

Optimal Branch Exchange for Feeder Reconfiguration in Distribution Networks

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Abstract—The feeder reconfiguration problem chooses the on/off status of the switches in a distribution network in order to minimize a certain cost such as power loss. It is a mixed integer nonlinear program and hence hard to solve. A popular heuristic search consists of repeated application of branch exchange, where some loads are transferred from one feeder to another feeder while maintaining the radial structure of the network, until no load transfer can further reduce the cost. Optimizing each branch exchange step is itself a mixed integer nonlinear program. In this paper we propose an efficient algorithm for optimizing a branch exchange step. It uses an AC power flow model and is based on the recently developed convex relaxation of optimal power flow. We provide a bound on the gap between the optimal cost and that of our solution. We prove that our algorithm is optimal when the voltage magnitudes are the same at all buses. We illustrate the effectiveness of our algorithm through the simulation of real-world distribution feeders.

I. INTRODUCTION

A. Motivation

Primary distribution systems have sectionalizing switches that connect line sections and tie switches that connect two primary feeders, two substation buses, or loop-type laterals. In normal operation these switches are configured such that a distribution network is acyclic and every load is connected to exactly one substation. The topology of the network can be reconfigured by changing the on/off status of these switches, for the purpose of load balancing, loss minimization, or service restoration; see e.g., [1]–[4] and an early survey in [5].

For instance when a single feeder is overloaded, a currently open tie switch can be closed to connect the feeder to another substation. Since this will create a loop or connect some loads to two substations, a currently closed sectionalizing switch will be opened to maintain a radial topology in which every load is connected to a single substation. Following [3] we call this a “branch exchange” where the goal is to select the pair of switches for closing/opening that achieves the best load balancing. More generally one can optimize a certain objective over the topology of the entire distribution network by choosing the on/off status of all the switches, effectively selecting a best spanning tree

among all possible spanning trees of the network topology. Even though the problem of minimum spanning tree has been well studied [6], the problem here is different. Unlike the standard minimum spanning tree problem where the link costs are fixed and the minimization is only over the topology, in our case, the link costs result from an optimal power flow (OPF) problem that must be solved for each candidate spanning tree. This is therefore a mixed integer nonlinear program and can generally be NP-hard. As a result a large majority of proposed solutions are heuristic in nature [5]; see also references in [7] to some recent works. A heuristic search method is proposed in [2], [3] which we discuss in more detail below. The problem is formulated as a multi-objective mixed integer constrained optimization in [4] and solved using a simulated annealing technique. Ordinal optimization is proposed in [7] to reduce the computational burden through order comparison and goal softening. Unlike these heuristic methods, an interesting exhaustive search method is proposed in [8] to compute a globally optimal solution under the assumption that loads are constant-current, instead of constant-power as often assumed in load flow analysis. Starting from an initial spanning tree, the proposed method applies the branch exchange step in a clever way to generate all spanning trees exactly once and efficiently compute the power loss for each tree recursively in order to find a tree with the minimum loss. A constant-current load model is also used in [9] where the optimization problem becomes a mixed integer *linear* program. A global optimality condition is derived and an algorithm is provided that attains global optimality under certain conditions.

In this paper we study a single branch exchange step first proposed in [3]. Each step transfers some loads from one feeder to another if it reduces the overall cost. An efficient solution for a single branch exchange step is important because, as suggested in [3], a heuristic approach for optimal network reconfiguration consists of repeated application of branch exchanges until no load transfer between two feeders can further decrease the cost. This simple greedy algorithm yields a local optimal. The key challenge is to estimate the cost reduction for each load transfer. Specifically once a currently open tie switch has been selected for closing, the issue is to determine which one of several currently closed sectionalizing switches should be opened that will provide the largest cost reduction. Each candidate sectionalizing switch (together with the given tie switch) transform the existing spanning tree into a new spanning tree. A naive approach will solve an OPF for each of the candidate spanning

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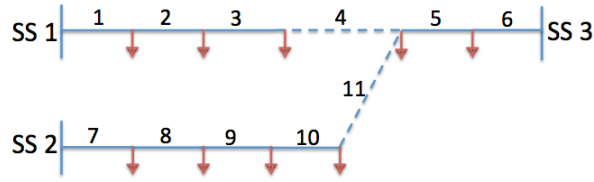


