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An Open-Source Optimization Tool for Microgrid Energy Systems

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Statutory declaration

Hiermit erkläre ich, dass ich die vorliegende Arbeit selbstständig und eigenhändig sowie ohne unerlaubte fremde Hilfe und ausschließlich unter der Verwendung der aufgeführten Quellen und Hilfsmittel angefertigt habe.

Berlin, Thursday 7th February, 2019

MAXIMILIAN EISSLER

Abstract

In this thesis an optimization tool for microgrid energy systems is designed and implemented in a modular, scalable architecture. It is then applied to a case study set in a village in Northrhine-Westphalia which is due to be destroyed by lignite mining. The results clearly show the commercial potential for the users of the proposed system, provided of course the village is not razed by RWE in 2023, which would impact the profitability of the investment very negatively. The case study furthermore indicates the potential of a decentralized energy system, especially as the investment costs of the applied technologies further decrease, and 'energy quality' factors, such as CO_2 emissions, are becoming increasingly more relevant.

Zusammenfassung

In dieser Abschlussarbeit wird ein Optmierungswerkzeug für Microgrid-Energiesysteme entworfen und in einer modularen und skalierbaren Software-Architektur implementiert. Es wird dann für eine Fallstudie an einem nordrhein-westfälischen Dorf angewendet, welches voraussichtlich durch Braunkohleabbau zerstört werden wird. Die Ergebnisse zeigen deutlich, dass die Nutzer des simulierten Systems wirtschaftlich davon profitieren würden, natürlich unter der Voraussetzung, dass es nicht mit dem Rest des Dorfes 2023 zerstört wird, was seiner Profitabilität erheblich schaden würde. Die Fallstudie zeigt außerdem das Potential einer gänzlich dezentralen Stromversorgung, besonders unter Berücksichtigung fallender Preise der eingesetzten Technologien und der steigenden Relevanz von Faktoren wie CO_2 -Ausstoß.

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1 Introduction

To achieve the highly ambitious goal of the Paris Agreement to keep global temperature increase ‘well below 2 degrees Celsius’ [“The Paris Agreement” 2018] in this century, an unparalleled transformation of the energy sector is required. Germany sees itself as a leader in this effort, as one of the first countries to encourage a large scale shift of electricity generation towards renewable sources. However, the country is presently set to miss its 2020 goals for CO_2 mitigation [Oei et al. 2018]. While Germany is phasing out relatively CO_2 -neutral nuclear power [“Nuclear Power in Germany” 2019], over 30 percent of electricity production still comes from coal and lignite plants, which are expected to remain running until 2035. Experts claim a coal exit by 2030 is necessary for the Paris goals to remain attainable [Yanguas Parra et al. 2018], and a severe reduction necessary to achieve the national goals for CO_2 -mitigation already set [Göke et al. 2018].

Despite this, the continually falling costs for small scale energy generation equipment, such as photovoltaics (pv) panels and fuel cells, and the rising cost of electricity have spawned a new movement favoring decentralized energy generation. This trend is further propelled by ever more affordable energy storage devices, which enable solar energy to be used throughout the night.

The concept of a microgrid proposes to take the idea of decentralization a step further by creating local autonomous grids that only interact with the main grid if necessary or advantageous. A microgrid in literature is defined as a small geographically bounded zone with clear electrical boundaries, which manages local loads and possibly contains generation units and storage. Furthermore it can be connected to the grid, from whose perspective it is seen as a single entity (see 2.4).

Proponents of the idea promise a more inclusive and resilient power supply with no single points of failure. In addition to this, the electricity grid needed to balance the few loads the microgrids cannot manage themselves would be significantly less expensive in investment and maintenance, and the consumption of locally produced electricity would lead to far smaller transmission losses.

However, critics point out that microgrid infrastructure and especially the required generation and storage units are expensive, while centralized generation and transmission infrastructure already exists. They argue that even if microgrid operators are able to be profitable it is by avoiding taxes and levies on power, which make up over half of the German retail power prices [“Monitoringbericht 2018” 2018], and not by actually being competitive with centralized generation [Heindl et al. 2014].

So far, the concept is being tested in a few case studies such as in Brooklyn [Mengelkamp et al. 2018] and on several North American university campuses [Chenoweth 2018], but still little is known about its performance in applications ranging from residential areas to hospitals.

Although there will no doubt be more pilot projects in the coming years, assuming the trends outlined above remain unbroken, there is a demand for a more thorough and intuitive understanding of the performance of such a piece of infrastructure under various conditions without substantial risk involved.

Mathematical models have proven a strong tool for a better understanding of many complex systems in the sciences but also in all engineering disciplines. Among them, Linear Programming (LP) has long been used to model electricity systems due to its high performance even for complex problems. Linear Programming (or Linear Optimization) is a method for solving mathematical models whose features can be expressed as linear relationships [Goodarzi et al. 2014]. The solution space of such a model is a convex polytope, which makes finding the optimal solution much faster, since there cannot be any local maximum, that is not also a global maximum. As subcategory of LP is Mixed Integer Linear Programming (MILP), which in addition to the linear relationships and variables used in LP introduces variables which can only take integer values. While this is slower, an optimal solution can be found reasonably quick by using the solution of the corresponding linear problem and applying the branch- and bound algorithm ["Mixed-Integer Programming (MIP) - A Primer on the Basics" 2018].

In this thesis my aim is to develop a flexible modeling tool suitable for optimizing a microgrid setup under diverse conditions. In addition to modeling any kind of small scale generation or storage unit, the modeling tool should be able to reflect trade between connected entities as well as behaviour such as load shifting and curtailment. The objective of the model will be to meet the demand of all connected entities at a minimum cost. In addition to these functionalities at a small solving time, the modeling tool should meet the criteria of being scalable, flexible in regard to the form and quality of input data and as user-friendly as possible.

To demonstrate the modeling tools performance, a case study will be conducted for a small community in the German state of Northrhine-Westphalia, where also the majority of the German lignite capacity is located. The robustness of the results will be confirmed through a sensitivity analysis. I expect the case study to show whether such a system is currently commercially viable in Northrhine-Westphalia and, to a lesser extent, if decentralized energy management in the form of microgrids is competitive compared to the prevailing centralized solution. All code written, as well as all other documents, including this one, will be published on github.com under an MIT license ["The MIT License" 2018]. In the following chapters a literature review will be conducted to reflect the current state of microgrid research. Then the model constructed will be described in mathematical terms, followed by a brief overview of the technical implementation. Finally, a case study including a sensitivity analysis is conducted, before conclusions are drawn from its result.

2 Literature Review

In this section I will give an overview over the current status quo regarding the optimization of microgrids and more specifically the tools available to do so. I will pursue this by answering a number of broad question, which I think are crucial regarding my subject. The procedure will be structured by utilizing a consistent and reproducible methodology described in detail below.

2.1 Key Questions & Methodology

This literature review will attempt to answer the following key questions in the context of my subject:

1. What are the most eminent scientific standards for modeling a microgrid allocation and dispatch?
2. What is usually within the scope of such a model (what types of generation assets, only electricity or also thermal energy, etc.)?
3. What are the common methods used to determine some of the key variables required in such a model (interest rates, CAPEX and OPEX of assets, etc.) ?
4. What are the most commonly used (commercial) tools for optimizing a microgrid? Are there any free or open source solutions?

To arrive at a reproducible dataset of scientific literature, I will use the methodology described in Petersen et al. 2008 and Petersen et al. 2018:

1. Define a search string.
2. Choose scientific databases to which to apply that search string.
3. Due to the possibly large amount of papers brought up by this kind of search I am only considering the 100 most relevant papers from each database.
4. Define keywords that have to occur in the abstracts of the publications. All publications that lack a keyword are discarded.
5. Define inclusion as well as exclusion criteria. A publication must satisfy all inclusion criteria as

well as none of the exclusion criteria to be included in the literature review.

Due to the nature of some of the questions I am trying to answer in my literature review it is additionally necessary to include further non-scientific sources at my discretion. The search strings, databases, and the result at each step is be included in the appendix.

2.2 Descriptive Analysis

After filtering the original dataset of 275 unique publications in the way described in the last chapter, I arrive at a set of 61 publications. The publication year, as can be observed in Figure 1, is for most publications quite recently: 27 out of 61 papers were published 2017 or later. This could indicate a rising interest in the subject, but is probably at least partly due to the way the different search engines employed compute relevancy.

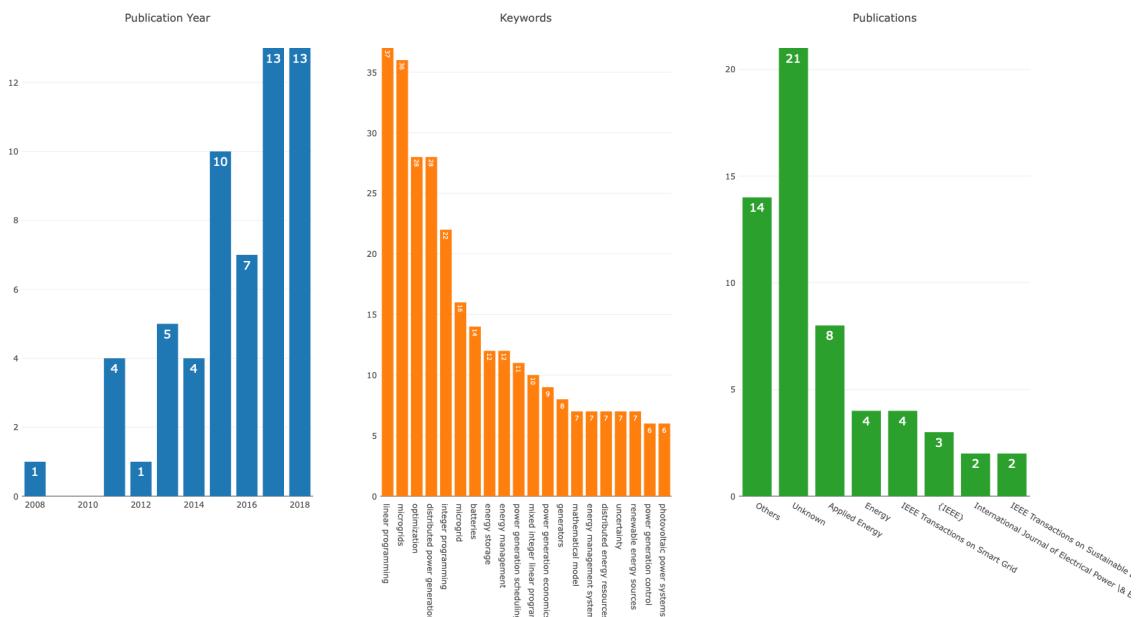


Figure 1: Results of an automated descriptive analysis

Source: Own illustration.

Looking at the keywords used to describe the publication shows that, unsurprisingly, the most frequently occurring keywords are the ones used in conducting the search: 'linear programming', 'microgrids' and 'optimization'. 'Distributed energy generation' and 'distributed energy resources' are mentioned 37 times, 'batteries' and 'energy storage' a total of 28 times, and 'renewable energy sources' and 'generators' a total of 16 times. This illustrates the focus on decentralized energy sources and storage, more specifically renewables and small scale combined heat and power generation that prevails

throughout the literature.

