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Voltage Robust Stability in Microgrid Power Management

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Abstract— **Voltage stability is of essential importance for power grids. The emergence of distributed energy generators, controllable loads, and local-area energy storage capabilities will introduce new scenarios for distribution networks in which classical frameworks for voltage stability may be inadequate. This paper introduces a control-theoretic framework for studying voltage stability and its robustness, as well as optimal power management in distribution systems composed of networked microgrids. The framework involves descriptions of the loads and generators by nonlinear state space models and the network connections by a set of topology-based algebraic equations. The combined system leads to a general nonlinear state space model for the microgrid systems. Four stability margins are introduced to capture different scenarios in microgrid power management capabilities and load disturbances. LMI (linear matrix inequality) methods are employed for computing stability margins. Illustrative examples are used to demonstrate the methods.**

I. Introduction

As the power industry has become more deregulated and power grids are expanded with distributed energy generators, the concept of microgrids [3], [8] has gained traction recently as a suitable framework to represent area distribution networks that include local generators of different types (solar, wind, micro-turbine, fuel cells), dynamic and manageable loads (PHEV, smart appliances), and energy storage systems (battery, super-capacitor, flywheel, heat pump, etc.). With more distributed renewable sources, controllable loads, and energy storage systems, voltage stability will become one of most critical issues in smart grid and microgrid implementations, due to more volatile generation and loads, and more sophisticated control requirements [9], [15], [18].

Traditionally, voltage stability analysis aimed for generation and transmission systems over a large area network under normal and contingent conditions [1], [2], [13]. Many methods have been proposed to address this important issue [10], [19], [20]. For local small-signal stability, state space models and Lyapunov indirect stability have been used as a basic framework in analyzing voltage stability [16] and other robust stability methods were also applied [6], [11], [12]. Robust stability of voltage against grid parameter uncertainties was investigated using tools from robust control methodologies [5], [7], [17]. These approaches treat a power grid as a linear time-invariant system and accommodate parameter uncertainties. Since load dynamics are mostly nonlinear and key uncertainties in microgrids are load disturbances, different approaches are needed for grid voltage robustness. Overall, frameworks that accommodate general grid network structures and load dynamics remain unavailable.

This paper introduces a framework in which voltage stability, network robustness, and optimal power management for sustained voltage stability can be rigorously studied and optimization strategies can be designed. In our framework, load and generator dynamics are represented by node dynamic systems and their connections by network topologies. Interactions of subsystems in such a grid-linked system result in a networked nonlinear dynamic system. The combined system leads to a general nonlinear state space model for microgrid systems. The main contributions of this paper are: (1) It introduces a framework to model microgrids for voltage stability analysis, including both grid links and local dynamics. Typical power flow analysis does not involved detailed local dynamics. (2) It derives stability conditions under grid constraints. (3) It introduces four different robust stability margins to capture typical scenarios in microgrid power management and disturbances. Traditional voltage stability analysis does not imply such stability margins. (4) It derives LMI-type (linear matrix inequalities) conditions for computing these stability margins for simplified microgrids. Most proofs are omitted due to page limitations. Also, to introduce stability margins, we use local stability concepts. For transient stability, Lyapunov direct methods will become more suitable and will be reported elsewhere.

Some earlier and preliminary results on voltage stability and robustness in microgrid systems were presented in ECC'13 [21]. This paper contains an improved problem formulation, more general stability results, and LMI conditions for computing stability margins.

II. Microgrid Models: Networked Nonlinear Dynamic Systems

A microgrid may be viewed as a networked cluster of dynamic local subsystems. The subsystems are loads, generators, storage devices, or combinations of these. Each subsystem is a dynamic system and forms a node in the cluster. Interconnections of the nodes are realized by the grid that consists of power lines, transformers, switches, and other grid components. In a scalable configuration, the subsystems may be microgrids themselves and the grid may represent a regional network of interconnected microgrids. As such, this structure is generic and modular in nature. A generic

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network of microgrids is shown in Fig. 1. In consideration of typical microgrid systems, we assume that the networked system consists of a main grid connecting bus ($\vec{V}_0$) and n nodes (subsystems). The main grid is also a dynamic system represented by an equivalent generator and an internal impedance. Each node may represent a motor, an electric vehicle, or a microgrid itself. In this structure, bus voltages and line currents are the network signals, and together with the states of the local subsystems, they collectively represent the state of this entire networked system.

