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On the Generalized-Proportional-Integral Multi-level Sigma-Delta Sliding Mode Control of a Buck-based Inverter

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Abstract— **This article considers the output feedback controller design for a multi-level dc-to-dc buck power converter structure. This converter is used as an inverter in an arbitrary bounded output voltage reference signal trajectory tracking task. The multi-level output feedback controller design is accomplished via a suitable combination of the Generalized-Proportional-Integral (GPI) feedback controller design technique along with a multi-level generalization of the sliding mode-based Sigma-Delta ($\Sigma - \Delta$) modulator. Simulations and physical implementation on a five Level $\Sigma - \Delta$ modulator, as well as the buck multi-level inverter prototype, depict the performance of the proposed output feedback controller scheme.**

I. INTRODUCTION

Multi-level switched power converters are rapidly becoming an object of interest in theoretical studies related to feedback controller design options in several power electronics problems [1], [6]. The fundamental feature of multi-level converters is constituted by switching strategies that occur between substantially reduced voltage levels thanks to a suitable arrangement of switching cells. This feature results in high power handling possibilities with substantially reduced switching losses and the versatility of classical "granular" nonlinear switching actions. Multi-level switching arrangements, in fact, evade the need for bulky transformers hardware, while substantially reducing harmonic distortion and Electro-magnetic interference. Thus, bestowing to these devices the possibilities of varied applications ranging from traction tasks, agricultural applications, ship maneuvering and multi-phase active filtering among many other applied fields [2], [18].

In this article, a particular class of multi-level converters is studied, constituted by a buck type converter combined with an arrangement of H-bridge cells in a cascaded structure. Here, we propose and solve the output feedback controller design for a multi-level dc-to-dc buck power converter structure, used as an inverter, in an arbitrary bounded output voltage reference signal trajectory tracking task. This class of multi-level switched power converters enjoys a unique feature: their infinite frequency average model is described by a linear input-output dynamics with a bounded average single control input. This key issue allows to propose a rather simple average linear feedback controller of the Generalized-Proportional-Integral (GPI) type written in classical compensation network form. It is shown that the required ultimate multi-level switching action, synthesized on the basis of the average feedback control input signal, may then be accomplished via a suitable generalization, to the multi-level arena, of the classical Sigma-Delta ($\Sigma - \Delta$) modulator. Extensively used in the early days of analog voice communications systems [15] and advantageously used today in many analog-to-digital conversion schemes [5]. A GPI output feedback regulation scheme is considered, for trajectory tracking tasks in combination with multi-level $\Sigma - \Delta$ modulation.

GPI controllers are, fundamentally speaking, linear classical compensation networks with an additional pure multiple integration structure that evade the need for on-line derivative calculations in linear pole-placement techniques [3]. GPI controllers have been successfully extended to deal with the robust precompensated control of nonlinear flat systems in [4]. For a power electronics application of GPI control see [9]. GPI and classical delta modulation is explored in [10]. Here, it is taken advantage of the fact that the infinite switching frequency average model of a multi-level buck converter is precisely described by a smooth second order linear dynamics. This allows to design an output feedback controller of the GPI type written in terms of a classical compensation network. Whose average output signal is processed by means of a multi-level $\Sigma - \Delta$ modulator. The discrete valued output of the modulator is used to trigger the appropriate switchings in the H-cell arrangement.

Section II is devoted to the problem formulation and states the main results of the article. In this section the multi-level output feedback controller is proposed using a suitable combination of the GPI feedback controller design technique, which here adopts the form of a classical output tracking error based compensator. The GPI controller is complemented with a multi-level generalization of the $\Sigma - \Delta$ sliding mode-based modulator in order to achieve the suitable switching action producing, in average, the desired output trajectory tracking. Section III contains digital computer simulations depicting the feasibility of the proposed output feedback controller scheme. Section IV presents the physical implementation of both the $\Sigma - \Delta$ modulator and the buck multi-level inverter prototype. Section V is devoted to the conclusions and some suggestions for further research in this area.