The publication chart is not very enlightening. This is due to the fact, that 28 out of 61 publications are conference papers are either classified as unknown or as others because there is only one instance of a paper from that particular conference. From a more general point of view, however, almost all papers are published either by Elsevier or IEEE, with only few exceptions.

2.3 Literature Overview

The first question in need of an answer in optimizing a microgrids design and dispatch is the question of the modeling approach itself. Although the dataset of literature derived from the methodology described in 2.2 is biased towards Linear Programming, which was one of the search and filter criteria, it contains multiple employed methodologies. For example, Lázár et al. 2017 employ both a Mixed Integer Linear Programming (MILP) model and a Genetic Algorithm (GA) and come to the conclusion that while both deliver accurate and robust results the MILP model is faster. Nemati et al. 2018 mostly concur. While in this case, the results of the GA were better in two scenarios, it was outperformed by the MILP model in the remaining three. However, a key problem of MILP seems to be its deterministic nature, which anticipates perfect knowledge of all the parameters involved. Especially for optimization problems concerned with short timeframes, such as electricity dispatch this is a significant problem, since actual parameters may be different. Several methods addressing this problem can be found in literature.

One such method are rolling time horizons. This approach optimizes the dispatch of a microgrid for a fixed time horizon based on steadily updated forecasts of the uncertain parameters. The optimization is repeated periodically to reflect the updated forecasts [Palma-Behnke et al. 2013, Silvente et al. 2015] and increase dispatch accuracy in the nearer future. But rolling horizon optimization is unfit to optimize investment, since it is not possible to adapt investment decisions ex post to changed conditions.

While the rolling time horizon method helps to limit uncertainty by reacting to changes of input parameters, Robust Optimization and Stochastic Optimization try to proactively account for a variety of possible scenarios. Robust Optimization achieves this by optimizing for a number of scenarios deemed equally likely, as in Craparo et al. 2017 using ensemble weather forecasts or in Hussain et al. 2018 and Zhang et al. 2015 using upper and lower boundaries for uncertain parameters. Stochastic Optimization on the other hand uses detailed probability distributions to weigh the probability of each scenario occurring as explained in Moshi et al. 2016. To arrive at these distributions, secondary tools are usually needed. Shams et al. 2018 use a simple Gaussian randomization to make their demand data reflect uncertainty, as well as more specific distributions for irradiation and wind speeds. A number of other methodologies appear in literature, such as employing a Monte Carlo simulation [Zheng et al. 2018b] or deriving multiple scenarios and corresponding realization probabilities from historical data [Nugraha

et al. 2015].

The scope of the microgrid models found in literature varies greatly. A constant is the definition of a microgrid as bounded, operating in a small geographical zone and with clear electrical boundaries [Tavakoli et al. 2018, Mashayekh et al. 2017, Kamboj and Chanana 2016]; managing local loads [Zhang et al. 2013, Zheng et al. 2018a]; possibly containing various generation units and storage [Silvente et al. 2015, Liu et al. 2016] and possibly being able to exchange power with the main grid [Nemati et al. 2018, Farsangi et al. 2018], from whose perspective it is seen as a single entity [Koltsaklis et al. 2018].

While most of the selected literature considers cases where a grid connection exists and can be used at all times, there are several publications, which consider either completely islanded microgrids [Zhang et al. 2018, Palma-Behnke et al. 2013, Sechilariu et al. 2014a, Santos et al. 2015] or microgrids that can sustain themselves in islanded mode for extended periods for example in the case of natural disasters [Tavakoli et al. 2018].

The clearest distinctions between microgrid models, after the object and applied methodology is properly defined, is based on the aspect or aspects that the model is supposed to optimize. The literature can be split in two groups, optimizing either dispatch only or both investment (e.g., the planning phase) and dispatch. The former group is definitely the larger, with only 12 out of 61 publications considering investment. It is notable, that only one of these, Moshi et al. 2016, uses any of the methods to model uncertainty discussed above.

There is also significant diversity in literature when it comes to the technologies considered in modeling. Modeling dispatchable (i.e. non-renewable + biofuels) as well as non-dispatchable (i.e. renewable) generation is rather common, but the specifics differ: While most models consider pv or wind as well as a CHP generators, the included generation technologies are as diverse as geothermal generators [Lázár et al. 2017] and gas turbines [Umeozor and Trifkovic 2016a, Nemati et al. 2018]. In addition to modeling electricity, some publications also consider heat generation and transmission [Lauinger et al. 2016, Wouters et al. 2015]. This is valuable, because the economic performance of a non-renewable generation unit (usually CHP) present in most models depends heavily on whether and how the heat is used, as Costa and Fichera 2014 point out.

The types of storage used in literature also vary. Although battery storage [Atia and Yamada 2016, Umeozor and Trifkovic 2016a, Wu et al. 2012] or an abstract storage device [Parisio and Glielmo 2013, Zhang et al. 2011] are the most common choices, there are a number of papers modeling heat storage [Zhang et al. 2013, Zhang et al. 2015, Lauinger et al. 2016, Wouters et al. 2015], and some with more exotic technology choices such as flywheels [Rigo-Mariani et al. 2013] or an electrolyzer [Gulin et al. 2015].

In addition to these generation and storage technologies some publications also model the network topology [Mashayekh et al. 2018, Chen et al. 2018, Shams et al. 2018] and some even consider electrical

phenomena such as active and reactive power losses [Liu et al. 2016, Mashayekh et al. 2017] and voltage deviation [Liu et al. 2016].

Demand side management, which is implemented in some models is usually represented by dividing loads into different categories. Chen et al. 2018 distinguish between 'critical loads', meaning loads that absolutely have to be satisfied, 'shiftable loads', meaning loads that have to be satisfied, although there is a time window, rather than an exact point in which they can be serviced, and 'adjustable' loads, which can be dropped if needed. Although the terms may vary, and many publications do not introduce the 'shiftable loads' category altogether, these conceptual distinctions are made by a number of other authors, such as Silvente et al. 2015, Zhang et al. 2015 and Zhaoxia et al. 2017.

The model results depend almost as much on the given input parameters used in case studies as on the modeling approach itself. There is, however, only a limited number of publications with detailed documentation of the parameters used. The parameters can be broadly categorized as either economic parameters, such as capital and operation and maintenance cost for the technologies used, generation related parameters such as irradiation and wind speeds, and load data representing consumer behavior. In publications documenting cost parameters there is a clear trend towards a split of technology cost into investment and maintenance cost. However, whereas Lauinger et al. 2016 define maintenance cost as a fixed cost per unit of time, Wouters et al. 2015 define it as a function of kWh produced, a measurement which Lauinger et al. 2016 call 'fuel costs'. Atia and Yamada 2016 choose a definition closer aligned with the former, but they define yearly maintenance cost simply as a flat percentage of the units' investment cost. Koltsaklis et al. 2018 seem to neglect maintenance costs altogether, focusing only on investment costs. While some models document economic parameters such as discount rates [Atia and Yamada 2016], many don't and it is unclear if future flows of value are discounted in these models. Furthermore many of the publications declare cost parameters as assumptions rather than trying to derive them from sources, although a few such as Lauinger et al. 2016 and Wouters et al. 2015 do so.

The selected literature offers multiple ways of obtaining input parameters of meteorological data. Henao-Muñoz et al. 2017 use historical meteorological data while Palma-Behnke et al. 2013 build a prediction model for forecasting irradiation and wind speeds. Similarly there are some authors who use historical data for load parameters such as Henao-Muñoz et al. 2017 and others who generate synthetic data based on appliance use and probabilistic models [Zhang et al. 2013, Zhang et al. 2011, Zhang et al. 2015] or neural networks using empirical data [Palma-Behnke et al. 2013].

When it comes to modeling tools, DER-CAM [Lauinger et al. 2016, Zheng et al. 2018b, Mashayekh et al. 2018] and HOMER [Amrollahi and Bathaee 2017, Zheng et al. 2018b, Lauinger et al. 2016] are the most often referenced. DER-CAM or Distributed Energy Resources Customer Adoption Model is an optimization tool developed at Berkeley Labs that uses Mixed Integer Linear Programming to optimize portfolio, placement, sizing and dispatch of Microgrid Energy Systems ["Distributed Energy Resources

- Customer Adoption Model (DER-CAM)" 2018]. HOMER Grid is a commercial optimization tool for behind the meter systems. Its main advantage is its large database of components and tariff rates in the United States and Canada ["HOMER Grid Behind-the-Meter Optimization Software for Demand Charge Reduction and Energy Arbitrage" 2018]. The most often mentioned solver is the CPLEX commercial solver [Sechilariu et al. 2014b, Nemati et al. 2018, Umeozor and Trifkovic 2016b, Liu et al. 2016]. Most of the models are written in GAMS [Silvente et al. 2015, Umeozor and Trifkovic 2016b, Craparo et al. 2017], although many are written in MATLAB [Liu et al. 2016, Chen et al. 2018]. Open source tools seem scarcely available. A search of 'microgrid optimization' on github.com, the major open source software platform, brings up only 3 items with more than one star, none of them having more than ten (popular libraries might have thousands)[["Github Search - 'Microgrid Optimization' , Sorted by Most Stars"](#) 2018]. The most up-to-date item has not been updated for a year, and none features a graphical user interface.

