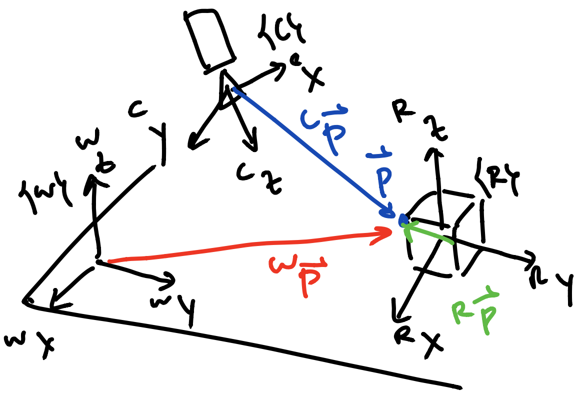
3D TRANSFORMATIONS

① Coordinate systems



- The point $\vec{P}$ can be observed and expressed from/in different coordinate systems — but it's always the same point!
- Coordinate systems or frames {W}, {C}, {R}

② Homogeneous transformations

$$^C\vec{P} = \begin{bmatrix} ^Cx_p \\ ^Cy_p \\ ^Cz_p \\ 1 \end{bmatrix} \longrightarrow {}^R\vec{P} = \begin{bmatrix} ^Rx_p \\ ^Ry_p \\ ^Rz_p \\ 1 \end{bmatrix}$$

$$\underbrace{\underset{\sim}{H}}_{4\times4} \cdot \underbrace{\vec{P}}_{4\times1} \quad !!$$

$$^RP = {}^R\underset{\sim}{H}_C \, {}^CP$$

$^R\underset{\sim}{H}_C$: Frame C expressed in frame R

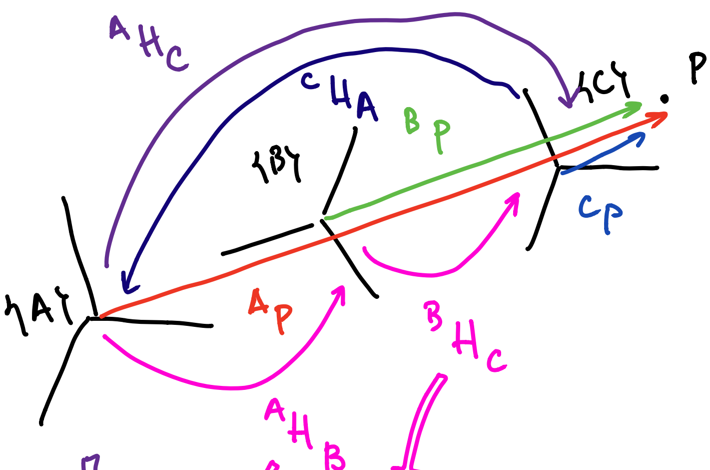
$$= {}^RR_C \cdot {}^CP + {}^Ct$$

$$\underbrace{^R\underset{\sim}{H}_C}_{4\times4/3\times4} = \left[\begin{array}{c|c} ^R\underset{\sim}{R}_C & ^R\vec{t} \\ \hline 0\,0\,0 & 1 \end{array}\right]$$

$^R\vec{t}$ → translation necessary from C to R, expressed in R

$$ {}^{R}\underset{\sim}{R}_c = \left[{}^{R}\vec{x}_c \,\middle|\, {}^{R}\vec{y}_c \,\middle|\, {}^{R}\vec{z}_c \right] \quad \begin{cases} * \ \det(R) = 1 \\ * \ \underset{\sim}{R} \cdot \underset{\sim}{R}^T = \underset{\sim}{I} \implies \underset{\sim}{R}^{-1} = \underset{\sim}{R}^T \\ * \ \underset{\sim}{R} \text{ is symmetric} \end{cases} $$

③ Chain & inverse rules



1) $${}^{A}H_{C} = {}^{A}H_{B} \cdot {}^{B}H_{C}$$

2) $${}^{C}H_{A} = \left({}^{A}H_{C}\right)^{-1}$$

④ Notes on homogeneous transformations

1) $$H = \left[\begin{array}{c|c} \underset{\sim}{R} & \vec{t} \\ \hline 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} r_{xx} & r_{xy} & r_{xz} & t_x \\ & \cdots & & t_y \\ & & & t_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & & \emptyset & t_x \\ & 1 & & t_y \\ \emptyset & & 1 & t_z \\ \hline & 0 & & 1 \end{array}\right] \cdot \left[\begin{array}{c|c} \underset{\sim}{R} & 0 \\ \hline 0 & 1 \end{array}\right] =$$

$$= \underset{\sim}{H}(\vec{t}) \cdot \underset{\sim}{H}(\underset{\sim}{R})$$

Translation pre-multiplied to rotation

2) $\underset{\sim}{H}_1 \cdot \underset{\sim}{H}_2 = \left[\begin{array}{c|c} \underset{\sim}{R}_1 & \vec{t}_1 \\ \hline 0 & 1 \end{array}\right] \cdot \left[\begin{array}{c|c} \underset{\sim}{R}_2 & \vec{t}_2 \\ \hline 0 & 1 \end{array}\right] =$

