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Voltage and Reactive Power Control Using Approximate Stochastic Annealing

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***Abstract*— One of the objectives of smart grids is to optimize the performance of power devices to improve energy efficiency by utilizing additional data from smart meters. In particular, this is important for the transmission and distribution network, in which approximately 7% of the total energy generated is wasted. Efficient management of voltage profiles and reactive power in power distribution systems plays an important role towards this goal. In this paper, we consider voltage and reactive power control (VVC) problem with the objective to determine the proper settings of capacitor banks and transformer taps in power distribution systems to minimize daily energy losses. Voltage constraints and operation limits constraints on transformer load tap changers (LTCs) and shunt capacitors (SCs) are considered in our model. We propose a stochastic search algorithm called Approximate Stochastic Annealing (ASA) for solving this VVC problem. The algorithm searches the optimal control schedule by randomly sampling from a sequence of probability distributions over the space of all possible settings of LTCs and SCs. A Lagrangian Relaxation-Dynamic Programming (LR-DP) algorithm is also proposed to obtain upper and lower bounds on the performance of the optimal solution. Our testing results on the well-known PG&E 69-bus distribution network indicate that the ASA algorithm may yield solutions very close to optimum within a modest amount of computational time.**

NOMENCLATURE

N_{sc}	number of SCs in a power distribution system
N_{ltc}	number of LTCs in a power distribution system
x^{sc}	an $N_{sc} \times T$ matrix with its $(i,t)th$ entry $x^{sc}(i,t)$ representing the setting of the ith SC at time t
x^{ltc}	an $N_{ltc} \times T$ matrix with its $(i,t)th$ entry $x^{ltc}(i,t)$ representing the setting of the ith LTC at time t
x	the setting of SCs and LTCs of a power distribution system, i.e. (x^{sc}, x^{ltc})
x_t	the setting of SCs and LTCs at time t
L	the number of branch lines in a power distribution system
N_{node}	the numbers of nodes in a power distribution system
$P(x_t)$	function to calculate the power loss in a power distribution system given x_t
V	an $N_{node} \times T$ matrix with $V(i,t)$ representing the voltage of the ith load position at time t
I	an $L \times T$ matrix with $I(i,t)$ representing the current of the ith branch at time t
$\bar{\Delta}^{ltc}$	the operation limit of an LTC
$\bar{\Delta}^{sc}$	the operation limit of an SC
$\overline{V}$	the upper bound of the voltage of a node in a distribution system
$\underline{V}$	the lower bound of the voltage of a node in a distribution system

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I. INTRODUCTION

Voltage and reactive power control (VVC) is an important practical problem in smart grid, especially in power distribution systems. The current practice of VVC is focused on maintaining the voltage profiles from violating voltage constraints and pays little attention to the minimization of power losses. Because of insufficient measurements of system parameters in traditional distribution systems, it is not realistic to have accurate state estimations, making the development of an efficient algorithm for minimizing power losses difficult. However, with a large number of smart meters being installed, accurate state estimation or even state forecasting now becomes feasible. Therefore, designing efficient VVC algorithms to minimize power losses becomes an important issue. In smart grid, one of the objectives of VVC is to determine the transformer tap positions and on/off states of capacitors over a certain time period to reduce power losses in power distribution systems. The transformer load tap changers (LTCs) and circuit breakers for shunt capacitors (SCs) are expensive devices, and cannot be operated frequently. To increase the life expectancy and save maintenance costs, the number of operations on these devices are not allowed to exceed some pre-specified operations limits within a single day. So optimal operations schedules and optimal settings of LTCs and SCs need to be computed to minimize the power losses within 24 hours given the day-ahead load prediction.

The VVC problem is usually formulated as a mixed-integer nonlinear programming problem (MINLP), where tap positions of LTCs and on/off states of SCs are modeled as discrete variables while other variables (e.g., voltages and power flows) are continuous. Many algorithms have been proposed for solving VVC problems. Dynamic programming approaches were proposed in [4]–[6], [9], [14]. A standard dynamic-programming algorithm was used in [4] for solving a problem with 4 capacitors. However, the complexity of the algorithm increases exponentially with respect the number of

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capacitors due to the well-known "curse of dimensionality". To address this issue, a heuristic method was used to reduce the state space and action space in [9] and [14], whereas in [5] and [6], artificial neural network was used to play a similar role. In [13], the VVC problem was decomposed into two sub-problems to compute the optimal setting of LTCs and SCs separately. These two sub-problems were solved by dynamic programming and fuzzy control algorithms, where a coordination algorithm between those two sub-problems was proposed and a heuristic procedure was used to reduce the solution space.