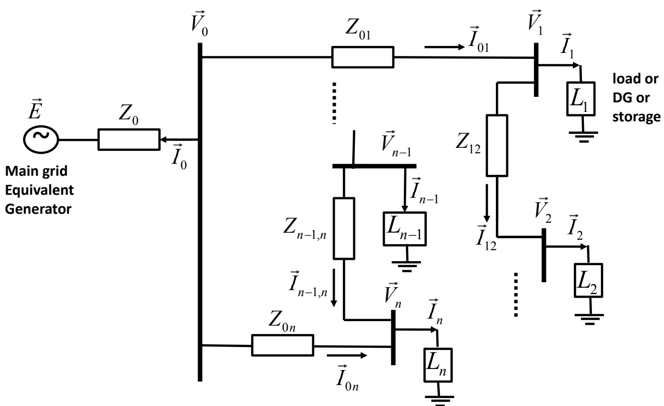

Fig. 1. A network of microgrids

The current directions are marked in Fig. 1. In principle, the current $\vec{I}_{kl}$ has the nominal direction from node k to node l. Hence, $\vec{I}_{kl} = -\vec{I}_{lk}$. The node currents flow from the buses to the loads/generators. By Kirchhoff's current law (applied to networks), the node currents satisfy $\sum_{k=0}^{n} \vec{I}_k = 0$.

This paper will not deal with the issue of frequency regulation. Hence, in the entire network the frequency on all buses and lines is assumed to be a fixed constant. We only consider three-phase and balanced systems in this paper. Consequently, all voltages and currents can be represented by their one-phase phasors. Also, we assume that the lines between buses are passive and represented by line impedances or equivalently line admittance, as indicated in Fig. 1.

The $\vec{E} = E\angle 0$ is the equivalent generator voltage from the utility grid to the network and is used as a reference point.[1] Generally, we will use $\vec{V}_k = V_k\angle\delta_k$ to represent a voltage phasor; and $\vec{I}_k = I_k\angle\theta_k$ to represent a current phasor. During transient time intervals when magnitudes or angles of voltages and currents change with time, we use the notation $\vec{V}_i(t) = V_i(t)\angle\delta_i(t)$. This should not be confused with the actual voltage time function which is a sine wave of a given frequency.

For a unified system treatment, we will use state space systems to model the microgrid. To facilitate this formulation, in dynamic system representations which are traditionally confined to real values, to avoid complex values we will use lower-case letters to represent vectors, that are equivalent to the phasor representations,

$$e = \begin{bmatrix} E \\ 0 \end{bmatrix}, v_k = \begin{bmatrix} V_k\cos\delta_k \\ V_k\sin\delta_k \end{bmatrix}, i_k = \begin{bmatrix} I_k\cos\theta_k \\ I_k\sin\theta_k \end{bmatrix}. \quad (1)$$

We will use the notation

$$i_k^+ = \begin{bmatrix} -I_k\sin\theta_k \\ I_k\cos\theta_k \end{bmatrix}$$

Under this notation, the real power is $P = v'i$, the reactive power is $Q = v'i^+$, and the complex power is $S = [P, Q]'$.

Each node will be a dynamic system itself. It can represent a load, a power source, or an energy storage system. For studies of voltage stability, for the kth-node dynamic system the input is v_k and the output is i_k, both are vectors as defined in (1), and its dynamics are expressed by a nonlinear state space model of dimension n_k and state x_k

$$\left\{ \begin{array}{lcl} \dot{x}_k & = & f_k(x_k, v_k), \\ i_k & = & g_k(x_k, v_k) \end{array} \right. \qquad k = 0, 1, \ldots, n. \quad (2)$$

For example, the state variables of a BLDC (Brushless DC) motor will typically include the rotor speed and armature current.

Denote $i = [i_0, \ldots, i_n]'$, $v = [v_0, \ldots, v_n]'$, $x = [x_0, \ldots, x_n]'$. Then (2) may be expressed compactly as

$$\dot{x} = F(x, v), i = G(x, v). \quad (3)$$

The network connections in the microgrid relate the load bus voltages $v_0, \ldots, v_n$, the bus load currents $i_0, \ldots, i_n$, and the link currents i_{kl} by using Kirchhoff's laws. The grid network is described by a directed graph $\mathcal{G}$. $(k, l) \in \mathcal{G}$ indicates a line connecting the bus k to bus l with link current i_{kl}. $\mathcal{G}$ is assumed to be symmetric, namely $(k, l) \in \mathcal{G}$ implies $(l, k) \in \mathcal{G}$, and $i_{kl} = -i_{lk}$.

