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Planning and Control of Electric Vehicles Using Dynamic Energy Capacity Models

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***Abstract*— This paper focuses on energy management for a large population of Plug-in Electric Vehicles (PEVs) for demand response applications. We consider both real time charging control as well as energy planning optimizations. The main contribution of the paper lies in the development of a novel dynamic energy capacity model in which the energy variation range of the aggregated loads available at each time step is a function of the past energy management decisions. Such a model enables systematic yet simple design of planning strategies that minimize energy costs while respecting the dynamic energy shifting capacity of the load aggregation. A further contribution is on the development of a novel stochastic hybrid system model that can fully characterizes the dynamics and stochasticity of individual charging demands for real time implementation of the planning decisions. Simulation results show that the proposed energy capacity model closely captures the capabilities of the real system. Additionally, we show how the model could be used to achieve a specific objective: minimization of daily energy costs.**

I. INTRODUCTION

The number of Plug-in Electric Vehicles (PEVs) is expected to be more than one million by the year 2015 [1]. Aggregations of large populations of PEVs with flexible charging schedules can be used for a variety of purposes [2], [3], [4], [5], [6], [7], [8].

Due to their diverse applications, energy management of PEVs have been studied extensively in the literature [9], [10], [11], [6], [7], [12], [8]. The problem involves two decision making stages: energy planning (or scheduling) based on uncertain fleet statistical information, and real-time charging control that coordinates the PEV charging online to make the actual energy consumption match the planned energy budget. The planning problem is often formulated as an optimization problem to minimize the energy cost of the fleet while respecting charging deadlines and limitations of distribution systems. While many planning and scheduling algorithms have been proposed in the literature, their coupling and interactions with the real time charging controller has not been adequately addressed. A key challenge lies in developing models that are sufficiently detailed to capture the dynamics, time varying constraints, and control capabilities of individual charging loads, while at the same time computationally tractable at the aggregated planning level [13]. One solution is to develop different models for different management tasks. In [14], the authors proposed a three step approach to PEV energy management: (i) constraint aggregation, (ii) optimization of the fleet, and (iii) real-time control. Importantly, the effectiveness of the real-time controller is a function of the accuracy of the planning algorithm. For example, if the aggregate model is inaccurate and causes an overestimation of the resource, then the real-time controller will not be able to achieve its objective. Therefore, the accuracy of the aggregate model is integral to our efficient use of charging demand flexibilities.

This paper presents a general framework that enables systematic design of both planning and real time controllers for a fleet of PEVs to achieve system level objectives. Our key contribution is on the development of a dynamic energy capacity model that accurately tracks the energy consumption flexibility of the aggregated charging loads. In contrast to previous work [14], [15], [16], we explicitly model the relation between past control actions and current resource capabilities. A further contribution is on the development of a stochastic hybrid model that allows us to fully capture the stochasticity of the charging demand and the information structure for making real time charging decisions. Together these models allow us to characterize the energy shifting potential of vehicle aggregations in a way that respects the control capabilities and dynamic constraints of individual charging demands. We demonstrate the effectiveness of this approach through a number of simulations.

The rest of the paper is organized as follows. Section II introduces a stochastic hybrid system model for individual charging loads. Section III describes the two-layer energy management structure. In Section IV, we present our energy capacity model to describe demand shifting dynamics of the PEV fleet. Realistic simulation results are provided in Section V, and concluding remarks are given in Section VI.

II. HYBRID SYSTEM MODEL FOR INDIVIDUAL VEHICLES

This paper considers a scenario where an aggregator jointly manages the charging for a large number $n \in \mathbb{Z}_+$ of PEVs. Each PEV will plug in and unplug at random times with random energy request from the grid. Let a_j^i be the j^{th} arrival (plug-in) time of the i^{th} vehicle. Assume the vehicle needs e_j^i amount of energy from the grid before a deadline time $a_j^i + \tau_j^i$ specified by the user, where τ_j^i is the maximal allowable charging time. The required energy e_j^i depends on