Stochastic search algorithms are also popular for solving VVC problems. Genetic algorithm was used in [1] and [8] and yielded promising results; however, operations limits of transformers and capacitors were not considered. The simulated annealing algorithm was proposed in [10]. The algorithm was tested on a distribution system with 1 LTC and 11 capacitors, and test results indicated improvement of simulated annealing over conventional methods. Interior Point Method (IPM) was proposed in [11], [12], where discrete control variables were relaxed to form a problem with only continuous problems. The IPM algorithm is a very efficient algorithm;. however, relaxing the discrete requirement of variables may compromise the optimality of the solution. Additionally, several heuristic methods were used to reduce the complexity of the VVC problem. A time-interval based approach was proposed in [8], where the genetic algorithm was used to partition the 24-hour scheduling horizon into several intervals, and switching operation was not allowed within a partition. Similarly, the work in [3] used heuristics to build priority lists of operations hours, based on which a simplified VVC model was formulated and solved. These heuristics were effective in both reducing the computational time and obtaining acceptable results.

In this paper, we present a simulation-based algorithm called Approximate Stochastic Annealing (ASA) for solving the VVC problem. The ASA algorithm was initially proposed in [7] for solving finite horizon Markov Decision Processes (MDPs). ASA iteratively samples candidate solutions from a probability distribution over the set of all admissible solutions, and then the distribution function is modified using a Boltzmann scheme based on the performance of sampled solutions. The hope is that the probability distribution will gradually converge to a degenerate distribution assigning unit probability mass to the optimal solution. The ASA algorithm can be viewed as a stochastic search algorithm; however, compared with some well-known stochastic search algorithms such as genetic algorithm and particle swarm optimization, ASA is shown to converge to the global optimal solution [7].

In most previous work, the performance of proposed algorithms was either compared with those of naive approaches (pure heuristic) or those of existing optimization algorithm. There is few investigation on how close the computational result of the proposed algorithm is to the exact optimal solution. Because of the complexity of VVC, it is difficult to get the exact optimal solution. In this work, we propose an LR-DP approach, which combines lagrangian relaxation and dynamic programming, to get lower and upper bounds on the performance of the optimal solution. By relaxing the operations limits, we get a dual problem which can be solved using a combination of the sub-gradient method and the dynamic programming algorithm. The solution of the dual problem can be viewed as the lower bound of the original VVC problem, which can be compared with the result of ASA to measure the effectiveness of ASA.

The rest of the paper is organized as follows. We provide a detailed formulation of the voltage and reactive power control problem in section II. In section III, we briefly describe the ASA algorithm for the VVC problem. We describe the LR-DP algorithm for VVC in section IV. We provide a numerical example to test the performances of ASA and LR-DP in section V.

II. PROBLEM FORMULATION

A. Objective function

The objective of the VVC problem in this paper is to find the optimal settings of shunt capacitors and transformer load tap changers to minimize power losses in power distribution systems, which can be formulated as follows.

$$\min_{x} \sum_{t=1}^{T} \sum_{l=1}^{L} P(x_t) \tag{1}$$

where $P(x_t) = |I(l,t)|^2 R_l$. $I(l,t)$ denotes the power flow on branch l at time t , and can be calculated given the value of x_t using power flow equations.