We assume that the network links involve only line impedance. For example, for the system in Fig. 1 we have, in the phasor notation, $\vec{I}_{kl} = Y_{kl}(\vec{V}_k - \vec{V}_l)$, where $Y_{kl} = Y_{kl}^R + jY_{kl}^I$ is the line admittance. Let $M_{kl} = \begin{bmatrix} Y_{kl}^R & -Y_{kl}^I \\ Y_{kl}^I & Y_{kl}^R \end{bmatrix}$. Then, M_{kl}^{-1} is the matrix representation of the line impedance. Under the vector notation (1), $i_{kl} = M_{kl}(v_k - v_l)$. Suppose the total number of the links in the grid is L_s. Choose a certain order of the links and let i_{link} (of dimension $2L_s$) be the vector containing all the link currents i_{kl}. In this case, we should use kl as its location index. Under the same order, $M = \mathrm{diag}(M_{kl})$ is of dimension $2L_s \times 2L_s$. The matrix H_1 is the network incident matrix and defined as follows. H_1 is a block matrix of $L_s \times (n+1)$ blocks whose elements are 2×2 matrices. If $(k, l) \in \mathcal{G}$, then its (kl, k) block is I_2 and its (kl, l) block is $-I_2$, where I_2 is the 2×2 identity matrix. All other blocks are 0 (2×2 matrix). It follows that $i_{\mathrm{link}} = MH_1 v$.

Furthermore, by the basic node current relationship, for node l, $i_l = \sum_{k,(k,l)\in\mathcal{G}} i_{kl} = H_2 i_{\mathrm{link}}$. Consequently, we have $i = H_2 i_{\mathrm{link}}$ and

$$i = H_2 M H_1 v = Hv, \quad (4)$$

with $H = H_2 M H_1$.

[1]Physically, the main grid, generators, loads have specific physical constraints such as power limits, voltage ratings, current ratings, switching tripping thresholds. In our general formulation, we do not explicitly include these constraints. But they can be imposed when analyzing specific systems.

This, together with (3), leads to the networked system $\dot{x} = F(x, v)$, $i = G(x, v)$, $i = Hv$. Or equivalently,

$$\dot{x} = F(x, v), Hv = G(x, v). \tag{5}$$

Let $\mathbb{1} = [1, \ldots, 1]'$ of an appropriate dimension. Then, it can be verified that $\mathbb{1}'H = 0$, $\mathbb{1}'G(x, v) = 0$ for any x and v. If the local subsystems are linear and time invariant (LTI), then $\dot{x} = Ax + Bv$, $i = Cx + Dv$, $i = Hv$. Moreover, we have $\mathbb{1}'C = 0$, $\mathbb{1}'D = 0$. We note that $Hv = G(x, v)$ is an implicit state feedback. Let solutions to this equation be denoted by $v = S(x)$. Consequently,

$$\dot{x} = F(x, v), v = S(x). \tag{6}$$

This state space model for the entire network of microgrids may be viewed as the closed-loop system in which the node dynamics $\dot{x} = F(x, v)$ define the open-loop unconnected subsystems and the network connections insert a state feedback by $v = S(x)$. In applications, certain variables are used as references and then the solution to $Hv = G(x, v)$ will become unique.

III. Stability Analysis

We now investigate stability of the networked system (5). It is noted that load types and load control strategies affect the open-loop dynamics and the network topology and link impedances define the state feedback. They jointly determine the network stability.

Due to intrinsic power system constraints, stability analysis is more involved than the standard Lyapunov linearization conclusions. This complication will be reflected both in equilibrium point calculations and derivations of stability conditions. For stability analysis, we first solve for the equilibrium points.[2] Suppose that e is the reference value $e = [E, 0]'$. Given the load real powers P_k and power factors γ_k, the load complex powers $S_k = [P_k, \frac{1-\gamma_k}{\gamma_k} P_k]'$, $k = 1, \ldots, n$, are obtained. The equilibrium point is calculated as follows. First, from the network equations, $i = Hv$, $i_0 = M_0(v_0 - e)$, $\mathbb{1}'i = 0$, $S_k = [P_k, \frac{1-\gamma_k}{\gamma_k} P_k]'$, $k = 1, \ldots, n$, the bus voltages v can be computed. Under these conditions, the solution denoted by v^0 is unique [1]. Then, we solve for x^0 from

$$F(x^0, v^0) = 0. \tag{7}$$

We assume that $F_x = \frac{\partial F(x,v)}{\partial x}$ is full rank. Consequently, for the given v^0, the solution x^0 is unique. Together, (x^0, v^0) is the equilibrium point. We note that if all the loads have constant power factors γ_k, then the equilibrium point is a function of load powers $P = [P_1, \ldots, P_n]'$. This understanding will be carried in the rest of the paper and used in discussions of robust stability with respect to load power disturbances.