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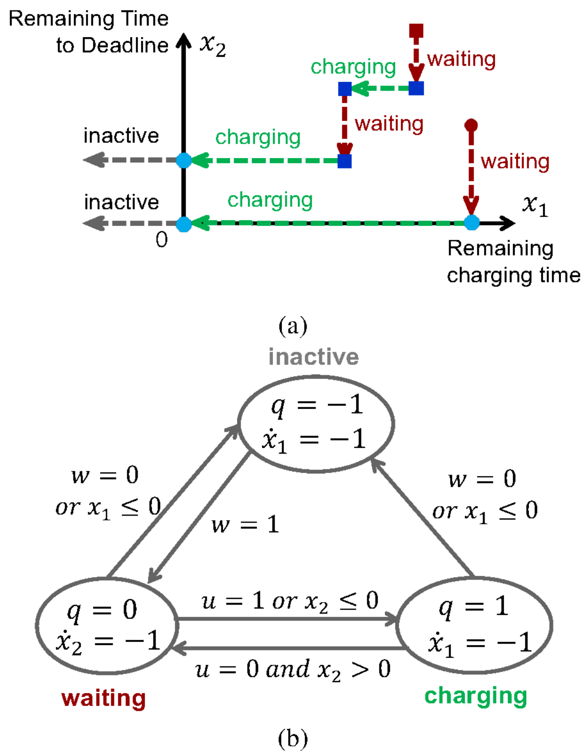

Fig. 1. (a) Examples of hybrid state trajectories; (b) Mode transition diagrams for proposed the stochastic hybrid system

the initial State of Charge (SOC), the battery size, and the desired final SOC level. We assume that the aggregator has to finish charging before the specified deadline because the PEVs have to unplug after the deadline. Battery discharging is not allowed. Hence, when the battery reaches the SOC designated by the user it is effectively unplugged. A PEV charging load will be controlled through direct on/off control signals and the charging rate r^i is constant which allow for a simple practical implementation. The time required to complete the charging load is thus given by e^i_j/r^i.

Consider the i^{th} PEV. Its discrete mode $q^i(t)$ can take three possible values in $\mathcal{Q} \triangleq \{-1, 0, 1\}$, representing that the load is inactive (not plugged in or has been completed[1]), is waiting to be served, and is actively charging, respectively. When $q^i(t) = 0$ or 1, the timing dynamics of the load are captured by a 2D continuous state variable $x^i(t) = [x^i_1(t), x^i_2(t)]^T$, where $x^i_1(t)$ represents the remaining time to complete the load if it is continuously being served and $x^i_2(t)$ represents the amount of time the load can be further deferred. Therefore, the load can be viewed as a particle in the 2D space as shown in Fig. 1. The particle moves down when $q^i(t) = 0$ and moves left when $q^i(t) = 1$. The transition between these two modes can be controlled by the aggregator through the control signal $u^i(t) \in U\{0, 1, -\infty\}$, representing waiting, charging, and "null" (no change) commands. To respect the charging deadline, whenever the particle hits the x_1-axis, it must switch to the charging mode and cannot be deferred anymore. The output $y(t)$ of the load model is

[1]A charging load is called *completed* if the SOC designated by the user has been reached.

the power consumption of the PEV, which is given by $y^i(t) = h^i(q^i(t))$. The function $h^i : Q \to \mathbb{R}_+$ is defined as

$$h^i(q) = \begin{cases} r^i & \text{if } q = 1 \\ 0 & \text{otherwise} \end{cases}$$

The arrival and departure process of the PEV is denoted by $w^i(t)$, where $w^i(t) = 1$ if $t \in [a^i_j, a^i_j + \tau^i_j]$ for some $j \in Z_+$, and $w^i(t) = 0$ otherwise. The overall charging demand dynamics can be precisely described by the following (stochastic) hybrid system model:

$$\begin{cases} \dot{x}^i(t) = f(x^i(t), q^i(t)) \\ q^i(t^+) = g(x^i(t), q^i(t), u^i(t), w^i(t)) \\ x^i\left((a^i_j)^+\right) = [e^i_j/r^i, a^i_j + \tau^i_j - e^i_j/r^i] \\ y^i(t) = h^i(q^i(t)), \end{cases} \tag{1}$$

