Data augmentation is worth a thousand sample

model, $f_\theta : \theta \in \Theta$

Parameter space: $\mathcal{F} \triangleq \{f_\theta : \forall \theta \in \Theta\}$

Coordinate space transformation :-

2D image : $I(x, y)$

mapping function : $t : \mathbb{R}^2 \to \mathbb{R}^2$

$T(u, v) = I(t(u, v))$ → coordinate position.

Translation : $t_\theta(x, y) = [x - \theta_1, \; y - \theta_2]^T$

Rotation : $t_\theta(x, y) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Zoom : $t_\theta(x, y) = [\theta_1 x \quad \theta_2 y]^T$

However, $\mathbb{E}_\theta [I \circ t_\theta]$ // Expectation

is not tractable.

current DA training : Monte-carlo Sampling

Expected loss :

$$\sum_{n=1}^{N} \mathbb{E}_{\theta}\left[\mathcal{L}\left(f_y(\underline{\underline{T}}_{\theta}(x_n))\right)\right] = \sum_{n=1}^{N} \mathcal{L}\left(f_y(\underline{\underline{T}}_{\theta_n}(x_n))\right)$$

θ_n i.i.d sample of θ

Data Space Transform :

$$I(u,v) = \int I(x,y)\,\delta(u-x,\ v-y)\,dx\,dy$$

↳ general formulation for previous one

Translation : $I(u,v) = \int I(x,y)\,\delta(u-x-\theta,\ v-y)\,dx\,dy$

Data-space Tx formulation :

$$T(u,v) = \int I(x,y)\ h(u,v,x,y)\,dx\,dy$$

↳ general to for any Augmentation.

DS → CS by setting :

$$h(u, v, x, y) = \delta(t(x, y) - [u, v])$$