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Distributed Adaptive Leader-following control for multi-agent multi-degree manipulators with Finite-Time guarantees

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***Abstract*— A robust distributed adaptive leader-following control for multi-degree-of-freedom (multi-DOF) robot manipulator-type agents is proposed to guarantee finite-time convergence for leader-following tracking and parameter estimation via agent-based estimation and control algorithms. The dynamics of each manipulator agent system of *n* degrees including the leader agent are assumed unknown. For a specific leader-following network Laplacian, the agents' position, velocity and some switched control information can be fed back to the communication network. In contrast to the current multi-agent literature for robotic manipulators, the proposed approach does *not* require *a priori* information of the leader's joint velocity and acceleration to be available to *all* agents due to the use of agent-based robust adaptive control elements. Due to the multi-DOF character of each agent, matrix theoretical results related to M-matrix theory used for multi-agent systems needs to be extended to the multi-degree context in contrast to recent scalar double integrator results. A simulation example of two-degree of freedom manipulators exemplifies the effectiveness of the approach.**

I. INTRODUCTION

Distributed control of multi-agent systems have sparked a substantial interest due to its significantly broad applications in many fields such as swarming, flocking, rendezvous and formation in mobile robots, unmanned aerial vehicles (UAV) and multi-manipulators. Prominent work shows that consensus control of multi-agent systems involves not just single-integrator and double-integrator dynamics type systems [1], [2], [3], [4], [5], [6], [7], [8], [9] but also a group of interconnected multiple degree of freedom (multi-DOF) systems [4], [10], [11], [12], [13]. The leader-following distributed consensus multi-agent problem for multi-manipulators saves the computational effort and simplifies the control implementation [13]. The field of cooperative control of multiple manipulators has introduced a distributed and cooperative control structure different from the centralised [14] or a pure master-slave structure [15].

In this paper, leader-following distributed control of robotic manipulators of n degrees-of-freedom acting as agents is considered. In particular, this paper considers finite-time convergence for synchronization between leader and follower, but also for parameter adaptation. According to Wang and Xiao in [16], [5], finite-time consensus allows better disturbance rejection, enhances robustness against uncertainties and increase control accuracy [16].

Recent work on cooperative control of multi-manipulator systems has advanced from scalar [1], [2], [4] to multi-degree-of-freedom agents [10], [11], [12]. The use of neural-networks [10] to estimate the agent's nonlinearities showed to be beneficial to aid the network consensus, which provides exponential convergence and ultimate boundedness guarantees of the synchronization error. The work in [11] requires each agent to know the leader's joint velocity. In contrast, work in [4] demonstrates the finite-time consensus reaching of double-integrator systems and multi-robot systems, in particular, for a leader-following objective. Here the multi-robot systems are of single DOF in nature and each agent requires its own and its neighbors' mass/inertia parameters, which simplifies the construction of the control law and the stability analysis. Another consensus control algorithm [12] introduces a constant position demand or with the requirement for enhanced synchronization error information of not only direct neighbors. This introduces a 'two-hop neighbor' information. This information is in addition to the requirement for the leader's initial joint position by *all* agents [12].

In contrast to recent work, it is of interest of this paper to propose a distributed, adaptive, finite-time leader-following consensus control algorithm for a robotic manipulator multi-agent system, extending recent results [17] for scalar agents to the context of multi-degree-of-freedom agents. Leader information (e.g. velocity) for agents which are not connected to the leader is avoided. To allow for this extension, matrix theory [18] usually developed for scalar agent and leader systems (e.g. [19]) has to be extended to the context of multi-degree-of-freedom agents. This facilitates the formulation of a distributed controller for multi-degree-of-freedom systems and the stability analysis, e.g. the construction of a Lyapunov function suited to this context. The control law provides finite-time convergence of the synchronization error and an adaptive parameter estimation error: This is based on an extension of [17], where also strong inspiration is taken from [4]. However, it is to note here that the leader provides information only to particularly pinned agents, while agents obtain information only from their neighbors in terms of position, velocity and a switched control component of the neighbor.

The next section introduces a generic communication network concept, necessary to define neighboring agents and the leader-agent communication.

II. LEADER - AGENT COMMUNICATION STRUCTURE

Consider a directed *tree* $\mathcal{G} = (\mathbb{V}, \mathbb{E})$ with nonempty finite set of N nodes $\mathbb{V} = \{v_1, \ldots, v_i, \ldots, v_N\}$ where node i represents the i-th agent. The defined graph is strongly connected and fixed consisting of directed edges or arcs $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ with no repeated edges and no self loops $(v_i, v_i) \notin \mathbb{E}, \forall i$. The connectivity matrix is denoted as $A = [a_{ij}]$ with $a_{ij} > 0$. The in-degree matrix is a diagonal matrix $D = [d_i]$ with $d_i = \sum_{j \neq i} a_{ij}$ the weighted in-degree node i (i.e. i-th row sum of A). The graph Laplacian matrix which is defined as $L = D - A,\ L = [l_{ij}],\ i, j = 1, \cdots, N,$ has all row sums equal to zero. The connectivity matrix A and L are irreducible [1], [19]. The leader communication

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is again directed from leader (node 0) to agent manipulator only, which is identified through the pinning gain $b_i \geq 0$. Thus, in case, an agent i, $(0 < i \leq N)$, is pinned, then $b_i > 0$. Thus, $b_i \neq 0$ if and only if there exists an arc from the leader node to the i-th node in $\mathcal{G}$.