3 Mathematical Formulation

Table 1: Overview of Parameters

| Name | Shape | Description |
|----------------------------------|--|---|
| S | $\in \mathbb{N}$ | number of discontinuous sets of time-steps |
| T | $\in \mathbb{N}$ | number of time-steps per discontinuous set |
| U | $\in \mathbb{N}$ | number of households |
| K | $\in \mathbb{N}$ | number of linear generation investment options |
| L | $\in \mathbb{N}$ | number of linear storage investment options |
| M | $\in \mathbb{N}$ | number of discrete generation investment options |
| N | $\in \mathbb{N}$ | number of discrete storage investment options |
| $H \hookrightarrow DEM$ | $\geq 0, \in \mathbb{R}^{S \times T \times 3}$ | demand series of a household |
| $H \hookrightarrow shiftP$ | ≥ 0 | demand shift ‘price’ of a household |
| $H \hookrightarrow curtP$ | ≥ 0 | curtailment ‘price’ of a household |
| $GEN \hookrightarrow C_{Cap}$ | ≥ 0 | investment cost of a linear generation option |
| $GEN \hookrightarrow C_{OpFix}$ | ≥ 0 | yearly operation cost of a linear generation option |
| $GEN \hookrightarrow C_{OpVar}$ | ≥ 0 | marginal operation cost of a linear generation option |
| $GEN \hookrightarrow T_{Life}$ | ≥ 0 | life expectancy of a linear generation option |
| $GEN \hookrightarrow EFF_{el}$ | ≥ 0 | electrical efficiency of a linear generation option |
| $GEN \hookrightarrow PR$ | ≥ 0 | performance ratio of a linear generation option |
| $GEN \hookrightarrow maxFl$ | $\geq 0, \in \mathbb{R}^{S \times T}$ | maximal available input flow series of a linear generation option |
| $dGEN \hookrightarrow CAP$ | $\in \mathbb{N}$ | capacity of a discrete generation option |
| $dGEN \hookrightarrow C_{Cap}$ | ≥ 0 | investment cost of a discrete generation option |
| $dGEN \hookrightarrow C_{OpFix}$ | ≥ 0 | yearly operation cost of a discrete generation option |
| $dGEN \hookrightarrow C_{OpVar}$ | ≥ 0 | marginal operation cost of a discrete generation option |
| $dGEN \hookrightarrow T_{Life}$ | ≥ 0 | life expectancy of a discrete generation option |
| $dGEN \hookrightarrow EFF_{el}$ | ≥ 0 | electrical efficiency of a discrete generation option |
| $dGEN \hookrightarrow PR$ | ≥ 0 | performance ratio of a discrete generation option |
| $dGEN \hookrightarrow maxFl$ | $\geq 0, \in \mathbb{R}^{S \times T}$ | maximal available input flow series of a discrete generation option |
| $ST \hookrightarrow C_{Cap}$ | ≥ 0 | investment cost of a linear storage option |
| $ST \hookrightarrow T_{Life}$ | ≥ 0 | life expectancy of a discrete generation option |
| $ST \hookrightarrow EFF_{+-}$ | ≥ 0 | roundtrip efficiency of a linear storage option |
| $ST \hookrightarrow maxP_+$ | ≥ 0 | maximum charging power of a linear storage option |
| $ST \hookrightarrow maxP_-$ | ≥ 0 | maximum discharging power of a linear storage option |
| $dST \hookrightarrow CAP$ | $\in \mathbb{N}$ | capacity of a discrete storage option |
| $dST \hookrightarrow C_{Cap}$ | ≥ 0 | investment cost of a discrete storage option |
| $dST \hookrightarrow T_{Life}$ | ≥ 0 | life expectancy of a discrete storage option |
| $dST \hookrightarrow EFF_{+-}$ | ≥ 0 | roundtrip efficiency of a discrete storage option |
| $dST \hookrightarrow maxP_+$ | ≥ 0 | maximum charging power of a discrete storage option |
| $ST \hookrightarrow maxP_-$ | ≥ 0 | maximum discharging power of a discrete storage option |
| $maxS$ | $\geq 0, \in \mathbb{R}^{S \times T}$ | series of maximum available power suppliable by the grid |
| $maxD$ | $\geq 0, \in \mathbb{R}^{S \times T}$ | series of maximum power feed-in into the grid |
| $gridC$ | $\geq 0, \in \mathbb{R}^{S \times T}$ | series of cost of one unit of electricity from the grid |
| $feedC$ | $\geq 0, \in \mathbb{R}^{S \times T}$ | series of value of one unit of electricity fed into the grid |

Table 2: Overview of Internal Variables

| Name | Shape | Description |
|--------------------------|---|---|
| $toTR$ | $\geq 0, \in \mathbb{R}^{U \times S \times T}$ | power received from the microgrid's internal trading pool |
| $fromTR$ | $\geq 0, \in \mathbb{R}^{U \times S \times T}$ | power sent to the microgrid's internal trading pool |
| $genS$ | $\geq 0, \in \mathbb{R}^{U \times K \times S \times T}$ | power generated by linear generation |
| $dgenS$ | $\geq 0, \in \mathbb{R}^{U \times M \times S \times T}$ | power generated by discrete generation |
| $fromST$ | $\geq 0, \in \mathbb{R}^{U \times L \times S \times T}$ | power taken out of linear storage devices |
| $toST$ | $\geq 0, \in \mathbb{R}^{U \times L \times S \times T}$ | power sent into linear storage devices |
| $fromDST$ | $\geq 0, \in \mathbb{R}^{U \times N \times S \times T}$ | power taken out of discrete storage devices |
| $toDST$ | $\geq 0, \in \mathbb{R}^{U \times N \times S \times T}$ | power sent into discrete storage devices |
| $fromSC$ | $\geq 0, \in \mathbb{R}^{U \times S \times T}$ | demand shifted in the previous time-step due to be consumed now |
| $toSC$ | $\geq 0, \in \mathbb{R}^{U \times S \times T}$ | demand shifted in the current time-step |
| ncS | $\geq 0, \in \mathbb{R}^{U \times S \times T}$ | demand curtailed |
| $fromGR$ | $\geq 0, \in \mathbb{R}^{U \times S \times T}$ | power imported from main grid |
| $toGR$ | $\geq 0, \in \mathbb{R}^{U \times S \times T}$ | power fed into main grid |
| $H \hookrightarrow GEN$ | $\geq 0, \in \mathbb{R}^K$ | installed linear generation capacity |
| $H \hookrightarrow ST$ | $\geq 0, \in \mathbb{R}^L$ | installed linear storage capacity |
| $H \hookrightarrow dGEN$ | $\in \mathbb{N}^M$ | installed discrete generation capacity |
| $H \hookrightarrow dST$ | $\in \mathbb{N}^N$ | installed discrete storage capacity |

3.1 Time

The range of scenarios and complexity the model can handle is, among other things, dependent on its implementation of discrete time. More and smaller time-steps and a representative sample of diverse expectable conditions regarding the parameters lead to higher model accuracy. Of course the amount of time-steps is limited by processing power and therefore by model complexity. To enable both diverse and therefore potentially discontinuous parameter sets and a reasonable length for each set, the length of 1 hour is chosen for the basic time-step $t; t \in [1, T]; T \in \mathbb{N}$. T denotes the number of hours in each discontinuous set of supply and demand parameters. Furthermore, the ability of including multiple discontinuous sets of supply and demand parameters of the same length in one scenario is gained by introducing a second iterator $s; s \in [1, S]; S \in \mathbb{N}$. S denotes the number of discontinuous sets of supply and demand parameters. In addition a vector $SW \in \mathbb{N}^S$ can be defined, which will be used by the model for weighing the costs incurred in each season S differently. The vector will be normalized to S to not interfere with the scaling of the results. This normalized weight vector is called SWN .

3.2 Households

To properly conduct trading and for the future possibility of adding voltage and topological constraints to the model, the households are modeled as independent entities, each with its own demand, generation

and storage. Therefore, a third iterator $u; u \in [1, U]; U \in \mathbb{N}$; is introduced. U denotes the number of households modeled. An abstract household H is defined as a tuple of storage and generation capacities as well as demand curves. For discrete storage and generation devices several sets of separate sets of positive integers define how much of each option available in the scenario was installed. Additionally each household has a price $shiftP$ and $curtP$ at which they are willing to shift or curtail a unit of their load respectively.

$$H := (GEN, ST, dGEN, dST, DEM, shiftP, curtP) \quad (1)$$

A concrete household H_u implements these values:

$$GEN \in \mathbb{R}^K; GEN_k \geq 0 \forall k; k \in [0, K] \quad (2)$$

$$ST \in \mathbb{R}^L; ST_l \geq 0 \forall l; l \in [0, L] \quad (3)$$

$$dGEN \in \mathbb{N}^M \quad (4)$$

$$dST \in \mathbb{N}^N \quad (5)$$

with

k, l in \mathbb{N}

and

$K, L, M, N \in \mathbb{N}$

and

$shiftP, curtP \geq 0$

K and L denote the amount of generation and storage devices in household H_u . This, in practice, is equal to the amount of linear scaling investment options for each category, since a zero capacity device is still modeled as a device. M and N denote the amount of discrete investment options in their respective category. GEN_k and ST_l are the capacities installed of the k -th and l -th linear investment option respectively. GEN_m and dST_n are the amount installed of the m -th and n -th discrete investment option available. DEM is further defined in the subsection demand.

A list of instances of the structure H and its length U are both model parameters.

3.3 Internal Variables

The objective of the model is to find optimal investment, dispatch and trade. Therefore, the following sets of internal parameters are introduced:

$\forall u, s, t$:

$$toTR_{u,s,t}, fromTR_{u,s,t} \geq 0 \quad (6)$$

The trade supply variables $toTR_{u,s,t}$ and $fromTR_{u,s,t}$ reflect the amount of traded power to the trading pool and from the trading pool respectively. They are defined for each household at each time-step.

$\forall u, k, m, s, t$:

$$genS_{u,k,s,t}, dgenS_{u,m,s,t} \geq 0 \quad (7)$$

The generation supply variables $genS_{u,k,s,t}$ and $dgenS_{u,m,s,t}$ describe the amount of energy produced by each linear or discrete generation option respectively. They are defined for each generation option in each household and at each time-step. It is useful to keep in mind, that not every household needs to implement all the generation options. In case of zero instances of the discrete generation option $dGEN_2$ installed in household H_5 , for example, $dgenS_{5,2,s,t} = 0$ for all s and t .

$\forall u, l, n, s, t$:

$$fromST_{u,l,s,t}, toST_{u,l,s,t}, fromDST_{u,n,s,t}, toDST_{u,n,s,t} \geq 0 \quad (8)$$

The storage supply variables $fromST_{u,l,s,t}$ and $toST_{u,l,s,t}$ indicate the amount of power fed into or withdrawn from a linear storage investment option. They are defined for each linear investment option and each household at each time-step. The variables $fromDST_{u,n,s,t}$ and $toDST_{u,n,s,t}$ have the equivalent function for all discrete storage investment options.

$\forall u, s, t$:

$$toSC_{u,s,t}, fromSC_{u,s,t}, ncS_{u,s,t} \geq 0 \quad (9)$$

The shifted consumption supply variable $toSC_{u,s,t}$ represents the amount of power consumption shifted by each household at each time-step. $fromSC_{u,s,t}$ indicates the amount of power consumption that has previously been shifted and is now consumed. The non-consumption supply variable $ncS_{u,s,t}$ designates the amount of power consumption curtailed for each household at each time-step.

$\forall u, s, t$:

$$fromGR_{u,s,t}, toGR_{u,s,t} \geq 0 \quad (10)$$

Finally, the grid supply variables $fromGR_{u,s,t}$ and $toGR_{u,s,t}$ denote the amount of power supplied by and to the grid respectively by each household at each time-step. Only power flowing out of and into the microgrid is included in this definition, power traded internally, while using the (micro)grid, is reflected in the trade variables. (see formula (6))

$\forall u, k:$

$$H_u \hookrightarrow GEN_k \geq 0 \quad (11)$$

$\forall u, l:$

$$H_u \hookrightarrow ST_l \geq 0 \quad (12)$$

$\forall u, m$

$$H_u \hookrightarrow dGEN_m \in \mathbb{N} \quad (13)$$

$\forall u, n$

$$H_u \hookrightarrow dST_n \in \mathbb{N} \quad (14)$$

$H_u \hookrightarrow GEN_k$, $H_u \hookrightarrow ST_l \geq 0$, $H_u \hookrightarrow dGEN_m$, $H_u \hookrightarrow dST_n$ are variables describing the realized capacities of each linear and discrete investment option in each household. They are explained in more detail in section 3.2.

