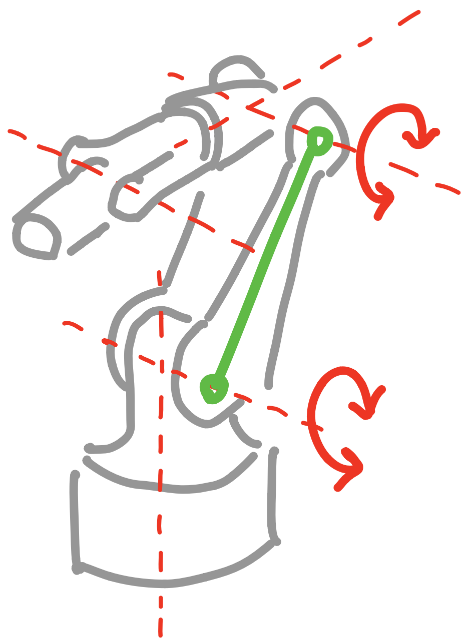
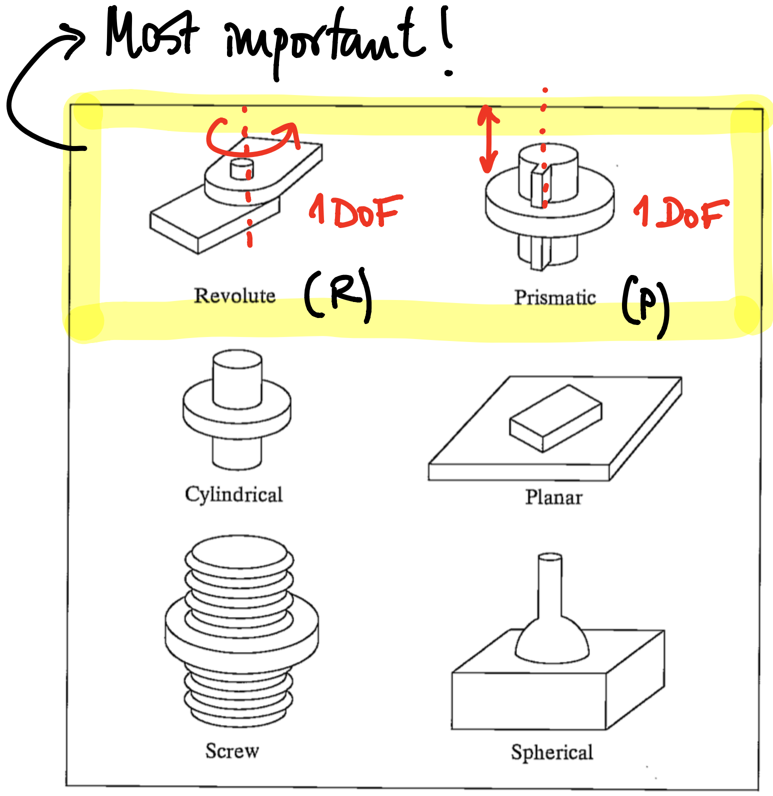
① Links and Joints





Craig - Fig. 3.1 Six possible joints

ABB robot with joints
and links between joints

- A general purpose robot has 6 DoFs; if it has more, it's redundant, if less deficient
- Rule of thumb: n DoF ⟷ n joints (each with 1 DoF)
  n-1 links

② Conventions for fixing frames to joints

There is a list of conventions to define the frames in the joints for later using DH. These sum up to:

- The axis of movement is z with z+ increasing (right hand)
- The axis x is the intersection of z's or the link axis
- y is derived using the ~~left~~ right-hand rule.

