Effects of loss models on locational reserve policies in uncertain power systems

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Abstract— Renewable energy sources (RES) often provide intermittent power at distributed locations in transmission and distribution networks. Storage devices distributed across the network are recognized as an important tool for compensating such power fluctuations. Operational policies of storage devices considered in literature focus solely on network balance. Here, the consumed network capacity is also considered. A linear network model with uncertain power and storage elements is supplemented with three possible models of line losses. Policy computation is formulated in each case as a convex optimization problem. The different solutions are compared in terms of computational complexity and policy structure. A single solution shows preference for local energy storage and remains computationally efficient. Other solutions either do not respond to uncertainty or attempt to reduce uncertainty using unnecessary power transmissions. Hence, the results yield a useful loss model as well as characterize undesirable behaviors that should be avoided with future models.

I. INTRODUCTION

In the global effort to curb carbon emissions, governments have put forth ambitious targets for renewable energy utilization [1]. In order to meet the proposed penetration levels, intermittency of renewable energy sources (see [2, 3]) must be efficiently managed. Storage devices at the transmission and distribution levels have been envisioned for this purpose. The real-time operational policies for these devices under uncertainty have only recently started to be considered.

Risk limiting dispatch introduced in [4, 5] considers dynamic allocation of different layers of reserves using the latest predictions of production and demand. Allocation policies of spinning reserves, supplemental reserves, and replacement reserves are optimised to maximise the maximum likelihood of the overall network balance. In [6], robust control theory is used to compute reserve policies that minimise the expected generation costs. In addition the policies are subject to overall network balance constraints and line flow limits for all system perturbations. In other works (e.g., see [7, 8]) policies over selected contingency scenarios are computed using dynamic programming principles. Herein we're interested in modelling the continuous nature of RES intermittencies and hence will not further reference these methods.

The frameworks in [4, 6] serve as bases for our study. The motivation are storage policies that consider the transmission costs as well as the benefits of overall network balance. In both [4] and [6], line losses are omitted in the network model and hence transmission costs are ignored. As will be shown below, this qualitatively changes the resulting behaviour.

The traditional deterministic direct current (DC) optimal power flow (OPF) framework is lossless. Different deterministic loss models have been explored in literature and adjoined to the traditional flow equations to increase the network model accuracy [9, 10]. In [9] quadratic losses are approximated using Taylor expansion. In [10] piecewise linear models are developed and compared to linear and quadratic approximations.

Herein models of uncertain losses are proposed and analysed. Taylor expansion is used to construct linear and quadratic approximations. The resulting optimization problems are formalized and characterized in terms of complexity and the storage policy dependence on system parameters. In this process, additional modeling and optimization aspects were encountered and appended to the evaluation criteria. The specific model characteristics investigated either explicitly or through a transparent case study are as follows:

Computational complexity of the generated optimization problem in terms of

1) convexity of the loss equations or amenability of the loss equations to convex relaxation,
2) a priori flow information (e.g., flow directions or operational points) required to compute the losses.

Physical manifestation of the optimal power flows in terms of their

1) feasibility and accuracy with respect to the real world model,
2) sensitivity to changes in uncertainty and cost parameters.

This work is potentially applicable in guiding the incorporation of line losses into OPF problems under uncertainty. Benefits and shortcomings of the different approaches are explained. Hence future works can use the results to implement the proposed loss models, construct new loss models, and test their models with respect to the listed criteria. Optimal power flow problems are widely used in intra-day and inter-day operations of existing transmission and distribution systems as well as proposed active distribution systems. Hence, the results have a wide application spectrum.

The following are the specific contributions of this work for power networks with intermittent power injections:

1) Three different models of losses are presented;
2) the general OPF problem is defined for lossy systems;
3) the importance of flow directionality and the presence of cyclic flows for given loss models is described;
4) sensitivity criteria for analyzing loss model fidelity are proposed;
5) formalization of power storage [11] as a power flow regulation tool within the OPF framework;
6) the proposal of a simple case study system for determining the fidelity of a given loss model.

The remainder of the paper is organized as follows: the network model is presented in section II. The optimization framework is defined in section III. Models of losses and additional considerations are formulated in section IV. The case study for a small 3-bus distribution network is presented in section V. The paper concludes in section VI.