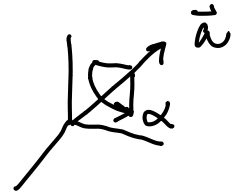
$$= \left[\begin{array}{c|c} \underset{\sim}{R}_1 \cdot \underset{\sim}{R}_2 & \underset{\sim}{R}_1 \cdot \vec{t}_2 + \vec{t}_1 \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

Second translation rotated with first transformation

⇨ Don't assume scalar operation style !!

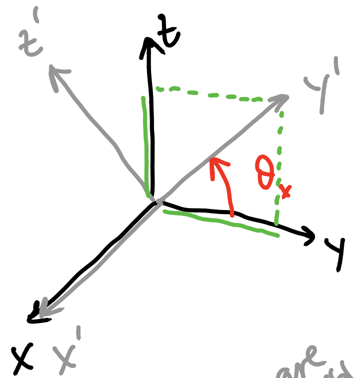
5 Rotations : Representations

- Only 3 values are enough for describing a rotation in 3D space, BUT order of rotation is important
- Types of representations
  - $\underset{\sim}{R}$: 9 values
  - Quaternions: 4 values
  - Dual quaternions : 8
  - Angle-axis representation: 4
  - Poses (HALCON) : 3+1



(+3 translation values)

⑥ Rotation order



$$R_x(\theta_x) = \begin{array}{cc} \begin{matrix} \;x' & \;\;y' & \;\;z' \end{matrix} & \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\vartheta & -s\vartheta \\ 0 & s\vartheta & c\vartheta \end{bmatrix} & \begin{matrix} x \\ y \\ z \end{matrix} \end{array}$$

$R_y(\theta_y)$, $R_z(\theta_z)$: computed analogously

$\underset{\sim}{R} = R_z(\alpha) \cdot R_y(\beta) \cdot R_x(\gamma)$ 'abg'

Yaw-Pitch-Roll

there are many possible order combinations!

$\underset{\sim}{R} = R_x(\gamma) \cdot R_y(\beta) \cdot R_z(\alpha)$ 'gba'

Roll-Pitch-Yaw

...

⑦ Poses

α, β, γ: rotation

$$\text{create\_pose}(\overbrace{t_x, t_y, t_z}^{\text{translation}}, \underbrace{r_x, r_y, r_z}, c)$$

it encodes order of rotation and type of transformation

$C = 0, \dots, 13$: differs depending on:

- order of transform: '$R \cdot p + t$' / $R(p - t)$
- order of rotation: 'abg', 'gba', ..
- View of transform: 'point', 'coord-system'

