· Distribution relation : without proof :

Geometric / Negative Binomial:

$f(x) = p(1-p)^{x}$ / $f(x) = C(r+x-1, x)\, p^{r} (1-p)^{x}$ // no. of failure before r success. ;

clearly when, when $r = 1$ // means 1st success. then

Negative binomial ⇒ Geometric

Again, sum of n geometric x_i ⇒ Neg binomial with (n, p)

Negative Binomial / Poisson

$\exp(-\lambda) \dfrac{\lambda^{x}}{x!} \leftarrow \binom{r+x-1}{x} p^{r} (1-p)^{x}$

r should be large, $p \to 1$. and $\{r(1-p) = \lambda\}$ then CDF $F_X \approx F_Y$; Y poisson random with $\lambda = r(1-p)$

Beta Binomial / Discrete uniform

$\dfrac{\binom{n}{x} B(\alpha + x, n + \beta - x)}{B(\alpha, \beta)}$ / $f(x) = \frac{1}{n}$ // for $x = 1 \ldots n$

$\alpha = 1$, $\beta = 1$; $n = n$ then, Beta binomial ⇒ Discrete uniform

Hypergeometric / geometric

$\dfrac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}$ / $p(1-p)^{x}$

Has to be with ~~geometric~~ sample size, & bootstrapping. When population size increases relative to sample size, Binomial / Poisson:

$\binom{n}{x} p^x (1-p)^{n-x}$ $\exp(-\lambda) \frac{\lambda^x}{x!}$

For binomial $P(X = n) \approx P(Y = n)$; when n is large & np is small.

(Y: poisson (np))

Binomial / Bernoulli

$\binom{n}{x} p^x (1-p)^{n-x}$ p

$\cdot$ $n = 1$ & p's bernoulli prob $\Rightarrow$ Bernoulli

Bernoulli / Binomial

sum of n Bernoulli(p) r.v. is binomial(n, p)

Poisson / Normal

$\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{\sigma^2}\right)$

$P(j \leq X \leq k) \approx P(j - \frac{1}{2} \leq Y \leq k + \frac{1}{2})$

X poisson with large mean || Y is normal same m, σ^2 as X.

Binomial / normal

$P(j \leq X \leq k) \approx P(j - \frac{1}{2} \leq Y \leq k + \frac{1}{2})$

Y normal with mean np & var $np(1-p)$

Better result with $p = \frac{1}{2}$.

(iv)

Beta / uniform

$$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\,du} \quad ; \quad \alpha = 1 \ \& \ \beta = 1 \ ; \ \text{easy}$$

standard Normal / chi-square

By definition sum of k standard Normal R.V. $\sim \chi_k^2$

$k \downarrow$ Degree of fre

Gamma / chi-square

$$\frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \qquad n \ \cancel{\text{Norm}}\ N(\mu, \sigma)$$

if $\alpha = \frac{v}{2}$ & $\beta = 1$; Gamma $\sim$ v degree of χ_v^2

cauchy / Normal - standard $(\mu, \sigma^2) = (0, 1)$

$$\frac{\sigma}{\pi\left((x-\mu)^2 + \sigma^2\right)}$$

$(x-\mu)^2$ → location, σ^2 → scale

$$\rightarrow \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right)$$

$$\frac{x}{y} \sim \text{caucy}(0, 1)$$

student - t / standard Normal

$$f(t) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{t^2}{v}\right)^{-\frac{v+1}{2}}$$

$f_X \approx f_Y$ → standard Normal

Large No. of Degree of freedom v

$$\frac{\bar{x} - \mu}{S/\sqrt{n}} \rightarrow$$ t - distribution with $N-1$ degree of freedom

<u>Normal</u> / <u>log Normal</u>

↙ easily

$\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$ $\Rightarrow$ $f_x(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(\log(x)-\mu)^2}{2\sigma^2}\right)/x$

the $\log(x)$ is normal (μ,σ^2) r.v.

<u>Beta</u> / <u>Normal</u>

$f_X = f_Y$ where, y is normal with $\mu = \frac{\alpha}{\alpha+\beta}$

$\Downarrow$

recetadiet.

$\sigma^2 = \frac{\alpha\beta}{(\beta+\alpha)^2(\alpha+\beta+1)}$

$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\,du}$;

<u>Gamma</u> / <u>Beta</u>

$\Gamma(\alpha,\beta)$ $\rightarrow$ $\frac{x^{\alpha-1}(1-x)^{\beta-1}}{\text{N. constant}}$

$\Downarrow$ $\times$ $1/\theta \rightarrow$ scale

k

shape

$X_1 \sim \text{gamma}(\alpha_1,\beta_1)$

$X_2 \sim \text{gamma}(\alpha_2,\beta_2)$

$f(x;\alpha,\beta) = \frac{\beta^\alpha x^{\alpha-1}e^{-\beta x}}{\Gamma(\alpha)}$

$\Downarrow$

$(\alpha-1)!$ / α positint.

the $\frac{X_1\beta_2}{\beta_2 X_1 + X_2\beta_1} \sim \text{Beta}(\alpha_1,\alpha_2$

Chi - Square / Exponential

$$\frac{1}{\mu} \exp\left(-\frac{x}{\mu}\right)$$

$\chi^2_2 \rightarrow \frac{1}{2} \exp\left(-\frac{x}{2}\right)$; $x>0$; $\mu = 2$ & vice versa.

Gamma / Exponential ; $\frac{1}{\mu} \exp\left(-\frac{x}{\mu}\right)$; $x \geq 0$

$$\frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}$$

:) If $\alpha = 1$ then Gamma(α, β) = exp(β)

sum of n exp(β) R.V. $\rightarrow \gamma(n, \beta)$

student - t / Cauchy

$$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}} \Big/ \frac{\sigma}{\pi\left((x-\mu)^2+\sigma^2\right)}$$

// $\left[\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\right]$

if $\nu = 1$ // 1 degree of freedom of student-t $\rightarrow$ Caucy(0,1)