_g/τ makes the thermal transfer faster. These two factors in turn facilitate the state transition to provide larger instant-by-instant reserve.

Remark 3 The advantage of using small Δ_{band} is that we can provide large short term reserve while controlling room temperatures to remain close to their set points but with the potential that we shorten the appliances' duty cycle. This trade off between system performance and appliances' duty cycle functioning is consistent with what we reported in [8] where electricity consumption was shown to be smoothed by shortening appliances duty cycle.

Remark 4 The fraction τ/T_g affects both long and short term reserve limitation. Based on (9) and (13), we find that an appliance that is able to provide a large amount of short term ramping regulation is less capable of providing long term accumulated regulation. The opposite also holds. Thus the two capabilities restrict each other.

C. Buildings in the Regulation Reserve Market

Based on the above propositions, we determine the maximum regulation reserve that a building can provide to the power market. To become a qualified provider in the U.S., a building needs to pass the T-50 qualifying test [20]. Fig.3 shows the test signal from PJM. The dotted line is the ideal consumption response of the building to the test signal. Two key requirements are needed to pass the test:

- Rate of response: the building needs to reach the maximum (minimum) consumption level within k minutes.
- Sustained response: the building needs to remain at the maximum (minimum) consumption level for 5 minutes.

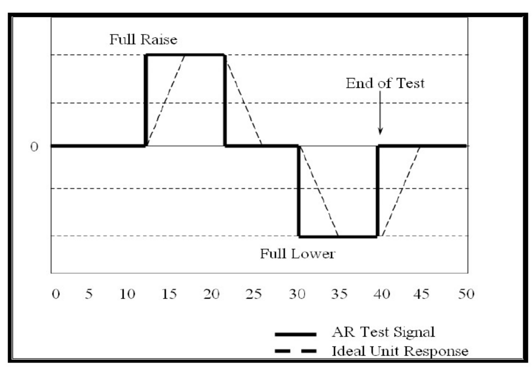

Fig. 3. T-50 test by PJM where regulation providers are obliged to ramp their consumption up and down to meet the test requirements [20].

The following corollary gives an upper bound on the regulation reserve that a building can provide.

Corollary [18] To pass the T-50 qualifying test with a response rate in k minutes ($k \leq 5$), the maximum regulation reserve $R_{\text{r,max}}$ that a building can sell to the market is

$$R_{\text{r,max}} = \min\left\{\frac{N_c \tau \Delta_{\text{set}}}{20 T_{\text{g}}}, \frac{k N_c}{\max(t_{\text{on}}, t_{\text{off}})}\right\}.$$

D. Real Time Spinning Reserve

When the real time consumption ramps up or down by ΔP due to stochastic demand, the ISO has to use spinning reserves to compensate for the demand that cannot be covered by the regulation reserve from buildings. In such cases, a feedback signal from the provider to the ISO is beneficial so that the ISO can determine the needed spinning reserve P_{spin}. The proposition below illustrates the relation between P_{spin} and ΔP when building information is available to the ISO.

Proposition 3 Denote ΔP as the stochastic demand change. Then the spinning reserve P_{spin} needed to maintain grid balance is given by,

$$\begin{aligned} P_{\text{spin}} = \quad & (\Delta P - \Delta R_{\text{max}}) \mathbb{1}_{\{\Delta P > \Delta R_{\text{max}}\}} + \\ & (\Delta P - \Delta R_{\text{min}}) \mathbb{1}_{\{\Delta P < \Delta R_{\text{min}}\}}, \end{aligned} \tag{15}$$

where $\mathbb{1}_{\{\cdot\}}$ is the indicator function, and

$$\begin{aligned} \Delta R_{\text{max}} = \quad & N_{\text{c}} \Big\{ (r_{\text{off}} x_{\text{N}} - r_{\text{on}} x_{2\text{N}}) + \\ & \min(\frac{T_{\text{set}} - T_{\text{set}}^{\text{min}}}{\Delta t \delta}, r_{\text{on}})(x_{\text{N}} + x_{2\text{N}}) \Big\}, \\ \Delta R_{\text{min}} = \quad & N_{\text{c}} \Big\{ (r_{\text{off}} x_{\text{N}} - r_{\text{on}} x_{2\text{N}}) - \\ & \min(\frac{T_{\text{set}}^{\text{max}} - T_{\text{set}}}{\Delta t \delta}, r_{\text{off}})(x_{\text{N}} + x_{2\text{N}}) \Big\}, \end{aligned} \tag{16}$$

are the maximum and minimum reserve capacity threshold from the provider.