We observe that i_0 and v_0 are not independent variables. From $i_0 = -\sum_{k=1}^n i_k$ and $i_0 = M_0(v_0 - e)$, the first subsystem of the state space model (5) is inherently algebraic (no dynamics).

[2]If one does not consider the local dynamic systems, then the equilibrium point can be obtained from the standard steady-state power flow analysis, for which many software packages exist.

Denote $\overline{i} = [i_1, \ldots, i_n]'$, $\overline{v} = [v_1, \ldots, v_n]'$, $\overline{x} = [x_1, \ldots, x_n]'$. Correspondingly,

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}; \; F(x, v) = \begin{bmatrix} F_1 \\ \overline{F}(\overline{x}, \overline{v}, v_0) \end{bmatrix};$$
$$G(x, v) = \begin{bmatrix} G_1 \\ \overline{G}(\overline{x}, \overline{v}, v_0) \end{bmatrix}.$$

Then (5) may be simplified as

$$\begin{cases} \dot{\overline{x}} = \overline{F}(\overline{x}, \overline{v}, v_0) \\ H_{21}v_0 + H_{22}\overline{v} = \overline{G}(\overline{x}, \overline{v}, v_0). \end{cases} \tag{8}$$

In addition,

$$v_0 = e + M_0^{-1} i_0 = e - M_0^{-1} \mathbb{1}'\overline{i} = e - M_0^{-1} \mathbb{1}'\overline{G}(\overline{x}, \overline{v}, v_0).$$

Define

$$\overline{F}_x = \frac{\partial \overline{F}(\overline{x}, \overline{v}, v_0)}{\partial \overline{x}} |_{(x^0,v^0)}, \quad \overline{F}_v = \frac{\partial \overline{F}(\overline{x}, \overline{v}, v_0)}{\partial \overline{v}} |_{(x^0,v^0)},$$
$$\overline{F}_{v_0} = \frac{\partial \overline{F}(\overline{x}, \overline{v}, v_0)}{\partial v_0} |_{(x^0,v^0)}, \quad \overline{G}_x = \frac{\partial \overline{G}(\overline{x}, \overline{v}, v_0)}{\partial \overline{x}} |_{(x^0,v^0)},$$
$$\overline{G}_v = \frac{\partial \overline{G}(\overline{x}, \overline{v}, v_0)}{\partial \overline{v}} |_{(x^0,v^0)}, \quad \overline{G}_{v_0} = \frac{\partial \overline{G}(\overline{x}, \overline{v}, v_0)}{\partial v_0} |_{(x^0,v^0)}.$$

Let

$$\begin{aligned} \Phi_1 &= (I + M_0^{-1}\mathbb{1}'\overline{G}_{v_0})^{-1} M_0^{-1}\mathbb{1}'\overline{G}_x; \\ \Phi_2 &= (I + M_0^{-1}\mathbb{1}'\overline{G}_{v_0})^{-1} M_0^{-1}\mathbb{1}'\overline{G}_v; \\ \Psi_1 &= (H_{21} - H_{22}\Phi_2 + \overline{G}_{v_0}\Phi_2 - \overline{G}_v)^{-1} \\ &\quad \times (H_{22}\Phi_1 + \overline{G}_x - \overline{G}_{v_0}\Phi_1); \\ \Psi_2 &= -(\Phi_1 + \Phi_2\Psi_1). \end{aligned}$$

Theorem 1: The equilibrium point (x^0, v^0) is locally asymptotically Lyapunov stable if and only if all eigenvalues of

$$J = \overline{F}_x + \overline{F}_v\Psi_1 + \overline{F}_{v_0}\Psi_2 \tag{9}$$

are in the open left-half plane.

IV. Power Management and Robust Stability

Suppose that the loads are of power types. Let the target power consumptions for the loads $L_1, \ldots, L_n$ be $P_1, \ldots, P_n$, respectively. Then generically, we may represent the load dynamics by

$$\dot{i}_k = -f_k(v_k)((v_k' i_k) - P_k), k = 1, \ldots, n. \tag{10}$$

The functions $f_k(v_k) > 0$ are positive functions (generators of constant power types will have the opposite signs for P_k), and their forms depend on the actual load features and their control systems. The load subsystems (10) simply indicate that when network variations and disturbances cause actual power changes, these constant-power loads will adjust the current intakes to maintain their desired power levels.

