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Optimal Load Following in Power Grids in the Presence of Battery-Powered Agents

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***Abstract*— We address the modeling and optimal control problems for load following in power grids in the presence of unconventional battery-powered agents. Compared to the traditional large-scale generators such as thermal and hydro-electric plants, these agents have significantly lower short-term response costs but higher long-term generation costs. Moreover, a battery-powered agent can act both as a supplier and as a load by operating in discharge or charge modes, respectively. In this work, we provide a tractable model that captures the multi-timescale and heterogeneous cost structures of conventional and unconventional systems, as well as the dual role of unconventional ones. Then, we pose and solve the problem of optimal finite-horizon control of the joint system to balance unpredicted fluctuations in the net load. The optimal controller takes a simple and intuitive form that is explicitly described in terms of cost parameters, the net load shift, and the time horizon of control. We further investigate numerically several characteristics of the controller to reveal the gains and dynamics associated with it under randomly changing net load.**

I. INTRODUCTION

One of the main objectives in the operation of electric power grids is the low-cost generation of power that matches the varying load at all times to maintain a reliable and stable service. Discrepancies in load supply matching caused by unpredicted demand, losses of generators, or other contingencies are compensated by operating reserves that provide ancillary services such as load following, load regulation, and frequency regulation [1]. However, the emergence of variable and uncertain *renewable energy sources*, such as wind and solar generators, together with the stochastic nature of *conventional consumer loads* create new challenges in the reliable and low-cost operation of power grids.

Conventional power plants, such as nuclear, fuel, and hydro generators that support the majority of the grid load, are slow and operationally costly for responding to continuous variations in the demand while they are highly optimized for sustaining a constant load level in the long run. This opens a significant market to new *unconventional technologies* that can help the grid react to variabilities by possessing relatively low ramp up/down costs, despite the long-term generation inefficiencies they may possess.

New technologies aiming to address the expanding ancillary services market vary from motor generators [2] to the use of battery-powered or hybrid vehicles that will be connected to the grid [3], [4]. In this paper, we particularly investigate the aid of battery-powered storage units in load following, which is the process of tracking the changes in the net load over time intervals ranging from five minutes to an hour [5]. The economical value of battery units and their impact on the grid operation have been extensively studied [6], [7]. Furthermore, previous literature suggests that storage units are viable candidates for load following applications due to their high power capacity, short response time, and capability to act both as generators and loads [7]–[10].

In the setup we consider, the load following system controls a conventional generator and a battery-powered storage unit. The optimal control strategy depends on the multi-timescale cost structures of these units that measure the operational/economic costs of changing power level in the short run or sustaining it at a constant level in the long run. In particular, changing rapidly the generation level of the unconventional system is significantly less costly, while the conventional supplier is more efficient in the consistent support of a given load. Another striking difference between the two systems is that the conventional generator purely supplies power whereas the unconventional system can either supply or consume power. In view of the described characteristics, our objective is to develop the optimal policy for load following under a tractable and reasonable model.

Our main contributions along with the outline of the paper are as follows: In Section II, we provide a tractable formulation that models the aforementioned heterogeneities in the cost structures of both the conventional and unconventional generators. In Section III, we fully develop the finite-horizon optimal controller in closed form as a function of the cost parameters and the net load disruptions, and exhibit its behavior. In Section IV, we investigate the optimal controller through simulations to demonstrate the gains achievable with the aid of the unconventional system, and the dynamics of the battery power evolution in it.

II. SYSTEM MODEL

We model the system operation to be in slotted-time. Time slots are indexed with $t \in \mathbb{N}$ and by convention $t = 0$ denotes the last time slot before control action starts. The decision on the amount of power to be generated or consumed at a time slot is given at the beginning of that time slot.

Referring to Figure 1, the amount of power produced by generator $i \in \{G, U, R\}$ at time slot t is $P_i(t) \in \mathbb{R}_+$, and the amount of power consumed by load $j \in \{U, C\}$ at time slot t is $L_j(t) \in \mathbb{R}_+$, where $\mathbb{R}_+$ denotes the set of non-negative real numbers. We let $u_i(t) \in \mathbb{R}$ (or $u_j(t)$) represent the