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II. PROBLEM FORMULATION AND MAIN RESULTS

A. The multi-level buck converter model

Consider the multi-level inverter of the buck type shown in Fig. 1. The H-bridge cell arrangement, bestowing the granularity type of stepped nonlinearity to the piecewise constant DC voltage feeding options to the circuit, is shown in detail in Fig. 2. Typically, this buck converter along with the portrayed H-cell structure are used for DC to AC conversion tasks via the suitable activation of the signals T_i in each cell [6].

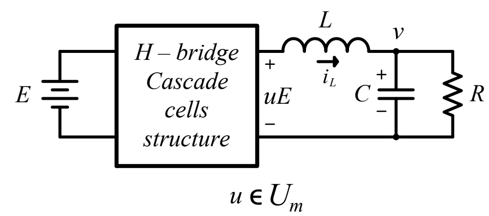

Fig. 1. Buck multi-level inverter

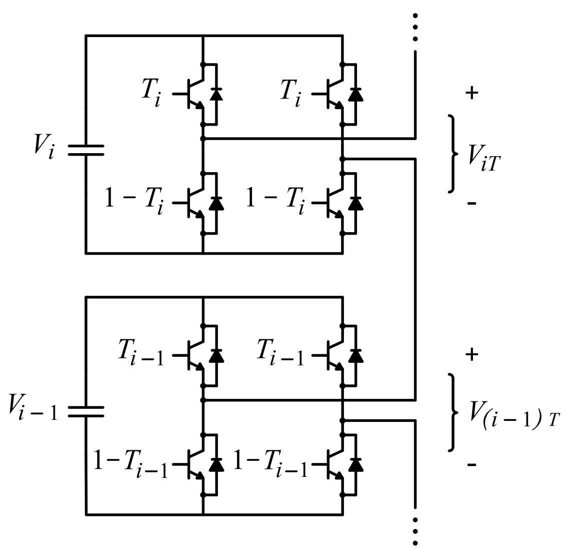

Fig. 2. H-cell arrangement

Let m be an integer and consider the following partition of the interval Γ =$[-1, 1]$ of the real line:

$$U_m = \left\{-1, -\frac{m-1}{m}, \cdots, -\frac{1}{m}, 0, \frac{1}{m}, \cdots, \frac{m-1}{m}, 1\right\} \quad (1)$$

expressed as a finite discrete set of rational values in Γ. The family of closed subintervals of the real line formed by any two consecutive elements of this set is said to constitute the switching levels. The limits of the switching levels will be called switching values. The set of switching values coincides with U_m.

Let E be the constant input voltage to a classical buck converter structure comprising an H-cell arrangement providing $2m+1$ switching levels in the interval $[-E, E]$. The switched model of the buck inverter with $2m+1$, $m \geq 1$ switching values, taken from the extreme values of adjacent uniformly distributed intervals within the set $[-E, E]$, can be written as follows:

$$\begin{aligned} L\frac{di_L}{dt} &= -v + uE \\ C\frac{dv}{dt} &= i_L - \frac{v}{R} \end{aligned} \quad (2)$$

where $u \in U_m \subset \Gamma$ is a multi-level switch position function.

The state average model of the buck inverter is readily given by

$$\begin{aligned} L\frac{dx_1}{dt} &= -x_2 + \mu E \\ C\frac{dx_2}{dt} &= x_1 - \frac{x_2}{R} \end{aligned} \quad (3)$$

where x_1 represents the inductor current, x_2 is the capacitor voltage, μ is the average control input continuously taking values in the closed interval Γ.

Taking the capacitor voltage as the output of the system y, the average input-output model of the multilevel buck inverter is thus given by the following linear dynamics:

$$\ddot{y} + \frac{1}{RC}\dot{y} + \frac{1}{LC}y = \frac{E}{LC}\mu \quad (4)$$

Clearly, under equilibrium conditions $\bar{y} = \bar{\mu}E$ thus establishing the attenuation properties of the given buck converter. The maximum, in absolute value, achievable DC output voltage for this system is, therefore, $\bar{y} = E$. The buck converter output voltage y is then necessarily restricted to lie in the interval $[-E, E]$ of the real line.

B. Problem statement

Given the switched system (1)-(2), and given a bounded average output reference signal $y^*(t) \in [-E, E]$, find an output error feedback control law for $u = u(e_y)$, $e_y = y - y^*$, taking values in the discrete set U_m, such that the average output voltage y asymptotically tracks the given reference signal y^*.