B. Constraints

1) Power Flow Equations: The power flows, voltages, reactive power and reactive power should satisfy a set of power flow equations, which can be represented abstractly in the form of Equation (2).

$$g(V, I, x) = 0 \tag{2}$$

2) Voltage Constraints: In a power distribution system, the voltage level of each node should be within a given range to avoid damages to electric devices and appliances. Theses constraints are modeled in Equation (3) below.

$$\underline{V} \leq V(i,t) \leq \overline{V} \qquad \forall i \in \{1, \ldots, N_{node}\}, t \in \{1, \ldots T\} \tag{3}$$

3) Operation Limits on LTCs and SCs: In a power distribution system, there are limitations on how frequently LTCs and SCs can be operated within a finite planning horizon (24 hours in this case). Our VVC model sets maximum numbers of operations for LTCs and SCs. The number of switching operations of a LTC at time t is the difference between tap positions at time t and $t-1$, and the total number of switching operations is the sum of switching operations of all time periods. This paper models these constraints in Equations (4) and (5) below.

$$\sum_{t=1}^{T} |x^{ltc}(i,t) - x^{ltc}(i_{ltc}, t-1)| \leq \bar{\Delta}^{ltc} \tag{4}$$

$$\sum_{t=1}^{T} |x^{sc}(i,t) - x^{sc}(i, t-1)| \leq \bar{\Delta}^{sc} \tag{5}$$

III. Solution Algorithm: Approximate Stochastic Annealing

The ASA algorithm is a simulation-based optimization algorithm developed in [7], and can be used for solving finite horizon MDPs. It was proven in [7] that under some assumptions, the ASA algorithm converges to the global optimal solution.

For each time period t, we let $q^{sc}(i,t)$ denote the probability of switching the ith capacitor on, and $q^{ltc}(i,j,t)$ denote the probability of moving the transformer tap of the ith LTC to the jth position. Thus two stochastic matrices are constructed for SCs and LTCs respectively. The dimension of matrix q^{sc} for SCs is $N_{SC} \times T$, and we denote its $(i,t)th$ element by $q^{sc}(i,t)$. The dimension of matrix q_{ltc} for $LTCs$ is $N_{LTC} \times N_{tap} \times T$, and we denote its $(i,j,t)th$ element by $q^{ltc}(i,j,t)$. In the ASA algorithm, a number of settings of LTCs and SCs are sampled according to these two stochastic matrices and the objective function value of each setting is calculated. These stochastic matrices are iteratively updated based on the performance of sampled settings. Since some of these sampled settings may not satisfy all constraints, penalty terms are added to the objective function. The augmented objective function thus becomes:

$$\begin{aligned} h(x) &= \sum_{t=1}^{T}\sum_{l=1}^{L} P_{t,l}(x_t) \\ &+ \eta_{ltc} \sum_{i=1}^{N_{ltc}} \max(\sum_{t=1}^{T} |x^{ltc}(i,t) - x^{ltc}(i,t-1)| - \bar{\Delta}^{ltc}, 0) \\ &+ \eta_{sc} \sum_{i=1}^{N_{sc}} \max(\sum_{t=1}^{T} |x^{sc}(i,t) - x^{sc}(i,t-1)| - \bar{\Delta}^{sc}, 0) \\ &+ \eta_{v} \sum_{i=1}^{N_{node}} \sum_{t=1}^{T} (\max(V(i,t) - \bar{V}, 0) + \max(\underline{V} - V(i,t), 0)), \end{aligned} \tag{6}$$

where the second term on the right hand side is the penalty on the violation of operation limits on LTCs, the third term is the penalty on the violation of operation limits on SCs, and the fourth term is the penalty on the violation of voltage constraints. η_{ltc}, η_{sc} and η_v are penalty coefficients. The ASA algorithm in this work is given below.

1) Specify a non-negative decreasing sequence $\{T_k\}$ and a sequence $\{\beta_k\}$ satisfying $0 \leq \beta_k \leq 1\ \forall k$. Specify two non-negative decreasing sequences $\{\alpha_k^{ltc}\}$ and $\{\alpha_k^{sc}\}$ satisfying $0 \leq \alpha_k^{ltc}, \alpha_k^{sc} \leq 1\ \forall k$. Select a sample size sequence $\{N_k\}$. Initialize $q_{sc,0}(i,t) = 0.5$, $q_{ltc,0}(i,j,t) = 1/N_{tap}$, $\forall i,j,t$.
2) Sample N_k settings of SCs and LTCs from matrices q_k^{sc} and q_k^{ltc} with probability $1-\beta_k$ and from q_0^{sc} and q_0^{ltc} with probability β_k. These settings form a set $X_k := \{x^1, x^2, \ldots, x^{N_k}\}$. Calculate the probability mass function $\hat{\phi}(x; q_k^{sc}, q_k^{ltc})$ for each setting x as follows:

$$\phi_{sc,t}(x; q_{sc}) = \prod_{i=1}^{N_{sc}} q_{sc}(i,t)^{x^{sc}(i,t)} (1 - q_{sc}(i,t))^{1-x^{sc}(i,t)}$$

$$\phi_{ltc,t}(x; q_{ltc}) = \prod_{i=1}^{N_{ltc}} q_{ltc}(i, x^{ltc}(i,t), t)$$

$$\phi(x; q_{sc}, q_{ltc}) = \prod_{t=1}^{T} \phi_{sc,t}(x; q_{sc}) \phi_{ltc,t}(x; q_{ltc})$$

$$\begin{aligned} \hat{\phi}(x; q_{sc,k}, q_{ltc,k}) =& (1-\beta_k)\phi(x; q_{sc,k}, q_{ltc,k}) \\ &+ \beta_k \phi(x; q_{sc,0}, q_{ltc,0}) \end{aligned}$$

3) For each setting x, calculate the the value of the augmented objective function $h(x)$ in Equation (6).
4) Let
$$f(x) = e^{-h(x)/T_k} \hat{\phi}^{-1}(x; q_{sc,k}, q_{ltc,k}).$$
Update the matrices q_k^{sc} and q_k^{ltc} by
$$\begin{aligned} q_{k+1}^{sc}(i,t) =& \alpha_k^{sc} \frac{\sum_{x \in X_k} f(x) I\{x^{sc}(i,t) = 1\}}{\sum_{x \in x_k} f(x)}, \\ &+ (1-\alpha_k^{sc}) q_k^{sc}(i,t) \end{aligned} \tag{7}$$
$$\begin{aligned} q_{k+1}^{ltc}(i,j,t) =& \alpha_k^{ltc} \frac{\sum_{x \in X_k} f(x) I\{x^{ltc}(i,t) = j\}}{\sum_{x \in x_k} f(x)}. \\ &+ (1-\alpha_k^{ltc}) q_k^{ltc}(i,j,t) \end{aligned} \tag{8}$$
5) If a stopping rule is satisfied, then stop; otherwise, set $k = k+1$ and go to step 1.

Note that some parameters should satisfy the following conditions to guarantee the asymptotic convergence property of the ASA algorithm:

1) $\alpha_k^{sc} > 0\, \forall k$, $\sum_{k=0}^{\infty} \alpha_k^{sc} = \infty$ and $\sum_{k=0}^{\infty} (\alpha_k^{sc})^2 < \infty$;
2) $\alpha_k^{ltc} > 0\, \forall k$, $\sum_{k=0}^{\infty} \alpha_k^{ltc} = \infty$ and $\sum_{k=0}^{\infty} (\alpha_k^{ltc})^2 < \infty$;
3) $T_k \to 0$ as $k \to \infty$;
4) $N_k \beta_k \to \infty$ as $k \to \infty$.

IV. Solution Algorithm: Lagrangian Relaxation-Dynamic Programming

Dynamic programming has been used for solving the VVC problem in [4] and [9]. As noted in the previous work, the complexity of the dynamic programming algorithm grows exponentially with respect to the planning horizon and problem size, and suffers from the curse of dimensionality. By relaxing the operation limits constraints, we can, to some extent, reduce the computational complexity. We can relax the operation limits constraints in Equations (4) and (5) to get a dual problem with objective function as follows:

$$
\begin{aligned}
D(\mu_{ltc}, \lambda_{sc}, x) = &\sum_{t=1}^{T}\{\sum_{l=1}^{L} P_{t,l}(x_t) \\
&+ \sum_{i=1}^{N_{ltc}} \mu_{ltc,i}|x^{ltc}(i,t) - x^{ltc}(i,t-1)| \\
&+ \sum_{i=1}^{N_{sc}} \lambda_{sc}|x^{sc}(i,t) - x^{sc}(i,t-1)|\} \\
&- \sum_{i=1}^{N_{ltc}} \mu_{ltc,i}\bar{\Delta}^{ltc} - \sum_{i=1}^{N_{sc}} \lambda_{sc,i}\bar{\Delta}^{sc}.
\end{aligned} \tag{9}
$$

