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Tracking Controller Design Methodology for Passive Port-Controlled Hamiltonians with Application to Type-2 STATCOM Systems

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Abstract— **We propose a general framework for the exponentially stable tracking controller design for passive port-controlled Hamiltonian systems with a single input and a single output. We use the Dynamic Extension Algorithm to the system. The dynamic extended system becomes an input affine system so that the tracking controller is obtained in input-output linearization framework. The tracking control law is generated considering the stability and performance of the input output linearized dynamics. We apply it to a static synchronous compensator (STATCOM) system, which is not an input affine system. We make a dynamic extension of the STATCOM system to conveniently design a reference output and then put the dynamics into the form of port-controlled Hamiltonian to apply the proposed tracking controller. Simulation results show that the proposed method improves the transient performance of the system over the previous results even in the lightly damped operating range.**

I. Introduction

Port-based network modeling of physical systems leads to a model class of nonlinear systems generally known as port-Hamiltonian systems [1]. Recently, the class of Port-Hamiltonian systems has received an increasing amount of interest from the control engineering. Interconnection and damping assignment passivity-based control (IDA-PBC) is a methodology that regulates the behavior of dynamical systems assigning a desired port-controller Hamiltonian structure to the closed-loop [2]. IDA-PBC has proven successful for a wide range of applications, e.g. in mechanical systems, electro-mechanical systems, power converters, etc. [3]–[8]. Recently, IDA-PBC method was generalized and strengthened for the stabilization of mechanical systems, and a necessary and sufficient condition for Lyapunov/exponential stabilizability was provided for the class of all linear mechanical systems [8], [9]. In [2], a tutorial of IDA results was presented using only basic linear algebra methods. Moreover, some proofs for mechanical systems were given.

In this paper, we propose a general framework for a passive port-controlled Hamiltonian systems with a single input and a single output to design a tracking controller. The proposed tracking controller guarantees the origin of the error dynamics of the system exponentially stable by using the properties of the interconnection structure and the dissipation. In this study, we consider a system that has relative degree but does not an input-affine system, that means the internal dynamics will be included the control input which is not helpful to guarantee the stability when the control input does not go to zero at infinite time [10]. For this system, we remodel the system into an input-affine system via Dynamic Extension Algorithm in order to make sure the internal dynamics of the dynamic extended system has no control input. Then, a framework of designing a tracking controller in the dynamic extended system which has a relative degree is proposed. The tracking control law is designed in the framework of input-output linearization by using the original states and the extended states. Finally, with the proposed method, the performance of the output could be expected to track the trajectory which has been generated.

The methodology is illustrated with a case study, as type 2 static synchronous compensator (STATCOM) with a single input and a single output. The proportional-plus-integral (PI), input-output feedback linearization (IOL), and passivity-based controller (PBC) for the system with the approximation model were designed for improving the performance in the lightly damped operating range [11]–[14]. In order to design a tracking controller based on a non-approximation model, the STATCOM system is rewritten in the port-controlled Hamiltonian form. Unfortunately, the system does not satisfy the condition of flatness property, and the internal dynamics includes the control input without the approximation [13]. In this study, we use the Dynamic Extension Algorithm to achieve an input-affine system, and generate the tracking control law based on the dynamic extended model. Finally, the local exponential stability of the error dynamics of the system is guaranteed in the whole operating range via ported-controller Hamiltonian method.

II. General Theory for Hamiltonian Systems

Let $x \in \mathbb{R}^n$ denote the state vector and $u \in \mathbb{R}$ denote the input. Consider a port-controlled Hamiltonian system:

$$\dot{x} = (\mathfrak{J}(u) - \mathfrak{R})\frac{\partial H(x)}{\partial x} + G(u), \tag{1}$$

where $H(\cdot)$ is the Hamiltonian function given by

$$H(x) = \frac{1}{2}x^T S x, \quad S = S^T \succ 0 \tag{2}$$

and

$$\mathfrak{J}^T(u) = -\mathfrak{J}(u), \quad \mathfrak{R} = \mathfrak{R}^T \succ 0.$$

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Here, S and $\mathfrak{R}$ are constant $n \times n$ matrices.