III. Manipulator Dynamics

A. Agent Dynamics

We assume the general structure of the robot dynamics of each agent [20] as:

$$M_i(q_i)\ddot{q}_i + c_i(q_i, \dot{q}_i) + G_i(q_i) = \tau_i \tag{1}$$

where $q_i = q_i(t), \dot{q}_i = \dot{q}_i(t), \ddot{q}_i = \ddot{q}_i(t) \in \mathbb{R}^n$ are the robot arm joint position, velocity and acceleration vectors respectively; $\tau_i \in \mathbb{R}^n$, the input torque vector of the i-th manipulator; $M_i(q_i) \in \mathbb{R}^{n\times n}$ and $M_i(q_i) > 0$, is the inertia matrix, a function of the n joint positions q_i, $c_i(q_i, \dot{q}_i) \in \mathbb{R}^n$ which represents the Coriolis/centrifugal torque, viscous, and nonlinear damping. $G_i(q_i) \in \mathbb{R}^n$ is the gravity torque vector. Several essential properties for (1) facilitate the distributed adaptive motion synchronisation control system design:

Property 1: The left hand side of (1) can be linearly parameterised as such,

$$M_i(q_i)\ddot{q}_i + V_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \phi_i(q_i, \dot{q}_i, \ddot{q}_i)\Theta_i \tag{2}$$

where $\Theta_i \in \mathbb{R}^l$ is the system parameter vector containing l parameters to be estimated, $\phi_i(q_i, \dot{q}_i, \ddot{q}_i) \in \mathbb{R}^{n\times l}$ is the known dynamic regression matrix [22]. The Corriolis/centrifugal matrix and the gravity matrix in the left hand side of (1) can be also linearly parameterised as such,

$$c_i(q_i, \dot{q}_i) + G_i(q_i) = V_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \phi_{vg_i}(q_i, \dot{q}_i)\Theta_i \tag{3}$$

Property 2: The inertia matrix $M_i(q_i)$ is symmetric and positive definite, satisfying the following inequalities:

$$c_1\|\xi\|^2 \leq \xi^T M_i(q_i)\xi \leq c_2\|\xi\|^2, \forall \xi \in \mathbb{R}^m, \tag{4}$$

where c_1 and c_2 are known positive constants.
The regression matrix, ϕ_i is given in Property 1. It has the acceleration as argument. Note that in our proposed adaptive control algorithm, the regression matrix will not use joint acceleration unlike in [23]. This is inspired by [20] where similar approaches are used to avoid acceleration measurements.

Denote the following states for joint variables for each agent manipulator,

$$q_i = q_{1i}, \quad \dot{q}_i = \dot{q}_{1i} = q_{2i} \tag{5}$$

where $q_{1i} \in \mathbb{R}^n$ and $q_{2i} \in \mathbb{R}^n$ are the agent manipulator's joint position and velocity respectively. Then, express the agent manipulator dynamics in (1) in a Brunovsky form,

$$\dot{q}_{1i} = q_{2i},\ \dot{q}_{2i} = M_i^{-1}\left(-V_i(q_{1i}, q_{2i})q_{2i} - G_i(q_{1i}) + \tau_i\right) \tag{6}$$

By the linearity-in-the-parameter assumption as stated in Property 1, (6) can be expressed as,

$$\dot{q}_{1i} = q_{2i}, \quad \dot{q}_{2i} = M_i^{-1}\left(-\phi_{vgi}\Theta_i + \tau_i\right) \tag{7}$$

where Θ_i is the agent manipulator's parameters associated with the Coriolis/centrifugal and gravity matrix to be estimated by the novel parameter estimation algorithm presented in this note.

The overall agent manipulator dynamics can be expressed as,

$$\dot{q}_1 = q_2,\ \dot{q}_2 \;=\; \mathcal{M}\left(-\Phi_{vg}\bar{\Theta} + \tau\right) \tag{8}$$

where $q_1 = [q_{11}^T, \ldots, q_{1i}^T, \ldots, q_{1N}^T]^T \in \mathbb{R}^{nN}$ and $q_2 = [q_{2i}^T, q_{21}^T, \ldots, q_{2N}^T]^T \in \mathbb{R}^{nN}$, $\tau = [\tau_1^T, \ldots, \tau_i^T, \ldots, \tau_N^T]^T \in \mathbb{R}^{nN}$, $\bar{\Theta} = [\Theta_1^T, \ldots, \Theta_i^T, \ldots, \Theta_N^T]^T \in \mathbb{R}^{lN}$, $\mathcal{M} = diag([M_1^{-1}, \ldots, M_i^{-1}, \ldots, M_N^{-1}]) \in \mathbb{R}^{nN\times nN}$ and $\Phi_{vg} = diag([\phi_{vg1}, \ldots, \phi_{vgi}, \ldots, \phi_{vgN}]) \in \mathbb{R}^{nN\times Nl}$.

B. Manipulator Dynamics of the leader

The leader manipulator satisfies the following general nonautonomous dynamics in a second order Brunovsky form,

$$\dot{q}_{1_0} = q_{2_0}, \dot{q}_{2_0} = M_0^{-1}\left(-V_0(q_{1_0}, q_{2_0})q_{2_0} - G_0(q_{1_0}) + \tau_0\right) \tag{9}$$

where $q_0 = [q_{1_0} \quad q_{2_0}]^T \in \mathbb{R}^n$ is the leader's corresponding joint position and velocity. It is assumed that the dynamics of the leader manipulator remain bounded, i.e. the leader state q_0 remains bounded.

The leader manipulator dynamics can be regarded as a command generator:

Property 3: It is assumed that the reference trajectory of the leader manipulator q_0 is at least twice continuously differentiable with time t and q_0 is sufficiently rich (SR) over any finite interval $[t, t+T]$ of the specific length $T > 0$ with respect to $\phi_i(q_0, \dot{q}_0)$, so that

$$\int_t^{t+T} \phi_i^T(q_0(\nu), \dot{q}_0(\nu))\phi(q_0(\nu), \dot{q}_0(\nu))d(\nu) > \tilde{\delta} I \tag{10}$$

for some $\tilde{\delta} > 0$.
The leader-following problem is to design a set of decentralized torque control laws τ_i for the i-th manipulator to drive each maniputlator to move in synchrony whilst following a virtual leader, i.e. $q_i = q_j = q_0$. The relevant inter-agent communication is specified in the next section.

