Quantum nonlocality on the quantum circuit

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Abstract

Quantum nonlocality is a peculiar relation of quantum observables that differentiates classical and quantum worlds. Meanwhile, the development of quantum computers gives us a powerful tool to simulate quantum physics. In this work, we show how quantum circuits - quantum computing's natural language - can be used to construct Hardy's nonlocal states and test them out. We build circuits for n-particle states with non-vanishing nonlocal probability as n grows, then run them on IBMQ real device and Qiskit simulator. The results show agreement with theoretical predictions of quantum mechanics.

The nonlocal problem

Consider two bases on n qubits, $\Omega = \{1, \cdots n\}$,

$$|u_k\rangle = A_k^*|c_k\rangle - B_k|d_k\rangle,\ \ |v_k\rangle = B_k^*|c_k\rangle + A_k|d_k\rangle,$$
$$|c_k\rangle = A_k|u_k\rangle + B_k|v_k\rangle,\ \ |d_k\rangle = -B_k^*|u_k\rangle + A_k^*|v_k\rangle.$$

With N be some normalization, let

$$|\Psi_n\rangle = N\Big[|c_1c_2\cdots c_n\rangle - \prod_{i=1}^{n} A_i|u_1u_2\cdots u_n\rangle\Big].$$

We verify $|\Psi_n\rangle$ to satisfy nonlocal conditions

$$P(U_1U_2\cdots U_n) = 0,$$
$$P(U_1\cdots U_{k-1}U_{k+1}\cdots U_n|D_k) = 1,\ \ \forall k \in \Omega,$$
$$P\Big(\prod_{k=1}^{m} D_i\Big) > 0,\ \ 2 \le m \le n$$

both theoretically and experimentally. The last condition yields the nonlocal probability,

$$P_{\text{nonlocal}} = A^{2n} - n\frac{A^{4n-2}(1-A^2)}{1-A^{2n}},$$

where $|A_k| = |A_l| = A\ \forall k \neq l \in \Omega$. In the case $n = 2$, we return to the traditional Hardy's paradox [1], where the optimal $A = \sin\frac{\theta}{2} = \frac{\sqrt{\alpha\beta}}{\sqrt{1-|\alpha\beta|}} \approx 0.785$ gives $P_{\text{nonlocal}} \approx 0.09$. For high n, optimization in Python indicates $P_{\text{nonlocal}} \approx 0.156$ for $n = 100000$ faster than exponential, with optimal A inching closer to 1 as n grows.

The IBMQ devices

Name	CX error	Readout error	Network
x2	1.82%	3.18%	⧖
Quito	1.09%	4.01%	T

The IBMQ computers still yield noticable error rates. Still, we were able to recover results that behave similarly to theoretical predictions.

References

[1] L. Hardy, Nonlocality for two particles without inequalities for almost all entangled states, Physical Review Letters **71**, 1665 (1993).

[2] Tran, D.M., Nguyen, D.V., Le, B.H. et al. Experimenting quantum phenomena on NISQ computers using high level quantum programming. EPJ Quantum Technol. **9**, 6 (2022).