Fig. 1: A distribution network. Solid Lines are closed and dash lines are open. The red arrows are load buses.

tree and choose one that has the smallest cost. This may be prohibitive both because the number of candidate spanning trees can be large and because OPF is itself a nonconvex problem and therefore hard to solve. The focus of [2], [3], [10] is to develop much more efficient ways to approximately evaluate the cost reduction by each candidate tree without solving the full power flow equations. The objective of [2] is to minimize loss and it derives a closed-form expression for approximate loss reduction of a candidate tree. This avoids load flow calculation altogether. A new branch flow model for distribution systems is introduced in [3] that allows a recursive computation of cost reduction by a candidate tree. This model is extended to unbalanced systems in [10].

B. Summary

We make two contributions to the solution of branch exchange. First we propose a new algorithm to determine the sectionalizing switch whose opening will yield the largest cost reduction, once a tie switch has been selected for closing. We use the full AC power flow model introduced in [11] for radial systems, but unlike [3], [4], [10], we solve them through the method of convex relaxation developed recently in [12], [13]. Moreover the algorithm requires solving at most three OPF problems regardless of the number of candidate spanning trees. Second we bound the gap between the cost of our algorithm and the optimal cost. We prove that when the voltage magnitude of each bus is the same our algorithm is optimal. We illustrate our algorithm on two Southern California Edison (SCE) distribution feeders, and in both cases, our algorithm has found the optimal branch exchange.

The rest of the paper is organized as follows. We formulate the optimal feeder reconfiguration problem in Section II and propose an algorithm to solve it in Section III. The performance of the algorithm is analyzed in Section IV. The simulation results on SCE distribution circuits are given in Section V. We conclude in Section VI. All the proof are skipped due to space limitation and can be found in [14].

II. MODEL AND PROBLEM FORMULATION

A. Feeder reconfiguration

A distribution system consists of buses, distribution lines, and (sectionalizing and tie) switches that can be opened or closed. There are two types of buses. *Substation buses* (or just *substations*) are connected to a transmission network from which they receive bulk power, and *load buses* that receive power from the substation buses. During normal operation the switches are configured so that

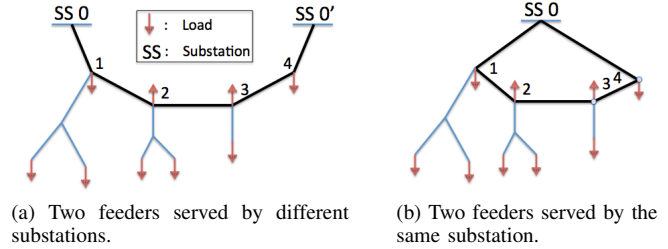


Fig. 2: Feeders after step 1 of a branch exchange.

- 1) The network is radial, i.e., has a tree topology.
- 2) Each bus is connected to a single substation.

We will refer to the subtree that is rooted at each substation bus as a feeder; hence each feeder is served by a single substation. Optimal feeder reconfiguration is the problem of reconfiguring the switches to minimize a certain cost subject to the two constraints above, in addition to operational constraints on voltage magnitudes, power injections, and line limits.

We assume that there is an on/off switch on each line (i.e., modeling the subsystem between each pair of switches as a single line), and focus on an iterative greedy algorithm first proposed in [3]. We illustrate this algorithm on the simple network shown in Fig. 1 where solid and dash lines represent closed and open switches respectively.

There are 3 feeders, each of which connects to one substation, SS1, SS2, or SS3. Suppose lines 4 and 11 are open in the current iteration. In each iteration one of the open switches is selected and closed, say, that on line 4. This joins two feeders so that every bus along lines 1 to 6 are now connected to both substations SS1 and SS3. To restore the property that each bus is connected to a single substation, we then choose one line among $\{1, 2, 3, 4, 5, 6\}$ to open that minimizes the cost. This two-step procedure is called a branch exchange. This procedure is repeated until the configuration stabilizes, i.e., the line that is chosen to open in step two is the original open line selected in step one, for all open switches. In summary, each iteration of the algorithm consists of two steps:

- 1) Chooses a line e_1 with an open switch and close the switch.
- 2) Identify a line e_2 in the two feeders that was joined in Step 1 to open that minimizes the objective.