Theorem 1: Let $x^d(t)$ be a reference trajectory and $u^d(t)$ the input that generates $x^d(t)$ such that $x^d(t)$ and $u^d(t)$ satisfy

$$\dot{x}^d = \left(\mathfrak{J}(u^d) - \mathfrak{R}\right) \frac{\partial H(x^d)}{\partial x} + G(u^d). \tag{3}$$

If $u = u^d$ is applied to the system (1), then the origin becomes a globally and exponentially stable equilibrium point in the tracking error dynamics. Namely, $\lim_{t \to \infty} \| x(t) - x^d(t) \| = 0$, where $x(t)$ is the trajectory of the system. ◇

Proof: Let $x(t) \in \mathbb{R}^n$ be the trajectory of (1) corresponding to $u = u^d \in U \subset \mathbb{R}$ such that

$$\dot{x} = \left(\mathfrak{J}(u^d) - \mathfrak{R}\right) \frac{\partial H(x)}{\partial x} + G(u^d).$$

Although $x(t)$ and $x^d(t)$ satisfy the same dynamics, they are not the same signals because their initial conditions are different in general. Defining error as $e := x - x^d$, we obtain the following tracking error dynamics:

$$\begin{aligned} \dot{e} &= \left(\mathfrak{J}(u^d) - \mathfrak{R}\right) \left(\frac{\partial H(x)}{\partial x} - \frac{\partial H(x^d)}{\partial x} \right) \\ &= \left(\mathfrak{J}(u^d) - \mathfrak{R}\right) \frac{\partial H(e)}{\partial e}, \end{aligned}$$

where H is defined in (2). The total time derivative of $H(e)$ is given by

$$\begin{aligned} \dot{H}(e) &= \frac{1}{2} \left(\dot{e}^T \left(\frac{\partial H(e)}{\partial e} \right) + \left(\frac{\partial H(e)}{\partial e} \right)^T \dot{e} \right) \\ &= -e^T S^T \mathfrak{R} S e. \end{aligned}$$

Since $S = S^T \succ 0$ and $\mathfrak{R} \succ 0$, the zero equilibrium point of the error dynamics is globally exponentially stable. Hence, $\lim_{t \to \infty} \| x(t) - x^d(t) \| = 0$. ■

Notice that the tracking controller proposed in the above theorem is an open-loop controller taking advantage of passivity of a port-controlled Hamiltonian system.

Assumption 1: *The system (1) with output*

$$y = h(x)$$

is input-output linearizable via the Dynamic Extension Algorithm. ◇

Under this assumption, by adding some auxiliary state variables, we can put the system (1) in the form

$$\begin{aligned} \dot{x}_a &= f_a(x_a) + g_a(x_a) u_a \\ y &= h_a(x_a), \end{aligned} \tag{4}$$

where $x_a = [x,\ \xi]^T \in X_a \subset \mathbb{R}^{n_a}$, is the extended state vector, $\xi \in \mathbb{R}^k$ is the extended state. and $u_a \in U_a \subset \mathbb{R}$, is the new control input, The functions $f_a : X_a \to \mathbb{R}^{n_a}$, $g_a : X_a \times U_a \to \mathbb{R}^{n_a}$, and $h_a : X_a \to \mathbb{R}$ are sufficiently smooth. Suppose that the extended system (4) has relative degree r. Then, the system is input-output linearizable via the state feedback controller [10], [15] as

$$u_a = \frac{\nu - L_{f_a}^{\gamma} h(x_a)}{L_{g_a} L_{f_a}^{r-1} h_a(x_a)}. \tag{5}$$

Suppose the system has relative degree r at x_{a_o}. Set $\phi_1(x_a) = h(x_a), \ldots, \phi_r(x_a) = L_{f_a}^{r-1} h_a(x_a)$. If r is strictly less than n_a under Assumption 1, it is always possible to find $n_a - r$ more functions $\phi_{r+1}(x_a), \ldots, \phi_r(x_a)$ such that the mapping

$$\Phi(x_a) = \left[\begin{array}{c} \phi_1(x_a), \ldots, \phi_{n_a}(x_a) \end{array} \right]^T$$

has a Jacobian matrix which is nonsingular at x_{a_o} and therefore qualifies as a local coordinates transformation in a neighborhood of x_{a_o}. The value of x_{a_o} of these additional functions can be fixed arbitrarily [10].

