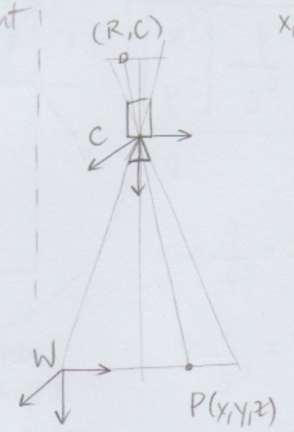
CAMERA MODEL & PARAMETERS

- Camera types { · area scan — whole area at once
  · line scan — lines captured + movement
- Lens types { · pin-hole: like human eye
  · telecentric: far objects as close ones
  · hypercentric: sides of objects captured

how a point is transformed from 3D point to R,C pixel

Mapping: World ⟷ Pixel
3D — 2D
X,Y,Z — r,c



^{W}P — World coord
$^{C}H_{W}$
^{C}P — Camera coord
projection
^{C}q — 2D in cam
distorsion model
$^{C}\tilde{q}$ — 2D distorted
tilt transformation
^{t}q — tilted
discretization
^{i}q — R,C pixel

This summary explains each step ↑

→ AREA SCAN (+ PIN-HOLE MODEL)

image is inverted!

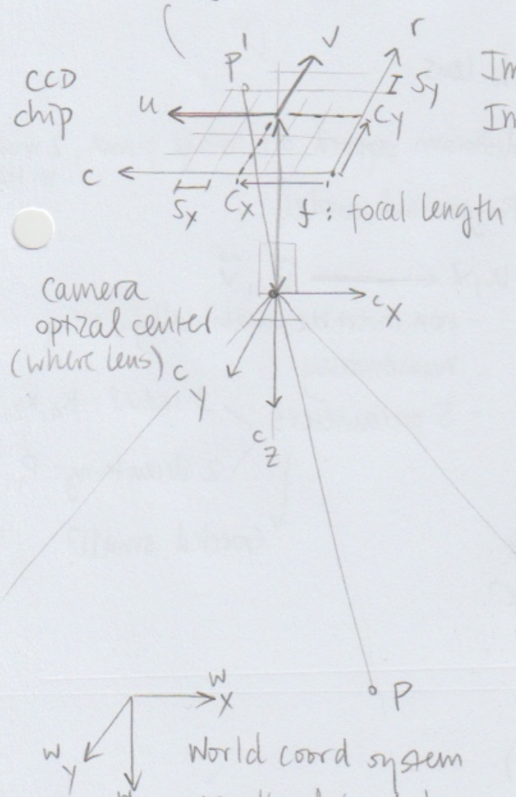


Image plane coord system (u,v)
Image coord system (r,c)

f: focal length

World coord system usually defined by pose of first cal plate

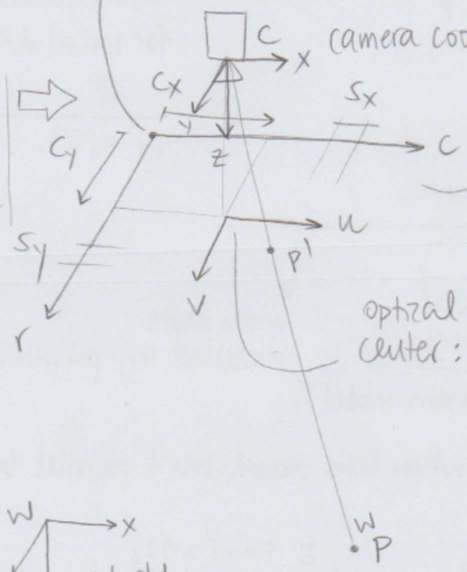
- For simplicity, the image plane is represented below the optical center (although it is behind)

Origin of image coords (r,c): top-left corner

from camera sheet:

C_x, C_y: Image/2
S_x, S_y: pixel size
f: focal distance

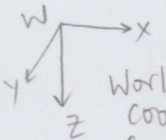
⇒



camera coord system

Virtual image plane

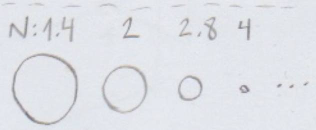
optical center: image center

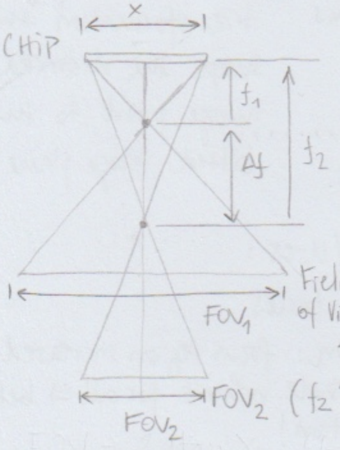


World coord system (first cal plate)