The vector field $f : \mathbb{R}^2 \times Q \to \mathbb{R}^2$ is given by

$$f(x, 0) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, f(x, 1) = f(x, -1) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

Notice that there are many ways to define the continuous dynamics in mode $q^i(t) = -1$. Here, our definition assumes $x^i_1(t)$ becomes negative once the load has been completed. The mode transition is governed by the transition function $g : \mathbb{R}^2 \times \mathcal{Q} \times U \times W \to \mathcal{Q}$, which can be viewed as a slight generalization of the state transition function used in formal descriptions of finite automata and discrete event systems (see for example [17]). The function g does not depend on the particular load and is given by the transition diagram shown in Fig. 1-(b). At each plug-in time a^i_j, the continuous state will reset to a random value e^i_j/r_i and then continue with its continuous evolution. Examples of two hybrid state trajectories are given in Fig. 1-(a), where square markers represent a controlled mode transition event while the circle markers represent the forced mode transition due to certain guard conditions.

Details on the execution (simulation) of a stochastic hybrid system like the one defined in (1) can be found in [18], [19]. The benefit of this hybrid system viewpoint lies in its complete characterization of the demand dynamics, and its explicit characterization of the effect of control under all the possible operation modes of the vehicle. In addition, the charging deadline constraint is encoded naturally by the guard condition (forced transition when hitting the x_1 axis).

III. Energy Management Framework

The overall demand dynamics of the PEV fleet can be represented by the collection of individual PEV hybrid system models. We define the overall fleet continuous state as $\mathbf{x}(t) = (x^1(t), \ldots, x^n(t))$, the fleet discrete mode as $\mathbf{q}(t) = (q^1(t), \ldots, q^n(t))$, the fleet charging control vector as $\mathbf{u}(t) = (u^1(t), \ldots, u^n(t))$. To simplify notation, we shall denote $\mathbf{X} = \mathbb{R}^{2\times n}$, $\mathbf{Q} = Q^n$, $\mathbf{U} = U^n$. The total power consumption of the fleet can be varied to a certain extent through the control $\mathbf{u}(t)$. This property makes the

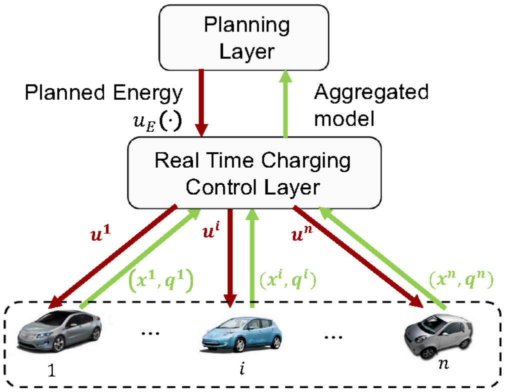

Fig. 2. The decision and information architecture of the energy management system.

fleet a valuable asset for various types of demand response programs.

The rest of this section will provide technical details regarding the two layer decision energy management problem.

A. Real Time Charging Control

At any given time $t \geq 0$, the information available to the aggregator is the hybrid state $(\mathbf{x}(t), \mathbf{q}(t))$. Let $\mu : \mathbf{X} \times \mathbf{Q} \times \mathbb{R}_+ \to \mathbf{U}$ be the charging control policy that maps the hybrid state $(\mathbf{x}(t), \mathbf{q}(t))$ at time $t \in \mathbb{R}_+$ to a fleet control action $\mathbf{u}(t) \in \mathbf{U}$. For each $x \in \mathbf{X}$ and $q \in \mathbf{Q}$, denote by $\mu^i(x, q, t)$ the i^{th} entry of the vector $\mu(x, q, t) \in \mathbf{U}$. In general, the control policy can be time-varying. For time-invariant policies, we will drop the time component and simply write $\mu(x, q)$.

