(Deep Infomax) : Contrastive Setup!

$$\boxed{ \begin{array}{l} MAP \rightarrow \text{feature vector} \\ M \times M \end{array} }$$

input x

output y

family of <u>encoder</u> : $\mathcal{E}_\phi = \{ E_{\phi,\psi} \}_{\psi \in \Psi}$

$U_{\psi, P} \rightarrow$ distribution over encoding.

for input x.

[Target] :
$$y \sim E_\psi(x)$$

<u>for some input</u>

$\begin{cases} \text{① maximize: } I(X, E_\psi(x)) \quad (MI) \\ \\ \text{⑪ constraint: } U_{\psi, P} = P \quad \text{match } \underline{V} \rightarrow \text{Another prior distribution}) \end{cases}$

(infomax)

[Figure ③ mother of all..
(global vs local)
($\underline{\text{All}}$) ✓
take one line

So, positive & Negative Samples:

MI estimation & maximization

$$I_w(x;y) := E_J\left[T_w(x,y)\right] - I_g\, \underline{E_M}\left[e^{T_w(x,y)}\right] \quad //\text{DV}$$

$$\underset{\text{joint}}{\downarrow} \qquad \underset{\text{marginal}}{\downarrow}$$

$$\downarrow T_{w+} \text{Discriminator}$$

so maximization

$$(\hat{w},\hat{\psi})_G := \arg\max_{w,\psi}\ \hat{I}_w\left(x,\, E_\psi(x)\right)$$

$\rightarrow$ low label features.

$$E_\psi = f_\psi \circ C_\psi$$

$$T_{w,\psi} = D_w \circ g \circ (C_\psi, E_\psi)$$

Instead using JSD.

InfoNCE

$$I_{w,\psi}^{\leftrightarrow}(x, E_\psi(x)) := E_P\left[T_{\psi,w}(x, E_\psi(x))\right] - E_{\tilde{P}}\left[\log \sum_K e^{T_{\psi,w}(x, E(x))}\right]$$

This paper

needs to be high   should be low

JSD

$$I_{w,\psi}(x, E_\psi(x)) := E_P\left[-sp\left(-T_{\psi,w}(x, E_\psi(x))\right)\right] - E_{P\times\tilde{P}}\left[sp\left(T_{\psi,w}(x, E(y))\right)\right]$$

$$sp(z) = \log(1 + e^z)$$

↳ only maximize not really the precise value

(requires smaller negative sample)

better than NCE

Local MI maximization

feature map

$$C_\psi(x) := \left\{ C_\psi^{(i)} \right\}_{i=1}^{M \times M}$$

$$E_\psi(x) = f_\psi \circ C_\psi(x)$$

$$(\hat{\omega}, \hat{\psi})_L = \underset{\psi, \omega}{\arg\max} \; \frac{1}{M^2} \sum_{i=1}^{M} \hat{I}_{\omega, \psi} \left(C_\psi^{(i)}(x) ; f_\psi(x) \right)$$

matching representation with prior

$$(\hat{\omega}, \hat{\phi})_P = \underset{\psi}{\arg\min} \; \underset{\phi}{\arg\max} \; \hat{D}_\phi \left(V \| U_{\psi, P} \right)$$

$$= E_V \left[\log D_\phi(y) \right] + E_P \left[\log \left(1 - D_\phi \left(E_\phi(x) \right) \right) \right]$$

$$\Updownarrow$$

Adversarial Auto encoder.

final loss = weighted sum of the previous three