- Focal length / focus: f
distance from lens to chip on which image is projected — you can adjust it manually, but never change it after calibration!

to improve focus





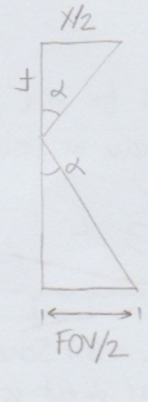
Field of View (FOV)

↑ (f_1↓ → FOV_1↑)

FOV_2 (f_2↑ → FOV_2↓)

$$FOV = 2\,\mathrm{atan}\left(\frac{x}{2f}\right)$$

for a pin-hole lens



- f-number: f/N: ratio of the system's focal length (f) to the diameter of the entrance pupil.
  - dimensionless, measure of lens speed
  - small number ⇒ large aperture ⇒ fast lens, because it can achieve same exposure with faster shutter speed

— **Projection**: from cam coords in 3D to image plane coord system 2D

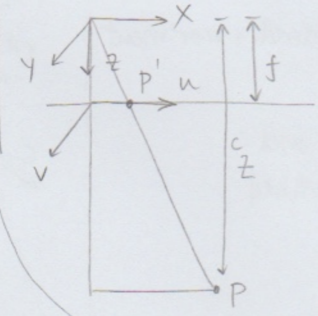
· Pin-hole (human eye)

$$\boxed{{}^{c}\vec{q} = \begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{{}^{c}z} \begin{bmatrix} {}^{c}x \\ {}^{c}y \end{bmatrix}}$$

P'   P

$$\frac{{}^{c}x}{{}^{c}z} = \frac{u}{f} \rightarrow u = \frac{f}{{}^{c}z} x$$

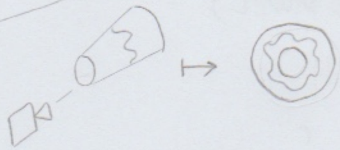
$$\frac{{}^{c}y}{{}^{c}z} = \frac{v}{f} \rightarrow v = \frac{f}{{}^{c}z} y$$



· Hypercentric lens (= pericentric): top and side seen simultaneously, with a converging view

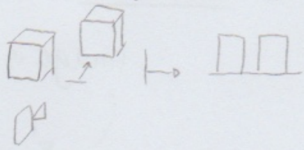
$$\boxed{{}^{c}\vec{q} = \begin{bmatrix} u \\ v \end{bmatrix} = \frac{-f}{{}^{c}z} \begin{bmatrix} {}^{c}x \\ {}^{c}y \end{bmatrix}}$$

Note: the only difference is $-f$, because optical center below lens & object between optical center & lens

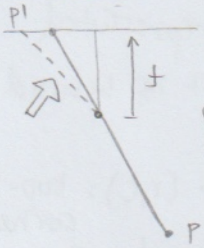


· Telecentric lens: parallel projection — far objects seen as close ones

$$\boxed{{}^{c}\vec{q} = \begin{bmatrix} u \\ v \end{bmatrix} = \underbrace{m}_{\text{magnification factor}} \cdot \begin{bmatrix} {}^{c}x \\ {}^{c}y \end{bmatrix}}$$



— **Distorsion**: due to lens distorsion P & P' don't lie on same line; distorsion applied on image plane, 2 models in HALCON



① Division model

$u, v \longleftrightarrow \tilde{u}, \tilde{v}$

- invertible analitically
- one parameter for radial distorsions: kappa (κ) → expected large?

$$u = \frac{\tilde{u}}{1 + \kappa(\tilde{u}^2 + \tilde{v}^2)}, \dots$$

② Polynomial model

$u, v \longleftarrow \tilde{u}, \tilde{v}$

- non-invertible analitically, only numerically
- 5 parameters → expected small?
  - 3 radial: K_1, K_2, K_3
  - 2 decentering: P_1, P_2

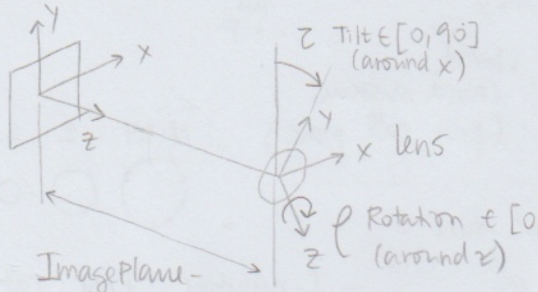
Typical distorsions:

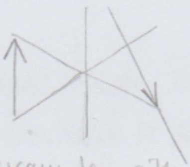
pincushion   barrel   ...

modelled by division model

much more modelled by polynomial model (see table)

— **Tilt lenses**: when lens plane isn't parallel to image plane (chip) — two rotation variables: ρ, τ



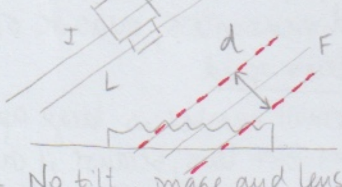


example with area scan and pin-hole — many other combinations possible!

$${}^{t}\vec{q} = \underset{\sim}{H}\, {}^{c}\tilde{\vec{q}}$$

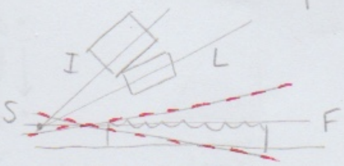
↳ homogeneous affine transformation which embodies the projective transformation that maps the distorted image point to the tilted image plane.

Why tilt lenses? Because thanks to the Scheimpflug principle planar objects can be completely in focus!



- No tilt, image and lens plane parallel.
- Depth-of-field = d, small focus region

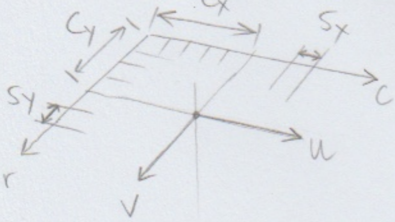
→ depends on f and f-number...



- $I \cap L = S$: Scheimpflug line
- planar object completely in focus!

Applications / Uses:
- Physical obstacles
- Stereo vision: focus region increased to cover whole volume for all ⇒ better reconstruction!
- Sheet-of-line: align with sheet of light

— Transformation to pixel coordinates



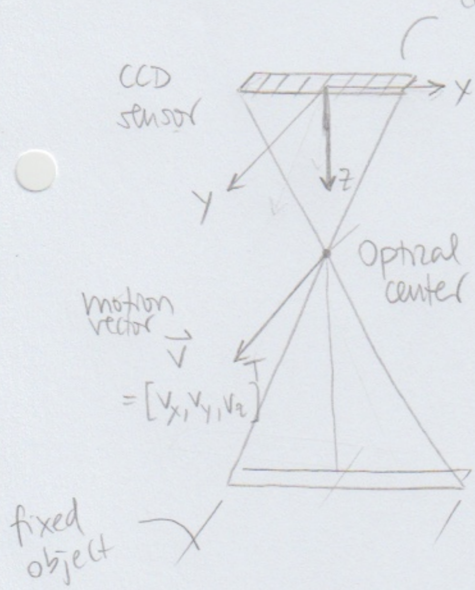
tilted image coord.

$$\vec{{}^{i}q} = \begin{bmatrix} r \\ c \end{bmatrix} = \begin{bmatrix} \frac{\hat{v}}{S_y} + C_y \\ \frac{\hat{u}}{S_x} + C_x \end{bmatrix}$$

C_x, C_y : ImageSize/2
principal point of image

S_x, S_y : scaling factors,
pixel size

→ LINESCAN



- $\vec{V}$: meters / scanline
- assumed object is fixed and cam moving if not, sign of V changes and that's it
- transformations from world to cam coords based on the pose of first image only!
- equations in Solution Guide, but are not necessary for using operators...

CAMERA PARAMETERS | Determined during CALIBRATION!!!

* Default (always necessary)
  CameraType, ImageWidth, ImageHeigth
  ImagePlaneDist → area / line
* Internal cam. parameters
  f, Magnification (m),
  Kappa (K),
  K_1, K_2, K_3, P_1, P_2,
  τ, ρ,
  S_x, S_y, C_x, C_y
* External cam. parameters
  Pose of cam in world (usually defined with first cal plate pose)
  $t_x, t_y, t_z, \alpha, \beta, \gamma$: ${}^{c}H_w$

Note: depending on cam type, lens type, tilt, model, etc. a subset is taken

Camera parameter subsets (example)

'area_scan_division'
  f, K, S_x, S_y, C_x, C_y, ImW, ImH
'area_scan_polynomial'
  ...
'*_tilt_*'
'*_telecentric_*'
  ...
'line_scan'] → no telecentric lens supported for calibration
  ...