The algorithm terminates when $e_1 = e_2$ for all the open switches. The greedy search only guarantees a local optimum since it may terminate before searching through all spanning trees. In this paper we propose an efficient and accurate method to accomplish Step 2 in each branch exchange (iteration). We will use the nonlinear (AC) power flow model and apply convex relaxations developed recently for its solution. Most existing algorithms that we are aware of perform Step 2 based on linearized power flow equations and the assumption that the power injection at all the buses are fixed, [2], [3]. Linearized power flow (called DC power flow)

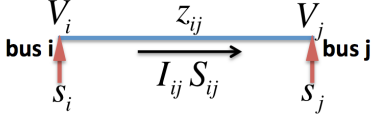


Fig. 3: Notations.

model is reasonable in transmission networks but is less so in distribution networks.

After we close the switch on a line there are two possible cases (see Fig 2): (1) The two connected feeders are served by different substations; or (2) The two connected feeders are served by the same substation. In both cases the switch on one of the lines needs to be opened. Case (2) can be reduced to case (1) by replacing the substation 0 by two virtual substations 0 and 0' as shown in Fig. 2a.

We now describe our model and formulate the problem of determining the optimal line to open along the path that connects two substations.

B. Network model

We consider an AC power flow model where all variables are complex. A distribution network is denoted by a graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$, where nodes in \mathcal{N} represent buses and edges in \mathcal{E} represent distribution lines. For each bus $i \in \mathcal{N}$, let $V_i = |V_i|e^{i\theta_i}$ be its complex voltage and $v_i := |V_i|^2$ be its magnitude squared. Let $s_i = p_i + iq_i$ be its net power injection which is defined as generation minus consumption. We associate a direction with each line $(i, j) \in \mathcal{E}$ represented by an ordered pair of nodes in \mathcal{N} . For each line $(i, j) \in \mathcal{E}$, let $z_{ij} = r_{ij} + ix_{ij}$ be its complex impedance and $y_{ij} := 1/z_{ij}$ its admittance. We have $x_{ij} > 0$ since lines are inductive. Let I_{ij} be the complex branch current from buses i to j and $\ell_{ij} := |I_{ij}|^2$ be its magnitude squared. Let $S_{ij} = P_{ij} + iQ_{ij}$ be the branch power flow from buses i to j . For each line $(i, j) \in \mathcal{E}$, define S_{ji} in terms of S_{ij} and I_{ij} by $S_{ji} := -S_{ij} + \ell_{ij}z_{ij}$. Hence $-S_{ji}$ represents the power received by bus j from bus i . The notations are illustrated in Fig. 3. A variable without a subscript denotes a column vector with appropriate components, as summarized below.

$p := (p_i, i \in \mathcal{N})$	$q := (q_i, i \in \mathcal{N})$
$P := (P_{ij}, (i, j) \in \mathcal{E})$	$Q := (Q_{ij}, (i, j) \in \mathcal{E})$
$v := (v_i, i \in \mathcal{N})$	$\ell := (\ell_{ij}, (i, j) \in \mathcal{E})$

In radial network, it suffices to work with a ‘relaxed’ model, first proposed in [11] to model radial network, where we ignore the phase angles of voltages and currents and use only the variables $x := (p, q, P, Q, \ell, v)$. These variables satisfy:

$$p_i = - \sum_{(k,i) \in \mathcal{E}} (P_{ki} - \ell_{ki}r_{ki}) + \sum_{(i,j) \in \mathcal{E}} P_{ij}, \quad i \in \mathcal{N} \quad (1)$$

$$q_i = - \sum_{(k,i) \in \mathcal{E}} (Q_{ki} - \ell_{ki}x_{ki}) + \sum_{(i,j) \in \mathcal{E}} Q_{ij}, \quad i \in \mathcal{N} \quad (2)$$

$$v_j = v_i - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) + \ell_{ij}|z_{ij}|^2, \quad (i, j) \in \mathcal{E} \quad (3)$$

$$\ell_{ij}v_i = P_{ij}^2 + Q_{ij}^2, \quad (i, j) \in \mathcal{E} \quad (4)$$

Given a vector x that satisfies (1)–(4) the phase angles of the voltages and currents can be uniquely determined for a radial network, and therefore this relaxed model (1)–(4) is equivalent to the full AC power flow model for a radial network; See [12] for details.