where $\mu_{ltc,i}, \lambda_{sc}$ are non-negative lagrangian multipliers. Then the dual problem is $\max_{\mu_{ltc},\lambda_{sc}} \min_x D(\mu_{ltc}, \lambda_{sc}, x)$ subject to constraints in Equations (2) and (3). The dual problem is convex and can be solved by the sub-gradient method. We can iteratively update the values of μ_{ltc} and λ_{sc} by solving $\min_x D(\mu_{ltc}, \lambda_{sc}, x)$ in each iteration. The optimization problem $\min_x D(\mu_{ltc}, \lambda_{sc}, x)$ can be formulated as a finite horizon MDP (X, A, R, P, T), where

- $X = \{x_t\}$ is the set of all possible settings of LTCs and SCs at time t;
- $A = \{a := (a_{ltc}, a_{sc})\}$ is the set of all feasible operations on the LTCs and SCs;
- $R = \sum_{l=1}^{L} Pl_{t,l}(x_t) + \sum_{i=1}^{N_{ltc}} \mu_{ltc,i}|x^{ltc}(i_{ltc}, t) - x^{ltc}(i_{ltc}, t-1)| + \sum_{i=1}^{N_{sc}} \lambda_{sc}|x^{sc}(i_{sc}, t) - x^{sc}(i_{sc}, t-1)|)$;
- $x_t = x_{t-1} + a$, i.e. $P(x_t + a|x_t) = 1$;
- T is a pre-defined study horizon, 24 hours in this case.
- Final cost is $-\sum_{t=1}^{N_{ltc}} \mu_{ltc,i}\bar{\Delta}^{ltc} - \sum_{t=1}^{N_{sc}} \lambda_{sc,i}\bar{\Delta}^{sc}$.

The standard dynamic programming method with complexity $O(|X||A|T)$ can be applied for solving this MDP. Note that an action a is feasible only if the setting $x_t = x_{t-1} + a$ is feasible, i.e. voltage constraints should be satisfied at state x_t. The LR-DP algorithm is given as follows:

1) Initialize μ_{ltc}, λ_{sc}, and set iteration number $k = 1$
2) Solve the optimization problem $\min_x D(\mu_{ltc}, \lambda_{sc}, x)$ and set $x^* = \text{argmin}_x D(\mu_{ltc}, \lambda_{sc}, x)$
3) Calculate the primal value $\sum_{t=1}^{T} \sum_{l=1}^{L} P_{t,l}(x^*)$ if x^* satisfy the operation limits constraints. Let y_{primal} be the best primal value obtained thus far.
4) Calculate the duality gap
$$\epsilon = \frac{y_{primal} - D(\mu_{ltc}, \lambda_{sc}, x^*)}{y_{primal}}$$
If the duality gap is less than a given threshold, return x with respect to the best primal value; otherwise, got to step 5 .
5) update the value of μ_{ltc}, λ_{sc} as below:
$$\delta^{ltc}_{i_{ltc}} = \sum_{t=1}^{T} |x^{ltc}(i_{ltc}, t) - x^{ltc}(i_{ltc}, t-1)| - \bar{\Delta}^{ltc}$$
$$\delta^{sc}_{i_{sc}} = \sum_{t=1}^{T} |x^{sc}(i_{sc}, t) - x^{sc}(i_{sc}, t-1)| - \bar{\Delta}^{sc}$$
Let $\delta^{ltc} = \{\delta^{ltc}_{i_{ltc}}\}$, $\delta^{sc} = \{\delta^{sc}_{i_{sc}}\}$ and $\delta = (\delta^{ltc}, \delta^{sc})$
$$\mu_{ltc}(i_{ltc}) := \max(\mu_{ltc}(i_{ltc}) + \alpha_k \frac{\delta^{ltc}_{i_{ltc}}}{||\delta||},\ 0)$$
$$\mu_{sc}(i_{sc}) := \max(\mu_{sc}(i_{sc}) + \alpha_k \frac{\delta^{sc}_{i_{sc}}}{||\delta||},\ 0),$$
where $\{\alpha_k\}$ is a sequence satisfying $\sum_{k=1}^{\infty} \alpha_k = \infty$ and $\sum_{k=1}^{\infty} \alpha_k^2 < \infty$. Go to step 2.