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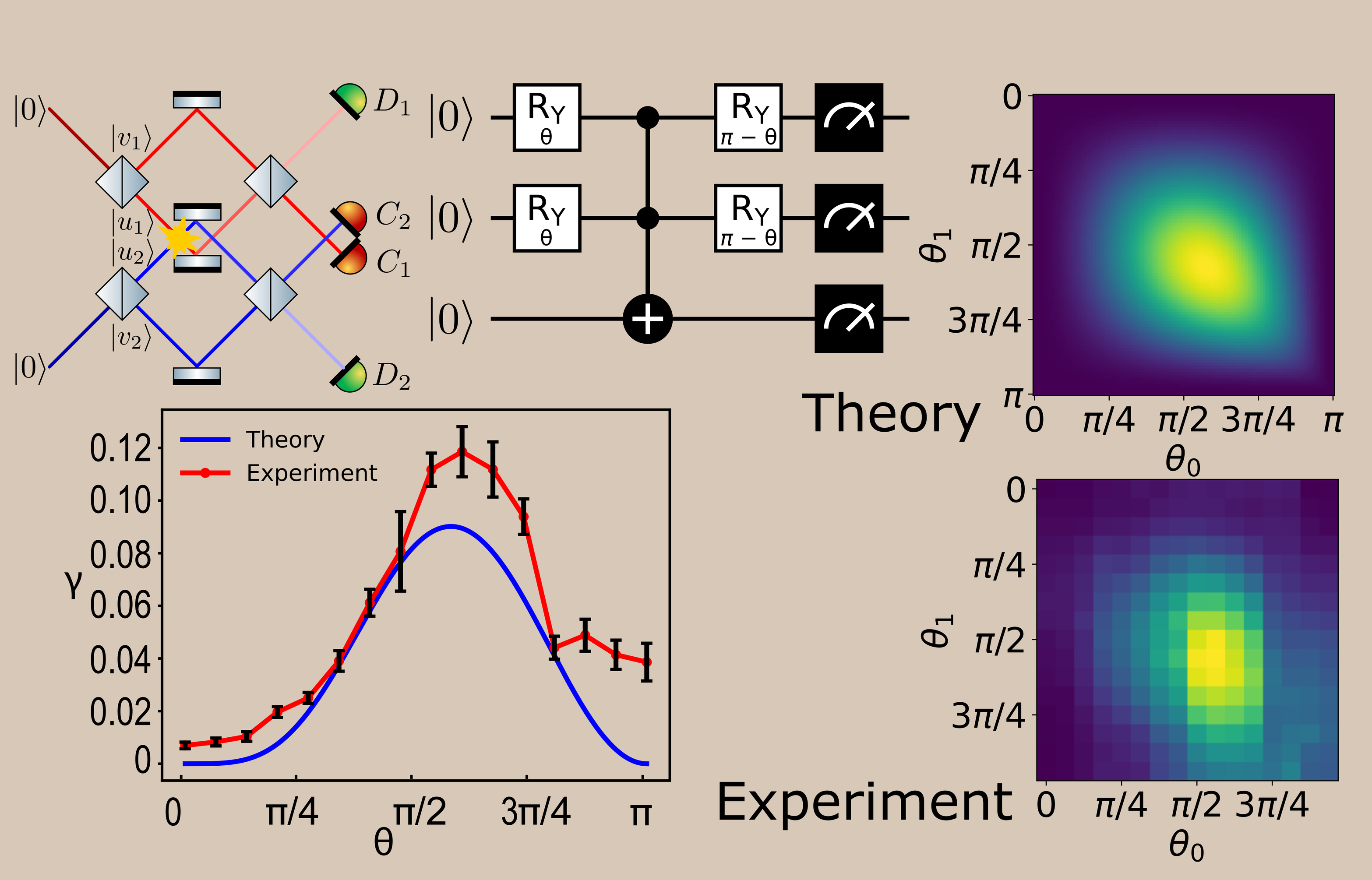
Hardy's paradox on the IBMQX2

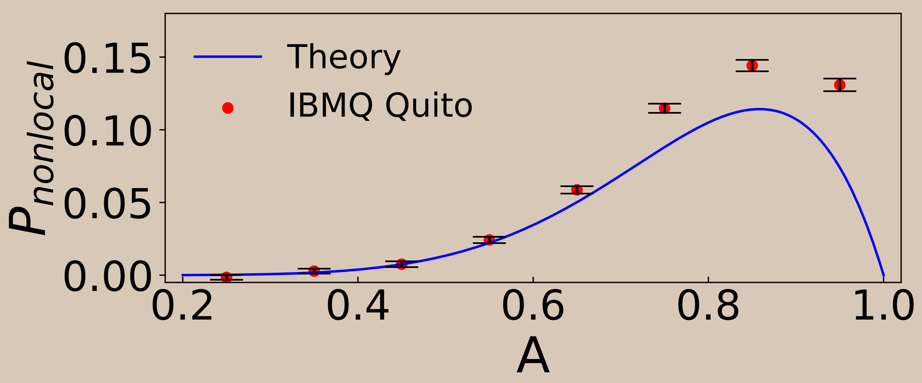
An electron-positron pair is lead into two Mach-Zehender interferometers (MZI) arranged so that one of each path crosses to an annihilation event. Without one, the other would always go to its constructive interference detector C_1 or C_2. A detection at the destructive interference detectors D_1 or D_2 indicates that a counterpart was sent through the crossing point. The interesting case occurs when both D_1 and D_2 click, which indicates that the electron and the positron met at the crossing point [1, 2]].