Under a given charging control policy μ, the closed-loop dynamics of the fleet are given by the following equations

$$\begin{cases} \dot{x}^i(t) = f(x^i(t), q^i(t)); \\ q^i(t^+) = g(x^i(t), q^i(t), \mu^i(x^i(t), q^i(t), t), w^i(t)) \\ y^{all}(t) = \sum_{i=1}^n h^i(q^i(t)) \end{cases} \tag{2}$$

Rigorously speaking, the total power consumption $y^{all}(t)$ is stochastic due to its dependence on the stochastic processes $\{w^i\}_{i \leq n}$ and the random variables $\{e^i\}_{i \leq n}$.

A key appealing feature of the PEV charging loads lies in the high controllability of its total charging power. The maximum and minimum achievable power at time t is given by

$$\begin{cases} p_{\max}(t) = \sum_{i \in \mathcal{I}_c(t)} r^i \\ p_{\min}(t) = \sum_{i \in \mathcal{I}_d(t)} r^i \end{cases} \tag{3}$$

Here, $\mathcal{I}_c(t) = \{i \leq n : q^i(t) \neq -1\}$ is the index set of PEVs that can actively charge or wait at time t while $\mathcal{I}_d = \{i \leq n : q^i(t) \neq -1 \wedge x_2(t) \leq 0\}$ is the index set of the PEVs that cannot be deferred. Therefore, the range of the fleet power consumption at time t depends nontrivially on the hybrid state vector $(\mathbf{x}(t), \mathbf{q}(t))$.

Various charging control policies can be designed for demand response applications. Let $\mu_L : \mathbf{X} \times \mathbf{Q} \to \mathbf{U}$ and $\mu_I : \mathbf{X} \times \mathbf{Q} \to \mathbf{U}$ be the *last minute charging policy* and *immediate charging policy*, respectively. Both of them are time-invariant and their i^{th} elements are given as follows:

$$\mu_L^i(\mathbf{x}(t), \mathbf{q}(t)) = \begin{cases} 0, & \text{if } q^i(t) = 1 \wedge x_2^i(t) > 0 \\ -\infty, & \text{otherwise} \end{cases}$$

$$\mu_I^i(\mathbf{x}(t), \mathbf{q}(t)) = \begin{cases} 1, & \text{if } q^i(t) = 0 \\ -\infty, & \text{otherwise} \end{cases}$$

The last minute charging policy will pause all the loads that can be deferred, while the immediate charging policy will allow all the loads connected to the system to charge.

For a given reference power $\hat{p} > 0$, let $\mu_T(\cdot, \cdot; \hat{p}) : \mathbf{X} \times \mathbf{Q} \to \mathbf{U}$ be the *power tracking policy* that allocates the power budget $\hat{p}$ to the PEVs with the smallest remaining time to deadline. The policy makes the best effort to match $y^{all}(t)$ with $\hat{p}$. Under this policy, $y^{all}(t) = p_{\max}(t)$ if $\hat{p} > p_{\max}(t)$, and $y^{all}(t) = p_{\min}(t)$ if $\hat{p} < p_{\min}(t)$. For $\hat{p} \in (p_{\min}(t), p_{\max}(t))$, the resulting $y^{all}(t)$ is approximately $\hat{p}$ with error smaller than the power of the load whose x_2 value is the largest among all the charging loads at time t.

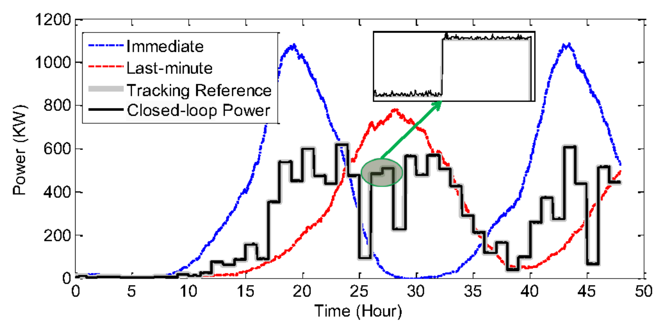

Fig. 3. Closed-loop trajectories under the immediate charging policy, the last minute charging policy, and a power tracking policy with a properly designed tracking reference.