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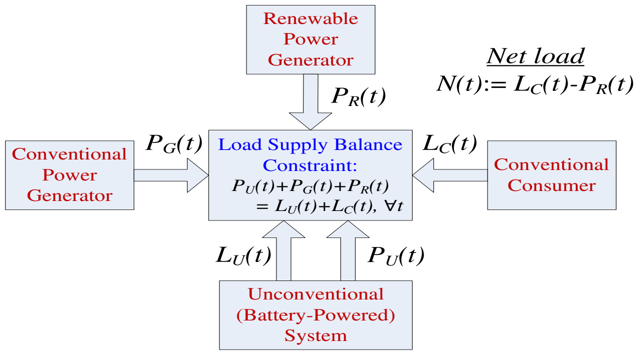

Fig. 1. Electricity grid components and load supply matching constraint.

change in generation (or in consumption) level of generator i (or load j) at one time step. Note that, we use the term 'generator' for the unconventional system regardless of its charge or discharge state, and set $u_U(t) = P_U(t) - P_U(t-1)$ or $u_U(t) = L_U(t) - L_U(t-1)$ depending on its operation mode. Furthermore, $u_i(t)$ is constrained by the 'hard' ramp up/down limits such that $u_{i,min} \leq u_i(t) \leq u_{i,max}$.

In the remaining of this section we first define the different types of costs, then describe the system entities in detail, and finally pose the finite-horizon cost minimization problem.

(i) Cost Structure: We adopt a multi-timescale cost structure which models two different types of costs associated with a generator: (i) *Base Generation Cost* is the long-term cost of generating or consuming a unit of power, and it is modeled as a function $C_\cdot^b : \mathbb{R}_+ \to \mathbb{R}_+$. (ii) *Ramp Up/Down Cost* is the short-term cost that reflects the wear and tear of the units due to changing generation or consumption frequently. It is a function $C_\cdot^r : \mathbb{R} \to \mathbb{R}_+$ of the difference in generation levels between two consecutive time slots [1].

(ii) System Blocks: In view of the system depicted in Figure 1, we can distinguish three entities: renewable generators and conventional consumers, conventional generator, and unconventional system.

1) Renewable Generators and Conventional Consumers: This block represents the net effect of the conventional consumers' load and intermittent renewable generation, and it is denoted by $N(t) \triangleq L_C(t) - P_R(t)$, that we call *net load*. We note that $N(t)$ is a random quantity, and it is the cause of fluctuations in the grid load. We also assume that the amount of renewable generation is not enough to serve the consumer load solely by itself, so $N(t) \in \mathbb{R}_+$

2) Conventional Generator: This block stands for the large scale conventional generator, such as a fuel-powered or hydro plant. These typically have low baseline cost per unit of power produced, but are ineffective in ramping up/down their generation due to high capital and maintenance costs [3], [11], leading to ramp up/down costs that are considerably higher than base generation costs.

We choose the base generation cost $C_G^b(\cdot)$ to be an affine function of $P_G(t)$. The reason is that the fluctuations in demand that are responded by the load following mechanism correspond to $5-10\%$ of the daily scheduled load, [12]. Therefore, we assume that the conventional generator operates in a limited region of its supply curve with fixed unit commitment. The ramp up/down cost $C_G^r(\cdot)$ is chosen to be quadratic function of $u_G(t)$. This choice is quite reasonable since it differentiates changes in generation within the ramp up/down limits, $u_{G,min} \leq u_G(t) \leq u_{G,max}$, by penalizing large changes greater than small changes.

Dropping, for convenience, the time index and generator subscripts of $u_G(t)$ and $P_G(t)$, we write down the cost for conventional generator as

$$C_G(u, P) = c_G^r(u)^2 + c_G^b P + d_G^b, \tag{1}$$

where $c_G^b > 0$, $d_G^b \geq 0$ are the parameters of the base generation cost function $C_G^b(\cdot)$, and $c_G^r > 0$ is the parameter of the quadratic ramp up/down cost function $C_G^r(\cdot)$.

3) Unconventional System: The unconventional system represents the battery-powered storage unit that is specifically designed for regulation purposes with the capability of both consuming and generating power [13]. Thus, we formulate the operational cost of the unconventional system separately for the charge and discharge modes although we use the same structure as that of the conventional generator:

$$C_U(u, P, L) = \begin{cases} c_U^r\,(u)^2 + c_U^b P + d_U^b, & \text{if Discharging } (P>0), \\ \tilde{c}_U^r\,(u)^2 + \tilde{c}_U^b L + \tilde{d}_U^b, & \text{if Charging } (L>0), \end{cases} \tag{2}$$

where $c_U^b > 0$, $d_U^b \geq 0$ are the parameters of the base generation cost function $C_U^b(\cdot)$, and $c_U^r > 0$ is the parameter of the ramp up/down cost function $C_U^r(\cdot)$. The parameters $\tilde{c}_U^b \in \mathbb{R}$, $\tilde{d}_U^b \geq 0$, and $\tilde{c}_U^r > 0$ are for the corresponding cost functions in charging mode, and as in the case of conventional generators $u_{U,min} \leq u \leq u_{U,max}$.