We can observe that the computational time in step 2 grows exponentially with respect to the number of LTCs and SCs, and may become prohibitive when the system under study is very large. However, in some realistic systems, the numbers of SCs and LTCs are small, which makes the LR-DP algorithm an effective method for solving the VVC problem. Moreover, unlike some stochastic search algorithms such as genetic algorithm and particle swarm optimization, which have difficulties in evaluating the accuracy of computational results, the LR-DP algorithm provides a lower bound on the optimal value and a duality gap, which can be used for evaluating the performance of the algorithm.

V. Numerical Results

We use the well-known PG&E 69-bus distribution network with 69 buses, 68 branches and 48 loads to illustrate the performance of both the ASA and LR-DP algorithms. The base power and base voltage are set to 10MVA and 12.66 kV, respectively. The data of branch impedance and maximum daily active and reactive powers for all loads can be found in [2]. The diagram of the system is given in Figure 1. Seven different load profiles in Figure 2 are assigned to different nodes. We assume that there are 10 capacitors installed on buses 9, 19, 31, 37, 40, 47, 52, 55, 57 and 65 as in [11] with size 0.3MVar, and that a LTC with tap setting $1 \pm 0.02 \times 3$ is installed at bus 1.

A. Implementation of the ASA algorithm

The ASA algorithm is implemented using MATLAB 7.10.0 on a computer with an Intel Core™2 CPU (2.40GHz × 2), 2.0GB RAM and Windows 7 OS. The smoothing parameters are set to $\alpha_k^{sc} = 1/(k + 100)^{0.51}$ and $\alpha_k^{ltc} = 1/(k + 100)^{0.6}$. The annealing schedule is set to $T_k = c/(k + 1)^{0.5}$, where c is the positive difference between the minimum value and median of all objective values calculated in the step 3 of the ASA algorithm. The penalty coefficient for violating voltage constraints is set to $\eta_v = 0.05$. The penalty coefficients for violating operation limits are set to $\eta_{ltc} = 1$ and $\eta_{sc} = 3$. The sample size is set to $N_k = \max(100, \sqrt{k})$. The algorithm stops when either $k = 10000$ or the following equations are satisfied:

$$\min(1 - q_{sc,k}(i_{sc}, t), q_{sc,k}(i_{sc}, t)) < 0.001 \forall i, t,$$

$$\min(1 - q_{ltc,k}(i_{ltc}, j, t), q_{sc,k}(i_{ltc}, j, t)) < 0.001 \forall i, t,$$

which indicate that all elements in these two stochastic are getting very close to 0 or 1.

B. Implementation of LR-DP algorithm

The LR-DP algorithm is implemented using MATLAB 7.10.0 on the same platform as that for the ASA algorithm. The initial lagrangian multipliers are all set to 1. The step-size is set to $\alpha_k = 20/(1+k^{0.75})$. The algorithm stops when the iteration number k exceeds 100 or when the duality gap $\epsilon < 0.5\%$.

C. Result

We divide our experiments into two cases. Case 1: a constant load model is used and electricity loads are independent of voltage profiles; case 2: electricity loads are correlated with voltage profiles and the well-known ZIP load model in [15] is used to model the correlation between loads and voltage profiles.

The computational results of the ASA algorithm are given in Table I for case 1 and Table III for case 2. We can see the convergence of the ASA algorithm in Figure 3. The figure plots the power loss of the current best sampled candidate solution as a function of the number of algorithm iterations. Power losses and CPU times in Table I and Table III are the average of 10 independent replication runs of the ASA algorithm. The computational results of LR-DP algorithm are provided in Table II for case 1 and Table IV for case 2. Unlike ASA algorithm, the LR-DP algorithm is deterministic, so a single run is performed.

In case 1, we observe from results of both ASA and LR-DP that the transformer taps are at positions leading to high voltage profiles. This is consistent with our intuition that given constant loads, higher voltage profile brings lower electricity current, which results in less power loss. In case 2, we observe different movements of transformer taps from those in case 1: under ZIP load model, an increase in voltage level may result in more power consumption, which may bring more power loss. Thus in case 2, the optimal tap position my not be the highest feasible position and may vary as electricity loads fluctuate.