IV. Leader-follower Consensus protocol

The control protocol proposed here is for the case of multi-agent MIMO systems instead of multi-agent SISO systems as in the authors' previous work in [17]. To solve this particular leader-following consensus problem, the synchronisation errors (position and velocity) for the i-th agent are defined as

$$e_{Ai} \;=\; \sum_{i=1}^{N} a_{ij}(q_{1j} - q_{1i}) + b_i(q_{1_0} - q_{1i}) \tag{11}$$

$$e_{Bi} \;=\; \sum_{i=1}^{N} a_{ij}(q_{2j} - q_{2i}) + b_i(q_{2_0} - q_{2i}) \tag{12}$$

The synchronization errors (11)-(12) are influenced only by their corresponding direct neighbour's dynamics whose connections depend on the graph description of L. This error (11)-(12) and a later introduced bounded switched term specific to an agent perceived by its direct neighbour can only be used by a particular agent for control purposes. The consensus error in (11) and (12) can be also expressed in terms of the overall network as

$$\mathcal{E}_A \;=\; -\left[(L+B)\otimes I_n\right](q_1 - \bar{q}_{1_0}) \tag{13}$$

$$\mathcal{E}_B \;=\; -\left[(L+B)\otimes I_n\right](q_2 - \bar{q}_{2_0}) \tag{14}$$

where $L + B \in \mathbb{R}^{N\times N}$ describes the communication topology of the leader-following multi-agent network. The pinning gains are $B = diag(b_1, \cdots, b_i, \cdots, b_N) \in \mathbb{R}^{N\times N}$, $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix, $\bar{q}_{h_0} = \underline{1}_N \otimes q_0 \in \mathbb{R}^{nN}, h \in \{1, 2\}$ (Noting that $\underline{1}_N = (1, 1, \cdots, 1_N)^T \in \mathbb{R}^N$).

Property 4: The communication topology L can be framed so that the irreducible L is upper triangular.

This property avoids loops but also introduces a specific leader-agent structure.

Suppose $\delta_h = q_h - \bar{q}_{h_0}$, $(h \in \{1,2\})$ represents the disagreement vector to be used only for analysis. Then, the synchronisation error vector $\mathcal{E}_k = [e_{k1}, e_{ki}, \ldots, e_{kN}]^T \in \mathbb{R}^{2N}, k \in \{A, B\}, \forall i$ is assumed to be bounded by

$$\|\delta_h\| \leq \|e_k\| / \underline{\sigma}((L+B)\otimes I_n), \quad k \in \{A,B\}, \quad h \in \{1,2\} \tag{15}$$

where $\underline{\sigma}(\cdot)$ denotes the minimum singular value of a matrix and $e = 0$ if and only if the nodes synchronise, i.e.

$$q_{1i}(t) = q_0(t), \quad \forall i = 1, \ldots, N \tag{16}$$

The errors (11)-(12) are utilized in the novel adaptive law proposed in this note: Through the inclusion of auxiliary filters in each of the agent manipulator system, it can be shown that finite-time convergence of the leader-following synchronization error and finite-time adaptation can be achieved. The next section focusses on the introduction of an adaptive law as used by each agent.

V. Finite-Time Parameter Estimation Algorithm

A. Auxiliary Torque Filters

In this section, an auxiliary filtered regression matrix and suitable filtered vectors for the adaptation algorithm will be formulated for each agent manipulator, based on its torque measurement τ_i. By having the torque measurement filtered, acceleration measurements for the regressor $\phi_i(q_{1_i}, \dot{q}_i, \ddot{q}_i)$ can be avoided [20], [21]. Indeed, the regressor $\phi_i(q_{1_i}, q_{2_i}, \ddot{q}_i)$ in (2) uses joint accelerations which generally is not practical. Hence, the equation (1) can be written as,

$$\tau_i = \dot{f}_i + h_i \tag{17}$$

The components of torque can be split and defined as,

$$\begin{aligned} \dot{f}_i &= \frac{d}{dt}[M_i(q_{1_i})q_{2_i}] && (18)\\ h_i &= -\dot{M}_i(q_{1_i})q_{2_i} + V_i(q_{1_i}, q_{2_i})q_{2_i} + G_i(q_{1_i}) && (19)\\ &= h_{1i} + h_{2i} && (20) \end{aligned}$$

where $h_{1i} = -\dot{M}_i(q_{1_i})q_{2_i}$ and $h_{2i} = V_i(q_{1_i}, q_{2_i})q_{2_i} + G_i(q_{1_i})$. By virtue of the linearity-in-the-parameter assumption, the split terms can be parameterised as such,

$$f_i = M_i(q_{1_i})q_{2_i} = \varphi_{m1i}(q_{1_i}, q_{2_i})\Theta_i \tag{21}$$

$$h_{1i} = -\dot{M}_i(q_{1_i})q_{2_i} = \varphi_{m2i}(q_{1_i}, q_{2_i})\Theta_i \tag{22}$$

$$h_{2i} = V_i(q_{1_i}, q_{2_i})q_{2_i} + G_i(q_{1_i}) = \varphi_{vgi}(q_{1_i}, q_{2_i})\Theta_i \tag{23}$$

Filtering the terms $\varphi_{m1i}, \varphi_{m2i}, \varphi_{vgi}$ and τ through an impulse response filter $\mathfrak{f} = \frac{1}{\kappa}e^{-1/kt}$ to produce $\varphi_{m1fi} = \mathfrak{f} * \varphi_{m1i}$, $\varphi_{m2fi} = \mathfrak{f} * \varphi_{m2i}$, $\varphi_{vgfi} = \mathfrak{f} * \varphi_{vgi}$ and $\tau_f = \mathfrak{f} * \tau$ respectively. The filtered computed-torque equation can be rewritten as,

$$\begin{aligned} [\frac{\varphi_{m1_i}(q_{1_i}, q_{2_i}) - \varphi_{m1f_i}(q_{1_i}, q_{2_i})}{\kappa_i} &+ \varphi_{m2f_i}(q_{1_i}, q_{2_i}) \\ +\varphi_{vgf_i}(q_{1_i}, q_{2_i})]\Theta_i &= \tau_{f_i} && (24)\\ \phi_{f_i}(q_{1_i}, q_{2_i})\Theta_i &= \tau_{f_i} \end{aligned}$$

where $\phi_{f_i}(q_{1_i}, q_{2_i}) \in \mathbb{R}^{n\times l}, \Theta_i \in \mathbb{R}^l$. By comparison to (1), the filtered system equation of (24) clearly avoids the acceleration measurements which are sometimes practically unavailable. Note that $\phi_i(q_{1_i}, q_{2_i}, \dot{q}_{2_i})$ is the unfiltered regressor for $\phi_{f_i}(q_{1_i}, q_{2_i})$.