Unconventional system's base generation cost is higher than the conventional generator's base cost, $c_G^b < c_U^b$, because traditional generators are optimized for steady long-term generation [3]. Conversely, ramp up/down cost for unconventional system is assumed to be lower than that of conventional generator, $c_U^r < c_G^r$, since batteries have lower maintenance and capital costs than large-scale plants [3].

Moreover, baseline cost for charging, $\tilde{c}_U^b$, might be negative due to the difference between the payment for participating in regulation and the cost of consuming electricity. However, it should be sufficiently high such that $-c_G^b < \tilde{c}_U^b$, because the opposite case results in a degenerate situation where charging the battery using the generated power from the conventional plant reduces total cost.

The aforementioned relations between the cost parameters of the generators are summarized in the next assumption.

Assumption 1. *The parameters of the cost functions of the conventional and the unconventional generators satisfy:*

$$0 < c_G^b < c_U^b, \qquad -c_G^b < \tilde{c}_U^b,$$
$$0 < c_U^r, \tilde{c}_U^r < c_G^r, \qquad c_G^b < c_G^r.$$

Remark 1. *Dynamics of the optimal controller can be inferred from Assumption 1.* $c_U^r, \tilde{c}_U^r < c_G^r$ *and* $c_G^b < c_G^r$

[1]Ramp up and ramp down costs can be differentiated in our formulation with additional notation, if these processes incur different operational costs.

imply that after the net load changes the conventional generator shares the load balancing job with the unconventional generator instead of changing suddenly its generation to match the new load by only itself. Moreover, $c_G^b < c_U^b$ and $-c_G^b < \tilde{c}_U^b$ imply that at the end of a regulation period the conventional generator should eventually supply the whole net load and the unconventional system should stop generation or consumption.

Now, we pose the control problem in order to find the minimum cost response of the joint system to a change in the net load. To set the problem up, we first assume that at $t = 0$ the whole system is in a stable state whereby the conventional generator supplies the net load alone and the unconventional system neither generates nor consumes power, i.e., $P_G(0) = N(0) > 0$ and $P_U(0) = L_U(0) = 0$. Suppose a (random) change of $N_\delta \in \mathbb{R}$ in the net load occurs at time 0 that shifts the net load to $N \in \mathbb{R}_+$, i.e. $N_\delta \triangleq N - N(0)$. Then, the control problem over a finite time horizon T is given by

$$\min_{\substack{u_G(t),u_U(t)\\t=1,\dots,T}} \frac{1}{T}\sum_{t=1}^{T} C_G\left(u_G(t), P_G(t)\right) + C_U\left(u_U(t), P_U(t), L_U(t)\right) \quad (3)$$

$$\text{s.t.}\quad P_G(t) + P_U(t) - L_U(t) = N, \quad t = 1,\dots,T \quad (4)$$

$$u_i(t) \in [u_{i,min}, u_{i,max}],\ i \in \{G, U\},\ t = 1,\dots T \quad (5)$$

where (4) reflects the load-supply balance constraint, and (5) represents the physical ramp up and down constraints.

III. OPTIMAL CONTROLLER DESIGN

In this section, we develop the optimal controller in closed form for the problem (3)-(5). First, we assume that the ramp constraints satisfy $u_i(t) \in [-u_m, u_m]$ without loss of generality (w.l.o.g.). Second, we divide the process into two phases: (i) the *initial response phase* taken in the first time slot where the conventional generator and the unconventional system together balance the net load N that has changed at time zero; (ii) the *transition phase* containing the rest of the slots $2, 3, \dots, T$, in which the balanced load is gradually shared between the two systems to reduce operational costs. Then, we combine the optimal solution of the two phases to obtain the end-to-end optimal control rule as a function of the change in the net load and the system parameters.

A. Optimal Control in the Transition Phase

Since the unconventional system may operate in charge and discharge modes with different cost parameters, we first consider the case when the change in the net load, N_δ, is positive. In this case, the system is overloaded and the unconventional system must act as a supplier, which implies that $L_U(t) = 0$, $t = 1, 2, \cdots, T$. The structure of the solution in this case is similar in the oversupplied scenario of $N_\delta < 0$, where the unconventional system acts as a load, setting $P_U(t) = 0$, $t = 1, 2, \cdots, T$.