C. Multi-level $\Sigma - \Delta$ modulation

$\Sigma - \Delta$ modulation has been extensively used in communication systems since the 1960's. A classical account of the many variants of delta (Δ) modulation systems, including adaptive schemes for binary analog signal encoding, is contained in the book by [15]. An early work casting $\Delta -$ modulation systems into the sliding mode control approach is given in [8]. More recently, the $\Sigma - \Delta$ modulation variant of $\Delta -$ modulation systems has been proposed as part of a direct output feedback control scheme for switched power electronic systems [12], [13]. See also [14] for the case of fractional derivative systems.

It is briefly considered the fundamentals of three level switching $\Sigma - \Delta$ modulation taking values in the set

$\{-1, 0, 1\}$. The fundamental issue in this "ternary" valued $\Sigma - \Delta$ modulation is that of translating a bounded continuous signal $\mu \in [-1, 1]$ into a switched signal u taking values in the discrete set: $\{-1, 0, 1\}$, such that the input signal $\mu(t)$ precisely coincides with the average value of the switched output u defined in a sliding mode sense, i.e., such that the equivalent value, under ideal sliding mode conditions, of the signal u, denoted by u_{eq}, coincides, precisely, with $\mu(t)$. This is accomplished via the following sliding mode dynamics:

$$\dot{e} = \mu - u$$

$$u = \frac{1}{2}[sign(\mu) + sign(e)] \tag{5}$$

Clearly, a sliding mode exists on the "encoding" surface $e = 0$, since, for $\mu \in [-1, 1]$, if $e > 0$, then $u = \frac{1}{2}[sign(\mu) + 1]$. Hence, $\dot{e} = \mu - \frac{1}{2} - \frac{1}{2}sign(\mu)$. It follows that $\dot{e} < 0$ regardless of the sign of μ. Similarly, for $e < 0$, it follows that $\dot{e} > 0$ for any sign of μ. In all cases $e\dot{e} < 0$, and a sliding regime exists on $e = 0$ (see [17]). Also, notice that, under the invariance conditions $e = 0$; $\dot{e} = 0$, the ideal sliding dynamics is characterized by $\dot{e} = \mu - u_{eq} = 0$, i.e., by $\mu(t) = u_{eq}(t)$ for all t where ideally $e = 0$.

A generalization of $\Sigma - \Delta$ modulation to a multi-level switching environment, where u takes values in the previously defined set U_m (see (1)), and the average input μ continuously takes values, in $[-1, 1]$, is given by the following sliding mode dynamics:

$$\dot{e} = \mu - u$$

$$u = \frac{1}{4m} \sum_{k=-m+1}^{m} (2k - 1 + sign(e)) \times \tag{6}$$

$$\left[sign\left(\mu - \frac{k-1}{m}\right) - sign\left(\mu - \frac{k}{m}\right)\right]$$

Notice that, for a fixed value of k, say, $k = j > 0$, with $j \leq m$, only one summand is active in (6) with all other summands being identically zero. It follows that, whenever $\mu \in \left[\frac{j-1}{m}, \frac{j}{m}\right]$, the following discontinuous dynamics is valid:

$$\dot{e} = \mu - u$$

$$u = \frac{1}{2m}[2j - 1 + sign(e)] \tag{7}$$

If $e > 0$ then $u = \frac{j}{m}$ and $\dot{e} < 0$. Also, if $e < 0$, then $u = \frac{j-1}{m}$ and $\dot{e} > 0$. Notice that $e\dot{e} < 0$, and a sliding regime exists on $e = 0$. As a consequence, the average value of the switching signal u, designed by u_{eq}, ideally coincides with μ. A similar reasoning is valid for $j < 0$.