3.4 Generation

For a model that is explicitly built to optimize decentralized generation in microgrids a robust and flexible mathematical representation of generation options is required. Therefore, an abstract linear generation device GEN and an abstract discrete generation device $dGEN$, which can represent a range of technology options, are defined:

$$GEN := (C_{Cap}, C_{OpFix}, C_{OpVar}, T_{Life}, EFF_{el}, PR, maxFl) \quad (15)$$

$$dGEN := (CAP, C_{Cap}, C_{OpFix}, C_{OpVar}, T_{Life}, EFF_{el}, PR, maxFl) \quad (16)$$

$CAP \geq 0$, which is only required for discrete investments, defines the generation capacity of the discrete investment option. $C_{Cap} \geq 0$ defines the total investment cost for discrete generation devices. For linear generation devices the investment cost is expressed in terms of one unit of capacity. $C_{OpFix} \geq 0$ expresses the fixed maintenance cost per year. In the case of linear generation they are expressed in proportion to capacity, while for discrete generation they express the maintenance cost of one discrete device. $C_{OpVar} \geq 0$ expresses the variable maintenance cost per unit of input consumed. $T_{Life} \in \mathbb{N}$ defines the life expectancy of a generation device, i.e. the time before it is

replaced. $EFF_{el} \in [0, 1]$ defines the ratio of supplied energy, for example sunlight or gas, to produced electricity. $PR \in [0, 1]$ is another definable penalty to electricity production, which can, for example, express the degradation of solar cells. $maxFl \in \mathbb{R}^{S \times T}$, $maxFL \geq 0 \forall s, t$ denotes the maximal possible flow of input energy, e.g., sunlight in case of solar pv or gas in case of a fuel cell. This enables the representation of intermittent availability for example for pv installations. It could nevertheless be used to model any condition that would restrict a conventional generation device from running at capacity at certain times, for example air quality regulation.

All instances of the GEN structure are collected in one list. The same is true for all instances of the $dGEN$ structure. These lists and their sizes K and M are parameters. The model uses the iterators k and m to represent individual instances of these structures (see section 3.2).

In addition, to model generation appropriately, several sets of constraints need to be defined:

$\forall u, k, s, t:$

$$\begin{aligned} genS_{u,k,s,t} &\leq \min(GEN_k \hookrightarrow EFF_{el} * GEN_k \hookrightarrow maxFl_{s,t}, 1) \\ &\quad * H_u \hookrightarrow GEN_k * GEN_k \hookrightarrow PR \end{aligned} \tag{17}$$

$\forall u, m, s, t:$

$$\begin{aligned} dgenS_{u,m,s,t} &\leq \min(GEN_m \hookrightarrow EFF_{el} * GEN_m \hookrightarrow maxFl_{s,t}, 1) \\ &\quad * H_u \hookrightarrow dGEN_m * dGEN_m \hookrightarrow CAP * dGEN_k \hookrightarrow PR \end{aligned} \tag{18}$$

The capacity constraints (17,18) restrict the use of any linear generation device GEN_k or any discrete generation device $dGEN_m$ to its capacity rating and the maximum flow of input energy supplied to it. The constraints differ due to the capacity of linear generation devices being stored in the household structure directly, while for discrete generation devices the household structure only contains an integer representing the amount implemented. This value is then multiplied with the capacity of one unit of the concerned discrete generation device to compute the total installed capacity. In both cases the result is multiplied with the performance ratio PR of the concerned device, which acts as a penalty to capacity (see above). The minimized term calculates the ratio of maximum output achievable with the available maximum input to the capacity of the device. If the input suffices for running at capacity, the term returns a maximum of 1, not restricting supply further. Otherwise it restricts capacity to the maximum possible under current resource input.

3.5 Storage

The introduction of the concept of energy storage is necessary if the use of a large proportion of intermittent power production while maintaining intermittent power demand is to be seriously explored. Therefore, an abstract linear storage device ST and an abstract discrete storage device dST , which can represent a range of technology options, are defined:

$$ST := (C_{Cap}, T_{Life}, EFF_{+-}, maxP_+, maxP_-) \quad (19)$$

$$dST := (CAP, C_{Cap}, T_{Life}, EFF_{+-}, maxP_+, maxP_-) \quad (20)$$

$CAP \geq 0$, which is only required for discrete investments, defines the usable capacity of the discrete investment option. $C_{Cap} \geq 0$ defines the total investment cost for discrete storage devices. For linear storage investment options the investment cost is expressed in terms of one unit of capacity. $T_{Life} \in \mathbb{N}$ defines the life expectancy of a storage device, i.e. the time before it is replaced. To be able to better estimate this value it is assumed, that storage is, on average, cycled once per day. $EFF_{+-} \in [0, 1]$ defines the roundtrip efficiency, i.e., what proportion of one unit of power is left after charging and discharging inefficiencies. It is used to compute the charge and discharge efficiencies. The charge efficiency EFF_+ is defined as $EFF_+ = \sqrt{EFF_{+-}}$ and the discharge efficiency EFF_- as $EFF_- = 1/EFF_+$ [Lauinger et al. 2016].

$maxP_+ \geq 0$ and $maxP_- \in [0, 1]0$ denote the maximum charge rate and discharge rate respectively, relative to the devices capacity. All instances of the ST structure are collected in one list. The same applies for all instances of dST . The two lists and their sizes L and N are model parameters. The model uses the iterators l and n to represent individual instances of these structures (see section 3.2). To model storage appropriately several sets of constraints need to be specified:

$\forall u, l, s, t:$

$$toST_{u,l,s,t} \leq ST_l \hookrightarrow maxP_+ * H_u \hookrightarrow ST_l \quad (21)$$

$$fromST_{u,l,s,t} \leq ST_l \hookrightarrow maxP_- * H_u \hookrightarrow ST_l \quad (22)$$

The flow constraints for linear storage (21,22) constrain the maximum in- and outflow for each linear storage device to its maximum in- and outflow rate relative to capacity.

$\forall u, n, s, t:$

$$toDST_{u,n,s,t} \leq dST_n \hookrightarrow maxP_+ * H_u \hookrightarrow dST_n \quad (23)$$

$$fromDST_{u,n,s,t} \leq dST_n \hookrightarrow maxP_- * H_u \hookrightarrow dST_n \quad (24)$$

The flow constraints for discrete storage (23,24) implement the same functionality as the linear flow constraints for discrete storage devices.

$\forall u, l, s:$

$$\sum_{t=1}^T (ST_l \hookrightarrow EFF_+ * toST_{u,l,s,t'} - ST_l \hookrightarrow EFF_- * fromST_{u,l,s,t'}) = 0 \quad (25)$$

$\forall u, n, s:$

$$\sum_{t=1}^T (dST_n \hookrightarrow EFF_+ * toDST_{u,n,s,t'} - dST_n \hookrightarrow EFF_- * fromDST_{u,n,s,t'}) = 0 \quad (26)$$

The model assumes the storage to be full at the start of each simulated discontinuous period s and requires it to be full at the end of each period. This means inflows minus outflows weighted by the respective efficiencies need to sum up to zero for the entirety of each period s (25,26).

$\forall u, l, s, t:$

$$toST_{u,l,s,t} * ST_l \hookrightarrow EFF_+ \leq \sum_{t'=1}^{t-1} (-ST_l \hookrightarrow EFF_+ * toST_{u,l,s,t'} + ST_l \hookrightarrow EFF_- * fromST_{u,l,s,t'}) \quad (27)$$

$\forall u, n, s, t:$

$$toDST_{u,n,s,t} * dST_n \hookrightarrow EFF_+ \leq \sum_{t'=1}^{t-1} (dST_n \hookrightarrow EFF_+ * toDST_{u,n,s,t'} - dST_n \hookrightarrow EFF_- * fromDST_{u,n,s,t'}) \quad (28)$$

The storage capacity constraints (27,28) prohibit storage usage beyond the storage device's capacity. The remaining storage capacity at time (s, t) is in theory calculated as the capacity minus current storage level, which is the sum of all inflows and outflows weighted by in- and outflow efficiency. Since the storage at each time-step $(s, 0)$ is required to be full, the capacity is subtracted once, which eliminates it and leaves only the sum. The constraint is implemented equivalently for discrete storage.

$\forall u, l, s, t:$

$$fromST_{u,l,s,t} * ST_l \hookrightarrow EFF_- \leq H_u \hookrightarrow ST_l + \sum_{t'=1}^{t-1} (ST_l \hookrightarrow EFF_+ * toST_{u,l,s,t'} - ST_l \hookrightarrow EFF_- * fromST_{u,l,s,t'}) \quad (29)$$

$\forall u, n, s, t:$

$$\begin{aligned} & fromDST_{u,n,s,t} * dST_n \hookrightarrow EFF_- \leq H_u \hookrightarrow dST_n * dST_n \hookrightarrow CAP+ \\ & \sum_{t'=1}^{t-1} (dST_n \hookrightarrow EFF_+ * toDST_{u,n,s,t'} - dST_n \hookrightarrow EFF_- * fromDST_{u,n,s,t'}) \end{aligned} \quad (30)$$

Finally, the storage level constraints (29,30) prohibit the storage level to fall below zero. This means there can be no more power withdrawn, weighted by outflow efficiency, than is currently stored in the device. Because we assume the storage to be full at the start of each period s , the current charge amounts to one full charge (storage capacity) plus the sum of all transactions during the current period up to the current time-step t . The only distinction for discrete storage devices is the way capacity is calculated (see 3.2).

3.6 Trade

Trade is implemented as an exchange platform, which sums up deposits and withdrawals of power at each time-step and always needs to be balanced (31). This approach has the advantage of greatly reducing the number of constraints and therefore complexity, although it is not very well suited to handle grid topology constraints, which might be added in the future.

The trade constraint is defined as:

$\forall s, t:$

$$\sum_{u=1}^U (toTR_{u,s,t} - fromTR_{u,s,t}) = 0 \quad (31)$$

3.7 Demand

Abstract demand data DEM is defined as a set of the size S of demand profile sets of the size T :

$$DEM := \{DEM_s | s \in [1, S]; s \in \mathbb{N}; \forall DEM_s, DEM_s := \{DEM_{s,t} | t \in [0, T]; t \in \mathbb{N}\}\} \quad (32)$$

As frequently applied in literature, $DEM_{s,t}$ is not a demand value, but rather a tuple of three demand values, critical demand, shiftable demand and curtailable demand Silvente et al. 2015Zhang et al. 2015Zhaoxia et al. 2017Chen et al. 2018 :

$$DEM_{s,t} := (critDEM_{s,t}, shiftDEM_{s,t}, curtDEM_{s,t}) \quad (33)$$

$\forall u, s, t:$

$$H_u \hookrightarrow critDEM_{s,t}, H_u \hookrightarrow shiftDEM_{s,t}, H_u \hookrightarrow curtDEM_{s,t}, t \geq 0$$

Critical demand will be met by the model as it is seen as a constraint. Shiftable demand can be shifted by one hour at a fixed rate defined for each household. Curtailable demand can be dropped at a fixed rate defined for each household. The restrictions on shiftable and curtailable demand are expressed by the following sets of constraints:

$\forall u, s, t:$

$$toSC_{u,s,t} \leq H_u \hookrightarrow shiftDEM_{s,t} + fromSC_{u,s,t} \quad (34)$$

$$ncS_{u,s,t} \leq H_u \hookrightarrow curtDEM_{s,t} \quad (35)$$

The shifted demand limit constraint (34) restricts the amount of shiftable demand to the shiftable part of household demand for the current time-step. To this the demand shifted in previous time-steps (36), which can be shifted further, is added. The curtailable demand limit constraint (35) restricts the amount of curtailable demand to the curtailable part of household demand for the current time-step.