B. Auxiliary Integrated Regressors

The filtered torque formulation is now considered for an auxiliary regressor used for the adaptation algorithm. Define a filtered regressor matrix $W_i(t)$ and vector $N_i(t)$ as,

$$\begin{aligned} \dot{W}_i(t) &= -k_{FF_i}W_i(t) + k_{FF_i}\phi_{f_i}^T(q_{1_i}, q_{2_i})\phi_{f_i}(q_{1_i}, q_{2_i}),\\ W_i(0) &= wI_l, && (25)\\ \dot{N}_i(t) &= -k_{FF_i}N_i(t) + k_{FF_i}\phi_{f_i}^T(q_{1_i}, q_{2_i})\tau_{f_i}, && (26)\\ N_i(0) &= 0 \end{aligned}$$

where, $k_{FF_i} \in \mathbb{R}^+$, can be interpreted as a forgetting factor. The solution of $W_i(t)$ (25) shows that $W_i(t) \geq wI_l e^{-k_{FF_i}t}$ for $w > 0$. This bound will be exploited in the Lyapunov analysis section later. Having formulated the auxiliary torque filters and filtered regressors, (24) can be expressed in an overall expression for the network as

$$\Phi_f(q_1, q_2)\bar{\Theta} = \tau_f \tag{27}$$

where $\Phi_f(q_1, q_2) = diag(\phi_{f_1}(q_{1_1}, q_{2_1}), \ldots, \phi_{f_i}(q_{1_i}, q_{2_i}), \ldots, \phi_{f_N}(q_{1_N}, q_{2_N})) \in \mathbb{R}^{N(n\times l)}$. $\bar{\Theta} = [\Theta_1^T, \ldots, \Theta_i^T, \ldots, \Theta_N^T]^T \in \mathbb{R}^{Nl}$. Moreover,

$$\bar{N}(t) = \bar{W}(t)\bar{\Theta} - (I_N \otimes e^{-k_{FF}t}wI_l)\bar{\Theta} \tag{28}$$

where $\bar{N} = [N_1^T, N_i^T, \ldots, N_N^T]^T \in \mathbb{R}^{Nl}$, and $\bar{W}(t) = diag(W_1, W_i, \ldots, W_N) \in \mathbb{R}^{Nl\times Nl}$.

C. Parameter Estimation Laws

The parameter estimation algorithm comprises of a switched parameter R_i for each agent manipulator i:

$$\dot{\hat{\Theta}}_i = -\Gamma_i R_i \tag{29}$$

$$\begin{aligned} R_i &= \omega_{1_i}\frac{W_i(t)\hat{\Theta}_i - N(t)_i}{\|W_i(t)\hat{\Theta}_i - N_i(t)\|} + \omega_{2_i}(W_i(t)\hat{\Theta}_i - N(t)_i),\\ & \quad i = 1, \ldots, N. && (30) \end{aligned}$$

where ω_{1_i} and ω_{2_i} are positive scalars which are to be chosen large enough in the Lyapunov based design to achieve robust stability. Γ_i is a diagonal positive definite matrix.

Remark 1: In [17], it has been shown that (10) implies that $W_0(t)$ is invertible with well defined bounds for the smallest and largest singular value. ◦

The agent-specific adaptive law (29) will be now used as part of a distributed control law for each agent.

VI. Distributed Adaptive Control Law

The concept of robust sliding mode control for a finite time sliding plane is introduced to allow for finite-time convergence of the synchronisation error. The approach presented here is suitably combined with an adaptive control element to enhance consensus control performance by incorporating finite-time parameter estimation.

A. Sliding variable Definition

Note that $\dot{\mathcal{E}}_A = \mathcal{E}_B$. Denote m as the index for one of n joints. Thus, the sliding variable r_{i_m} is defined for each joint $m, m \in \{1, 2, \ldots, n\}$ of agent manipulator i,

$$r_{i_m} = |e_{B_{i_m}}|^\rho sign(e_{B_{i_m}}) + \lambda_{i_m}e_{A_{i_m}} \in \mathbb{R}^1 \tag{31}$$

where $r_i = [r_{i_1}, \cdots, r_{i_m}, \cdots, r_{i_n}]^T \in \mathbb{R}^n$ is the sliding error for agent manipulator i. The scalar ρ satifies $1 < \rho < 2$ and $\lambda_m > 0$. It can be shown that the sliding variable

r_{i_m} leads to finite time convergence of the closed-loop, i.e. $r_{i_m} = 0$ is governed by

$$\dot{e}_{A_{i_m}} = -\lambda_i^{1/\rho} |e_{A_i}|^{1/\rho} sign(e_{A_i}), \quad (32)$$

The sliding variable r_i for each manipulator i is therefore

$$r_i \quad = \quad \varepsilon_{B_i} + \Lambda e_{A_i} \in \mathbb{R}^n \quad (33)$$

where $\varepsilon_{B_i} = [|e_{Bi_1}|^\rho sign(e_{Bi_1}), |e_{Bi_2}|^\rho sign(e_{Bi_2}), \ldots, |e_{Bi_n}|^\rho sign(e_{Bi_n})]^T \in R^n$ and $\Lambda = diag(\lambda_{i_1}, \lambda_{i_2}, \ldots, \lambda_{i_n})$. The sliding variable in (33) can be expressed for the overall network,

$$\bar{r} = \bar{E}(\mathcal{E}_B) + (I_N \otimes \Lambda)\mathcal{E}_A \in \mathbb{R}^{nN} \quad (34)$$

where $\bar{E}(\mathcal{E}_B) = [\varepsilon_{B_1}^T, \varepsilon_{B_i}^T, \ldots, \varepsilon_{B_N}^T]^T$. Differentiating $\bar{r}$ yields,

$$\dot{\bar{r}} = \rho \check{E} \dot{\mathcal{E}}_B + (I_N \otimes \Lambda)\dot{\mathcal{E}}_A \quad (35)$$

where

$$\check{E} = diag(\check{E}_1, \ldots, \check{E}_i, \ldots, \check{E}_N) \in \mathbb{R}^{(Nn \times Nn)} \quad (36)$$

with $\check{E}_i$ defined as

$$\check{E}_i = diag(|e_{B_1}|^{\rho-1}, |e_{B_2}|^{\rho-1}, \cdots, |e_{B_n}|^{\rho-1}) \in \mathbb{R}^{n \times n} \quad (37)$$

B. Leader following control law

A set of adaptive control laws are to be defined in this section, which will solve the leader following control problem within a finite time. To facilitate the analysis and design a result known from the cooperative control literature, e.g. [19], is extended to the context of multi-degree-of-freedom systems.

Lemma 1: Let $L \in \mathbb{R}^{N\times N}$ be an irreducible and upper triangular matrix and $B \in \mathbb{R}^{N\times N}$ may have at least one diagonal element. Moreover, there is a matrix $\mathcal{N} = diag(\mathcal{N}_1, \mathcal{N}_2, \cdots, \mathcal{N}_N) \in \mathbb{R}^{nN\times nN}$ for which $\mathcal{N}_i \in \mathbb{R}^{n\times n}$, $i = 1, \cdots, N$, are positive definite and the following inequalities hold

$$\underline{\sigma}(\mathcal{N}_i) > \|\mathcal{N}_{i+1}\| > 0, \; i = 1, \cdots, N-1, \quad (38)$$

then there exists a matrix $\bar{P}$

$$\begin{aligned}
&\bar{P} = P_\sigma P \otimes I_n, \; P = \mathrm{diag}(x_1/y_1, x_2/y_2, \cdots, x_N/y_N), \\
&x = (L+B)^{-1} 1_N, \; y = (L+B)^{-T} 1_N, \\
&1_N = [1, 1, \cdots, 1]^T, \; 1_N \in \mathbb{R}^N, \; \kappa_{max} > \max_{i=1,\ldots,N} \frac{\|\mathcal{N}_i\|}{\underline{\sigma}(\mathcal{N}_i)} \\
&P_\sigma = \mathrm{diag}(1, \kappa_{max}, \cdots, \kappa_{max}^{(N-1)}), \quad (39)
\end{aligned}$$

so that:

$$\bar{P}((L+B)\otimes I_n)\mathcal{N} + (((L+B)\otimes I_n)\mathcal{N})^T \bar{P} > 0 \quad (40)$$

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The proof of this Lemma can be found in the appendix. The diagonal matrix $\bar{P}$ from Lemma 1 will be used in the leader following control law in the following Theorem:

Theorem 1: Consider the multi-manipulator system with dynamics defined by (1), adaptive parameter estimation algorithms (29) and communication interconnections between manipulator agents and its corresponding virtual leader defined through the given Laplacian matrix L. The adaptive control law τ_i, for each agent manipulator i is:

$$\tau_i = \phi_{vg_i} \hat{\Theta}_i + \eta_i \tau_{c_i} + \eta_i \check{E}_i r_i, \quad (41)$$

where the auxiliary torque input τ_{ci} is:

$$\tau_{ci} \quad = \quad \left[\frac{1}{(d_i + b_i)}\right] \left[\sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij}\tau_{cj} + k_i \frac{\tilde{r}_i}{\|\tilde{r}_i\|} \right] \quad (42)$$

and $\tilde{r}_i = \check{E}_i r_i$. The control gains η_i are chosen so that the matrix $diag(\eta_1 M_1^{-1}, \eta_2 M_2^{-1}, \cdots, \eta_N M_N^{-1})$ satisfies the conditions (38) for matrix $\mathcal{N}$. This implies for suitable choice of $k_i > 0$ that the parameter estimation errors $W_i(t)\hat{\Theta}_i - N(t)_i$, $(i = 1, \cdots, N)$ and synchronisation errors $\mathcal{E}_A$ and $\mathcal{E}_B$ converge to 0 in finite time in an arbitrarily large compact set of $\mathcal{E}_A$, $\mathcal{E}_B$ and $\hat{\Theta}_i$ determined by k, ω_{1_i} and ω_{2_i}. The parameter estimates converge to its true values. ♢

The overall combined control law can be written as

$$\tau = \Phi_{vg}\hat{\bar{\Theta}} + \bar{\eta}\tau_c + \eta\tilde{r} \quad (43)$$

where $\bar{\eta} = \eta \otimes I_n$ with $\eta = diag(\eta_1, \eta_i, \ldots, \eta_N) \in \mathbb{R}^{N\times N}$ and $\hat{\bar{\Theta}} = [\hat{\Theta}_1^T, \hat{\Theta}_i^T, \ldots, \hat{\Theta}_N^T]^T$. The corresponding auxiliary torque input which encapsulates the decentralized switching laws for all agents is:

$$\tau_c = [[(D+B)\otimes I_n]]^{-1}\left[(A \otimes I_n)\tau_c + KSIGN(\tilde{r})\right] \quad (44)$$

where $K = diag([k_1, k_i, \ldots, k_N])$ and

$$SIGN(\tilde{r}) = \left[\frac{\tilde{r}_1}{\|\tilde{r}_1\|}, \frac{\tilde{r}_i}{\|\tilde{r}_i\|}, \ldots, \frac{\tilde{r}_N}{\|\tilde{r}_N\|}\right]^T \quad (45)$$