From our computational results, we can observe that, generally, the power losses increase as the operation limits decreases. From Table II and Table IV, we observe that the duality gap is less than 0.2%, which indicates that the computational result of LR-DP algorithm is very close to the optimal value. Additionally, we can see that the LR-DP algorithm is time consuming. The computational results of the ASA algorithm are compared with that of the LR-DP algorithm. We observe that the performance of the ASA algorithm is not as good as LR-DP algorithm. However, the differences between calculated power losses are less than 0.4%, which is very small. Moreover, the ASA algorithm is significantly faster than the LR-DP algorithm; the algorithm takes much less time to compute the result with only very little compromise on the quality of solution.

VI. Conclusion

A novel Approximate Stochastic Annealing algorithm is proposed for solving the voltage and reactive power control problem with operation limits on shunt capacitors and LTCs. The proposed ASA algorithm iteratively samples settings of SCs and LTCs according to a probability distribution, which is iteratively updated using a Boltzmann selection scheme. A Lagrangian Relaxation-Dynamic Programming algorithm is also used for solving the problem, and lower and upper bounds of the optimal solution are obtained. Test results on a well known PG&E 69 bus system indicate that the ASA algorithm can solve the problem very quickly with a little compromise on the optimality of the solution. Therefore, it is concluded that the ASA algorithm is an effective method for solving the voltage and reactive power control problem.

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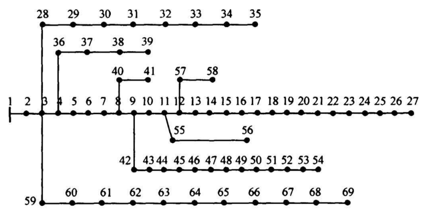

Fig. 1. Diagram of PG&E 69-bus System

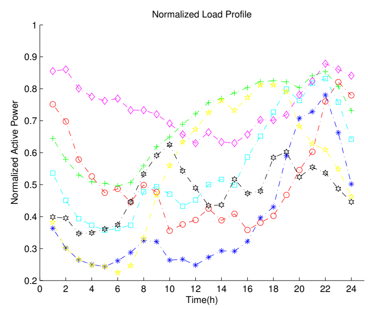

Fig. 2. Load Profiles

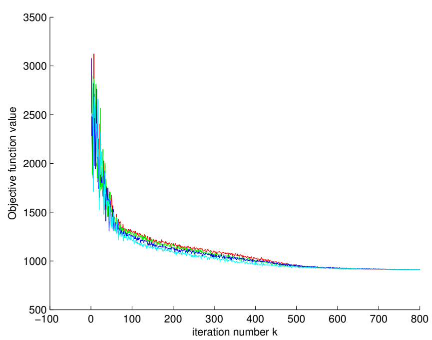

Fig. 3. Trajectory of objective function value of ASA

TABLE I

COMPUTING RESULT OF THE ASA ALGORITHM: CASE 1

Operation Limits LTC	SC	Power Losses (kWh)	CPU time (s)
4	2	901.09	1003s
4	3	899.29	1093s
4	4	898.33	1206s
4	5	898.01	1262s

TABLE II

COMPUTING RESULT OF THE LR-DP ALGORITHM: CASE 1

Operation Limits LTC	SC	Power Losses (kWh)	CPU time (s)	Duality Gap
4	2	897.83	25395	0.15%
4	3	897.09	12693	0.03%
4	4	897.09	12684	0.06%
4	4	896.82	6346	0

TABLE III

COMPUTING RESULT OF THE ASA ALGORITHM: CASE 2

Operation Limits LTC	SC	Power Losses (kWh)	CPU time (s)
2	2	913.18	1585
4	2	912.44	1490
4	4	911.30	1290
8	4	910.64	1084

TABLE IV

COMPUTING RESULT OF THE LR-DP ALGORITHM: CASE 2

Operation Limits LTC	SC	Power Losses (kWh)	CPU time (s)	Duality Gap
2	2	911.06	12442	0.09%
4	2	911.00	18914	0.15%
4	4	910.15	12277	0.10%
8	4	910.07	12214	0.09%

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