Debiased Contrastive Learning:

contrastive Setup

$$\mathbb{E}_{x, x^+, \{x_i^-\}_{i=1}^N} \left[-\log \frac{e^{f(x)^T f(x^+)}}{e^{f(x)^T f(x^+)} + \sum_{i=1}^N e^{f(x)^T f(x_i^-)}} \right]$$

↓

replaced by Summation

whatif x^- is (uniformly Sampled from dataset)

may be same as !! → Sample bias

⇒ supervised Case → Straight forward II

for unlabelled → need unbiased sampling

$$L^N_{unbiased}(f) = \mathbb{E}_{x \sim p,\, x^+ \sim p_x^+,\, x^- \sim p_x^-} \left[-\log \frac{e^{f(x)^T f(x^+)}}{e^{f(x)^T f(x^+)} + \frac{Q}{N} \sum_{i=1}^N e^{f(x)^T f(x_i^-)}} \right]$$

$p_x^- = p(x_i' \mid h(x_i') \neq h(x))$ not accessible??

so biased x^- from p!!

How to debias:

$p(x') = \tau^+ p_x^+(x') + \tau^- p_x^-(x')$ // total prob rule.

$p_x^-(x') = (p(x') - \tau^+ p_x^+(x')) / \tau^-$

So equation ⑥ fairly conclusion

$$\mathbb{E}_{x \sim p,\, x^+ \sim p^+} \left[-\log \frac{e^{f(x)^T f(x^+)}}{e^{f(x)^T f(x^+)} + \frac{Q}{\tau^-} \left(\mathbb{E}_{x^- \sim p} \left[e^{f(x)^T f(x^-)} \right] - \tau^+ \mathbb{E}_{v \sim p_x^+} \left[e^{f(x)^T f(v)} \right] \right)} \right]$$

$$g\left(x, \{u_i\}_{i=1}^N, \{v_i\}_{i=1}^M\right) = \max \left\{ \frac{1}{\tau^-} \left(\frac{1}{N} \sum_{i=1}^N e^{f(x)^T f(u_i)} - \tau^+ \frac{1}{M} \sum_{i=1}^M e^{f(x)^T f(v_i)} \right), e^{-1/t} \right\}$$

Eq 8

contrast. learning with hard Sampling:

cracked: $\mathbb{E}_{x \sim p,\ x^+ \sim p_x^+,\ \{x_i^-\}_{i=1}^N \sim q} \left[-\log \frac{e^{f(x)^T f(x^+)}}{e^{f(x)^T f(x^+)} + \frac{Q}{N}\sum_{i=1}^N e^{f(x)^T f(x_i^-)}} \right]$

1. (q) distribution biased case (unbiased) (f dependent) or negative itself.

design this guy. not same label

$q_\beta^-(x^-) = q_\beta(x^- \mid h(x) \neq h(x^-)) \Rightarrow q_\beta(x^-) \propto \underbrace{e^{\beta f(x)^T f(x^-)}}_{\text{(concentration term)}} \cdot \underbrace{p(x^-)}$ (Depends on f)

~~conditioned~~

How to get this → Debiased contrastive learning?

$q_\beta(x^-) = q_\beta^-(x^-)\, \tau^- + q_\beta^+(x^-)\, \tau^+$ // total prob.

$\Rightarrow q_\beta^-(x^-) = \left(q_\beta(x^-) - \tau^+ q_\beta^+(x^-)\right) / \tau^-$

similarly.

$\mathbb{E}_{x \sim p,\ x^+ \sim p^+} \left[-\log \frac{e^{f(x)^T f(x^+)}}{e^{f(x)^T f(x^+)} + \frac{Q}{\tau^-}\left(\mathbb{E}_{x^- \sim q_\beta}\left[e^{f(x)^T f(x^-)}\right] - \mathbb{E}_{v \sim q_\beta^+}\left[e^{f(x)^T f(v)}\right]\right)} \right]$

now use potential function!!

↳ importance Sampling: