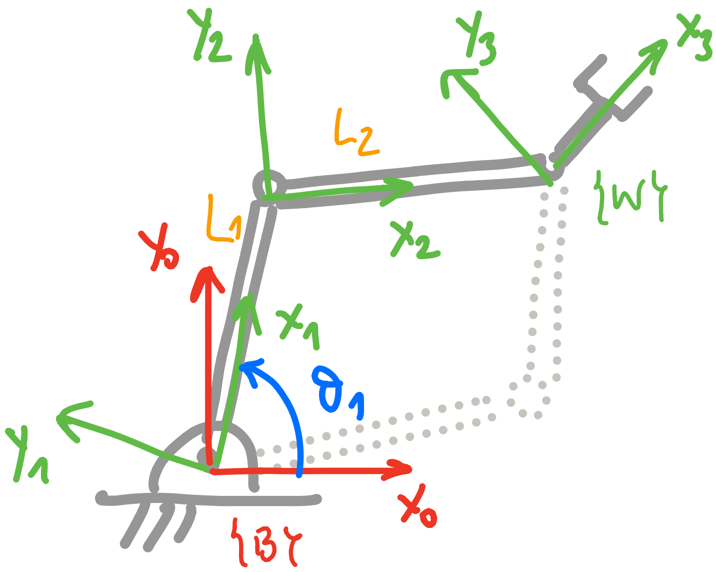
① Solvability Issues

- Direct kinematics takes θ_i joint values and computes the transformations ${}^{i-1}\underset{\sim}{H}_i$ so that ${}^{0}\underset{\sim}{H}_n$ is numerically determined
- Inverse kinematics takes ${}^{i-1}\underset{\sim}{H}_i$ and tries to derive θ_i
- Important issues for solving the inverse kinematics
  - Often non-linear transcendental equations arise, which are difficult to solve
    - sometimes only numerical solutions are possible: bad, because these are iterative (slow) and solution is not guaranteed
    - preferably, closed-form solutions are desired, and manipulators should be designed for that
      - a sufficient condition for $n = 6$ R joints robot is that 3 neighboring axes intersect at a point
      - closed-form solutions can be obtained geometrically or algebraically

that's Pieper's solution

  - We need to evaluate whether our goal is in the workspace first, if so, is it in the dextrous workspace (ie, reachable from all orientations) or in the reachable workspace (ie., at least one orientation)?
  - We can easily have multiple solutions! Solutions tend to increase with params $\neq 0$.

② Example with a planar manipulator



DH table:

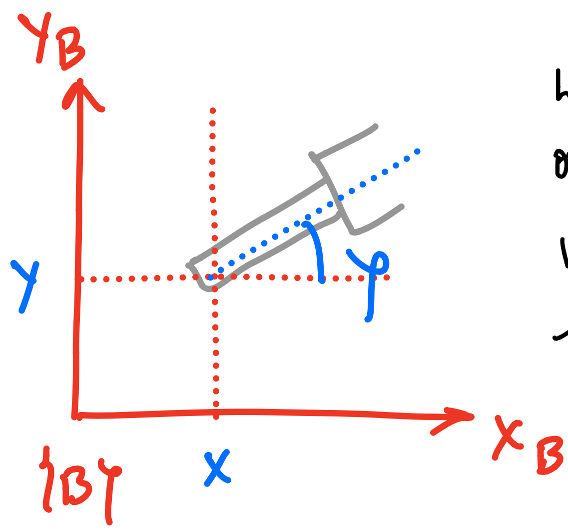
Axis (i)	a_i	α_i	d_i	θ_i
$0 \to 1$	0	0	0	θ_1
2	L_1	0	0	θ_2
3	L_2	0	0	θ_3

Planar manipulator with 3 links / joints. Note that 2 solutions exist for the final configuration drawn!

$$
{}^{B}\underset{\sim}{H}_{W} = \left[\begin{array}{ccc|c} c_{123} & -s_{123} & 0 & L_1 c_1 + L_2 c_{12} \\ s_{123} & c_{123} & 0 & L_1 s_1 + L_2 s_{12} \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] =
$$

obtained applying direct / forward kinematics

$$
= \left[\begin{array}{ccc|c} c_\varphi & -s_\varphi & 0 & x \\ s_\varphi & c_\varphi & 0 & y \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$



We want to obtain the θ_i values to put the wrist in a position (x, y) with orientation φ

- We want to solve the 3 θ_i unknowns parametrized on x, y, φ.
- Generally 2 approaches are possible:
  1) Algebraically: write down equations and try to find relationships which isolate θ_i
  2) Geometrically: try to draw and perform link projections that reveal relationships (= equations) which lead to 1)
- Once equations are obtained, we need to work on them — each mechanism has its own equations!
- Often transcendental equations (equations which contain sin & cos) arise, which are difficult to solve
  - Variable changes like the following convert these transcendental eq.s to polynomials:

$$\left.\begin{aligned} u &= \tan\frac{\theta}{2} \\ \cos\theta &= \frac{1-u^2}{1+u^2} \\ \sin\theta &= \frac{2u}{1+u^2} \end{aligned}\right\} \longrightarrow \begin{aligned} a\cos\theta + b\sin\theta &= c \\ &\Downarrow \\ (a+c)u^2 - 2bu + (c-a) &= 0 \end{aligned}$$

3 Repeatability & Accuracy

- Industrial robots have usually a very good REPEATABILITY
  - Given a set of joint values θ_i, they can repeatedly go to them with a high precission
  - Note that for classical industrial applications that is enough: we record the joint values at target positions and can play them without even inverse kinematics

- However, the ACCURACY of the manipulator is usually not as good:
  - Accuracy refers to the precission with which any cartesian point can be reached
  - We need the inverse kinematics for that and a precise DH table — if anything fails a bit, an error is accumulated...
  - In order to have a good enough accuracy, calibration is often needed!