II. Network model

We use following notation throughout the article. Capital letters are used to denote matrices and random variables. Vectors are defined by lowercase letters and $x(i)$ is the i-th element of the vector. The expectation values of random variable X are written as $\langle X \rangle$. The variance is written as $\langle\langle X \rangle\rangle$. The power network is described by an oriented graph where the lines represent the branches and nodes represent the buses. We denote the set $\mathcal{N} = 0, 1, \ldots, N$ as set of nodes and set of branches $\mathcal{B} \subseteq \mathcal{N} \times \mathcal{N}$, where each $(k, m) \in \mathcal{B}$ is directed from node k to node m. From each node $k \in \mathcal{N}$, the set of outgoing branches $\mathcal{O}_k$ is defined as $\{b \in \mathcal{B} | b = (k, j), j \in \mathcal{N}\}$ and the set of incoming branches $\mathcal{I}_k$ as $\{b \in \mathcal{B} | b = (j, k), j \in \mathcal{N}\}$.

The network is described by a set of variables. For each node $k \in \mathcal{N}$ we define a phase angle $\Theta(k)$, a voltage $V(k)$, a net power injection $P(k)$. In addition, $P(k)$ comprises the generator injection $e_g(k) \in \mathbb{R}^N$ with its limits $e_g(k)^+, e_g(k)^-$, the power demand $e_d(k) \in \mathbb{R}^N$, the intermittent source injection $W(k)$, and the power storage $S(k)$. For each branch $b \in \mathcal{B}$ there is the impedance $z(b) = r(b) + ix(b)$, the power flow $F(b)$, branch limits $f^+(b)$ and $f^-(b)$, the transmission loss $L(b)$, and the current $I(b)$.

Earlier works [4] and [6] consider the power uncertainty but do not model network topology. With the integration of a network topology, additional assumptions have to be made to preserve network model consistency. For one, the power balance has to be maintained at each node. To balance the uncertainty injected by the network participants, uncertain power flows are added to the network model.

The uncertainty is modelled by the random variable Δ that takes values in $\Omega \subseteq \mathbb{R}^N$ with a zero mean and defined covariance matrix. The variable Δ can be described by any chosen probability distribution. The set $\Omega = \mathsf{X}_{i \in \mathcal{N}} \Omega_i$, where $\Omega_i = \{\delta_i \in \mathbb{R} | S\delta_i \leq h_i, S = [1, -1]^T, h_i = [\delta_i^+, -\delta_i^-]^T \in \mathbb{R}^2\}$ and δ_i^+, δ_i^- are pre-defined bounds.

The uncertainty enters the framework through the intermittent power $W = e_w + D_w\Delta$, where $e_w \in \mathbb{R}^N$ and $D_w \in \mathbb{R}^{N \times N}$. Storage devices, used to balance the intermittent injections, are automatically controlled by the storage

policies $f_k : \mathbb{R}^N \to \mathbb{R}$, i.e. $S(k) = f_k(\Delta)$. Computation of an optimal storage policy is often intractable. We follow the approach introduced in [6], where storage policies are assumed to be affine functions of Δ, i.e., $S = e_s + D_s\Delta$ with its limits d_s^+, d_s^-. For the sake of simplicity, we absorb the constant part e_s into the deterministic generation e_g. In other words, we let $e_s = 0$. Similarly the power flows and phase angles are taken to be affine with respect to Δ. Therefore, the vector of the flows $F = e_f + D_f\Delta$ and the vector of the phase angles $\Theta = e_\theta + D_\Theta\Delta$. Note, if the losses are approximated by affine functions, then the restrictions of F and Θ are without loss of generality. Hence, in the most general form of losses considered herein $L = e_l + D_l\Delta$.

To access elements of the random vectors, the following notation is used. Take the vector of flows as an example,

$$F(b) = e_f(b) + D_f(b)\Delta,$$

where $e_f(b) \in \mathbb{R}$ and $D_f(b) \in \mathbb{R}^N$ are the corresponding rows of e_f and D_f.

To simplify the notation, all variables are assumed to be in per unit scale. Note, that additional network participants, e.g., regulation or deterministic power storage devices can be easily added to this framework.

III. Optimization framework

The non-linear OPF problem is non-convex and therefore computationally intractable [12, 13]. Branch flows are generally non-linear functions of voltage phase angles and the losses are quadratic functions of the branch currents. We use the standard DC approximation, see [14], [15]. The DC OPF is computationally tractable and has proven economic value. For these reasons we choose to build the analysis on the same set of physical assumptions. To simplify the notation, we define the net power injections as

$$P = e_g + e_d + e_w + D_w\Delta + D_s\Delta,$$

and we further split P into its deterministic part p_d and stochastic part $P_u\Delta$.

Under these assumptions, we define the lossless probabilistic optimal power flow (ℓ-POPF) problem as follows.