Fig. 3 shows a realization of the total power of a fleet under each of the three policies μ_L, μ_I, and μ_T, defined above. The generation of the fleet data can be found in Section V. The figure indicates that the total power can be varied significantly without violating deadline constraints, and with a properly selected tracking reference, the power consumption can be controlled fairly accurately.

B. Energy Planning

The planning decision layer determines the energy budget $u_E(k)$ allocated to each decision period $[t_{k-1}, t_k]$, $k = 1, \ldots, N$ to achieve various demand response objectives. The vector $\mathbf{u}_E = [u_E(1), \ldots, u_E(N)]^T$ is called a planning control sequence. Once the planning decision $\mathbf{u}_E$ is made, we assume the tracking control policy $\mu_T(\cdot, \cdot; p_k)$ is used for the k^{th} period, where the power reference is given by $p_k = u_E(k)/\Delta t$. In the rest of this paper, the hybrid trajectory $(\mathbf{x}(t), \mathbf{q}(t))$ and output $y^{all}(t)$ are assumed to be the closed-loop trajectories under the policy $\mu_T(\cdot, \cdot; p_k)$, unless otherwise specified.

Let $E(k)$ be the actual closed-loop energy consumption over the k^{th} period, i.e.,

$$E(k) = \int_{t_{k-1}}^{t_k} y^{all}(t)dt$$

If planning control sequence is not selected properly, the actual energy consumption $E(k)$ can be quite different from the planned value $u_E(k)$. Therefore, the planning decision needs to respect the actual variability of the fleet energy. Let $E_{\max}(k)$ and $E_{\min}(k)$ be the maximum and minimum achievable energy of the closed-loop system trajectory under policy $\mu_T(\cdot,\cdot;p_k)$ during the k^{th} period, i.e.,

$$E_{\max}(k) = \int_{t_{k-1}}^{t_k} p_{\max}(t)dt \text{ and } E_{\min}(k) = \int_{t_{k-1}}^{t_k} p_{\min}(t)dt.$$

Then the energy planning problem can be formulated as

$$\begin{cases} \min_{u_E} \sum_{k=1}^{N} L(u_E(k)) \\ \text{s.t.: } E_{\min}(k) \le u_E(k) \le E_{\max}(k), k = 1,\dots,N \end{cases} \tag{4}$$

where L is an arbitrary cost function of $u_E(k)$. The main challenge for solving the above problem lies in the dynamic dependence of $E_{\min}$ and $E_{\max}$ on the control sequence u_E. For each given control u_E, one can use Monte Carlo simulations of the closed-loop trajectories to compute the actual $E_{\min}$ and $E_{\max}$; thus the feasibility of a given control can be checked numerically. However, this approach is not fully scalable, and makes it hard (if not impossible) to find the globally optimal solution. In the next section, we will develop a novel dynamic model to quantify how the energy allocation at step k will affect the energy variation range over later steps. This will transform the energy planning problem to a discrete time optimal control problem, admitting a simple and scalable solution (with respect to the number of PEVs) through dynamic programming.

IV. Modeling Demand Shifting Dynamics for Energy Planning

The energy variation range $[E_{\min}(k), E_{\max}(k)]$ has been mostly treated as static in the literature [14], [15], [16]. Our key observation here is that the energy variation range $[E_{\min}(k), E_{\max}(k)]$ depends dynamically on the planning decision $\mathbf{u}_E(k)$. This phenomenon will be referred to as the demand shifting dynamics. In this section, we will first illustrate the shifting dynamics using a simple example, and then develop a dynamic model to capture the relationship between the variation range and the planning control sequence.

A. Simple Illustrating Example

For illustration purposes, we consider a simplified scenario, where the current and future charging loads are divided into 4 groups as shown in Fig. 4. The first three groups are already in the system at time 1 while Group 4 will arrive at time 2. The shaded squares in the figure represent the amount of required energy, while the blank squares represent the extra times available for serving the loads. For example, the figure indicates that Group 2 demands 2 units of energy from the grid before time 4.