D. Flatness based off-line trajectory planning

Given a desired average output voltage signal $y^*(t)$, the normalized average input-output dynamics leads to the following parametrization of the average input signal $\mu^*(t)$,

$$\mu^*(t) = \frac{LC}{E}\left[\ddot{y}^*(t) + \frac{1}{RC}\dot{y}^*(t) + \frac{1}{LC}y^*(t)\right] \tag{8}$$

Clearly, for an arbitrary bounded average output reference signal $y^*(t) \in [-E, E]$, it does not necessarily follows that $\mu^*(t) \in [-1, 1]$ for all t. An off-line trajectory planning, which guarantees $\mu^*(t) \in [-1, 1]$ can be carried out by suitable "time stretching" and "amplitude scaling" of the desired signal waveform $y^*(t) \in [-E, E]$. This off-line procedure is quite well known in flatness based trajectory planning [11].

E. Solution to the problem

It is adopted the following output feedback controller design strategy: Based on the average model, first it is specified a continuous feedback controller of the GPI type for the SISO average bounded control input $\mu \in \Gamma = [-1, 1]$. To specify the feedforward part of the possibly saturating average output feedback controller $\mu^*(t)$ the flatness property of the average normalized dynamics is used to carry out a suitable off-line trajectory planning of the required output reference signal. Once the bounded continuous average control input is obtained as a feedback signal $\mu = \mu(e_y)$, a multi-level $\Sigma - \Delta$ modulator it is designed, as a generalization of the classical binary $\Sigma - \Delta$ modulator. Which properly processes the average feedback input signal $\mu(t)$, producing a switched output to be delivered to the actual system input u. This input actively takes values from the required adjacent voltage levels of the uniform partition induced on the overall input voltage interval, so that the average output reference trajectory tracking is dully guaranteed. The proposed controller design method is summarized in the block diagram of Fig. 3.

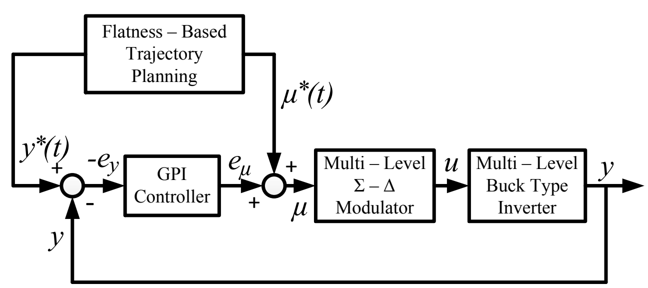

Fig. 3. Proposed controller design method comprising a GPI controller, a flatness based trajectory planning scheme, and a multi-level $\Sigma - \Delta$ modulator.

Proposition 1. Let

$$e_y^{(4)} + \gamma_3 e_y^{(3)} + \gamma_2 \ddot{e}_y + \gamma_1 \dot{e}_y + \gamma_0 e_y = 0 \tag{9}$$

be a given desired closed-loop dynamics for the average output tracking error $e_y = y - y^*(t)$, which is characterized by the Hurwitz characteristic polynomial: $p_d(s) = s^4 + \gamma_3 s^3 + \gamma_2 s^2 + \gamma_1 s + \gamma_0$. Then the switched multi-level output feedback controller given by

$$\dot{e} = \mu - u$$

$$u = \frac{1}{4m} \sum_{k=-m+1}^{m} (2k - 1 + sign(e)) \times$$

$$\left[sign\left(\mu - \frac{k-1}{m}\right) - sign\left(\mu - \frac{k}{m}\right) \right]$$

$$\mu = \mu^* - \frac{LC}{E} \left[\frac{k_2 s^2 + k_1 s + k_0}{s(s + k_3)} \right] (y - y^*) \qquad (10)$$

produces the desired average output trajectory tracking error dynamics (9) on the buck converter (1)-(2), provided the GPI controller coefficients k_3, k_2, k_1, k_0 are chosen as:

$$k_3 = \gamma_3 - \frac{1}{RC}$$

$$k_2 = \gamma_2 - \frac{k_3}{RC} - \frac{1}{LC} \qquad (11)$$

$$k_1 = \gamma_1 - \frac{k_3}{LC}$$

$$k_0 = \gamma_0$$

Proof. The proof follows from the fact that the average value, in the sliding mode sense, of the switched control input u, coincides with the feedback generated input μ. Let $e_\mu = \mu - \mu^*$. Hence, the output tracking error satisfies:

$$e_y = \left[\frac{\frac{E}{LC}}{s^2 + \frac{1}{RC} s + \frac{1}{LC}} \right] e_\mu$$

$$e_\mu = -\frac{LC}{E} \left[\frac{k_2 s^2 + k_1 s + k_0}{s(s + k_3)} \right] e_y \quad ,$$

i.e.,

$$[\ s^4 + \left(k_3 + \frac{1}{RC}\right) s^3 + \left(k_2 + \frac{k_3}{RC} + \frac{1}{LC}\right) s^2$$

$$+ \left(k_1 + \frac{k_3}{LC}\right) s + k_0\ ]\ e_y = 0$$

Notice that (9) is recovered if the design parameters are chosen according to (11).

III. SIMULATION RESULTS

This section presents the particular case of sinusoidal tracking also referred as DC-to-AC inversion. In other words, it is desired to track the following bounded voltage signal:

$$y^*(t) = Asin(\omega t), \qquad A < E \qquad (12)$$

To assess the amplitude-frequency trade-off for the desired nominal sinusoidal output voltage reference signal (12), it is made use of the flatness based differential parametrization of the average control input, $\mu(t)$ in equation (4) in terms of the equation (8) with $\mu^*(t)$.

Setting $y^*(t) = Asin(\omega t)$ in (8) and expressing it in terms of a single sinusoid at a frequency ω, and a frequency-dependent phase. From the fact that $\mu^*(t) \in [1, 1]$, it is obtained that the amplitude A and the frequency ω must satisfy the following restriction:

$$\frac{A}{E} \leq \frac{1}{\left[\sqrt{(1 - LC\omega^2)^2 + \left(\frac{L\omega}{R}\right)^2} \right]} \qquad (13)$$

The parameters of the buck converter operating as a DC-to-AC converter (inverter) are shown in Table I.

TABLE I

PARAMETERS OF THE SYSTEM

ω	377 rad (60 Hz)
A	40 V
E	48.6 V
L	18 mH
C	10 μF
R	100 Ω

The closed-loop poles have been located at $(s + 475 + 2310j)(s + 475 - 2310j)(s + 70)(s + 7) = 0$ to avoid overflow in the physical implementation of the controller. The GPI controller coefficients are obtained from (11) and a simplification from (10) as follows:

$$\begin{aligned} k_3 &= 27 \\ \tilde{k}_2 &= \frac{LC}{E} k_2 = 0.0002 \\ \tilde{k}_1 &= \frac{LC}{E} k_1 = 1.03 \\ \tilde{k}_0 &= \frac{LC}{E} k_0 = 10.08 \end{aligned} \qquad (14)$$

A. Sinusoidal tracking with 5-Level topology

A 5-level topology, which includes two H-bridge cells, provides four equally spaced sub-intervals of the interval $[-1, 1]$, thus providing the following five switching values:

$$U_4 = \left\{ -1, -\frac{1}{2}, 0, \frac{1}{2}, 1 \right\}$$

Figure 4 depicts the closed loop performance of the proposed GPI plus multilevel $\Sigma - \Delta$ modulator for a 5-Level buck converter used as an inverter with the parameters described in Table I.

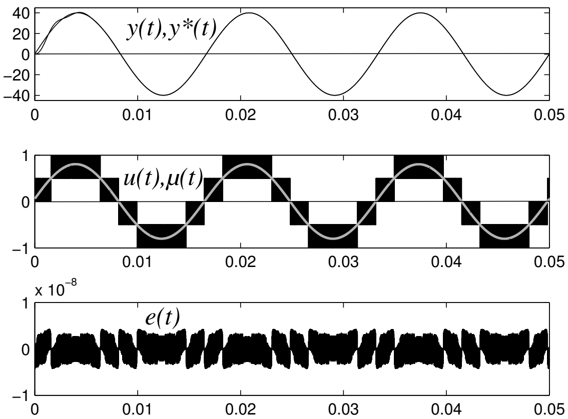

Fig. 4. Closed loop performance of a 5-Level buck converter controlled with the proposed GPI plus a multilevel $\Sigma - \Delta$ modulator considering a sinusoidal output reference tracking.