The state-space of the system in the new coordinate, $(z_1, \ldots, z_{na})^T$, will be described as follows

$$\begin{aligned} \frac{dz_1}{dt} &= \frac{\partial \phi_1}{\partial x_a} \frac{\partial x_a}{\partial t} = L_{f_a} h_a(x_a(t)) = \phi_2(x_a(t)) = z_2(t) \\ &\vdots \\ \frac{dz_{r-1}}{dt} &= \frac{\partial \phi_{r-2}}{\partial x_a} \frac{\partial x_a}{\partial t} = L_{f_a}^{r-1} h_a(x_a(t)) = \phi_r(x_a(t)) = z_r(t). \end{aligned}$$

For z_r, we obtain

$$\frac{dz_r}{dt} = L_{f_a}^r h_a(x_a(t)) + L_{g_a} L_{f_a}^{r-1} h_a(x_a(t)) u_a(t).$$

Replacing $x_a(t)$ with its expression as a function of $z(t)$, i.e. $x_a(t) = \Phi^{-1}(z(t))$. Thus,

$$\frac{dz_r}{dt} = b(z(t)) + a(z(t)) u_a(t), \tag{6}$$

where

$$\begin{aligned} a(z(t)) &= L_{g_a} L_{f_a}^{r-1} h_a(\Phi^{-1}(z(t))), \\ b(z(t)) &= L_{f_a}^r h_a(\Phi^{-1}(z(t))). \end{aligned}$$

Note that at the point $z_o = \Phi(x_o)$, $a(z_o) \neq 0$. Thus, the coefficient $a(z)$ is nonzero for all z in a neighborhood of z_o. If $\phi_{r+1}(x_a), \ldots, \phi_{n_a}(x_a)$ have been chosen in such a way that $L_{g_a} \phi(x_a) = 0$, then

$$\frac{dz_i}{dt} = L_{f_a} \phi_i(x_a(t)) + L_{g_a} \phi_i(x_a(t)) = \phi_r(x_a(t)) = q_i(z)$$

for all $r + 1 \leq i \leq n_a$. where

$$q_i(z) = L_{f_a} \phi_i(\Phi^{-1} x(t)).$$

Theorem 2: If $z_1^d = y^d$ is given, then the desired control input can be obtained by using state feedback form as

$$u_a^d = \frac{1}{a(z^d)} (-b(z^d) + \nu), \tag{7}$$

where ν is the external control input that stabilizes the dynamics $\dot{z}_1^d, \ldots, \dot{z}_r^d$. ◇

Due to page limit, we omit the proof. Figure 1 illustrates the structure of the desired control input.

Assumption 2: *If the desired control input* (7) *is taken into the system* (4)*, the rest of $n - r$ desired zero dynamics*

$$\dot{z}_{r+1}^d = q_{r+1}(z^d), \ldots,\ \dot{z}_n^d = q_n(z^d)$$

are stable in the set $\Omega_{z^d} = \{z^d \in \mathbb{R}^{n_a} : z_1^d = z_2^d = \ldots = z_r^d = 0\}$. ◇