We now compute the $E_{\min}$ and $E_{\max}$ for this simple scenario. Since all the first 3 groups can be deferred for at least 1 time step, we can easily find out that $E_{\min}(1) = 0$ and

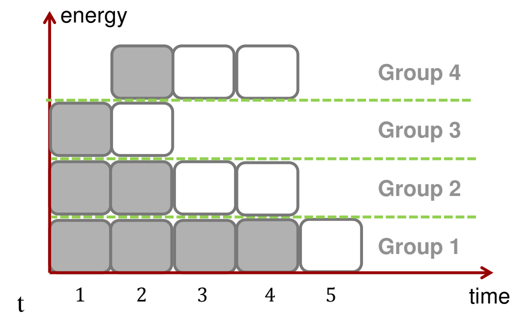

Fig. 4. Illustration of demand shifting dynamics.

$E_{\max}(1) = 3$. Suppose we choose a feasible energy budget for time step 1 as $u_E(1) = 0$. This implies all three groups will be postponed by 1 time step. Therefore, at time step 2, Group 1 and Group 3 can no longer be deferred, indicating that $E_{\min}(2) = 2$ and $E_{\max}(2) = 4$. However, if we choose $u_E(1) = 3$ at time step 1, then at time step 2, Group 3 is completed while Group 1 and Group 2 can still be deferred, indicating that $E_{\min}(2) = 0$ and $E_{\max}(2) = 3$.

Based on the above discussion, the relationship between energy variation range and the planning control sequences is dynamic and depends on the states of all the PEVs[2]. Since the evolution of the states is rather complex, the exact and complete characterization of the demand shifting dynamics can be computationally intensive. To enable real world applications, a simple yet effective means to capture the shifting dynamics is desirable.

B. Dynamics of E_{min}

We first derive the dynamics of $E_{\min}(k)$. For each $k \le N$, let $d(k)$ be the energy consumption of the fleet in period k under the condition that the last minute charging strategy is applied from time 1 to time k. The sequence $d(k)$ can be estimated accurately at the beginning of the entire planning horizon through simulations. Referring again to the example in the last subsection, $[d(1),\dots,d(5)] = [0,2,2,3,1]$.

It is easy to verify that the sequence $d(k)$ is not a lower bound for feasible control $u_E(k)$, but it does provide a lower bound in terms of its partial sum. Therefore, it is often possible to have $u_E(k) < d(k)$ for a general $k > 1$. However, $\sum_{m=1}^{k} u_E(m) \le \sum_{m=1}^{k} d(m)$ must be satisfied for all $k = 1,\dots,N$. At step k, the quantity $\sum_{m=1}^{k} u_E(m) - \sum_{m=1}^{k} d(m)$ represents the extra load completed as compared to the last minute charging strategy. This extra amount will reduce the minimum required energy for step $k+1$ or later. Therefore, an estimated value of $E_{\min}(k)$, denoted by $\hat{E}_{\min}(k)$, after applying the energy allocation sequence $u_E(1),\dots,u_E(k)$, is given by

$$\hat{E}_{\min}(k) = \max\left\{0, d(k) - \left(\sum_{m=1}^{k-1} u_E(m) - \sum_{m=1}^{k-1} d(m)\right)\right\} \tag{5}$$

[2]It is worth mentioning that the effect of demand shifting dynamics typically does not decrease as the population size increases.

C. Dynamics of $E_{\max}$

The dynamics of $E_{\max}$ can be analyzed similarly. We define $b(k)$ as the energy consumption of the fleet in period k under the condition that the immediate charging strategy is applied from time 1 to time k. Similar to $d(k)$, the sequence $b(k)$ can be determined offline through simulations. Notice that it is possible to have $u_E(k) \geq b(k)$ for some $k > 1$, but $\sum_{m=1}^{k} u_E(m) \leq \sum_{m=1}^{k} b(m)$ must be satisfied for all $k \geq 1$. The net difference $\sum_{m=1}^{k} b(m) - \sum_{m=1}^{k} u_E(m)$ represents the total postponed energy up to time k as compared with the immediate charging case. Unfortunately, the idea used for deriving $E_{\min}$ does not apply to $E_{\max}$ since the total postponed energy at k does not directly add up to that of the next step $E_{\max}(k+1)$. Due to this complication, simple analyical expression as in (5) is not available for $E_{\max}$.