A. Optimal Power Generation

For the network in Fig. 1, what is the maximum load power that can be supported on this distribution system? This can be formally stated in the following optimization problem. Let power dependence of J in (9) be explicitly denoted by $J(P)$ and Δ be the set of P for which $J(P)$ is stable.

$$\begin{cases} p_{\max} = \max \sum_{i=1}^n P_i \\ s.t. \; P \in \Delta. \end{cases} \tag{11}$$

B. Stability Margins

Suppose that the loads of the network are $P = [P_1, \ldots, P_n]'$. The total load is $p = P'\mathbb{1} = \sum_{i=1}^{n} P_i$, where $\mathbb{1} = [1, 1, \ldots, 1]'$ of compatible dimension. For a maximum load increase $\delta > 0$, the set $B(\delta)$ denotes all possible load assignments whose total load increase is bounded by δ,

$$B(\delta) = \{d = [d_1, \ldots, d_n]' : d_i \geq 0, i = 1, \ldots, n; d'\mathbb{1} \leq \delta\}.$$

We use the set notation $P + B(\delta) = \{P + d : d \in B(\delta)\}$. Consider the following stability margins.

1)
$$\delta(P) = \sup\{\delta : P + B(\delta) \subseteq \Delta\}. \qquad (12)$$

$\delta(P)$ is called the "voltage stability margin" at P.

2) Given the total load p,
$$\delta_{opt}(p) = \sup\{\delta(P) : P'\mathbb{1} = p\}. \qquad (13)$$

$\delta_{opt}(p)$ is called the "optimal voltage stability margin" of load p. The condition means that the current loads, but not the future loads, can be optimally re-assigned.

3) Given the current load assignment P with the total load $p = P'\mathbb{1}$,
$$\delta_{\max}(p) = p_{\max} - p. \qquad (14)$$

$\delta_{\max}(p)$ is called the "maximum load margin" of load p. The condition means that both the current and future loads can be optimally re-assigned.

4) Given the current load assignment P,
$$\delta_k(P) = \sup\{\delta_k > 0 : P + e_k\delta_k \subseteq \Delta\}, \quad k = 1, \ldots, n \qquad (15)$$

where e_k is the kth elementary vector $e_k = [0, \ldots, 0, 1, 0, \ldots, 0]'$ with the 1 at the kth position. $\delta_k(P)$ is called the "kth load stability margin" at P. This characterizes the tolerable load increase at the load L_k when all other load powers are unchanged.

V. Radial Networks of Resistance Microgrids

To convey the basic ideas of power management and robust stability more concisely and provide explicit solutions, we now consider the special case of a resistive network of radial topology, shown in Fig. 2. Here the line losses and loads are all resistive. Consequently, all voltages and currents are in phase, and hence are represented by their RMS values only. Since all variables are real, in this section the voltages v_k and currents i_k are scalars.

A. Models and Stability Conditions

The network equations are

$$v_k = e - (i_1 + \cdots + i_n)R_E - i_k R_k, \quad k = 1, \ldots, n. \qquad (16)$$

Or equivalently,

$$v_k = e - (R_k + R_E)i_k - R_E \sum_{l \neq k} i_l, \quad k = 1, \ldots, n. \qquad (17)$$

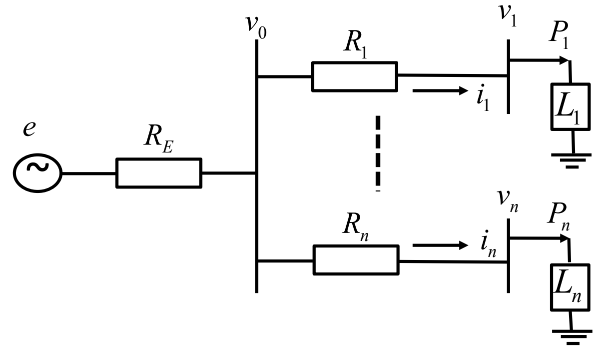

Fig. 2. A resistive radial network

Define $v = [v_1, \ldots, v_n]'$, $i = [i_1, \ldots, i_n]'$, $B = [1, \ldots, 1]'$,

$$M = \begin{bmatrix} R_1 + R_E & R_E & \cdots & R_E \\ R_E & R_2 + R_E & \cdots & R_E \\ \vdots & & & \vdots \\ R_E & R_E & \cdots & R_n + R_E \end{bmatrix}.$$

We have $v = -Mi + Be$. Further denote $F(v) = \mathrm{diag}[f_k(v_k)], C_k = [0, \ldots, 1, \ldots, 0]$ with the 1 at the kth position. It follows that

$$\begin{aligned} \dot{v} &= -M \begin{bmatrix} -f_1(v_1)(v_1 i_1 - P_1) \\ \vdots \\ -f_n(v_n)(v_n i_n - P_n) \end{bmatrix} \\ &= MF(v) \begin{bmatrix} v'Q_1 v + v'K_1 - P_1 \\ \vdots \\ v'Q_n v + v'K_n - P_n, \end{bmatrix} \end{aligned}$$

where $Q_k = -C_i' C_k M^{-1}$, $K_k = C_k' C_k M^{-1} Be$, $k = 1, \ldots, n$.