For the overloaded scenario of $N_\delta > 0$, w.l.o.g. we take $P_G(0) = 0$ along with $P_U(0) = L_U(0) = 0$ so that the balance constraint can be simply written as $P_G(t)+P_U(t) = N_\delta$, $t = 1,\dots,T$, i.e. the net load increases from 0 to N_δ at time 0. Let the transition phase start at the end of time $t = 1$ with the power distribution of $P_G(1) = p = N_\delta - P_U(1)$ where $p \in [0, N_\delta]$. Since the balance constraint requires that $P_U(t) = N_\delta - P_G(t)$, it is sufficient to describe the system state by $P_G(t)$ in the transition phase. Furthermore, $u_U(t) = -u_G(t)$ because as one of the generators' output increases the other generator's output should decrease to match the load. For any $k = 2, \cdots, T-1$ let the optimal cost-to-go function starting from $P_G(k) = p$ be denoted as $J_k^+(p)$ with boundary condition $J_T^+(p) = 0$, for all $p \in [0, N_\delta]$. Here, the superscript $^+$ is meant to indicate the overloaded scenario of $N_\delta > 0$. Then, $J_k^+(p)$ must satisfy the Bellman's Equation:

$$J_k^+(p) = \min_{\substack{u\in[-p,N_\delta-p]\\u\le u_m}} \{C_G(u, p+u) + C_U(-u, N_\delta - p - u, 0) + J_{k+1}^+(p+u)\}, \quad k = 2,\cdots,T-1.$$

Similarly, in the scenario of the oversupplied system when $N_\delta < 0$, suppose w.l.o.g. that $P_G(0) = -N_\delta$ and $P_G(t) - L_U(t) = 0$, i.e. at time 0 the net load decreases from $-N_\delta$ to 0. We assume the transition phase starts at the end of time $t = 1$ with $P_G(1) = p \in [0, -N_\delta]$. Again, $P_G(t)$ is the state variable due to the balance constraint. Furthermore, $u_U(t) = u_G(t)$ because as conventional generator changes its power output the unconventional system should change its power consumption in the same amount in order to satisfy the balance constraint. Then, we can write the corresponding Bellman's Equation for the optimal cost-to-go function $J_k^-(p)$ in the oversupplied scenario when $P_G(k) = p$ as:

$$J_k^-(p) = \min_{\substack{u\in[-p,-N_\delta-p]\\u\ge -u_m}} \{C_G(u, p+u) + C_U(u, 0, p+u) + J_{k+1}^-(p+u)\}, \quad k = 2,\cdots,T-1,$$

with the boundary condition $J_T^-(p) = 0$, for all $p \in [0, -N_\delta]$.

The next proposition provides the optimal control of the transition phase for both scenarios that follows by solving the corresponding Bellman's Equations. The result is presented in terms of $\{u_G(t)\}_{t=2}^{T-1}$ for a given $P_G(1)$ since this is sufficient to determine $\{P_G(t)\}_{t=2}^{T}$ and, from the balance constraint, $\{P_U(t)\}_{t=2}^{T}$.

Proposition 1. *The transition phase optimal control $\{u_G(t)\}_{t=2}^{T-1}$ of the conventional generator for solving (3) under the overloaded and oversupplied scenarios are:*
For $k \in \{2,\dots,K+2\}$ define $\tau_1(k) \triangleq \frac{k(k-1)}{2}$ and $\tau_2(k) \triangleq \frac{(k-1)(k-2)}{2}$. Define the critical parameters

$$\alpha^+ \triangleq \frac{c_U^b - c_G^b}{2\,(c_G^r + c_U^r)},\ \text{and}\ \alpha^- \triangleq \frac{-\tilde{c}_U^b - c_G^b}{2\,(c_G^r + \tilde{c}_U^r)}$$

where $\alpha^+ > 0$ and $\alpha^- < 0$ due to Assumption 1.