In addition, x must also satisfy the following operational constraints:

- *power injection constraints*: for each bus $i \in \mathcal{N}$

$$\underline{p}_i \leq p_i \leq \bar{p}_i \quad \text{and} \quad \underline{q}_i \leq q_i \leq \bar{q}_i \quad (5)$$

- *voltage magnitude constraints*: for each bus $i \in \mathcal{N}$

$$\underline{v}_i \leq v_i \leq \bar{v}_i \quad (6)$$

- *line flow constraints*: for each line $(i, j) \in \mathcal{E}$

$$|S_{ij}| \leq \bar{S}_{ij} \quad (7)$$

C. Problem formulation

As described in section II-A, we focus on reconfiguring a network path where two feeders are served by two different substations as shown in Fig. 2a. Consider a (connected) tree network $\mathcal{G}(\mathcal{N}, \mathcal{E})$. $\mathcal{N} := \{0, 1, \dots, n, 0'\}$ denote the set of buses, where the two substations are indexed by 0, 0' and the load buses are indexed by $\{1, \dots, n\}$.

Since \mathcal{G} is a tree there is a unique path between any two nodes in \mathcal{G} . For every pair of buses $i, j \in \mathcal{N}$ let $\mathcal{E}(i, j) \subseteq \mathcal{E}$ be the collection of edges on the unique path between i and j . Given any subgraph \mathcal{G}' of \mathcal{G} let $x^{\mathcal{G}'} := (p^{\mathcal{G}'}, q^{\mathcal{G}'}, P^{\mathcal{G}'}, Q^{\mathcal{G}'}, \ell^{\mathcal{G}'}, v^{\mathcal{G}'})$ denote the set of variables defined on \mathcal{G}' with appropriate dimensions. For notational simplicity we often ignore the superscript \mathcal{G}' and write $x := (p, q, P, Q, \ell, v)$ instead when the meaning is clear from the context. Given any subgraph \mathcal{G}' of \mathcal{G} , let $\mathbb{X}(\mathcal{G}') := \{x^{\mathcal{G}'} \mid x^{\mathcal{G}'}$ satisfies (1)–(7)\} be the feasible set of variables x defined on \mathcal{G}' . In particular, $\mathbb{X}(\mathcal{G})$ is the feasible set for the entire distribution network represented by \mathcal{G} .

Given a (connected) tree $\mathcal{G}(\mathcal{N}, \mathcal{E})$ and a path $\mathcal{E}(0, 0')$ between node 0 and 0', denote by $\mathcal{G}_0^i(\mathcal{N}_0, \mathcal{E}_0)$ and $\mathcal{G}_{0'}^j(\mathcal{N}_{0'}, \mathcal{E}_{0'})$ the two subtrees after we remove line $(i, j) \in \mathcal{E}(0, 0')$, where $0 \in \mathcal{N}_0$ and $0' \in \mathcal{N}_{0'}$. The minimum power injections for \mathcal{G}_0^i and $\mathcal{G}_{0'}^j$ are defined as

$$p_0^i := \min_{x \in \mathbb{X}(\mathcal{G}_0^i)} p_0 \quad (8)$$

$$p_{0'}^j := \min_{x \in \mathbb{X}(\mathcal{G}_{0'}^j)} p_{0'} \quad (9)$$

The optimal branch exchange for feeder reconfiguration problem is defined as:

$$\text{OFR-branch (OFR):} \quad \min_{(i,j) \in \mathcal{E}(0,0')} \Gamma(p_0^i, p_{0'}^j)$$

where $\Gamma(p_0, p_{0'})$ can be any convex increasing cost function. When $\Gamma(p_0, p_{0'}) = p_0 + p_{0'}$, our goal is to minimize the aggregate power injection from the substations. Since $p_0 + p_{0'}$ equals the aggregate load (real power consumption) in the network and the total real power loss, if the loads are fixed, then minimizing $p_0 + p_{0'}$ also minimizes power loss. For

simplicity we will also refer to OFR-branch as OFR in this paper.

A naive solution to OFR is to enumerate all the lines in $\mathcal{E}(0, 0')$ and compare the objective value for each case. It is inefficient as it requires solving two optimal power flow (OPF) problems (8) and (9) for each line. This can be computationally expensive if the size of $\mathcal{E}(0, 0')$ is large. In the following we will develop an algorithm to solve OFR that involves solving at most three OPF problems regardless of the size of $\mathcal{E}(0, 0')$.

We start by briefly describing SOCP (second-order cone program) relaxation of OPF recently developed in [12], [13].

D. OPF and convex relaxation

The optimal power flow problem seeks to optimize a certain objective over the feasible set $\mathbb{X}(\mathcal{G})$ specified by the power flow equations (1)-(4) and the operation constraints (5)-(7):

$$\text{OPF-}\mathcal{G}: \min_{x \in \mathbb{X}(\mathcal{G})} \Gamma(p_0, p_{0'})$$

It is a non-convex problem due to the quadratic equalities (4). Relaxing (4) to inequalities:

$$\ell_{ij} v_i \geq P_{ij}^2 + Q_{ij}^2 \quad (10)$$

leads to a second order cone program (SOCP) relaxation. Formally define $\mathbb{X}_c(\mathcal{G}) := \{x \mid x \text{ satisfies (1) - (3), (5) - (7), (10)}\}$. The SOCP relaxation of OPF- \mathcal{G} is:

$$\text{SOPF-}\mathcal{G}: \min_{x \in \mathbb{X}_c(\mathcal{G})} \Gamma(p_0, p_{0'})$$

SOPF- \mathcal{G} is convex and can be solved efficiently. Clearly SOPF- \mathcal{G} provides a lower bound for OPF- \mathcal{G} since $\mathbb{X} \subseteq \mathbb{X}_c$. It is called *exact* if every solution x^* of SOPF- \mathcal{G} attains equality in (10). For radial networks SOCP relaxation is exact under some mild conditions [12], [13], [15].

Throughout this paper we will assume that the SOCP relaxation of OPF is always exact. In that case we can solve SOPF- \mathcal{G} and recover an optimal solution to the original non-convex OPF- \mathcal{G} . A similar approach can be applied to the OPF problems defined in (8) and (9).

III. OPTIMAL FEEDER RECONFIGURATION ALGORITHM

OFR seeks to minimize $\Gamma(p_0, p_{0'})$ by opening the switch on a line in $\mathcal{E}(0, 0')$. Let $k_0, k_{0'} \in \mathcal{N}$ denote the buses such that $(0, k_0), (k_{0'}, 0') \in \mathcal{E}(0, 0')$. The algorithm for OFR is given in Algorithm 1.

The basic idea of Algorithm 1 is simple and we illustrate it using the line network in Fig. 4. After we solve OPF- \mathcal{G} with x^* :

- 1) if bus 0 receives positive real power from bus 1 through line $(0, 1)$, open line $(0, 1)$.
- 2) if bus $0'$ receives positive real power from bus n through line $(n, 0')$, open line $(n, 0')$.
- 3) if there exists a line $(k, k+1)$ where positive real power is injected from both ends, open line $(k, k+1)$.
- 4) if there exists a bus k that receives positive real power from both sides, open either line $(k-1, k)$ or $(k, k+1)$.

Algorithm 1: Optimal Feeder Reconfiguration Algorithm

Input: objective $\Gamma(p_0, p_{0'})$, network constraints $(\bar{p}, \bar{q}, \bar{g}, \bar{S}, \bar{v}, \bar{v})$.

Output: line e^* .

Solve OPF- \mathcal{G} ; let x^* be an optimal solution.

- 1) $P_{0, k_0}^* \leq 0$: $e^* \leftarrow (0, k_0)$.
- 2) $P_{k_{0'}, 0'}^* \geq 0$: $e^* \leftarrow (k_{0'}, 0')$.
- 3) $\exists (k_1, k_2) \in \mathcal{E}(0, 0')$ such that $P_{k_1 k_2}^* \geq 0$ and $P_{k_2 k_1}^* \geq 0$: $e^* \leftarrow (k_1, k_2)$.
- 4) $\exists (k_1, k_2), (k_2, k_3) \in \mathcal{E}(0, 0')$ such that $P_{k_2, k_1}^* \leq 0$ and $P_{k_2, k_3}^* \leq 0$. Calculate $p_0^{k_1}, p_{0'}^{k_2}, p_0^{k_2}$ and $p_{0'}^{k_3}$.
 - $\Gamma(p_0^{k_1}, p_{0'}^{k_2}) \geq \Gamma(p_0^{k_2}, p_{0'}^{k_3})$: $e^* \leftarrow (k_2, k_3)$.
 - $\Gamma(p_0^{k_1}, p_{0'}^{k_2}) < \Gamma(p_0^{k_2}, p_{0'}^{k_3})$: $e^* \leftarrow (k_1, k_2)$.