Since M and $F(v)$ are positive definite, for the given $P_1, \ldots, P_n$, the equilibrium point of the above voltage dynamic system can be solved from $v'Q_k v + v'K_k = P_k$, $k = 1, \ldots, n$. Let the equilibrium point be still denoted by v.

Theorem 2: The equilibrium point v is local asymptotically stable in the Lyapunov sense if and only if all eigenvalues of

$$J_v = MF(v) \begin{bmatrix} 2v'Q_1 + K_1' \\ \vdots \\ 2v'Q_n + K_n' \end{bmatrix} \qquad (18)$$

are in the open left half plane.

B. Stability Margins

We now present more concrete expressions for computing stability margins. In general, the closed-form solutions may not be available, except for some simple special cases. As a result, we are seeking suitable expressions for numerical solutions. The notations follow those in Section IV-B.

We will have a general discussion on suitable expressions and related numerical computational complexity, followed by expressions for different stability margins. The goal is to express the conditions in terms of linear matrix inequalities (LMI) so that efficient numerical algorithms can be used and their computational complexity can be better understood.

For a given load distribution $P = [P_1, \dots, P_n]'$, the voltage profile $v = [v_1, \dots, v_n]'$ is the equilibrium point calculated from

$$v'Q_1 v + v'K_1 = P_1, \dots, v'Q_n v + v'K_n = P_n. \qquad (19)$$

This is a quadratic form and there are multiple solutions. However, in this setup, due to physical meanings all voltages are positive. So the equilibrium point may be expressed as the unique solution to the LMI problem

$$\left\{ \begin{array}{rcl} v'Q_1 v + v'K_1 & = & P_1 \\ & \vdots & \\ v'Q_n v + v'K_n & = & P_n \\ v_1 & > & 0 \\ & \vdots & \\ v_n & > & 0. \end{array} \right. \qquad (20)$$

First, by the Lyapunov stability theory, for a given v, the stability of the matrix J_v can be equivalent evaluated from the Lyapunov equation. Suppose that for any given $Q > 0$, the symmetric matrix $H' = H$ is a solution to the Lyapunov equation

$$J'H + HJ = -Q, \qquad (21)$$

then J is exponentially stable if and only if $H > 0$. As a result, the stability condition of Theorem 2 will be expressed as an LMI condition. Since $Q > 0$ in (21) is arbitrary, without loss of generality we choose $Q = I$. In this paper, we use the convention that $H > 0$ means that H is symmetric and positive definite.

Lemma 1: J_v in Theorem 2 is exponentially stable if and only if the following LMI conditions are satisfied

$$\left\{ \begin{array}{rcl} J_v'H + HJ_v & = & -I \\ H & > & 0. \end{array} \right. \qquad (22)$$

It is noted that since the Lyapunov equation is linear and testing condition $H > 0$ is of finite step, the numerical complexity of testing the condition (22) is finite. For a total load p_0, the load allocation to the network nodes is the following operation: For $\alpha_1 > 0, \dots, \alpha_n > 0$ and $\alpha_1 + \alpha_n = 1$, the load distribution $P = [P_1, \dots, P_n]'$ satisfies

$$P_k = \alpha_k p_0.$$

Let $\alpha = [\alpha_1, \dots, \alpha_n]'$. In the following, $\alpha > 0$ is a component-wise condition. Then these conditions become

$$\alpha > 0, \mathbb{1}'\alpha = 1, P = p_0 \alpha. \qquad (23)$$

The set of all α satisfying these conditions is a convex set. As a result, a search over this set is a convex optimization problem.

We will illustrate only the voltage stability conditions in an LMI form. For a comprehensive exploration and historical perspective on the LMI methods, we refer the reader to [4]. The voltage stability margin characterizes the total tolerable load increase of the network under any possible assignment without destabilizing the network. Given the current load assignment $P^0 = [P_1^0, \dots, P_n^0]'$,

$$\delta(P^0) = \sup\{\delta : P^0 + B(\delta) \subseteq \Delta\}. \qquad (24)$$

Theorem 3: The voltage stability margin can be expressed in the following LMI-type conditions.

$$\delta(P^0) = \sup \quad \delta$$

such that for all $\alpha > 0$, $\mathbb{1}'\alpha = 1$, $P = P^0 + \delta\alpha$, it is feasible that

$$\left\{ \begin{array}{rcl} v'Q_1 v + v'K_1 & = & P_1 \\ & \vdots & \\ v'Q_n v + v'K_n & = & P_n \\ v_1 & > & 0 \\ & \vdots & \\ v_n & > & 0 \\ J_v'H + HJ_v & = & -I \\ H & > & 0. \end{array} \right.$$

