

# CBD

Problem Setup:  $x \sim p \mid u$  ← data

$$S_{\text{enc}} = f^S(x)$$

$$T = f^T(x)$$

key pair

Two distributions:  $q(T, s | c=1) = p(T, s)$  joint distribution

$$q(T, s | c=0) = p(T) p(s)$$

$$q(c=1) = \frac{1}{N+1}$$

$$q(c=0) = \frac{N}{N+1}$$

Same input for both  $T$  &  $S$ .

congruent and  $N$  is congruent

different input for  $T$  &  $S$

$$q(c=1 | T, s) = \frac{q(T, s | c=1) q(c=1)}{q(T, s | c=1) q(c=1) + q(T, s | c=0) q(c=0)}$$

$$= \frac{p(T, s)}{p(T, s) + N p(T) p(s)}$$

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connection to MI:  $\log q(c=1 | T, s) = \log \frac{p(T, s)}{p(T, s) + N p(T) p(s)}$

$$= -\log \left( 1 + \frac{N p(T) p(s)}{p(T, s)} \right) \leq -\log N + \log \frac{p(T) p(s)}{p(T, s)}$$

(convexity of log)

Taking expectation  $p(T, s)$

$$I(T, s) \geq \log N + E_{q(T, s | c=1)} \log q(c=1 | T, s)$$

← Bound MI



$$E_{q(\tau, s | c=1)} \log q(c=1 | \tau, s) \Rightarrow ? \text{ how}$$

maximization

$$h\{\tau, s\} \rightarrow [0, 1]$$

Domain

P-cells

$$h^*(\tau, s) = q(c=1 | \tau, s) \text{ Bernoulli distribution}$$

$$L_{criterion}(h) = E_{q(\tau, s | c=1)} \log h(\tau, s)$$

$$+ E_{q(\tau, s | c=0)} \log (1 - h(\tau, s))$$

$$f^{st} = \arg \max_{f, s} \max_h L_{criterion}(h)$$

(Contrastive loss)

$h$  chosen from family of function  $h\{\tau, s\} \rightarrow [0, 1]$

$$h(\tau, s) = \frac{\exp(g^T(\tau) \cdot g^s(s))}{\exp(g^T(\tau) \cdot g^s(s)) + \frac{n}{m}}$$

$$\text{WCE loss}$$



KD objective

$$L_{KD} = (1 - \alpha) H(y, y_s) + \alpha p^2 H(\sigma(z/p), \sigma(z_s/p))$$

↓ weight
↑ temperature

↓ softmax

cross entropy

$$L_{KD-EN} = H(y, y_s) - \beta \sum_i L_{KL}(T_i, s)$$

↓  
 cross entropy