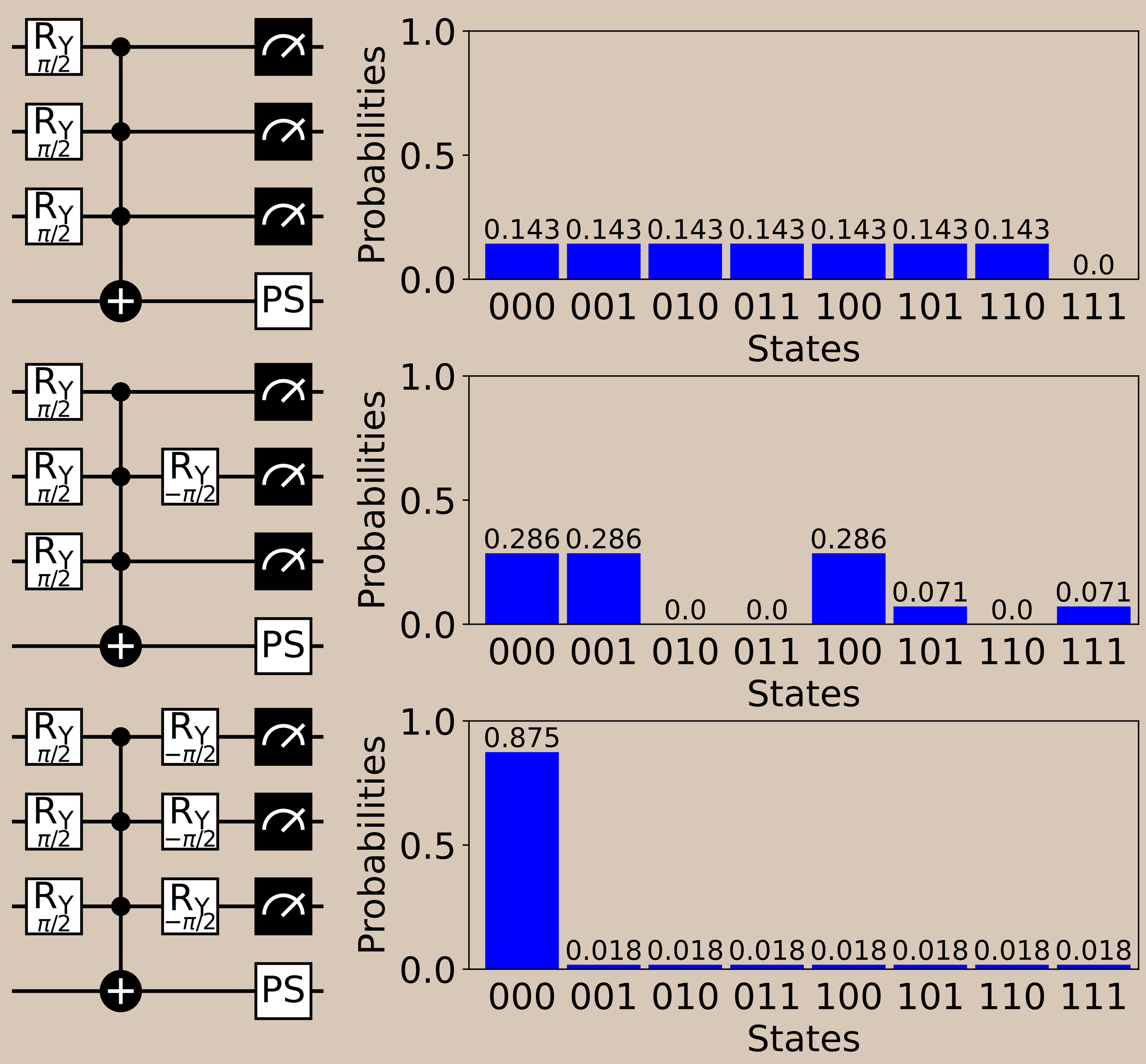


Experiment from the IBMQ Vigo shows a maximal nonlocal probability of $\gamma = 0.119$ at $\theta = 0.533\pi$, close to Hardy's result.