Note that $\left[I + [(D+B)\otimes I_n]^{-1}(-A\otimes I_n)\right] = [(D+B)\otimes I_n]^{-1}\left[[(D+B)\otimes I_n] + (-A\otimes I_n)\right]$. Invoking the associative property of the Kronecker product, i.e. $\mathcal{A}\otimes\mathcal{F} + \mathcal{B}\otimes\mathcal{F} = (\mathcal{A}+\mathcal{B})\otimes\mathcal{F}$ and since $L = D - A$, then (44) can be simplified as,

$$\tau_c = [[(L+B)\otimes I_n]]^{-1}\left[KSIGN(\tilde{r})\right] \quad (46)$$

Proof of Theorem 1: The following Lyapunov function is proposed,

$$\begin{aligned}
\mathcal{V} \quad &= \quad \mathcal{V}_r + \mathcal{V}_\Theta \quad (47) \\
&= \quad \frac{1}{2}\bar{r}^T\bar{P}\bar{r} + \frac{1}{2}\tilde{N}^T\bar{W}^{-1}\Gamma^{-1}\bar{W}^{-1}\tilde{N} \quad (48)
\end{aligned}$$

where $\tilde{\bar{N}}$ is defined as

$$\begin{aligned}
\tilde{N}(t) \quad &= \quad \bar{N}(t) - \bar{W}(t)\hat{\bar{\Theta}} \quad (49) \\
&= \quad \bar{W}(t)\tilde{\bar{\Theta}} - K_{Ieff}\bar{\Theta}, \quad (50)
\end{aligned}$$

$K_{Ieff} = (I_N \otimes e^{-k_{FF}t}w) \in \mathbb{R}^{Nl\times l}$, $\tilde{\bar{\Theta}} = \bar{\Theta} - \hat{\bar{\Theta}}$, $\bar{W}(t) = diag(W_1(t), W_2(t), \ldots, W_N(t))$ and $N(t) = [N_1^T, N_2^T, \cdots, N_N^T]^T$. Note that,

$$\bar{W}(t) \geq e^{-k_{FF}t}wI_l \quad \Rightarrow \quad \left\|\bar{W}^{-1}(t)\right\| \leq e^{k_{FF}t}\frac{1}{w}. \quad (51)$$

To compute $\dot{\mathcal{V}}_r$ in our analysis, (36) is used to denote the following:

$$\tilde{r} = \check{E}\bar{r} \quad (52)$$

Differentiating (13),

$$\begin{aligned}
\dot{\mathcal{E}}_A \quad &= \quad -\left[(L+B)\otimes I_n\right](q_2 - \bar{q}_{2_0}) \quad (53) \\
\dot{\mathcal{E}}_B \quad &= \quad -\left[(L+B)\otimes I_n\right](\dot{q}_2 - \dot{\bar{q}}_{2_0}) \quad (54)
\end{aligned}$$

The derivative of the sliding mode term in (35) can be written as

$$\dot{\tilde{r}} = \rho\check{E}\left[-(L+B)\otimes I_n\right]\left(\mathcal{M}\left(-\Phi_{vg}\bar{\Theta}+\tau\right)-\bar{\mathcal{Q}}_0\right)+\tilde{\Lambda}\mathcal{E}_B \tag{55}$$

where $\bar{\mathcal{Q}}_0 = 1_N \otimes [M_0^{-1}(-V_0(q_{1_0}, q_{2_0})q_{2_0} - G_0(q_{1_0}) + \tau_0)]$ and $\tilde{\Lambda} = (I_N \otimes \Lambda)$. Differentiating $\mathcal{V}$ yields,

$$\dot{\mathcal{V}} = \bar{r}^T\bar{P}\dot{\bar{r}} + \tilde{N}^T\bar{W}^{-1}\Gamma^{-1}\frac{\partial}{\partial t}\left[\bar{W}^{-1}\tilde{N}\right] \tag{56}$$

Computing the derivative of $\bar{W}^{-1}\tilde{\bar{N}} = \tilde{\bar{\Theta}} - \bar{W}^{-1}K_{Ieff}\bar{\Theta}$ provides

$$\frac{\partial}{\partial t}\left[\bar{W}^{-1}\tilde{\bar{N}}\right] = \dot{\tilde{\bar{\Theta}}} + \xi \tag{57}$$

where $K_{kIeff} = k_{FF}K_{Ieff}$ and $\xi = \bar{W}^{-1}K_{Ieff}\Theta\left[k_{FF} - \dot{\bar{W}}\bar{W}^{-1}\right] = [\xi_1, \cdots, \xi_N]$ for $\xi \in \mathbb{R}^n$. We may now define $\tilde{r}_i \stackrel{def}{=} \check{E}_i\bar{r}_i$, $\tilde{\mathcal{E}}_B \stackrel{def}{=} \check{E}^{-1}\mathcal{E}_B$, $\tilde{\xi}_i = \xi_{+}\omega_{2_i}\Gamma_i e^{-k_{FF}t}w\Theta_i$, $\tilde{\xi} = [\tilde{\xi}_1, \cdots, \tilde{\xi}_N]$ and $\Upsilon \stackrel{def}{=} \rho[(L+B)\otimes I_n]\bar{\mathcal{Q}}_0 + \tilde{\Lambda}\tilde{\mathcal{E}}_B + \bar{W}^{-1}K_{Ieff}\bar{\Theta}$, $\Upsilon \stackrel{def}{=} [\Upsilon_1, \ldots, \Upsilon_N]^T$. Note that $\tilde{\mathcal{E}}_B$ is not singular due to the choice of ρ, $1 < \rho < 2$. Exploiting the fact that $\bar{P}$ and $\check{E}$ are diagonal, adopting the control torque in (43) and incorporating the auxiliary torque input τ_c (44) and (50) it follows:

$$\begin{aligned}
\dot{\mathcal{V}} \;=\; & -\frac{1}{2}\rho\tilde{r}^T(\bar{P}[(L+B)\otimes I_n][\mathcal{M}\bar{\eta}] \\
& +[\mathcal{M}\bar{\eta}]^T[(L+B)\otimes I_n]^T\bar{P})\tilde{r} \\
& +\rho\tilde{r}^T\bar{P}[(L+B)\otimes I_n]\mathcal{M}\Phi_{vg}\bar{W}^{-1}\tilde{\bar{N}} \\
& -\rho\tilde{r}^T\bar{P}[(L+B)\otimes I_n]\mathcal{M}\bar{\eta}[(L+B)\otimes I_n]^{-1}\times \\
& [KSIGN(\tilde{r})] + \sum_{i=1}^{N}\tilde{r}_i^T\bar{P}_i\Upsilon_i + \sum_{i=1}^{N}\tilde{N}_i^T W_i^{-1}\Gamma_i^{-1}\tilde{\xi}_i \\
& -\sum_{i=1}^{N}\omega_{1_i}\tilde{N}_i^T W_i^{-1}\frac{\tilde{N}_i}{\|\tilde{N}_i\|} - \sum_{i=1}^{N}\omega_{2_i}\tilde{N}_i^T W_i^{-1}\tilde{N}_i
\end{aligned} \tag{58}$$

We may now analyse the matrix

$$\tilde{\mathcal{M}} = \tilde{r}^T\bar{P}[(L+B)\otimes I_n]\mathcal{M}\bar{\eta}[(L+B)\otimes I_n]^{-1}K \tag{59}$$

Note that $(L+B)$ is upper triangular, i.e. its inverse is also upper triangular. Thus, the structure of $\tilde{\mathcal{M}} = [\tilde{\mathcal{M}}_{ij}]$, ($\tilde{\mathcal{M}}_{ij} \in \mathbb{R}^{n\times n}$) follows also an upper triangular structure, i.e. $\tilde{\mathcal{M}}_{ij} = 0$ for $i > j$. Note also the diagonal structure of $\bar{P}$ (39) and the symmetry of M_i^{-1}, which implies that $\tilde{\mathcal{M}}_{ii}$ are all symmetric. The inverse matrix of $(L+B)$ can contain only non-negative elements. Thus, the definition of $\mathcal{M}\bar{\eta}$ implies that $\tilde{\mathcal{M}}_{ii}$ is also positive definite. We may now write the Lyapunov matrix $\bar{P}$ (39) as $\bar{P} = diag(\bar{P}_1 I_n, \bar{P}_2 I_n, \ldots, \bar{P}_N I_n)$. Employing now Lemma 1, it follows the following upper bound for $\dot{\mathcal{V}}$:

$$\begin{aligned}
\dot{\mathcal{V}} \;\leq\; & -\begin{bmatrix}\tilde{r} \\ \tilde{\bar{N}}\end{bmatrix}^T \Delta \begin{bmatrix}\tilde{r} \\ \tilde{\bar{N}}\end{bmatrix} \\
& -\sum_{i=1}^{N}\|\tilde{r}_i\|\left(\rho k_i\underline{\sigma}(\tilde{\mathcal{M}}_{ii}) - \sum_{j=i+1}^{N}\rho k_j\|\tilde{\mathcal{M}}_{ij}\| - \|\bar{P}_i\Upsilon_i\|\right) \\
& -\sum_{i=1}^{N}\|\tilde{N}_i\|\left(\omega_{1_i}\underline{\sigma}(W_i^{-1}) - \left\|W_i^{-1}\Gamma_i^{-1}\tilde{\xi}_i\right\|\right)
\end{aligned}$$

where for $\Omega_2 = diag((I_l\otimes\omega_{21}), (I_l\otimes\omega_{2i}), \ldots, (I_l\otimes\omega_{2N}))$, $\Omega_2 > 0$ the matrix Δ is:

$$\Delta = \begin{bmatrix}\rho Q & -\rho\bar{P}[(L+B)\otimes I_n]\mathcal{M}\Phi_{vg}\bar{W}^{-1} \\ * & \Omega_2\bar{W}^{-1}\end{bmatrix} \tag{60}$$

This leads after some manipulation to

$$\begin{aligned}
\dot{\mathcal{V}} \;\leq\; & -\epsilon\left(\underline{\sigma}(\check{E})^2\mathcal{V}_r + \mathcal{V}_\Theta\right) \\
& -\epsilon\underline{\sigma}(\check{E})\sqrt{N}\sqrt{\mathcal{V}_r} - \epsilon\sqrt{N}\sqrt{\mathcal{V}_\Theta},
\end{aligned} \tag{61}$$

for suitable choice of gains k_i and ω_i so that $\epsilon > 0$ exists. Following arguments of [4] and [17], this guarantees that all agent trajectories follow the persistently exciting demand trajectory of the leader by Property 3 within finite time, so that $\underline{\sigma}(W_i^{-1})$ and $\underline{\sigma}(W_i)$ remain finite and strictly larger than 0. The estimates, $\hat{\bar{\Theta}}$ converge to their true values. ■

VII. Leader-Following Multi-Manipulator Example

A simulation example is presented to illustrate the performance of the proposed distributed adaptive leader-following control algorithm: we consider a simple network of two manipulators of 2 DOF (revolute planar) with a leader whose communication topology is defined by a Laplacian matrix, $L = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$. The diagonal pinning matrix is B =diag$(0, 1)$. The manipulator leader is controlled by means of a feedback linearisation controller following sinusoidal/SR signals for both joints. Table I shows the masses of the manipulator agent's links to be estimated.

TABLE I
Manipulator System Parameters, $m_i = [m_{1i}, m_{2i}]^T$

Manipulator i	Link 1 mass (kg)	Link 2 mass (kg)
Leader	2.35	3
Agent 1	**1**	**1.35**
Agent 2	**0.5**	**1**

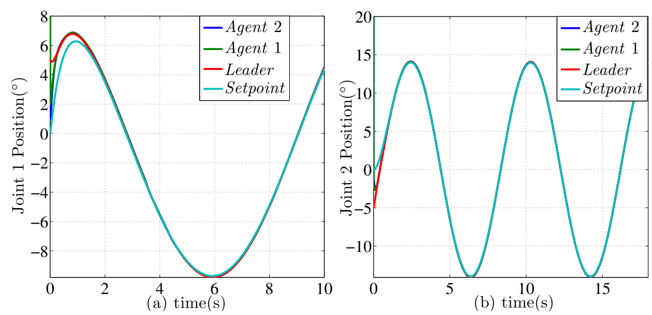

Fig. 1. Joint 1 and Joint 2 Position for for agent i and their leader with proposed finite-time distributed adaptive control.