$\forall u, s; t \in [2, T]:$

$$fromSC_{u,s,t} = toSC_{u,s,t-1} \quad (36)$$

Demand shifted in the previous time-step has to be consumed in the current time-step or shifted further. This is ensured by (36). It does not apply for the first time-step, but the model still does not set $fromSC_{u,s,0}$ to greater than 0, because it would raise cost.

$\forall u, s:$

$$toSC_{u,s,T} = 0 \quad (37)$$

(37) ensures there is no consumption shifted at the last time-step of each season s as it would not have to be consumed at all.

3.8 Grid

The modeled microgrid is connected to the grid via a common point of coupling. The grid is defined as:

$$GRID := (maxS, maxD, gridC, feedC) \quad (38)$$

with

$$maxS, maxD, gridC, feedC \in \mathbb{R}^{S \times T}$$

and $\forall s, t$:

$$maxS_{s,t}, max_{s,t}, gridC_{s,t}, feedC_{s,t} \geq 0$$

$maxS$ describes the maximum amount of power supplied by the grid at each time-step (s, t) , while $gridC_{s,t}$ expresses the price at which this power can be bought. $maxD$ describes the maximum amount power that can be fed into the grid at time-step (s, t) , while $feedC_{s,t}$ expresses the price at which the grid buys this power.

To model the expected behaviour of the grid several sets of constraints need to be introduced:

$\forall s, t$:

$$\sum_{u=1}^U (fromGR_{u,s,t} - toGR_{u,s,t}) \leq maxS_{s,t} \quad (39)$$

The first grid constraint (39) restricts the sum of each households ‘trade balance’ with the grid to the maximum power the grid can supply at each time-step.

$\forall s, t$

$$\sum_{u=1}^U (toGR_{u,s,t} - fromGR_{u,s,t}) \leq maxD_{s,t} \quad (40)$$

The second grid constraint (40) does the reverse, in that it restricts the sum of each households ‘trade balance’ with the grid to the maximum power the microgrid can feed into the grid at each time-step.

3.9 Power Balance

The power balance constraints describe the need for a balance between supply and demand at each node at each time-step:

$\forall u, s, t:$

$$\begin{aligned}
 H_u \hookrightarrow critDEM_{s,t} + H_u \hookrightarrow shiftDEM_{s,t} + H_u \hookrightarrow curtDEM_{s,t} \\
 = \\
 fromGR_{u,s,t} - toGR_{u,s,t} + \sum_{k=1}^K genS_{u,k,s,t} + \sum_{m=1}^M dgenS_{u,m,s,t} \\
 + fromTR_{u,s,t} - toTR_{u,s,t} + toSC_{u,s,t} - fromSC_{u,s,t} + ncS_{u,s,t} \\
 + \sum_{l=1}^L fromST_{u,l,s,t} - toST_{u,l,s,t} + \sum_{n=1}^N fromDST_{u,n,s,t} - toDST_{u,n,s,t}
 \end{aligned} \tag{41}$$

The power balance constraint (41) requires electricity supply and demand to be balanced for each household at each time-step. On the demand side, the three types of demand of the concerned household and the current time-step are summed up. The supply side is made up of several terms: $fromGR_{u,s,t} - toGR_{u,s,t}$ expresses the momentary trade balance with the grid, while $fromTR_{u,s,t} - toTR_{u,s,t}$ denotes the momentary trade balance with the microgrid's trading pool. $toSC_{u,s,t} - fromSC_{u,s,t}$ is the balance of consumption shifted from the current time-step into the future and previously shifted demand realized now. $ncS_{u,s,t}$ reflects loads dropped in the current time-step. Finally, the power generated by all linear and discrete generation devices is summed up, as well as the balance of storage use for all linear and discrete storage devices.

3.10 Cost Minimization Formula

The cost minimization formula calculates the cost of all investment and dispatch decisions made:

$$\min C_{total} = C_{Investment} + C_{Operation} + C_{Dispatch} \tag{42}$$

The total cost can be broken down into investment, operation and dispatch costs.

$$\begin{aligned}
 C_{Investment} = & \sum_{u=1}^U \left[\sum_{k=1}^K \frac{GEN_k \hookrightarrow C_{Cap} * H_u \hookrightarrow GEN_k * S * T}{8760 * GEN_k \hookrightarrow T_{Life}} \right. \\
 & + \sum_{l=1}^L \frac{ST_l \hookrightarrow C_{Cap} * H_u \hookrightarrow ST_l * S * T}{8760 * ST_l \hookrightarrow T_{Life}} \\
 & + \sum_{m=1}^M \frac{dGEN_m \hookrightarrow C_{Cap} * H_u \hookrightarrow dGEN_m * S * T}{8760 * dGEN_m \hookrightarrow T_{Life}} \\
 & \left. + \sum_{n=1}^N \frac{dST_n \hookrightarrow C_{Cap} * H_u \hookrightarrow dST_n * S * T}{8760 * dST_n \hookrightarrow T_{Life}} \right]
 \end{aligned} \tag{43}$$

The investment costs consist of all investment into generation and storage devices throughout the microgrid. In the context of the model, this means that for linear investment options the installed capacity times the costs of capital per unit of capacity and for discrete investment options the costs of capital for a single instance of a device multiplied with the number of instances installed. All values are multiplied by the total amount of time-steps $S * T$ and then divided by the entire lifetime of the device in hours. This is inaccurate to some degree as it resembles a leasing model with negligible interest rates rather than an investment, but is vastly easier to compute and very close to the truth for most of Europe and the US.

$$C_{Operation} = \sum_{u=1}^U [\sum_{k=1}^K \frac{GEN_k \hookrightarrow C_{OpFix} * H_u \hookrightarrow GEN_k * S * T}{8760} + \sum_{m=1}^M \frac{dGEN_m \hookrightarrow C_{OpFix} * H_u \hookrightarrow GEN_m * S * T}{8760}] \quad (44)$$

The operation costs are the sum of all fixed operation costs of all generation devices deployed. They are calculated for each investment option in each household by multiplying its fixed operation costs with the installed capacity (or pieces in the case of discrete investment options) and the total number of time-steps. This value is then divided by the length of a year in hours to adjust the yearly values to the timespan considered in the conducted optimization. The sum of all of these investment options are the total maintenance costs for generation in the microgrid. Since there is no fixed maintenance cost defined for storage this value is equal to all the maintenance costs incurred.

$$C_{Dispatch} = \sum_{s=1}^S SWN_s * \sum_{t=1}^T \sum_{u=1}^U [gridC_{s,t} * fromGR_{u,s,t} - feedC_{s,t} * toGR_{u,s,t} + H_u \hookrightarrow curtP * ncS_{u,s,t} + H_u \hookrightarrow shiftP * scS_{u,s,t} + \sum_{k=1}^K (GEN_k \hookrightarrow C_{OpVar} * genS_{u,k,s,t}) + \sum_{m=1}^M (dGEN_m \hookrightarrow C_{OpVar} * dgenS_{u,m,s,t})] \quad (45)$$

The dispatch cost consist of the fuel costs (or variable operation costs C_{OpVar}) incurred by the use of all deployed generation devices as well as the value of all shifted and curtailed loads. In addition all transactions with the main grid are valued at the power and feed-in rates for their respective time-step

and summed up. The result in each season is then weighted according the value of the normalized weight vector for that season SWN_s (see 3.1).

4 Technical Implementation

The performance of a modeling tool depends as much on its implementation, as on mathematical soundness. This section outlines the choices made implementing the model described in the previous section.

The implementation of the modeling tool pursues multiple objectives:

- Performance: The ability to solve the problem reasonably fast.
- Scalability: The ability for the system to potentially be used by many users concurrently, without encountering performance issues.
- Flexibility: The ability to receive a wide range of differently formatted input data and still function adequately. Sparse or otherwise inadequate input data is preprocessed and can be supplemented by default values.
- User-Friendliness: The tool offers a user-friendly way of inputting data and of viewing computation results, possibly through a graphical interface.

4.1 Architecture

To address these objectives, the modeling tool was designed to work as a multi-layer architecture, with each layer performing a distinct part of the work flow. It is important to note, that these layers can run on separate machines to provide scalability but they do not have to do so. In the setup used for conducting the case study all layers were run on a single machine.

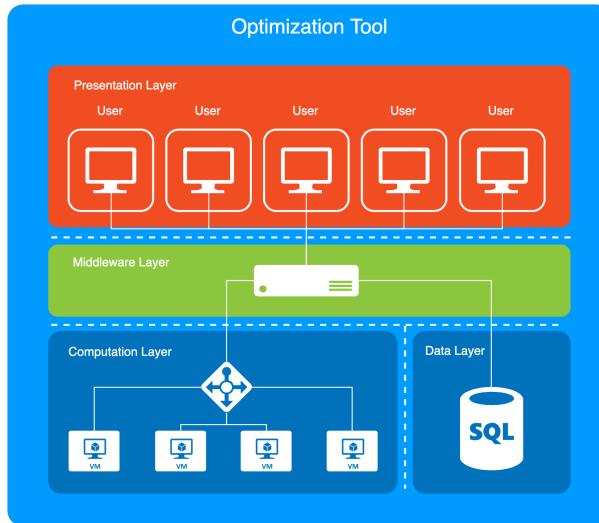


Figure 2: The proposed architecture for scaling the optimization tool

Source: Own Illustration.

The computation layer is where the actual model is built and solved. It ideally consist of several worker units, possibly virtualized or containerized, which run the model code, and a load balancing server queueing and assigning tasks received from the middleware layer via messages and relaying the returned solutions. It is currently implemented only rudimentarily as shell script, that relays input and output data and launches the model code. The model code is written in the Julia programming language, which was chosen for its high performance ["Julia Micro-Benchmarks" 2018] and its focus on scientific computing. The model code also includes the data preprocessing code, which grants the user higher flexibility regarding the form and quality of her input data. It is integrated directly with the model itself, so the model can be used with different interfaces without the need to implement data preprocessing first (see 4.2). This does however impact performance, as preprocessing in the presentation layer would be distributed.

The data layer should ideally consist of an SQL database, which allows the implementation of features in the presentation layer, requiring a persistent state, such as user authentication or saved requests and solutions. It is accessed via the middleware layer. For all but the largest scale implementations there is no need to physically separate it from the middleware layer. The data layer and corresponding functionality are currently not implemented.

The middleware layer coordinates communication between the users and the layers below. It is currently implemented as a simple node.js server ["About Node.Js" 2018] exposing a REST API [Fielding 2000] to the presentation layer.

