

# Final Project Report for Advanced Quantitative Methods of IS Research-Statistical Learning

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## Introduction

In this project, I apply statistical learning methods from ISLR book on the publicly available car price dataset kaggle website in the following link. My all works related to this project are also available at my github link.

## Chapter 2

I start the experiment by adding the dataset in the working environment. The summary and name would provide the variables related to dataset. By attaching them it is convenient to call them using their name directly.

```
card <- read.csv("CarPrice_Assignment.csv")
attach(card) # attaching the variable names
coln <- colnames(card)
print(coln)
```

```
## [1] "car_ID"      "symboling"   "CarName"     "fueltype"
## [5] "aspiration"  "doornumber"  "carbody"     "drivewheel"
## [9] "enginelocation" "wheelbase"   "carlength"   "carwidth"
## [13] "carheight"   "curbweight"  "enginetype"   "cylindernumber"
## [17] "enginesize"   "fuelsystem"  "bore ratio"   "stroke"
## [21] "compressionratio" "horsepower"  "peakrpm"     "citympg"
## [25] "highwaympg"  "price"
```

```
summary(card)
```

```
##      car_ID      symboling      CarName      fueltype      aspiration
## Min.   : 1      Min.   :-2.0000      peugeot 504 : 6      diesel: 20      std :168
## 1st Qu.: 52      1st Qu.: 0.0000      toyota corolla: 6      gas :185      turbo: 37
## Median :103      Median : 1.0000      toyota corona : 6
## Mean   :103      Mean   : 0.8341      subaru dl : 4
## 3rd Qu.:154      3rd Qu.: 2.0000      honda civic : 3
## Max.   :205      Max.   : 3.0000      mazda 626 : 3
##                                     (Other) :177
## doornumber      carbody      drivewheel      enginelocation      wheelbase
## four:115      convertible: 6      4wd: 9      front:202      Min. : 86.60
```

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```
## two : 90    hardtop    : 8    fwd:120    rear : 3    1st Qu.: 94.50
##           hatchback  :70    rwd: 76    Median : 97.00
##           sedan      :96    Mean    : 98.76
##           wagon      :25    3rd Qu.:102.40
##                                     Max.    :120.90
##
##      carlength      carwidth      carheight      curbweight      enginetype
## Min.    :141.1    Min.    :60.30    Min.    :47.80    Min.    :1488    dohc : 12
## 1st Qu.:166.3    1st Qu.:64.10    1st Qu.:52.00    1st Qu.:2145    dohcv: 1
## Median :173.2    Median :65.50    Median :54.10    Median :2414    l     : 12
## Mean    :174.0    Mean    :65.91    Mean    :53.72    Mean    :2556    ohc  :148
## 3rd Qu.:183.1    3rd Qu.:66.90    3rd Qu.:55.50    3rd Qu.:2935    ohcf : 15
## Max.    :208.1    Max.    :72.30    Max.    :59.80    Max.    :4066    ohcv : 13
##                                     rotor: 4
##      cylindernumber      enginesize      fuelsystem      boreratio      stroke
## eight : 5    Min.    : 61.0    mpfi    :94    Min.    :2.54    Min.    :2.070
## five  : 11    1st Qu.: 97.0    2bbl    :66    1st Qu.:3.15    1st Qu.:3.110
## four  :159    Median :120.0    idi     :20    Median :3.31    Median :3.290
## six   : 24    Mean    :126.9    1bbl    :11    Mean    :3.33    Mean    :3.255
## three : 1    3rd Qu.:141.0    spdi    : 9    3rd Qu.:3.58    3rd Qu.:3.410
## twelve: 1    Max.    :326.0    4bbl    : 3    Max.    :3.94    Max.    :4.170
## two   : 4    (Other): 2
##      compressionratio      horsepower      peakrpm      citympg
## Min.    : 7.00    Min.    : 48.0    Min.    :4150    Min.    :13.00
## 1st Qu.: 8.60    1st Qu.: 70.0    1st Qu.:4800    1st Qu.:19.00
## Median : 9.00    Median : 95.0    Median :5200    Median :24.00
## Mean    :10.14    Mean    :104.1    Mean    :5125    Mean    :25.22
## 3rd Qu.: 9.40    3rd Qu.:116.0    3rd Qu.:5500    3rd Qu.:30.00
## Max.    :23.00    Max.    :288.0    Max.    :6600    Max.    :49.00
##
##      highwaympg      price
## Min.    :16.00    Min.    : 5118
## 1st Qu.:25.00    1st Qu.: 7788
## Median :30.00    Median :10295
## Mean    :30.75    Mean    :13277
## 3rd Qu.:34.00    3rd Qu.:16503
## Max.    :54.00    Max.    :45400
##
```

## Basic codes from the chapter 2

The chapter two introduces as some of the important introductory concept in R. In this chapter I have run and understand the basics from the book and showed it here. I will be using the codes to my dataset to implement the introductory codes and check the lengths and summary.

In this section, I will be also plotting the statistical parameters like mean, vaiances of the output variable and some input predictor variables.