C. Illustrative Examples

Consider the special network of two loads and three buses in Fig. 3. It consists of one generator and two loads interconnected by a three-bus network. The target power consumptions for the loads L_1 and L_2 are P_1 and P_2, respectively. The load dynamics are

$$\left\{ \begin{array}{rcl} \dot{i}_1 & = & -f_1(v_1)(v_1 i_1 - P_1) \\ \dot{i}_2 & = & -f_2(v_2)(v_2 i_2 - P_2). \end{array} \right. \qquad (25)$$

Here, the functions $f_1(v_1) > 0$ and $f_2(v_2) > 0$.

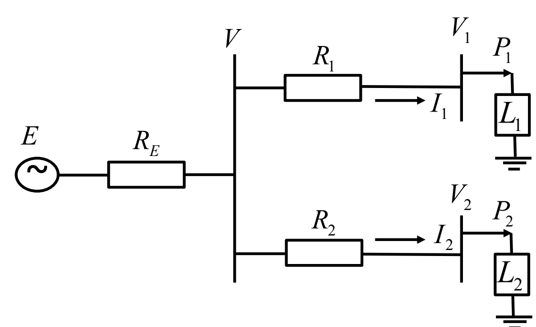

Fig. 3. A two-load three-bus network

For the given P_1 and P_2, the equilibrium point of the above voltage dynamic system can be solved from

$$\left\{ \begin{array}{rcl} -(R_2 + R_E)v_1^2 + R_E v_1 v_2 - R_2 E v_1 & = & P_1 \\ -(R_1 + R_E)v_2^2 + R_E v_1 v_2 - R_1 E v_2 & = & P_2. \end{array} \right. \qquad (26)$$

Let the equilibrium point be v. Then, the Jacobian matrix of the system at the equilibrium point is

$$J(P) = MF(v) \left[\begin{array}{c} 2v'Q_1 + K_1' \\ 2v'Q_2 + K_2' \end{array} \right]. \qquad (27)$$

Suppose that a network has the following parameters: $E = 10$ kV, $Re = 3$ ohm, $R_1 = 3$ ohm, $R_2 = 3$ ohm, $f_1 = 1$,

$f_2 = 5$. The load stability region for this network in terms of steady-state load powers is shown in Fig. 4. Also, the total load $P_1 + P_2$ is plotted with different values, on top of the stability region. It is seen that the maximum load that can be supported by this network is the line that is tangent to the boundary of the stability region, shown in Fig. 4.

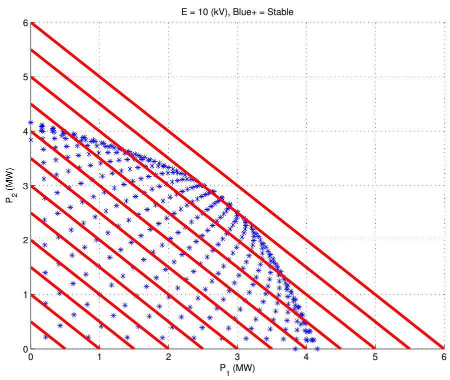

Fig. 4. Maximum total load for the network.

The first three stability margins are illustrated in Fig. 5 and the last one in Fig. 6. Clearly, $\delta \leq \delta_{opt} \leq \delta_{\max}$. Also, $\delta \leq \delta_i$, $i = 1, \ldots, n$.

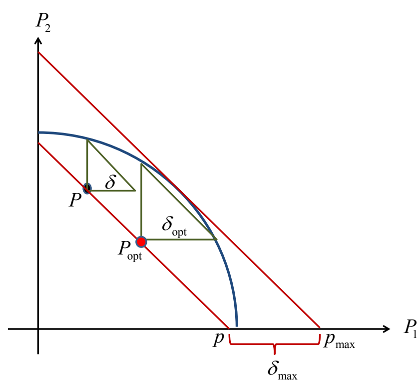

Fig. 5. Voltage stability margins.

VI. Concluding Remarks

This paper introduces a general framework for studying voltage stability and robustness in microgrid systems. There are many potential applications of this framework in exploring management, reliability, and optimality of microgrids. In our opinions, distribution and load allocations of PHEV charging, battery management, load management of microgrids can be studied within this framework. Although this paper uses resistive and constant-power loads as basic examples, the framework can be applied to loads of different types and more complicated link characterizations.