Problem 1: ℓ - POPF

Let $\delta \in \Omega$ and Γ_g and Γ_s be the penalization matrices for e_g and D_s, respectively. The problem is to

$$Minimize \ e_g^T\Gamma_g e_g + trace\left(\Gamma_s^T D_s \langle\langle \Delta \rangle\rangle D_s^T \Gamma_s\right),$$

subject to, $\forall b \in \mathcal{B}$ and $\forall n, m \in \mathcal{N}$,

$$e_g^- \leqq e_g \leqq e_g^+, \quad d_s^- \leqq D_s\delta \leqq d_s^+,$$

$$f^- \leq e_f + D_f\delta \leq f^+,$$

$$0 = p_d(n) + \sum_{b \in \mathcal{I}_n} e_f(b) - \sum_{b \in \mathcal{O}_n} e_f(b), \qquad (1)$$

$$0 = P_u(n)\delta + \sum_{b \in \mathcal{I}_n} D_f(b)\delta - \sum_{b \in \mathcal{O}_n} D_f(b)\delta, \qquad (2)$$

$$e_f(b) = \frac{1}{x(b)} \left(e_\theta(n) - e_\theta(m) \right), \tag{3}$$

$$D_f(b)\delta = \frac{1}{x(b)} \left(D_\Theta(n) - D_\Theta(m) \right) \delta, \ \forall \delta \in \Omega. \tag{4}$$

∎

The above problem is lossless. In other words, the power balance conditions (1) and (2) state that the flow into a branch equals the flow out of a branch. This model allows the deterministic and the uncertain flows to be balanced separately. The relation between Θ and F is also separated into its deterministic and stochastic parts.

The novelty of our approach lies in the modeling of deterministic and uncertain flows. The explicit representation of the flows $F = e_f + D_f \Delta$ is used in the next section to derive various loss models.

$$F(b) = e_f(b) + D_f(b)\Delta. \tag{5}$$

The solution of ℓ-POPF is non-trivial. Since the optimization is done $\forall \delta \in \Omega$, its computation involves an infinite number of optimization problems. In this paper, the duality principle, described in [16] and applied in [6], is used to transform ℓ-POPF to a computationally tractable convex optimization problem. Appendix A presents this problem for the lossy case defined in the next section.

IV. MODELS OF LOSSES UNDER UNCERTAINTY

A. General model of flows and losses

If we want do develop a more accurate network model, we need to take transmission losses into consideration. Losses are a function of the branch currents, i.e.,

$$L(b) = r(b)\|I(b)\|^2.$$

For any $b \in \mathcal{B}$, the power flowing into the line satisfies $\|F(b)\| = \|V(b)\|\|I(b)\|$, where $V(b)$ is the voltage at the line's origin (see [15]). Moreover, in the p.u. scale,

$$\|F(b)\| = \|I(b)\|,$$

which implies, the losses are given by

$$L(b) = r(b)\|I(b)\|^2 = r(b)\|F(b)\|^2. \tag{6}$$

B. Integration of losses into the network model

There are two options when integrating losses into the constraints (1) and (2) of ℓ-POPF. For the 2 node connected network shown in Figure 1 a), where $P(1) > 0$ and $P(2) > 0$, we can subtract the losses from the incoming flows, i.e.,

$$P(1) - F = 0, \quad -P(2) + F - L = 0.$$

or we can add the losses to the outgoing flows, i.e.,

$$P(1) - F + L = 0, \quad -P(2) + F = 0.$$

Figure 1 b) shows the power transferred from node 1 to node 2 as a function of power injected into node 1. We can see that adding losses to the outgoing flow leads to an unlimited maximum power transfer. Subtraction of losses from the
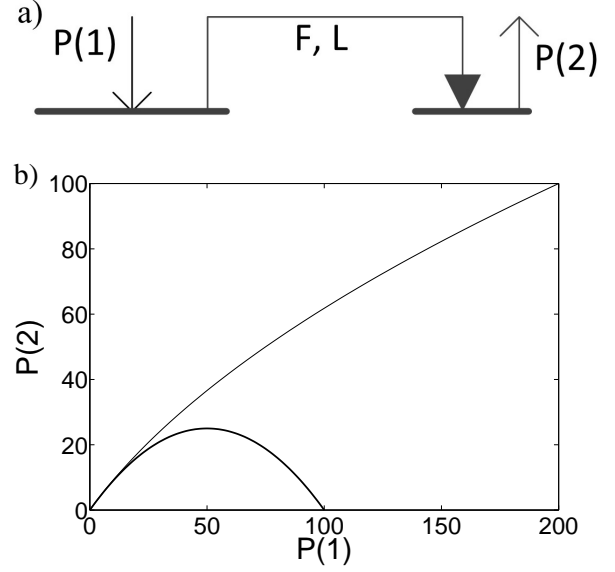

a)

b)