► ***Overloaded Case** ($N_\delta > 0$): Let $K \in \mathbb{Z}^+$ be s.t. $K\alpha^+ < u_m \le (K+1)\alpha^+$. Then, for any given $P_G(1) \in [0, N_\delta]$, the optimal control $\{u_G(t)\}_{t=2}^{T-1}$ is given by:*

• *if there exists a $k \in \{2,\cdots,K+2\}$ such that $P_G(t-1) \in (\max\{N_\delta - \tau_1\alpha^+, N_\delta + \tau_2\alpha^+ - (k-1)u_m\}, N_\delta - \tau_2\alpha^+]$,* *then*

$$u_G(t) = \frac{N_\delta - P_G(t-1)}{k-1} + \frac{(k-2)\alpha^+}{2}.$$

• *if no such k exists, then*

$$u_G(t) = \min\left\{(T-t)\alpha^+, u_m\right\}. \tag{6}$$

► ***Oversupplied Case*** *($N_\delta < 0$): Let $K \in \mathbb{Z}^+$ be s.t. $-K\alpha^+ < u_m \leq -(K+1)\alpha^+$. Then, for any given $P_G(1) \in [0, -N_\delta]$, the optimal control $\{u_G(t)\}_{t=2}^{T-1}$ is given by:*
• *if there exists a $k \in \{2, \cdots, K+2\}$ such that $P_G(t-1) \in (-\tau_2\alpha^-, \min\{-\tau_1\alpha^-, \tau_2\alpha^- + (k-1)u_m\}]$, then*

$$u_G(t) = \frac{-P_G(t-1)}{k-1} + \frac{(k-2)\alpha^-}{2}.$$

• *if no such k exists, then*

$$u_G(t) = \max\{(T-t)\,\alpha^-, -u_m\}. \tag{7}$$

Proof. The proof follows from the solution of Bellman's equations for $J^+(\cdot)$ and $J^-(\cdot)$ through backward recursion, and is omitted for brevity. □

While Proposition 1 provides the full description of the optimal transition phase controller, its structure is much more apparent from the visual illustration provided in Figure 2.

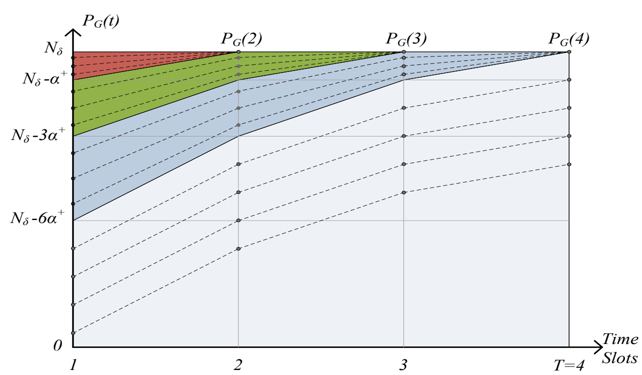

Fig. 2. Illustration showing optimum transition phase trajectories of $\{P_G(t)\}_{t=2}^T$ corresponding to possible values of $P_G(1)$ in the overloaded case $N_\delta > 0$ with $3\alpha < u_m$.

Figure 2 plots the optimal power output trajectories of the conventional generator over a duration of 4 slots starting from different levels of $P_G(1)$ for the overloaded case. This plot reveals the significance of the parameter α^+ in the optimal power allocation decision. For example, we see that if $P_G(1) \in [N_\delta - 3\alpha^+, N_\delta - \alpha^+)$, then $P_G(2) \in [N_\delta - \alpha^+, N_\delta]$ and $P_G(t) = N_\delta$ for $t = 3, 4$ i.e., the transfer of the load to the conventional supplier is complete at $t = 3$. However, if $P_G(1)$ is too small and the time horizon T is not long enough, the transfer may not be complete by time T. For example, in Figure 2, if $P_G(1) \in [0, N_\delta - 6\alpha^+)$ then it is increased by $3\alpha^+$, $2\alpha^+$, and α^+ in consecutive steps; but the horizon ends before $P_G(T)$ reaches N_δ.

These observations motivate us to define a critical time horizon, T_C, in which the system is stabilized by the load following control. We do so after we obtain the complete solution including the initial response phase.

B. Optimal Control in the Initial Response Phase

The structure of the cost function in the initial phase differs from that of the transition phase, since the latter assumes a balanced supply and demand.