IV. EXPERIMENTAL RESULTS

The implementation of the multi-level $\Sigma - \Delta$ modulator required the combination of diverse electronic devices. For the realization of the modulator low noise op-amps have been used. It is conformed by

Figure 5 shows the block diagram of the multi-level $\Sigma - \Delta$ modulator. Notice the combination between analog and TTL signals in the closed-loop scheme. To work with positive voltages, the continuous control signal μ is scaled to a voltage range between 0 to 5 volts. Then a differential stage provides the error of encoding signal μ. This error signal passes through an inverting integrator configuration.

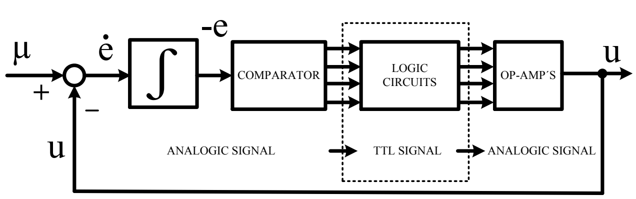

Fig. 5. Implementation of the $\Sigma - \Delta$ modulator.

Table II describes the mathematical expressions to get a sliding regime according to the interval in which the μ signal is found. In the first case, it is multiplied the expression (5) by $\frac{1}{4}$ instead of $\frac{1}{2}$ to generate the interval $\{-0.5, 0, 0.5\}$. In the second and third case, it is used (7). It is taken $j = m = -2$ for the second case, and similarly, $j = m = 2$ for the third case. Therefore the switching intervals are obtained as $\{-1, -0.5, 0, 0.5, 1\}$ with its equivalent voltage values $\{0V, 1.25V, 2.5V, 3.75V, 5V\}$. This voltages values comprise the comparators stage.

The comparators stage is used to detect in which voltage interval lies the signal μ. In the logic circuits stage, first a series of flip-flop D type stores and synchronizes the information at each clock cycle. The frequency of the clock is 51 kHz. This stage uses truth tables with the adequate outputs

TABLE II

MATHEMATICAL EXPRESSIONS FOR THE 5-LEVEL MODULATOR

Mathematical Expression	Interval	Voltage[V]
$u = \frac{1}{4}[sign(\mu) + sign(e)]$	$\{-0.5, 0, 0.5\}$	$\{1.25, 2.5, 3.75\}$
$u = \frac{1}{4}[-3 + sign(e)]$	$\{-0.5, -1\}$	$\{1.25, 0\}$
$u = \frac{1}{4}[3 + sign(e)]$	$\{0.5, 1\}$	$\{3.75, 5\}$

synthesized via Karnaugh maps so as to obtain suitable switching sequences for the H-Bridge cascade cells.

The op-amps stage converts the TTL signal into an analog signal with the correct voltage value, which depends on the interval where signal μ is located. This is the feedback signal to the first stage of the modulator, which closes the loop.

Figure 6 depicts the results of the 5-level $\Sigma - \Delta$ modulator.

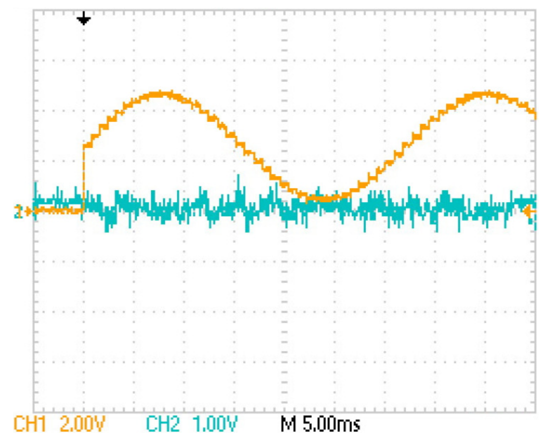

Fig. 6. Encoding error of the 5-level $\Sigma - \Delta$ modulator. (channel 1) Control signal μ. (channel 2) $(-e)$ signal.

Figure 7 shows the performance of the implemented 5-level $\Sigma - \Delta$ modulator, which validates the simulation results. It also verifies the encoding of the continuous control signal μ as compared with the multi-level switch position function u.