Fig. 1. Example illustrating 2 means of integrating losses into ℓ-POPF. (a) Example 2-node network where $P(1) > 0$ and $P(2) > 0$. Arrows indicate power flow directions. (b) Power transmitted P(2) as a function of power injected P(1) when losses are subtracted from incoming flows (heavy line) or added to outgoing flows (light line). Subtraction from incoming flows yields limited maximum power transfer.

incoming flow leads to a limited maximum power transfer, which is a more realistic scenario and the one implemented in the POPF reformulation.

Problem 2: POPF

Let $\delta \in \Omega$, and Γ_g, Γ_s, and Γ_l be the penalization matrices for e_g, D_s, and L, respectively. The problem is to

Minimize $e_g^T \Gamma_g e_g + L^T \Gamma_l L + trace \left(\Gamma_s^T D_s \langle\langle \Delta \rangle\rangle D_s^T \Gamma_s \right)$,

subject to, $\forall b \in \mathcal{B}$ and $\forall n, m \in \mathcal{N}$,

$$e_g^- \leqq e_g \leqq e_g^+, \ \ d_s^- \leqq D_s\delta \leqq d_s^+,$$

$$f^- \leqq e_f + D_f\delta \leqq f^+,$$

$$0 = p_d(n) + \sum_{b\in(I_n)} e_f(b) - e_l(b) - \sum_{b\in(O_n)} e_f(b), \tag{7}$$

$$0 = P_u(n)\delta + \sum_{b\in(I_n)} (D_f(b) - D_l(b))\delta - \sum_{b\in(O_n)} D_f(b)\delta, \tag{8}$$

$$e_f(b) = \frac{1}{x(b)} \left(e_\theta(n) - e_\theta(m) \right),$$

$$D_f(b)\delta = \frac{1}{x(b)} \left(D_\Theta(n)\delta - D_\Theta(m)\delta \right), \ \forall \delta \in \Omega.$$

∎

For POPF, the caviat is all flows must be positive in order to subtract losses from the incoming flows. This is achieved by splitting each branch into two opposite oriented lines with

non-negative flows. This creates the possibility of unrealistic loop flows, an issue considered in Section IV-D. As with ℓ-POPF, the above problem require solving an infinite number of optimization problems. Appendix A defines the equivalent finite dimensional problem used in the case studies.

C. Three models of transmission losses

In this section, three types of losses are presented: quadratic stochastic losses (QS), linearized deterministic losses (LD), and linearized robust losses (LR). Let (9) be the general model of transmission losses derived by substitution of (5) into (6).

$$L(b) = r(b)(e_f(b) + D_f(b)\Delta)^2. \tag{9}$$

This model presents a non-convex equality constraint and hence is not suitable for optimization. In the following text, parallel paths are used to approximate (9).

Recall, losses are constrained to the form $L(b) = e_l(b) + D_l(b)\Delta$. A loss model is taken to be a set of constraints on e_l and D_l that complete the power flow constraints (7) and (8) of Problem POPF. This presents a modular framework, in which any loss model can be easily implemented.

1) Quadratic stochastic losses (QS): In this approach, stochastic terms are considered. The model is derived by taking the expected values of both sides in (9). We get

$$\langle L(b)\rangle = r(b)\left\langle (e_f(b) + D_f(b)\Delta)^2 \right\rangle,$$

which, by $\langle\Delta\rangle = 0$, reduces to

$$e_l(b) = r(b)e_f(b)^2 + D_f(b) \ll \Delta \gg D_f(b)^T. \tag{10}$$

The QS model overapproximates the deterministic losses e_l and neglects the uncertain losses $D_l\Delta$. It favors operators that are averse to insufficient power supply.

Equation 10 is nonlinear in the optimization variables and hence not convex. The computational tractability is attained by relaxing the constraint in (10) to the inequality constraint

$$e_l(b) \geq r(b)e_f(b)^2 + D_f(b) \ll \Delta \gg D_f(b)^T. \tag{11}$$

The resulting expression (11) is convex. For this relaxation to be exact, the inequalities must be equalities at the optimal points. Indeed, this is the case for POPF, as is shown in [15].