The presentation layer handles all user input and presents the results of the calculations conducted by the computation layer. It also handles all data processing required for presenting the data to the user. It is currently implemented as a Web Application using the React.js framework ["React - A JavaScript

Library for Building User Interfaces" 2019].

4.2 Work Flow

When the user enters data in the presentation layer, basic type checks are conducted. The input is then encoded as a JSON string [[crockfordIntroducingJSON](#)], which is sent as the payload of an HTML GET-Request [Fielding 2000] to the middleware layer. The middleware layer transfers the JSON string into the environment the model code is run in and launches the model code. The model preprocesses the received input. The preprocessing is configured with option parameters which are included in the input. Currently the model supports a number of preprocessing options:

- Filling vector parameters, such as electricity prices or shiftable demand, with a constant.
- Adjusting the length of a number of vector parameters by cutting, if too long, and looping, if too short.
- Adding noise to a number of vector parameters by multiplying each value with a random number out of a normal distribution with a mean of 1.0. A variance can additionally be supplied. Checks that multiplier is not negative.
- Normalizing the weights given to the discontinuous periods s_i .

The model is then constructed and solved. For construction the Julia library 'JuMP' [["JuMP.JI - Modeling Language for Mathematical Optimization \(Linear, Mixed-Integer, Conic, Semidefinite, Nonlinear\)" 2019](#)] is used, for solving the Gurobi commercial solver [["Gurobi Optimization"](#)]. The model results are saved and transferred to the presentation layer via the middleware layer in a response to the GET-Request sent earlier. In the presentation layer the results are interactively visualized using the plotly visualization library [["Plotly Javascript Open Source Graphing Library" 2019](#)]. Currently a dispatch chart for each modeled household as well as two charts showing the generation and storage investment of each household are generated. In addition, Key Performance Indicators (KPIs) such as the levels of autarky and self-consumption or the average cost per kWh consumed are calculated to make different results easier to compare.

5 Case Study

5.1 Overview

To demonstrate the viability of both the designed model and of local microgrids in general, a limited case study is conducted. The location chosen is the small village of Morschenich in Northrhine-Westphalia, which is located in immediate proximity to the lignite surface mine of Hambach. According to current plans of RWE, the owner and operator of the mine and the state of Northrhine-Westfalia, the village is due to be razed and the population resettled by the year 2024, when the mining activities are expected to reach the village ["Rahmendaten - Morschenich, Gemeinde Merzenich (Kreis Düren)" 2018, "Umsiedlung Morschenich"]. Oei et al. 2019 have shown, that the demolition of Morschenich and a number of other villages is unnecessary if climate change mitigation goals are taken seriously.



Figure 3: The village of Morschenich

Source: Google Maps.

The resettlement process has already begun and the village 'Morschenich-Neu' is under construction several kilometers away. However, as of the date of writing, there are several hundred inhabitants remaining in Morschenich, including a number of Syrian war refugees, who have been accommodated in abandoned houses of families already resettled [Bauer 2018].

The case study will focus on the northern part of Morschenich, where several housing units and two

commercial units are located in a group, as indicated by the white line in Figures 3 and 4.



Figure 4: A detailed view of the chosen location.

Source: Google Maps.

In Figure 4 all buildings in the area designated for the case study's proposed microgrid have been marked with colored boxes. Blue boxes represent small residential units, while green boxes indicate larger residential units and red boxes commercial units.

The case study will examine the business case of the establishment of an autonomous microgrid in the selected area. Therefore investment decisions and dispatch are modeled for three discontinuous periods of one week each to reflect the different conditions encountered throughout the year. The designated buildings will be assumed to be the participants of the considered microgrid. The different classes of buildings are modeled with different parameters to accurately represent their distinct sizes and uses.

The following sections document the assumed input parameters the model requires for the calculation of a base scenario and the way in which they were derived. In order to function properly, the model requires meteorological data for constraints on intermittent power generation, as well as data on investment options, household demand and grid supply. The final section will introduce a number of

further scenarios in which some of the assumptions are changed.

5.2 Meteorological Data

An hourly profile of the capacity factor for a pv installation in Morschenich is sourced from Pfenninger and Staffell 2016. From the profile, which includes an entire year of data, three samples are extracted: One 144 hour sample representing the transitional season, i.e. spring and fall, and two 72 hour samples representing summer and winter. The difference in length is not relevant for use in the model, as the data preprocessing only uses these profiles as input for generating the profiles used in the optimization itself (see section 4). The resulting vector of values is used as the parameter $GEN_1 \hookrightarrow maxFl$, which is interpreted as the maximum input flow for the solar pv installation. In addition the parameter EFF_{el} is set to 1.0, which means the model effectively treats the vector as a capacity factor.

5.3 Investment Options

To test the different implementations of discrete and linear investment options the case study incorporates two linear-scaling generation devices as possible investments, as well as two discretely scaling storage devices. The generation devices consist of a photovoltaic system as a renewable option and a combined heat and power fuel cell as a conventional but decentralized and highly efficient option. It is also capable of using biofuels, which can be expressed through higher fuel prices.

Table 3: Parameters describing the generation technologies

| Parameter | Name | Unit | Value | Reference |
|----------------------------|-------------|-----------------------------|---------|--|
| <i>Photovoltaic System</i> | GEN_1 | | | |
| Installation Cost | C_{Cap} | $\text{€ } KW^{-1}$ | 1200 | estimate based on Wirth 2018, "Solarsystem eBay" 2018, "Module Price Index" 2018 |
| Maintenance Cost | C_{OpFix} | $\text{€ } KW^{-1} yr^{-1}$ | 50 | estimate |
| Fuel Cost | C_{OpVar} | $\text{€ } kWh^{-1}$ | 0.0 | |
| Lifetime | T_{Life} | yr | 25 | Wirth 2018 |
| Electric Efficiency | EFF_{el} | % | 100 | included in $maxFl$ |
| Performance Ratio | PR | % | 85 | Wirth 2018 |
| Maximum Input Flow | $maxFl$ | KW | see 5.2 | based on capacity factor |
| <i>CHP Micro Fuel Cell</i> | GEN_2 | | | |
| Installation Cost | C_{Cap} | $\text{€ } KW^{-1}$ | 3500 | estimate based on Lauinger et al. 2016 |
| Maintenance Cost | C_{OpFix} | $\text{€ } KW^{-1} yr^{-1}$ | 175 | estimate based on Lauinger et al. 2016 |
| Fuel Cost | C_{OpVar} | $\text{€ } kWh^{-1}$ | 0.0608 | "Natural gas prices for households in Germany from 2010 to 2018, semi-annually (in euro cents per kilowatt-hour)" 2018 |
| Lifetime | T_{Life} | yr | 15 | Lauinger et al. 2016 |
| Electric Efficiency | EFF_{el} | % | 60 | "BlueGEN - the Worlds Most Efficient Micro-CHP" 2018 |
| Performance Ratio | PR | % | 100 | no additional penalties |
| Maximum Input Flow | $maxFl$ | KW | 999 | no restrictions |

The storage investment options given are two differently sized Lithium Ion batteries: A 4 KWh option manufactured by Victron Energy and a 13.5 KWh option manufactured by Tesla. While the two generation devices are hypothetical, these storage devices can actually be bought, which leads to lower error due to false assumptions and linearization of non-linear relationships (such as storage size and price). This is the main advantage discrete modeling of investment options has over linear modeling.

Table 4: Parameters describing the storage technologies

| Parameter | Name | Unit | Value | Reference |
|--------------------------|------------|------|-------|---|
| <i>Li-Ion Battery 1</i> | dST_1 | | | <i>Tesla Powerwall</i> |
| Usable Capacity | CAP | KWh | 13.5 | "Powerwall - Tesla" 2018 |
| Installation Cost | C_{Cap} | € | 10000 | estimate based on "Powerwall - Tesla" 2018 |
| Lifetime | T_{Life} | yr | 10 | estimate based on "Powerwall - Tesla" 2018 |
| Roundtrip Efficiency | EFF_{+-} | % | 90 | "Powerwall - Tesla" 2018 |
| Max. Charging Power | $maxP_+$ | kW | 4.6 | "Powerwall - Tesla" 2018 |
| Max. Discharging Power | $maxP_-$ | kW | 4.6 | "Powerwall - Tesla" 2018 |
| Initial State of Charge | — | % | 100 | assumption |
| Terminal State of Charge | — | % | 100 | assumption |
| <i>Li-Ion Battery 2</i> | dST_2 | | | <i>Victron Energy Lithium HE Batterie</i> |
| Usable Capacity | CAP | KWh | 4 | based on "Lithium-Ionen HE (High Energy) Batterie und Lynx Ion BMS" 2018 considering depth of discharge |
| Installation Cost | C_{Cap} | € | 5000 | estimate based on "Victron Energy Lithium HE Batterie 24V 200Ah" 2018 |
| Lifetime | T_{Life} | yr | 10 | estimate based on "Lithium-Ionen HE (High Energy) Batterie und Lynx Ion BMS" 2018 |
| Roundtrip Efficiency | EFF_{+-} | % | 92 | Lauinger et al. 2016 |
| Max. Charging Power | $maxP_+$ | KW | 4.8 | "Lithium-Ionen HE (High Energy) Batterie und Lynx Ion BMS" 2018 |
| Max. Discharging Power | $maxP_-$ | KW | 7.2 | "Lithium-Ionen HE (High Energy) Batterie und Lynx Ion BMS" 2018 |
| Initial State of Charge | — | % | 100 | assumption |
| Terminal State of Charge | — | % | 100 | assumption |

5.4 Demand

As the case study distinguishes between three types of buildings, three demand profiles were compiled. Each profile is made up of three hourly load vectors spanning one or several days. Demand vectors for the two types of residential units are sourced from "Referenzlastprofile von Ein- und Mehrfamilienhäusern für den Einsatz von KWK-Anlagen" 2008. The choice was made because these profiles are specifically created for optimization of decentralized generation. They are samples of measurements taken on several houses and therefore feature a much higher variability than standard profiles, which are usually averages of many measurements. In the context of this case study it is critical to accurately reflect this high variability, which a microgrid can do a lot to manage. For the commercial units, profiles were sourced from the association of the German power industry (BDEW) [Fünfgeld and Tiedemann 2000, "Standardlastprofile Strom" 2017]. These are standard profiles specifically from commercial consumers. The samples were of varying sizes of one to several days, so as to reflect the approximate ratios of occurrences of the different model days provided for the chosen meteorological region. They were further preprocessed by the model itself. The noise filter of the model was provided with a variance value of 0.25 for the smaller residential households and a variance of 0.15 for the

larger residential and commercial buildings (see 4). This additional noise is crucial for distinguishing the demands of different instances of one household type from one another, as would occur in reality. Furthermore the noise filter makes the load profile more realistic when scaled beyond its original length, since it makes each repetition of the profile differ notably from the others (see Figure 6).