Figure 1 shows all the agents' joint position trajectories for joint 1 and joint 2 with different initial conditions. The leader tracking of all the agents is observed to be finite-time. All the agents successfully follow the leader within less than 1 sec. Thus, the leader-following task by each agent has been accomplished. Figure 2 shows the finite-time convergence of the respective agent's link masses estimates within less than 1.5 seconds. The exemplified result shows that the local finite-time parameter estimation algorithm by each agent also enhances the convergence of the network consensus in addition to the switching signal fed back to the network.

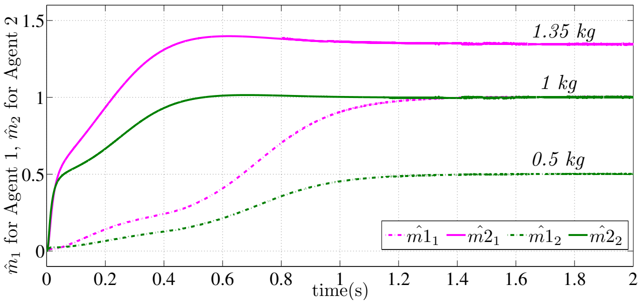

Fig. 2. Link Mass Estimation for for agent $i \in \{1, 2\}$

VIII. Conclusions

A novel distributed leader-following adaptive controller of multi-manipulators has been presented. It is shown that the proposed approach guarantees finite-time convergence for leader-following tracking and parameter estimation via agent-based estimation and control algorithms. It is shown that the extended matrix theoretical results related to M-matrix theory used for multi-agent systems is instrumental to the multi-degree context. This allows to prove the existence of a Lyapunov function which is used in the analysis for stability and finite-time convergence of the consensus error and parameter estimation error. Information on the leader's dynamics is only required by pinned agents and the dynamic interaction by all agents is fully defined by the communication network: Any unknown dynamics are compensated by the switching control which is fed-back to the communication network and therefore, the leader's joint position and velocity are not required *a priori*. Numerical examples of a two-degree of freedom two-agent system with one leader prove the feasibility of the results.

Appendix 1: Proof of Lemma 1

To facilitate the proof of Lemma 1, the following preliminary Lemma is a necessary extension of work for instance found in [18].

Lemma 2: Suppose $\mathcal{Q} \in \mathbb{R}^{nN\times nN}$ is defined as below:

$$\mathcal{Q} = \begin{bmatrix} \mathcal{Q}_{11} & \mathcal{Q}_{12} & \cdots & \mathcal{Q}_{1N} \\ \mathcal{Q}_{21} & \mathcal{Q}_{22} & \cdots & \mathcal{Q}_{2N} \\ \cdots & \cdots & \ddots & \vdots \\ \mathcal{Q}_{N1} & \mathcal{Q}_{N2} & \cdots & \mathcal{Q}_{NN} \end{bmatrix}$$

where $\mathcal{Q}_{ij} \in \mathbb{R}^{n\times n}$, $i, j = 1, \cdots, n$, are symmetric matrices satisfying:

$$\mathcal{Q}_{ii} > 0,\ \mathcal{Q}_{ij,i\neq j} \leq 0,\ \underline{\sigma}(\mathcal{Q}_{ii}) > \left\| \sum_{j=1,i\neq j}^{N} \mathcal{Q}_{ij,i\neq j} \right\| \quad (62)$$

then (i) the matrix $\mathcal{Q}$ is invertible and (ii) any real eigenvalue of $\mathcal{Q}$ is positive. •

From [19, Chapter 4, p.174] follows for P, $(L + B)$ that $P(L + B)$ is upper triangular and diagonally dominant in terms of row and column vectors, i.e. $P(L + B) + (L + B)^T P > 0$ is positive definite. From $\kappa_{max} > 1$ and the definition of P_σ (39), this easily also implies that $P_\sigma P(L+B)$ is again diagonally dominant in terms of row and column vectors so that $P_\sigma P(L + B) + (L + B)^T P P_\sigma > 0$. Now, we may investigate

$$\tilde{\mathcal{Q}} = \bar{P}((L + B) \otimes I_n)\mathcal{N} + (((L + B) \otimes I_n)\mathcal{N})^T \bar{P} \quad (63)$$

From (38) and the diagonal dominance of the upper triangular $P(L + B)$, it follows $\underline{\sigma}(\breve{\mathcal{Q}}_{ii}) > \left\| \sum_{j=i+1}^{N} \breve{\mathcal{Q}}_{ij} \right\|$ for and $\breve{\mathcal{Q}} = \bar{P}((L + B) \otimes I_n)\mathcal{N}$, $\breve{\mathcal{Q}} = [\breve{\mathcal{Q}}_{ij}]$, $\breve{\mathcal{Q}}_{ij} \in \mathbb{R}^{n\times n}$. The choice of P_σ and the diagonal dominance of $P(L+B)$ (and $P_\sigma P(L+B)$) guarantees $\underline{\sigma}(\breve{\mathcal{Q}}_{ii}) > \left\| \sum_{j=1}^{i-1} \breve{\mathcal{Q}}_{ji} \right\|$, where $\breve{\mathcal{Q}}_{ji} = 0$ for $j < i$. This implies that $\tilde{\mathcal{Q}} = \breve{\mathcal{Q}} + \breve{\mathcal{Q}}^T$ satisfies the conditions of Lemma 2, i.e. the symmetric matrix $\tilde{\mathcal{Q}}$ is positive definite.

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