2) Linearized deterministic losses (LD): In this approach, stochastic terms are considered after linearization. This model is derived by linearizing the right hand side of (9) around the nominal point, which is taken to be the solution of the corresponding ℓ-POPF. Variable $f_e(b)$ represents the resulting power flowing through the branch $b \in \mathcal{B}$ of Problem ℓ-POPF. Afterwards, the expectation of both sides is evaluated. This leads to the following set of equations:

$$\langle L(b)\rangle = r(b)\left\langle (f_e(b)^2 + 2f_e(b)(F(b) - f_e(b))) \right\rangle,$$
$$\langle L(b)\rangle = r(b)\left\langle -f_e(b)^2 + 2f_e(b)e_f(b) + 2f_e(b)D_f(b)\Delta \right\rangle,$$

which, since $\langle\Delta\rangle = 0$, reduces to,

$$\langle L(b)\rangle = r(b)(-f_e(b)^2 + 2f_e(b)e_f(b)). \tag{12}$$

This model only penalizes the deterministic flows. The effect of uncertain flows on the line losses is ignored. Structure of (12) is identical to loss models considered in deterministic OPF formulations, thereby requiring minimal additional modifications to existing architectures. Its usage in systems with high intermittent source penetration may be limited.

For small e_f, the model presented in (12) can generate unrealistic solutions. In the worst case, if $e_f(b) = 0$ and $f_e(b) > 0$, negative losses result.

To handle this problem, one can either eliminate the constant part $-r(b)f_e(b)^2$ or constrain the flow $e_f(b)$ to be sufficiently large. In this paper, the latter approach is followed to preserve the accuracy of the linearization and allow for fair comparison with the other loss models. The following inequality is appended to (12)

$$2f_e(b)e_f(b) \geqq f_e(b)^2. \tag{13}$$

3) Linearized robust losses (LR): This model is derived by linearizing the RHS of (9) around the nominal point, which is taken to be the solution of the corresponding ℓ-POPF Problem,

$$L(b) = r(b)(f_e(b)^2 + 2f_e(b)(F(b) - f_e(b))). \tag{14}$$

Unlike in previous models, this equation is considered at all values of Δ, i.e., the expression of losses is not limited to the expected value. In order to balance the equality at all values of Δ, the uncertain portion of the losses must be retained. The equality (14) decomposed into deterministic and uncertain parts is shown in (15) and (16).

$$e_l(b) = r(b)(-f_e(b)^2 + 2f_e(b)e_f(b)), \tag{15}$$

$$D_l(b)\Delta = 2r(b)f_e(b)D_f(b)\Delta. \tag{16}$$

Constraints can be added to the equations (7) and (8). Equations 15 and 16 are linear in the optimization variables and therefore convex; no further relaxation is needed.

The deterministic part (15) can produce unrealistic solutions, similarly to the LD model. For the same reasons as above, this issue is dealt with by constraining $e_f(b)$ to be sufficiently large using the inequality constraint (13).

The uncertain part (16) is new and can also cause problems. These lead to the need to a priori specify the directions of the uncertain flows $D_f\Delta$. This problem is further explained in the next section.

D. Additional considerations

In order to implement the LR model, the flow directions after perturbations must be known. This increase the computational complexity of the associated POPF Problem. Indeed, the problem of finding the uncertain flow directions is NP Hard. In this section, a short example will be used to show why the uncertain flow directions must be a priori specified.

Recall from IV-B that each line is split into two opposite oriented lines with non-negative flows. Consider the network illustrated in Figure 2, where the lines are explicitly drawn and the notation used in this section is denoted. We assume

that $L(i,j) > 0 \iff F(i,j) > 0$. The intermittent power injected into the system is denoted by W. The cost function, penalizing the storage utilization, is defined as $\langle S^2 \rangle$.

Now compare the solution when loop flows are either permitted or prevented. If we prevent loop flows, $F(2,1)$ and $F(1,2)$ must be necessarily zero and, hence, $L(1,2)$ and $L(2,1)$ are zero as well. The cost is then $\langle S^2 \rangle = \langle W^2 \rangle$.

Next consider the solution when loop flows are permitted. The power balance equations for this topology are,