References

[1] P.M. Anderson and A.A. Fouad, Power System Control and Stability, 2nd Ed., John Wiley & Sons, 2003.

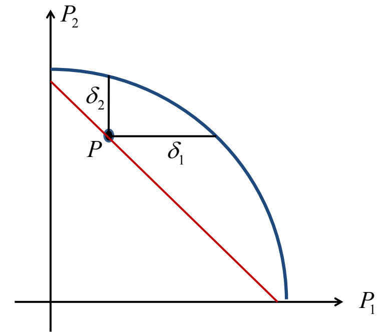

Fig. 6. Load stability margins.

[2] A. Araposthatis, S. Sastry, and P. Varaiya, Analysis of power flow equation, Electric Power & Energy Systems, vol. 3, no. 3, pp. 115-126, 1981.
[3] M.H. Bollen and F. Hassan, Integration of Distributed Generation in the Power System, John Wiley & Sons, 2011
[4] S. Boyd, L. El-Ghaoui, E. Feron, and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, SIAM, Studies in Applied and Numerical Mathematics, Vol. 9, Philadelphia, PA,1994.
[5] R. Castellanos and A.R. Messina, Robust Stability and Performance Analysis of Large Scale Power Systems With Parametric Uncertainty: A Structured Singular Value Approach, Nova Science, 2009.
[6] T. Van Cutsem, An approach to corrective control of voltage instability using simulation and sensitivity, IEEE Transactions on Power Systems, vol. 10, no. 2, pp. 616-22, May 1995.
[7] M. Djukanovic, M. Khammash, and V. Vittal, Application of the Structured Singular Value Theory for Robust Stability and Control Analysis in Multimachine Power Systems, Part I: Framework Development and Part II: Numerical Simulation and Results, IEEE Transactions on Power Systems, Vol. 13, pp. 1311-1316, November 1998.
[8] DOE Report: The Potential Benefits of Distributed Generation and Rate-Related Issues that May Impede Their Expansion, 2007.
[9] X. Fu and X. Wang, Determination of load shedding to provide voltage stability, International Journal of Electrical Power and Energy Systems, pp. 515-521, vol. 33, 2011.
[10] M. Glavic, and T. Van Cutsem, A short survey of methods for voltage instability detection, 2011 IEEE Power and Energy Society General Meeting, Detroit, USA, 2011.
[11] R.B.L. Guedes, F.H.J.R. Silva, L.F.C. Alberto, and N.G. Bretas, Large disturbance voltage stability assessment using extended Lyapunov function and considering voltage dependent active loads, 2005 IEEE Power Engineering Society General Meeting, pp. 1760-1767, vol. 2, 2005.
[12] I.A. Hiskens and D.J. Hill, Energy functions, transient stability and voltage behaviour in power systems with nonlinear loads, IEEE Transactions on Power Systems, vol. 4, no. 4, pp. 1525-1533, Nov. 1989.
[13] IEEE/CIGRE Joint Task Force on Stability Terms and Definitions, Definition and Classification of Power System Stability, IEEE Trans. on Power Syst., vol. 19, no.2, pp. 1387-1401, 2004.
[14] H.K. Khalil, Nonlinear Systems, 2nd Ed., Pretice-Hall, 1996.
[15] D. Khani, Ahmad Sadeghi Yazdankhah, and Hossein Madadi Kojabadi, Impacts of distributed generations on power system transient and voltage stability, International Journal of Electrical Power and Energy Systems, pp. 488-500, vol. 43, 2012.
[16] P. Kundur, Power System Stability and Control, EPRI Power System Engineering Series, McGraw-Hill, 1993.
[17] B. Pal and B. Chaudhuri, Robust Control in Power Systems, 2005, Springer, ISBN 978-0-387-25949-9
[18] A. Tamersi, G. Radman, and M. Aghazadeh, Enhancement of microgrid dynamic voltage stability using microgrid Voltage Stabilizer, IEEE SoutheastCon 2011. Building Global Engineers, pp. 368-373, 2011
[19] B. Venkatesh, R. Ranjan, and H.B. Gooi, Optimal reconfiguration of radial distribution systems to maximize loadability, IEEE Trans. on Power Syst., vol. 19, no. 1, pp. 260-266, 2004.
[20] K. Vu, M.M. Begovic, D. Novosel, and M.M. Saha, Use of local measurements to estimate voltage-stability margin, IEEE Trans. on Power Syst.,vol. 14, no.3, pp. 1029-1035, 1999.
[21] L.Y. Wang, M. Polis, C. Wang, and F. Lin, Voltage stability and robustness for microgrid systems, Proc. 2013 European Control Conference (ECC'13), pp. 2038-2043, 2013.