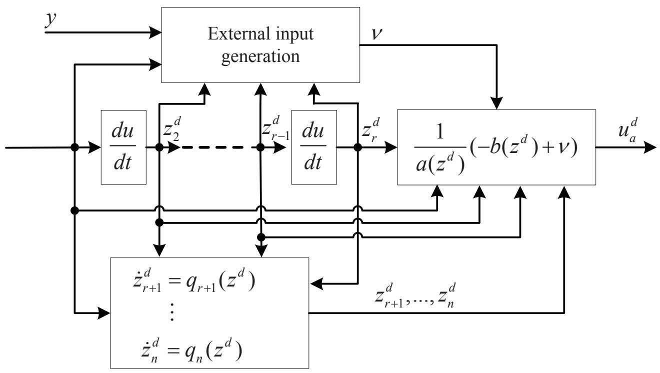

Fig. 1. Desired control input generator block

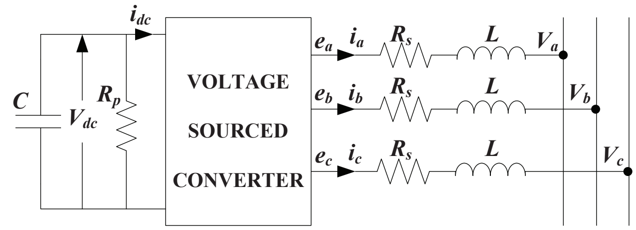

Fig. 2. Equivalent STATCOM circuit

III. CASE STUDY

In this study, we consider a type 2 STATCOM system with a single input and a single output. For this system, several controllers were designed considering the characteristics of the system [11]–[14]. A synthesized feedback controller was designed in order to improve the phase margin in the inductive load where the system has little phase margin [11]. Due to the nonlinearity of the STATCOM system, an IOL controller was designed such that the oscillation amplitude of the internal dynamics can be effectively decreased by adding a damping term [12]. Recently, a modified damping controller was designed to enhance the stability and performance of the internal dynamics [13]. The modified damping term move the poles of the internal dynamics further away from the imaginary axis than the IOL method is used [16]. Sufficient conditions with a parameter-dependent Lyapunov function were investigated for semi-globally exponential stability of damped internal dynamics [17]. Furthermore, in order to improve robustness, a passivity-based controller with the nonlinear damping was developed for this system [14]. However, all these controllers were designed based on an approximated model as the control input is a small value in the operating range. Without the approximation, the control input will still appear in the internal dynamics [13], [17], which affects the stability of the system. In this paper, we design a controller with non-approximated model to improve the transient performance. Firstly, a new state variable is added to get an input-affine system based on Dynamics Extension Algorithm. Then, based on this extended system, the control law is generated considering the stability of the linearized dynamics. Finally, the local exponential stability of the error dynamics of the system is guaranteed in the whole operating range via ported-controlled Hamiltonian method.

A. Mathematical model of STATCOM system

The equivalent circuit of a STATCOM connected to the transmission line through inductances L's in series which represent the leakages of the actual power transformers is shown in Fig. 2. $V_{a,b,c}$ and $e_{a,b,c}$ represent the line voltage vectors and the converter voltage vectors, respectively. i_a, i_b and i_c represent flows of three phase currents through the lines. V_{dc} and i_{dc} represent DC side of voltage and current, respectively. The resistances R_s's represent conduction losses between the inverter and the transformer, and the resistance R_p, which is connected parallelly to a capacitor C, represents switching losses in the system [11], [14], [18]. A mathematical averaged model of STATCOM system in state-space on d-q frame can be expressed as follows [11]:

$$\begin{aligned} \dot{x} &= f(x,\alpha) \\ &= \begin{pmatrix} -\frac{R_s'\omega_b}{L'}x_1 + \omega x_2 + \frac{k\omega_b}{L'}x_3\cos\alpha - \frac{\omega_b}{L'}|V'| \\ -\omega x_1 - \frac{R_s'\omega_b}{L'}x_2 + \frac{k\omega_b}{L'}x_3\sin\alpha \\ -\frac{3}{2}kC'\omega_b x_1\cos\alpha - \frac{3}{2}kC'\omega_b x_2\sin\alpha - \frac{\omega_b C'}{R_p'}x_3 \end{pmatrix}, \\ y &= h(x) = x_2, \end{aligned} \tag{8}$$

where

$$\left[x_1\ x_2\ x_3\right]^T = \left[I_d'\ I_q'\ V_{dc}'\right]^T.$$

Here $f: \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$ is a sufficiently smooth function, and $\alpha \in \mathbb{R}$ is the control input. I_d', I_q' and V_{dc}' represent active current, reactive current, and voltage of capacitor, respectively, which are defined as system states. The parameters and state variables with an apostrophe represent the per-unit values. The factor k is a constant which relates the DC voltage to the maximum amplitude of the phase-to-neutral voltage at the converter AC side terminals, and α is the phase shift angle by which the converter voltage vectors $e_{a,b,c}$ lead the line voltage vectors $V_{a,b,c}$ [13], [18]. When the reactive current has a positive sign, the STATCOM is working in inductive mode, i.e., it is consuming the reactive power. When the reactive current has a negative sign, it is operating in capacitive mode, i.e., it is supplying the reactive power to power system [13]. The system has an operating range as $-1 \le I_q' \le 1$, and the control input has a limitation as $-22.1^\circ \le \alpha \le 22.1^\circ$ [13].