Proof. The allowable control u is limited by two factors. The first is that the set point shift resulting from u should be within the allowable set point range $T_{\text{set}} + \Delta t \delta u \in [T_{\text{set}}^{\text{min}}, T_{\text{set}}^{\text{max}}]$. The second is the requirement of maintaining non-negative Markov rate $u \in [-\beta, \alpha]$. From the two constraints

$$u \in \left[\max(\frac{T_{\text{set}}^{\text{min}} - T_{\text{set}}}{\Delta t \delta}, -\beta), \min(\frac{T_{\text{set}}^{\text{max}} - T_{\text{set}}}{\Delta t \delta}, \alpha)\right]. \tag{17}$$

From (14), the instantaneous response can be expressed in terms of the control u,

$$\Delta R = N_{\text{c}} \big\{ (\alpha x_{\text{N}} - \beta x_{2\text{N}}) - u(x_{\text{N}} + x_{2\text{N}}) \big\} \tag{18}$$

Substituting (17) into (18) yields $\Delta R \in [\Delta R_{\text{min}}, \Delta R_{\text{max}}]$, where ΔR_{min} and ΔR_{max} take values in (16). For the grid balance, we have $\Delta P = \Delta R + P_{\text{spin}}$. We wish to use as little spinning reserve as possible, then P_{spin} will take value in (15). The indicator function in (15) gives the condition that ΔP is outside the range of ΔR.■

E. Real Time Regulation Signal Dispatch

In the power market, the ISO purchases reserves from a number m of providers. Assume that feedback signals are available between each provider and the ISO to exchange real time information. Upon receiving information, the ISO knows their individual capability limitations, and then dispatches regulation signals to each of the providers. The signals are dispatched such that the minimum spinning generation is used and such that the request is within the capability the providers to respond. The question is how can the ISO dispatch signals in an optimized way at time t. One criteria is to maximize the regulation response capability at time $t+1$. From proposition 3, the i^{th} building can provide regulation reserve with range $\Delta R \in [\Delta R_{\text{min}}^i, \Delta R_{\text{max}}^i]$. The range of the closed interval of regulation reserve is given by

$$\begin{aligned} W_{\text{d}}^i \quad & = \Delta R_{\text{max}}^i - \Delta R_{\text{min}}^i \\ & = N_{\text{c}}^i (x_{\text{N}}^i(t+1) + x_{2\text{N}}^i(t+1)) \\ & \Big\{ \min(\frac{T_{\text{set}}^i(t+1) - T_{\text{set}}^{\text{min},i}}{\Delta t \delta}, r_{\text{on}}^i) + \\ & \min(\frac{T_{\text{set}}^{\text{max},i}(t+1) - T_{\text{set}}^i}{\Delta t \delta}, r_{\text{off}}^i) \Big\}. \end{aligned}$$

Then the objective becomes a piecewise linear function with quadratic terms

$$\max \sum_{i=1}^{m} W_{\text{d}}^i - M P_{\text{spin}}^2, \tag{19}$$

which is to say that we are maximizing the sum of m regulation reserve ranges from the buildings at time $t+1$, minus the spinning generation penalty P_{spin} with positive M. The maximization problem is subject to the following constrains for each reserve provider i,

$$\begin{gathered} x_{\text{N}}^i(t+1) = x_{\text{N}}^i(t) + \Delta t(\alpha^i - u^i)(x_{\text{N-1}}^i(t) - x_{\text{N}}^i(t)), \\ x_{2\text{N}}^i(t+1) = x_{2\text{N}}^i(t) + \Delta t(\beta^i + u^i)(x_{2\text{N-1}}^i(t) - x_{2\text{N}}^i(t)), \\ N_{\text{c}}^i \Delta t(x_{\text{N}}^i(t) + x_{2\text{N}}^i(t)) u^i + \Delta R^i = N_{\text{c}}^i \Delta t(\alpha x_{\text{N}}^i(t) - \beta x_{2\text{N}}^i(t)), \\ \sum_{i=1}^{m} \Delta R^i + P_{\text{spin}} = \Delta P, \\ -\beta^i \leq u^i \leq \alpha^i, \\ R_{\text{b}} - R_{\text{r}} \leq Cx + \Delta R^i \leq R_{\text{b}} + R_{\text{r}}, \\ T_{\text{set}}^{\text{min},i} \leq T_{\text{set}}^i(t+1) = T_{\text{set}}^i(t) + u^i \Delta t \delta \leq T_{\text{set}}^{\text{max},i}, \end{gathered} \tag{20}$$

where the first two equations in (20) are the state dynamics, and the remaining five equations stand for the feedback controller design, supply demand balance, the allowable control set, the allowable regulation reserve range, and the allowable set point range respectively. We transform the quadratic objective function with piece-wise linear terms into a standard quadratic problem. Let

$$\begin{aligned} m_1^i &= \min(\frac{T_{\text{set}}^i(t+1) - T_{\text{set}}^{\text{min},i}}{\Delta t \delta}, r_{\text{on}}^i), \\ m_2^i &= \min(\frac{T_{\text{set}}^{\text{max},i} - T_{\text{set}}^i(t+1)}{\Delta t \delta}, r_{\text{off}}^i). \end{aligned} \tag{21}$$