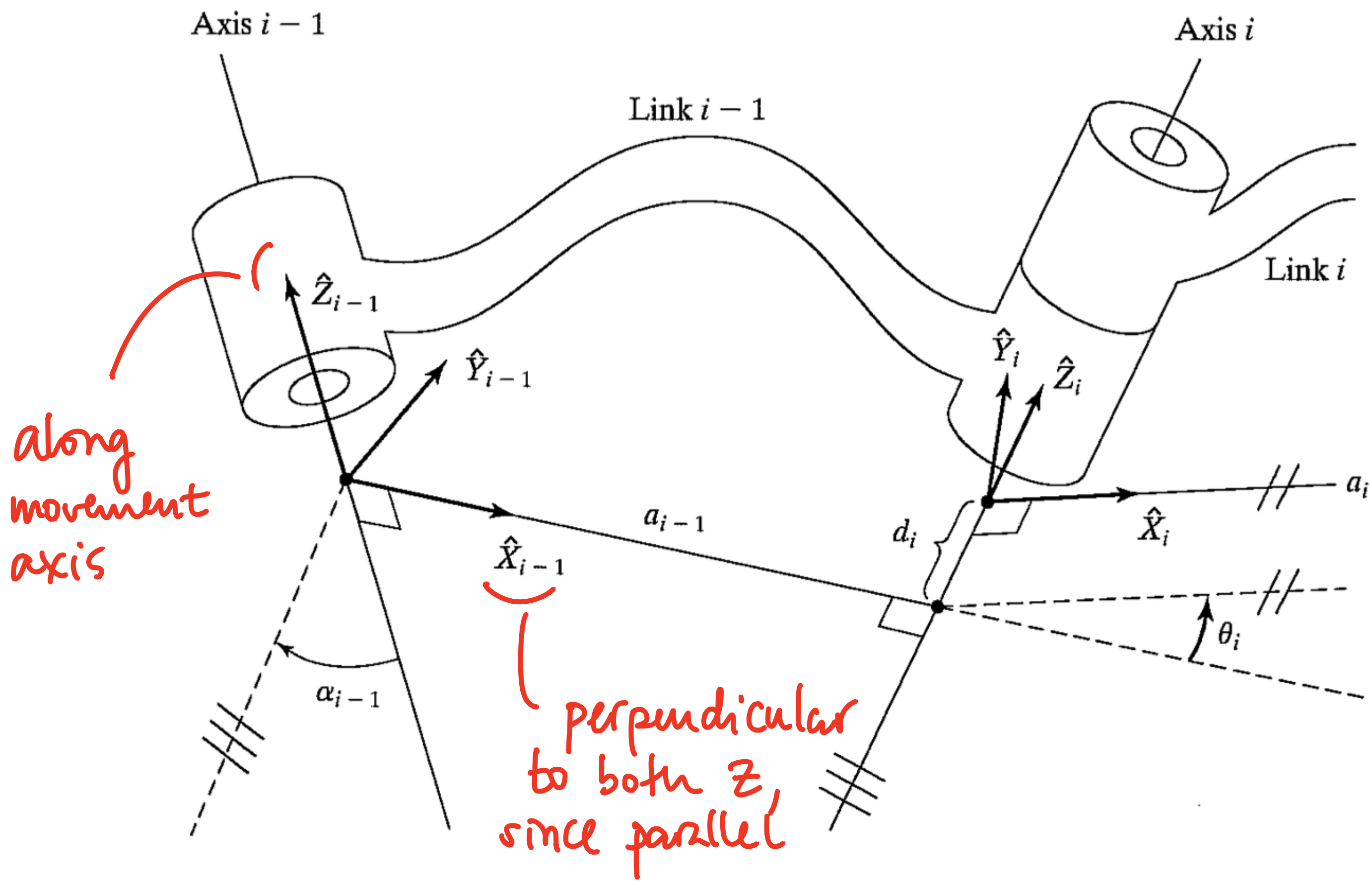
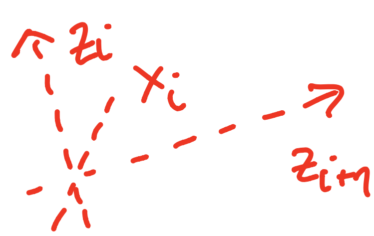


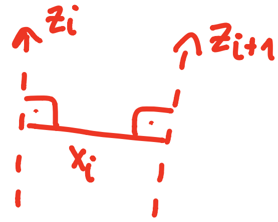
FIGURE 3.5: Link frames are attached so that frame $\{i\}$ is attached rigidly to link i.
Craig, 3rd Ed., p.68

Full list of conventions to fix frames to joints:

1. Joint Z axes are the movement axes
   - Prismatic: moving away is + direction
   - Revolute: right hand rule is + direction
2. Axis X_i is derived with Z_i & Z_{i+1}
   - if Z_i & Z_{i+1} intersect, the origin of i is the intersection point and X_i is $\perp$ to both Z_i & Z_{i+1}, i.e. cross product