B. Local coordinates transformations

Let us define an additional state x_4 and a new input u_a for the system (8) as follows:

$$\begin{aligned} x_4 &:= \sin\alpha, \\ u_a &:= \dot{\alpha}. \end{aligned} \tag{9}$$

With the extended state $x_a = (x_1, x_2, x_3, x_4)^T$ and the new control input u_a, the system can be rewritten as follows:

$$\begin{aligned} \dot{x}_a &= f_a(x_a) + g_a(x_a)u_a, \\ y &= h_a(x_a), \end{aligned} \tag{10}$$

where $f_a(x_a)$ and $g_a(x_a)$ are smooth vector fields, and $h_a(x_a)$ is a smooth function defined on an open set $\Omega_x \in \mathbb{R}^4$.

$$f_a(x_a) = \begin{pmatrix} -\frac{R_s'\omega_b}{L'}x_1 + \omega x_2 + \frac{k\omega_b}{L'}x_3\sqrt{1-x_4^2} - \frac{\omega_b}{L'}|V'| \\ -\omega x_1 - \frac{R_s'\omega_b}{L'}x_2 + \frac{k\omega_b}{L'}x_3x_4 \\ -\frac{3}{2}kC'\omega_b x_1\sqrt{1-x_4^2} - \frac{3}{2}kC'\omega_b x_2 x_4 - \frac{\omega_b C'}{R_p'}x_3 \\ 0 \end{pmatrix},$$

$$g_a(x_a) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sqrt{1-x_4^2} \end{pmatrix}, \; h_a(x_a) = x_2,$$

For the system (10) we have

$$\begin{aligned} \frac{\partial h_a}{\partial x_a} &= (0\ 1\ 0\ 0), \\ L_g h_a(x_a) &= 0, \\ L_f h_a(x_a) &= -\omega x_1 - \frac{R_s'\omega_b}{L'}x_2 + \frac{k\omega_b}{L'}x_3x_4, \\ \frac{\partial(L_f h_a)}{\partial x_a} &= \begin{pmatrix} -\omega & -\frac{R_s'\omega_b}{L'} & \frac{k\omega_b}{L'}x_4 & \frac{k\omega_b}{L'}x_3 \end{pmatrix}, \\ L_g L_f h_a(x_a) &= \frac{k\omega_b}{L'}x_3\sqrt{1-x_4^2}. \end{aligned}$$

Then the system has relative degree two in $\mathbb{R}^4$. In order to find the normal form, we set

$$\begin{aligned} z_1 &:= \phi_1(x_a) = h_a(x_a) = x_2, \\ z_2 &:= \phi_2(x_a) = L_f h_a(x_a) = -\omega x_1 - \frac{R_s'\omega_b}{L'}x_2 + \frac{k\omega_b}{L'}x_3x_4 \end{aligned} \tag{11}$$

and we seek for two functions ϕ_3 and ϕ_4 such that

$$\begin{aligned} \frac{\partial \phi_3}{\partial x_a}g_a(x_a) &= \frac{\partial \phi_3}{\partial x_4}\sqrt{1-x_4^2} = 0, \\ \frac{\partial \phi_4}{\partial x_a}g_a(x_a) &= \frac{\partial \phi_4}{\partial x_4}\sqrt{1-x_4^2} = 0. \end{aligned}$$

For simplification, two new variables are defined as follows:

$$\begin{aligned} z_3 &:= \phi_3 = x_3, \\ z_4 &:= \phi_4 = x_1 \end{aligned}$$

satisfy these conditions. These and the previous two function in (11) define a transformation $z = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 \end{bmatrix}^T = \Phi(x_a)$ whose Jacobian matrix

$$\frac{\partial \Phi}{\partial x_a} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega & -\frac{R_s'\omega_b}{L'} & \frac{k\omega_b}{L'}x_4 & \frac{k\omega_b}{L'}x_3 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

is nonsingular for all operating points due to $x_3 \neq 0$. Thus, $\Phi(x_a)$ is a local diffeomorphism [10]. The inverse transformation is given by

$$x_1 = z_4, \quad x_2 = z_1, \quad x_3 = z_3, \quad x_4 = \frac{z_2 + \omega z_4 + \frac{R_s'\omega_b}{L'}z_1}{\frac{k\omega_b}{L'}z_3}.$$