$$S = F(2,1) - L(2,1) + W - F(1,2), \qquad (17)$$

$$F(1,2) = L(1,2) + F(2,1). \qquad (18)$$

Solving these equations gives

$$S = W - L(2,1) - L(1,2). \qquad (19)$$

For any positive $F(1,2)$ and $F(2,1)$ that satisfy (17) and (18), S is strictly smaller than W. The resulting cost is equal to $\langle S^2 \rangle = \langle (W - L(1,2) - L(2,1))^2 \rangle$, which is less than the cost when loop flows are prevented. In other words, losses attenuate flow uncertainty. While in practice this may be beneficial, this attenuation is not possible in artificially split lines. Hence, the flow direction after perturbation must be a priori specified for the LR model. For any pair of artificially split lines, $b_1, b_2 \in \mathcal{B}$ where $b_1(1) = b_2(2)$ and $b_1(2) = b_2(1)$, either $D_f(b_1) = 0$ or $D_f(b_2) = 0$. In the case study below, the uncertain flow directions are resolved using the nominal flow directions from the ℓ-POPF Problem, i.e., $\forall b \in \mathcal{B}$, if $f_e(b) = 0$, then $D_f(b) = 0$.
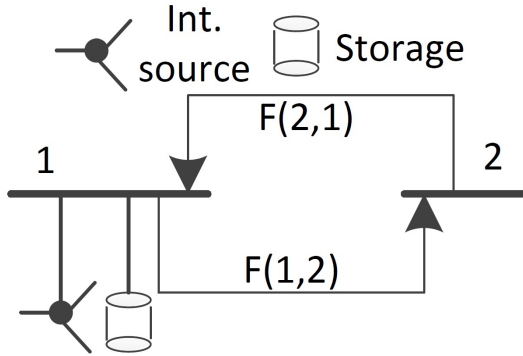

Fig. 2. An example network used to demostrate the occurence of loop currents. Network comprises of an intermittent source and a storage device. Branch arrows indicate power flow directions

V. CASE STUDY

The different types of loss models were analyzed on a small 3-bus test system. The optimization problem was implemented in Matlab and solved using the optimization tool CVX [17]. Figure 3 illustrates the network topology. The network consists of two energy storage devices ($S(1)$ and $S(3)$), a generator ($e_g(2)$), an intermittent power source ($W(1)$), and a consumer ($e_d(3)$). To isolate loss effects, various network capacity limitations were relaxed. Generators and storage devices were given unlimited capacity, and limits on branch flows were ignored to emphasize the effects

of the presented loss models. To simplify the notation, the local storage policy $D_s(1)$ and the distant storage policy $D_s(3)$ are denoted by D_1 and D_3, respectively. The injected intermittent power $W(1) = e_w(1) + D_w(1)\Delta(1)$, where the bounds of Δ are $\delta^+ = (\delta_1^+, 0, 0)^T$ and $\delta^- = (\delta_1^-, 0, 0)^T$. In addition to simplify the parametrization we let $\delta_1^- = -\delta_1^+$ and the variance $\langle\langle \Delta(1) \rangle\rangle = (\delta_1^+)^2$. Other parameter values used in the case study are listed in Table I.

Parameter	Value
Γ_g	1
Γ_r	1
Γ_l	3
$e_w(1)$	0.1
$e_d(3)$	-0.1
$r(b)$	1
$D_w(1)$	1

TABLE I

PARAMETERS OF THE 3-BUS TEST SYSTEM

The test system was chosen to demonstrate storage policy preferences for local versus distant storage devices. Moreover, the expected behaviors are studied on the defined test network. In general, following behaviors are expected:

B1: Rising uncertainty upper limit δ_1^+ will increase the preference for the local storage D_1.

B2: Uncertainty attenuates across network branches due to losses.

B3: Rising uncertainty upper limit δ_1^+ has no effect on the optimal generator output $e_g(2)$.

In the following study, the goal is twofold: 1) find the loss model that yields as many of the expected behaviors described in B1-B3 and to 2) find the loss model that yields reasonable deterministic and uncertain dispatch using a computationally tractable algorithm. Table II shows the quali-

Indicator	QS	LD	LR
B1	+	-	-
B2	-	-	+
B3	-	+	+
Range	+	-	-
Complexity	+	+[1]	-

TABLE II

QUALITATIVE EVALUATION OF THE LOSS MODELS.

tative result (+ or -) indicating how each model performed in various categories. Categories B1-B3 correspond to the above described behaviors. The category Range denotes the correctness of the computed solution away from the nominal point. The category Complexity denotes the existence of an solution algorithm with polynomial complexity. The results of Table II are quantitatively justified in the next sections.

A. Effects of uncertainty

The introduced optimization problem was solved for $\delta_1^+ \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. The results are summarized in Figures 4 and 5.

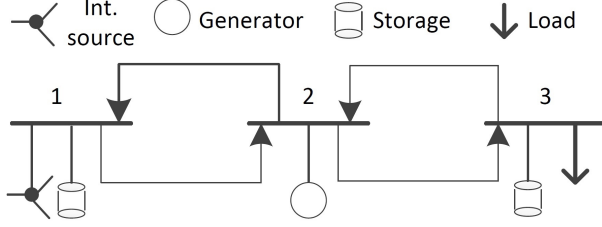[1]As long as the LD model is purely positive, e.g., equation 13 holds.