We are interested in the performance of Algorithm 1, specifically:

- Is the solution x^* to OPF- \mathcal{G} unique and satisfies exactly one of the cases 1) - 4)?
- Is the line e^* returned by Algorithm 1 optimal for OFR?

We next state our assumptions and answer these two questions under those assumptions.

IV. PERFORMANCE OF ALGORITHM 1

For ease of presentation we only prove the results for a line network as shown in Fig. 4. They generalize in a straightforward manner to radial networks.

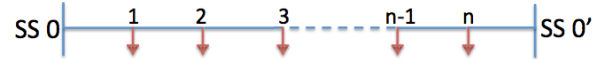


Fig. 4: A line Network

Our analysis is divided into two parts. First we show that, OPF- \mathcal{G} has a unique solution x^* and it satisfies exactly one of the cases 1) - 4) in Algorithm 1. This means that Algorithm 1 terminates correctly. Then we prove that the performance gap between the solution e^* given by Algorithm 1 and an optimum of OFR is zero when the voltage magnitude of every bus is fixed at the same nominal value, and bound the gap by a small value when the voltage magnitudes are fixed but different.

A. Assumptions

For the line network in Fig. 4, let the buses at the two ends be substation buses and buses in between be load buses. Hence the path between substations 0 and $0'$ is $\mathcal{E}(0, 0') = \mathcal{E}$; we sometimes use $0'$ and $n+1$ interchangeably for ease of notation. We collect the assumptions we need as follows:

- A1 : $\bar{p}_k < 0$ for $1 \leq k \leq n$ and $\bar{p}_k > 0$ for $k = 0, 0'$.
- A2 : $\bar{v}_k = \bar{v}$, $\bar{q}_k = -\bar{q}_k = \infty$ for $k \in \mathcal{N}$.
- A3 : $|\theta_i - \theta_j| < \arctan(x_{ij}/r_{ij})$ for $(i, j) \in \mathcal{E}$.
- A4 : The feasible set $\mathbb{X}(\mathcal{G})$ is compact.

A1 is a key assumption and it says that buses 0 and $0'$ are substation buses that inject positive real power while buses $1, \dots, n$ are load buses that absorb real power. A2 says that the voltage magnitude at each bus is fixed at

their nominal value. To achieve this we also require that the reactive power injections are unconstrained. This is a reasonable approximation for our purpose since there are Volt/VAR control mechanisms on distribution networks that maintain voltage magnitudes within a tight range around their nominal values as demand and supply fluctuate. Our simulation results on real SCE feeders show that the algorithm also works well without A2. A3 is a technical assumption and usually satisfied in distribution systems that, together with A2, guarantees that SOCP relaxation of OPF is exact [15].¹ A4 is an assumption that is satisfied in practice and guarantees that our optimization problems are feasible.

B. Main results

Algorithm 1 needs to solve up to three OPF problems. The result of [15] implies that we can solve these problems through their SOCP relaxation.

Theorem 1: Suppose A2 and A3 hold. Then, for any subgraph \mathcal{G}' of \mathcal{G} (including \mathcal{G} itself),

- 1) SOPF- \mathcal{G}' is exact provided the objective function $\Gamma(p)$ is a convex nondecreasing function of p .
- 2) OPF- \mathcal{G}' has a unique solution provided the objective function $\Gamma(p)$ is convex in p .

The next result says that Algorithm 1 terminates correctly because any optimal solution of OPF- \mathcal{G} will satisfy exactly one of the four cases in Algorithm 1.

Theorem 2: Suppose A1 holds. Given any solutions x^* of OPF- \mathcal{G} , exactly one of the following holds:

- C1 : $P_{0,1}^* \leq 0$.
- C2 : $P_{n,0'}^* \geq 0$.
- C3 : $\exists! k \in \mathcal{N}$ such that $P_{k,k+1}^* \geq 0$ and $P_{k+1,k}^* \geq 0$.
- C4 : $\exists! k \in \mathcal{N}$ such that $P_{k,k-1}^* \leq 0$ and $P_{k,k+1}^* \leq 0$.

When the voltage magnitude of all the buses are fixed at the same reference value, e.g. 1 p.u., Algorithm 1 finds an optimal solution to OFR.

Theorem 3: Suppose A1–A4 hold. If the voltage magnitudes of all buses are fixed at the same value, then the line e^* returned by Algorithm 1 is optimal for OFR.

When the voltage magnitudes are fixed but different at different buses, Algorithm 1 is not guaranteed to find a global optimum of OFR. However it still gives an excellent suboptimal solution. By nearly optimal, it means the suboptimality gap of Algorithm 1 is negligible.

Define L_k for each line $(k, k+1) \in \mathcal{E}$ as sequel.

$$L_k := \frac{\delta v_k^2 r_{k,k+1} / |z_{k,k+1}|^2}{(v_k + v_{k+1}) + \sqrt{(v_k + v_{k+1})^2 - \delta v_k^2 \left(\frac{r_{k,k+1}^2}{x_{k,k+1}^2} + 1 \right)}}$$

where $\delta v_k := v_k - v_{k+1}$. L_k represents the thermal loss of line $(k, k+1)$ when either $P_{k,k+1}$ or $P_{k+1,k}$ is 0. Conceptually it means all the real power sending from bus on one end of the line is converted to thermal loss and the other bus receives 0 real power, namely either $P_{k,k+1} =$

¹Although voltage phase angles θ_i are relaxed in the relaxed branch flow model (1)–(4), they are uniquely determined by $\theta_i - \theta_j = \angle(v_i - z_{ij}^* S_{ij})$ in a radial network [12].

TABLE I: The aggregate power injection from substation 1 for each configuration

Opened line	(1, 2)	(2, 4)	(4, 20)	(20, 23)
Power injection (MW)	3.8857	3.8845	3.8719	3.8718
Opened line	(23, 25)	(25, 26)	(26, 32)	(32, 1)
Power injection (MW)	3.8719	3.8721	3.8755	3.9550

$\ell_{k,k+1} r_{k,k+1}$ or $P_{k+1,k} = \ell_{k,k+1} r_{k,k+1}$. Then the expression of $L_k = \ell_{k,k+1} r_{k,k+1}$ can be obtained by substituting either $P_{k,k+1} = \ell_{k,k+1} r_{k,k+1}$ or $P_{k+1,k} = \ell_{k,k+1} r_{k,k+1}$ into (3) and (4). L_k is negligible compared to the power consumption of a load in a distribution system. Therefore the ratio of these two quantity, defined as $R_k := -\bar{p}_{k+1}/L_k$, is usually quite large. To obtain a suboptimality bound, we also need to define one OPF problem as follows.

OPF- \mathcal{G}_s : $f(p_0) := \min_{x \in \mathbb{X}(\mathcal{G})} p_{0'}$ s.t. p_0 is a constant

Let $I_{p_0} := \{p_0 \mid \exists x \in \mathbb{X}(\mathcal{G})\}$ represent the projection of $\mathbb{X}(\mathcal{G})$ on real line. I_{p_0} is compact since $\mathbb{X}(\mathcal{G})$ is compact by A4. In [14], we show that $f(p_0)$ is a strictly convex decreasing function of p_0 under assumption A2–A4. Thus it is right differentiable and denote its right derivative by $f'_+(p_0)$, which is monotone increasing and right differentiable and denote its right derivative by $f''_{++}(p_0)$. Let

$$\kappa_f := \inf_{p_0 \in I_{p_0}} f''_{++}(p_0) \geq 0. \quad (11)$$

κ_f represents the minimal value of the curvature on a compact interval if $f(p_0)$ is twice differentiable.

Let $R := \min R_k$ and κ_f as defined in (11). Let Γ^* be the optimal objective value of OFR and Γ_A be the objective value if we open the line e^* given by Algorithm 1.

Theorem 4: Suppose A1–A4 hold. Then

$$\Gamma^* \leq \Gamma_A \leq \Gamma^* + \max \left\{ \frac{c_0^2}{c_{0'}}, \frac{c_{0'}^2}{c_0} \right\} \frac{2}{R^2 \kappa_f},$$

if $\Gamma(p_0, p_{0'}) := c_0 p_0 + c_{0'} p_{0'}$ for some positive $c_0, c_{0'}$.