In the new coordinates, the system is described by

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= b(z) + a(z)u_a, \\ \dot{z}_3 &= -\frac{3}{2}kC'\omega_b z_4 q_c(z)^{\frac{1}{2}} - \frac{3}{2}kC'\omega_b z_1(1-q_c(z))^{\frac{1}{2}} - \frac{\omega_b C'}{R_p'}z_3, \\ \dot{z}_4 &= -\frac{R_s'\omega_b}{L'}z_4 + \omega z_1 + \frac{k\omega_b}{L'}z_3 q_c(z)^{\frac{1}{2}} - \frac{\omega_b}{L'}|V'|, \end{aligned} \tag{12}$$

where

$$\begin{aligned} b(z) = &-\omega\left(-\frac{R_s'\omega_b}{L'}z_4 + \omega z_1 + \frac{k\omega_b}{L'}z_3 q_c(z)^{\frac{1}{2}} - \frac{\omega_b}{L'}|V'|\right) \\ &- \frac{R_s'\omega_b}{L'}(z_2) + \frac{k\omega_b}{L'}(1-q_c(z))^{\frac{1}{2}}\left(-\frac{3}{2}kC'\omega_b z_3 q_c(z)^{\frac{1}{2}} \right. \\ &\left. - \frac{3}{2}kC'\omega_b z_1(1-q_c(z))^{\frac{1}{2}} - \frac{\omega_b C'}{R_p'}z_3\right), \end{aligned}$$

$$a(z) = \frac{k\omega_b}{L'}z_3 q_c(z)^{\frac{1}{2}}, \quad q_c(z) = 1 - \left(\frac{z_2 + \omega z_4 + \frac{k\omega_b}{L'}z_1}{\frac{k\omega_b}{L'}z_3}\right)^2.$$

C. Desired control input

If the desired output, z_1^d, is given, then the other desired values of the states can be obtained using the dynamics (12) in the new coordinates as follows:

$$\begin{aligned} z_2^d &= \dot{z}_1^d \\ \dot{z}_3^d &= -\frac{3}{2}kC'\omega_b z_4^d q_c(z^d)^{\frac{1}{2}} - \frac{3}{2}kC'\omega_b z_1^d(1-q_c(z^d))^{\frac{1}{2}} - \frac{\omega_b C'}{R_p'}z_3^d \\ \dot{z}_4^d &= -\frac{R_s'\omega_b}{L'}z_4^d + \omega z_1^d + \frac{k\omega_b}{L'}z_3^d q_c(z^d)^{\frac{1}{2}} - \frac{\omega_b}{L'}|V'|, \end{aligned} \tag{13}$$

where

$$q_c(z^d) = 1 - \left(\frac{z_2^d + \omega z_4^d + \frac{k\omega_b}{L'}z_1^d}{\frac{k\omega_b}{L'}z_3^d}\right)^2.$$

Thus, from (12) and (13) the desired new control input can be calculated as

$$u_a^d = \frac{-b(z^d) + \nu}{a(z^d)}, \tag{14}$$

where

$$\nu = \ddot{z}_1^d - K_1(\dot{z}_1 - \dot{z}_1^d) - K_2(z_1 - z_1^d).$$

Here, z_1^d is generated by 5^{th} profile, $K_1 > 0$ and $K_2 > 0$. Note that $a(z^d) \neq 0$ for all operating points, since $z_3^d = x_3^d \neq 0$ and $q_c(z^d) = \cos^2 \alpha^d \neq 0$.