In the overloaded scenario of $N_\delta > 0$, $P_G(1) = u_G(1)$ and $P_U(1) = u_U(1)$ because of the initial condition $P_G(0) = P_U(0) = 0$. In addition, $u_U(1) = N_\delta - u_G(1)$ due to the balance constraint. Then, the optimal initial response phase power allocation of the conventional generator is given by:

$$u_G(1) = \arg\min_{\substack{u\in[0,N_\delta]\\ u\leq u_m}} \{C_U(N_\delta - u, N_\delta - u, 0) + C_G(u, u) + J_2^+(u)\}.$$

Similarly, in the oversupplied scenario of $N_\delta < 0$, $P_G(1) = -N_\delta + u_G(1)$ and $L_U(1) = u_U(1)$ because of the initial conditions $P_G(0) = -N_\delta$ and $L_U(0) = 0$. Due to the balance constraint we also have $u_U(1) = -N_\delta + u_G(1)$. Then, the optimal initial response phase power allocation is:

$$u_G(1) = \arg\min_{\substack{u\in[N_\delta,0]\\ u\geq -u_m}} \{C_U(N_\delta - u, 0, -N_\delta + u) + C_G(u, -N_\delta + u) + J_2^-(-N_\delta + u)\}.$$

The following proposition provides the closed form solutions of these minimizations.

Proposition 2. *The initial response phase optimal control $u_G(1)$ of the conventional generator for solving (3) under the overloaded and oversupplied scenarios are: For $k \in \{2, \ldots, T\}$ define $\xi_1(N) = \frac{(c_G^r + c_U^r)u_m}{c_G^r(k-3)} - \frac{N}{k(k-3)}$, $\xi_2(N) = \frac{(c_G^r + c_U^r)u_m}{c_G^r(k-1)} - \frac{N(c_G^r + kc_U^r)}{k(k-1)c_G^r}$, and $\tilde{\xi}_1(N)$, $\tilde{\xi}_2(N)$ by the same way with $\tilde{c}_U^r$.*
► ***Overloaded Case*** *($N_\delta > 0$): Recall the critical parameter α^+ from Proposition 1. Then,*
• *if there exists a $k \in \{1, \cdots, T\}$ such that $\frac{\alpha^+(c_G^r + c_U^r)}{2c_G^r} \in \left(\frac{N_\delta}{k(k+1)}, \min\left\{\frac{N_\delta}{k(k-1)}, \xi_1(N_\delta), \xi_2(N_\delta)\right\}\right]$, then*

$$u_G(1) = \frac{(c_G^r + k\,c_U^r)\,N_\delta}{k\,(c_G^r + c_U^r)} + \frac{(k-1)\,\alpha^+}{2}.$$

• *if no such k exists, then*

$$u_G(1) = \min\left\{\frac{c_U^r N_\delta}{(c_G^r + c_U^r)} + T\alpha^+, u_m\right\}.$$

► ***Oversupplied Case*** *($N_\delta < 0$): Recall the critical parameter α^- from Proposition 1. Then,*
• *if there exists a $k \in \{1, \cdots, T\}$ such that $\frac{-\alpha^-(c_G^r + c_U^r)}{2c_G^r} \in \left(\frac{-N_\delta}{k(k+1)}, \min\left\{\frac{-N_\delta}{k(k-1)}, \tilde{\xi}_1(-N_\delta), \tilde{\xi}_2(-N_\delta)\right\}\right]$, then*

$$u_G(1) = \frac{(c_G^r + k\,\tilde{c}_U^r)\,N_\delta}{k\,(c_G^r + \tilde{c}_U^r)} + \frac{(k-1)\,\alpha^-}{2}.$$

• *if no such k exists, then*

$$u_G(1) = \min\left\{\frac{\tilde{c}_U^r N_\delta}{(c_G^r + \tilde{c}_U^r)} + T\alpha^-, -u_m\right\}.$$

Proof. The proof follows from the solution of the above minimizations after expressing $J_2^+(\cdot)$ and $J_2^-(\cdot)$ as explicit functions of their parameters. It is omitted for brevity. □

Now, with the complete solution to the problem in hand, we can identify the critical time horizon in which the conventional supplier will match the shifted load level.

Corollary 1. *For the overloaded case, let $K \in \mathbb{Z}^+$ be s.t. $K\alpha^+ < u_m \leq (K+1)\alpha^+$ and define*

$$T_{C,1} = \left\lceil \frac{1}{2}\left(\sqrt{1 + \frac{8c_G^r N_\delta}{\alpha^+ (c_G^r + c_U^r)}} - 1\right)\right\rceil,$$
$$T_{C,2} = \left\lceil \frac{2N_\delta - K(K+1)\alpha^+ + 2Ku_m}{2u_m}\right\rceil,$$

and also define the maximum allowable change in load $v(T) = \frac{c_U^r N_\delta}{(c_G^r + c_U^r)} + T\alpha^+$.
Then, we define the critical time as

$$T_C(N_\delta) \triangleq \begin{cases} T_{C,2}, & \text{if } v(T_{C,1}), v(T_{C,2}) > u_m \\ T_{C,1}, & \text{otherwise} \end{cases} \quad (8)$$

which gives the minimum time needed by the optimal controller to have the conventional generator match the net load. Critical time expression for the oversupplied case is identical with parameters α^- and $\tilde{c}_U^r$.