Remark: R is large, usually on the order of 10^3 , in a distribution system if there is no renewable generation. Although it is difficult to estimate the value of κ_f in theory, our simulation shows that κ_f is typically around $0.025 MW^{-1}$ for a feeder with loop size of 10, thus the bound is approximately 80W if $c_0 = c_{0'} = 1$, which is quite small. Moreover simulations of two SCE distribution circuits show that Algorithm 1 always finds the global optima of OFR problem; see section V. Therefore the bound in the theorem, already negligible, is not always tight.

V. SIMULATION

In this section we present an example to illustrate the effectiveness of Algorithm 1. The simulation is implemented using the CVX optimization toolbox [16] in Matlab. We use a 56-bus SCE distribution feeder whose circuit diagram is shown in Fig. 5. The network data, including line impedances and real power demand of loads, are listed in [14, Table II]. Since there is no loop in the original feeder we added a

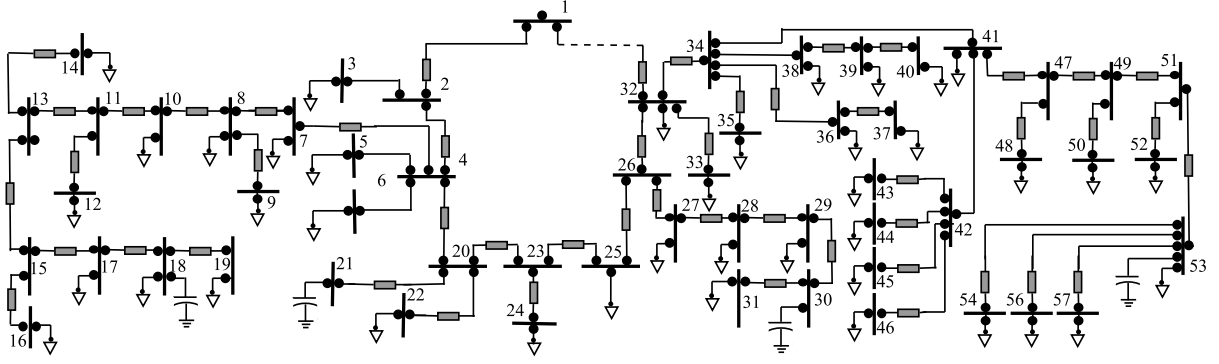


Fig. 5: A modified SCE 56 bus feeder. Bus 1 is the substation and line (1, 32) is added.

tie line between bus 1 and bus 32, which is assumed to be initially open.

In our simulation the voltage magnitude of the substation (bus 1) is fixed at 1 p.u.. We relax the assumption needed for our analysis that their voltage magnitudes at all other buses are fixed and allow them to vary within $[0.97, 1.03]$ p.u. The demand of real power is fixed for each load and described in [14, Table II]. The reactive power at each bus, which is kept within 10% of the real power to maintain a power factor of at least 90%, is a control variable, as in volt/var control.

We use the aggregate power injection as our objective, $\Gamma(p_0, p_{0'}) := p_0 + p_{0'}$. It also represents the power loss in this case since we have fixed real power demand of each load. Our addition of the line between buses 1 and 32 creates a loop 1-2-4-20-23-25-26-32-1 that must be broken by turning off the switch on one line from $\{(1, 2), (2, 4), (4, 20), (20, 23), (23, 25), (25, 26), (26, 32), (32, 1)\}$. In Table I we list the corresponding aggregate power injection for all the possible configurations. The optimal configuration is to open line (20, 23) at an optimal cost of 3.8718 MW. After we run Algorithm 1 bus 23 receives real power from both sides and our algorithm returns line (20, 23), which is the optimal solution to OFR.

Even though the voltage magnitudes in our simulation are not fixed at the nominal values as assumed in our analysis, Algorithm 1 still gives the optimal solution to OFR by solving a convex relaxation of OPF. The underlying reason is that the voltage magnitude does not vary much between adjacent buses in real network, hence the performance of the algorithm is not limited by the assumption of fixed voltage magnitudes.

We have also tested our algorithm in another SCE 47 bus distribution feeder and it again yields the optimal solution to OFR.

VI. CONCLUSION

We have proposed an efficient algorithm to optimize the branch exchange step in feeder reconfiguration, based on SOCP relaxation of OPF. We have derived a bound on the suboptimality gap and argued that it is very small. We have proved that the algorithm computes an optimal solution when all voltage magnitudes are the same. We have demonstrated

the effectiveness of our algorithm through simulations of real-world feeders.

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