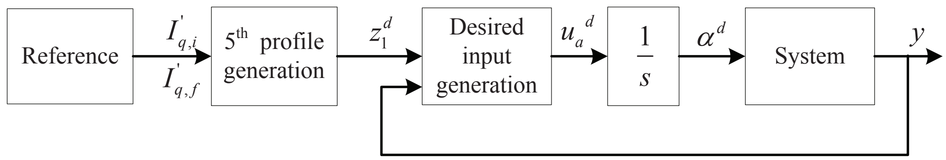

Fig. 3. Controller block using Dynamic Extension Algorithm

D. Hamiltonian form

For the original system (8), we take

$$H(x) = \frac{1}{2}x^T Sx, \tag{15}$$

where

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3L'C'} \end{bmatrix}.$$

Then the system (8) is put in the form (1) as follows:

$$\dot{x} = (\mathfrak{J}(\alpha) - \mathfrak{R})\frac{\partial H(x)}{\partial x} + G, \tag{16}$$

where

$$\mathfrak{J}(\alpha) = \begin{bmatrix} 0 & \omega & \frac{3}{2}kC'\omega_b \cos\alpha \\ -\omega & 0 & \frac{3}{2}kC'\omega_b \sin\alpha \\ -\frac{3}{2}kC'\omega_b \cos\alpha & -\frac{3}{2}kC'\omega_b \sin\alpha & 0 \end{bmatrix},$$

$$\mathfrak{R} = \begin{bmatrix} \frac{R'_s\omega_b}{L'} & 0 & 0 \\ 0 & \frac{R'_s\omega_b}{L'} & 0 \\ 0 & 0 & \frac{3\omega_b L' C'^2}{2R'_p} \end{bmatrix}, \quad G = \begin{bmatrix} -\frac{\omega_b}{L'}|V'| \\ 0 \\ 0 \end{bmatrix}.$$

Here, we take the desired control input based on (9) and (14) as

$$\alpha^d = \int u_a^d dt, \tag{17}$$

that satisfies

$$\dot{x}^d = \left(\mathfrak{J}(\alpha^d) - \mathfrak{R}\right)\frac{\partial H(x^d)}{\partial x} + G.$$

If we take $\alpha = \alpha^d$ for the system as Theorem 1, then the error dynamics of the system has an exponentially stable equilibrium point at the origin. The whole procedure of the proposed method is illustrated in Fig. 3. The desired initial and final values of I'_q are indicated by $I'_{q,i}$ and $I'_{q,f}$, respectively.

IV. Performance of Controller

To validate the proposed control strategies, simulations using the averaged model of (8) are performed in MATLAB/Simulink. Table I lists the parameters used in the simulation, and control specifications of reactive current are settling time, $T_s < 16$ [ms], steady state error, $e_{ss} < 0.05$ [pu], and overshoot, $OS < 0.1$ [pu]. The operating range of $I'_{q,ref}$ is from -1 [pu] to 1 [pu] [13]. In this study, z_1^d is generated by using 5th profile to make sure the generated reference reach the final value more smoothly. The proposed method is compared to the input-output feedback linearization with modified damping (IOLMD) used in [13].

TABLE I
System parameters used in simulation

Parameter	Value	Unit
R'_s	0.0071	pu
L'	0.15	pu
C'	2.78	pu
R'_p	727.5846	pu
k	0.6312	
ω	60	Hz

Figure 4 shows the performance of the system when the reactive current reference, $I'_{q,ref}$, is changed -1 [pu] to 0.5521 [pu] at 1 [sec] and indicated by the red dotted line. At this operating point, the system with the IOLMD has weakly controllable which means that the internal dynamics poles at this operating point are hardly moved by the IOLMD instead of using a large capacitor [13], [16]. In Fig. 4, the green solid line corresponds to the proposed method, the purple dash-dotted line corresponds to the IOLMD used in [13], and is indicated by IOLMD_1. From Fig. 4, the tracking performance of the reactive current, I'_q, is better than IOLMD_1. Moreover, in the active current, I'_d, and DC voltage, V'_{dc}, the overshoot and the magnitude of oscillation in the transient responses are smaller when using the proposed method than IOLMD_1.

Furthermore, we compare the proposed method to the IOLMD using 5th profile reference indicated by IOLMD_2 as shown in Fig. 5. The performance of I'_q using IOLMD_2 is better than before, but still worse than that using the proposed method. Moreover, the overshoot and the magnitude of oscillation using IOLMD_2 are larger than those using the proposed method.