Fortunately, the dynamics of $E_{\max}$ do not exhibit strong nonlinearity as in (5). We propose a simple linear function to approximate its relation with u_E

$$\hat{E}_{\max}(k) = \alpha_{\hat{k}} \left(\sum_{m=1}^{k-1} b(m) - \sum_{m=1}^{k-1} u_E(m) \right) + \beta_{\hat{k}}, \quad (6)$$

where $\hat{k}$ denotes the relative time step with respect to the beginning of the current day. For example, if the planning starts at 12pm of Day 2 with $\Delta t = 0.5$ hour. Then $k = 10$ will result in $\hat{k} = 12/0.5 + 10 = 34$. In this way, the coefficient vectors α and β will both have $24/\Delta t$ elements. Notice that for a large number of PEVs their daily energy demand is very similar over different days.

To estimate α and β, we can generate a large number of random u_E over a multi-day period and simulate the corresponding $E_{\max}$ sequence. The data can be used to calculate the coefficients α and β in (6) using a standard least squares approach. A key feature of the model is that once the coefficients are identified, the aggregator can keep using the same model for making planning decisions regardless of the actual starting time of the planning horizon.

D. Application to Planning Decision Making

With the models for $E_{\min}$ and $E_{\max}$, we can transform the planning problem (4) to an optimal control problem with a simple dynamic constraint. Define $z(k) = \sum_{m=1}^{k-1} u_E(m)$, for $k = 2, \ldots, N$, with $z(1) = 0$. The problem (4) can be solved through the following deterministic optimal control problem

$$\min_{u_E} \sum_{k=1}^{N} L(u_E(k)) \quad (7)$$

subject to:

$$\begin{cases} z(k+1) = z(k) + u_E(k) \\ u_E(k) \leq \alpha_{\hat{k}} \left(\sum_{m=1}^{k-1} b(m) - z(k) \right) + \beta_{\hat{k}} \\ u_E(k) \geq \max \left\{ 0, d(k) - \left(z(k) - \sum_{m=1}^{k-1} d(m) \right) \right\} \end{cases} \quad (8)$$

Notice that the dynamic constraint (8) is only 1-dimensional with a state-dependent nonlinear control constraint. This problem can be efficiently solved using dynamic programming.

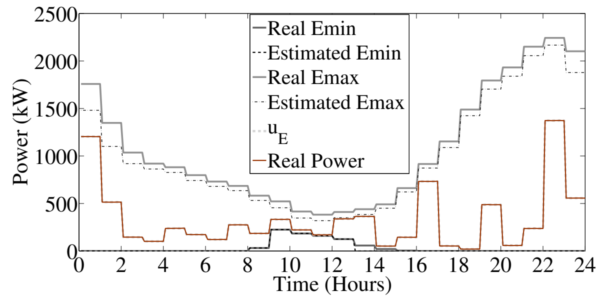

Fig. 5. Illustration of the estimated and real $E_{\min}$ and $E_{\max}$. The sequence u_E is randomly generated between its dynamic range

V. Simulation

In this section, we will validate the proposed model of $E_{\min}$ and $E_{\max}$ and show its application in a realistic energy planning problem. The charging profile of a fleet involving 2,000 PEVs will be simulated. The fleet data used in the simulation can be found in [20]. Notice that more realistic driving pattern data can be easily incorporated into our simulation.

A. Model Validation for Demand Shifting Dynamics

We first validate the accuracy of $\hat{E}_{\min}$ and $\hat{E}_{\max}$ in our model. In this simulation, we consider the case where $N = 24$ hours and $\Delta t = 1$ hour. At the beginning of the planning horizon, the last minute and immediate charging demand profile ($d(\cdot)$ and $b(\cdot)$) can be estimated based on historical data. Since $\hat{E}_{\min} = d(1)$ and $\hat{E}_{\max} = b(1)$, we generate $u_E(1)$ randomly within the range $[d(1), b(1)]$. This $u_E(1)$ in return allows us to compute $\hat{E}_{\min}(2)$ and $\hat{E}_{\max}(2)$ based on (5) and (6). Continuing with this process, we can obtain a control sequence $\mathbf{u}_E(k)$ and the corresponding real time charging policy $\mu_T(\cdot, \cdot; p_k)$.