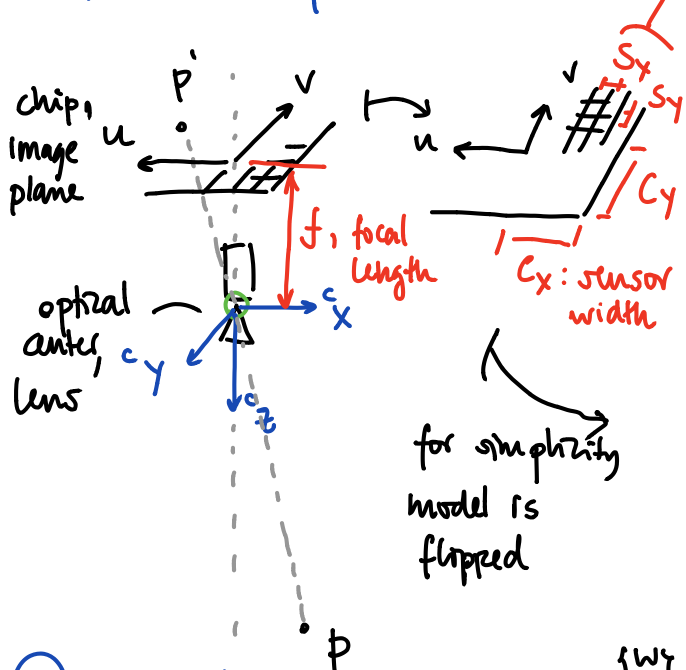
~ usual choice

CAMERA MODEL & PARAMETERS

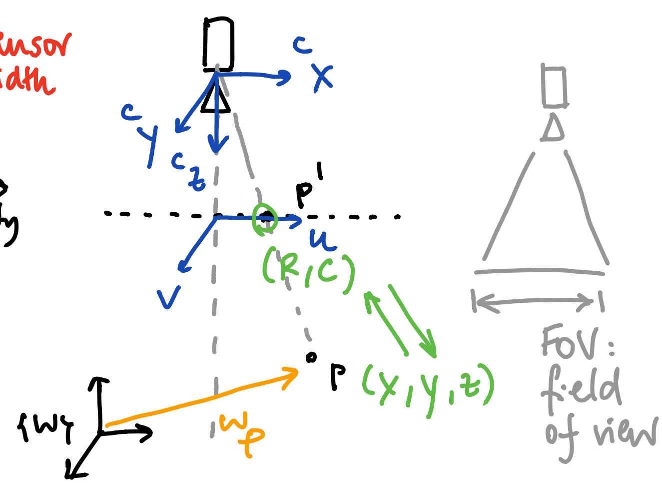
- camera types { · area scan · line scan
- Lens types { · pin-hole : "like human eye" · tellecentric · hypercentric

① Area scan + pin-hole model

scale factors / distance between sensor cells



Calibration: initial parameters
$f, C_x, C_y, S_x, S_y, \ldots$
INTERNAL PARAMETERS

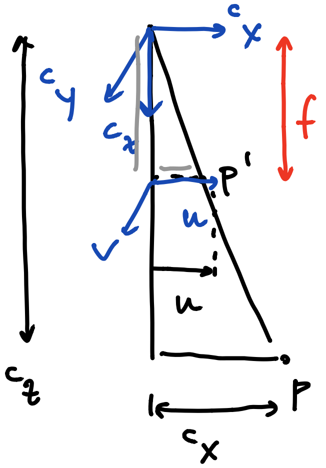


② Calibration

$$^{w}P \xrightarrow{\;^{c}H_{w}\;} {}^{c}P \xrightarrow{\text{projection}} p' \xrightarrow{\text{distorsion}} \tilde{q} \xrightarrow{\text{tilt?}} {}^{t}q \xrightarrow{\text{discretization}} {}^{i}q$$

(x,y,z) q(2D) (R,C)

③ Projection

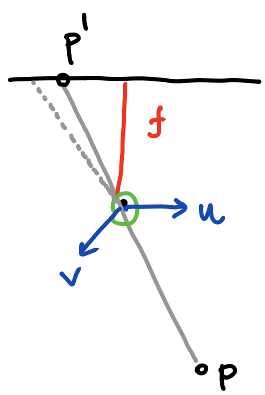


triangle similarity:

$$\frac{u}{f} = \frac{{}^cx}{{}^cz} \rightarrow u = f \cdot \frac{{}^cx}{{}^cz}$$

$${}^c\vec{q} = \begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{{}^cz} \begin{bmatrix} {}^cx \\ {}^cy \end{bmatrix}$$