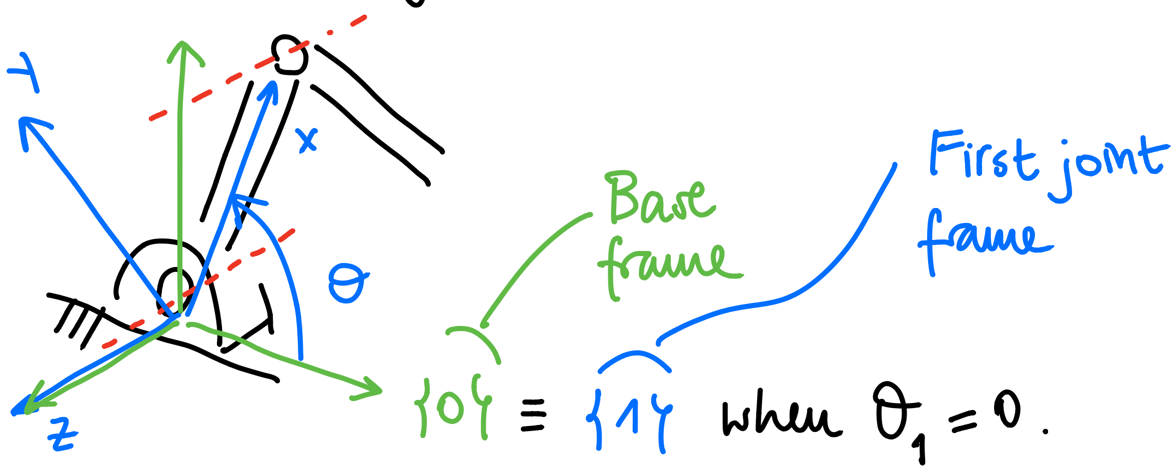


- if z_i & z_{i+1} are parallel, x_i is along the shortest line from z_i to z_{i+1}, ie., the common perpendicular



3. Assing Y_i to each frame according to the right hand rule.
   - Note that the Y axis is not relevant for DH, because it's automatic once we have X & Z.

4. Base frame. It could be chosen arbitrarily, but better define it so that the first joint frame matches it when the 1st joint variable is 0:



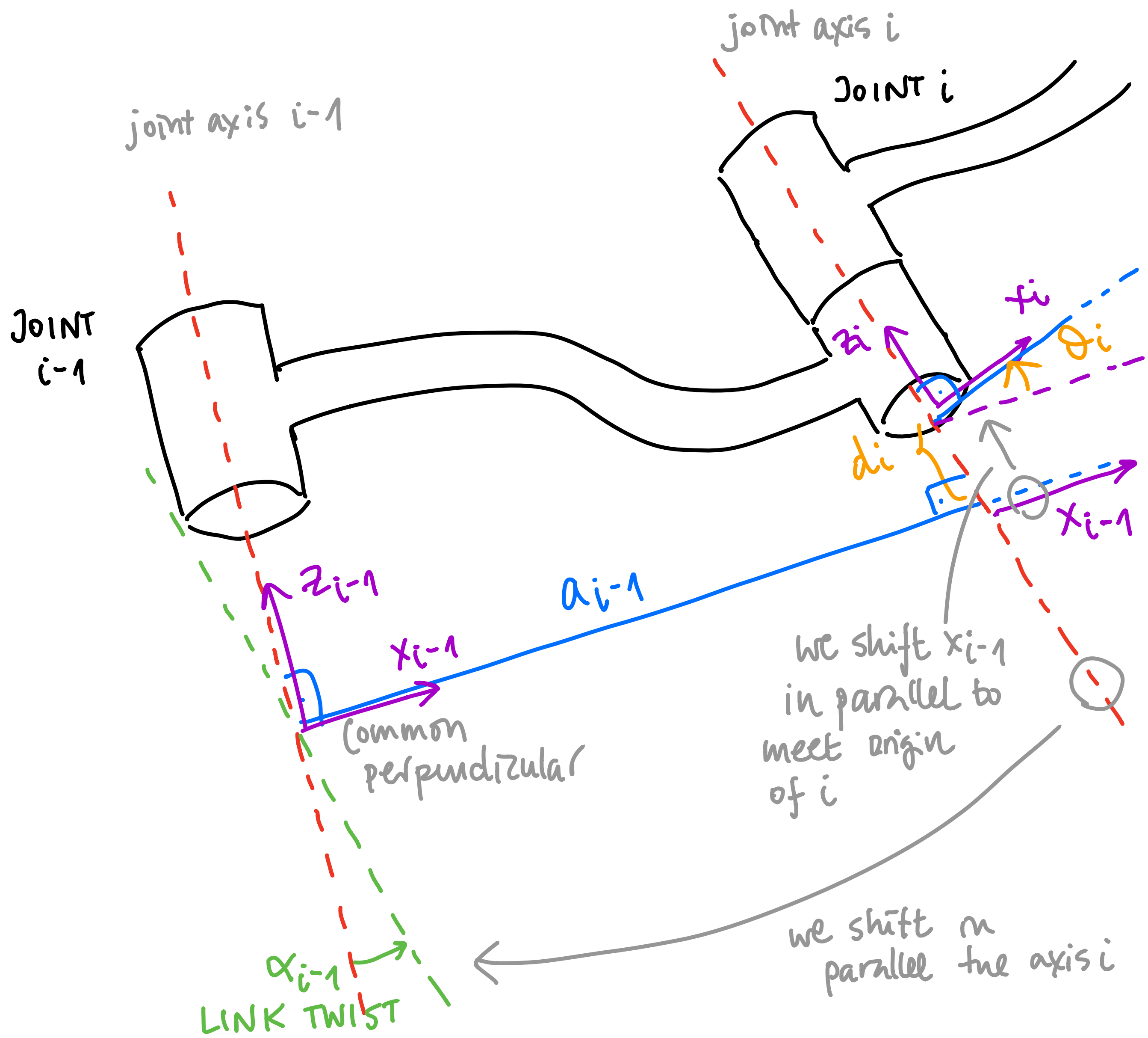
$\{0\} \equiv \{1\}$ when $\theta_1 = 0$.