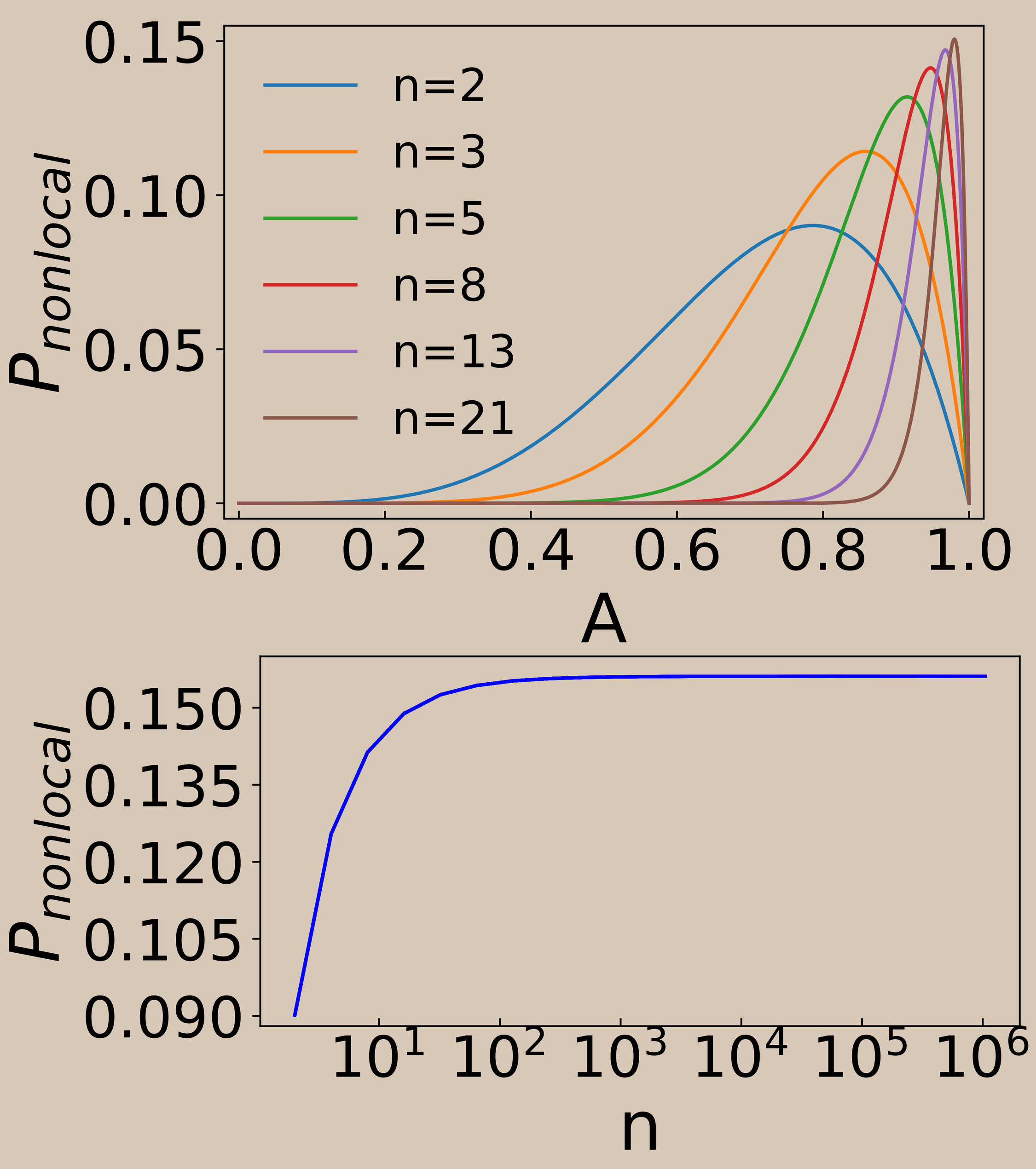
n-Hardy's paradox on the quantum circuit and IBM Quito