④ Distorsion



1) Division model

$u, v \longleftrightarrow \tilde{u}, \tilde{v}$

$$u = \frac{\tilde{u}}{1 + k(\tilde{u}^2 + \tilde{v}^2)}$$

$v = \ldots$

one parameter: radial
analitically invertible

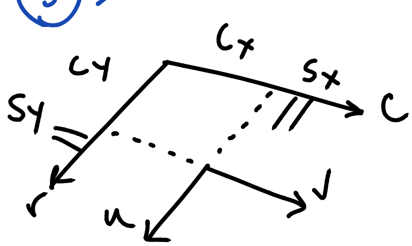
2) Polynomial model

$u, v \longleftarrow \tilde{u}, \tilde{v}$

5 parameters

- 3 (K_i): radial
- 2 (P_i): decentering

⑤ Discretization

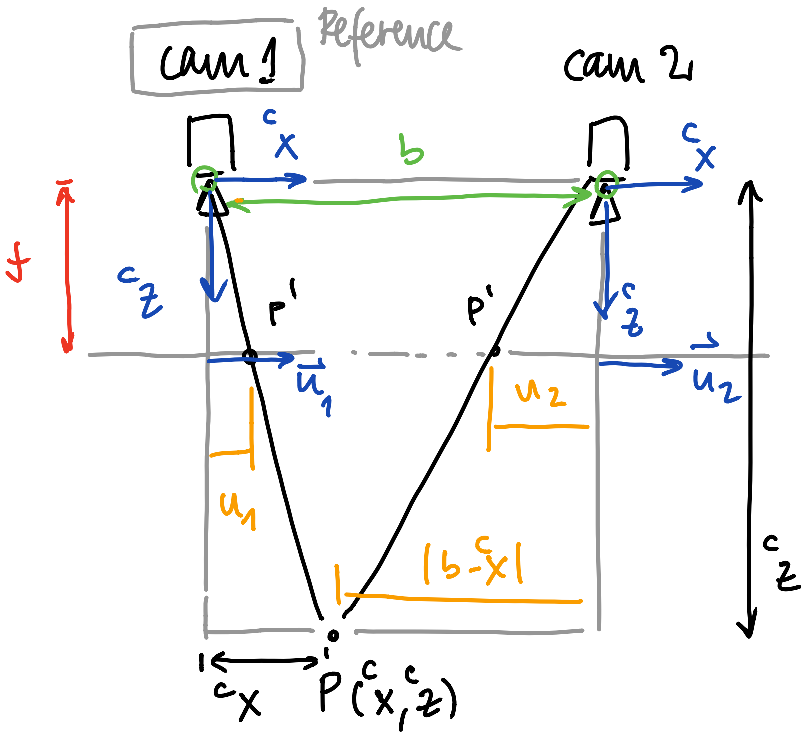


$${}^i\vec{q} = \begin{bmatrix} r \\ c \end{bmatrix} = \begin{bmatrix} \frac{v}{s_y} + c_y \\ \frac{u}{s_x} + c_x \end{bmatrix}$$

STEREO VISION

- Surfaces can be reconstructed, moving up in z direction
- 2 methods in HALCON
  - binocular: 2 cams, 1 view
  - stereo: >2 cams, 1+ views

① Principle



$$\frac{u_1}{f} = \frac{^cx}{^cz}$$

$$\boxed{u_1 = f \cdot \frac{^cx}{^cz}}$$

$$\boxed{u_2 = f \cdot \frac{^cx - b}{^cz}}$$

CONJUGATE POINTS

$$\boxed{d = u_2 - u_1 = -\frac{f \cdot b}{z_c}}$$

DISPARITY

$$\boxed{^cz = -\frac{fb}{d}} = \frac{-f \cdot b}{u_2 - u_1}$$

- Conjugate points are the same point observed by diff. cameras ⇒ they have different image coords
  ↳ difference = DISPARITY

- Stereo reconstruction: steps

1) Camera calibration : $P(\underbrace{X, Y}, 0) \longleftrightarrow (u, v), (R, C)$

2) stereo matching

2.1) Images are rectified to world plane.

After rectification all pairs of conjugate points have same ROW coords !!

2.2) for ROW

for COLUMN

$(R, C) \leftrightarrow u_1$

find a match in other image

3 methods :

1) Correlation based : compare textures

2) multigrid

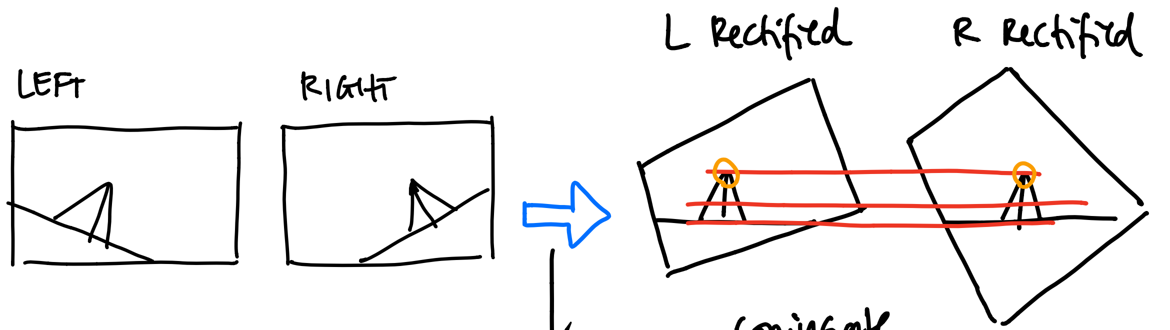
3) multi scanline

→ obtain best match for u_2

2.3) Assemble everything

$$u_1, u_2 \longrightarrow d \longrightarrow \underbrace{{}^{c}z} = -f \cdot \frac{b}{d} \rightarrow \overbrace{{}^{c}X, {}^{c}Y}, {}^{c}z$$

② Rectification



RECTIFIZATION

conjugate points are colinear!!

③ Resolution

$$\Delta z = \frac{z^2}{f \cdot b} \cdot \Delta d$$

distance resolution in $^c z$

accuracy with which disparity values can be determined

if calibration error < 0.1 pixels,
disparity accuracy ≈ [error, 2·error]
given 7.4μm pixel size ⇒ $\Delta d \approx 1\,\mu m$

- Resolution decreases quadratically as we go away from camera
- increase f & b — BUT too large b results in lower overlap of FOVs