Note1: Notize that the frame origins do not need to be at the center of the joint; instead, following convention 2), we could have the joint frames somewhere outside of the robot!

Note2: Sometimes we have feedom for choosing the X axis (e.g., with the base) — then, we choose the most sensible one.

③ Denavit-Hartenberg representation for describing Direct Kinematics

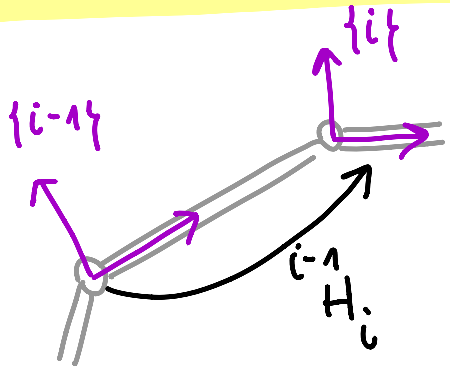
DH-parameters: 4 parameters define the transformation from link i to $i-1$; 3 are usually fixed by mechanical design, thus, a link transformation will have only 1 variable!



a_{i-1}	Link length	- we draw the i & i-1 link axis z and find the common perpendicular - the length of the perpendicular is a_{i-1} - distance from z_{i-1} to z_i measured along x_i
α_{i-1}	link twist	- we shift axis i (z_i) to axis i-1 (z_{i-1}) - α_{i-1} : angle from axis i-1 to i - angle from z_{i-1} to z_i measured about x_i
d_i	link offset often only PRISMATIC joints have $d_i \neq 0$, REVOLUTE joints: $d_i = 0$ usually	- offset distance from the common perpendicular to the link i origin - distance from x_{i-1} to x_i measured along z_i
θ_i	joint angle often only REVOLUTE joints have $\theta_i \neq 0$, PRISMATIC joints $\theta_i = 0$ usually	- angle from x_{i-1} to x_i about z_i

Then, with all these 4 values we can compute ${}^{i-1}\underset{\sim}{H}_i$:

$$
{}^{i-1}\underset{\sim}{H}_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



- This transformation is the frame i expressed in $i-1$.
- It is parametrized in θ_i or d_i, depending on the type of the joint:

REVOLUTE : ${}^{i-1}H_i(\theta_i)$, $d_i = 0$

PRISMATIC : ${}^{i-1}H_i(d_i)$, $\theta_i = 0$

The direct kinematics transformation chain from the endeffector to the base is a composition/concatenation:

$$
{}^{base}\underset{\sim}{H}_{endeffector} = \prod_{i = endeffector, \dots base} {}^{i-1}\underset{\sim}{H}_i = {}^{0}H_1 \cdot {}^{1}H_2 \cdots {}^{4}H_5
$$