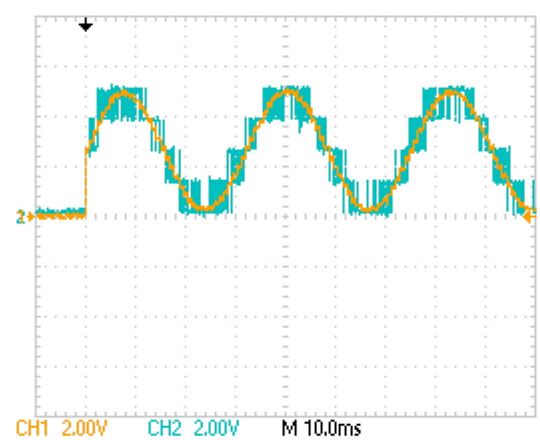

Fig. 7. Performance of the implementation of the five level $\Sigma - \Delta$ modulator. (channel 1) Control signal μ. (channel 2) Average signal u

A. Sinusoidal tracking (DC-to-AC inversion)

This section describes the realization of the particular sinusoidal tracking case described in Section III.

The GPI controller was implemented in a development board ezdsp-TMS320LF2407. A two H-Bridge cascade cells topology has been used as shown in Fig. 2. The parameters of the system are presented in Table I. Figure 8 shows the time response of the buck-based multi-level inverter during the tracking of a bounded voltage reference signal $y^*(t) = 40sin(377t)$.

These results exhibit the benefits of using $\Sigma - \Delta$ modulation multi-level where oversampling produces a high-resolution encoding of the control signal μ, and in addition reduces the LC filter required [7], [16]. Figure 9 presents a picture of the final prototype.

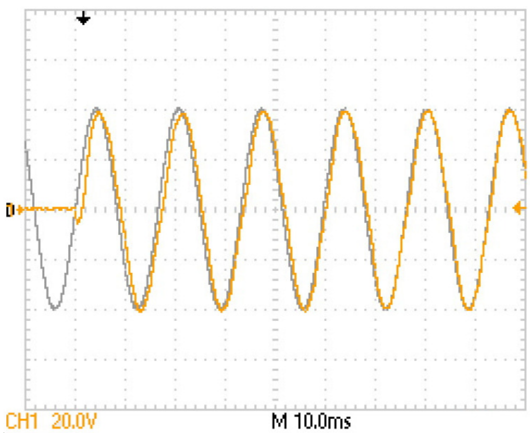

Fig. 8. Time response of the buck-based multi-level inverter. (channel 1) Output voltage of the system, and (gray) Its reference signal.

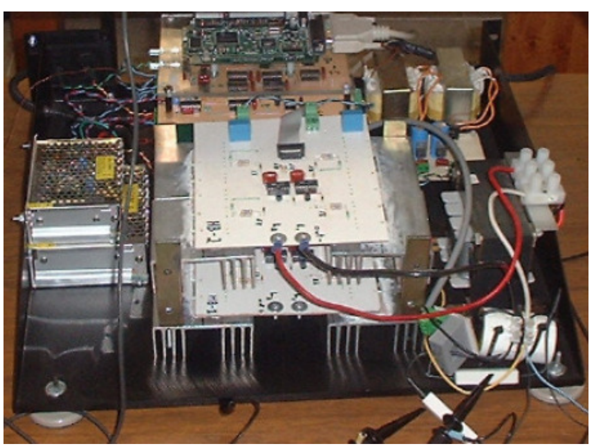

Fig. 9. Physical implementation of the buck-based multi-level inverter.

V. CONCLUSIONS

This article presented an output feedback controller design procedure for an arbitrary bounded output voltage reference signal trajectory tracking task in a multi-level dc-to-dc buck power converter structure, used as a voltage inverter. The proposed output feedback controller comprised a suitable combination of a Generalized-Proportional-Integral (GPI) feedback controller along with a multi-level generalization of the sliding mode $\Sigma - \Delta$ modulator. The design was based on the ideal sliding mode dynamics of the composite inverter and the $\Sigma - \Delta$ modulator dynamics. The last took place only on the modulators unidimensional state space. Computer simulations and physical implementation of the proposed discontinuous output feedback controller scheme were performed to assess the performance of the proposed scheme.

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