Interpreting it from the unconventional agent's perspective, Corollary 1 reveals exactly how much time he needs to be involved in the load following process as a function of the net load disturbance N_δ and the parameters α^+ and α^-.

Having the control problem solved for both phases, we now demonstrate the behavior of the optimal solution. In Figure 3, the trajectories of the system during a single regulation interval, in both overloaded and oversupplied scenarios, are demonstrated for different values of α^- and fixed α^+. For the overloaded and oversupplied cases, the same cost parameters that satisfy Assumption 1 are used and the change in the net load N_δ is equal in magnitude.

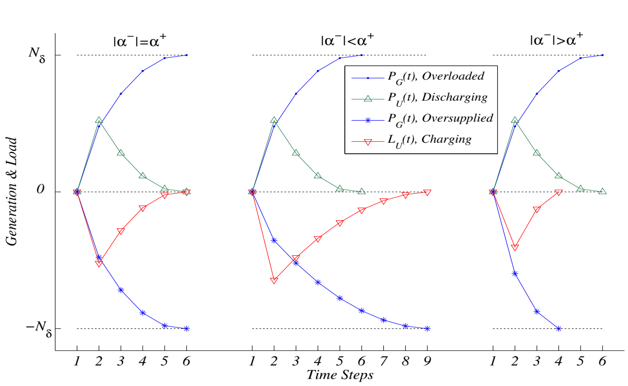

Fig. 3. Charging/discharging behavior of the unconventional system during a single regulation interval for three different values of α^- and fixed α^+.

In Figure 3, the area under a curve represents the amount of power generated or consumed by the corresponding unit. If $|\alpha^-| = \alpha^+$, the area under the unconventional system's trajectory is identical during oversupplied and overloaded cases. In the long run, this means the total charge and discharge amounts will be almost equal if the change in net load is a 0 mean random variable. For the other case where $|\alpha^-| < \alpha^+$ (resp. $|\alpha^-| > \alpha^+$), the unconventional system consumes more (resp. less) electricity by charging than it generates by discharging, consequently the battery level increases (resp. decreases) in the long run. The same interpretation applies if we assume $c_U^r = \tilde{c}_U^r$ and consider varying $\tilde{c}_U^b$ while keeping other cost parameters fixed; as $\tilde{c}_U^b$ decreases $|\alpha^-|$ decreases and unconventional system charges more than it discharges, and vice versa. The long-term behavior of the battery level is investigated numerically in Section IV.

IV. SIMULATIONS

In this section, we use simulations to investigate the behavior of the optimal controller of Section III (cf. Propositions 2 and 1) under various scenarios. We first investigate the cost reduction achievable with the presence of the unconventional system. Then, we introduce and study the battery dynamics under the optimal control rule [2].

(1) Performance Gains due to Unconventional System: The objective of this simulation study is to present the performance improvement due to the unconventional system. To that end, we compare the long-term cost experienced when only the conventional generator is operative to that when the unconventional system is also available. In both scenarios, we model the randomness in the change of net load $N(t)$ with a *normal distributed* random variable. In the latter scenario, a random load shift occurs with distribution $\sim \mathcal{N}(0, \sigma^2)$ when a control horizon ends [3]. Then, for a fair comparison, the same net load shift is assumed at the same time in the former scenario. The choice of normal distribution is justified by the central limit theorem as it is governed by the sum of many renewable energy sources and consumer demands. Yet, similar gains can be reasonably expected for other distributions.

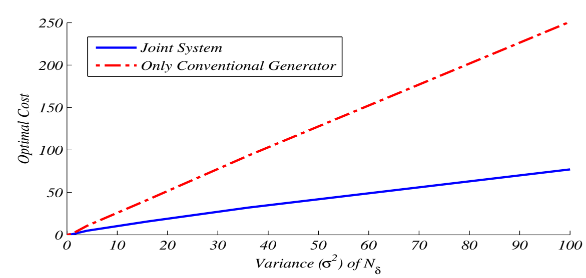

Fig. 4. Optimum cost of the proposed load following system compared to the cost of the system with only conventional generators.

