Spectral contrastive loss

Background: Graph spectral decomposition.

Similarity graph: $G = (V, E)$

[forming graph from the data points]

based on distance s_{ij}

undirected graph $G = (V, E)$

vertex set, $V = \{v_1, v_2 \cdots v_n\}$

weight $w_{ij} \geq 0$ [between v_i & v_j]

Degree, $d_i = \sum_{j=1}^{n} w_{ij}$

Degree matrix $D = \text{diag}\{d_1, d_2, \cdots d_n\}$

Subset of vertices $A \subset V$

complement $V \setminus A = \bar{A}$

indicator $\mathbb{1}_A = [f_1, f_2, \cdots f_n]' \in \mathbb{R}^n$

$f_i = 1$ if $v_i \in A$

$f_i = 0$; otherwise

shorthand: $i \in A$ s.t. $\{ i \mid v_i \in A \}$

$$W(A, B) := \sum_{i \in A, j \in B} w_{ij}$$

$|A| :=$ number of vertices in A

$$\mathrm{vol}(A) = \sum_{i \in A} d_i$$ //Total edges in set

Different similarity graphs:

→ ϵ - neighborhood

→ k - nearest neighbors

→ fully connected graph

Unnormalized Graph Laplacian:

$$L = D - W$$

Properties: i) $f \in \mathbb{R}^n$

$$\sum^{n} w_{ij} (f_i - f_j)^2$$

$$f^T L f = \frac{1}{2} \sum_{i,j=1} w_{ij} (f_i - f_j)$$

ii) $L \to$ symmetric & PSD

iii) smallest eigen value $\Rightarrow 0$ & eigen vector $[1 \ldots 1]$

(iv) n non negative eig value $0 = \lambda_1 \leq \lambda_2 \leq \lambda_n$

[Number of zero eig values = No of connected components.

[if $A_1 \ldots A_k$ is disjoint vertices

$\to$ k eig value $= 0$

$\to$ with eigen value $[1_{A_k}]$

Back to paper :

Proposed loss function.

$$\mathcal{L}(f) = -2 \cdot \mathbb{E}_{x, x^+} \left[f(x)^T f(x^+) \right]$$

$$+ \mathbb{E} \left[\left(f(x)^T f(x^-) \right)^2 \right]$$

x, x^- [,]

f ; Data dim $\rightarrow \mathbb{R}^k$