V. Conclusions

A general framework of designing of a tracking control law for the port-controlled Hamiltonian system was proposed. In this article, the main idea was that the desired control input is obtained by considering the system dynamics. Then, the origin of the error dynamic of the system with the desired control input could be guaranteed globally exponentially stable by using the properties of the interconnection structure and the dissipation. The Dynamic Extension Algorithm is used to achieve an input-affine system. With

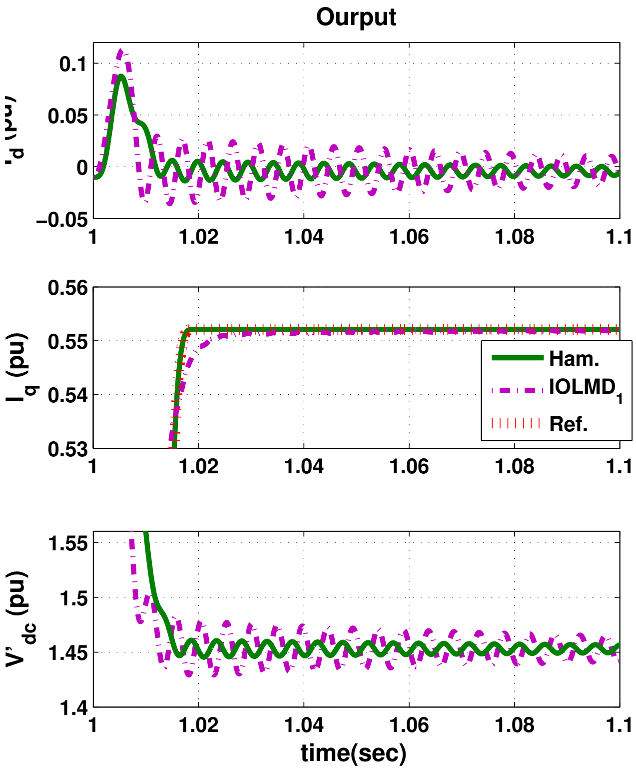

Fig. 4. Time responses of STATCOM when $I'_{q,ref}$ is changed from -1 [pu] to 0.5521 [pu] at 1 [sec] using ramp rate profile.

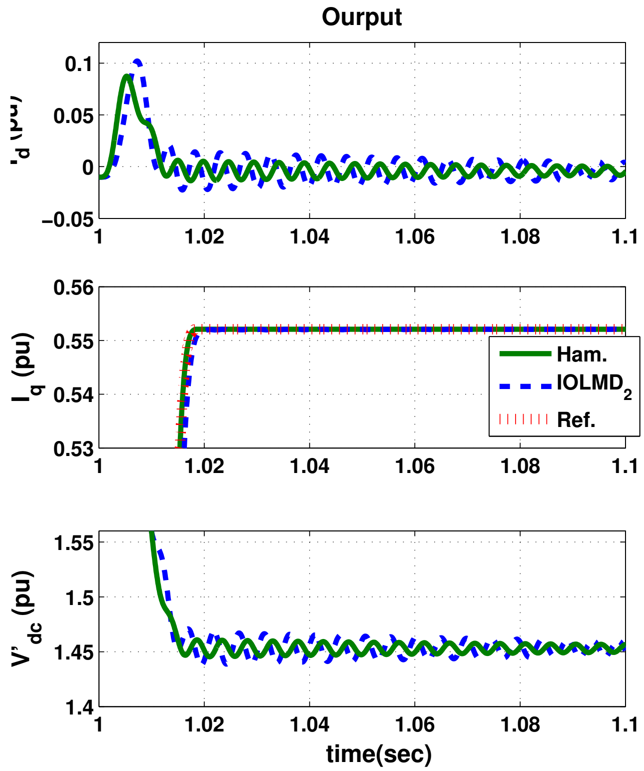

Fig. 5. Time responses of STATCOM when $I'_{q,ref}$ is changed from -1 [pu] to 0.5521 [pu] at 1 [sec] using 5^{th} order profile.

this extended system, a tracking controller is designed in the framework of input-output linearization based on the stability of the linearized dynamics. The methodology was illustrated with a type 2 STATCOM system. Due to characteristic of the system, a dynamic extended model was proposed in order to calculate the desired control input, which was designed by considering the non-approximated model to improve the transient performance. Simulation results showed that the proposed method improves the transient performance of the system in the lightly damped operating range.

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