For the residential buildings it is assumed that at any time 10 percent of loads can be curtailed at a flat renumeration rate of 0.35 € per kWh. A further 20 percent of total loads are assumed to be shiftable at a flat rate of 0.15 € per kWh per hour shifted. The remaining 70 percent of loads are deemed essential. The commercial units can also shift up to 20 percent of their load for 0.15€ per kWh per hour, but no curtailment is allowed.

The profiles are scaled for the total yearly demand of the smaller residential buildings to be around 5000 kWh. The multi-family residential and commercial buildings' profiles are scaled to a yearly demand of approximately 10000 kWh and 16000 kWh respectively.

5.5 Grid Supply

For the base scenario the grid supply is assumed to be capped at a constant value of 500KW, which guarantees that all supply can be met solely by the grid. It is further assumed that the price of this energy is set at 0.3 € at any time ["Monitoringbericht 2018" 2018]. The feed-in tariff is assumed to be constant at 0.08 € and the feed-in is capped at a constant 20 kW.

5.6 Further Assumptions

To keep the model from trading randomly inside the microgrid, a trade fee of 0.001 € per kWh is imposed on all trade within the microgrid.

To accurately reflect the prevalence of the weather conditions and demand profiles represented by the three seasonal periods modeled, the summer, winter and transitional period have been weighted with the factors 169, 81 and 115, respectively. These factors are normalized during preprocessing (see 4) and all cost incurred during each period is multiplied with the normalized corresponding weight. The weights have been chosen according to the prevalence of different weather conditions in the meteorological region of Morschenich "Referenzlastprofile von Ein- und Mehrfamilienhäusern für den Einsatz von KWK-Anlagen" 2008.

5.7 Results

Surprisingly, even in the base scenario, the microgrid seems to be commercially viable, as an autarky level of 94 percent is reached. Investment is heavily biased towards Fuel Cells (see figure 4), which are installed at a smaller capacity than pv installations but operate at a capacity factor of up to 100 percent versus around 12 for pv. Therefore, most power is generated by flexible gas fuel cells (dark blue) and only supplemented by a small amount of pv generation (light blue)(see Figures 6 and 7). The ratio of fuel cell versus pv installation varies depending on the total amount and temporal distribution of demand: While the small residential residences install mostly fuel cells, the larger residential units invest in an almost equal capacity of both and the commercial units install a much more pv capacity than fuel cells. This is probably to keep the self-consumption ratio high, while at the same time minimizing trade, which is cheap but not free. The commercial units have by far the highest demand during midday, which can be served by solar energy without demand shifting or storage.

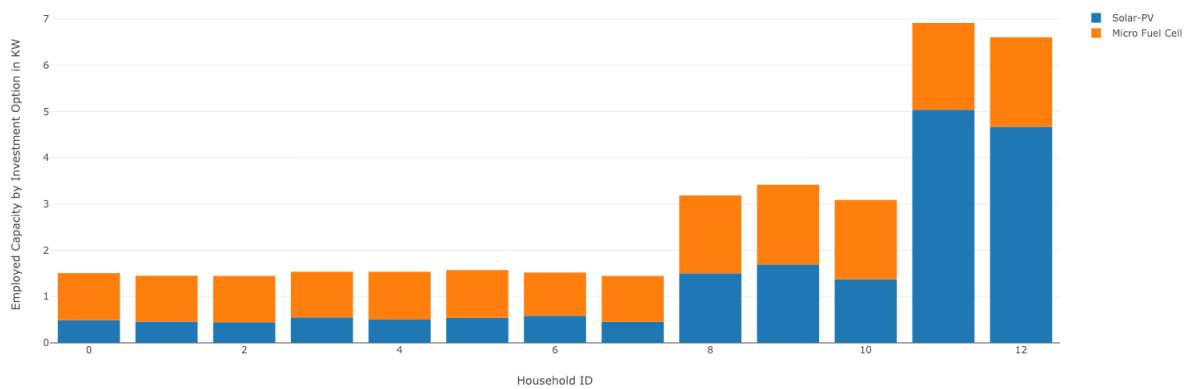


Figure 5: Generation investment in the base setup of the case study

Source: Own Model User Interface

With this setup the model stays at a self-consumption rate of 100 percent, as feed-in renumeration seems to be smaller than the marginal cost of generating power with the fuel cells. Furthermore no storage is needed to achieve these levels of autarky and self-consumption.

As can be observed in Figures 5 and 6 trading (orange and pink) is heavily utilized for balancing, as the load profiles of participants differ enough to enable this. Also load shifting (green and purple) is sporadically used (see Figures 5 and 6), but loads are rarely shifted for more than one hour.

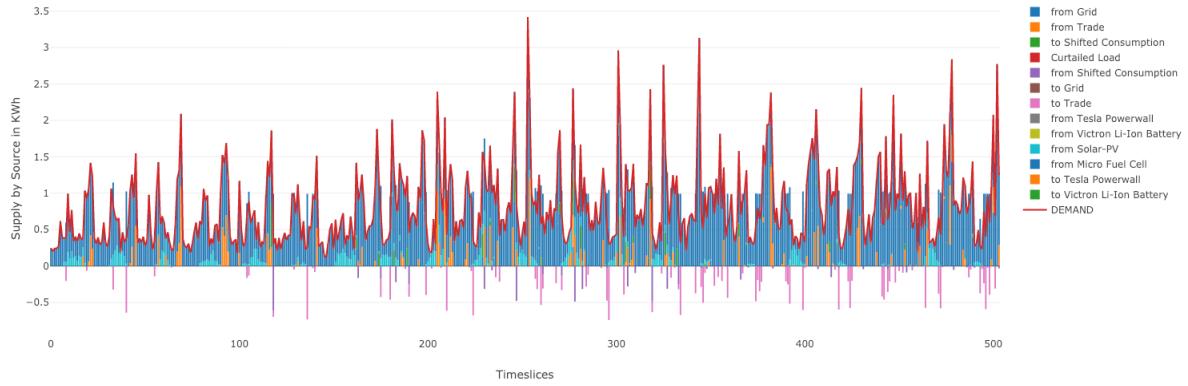


Figure 6: The dispatch of a residential building in the base setup of the case study

Source: Own Model User Interface

The use of grid energy (also dark blue, but below orange and light blue) occurs during the winter week (the last third of the dispatch chart), when generation and internal dispatch are insufficient to meet peak demand, most likely due to a lack of irradiation (see Figure 7). This indicates that the leveledized cost of solar energy is significantly below that of the electricity generated by fuel cells, as it would otherwise be cheaper to satisfy demand entirely with them instead of adding pv and as a consequence having to rely on some grid imports during winter.

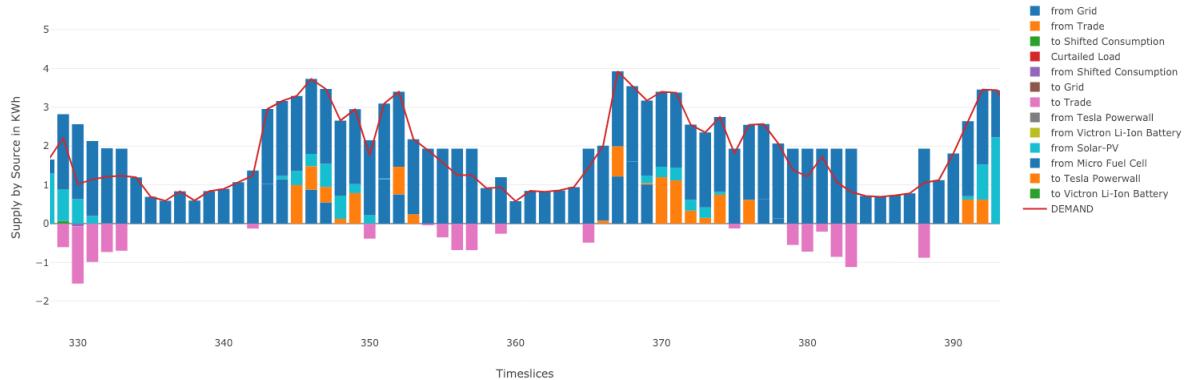


Figure 7: The dispatch of a commercial building in detail for two winter days

Source: Own Model User Interface

Overall, the result is surprisingly cheap, with the average cost per kWh being at about 0.164 € and the total cost at approx. 1259.624 €.

The model solves in approx. 4-6 minutes on a macOS 10.14 system with the Gurobi Commercial Solver and a 3.3Ghz Quad-Core Intel Core i5 processor with 24GB of 1867MHz DDR3 RAM.

5.8 Sensitivity Analysis

To test the validity and robustness of the solution obtained above, a sensitivity analysis is conducted.

To test the importance of the use of fuel cells for the result and its robustness, first the price of gas is raised by 50 percent. This could reflect the CO^2 neutral use of more expensive biogas or the insecurity of the price of fossil fuels in general, as the investment horizon considered in the case study is up to 25 years.

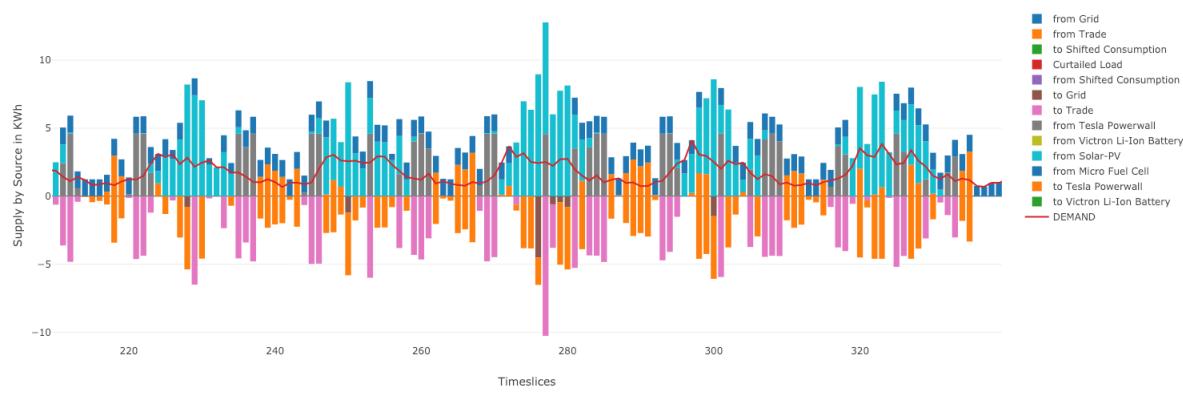


Figure 8: Effects of a gas price increase of 50 percent

Source: Own Model User Interface

The increased gas price results in a drop of installed fuel cell capacity by around a quarter. Pv deployment consequently increases by over 300 percent. One of the commercial buildings is equipped with both 17.7 kWp of pv and a Tesla Powerwall, which is utilized heavily to store solar energy traded to the microgrid during the evening (see Figure 7). Autarky remains high at 92 percent, while self-consumption drops slightly to 96 percent. The grid is still not utilized much since the average price per KWh at 0.199 € is still well below grid power prices. Still the objective value rises approx. 20 percent to 1524.547 €.