Then the objective becomes

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} N_{\mathrm{c}}^{i}(x_{\mathrm{N}}^{i}(t+1)+x_{2\mathrm{N}}^{i}(t+1))(m_1^i+m_2^i) \\ & -MP_{\text{spin}}^2. \end{aligned} \tag{22}$$

If we add the following constrains,

$$\begin{aligned} m_1^i \le r_{\text{on}}, m_1^i \le \frac{T_{\text{set}}^i(t+1)-T_{\text{set}}^{\min,i}}{\Delta t\delta}, \\ m_2^i \le r_{\text{off}}, m_2^i \le \frac{T_{\text{set}}^{\max,i}-T_{\text{set}}^i(t+1)}{\Delta t\delta}, \end{aligned} \tag{23}$$

and solve the QP,

$$\max \ (22) \quad \text{s.t. } (20),(23), \tag{24}$$

we will obtain the same optimal solution as solving the original problem. This is because when reaching the optimal solution, one of the inequality constraints for both m_1^i and m_2^i in (23) will be strict, otherwise the optimal solution is not reached since we can increase the value of m_1^i or m_2^i to increase the value of objective function due to positive coefficient $N_{\mathrm{c}}^{i}(x_{\mathrm{N}}^{i}(t+1)+x_{2\mathrm{N}}^{i}(t+1))$ in (22). Then the optimal solution for (24) will satisfy (21) and optimizing over (24) is equivalent to solving (19) subject to (20). Note that (24) has quadratic objective with linear constraints so that we can solve it efficiently.

IV. Time Varying Observer Design

In controller (8), it is assumed that $x(t)$ is measurable. If this is not true, especially when the temperature band is finely discretized, i.e. the width of the temperature bin δ is so small that the sensor cannot provide the required precision, we need an observer to estimate the state. Considering the following dynamics of the observer,

$$\dot{\hat{x}} = A\hat{x} + B\hat{x}u + L(t)(y - C\hat{x}),$$

where $L(t)$ is the time varying column vector to be designed. The last term is similar to the innovation term in a Luenberger filter. Define the estimation error, $e = x - \hat{x}$. Then

$$\dot{e} = [A + Bu(t) - L(t)C]e.$$

In the observer design, we restrict $u(t)$ to (25) to prevent control saturation with small constant positive margin $\hat{\epsilon} > 0$,

$$-\beta + \hat{\epsilon} \le u(t) \le \alpha - \hat{\epsilon}. \tag{25}$$

This guarantees an irreducible and ergodic Markov chain since all transition rates are positive in Fig.1(a). If the control signal is allowed to stay saturated for a long time, then the Markov chain breaks and some of the states become isolated, see Fig.1(a) when $u = -\beta$ or $u = \alpha$. Estimation error of the isolated states may not be reduced. The desired observer design is shown below.

Proposition 4 [18] If the control is restricted to (25), then (1) There exists a time varying $L(t) = [\underbrace{0,\ldots,0}_{2N-1}, L_{2\mathrm{N}}(t)]^T$ such that $e(t)^T[A+Bu(t)-L(t)C]e(t) < -\epsilon(t)\|e(t)\|^2$ for all $e(t)$ and $\epsilon(t) = \gamma\min[\beta+u(t), \alpha-u(t)]$ with $\gamma \in (0,1)$.
(2) $\lim_{t\to\infty} e(t) = 0$.

Remark 5 $\epsilon(t) = \gamma\min[\beta+u(t), \alpha-u(t)] \ge \gamma\hat{\epsilon}$ is the lower bound on the convergence rate. In real time control when the signal is distributed across the allowable set being away from saturation more than $\hat{\epsilon}$, the convergence rate is larger than this conservative bound. We note that as the duty cycle increases, the boundary values of $\alpha = N/t_{\text{off}}$ and $\beta = N/t_{\text{on}}$ decrease, which means we have smaller allowable control set. Consequently, the control with large duty cycle appliances would be closer to the boundary than the control with small duty cycle appliances given the same ISO signal. It is therefore anticipated that the convergence error will be faster for systems having short duty cycle appliances.

Remark 6 The restriction in (25) is conservative. A mild statement is the following:

Let S be the set of time that the control is bounded away from saturation with some constant value $\tilde{\epsilon} > 0$, i.e. $S = \{t : -\beta + \hat{\epsilon} \le u(t) \le \alpha - \hat{\epsilon}\}$. Then Proposition 4 holds if the measure of the set is infinity: $\mu(S) \to \infty$ as $t \to \infty$.