The real energy consumption $E(k)$ and the real $E_{\min}(k)$ and $E_{\max}(k)$ are shown in Fig. 5 in comparison with the planned energy budget $\mathbf{u}_E(k)$ as well as the estimated $\hat{E}_{\min}(k)$ and $\hat{E}_{\max}(k)$. It can be seen that the real energy follows the planned energy very closely. One can also see that the $E_{\min}$ and $E_{\max}$ are estimated accurately.

It is worth pointing out that the above validation is done in an open-loop sense for which the model did not refer to the true values of $E_{\min}$ and $E_{\max}$ throughout the entire planning horizon. Since the model can accurately capture the demand shifting dynamics it can be used for energy planning over a relatively large planning horizon (e.g., a day).

B. Energy Planning Application

In this section, we apply the models of $E_{\min}$ and $E_{\max}$ to a day ahead energy planning problem, in which we aim to minimize the cost of energy over a day. We consider a planning horizon of 24 hours, starting from 12PM on August 1, 2013 to 12PM on August 2, 2013. The real LMP (Locational Marginal Price) during this 24 hour period is obtained from the PJM website [21].

To evaluate the optimal planning decision, we choose the immediate charging policy as the baseline. To ensure a fair comparison, we impose an additional constraint that the total

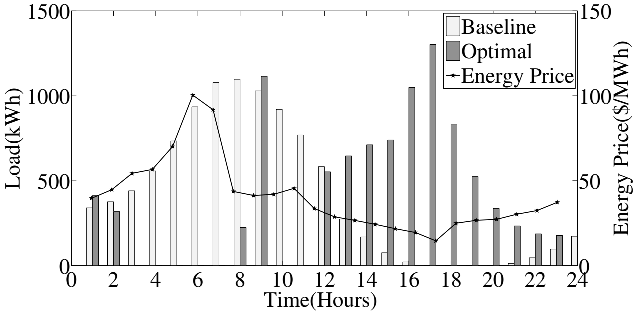

Fig. 6. Results of energy planning problem under real LMPs from PJM

charging energy must be the same as the baseline. That is, $\sum_{i=1}^{24} u_E(i) = \sum_{i=1}^{24} b(i)$. Denote the energy price as $Pr = [Pr(1), Pr(2), ..., Pr(24)]$, the charging cost can be minimized by solving the following optimization problem using dynamic programming:

$$\begin{cases} \min_{u_E} \sum_{k=1}^{24} Pr(k) \cdot u_E(k) \\ \text{subject to: (8) and } \sum_{i=1}^{24} u_E(i) = \sum_{i=1}^{24} b(i) \end{cases}$$

The simulation results are shown in Fig. 6. Our optimal charging policy defers the loads when energy prices are high and charges the loads when they are low. For the obtained optimal solution, the energy cost of charging the fleet is reduced by almost 52%. Moreover, this energy planning decision is verified through closed-loop simulations under the real time tracking policy $u_T(\cdot, \cdot; p_k)$. The result shows the energy budget is followed accurately.

VI. CONCLUSION

In this paper, we presented two models that allow us to coordinate aggregations of electric vehicles to achieve common objectives. The proposed stochastic hybrid system model can effectively capture the dynamics of individual charging loads for real time charging control, while the dynamic energy capacity model accurately estimates the demand shifting dynamics of aggregations of PEVs by explicitly modeling the effect of past control actions on current resource capabilities. Using these models, we are able to efficiently coordinate vehicles to achieve objectives like minimizing daily energy costs while respecting the dynamic constraints and control capabilities of individual vehicles. Simulations showed the effectiveness of the approach. Future research will investigate the use of this approach to achieve objectives such as energy balancing and frequency support.

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