We proposed a quantum simulation method to test the 3 sets of nonlocal conditions. n distinct agents are given n particle. They can measure in either $\{|u_k\rangle, |v_k\rangle\}$ or $\{|c_k\rangle, |d_k\rangle\}$ bases, then come together to verify the nonlocal conditions. At $\theta = \frac{\pi}{2} \Rightarrow A = \sin\frac{\theta}{2} = \frac{\sqrt{2}}{2}$, $P(U_1U_2U_3) = P(111) = 0$, $D_2 = 1 \Rightarrow U_1 = U_3 = 1$, and $P_{\text{nonlocal}} = 0.072$. Below is the experimental results of $|\Psi_3\rangle$ on IBMQ Quito, with error bars showing standard deviations of 100 tests on each A (all dots shifted down by the same systematic error).



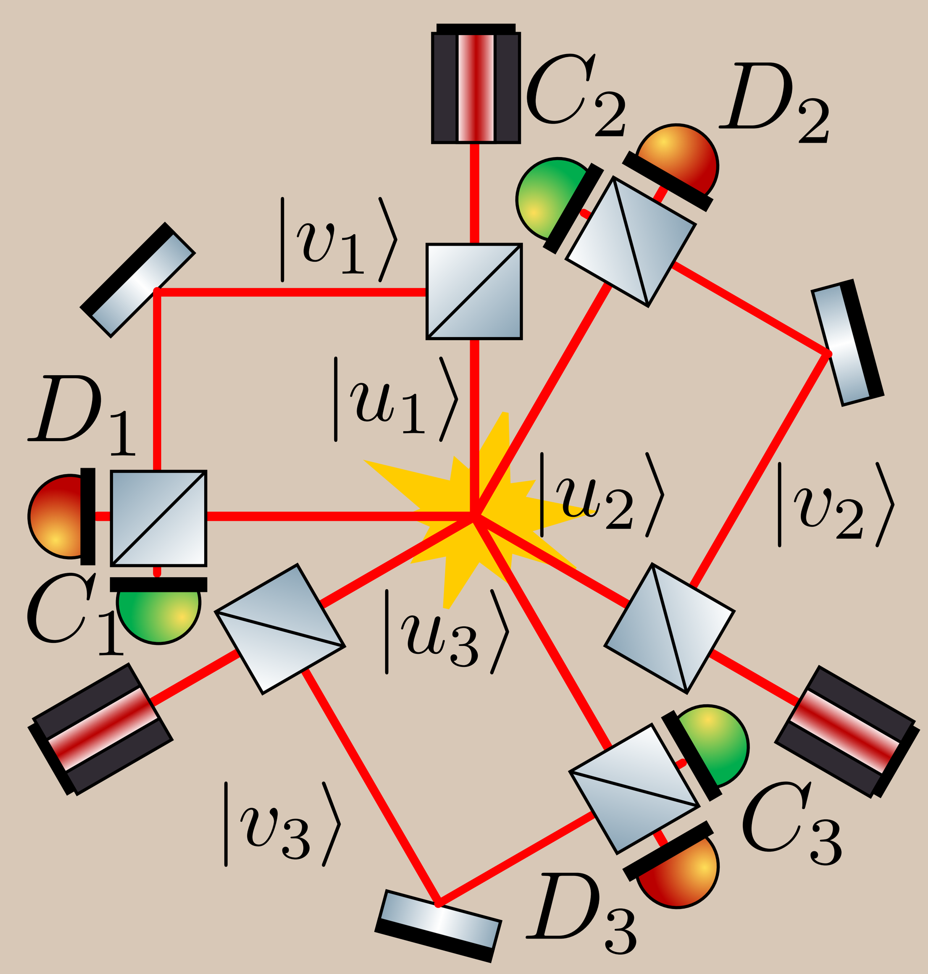


High n analysis



Optical schematic diagram

The experiment could be done by 3 Mach-Zhender interferometers with a coincidence counter at the $|u_1u_2u_3\rangle$ path.



Conclusion

We develop a quantum nonlocal scheme where the nonlocal probabilty oddly grows along n to infinity. The explicit nature of how $|\Psi_n\rangle$ is formed on qubits makes it easily realizable on quantum simulation systems as we have demonstrated on the quantum circuit, IBMQX2 and IBM Quito devices. Results indicate relative agreement with traditional quantum theory.