V. Simulation

A. Long and Short Term Reserve Limitation

Fig.4 shows that both the long and short term reserve limitation affect the performance in the T-50 test. Figures in the left column show the inability to provide long term sustained reserve. The set point hits the lower bound of the allowable range between 15 and 25 minutes which prevents the building from providing a sustained high level reserve by further adjusting the set point. Similarly, the set point hits the upper bound after 40 minutes which prevents the building from providing sustained low level reserve. Figures in the right column is to show the inability to provide fast ramping reserve. Although the set point shift is within the allowable set indicating large capacity potential, the response rate is smaller than the required rate for a given level R_{r} sold to the market. This indicates that thermostatic appliances with large duty cycle have limited instant-by-instant reserve capability. For example, the typical duty cycle of refrigerators is between 20 and 30 minutes reflecting their large effective thermal constant. These long duty cycle appliances have limited capacity to provide short term reserve but huge potential for long term reserve. On the other hand, appliances like air conditioners with shorter duty cycles around 10 minutes have limited capacity for sustained reserve but large potential for ramping reserve.

B. Optimal Regulation Signal Dispatch

We use PJM data [21] to verify the performance of the two level feedback system. Three smart buildings with aggregated air conditioners provide reserve to the ISO with contracted level R_{r} 200kW, 150kW, 400kW and base consumption level R_{b} 500kW, 300kW, 800kW, respectively. Fig.5 and Tab.1 compare the proposed two level feedback system and the system without higher level information exchange where the ISO assigns reserve signals proportionally to the individual building's contracted level $R_{\mathbf{r}}$. Simulations show that the real time aggregated spinning generation is reduced approximately by 50% with optimized dispatch signals.

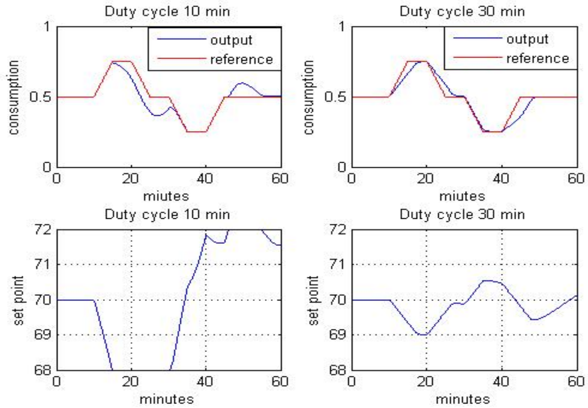

Fig. 4. Reserve provider fails to pass the T-50 qualifying test due to either a bounded allowable set point range or a limited ramping capability. Short duty cycle appliances are able to provide short ramping reserve, but limited sustained reserve. Long duty cycle appliances behaves oppositely.

The spinning generation's standard deviation, maximum, and minimum values are reduced by 30%, 30%, and 15% respectively. Clearly the higher information feedback results in significantly reduced operation costs.

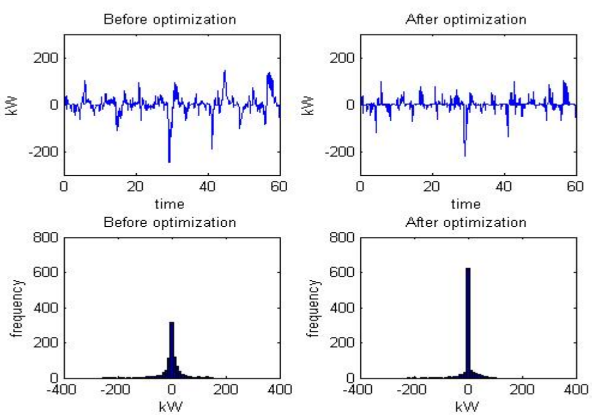

Fig. 5. Real time spinning generation before/after optimization. Spinning generation is more concentrated around 0 with reduced variation after regulation signals are optimally dispatched.

TABLE I
COMPARISON OF SYSTEMS' PERFORMANCE

Unit/kW	Spin. Gen.	Mean	Std. Dev.	Max	Min
Opt. Sig.	10216	0.18	27.20	100.27	-210.59
N-Opt. Sig.	20525	0.44	41.97	146.19	-247.65

VI. CONCLUSION

This paper proposes a two level feedback system design to provide high regulation reserve. The lower level building feedback controller allows the operator to track a given signal within certain limits, and the higher level information feedback allows the ISO to optimally dispatch real time regulation signals to multiple providers. We derive an analytic expression for the building's limitation in providing both long and short term regulation reserve and provide intuitive explanations. The necessary information feedback structure between the ISO and multiple providers is proposed to reduce real time spinning generation costs. Future work will integrate the energy packet framework [8], [9] into the two level feedback system to create a packetized solution.

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