In Figure 4, cost of a single regulation interval, which excludes the cost of sustaining the initial generation level

[2] In the simulations we used the following set of parameters that satisfy Assumption 1: $c_G^b = 1$, $c_G^r = 50$, $c_U^b = \tilde{c}_U^b = 10$, and $c_U^r = \tilde{c}_U^r = 10$.

[3] Similar behavior and reduction in cost can be observed in simulations with load shifts occurring at random times, for instance before previous shift is completely balanced.

$P_G(0)$ during the interval, is plotted by averaging over a large number of regulation periods for a range of σ^2 values. We observe that the costs of both systems increase with variance and the joint system under optimal control performs better as expected. Additionally, as σ^2 increases the gap between the regulation costs of the two systems increases. This is because the only-conventional system adjusts the generation level of the conventional generator as soon as the net load shifts and pays a quite large amount of ramp up/down cost.

(2) Battery Dynamics of the Unconventional System: In our design, we implicitly assumed that the unconventional system has sufficient battery charge/discharge capacity when its involvement is needed. While such an assumption may be acceptable in the short run, we must investigate whether (or under what conditions) the battery charge and discharge levels balance each other in the long run. This motivates us in this investigation to study the battery-level evolution under our optimal controller. To that end, we let $X_U(t)$ denote the total battery level of the unconventional system at time t. Then, the battery level evolves (also see [14]) as $X_U(t+1) = (\rho_s X_U(t) + \rho_c L_U(t) - \rho_d P_U(t))^+$, where $(y)^+ \triangleq \max(0, y)$, and $\rho_s, \rho_c, \rho_d \in (0, 1]$ are parameters that capture the storage, charge, and discharge losses of the battery.

The long-term evolution of the battery level is depicted in Figure 5, where we chose $\rho_s = 1$ and $\rho_c = \rho_d = 0.95$ in order to demonstrate the net effect of the optimal controller. The plot shows that when $|\alpha^-| > \alpha^+$ the battery level of the unconventional system decreases in the long-term as argued in the discussion on Figure 3.

However, in a practical implementation, it is desirable to keep the battery level close to a predetermined value X_D. Here, we present a simple dynamic mechanism to modify the degree of involvement of the unconventional system. In light of the observations on Figure 3 at the end of Section III, the idea is to dynamically adjust $\tilde{c}_U^b$, consequently α^-, at the end of each regulation interval while keeping other cost parameters and α^+ fixed. The simple update rule is:

$$\tilde{c}_U^b(t+1) = \tilde{c}_{U,pr}^b + \gamma \left(X_U(t) - X_D \right), \tag{9}$$

where $\tilde{c}_{U,pr}^b$ is the presumed baseline cost of the unconventional system while charging, and $\gamma \in \mathbb{R}_+$.

In Figure 5, battery level of the unconventional system both with dynamic baseline costs (9) and static baseline costs at level $\tilde{c}_{U,pr}^b$ is plotted for $|\alpha^-| > \alpha^+$. It is observed that the battery level is kept fairly close to the desired value by the dynamic cost adjustment scheme whereas it continuously decreases in the static cost scheme. The update rule pushes $\tilde{c}_U^b$ away from its presumed value $\tilde{c}_{U,pr}^b$ as the gap between the desired battery level and its current value increases.

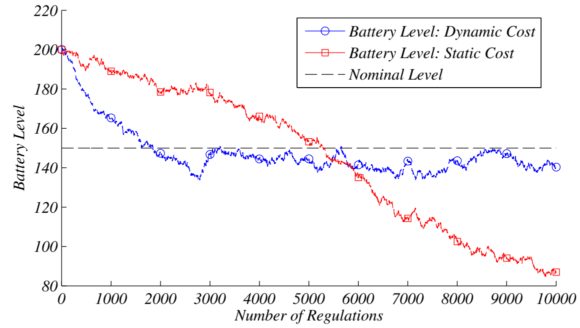

Fig. 5. Battery level evolution of the unconventional system for fixed and dynamic costs with $|\alpha^-| > \alpha^+$, and $\gamma = 0.2$ for dynamic costs.

V. Concluding Remarks

In this work, we proposed a tractable model and a closed form solution for the optimal control of a load following system with storage units. Our findings only scratch the surface of a much broader problem of the distributed and robust operation of these systems under additional dynamics and even selfish entities that react to pricing. The tractable nature of our formulation (cf. Section II), the intuitive and closed form description of our optimal controller(cf. Section III), and the preliminary results in its adaptive variant that incorporate battery dynamics (cf. Section IV) show promise in pursuing these ends.

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