Fig. 3. The 3-bus test network of the case study. Network comprises of an intermittent source , two storage devices, a consumer with a given power load, and a generator. Branch arrows indicate power flow directions.

Figure 4 shows the generator response $e_g(2)$ to increasing uncertainty δ_1^+. The LR and LD solution show to be insensitive to changes in uncertainty, while the QS solution is sensitive to this change. The generated power $e_g(2)$ in the QS solution increases with δ_1^+ to compensate for the possible stochastic losses. This approach is therefore suitable for network operators that are risk averse to power underproduction. In Figure 4, the QS solution allocates less $e_g(2)$ than the LR and LD solutions at lower uncertainty levels. Linearized models can overapproximate the losses leading to overestimation of required power. This illustrates the benefit of the quadratic model in better tracking the losses away from the nominal point.
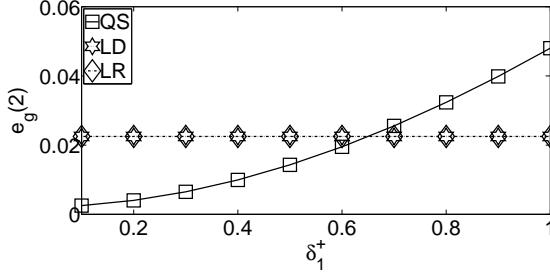

Fig. 4. Generator output $e_g(2)$ at different uncertainty levels δ_1^+. Plot contains POPF solution corresponding to the QS model (square), the LR model (diamond), and the LD model (star). Figure illustrates the LR and LD consistency with B3.

Figure 5 shows the utilization of distant and local storage devices. The QS solution yields the only uncertainty sensitive behavior B1. With increasing uncertainty, the local storage is favored over the distant one. The LD and LR are insensitive to the changes of δ_1^+.

The LR solution yields the only uncertainty attenuating behavior B2. The distant storage is favored because the power losses attenuate the uncertainty. The cost sensitivity analysis in Section V-B further supports this observation.

B. Effects of weights

In this section, the introduced optimization problem was solved for $\Gamma_l \in \{1, 2, 3, 4, 5\}$. The level of δ_1^+ was fixed at $\sqrt{0.5}$. The results are summarized in Figures 6 and 7.

Figure 6 shows the generator response $e_g(2)$ to the changes of Γ_l. Only the QS case shows sensitivity, while the LR and LD are insensitive and stay at the same level
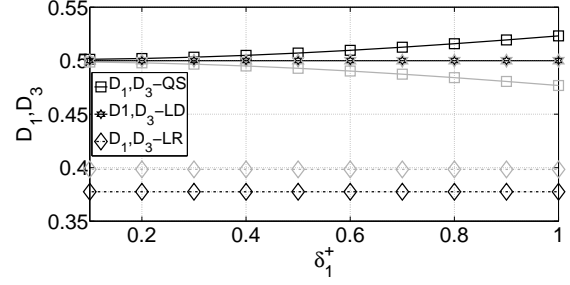

Fig. 5. Storage policies of devices at node 1 (D_1, dark line) and node 3 (D_3, light line) for different uncertainty levels δ_1^+. Plot contains POPF solution corresponding to the QS model (square), the LR model (diamond), and the LD model (star). Figure illustrates the QS model consistency with B1 and the LR model consistency with B2.

($e_g(2) = 0.0225$). As explained in IV-C, the QS model overapproximates the deterministic losses to compensate for the stochastic losses. Rising Γ_l shifts the stored power to node 1 decreasing the stochastic loss compensation. As a result, the deterministic power $e_g(2)$ decreases.

In Figure 7, the utilization of local and distant storage devices is shown. The QS and LR solutions yield the cost sensitive behavior, while the LD solution is insensitive. The LR solution shows whenever storage is more costly than losses, distant storage devices are preferred to attenuate uncertainty through lossy transmission. On the other hand, if the losses are more costly, the local storage device is favored. The break even point, where both storage devices are equally used, is between $\Gamma_l = 3$ and $\Gamma_l = 4$. These trends are consistent with the expected behavior B2.