To fully parametrize a robot, its DH table is provided:

Axis (i)	a_{i-1}	α_{i-1}	d_i	θ_i
JOINT $0 \rightarrow 1$ ($i=1$)				
$1 \rightarrow 2$ ($i=2$)				

④ DH Parameters Example

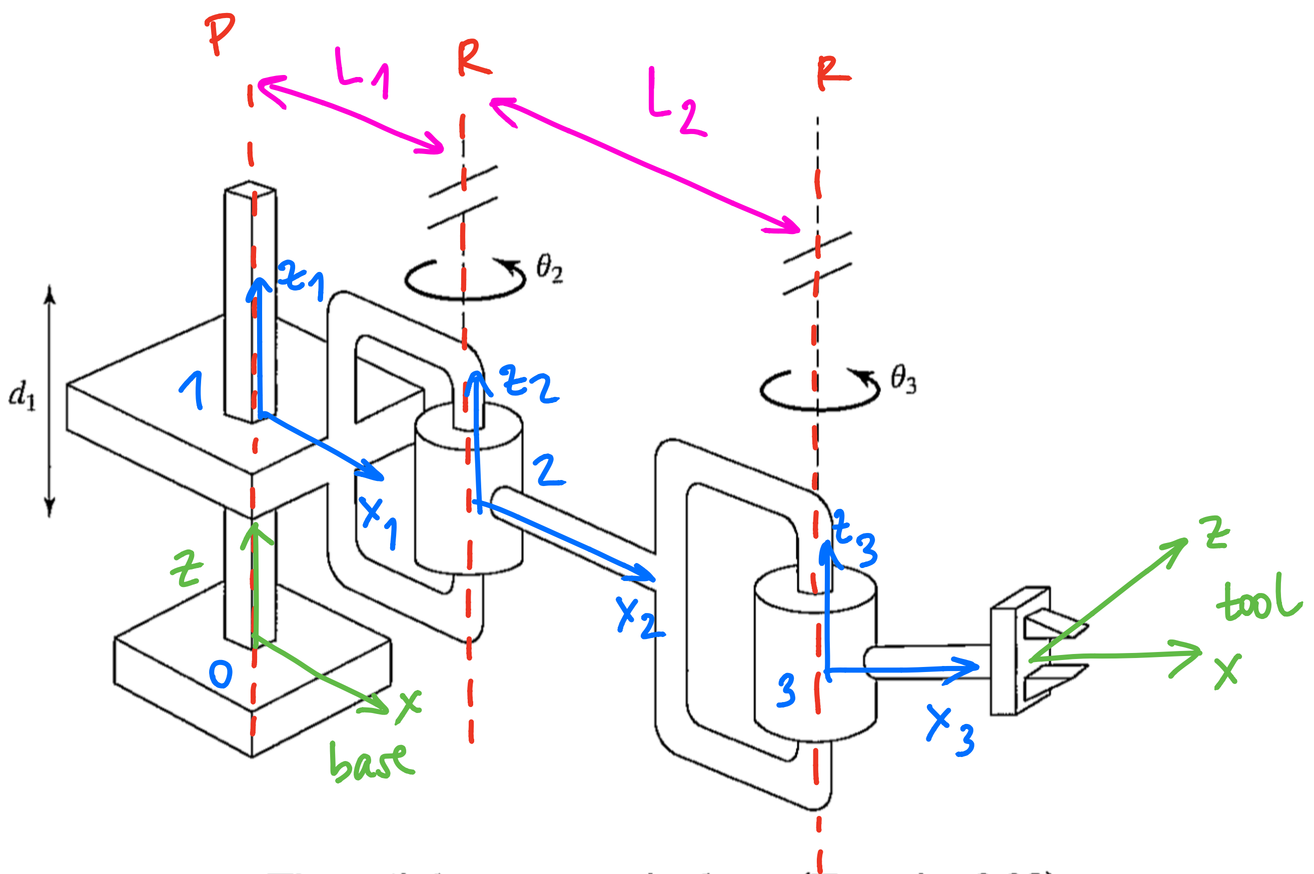
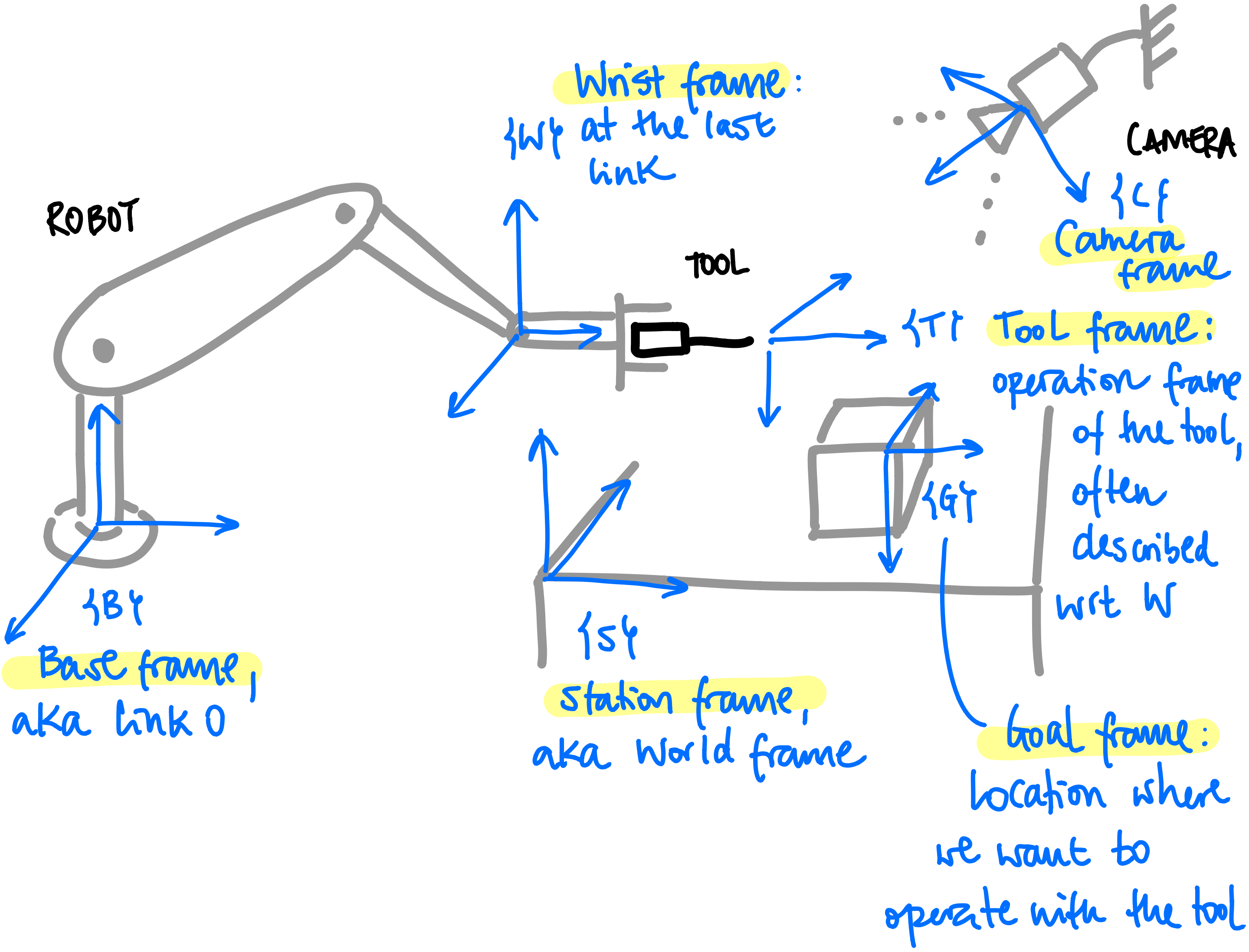