Another factor which seems critical to the base result is the reliance on trading. To test the robustness of trading, the price per traded KWh of electricity is raised from a merely symbolic 0.001 € to 0.03 €. This could be considered the cost of maintaining the infrastructure of the grid, or the energy lost in cross-microgrid transactions.

This has only a negligible effect, as the objective value is only raised by about 10 €. Raising the fee to 0.06 € per KWh has a similarly small effect, as it increases the objective value by another 18 €. A further increase to 0.09 € fails to have a much greater impact at an objective value of approx. 1293

€.

Another possibility is a change in the price of pv panels and batteries in the coming years. To test the effect these changes could have on the optimal result, the base case is altered by reducing the investment price of pv installations by 25 percent and the investment price of storage by 35 percent. This has a similar effect to increased gas prices, in that fuel cell deployment falls, whereas pv installations increase. Additionally, as before, one Tesla Powerwall is installed by a commercial entity. However, the effect on the objective value is not significant, as it is only decreased by 46 € or 0.005 € per KWh. Adding the 50 percent increased gas prices to the alterations from base case results in a total installation of 64 kWp pv, but only a single Powerwall, which seems to suffice, as self-consumption is still at 92 percent. The average electricity price rises to 0.19 €. Decreasing battery prices by 50 percent relative to the base case results in the installation of a second Powerwall in one of the multi-family residential buildings. The solution however, is hardly superior at 0.188 € per KWh.

Finally, a major weakness of the base cases result is the linear scaling of the price of generation technology, which leads to the installation of small capacities throughout the microgrid. In reality this would be very expensive, as the prices do not in fact scale linearly. To test the robustness to non-linear generation options, we convert both generation options to discrete investments. The pv installation will only be available in increments of 5 kWp and the Fuel Cell in increments of 2.5 kWp.

The computation is run with a 15 min time limit, as it does not terminate in a reasonable amount of time otherwise. The gap to optimality after this period is at 0.26 percent or approx. 4 €.



Figure 9: A household with a pv installation under an all-discrete constraint

Source: Own Model User Interface

While the objective value at approx. 1257 € does not greatly differ from the base case, the added

constraints led to a very different behavior. Households are now clearly separated into producers and consumers, some generating power with a pv installation, some with a fuel cell (see Figures 9 and 10). This leads to a strong increase in trading activity between the households as the microgrid tries to distribute the now more centralized generation.



Figure 10: A household with a fuel cell under an all-discrete constraint

Source: Own Model User Interface

This is conducted quite successfully, as autarky remains high at 94 percent. Electricity from the grid is only imported during the winter period (see Figure 11). The model still does not construct any storage.

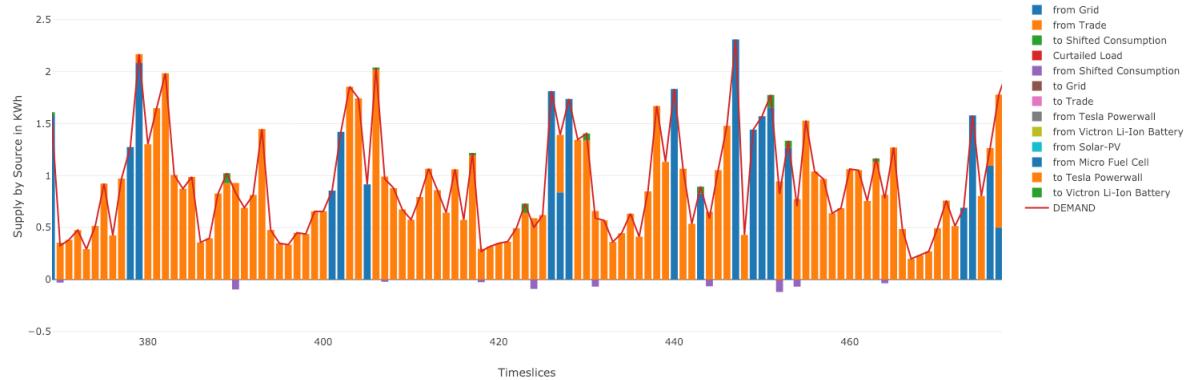


Figure 11: A household without generation under an all-discrete constraint during winter

Source: Own Model User Interface

6 Discussion

Although the sensitivity analysis showed the model result to be quite robust, there are weaknesses to point out: The cost computation is simplified and does not take into account borrowing rates and does not discount future earnings. The result rather resembles a situation in which all equipment is leased and in which interest rates on this leasing contract are negligible. While this is very close to the truth in most of Europe and the US, it is still a minor distortion and a major one anywhere else.

Another weakness is the modeling approach itself. By its very nature LP assumes perfect knowledge of the input parameters involved. This includes all parameters describing demand and supply, which in reality cannot be known at the point of an investment decision. Furthermore, dispatch decisions in an LP are also made with perfect knowledge of the future, so that every time storage is filled or demand shifted, the perfect amount is certain, which of course is not the case in a real-world scenario. This affects only dispatch decisions with an impact on future dispatch decisions, which do not feature heavily in the base case, since it does not install any storage. As a consequence, the fact that the results arrived at are the product of an LP should only imply a minor advantage in investment optimality, since this can be predicted with quite high accuracy by means of statistics. It also implies only a minor advantage in dispatch optimality, since demand shifting is the only dispatch method utilized that is impacted by perfect foresight. Moreover, demand shifting is mostly utilized for only one hour into the future, for which quite accurate forecasting tools are available, which further reduces the information disadvantage of a real-life scenario.

Any minor inaccuracies aside the model results clearly conclude that a microgrid, as described in the base case, if regulatorily feasible, would be highly profitable with a margin of over 0.13 € per KWh. The sensitivity analysis further shows that even under significant deterioration of some of the assumptions made the margin still remains higher than 0.10 € per KWh. There is, however, some uncertainty involved, as being razed by RWE would significantly worsen the profitability of the system. To the question of the bigger picture of decentralization, the answer is less clear: While clearly profitable in the context of German electricity retail prices, the generation costs of 0.164 € per KWh are still significantly higher than the current generation costs included in the retail price of 0.067 € per KWh ["Monitoringbericht 2018" 2018]. This would conclude, that decentralized generation is still far off, when it comes to large scale competitiveness, it however fails to consider some important factors: First, there is an additional cost of 0.0719 € per KWh incurred by transmission, when buying electricity ["Monitoringbericht 2018" 2018]. In a decentralized scenario of 94 percent autarky, as exhibited in the base results, these costs would most likely be vastly smaller, if at all significant. This brings us to a total of 0.1389 € per KWh which is significantly closer to the 0.164 € result of the

base case. Secondly, there are another 0.0917 € of levies (excluding taxes) included in the retail price, the majority of which is used to subsidize renewable generation ["Monitoringbericht 2018" 2018]. Still, the present German electricity mix is much dirtier, than the one used in the microgrid of the base case, which only consists of solar and highly efficient natural gas generation. Taking this 'power quality' consideration into account the total price of equivalent electricity delivered by a centralized system is (at least in Germany) closer to 0.2305 €, which is significantly higher than even the results reached in the sensitivity analysis. Furthermore it does not even take into account the heat output of the fuel cells, which can be used without losses for heating water or air.

In addition, it is highly likely that a major uptake of decentralized generation methods would reduce investment costs, especially of fuel cells, further bringing down the estimated price of electricity. Of course, a decentralized system would incur other costs, as a rudimentary grid would still be needed and industry would possibly suffer from the loss of access to subsidized electricity [Küchler and Wronski 2014]. But overall, the results arrived at in this case study suggest a bright future for decentralized generation systems, as they close, – or even have already closed – the the cost gap to current centralized systems.

7 Conclusion

In this thesis a mathematical modeling tool for microgrid energy systems has been designed, taking the current literature on the subject into account. The modeling tool has then been used for a case study on a village in Northrhine-Westphalia, in which several buildings of the village form a hypothetical microgrid. The investment choices available in the case study consisted of a pv installation and a fuel cell as generation technologies and two different discrete battery investments as energy storage. In addition the participants were able to trade energy within the microgrid as well as shift or curtail some of their demand. There also was the possibility to draw power from the main grid at standard retail rates. Special attention was given to the realism of the demand profiles, as standard profiles used elsewhere are averaged and would have distorted the model's results significantly (see 5.4).

The results showed the clear profitability of the base case at an average electricity cost of about 0.164 € per kWh. The microgrid had an autarky ratio of 94 percent, while 100 percent self-consumption was reached. This was due to significant investment in fuel cells, which produced the majority of electricity in the base case. There was no need for storage installation. The sensitivity analysis showed the astounding robustness of the base case. An increase in fuel prices of 50 percent lead to the biggest additional cost, with the price per kWh rising to 0.199 €. Even a full discretization of all investment options merely resulted in a vast increase of trading activity within the microgrid.

It has been shown clearly, that the base case, if regulatorily feasible, would be highly profitable. Based on these results a microgrid system architecture could even be considered on par with, or superior to the current grid performance excluding taxes and levies: It hardly needs any transmission infrastructure outside microgrid boundaries and the power generated comes exclusively from renewable or highly efficient conventional sources that could also use biofuels or hydrogen in the future (see 6). Considering these results, the concept of a decentralized multi-microgrid energy system should be explored further, e.g. by means of a larger scale pilot project. Furthermore a partially or fully decentralized option should be added to the plans to decarbonize the German electricity system by 2050, as only heavily centralized solutions are currently discussed by planners. Fuel cell technology as a small scale, flexible way of generating power and heat should receive more attention. It has the potential to be a key technology enabling a faster transition to renewable generation without relying on lignite and coal plants. The next steps in the development of the modeling tool include improvement of the user interface, including the possibility for users to add their own investment options and demand profiles. Another priority is the publication of a running instance of the tool on the internet, enabling everyone to experiment with the model.

My thesis demonstrated the enormous potential of decentralized microgrid energy systems, especially considering expected price decreases for all the investment options considered. Microgrid energy systems are becoming serious competitors for conventional centralized power systems.

8 Appendix

8.1 Literature Review

The used search string was: [microgrid AND (optimization OR optimisation) AND linear programming]

The used databases are ScienceDirect, IEEE Xplore and Google Scholar.

The original dataset consisted of 300 publications, of which 275 remained after duplicates were merged.

The search string for the abstract keyword search was [(microgrid OR micro-grid OR off-grid) AND (optimization OR optimisation OR optimise OR optimal OR optimally) AND (linear programming OR linear program OR mixed integer)]

After the abstract keyword search was conducted, 106 publications remained.

The inclusion criteria were:

- An optimization model is employed.
- There is some discussion of the design of the model.
- The objective of the model is the optimization of design or dispatch of a single microgrid.
- There is some sort of case study conducted.

The exclusion criteria were:

- The publication is not in English.
- The full text is not obtainable for this author with reasonable effort.
- The publication is a Work-In-Progress / Conference Paper version of a publication published in a journal and also included in this dataset.
- The mathematical model designed is non-linear.
- The mathematical model designed only considers a specific aspect of dispatch or design, not the entirety.
- The mathematical model is mostly focused on heat generation / distribution rather than

electricity.

After filtering the publications by inclusion and exclusion criteria, 58 publications remained.

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