Finally, none of the introduced loss models represent the perfect model of transmission losses under uncertainty. Each representation has advantages under specific conditions.
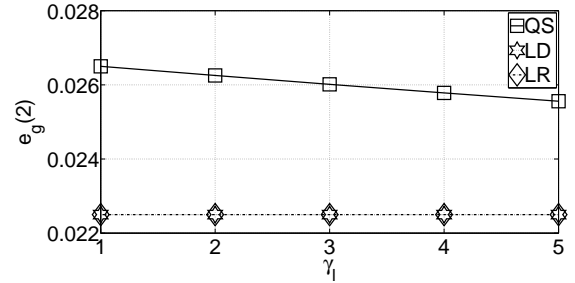

Fig. 6. Generator output $e_g(2)$ at different loss penalties Γ_l. Plot contains POPF solution corresponding to the QS model (square), the LR model (diamond), and the LD model (star).

VI. Conclusion

In this paper we extend the OPF problem under uncertainty to account for transmission losses. Given intermittent power injections, power flow balance equations are decomposed into their deterministic and uncertain parts. Three loss models with different means of modeling uncertainty are proposed together with a list of specific criteria the optimal solutions should obey. The criteria placed on the loss models
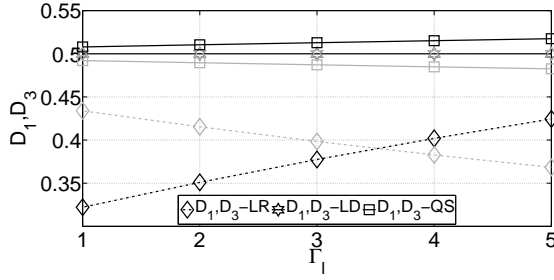
Fig. 7. Storage policies of devices at node 1 (D_1, dark line) and node 3 (D_3, light line) for different loss penalties Γ_l. Plot contains POPF solution corresponding to the QS model (square), the LR model (diamond), and the LD model (star). Figure illustrates the LR model consistency with B2.

are summarized in Table II. No one model conformed to the entire list, however, the QS model performed favorably with respect to all but two items. In addition, the QS model showed to be amenable to exact convex relaxation and required no additional information about flow directionality or nominal operation regimes. The LR model was computationally less tractable, requiring a priori information regarding power flows and the knowledge of the nominal operating point, but captured the uncertainty attenuating effect of power transmission and the insensitivity of thermal generation to changes in RES variances. The standard LD model served as a negative control in this study by confirming that the omission of uncertainty from the loss model fails to capture many of the expected behaviors.

APPENDIX A
UNCERTAINTY IMMUNIZED CONVEX POPF

The following problem is equivalent to problem POPF. In other words, the values of e_g and D_s that solve CPOPF solve the infinite number of problem formulated in the definition of POPF. Define the following composite matrices. For the set Ω, $S = diag(S_i)_{i\in\mathcal{N}}$, $h = (h_1^T, ..., h_{\mathbb{N}}^T)^T$, where $diag(S_i)_{i\in\mathcal{N}}$ denotes the block-diagonal matrix with S_i located in the i-th block.

Problem 3: CPOPF

Let Γ_g, Γ_s, and Γ_l be the penalization matrices for e_g, D_s, and L, respectively. The problem is to

Minimize $e_g^T \Gamma_g e_g + L^T \Gamma_l L + trace\left(\Gamma_s^T D_s \langle\langle\Delta\rangle\rangle D_s^T \Gamma_s\right)$,

subject to, $\forall b \in \mathcal{B}$ and $\forall n, m \in \mathcal{N}$,

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} D_s = Y_1^T S \qquad (20)$$

$$Y_1^T h \leqq \begin{pmatrix} D_s^+ \\ -D_s^- \end{pmatrix} \qquad (21)$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} D_f = Y_2^T S \qquad (22)$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} e_f + Y_2^T h \leqq \begin{pmatrix} f^+ \\ -f^- \end{pmatrix} \qquad (23)$$

$Y_1, Y_2 \geqq 0$, *element-wise*

$$e_g^- \leqq e_g \leqq e_g^+,$$

$$0 = p_d(n) + \sum_{b\in(I_n)} e_f(b) - e_l(b) - \sum_{b\in(O_n)} e_f(b), \qquad (24)$$

$$0 = P_u(n) + \sum_{b\in(I_n)} D_f(b) - D_l(b) - \sum_{b\in(O_n)} D_f(b), \qquad (25)$$

$$e_f(b) = \frac{1}{x(b)} \left(e_\theta(n) - e_\theta(m)\right),$$

$$D_f(b) = \frac{1}{x(b)} \left(D_\Theta(n) - D_\Theta(m)\right).$$

$\blacksquare$

Variables Y_1, Y_2 represent dual variables with corresponding dimensions. Note, h and S represent boundaries of the compact set Ω. For the full derivation, see [6, 16].

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