FIGURE 3.40: Three-link PRR manipulator (Exercise 3.20).

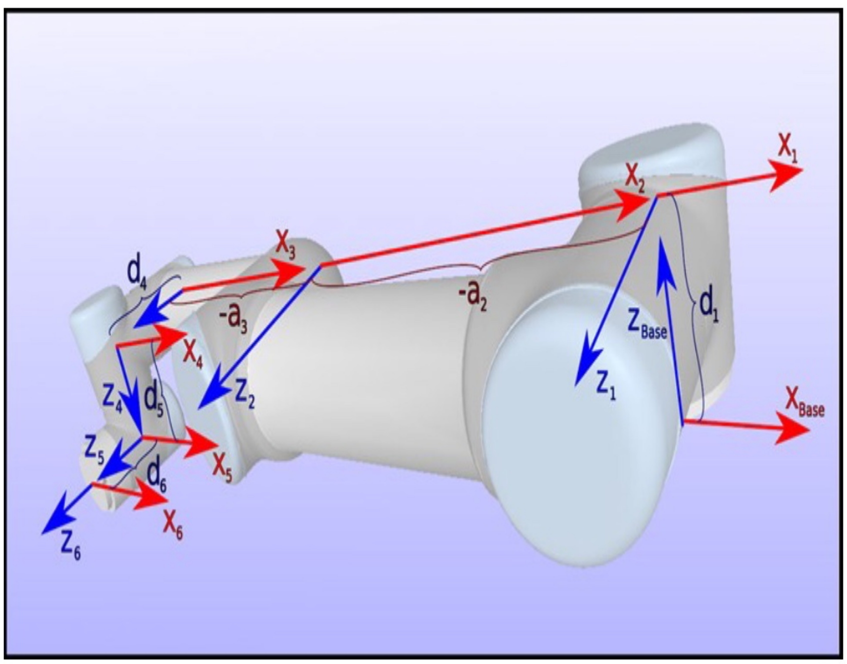
Craig, 3rd Ed., p. 98

Axis i	a_{i-1}	α_{i-1}	d_i	θ_i
joint 0 → 1 = i	0 no displacement between z_i & $i-1$	0 no twist, collinear	d_1 VARIABLE	0 both X pointing in same dir
1 → 2 = i	L_1 PARAMETER	0 no twist, parallel	0 Xs are co-planar	θ_2 VARIABLE
2 → 3 = i	L_2 PARAMETER	0 no twist, parallel	0 Xs are co-planar	θ_3 VARIABLE

(5) Frames with standard names



(6) DH Parameters of the UR5



DH tables are available online:

UR3e							
Kinematics	theta [rad]	a [m]	d [m]	alpha [rad]	**Dynamics**	Mass [kg]	Center of Mass [m]
Joint 1	0	0	0.15185	π/2	Link 1	1.98	[0, -0.02, 0]
Joint 2	0	-0.24355	0	0	Link 2	3.4445	[0.13, 0, 0.1157]
Joint 3	0	-0.2132	0	0	Link 3	1.437	[0.05, 0, 0.0238]
Joint 4	0	0	0.13105	π/2	Link 4	0.871	[0, 0, 0.01]
Joint 5	0	0	0.08535	-π/2	Link 5	0.805	[0, 0, 0.01]
Joint 6	0	0	0.0921	0	Link 6	0.261	[0, 0, -0.02]

UR5e							
Kinematics	theta [rad]	a [m]	d [m]	alpha [rad]	**Dynamics**	Mass [kg]	Center of Mass [m]
Joint 1	0	0	0.1625	π/2	Link 1	3.761	[0, -0.02561, 0.00193]
Joint 2	0	-0.425	0	0	Link 2	8.058	[0.2125, 0, 0.11336]
Joint 3	0	-0.3922	0	0	Link 3	2.846	[0.15, 0.0, 0.0265]
Joint 4	0	0	0.1333	π/2	Link 4	1.37	[0, -0.0018, 0.01634]
Joint 5	0	0	0.0997	-π/2	Link 5	1.3	[0, 0.0018,0.01634]
Joint 6	0	0	0.0996	0	Link 6	0.365	[0, 0, -0.001159]

UR10e							
Kinematics	theta [rad]	a [m]	d [m]	alpha [rad]	**Dynamics**	Mass [kg]	Center of Mass [m]
Joint 1	0	0	0.1807	π/2	Link 1	7.369	[0.021, 0.000, 0.027]
Joint 2	0	-0.6127	0	0	Link 2	13.051	[0.38, 0.000, 0.158]
Joint 3	0	-0.57155	0	0	Link 3	3.989	[0.24, 0.000, 0.068]
Joint 4	0	0	0.17415	π/2	Link 4	2.1	[0.000, 0.007, 0.018]
Joint 5	0	0	0.11985	-π/2	Link 5	1.98	[0.000, 0.007, 0.018]
Joint 6	0	0	0.11655	0	Link 6	0.615	[0, 0, -0.026]

With these tables, we can easily build our own simplified robot simulator using the formulas for ${}^{i-1}\underset{\sim}{H}_i$ in section 3 and concatenating them. However, note that the frame origins can be located outside from the robot.