```
library(ISLR) # making all dataset available
```

```
## Warning: package 'ISLR' was built under R version 4.0.0
```

```
x <- c(1,6,2)
y <- c(1,4,3)
x-y      #x+y, x*y
```

```
## [1] 0 2 -1
```

```
length(card) # length(y)
```

```
## [1] 26
```

```
ls() # check existing variables
```

```
## [1] "card" "coln" "x"    "y"
```

```
rm(x,y) # remove variables
x <- matrix(data=c(1,2,3,4), nrow=2, ncol=2)
matrix(c(1,2,3,4),2,2, byrow=TRUE)
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]    3    4
```

```
x^2
```

```
##      [,1] [,2]
## [1,]    1    9
## [2,]    4   16
```

```
sqrt(x)
```

```
##      [,1]      [,2]
## [1,] 1.000000 1.732051
## [2,] 1.414214 2.000000
```

```
x <- rnorm(50)
y <- x + rnorm(50, mean=50, sd=1)
cor(x,y)
```

```
## [1] 0.6638184
```

```
set.seed(1303)
rnorm(50)
```

```
## [1] -1.1439763145  1.3421293656  2.1853904757  0.5363925179  0.0631929665
## [6]  0.5022344825 -0.0004167247  0.5658198405 -0.5725226890 -1.1102250073
## [11] -0.0486871234 -0.6956562176  0.8289174803  0.2066528551 -0.2356745091
## [16] -0.5563104914 -0.3647543571  0.8623550343 -0.6307715354  0.3136021252
## [21] -0.9314953177  0.8238676185  0.5233707021  0.7069214120  0.4202043256
```

```
## [26] -0.2690521547 -1.5103172999 -0.6902124766 -0.1434719524 -1.0135274099
## [31]  1.5732737361  0.0127465055  0.8726470499  0.4220661905 -0.0188157917
## [36]  2.6157489689 -0.6931401748 -0.2663217810 -0.7206364412  1.3677342065
## [41]  0.2640073322  0.6321868074 -1.3306509858  0.0268888182  1.0406363208
## [46]  1.3120237985 -0.0300020767 -0.2500257125  0.0234144857  1.6598706557
```

```
set.seed(3)
y <- rnorm(100)
mean(y)
```

```
## [1] 0.01103557
```

```
var(y)
```

```
## [1] 0.7328675
```

```
sqrt(var(y))
```

```
## [1] 0.8560768
```

```
sd(y)
```

```
## [1] 0.8560768
```

Now implementing the commands to get the dataset description on output variables.

```
mean(price)
```

```
## [1] 13276.71
```

```
var(price)
```

```
## [1] 63821762
```

```
sqrt(var(price))
```

```
## [1] 7988.852
```

```
sd(price)
```

```
## [1] 7988.852
```

```
mode(price)
```

```
## [1] "numeric"
```

We can also implementing the commands to get the statistical parameters for the input variables too.

```
mean(engineSize)
```

```
## [1] 126.9073
```

```
var(engineSize)
```

```
## [1] 1734.114
```

```
sqrt(var(engineSize))
```

```
## [1] 41.64269
```

```
sd(engineSize)
```

```
## [1] 41.64269
```

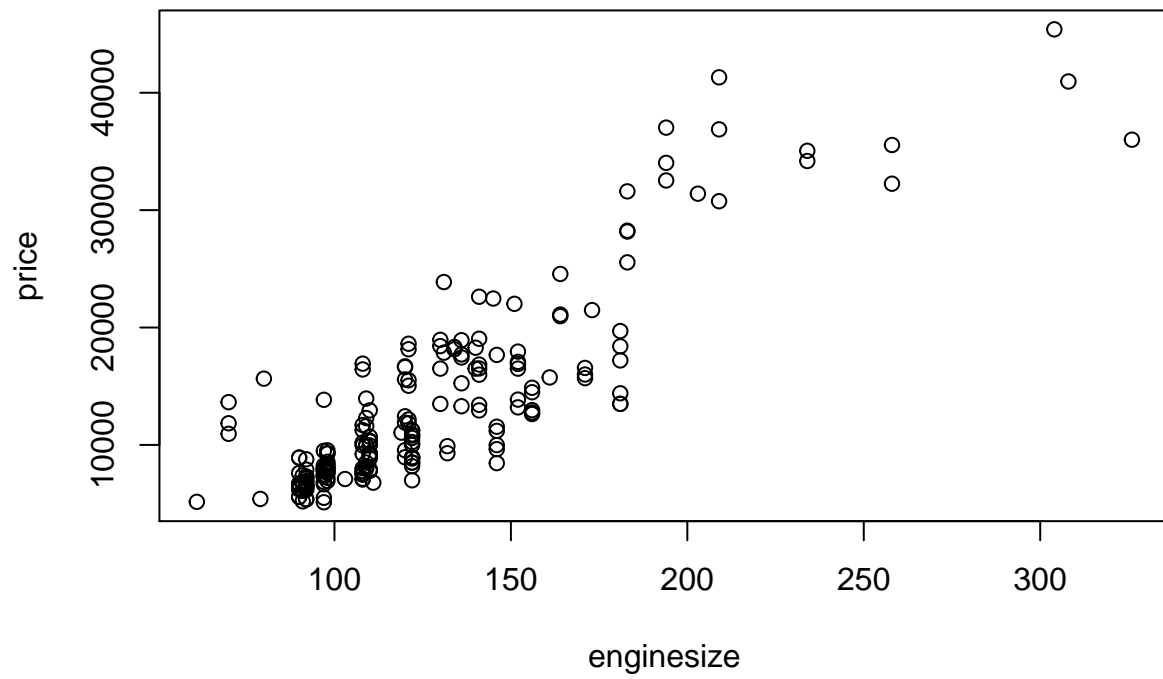
```
mode(engineSize)
```

```
## [1] "numeric"
```

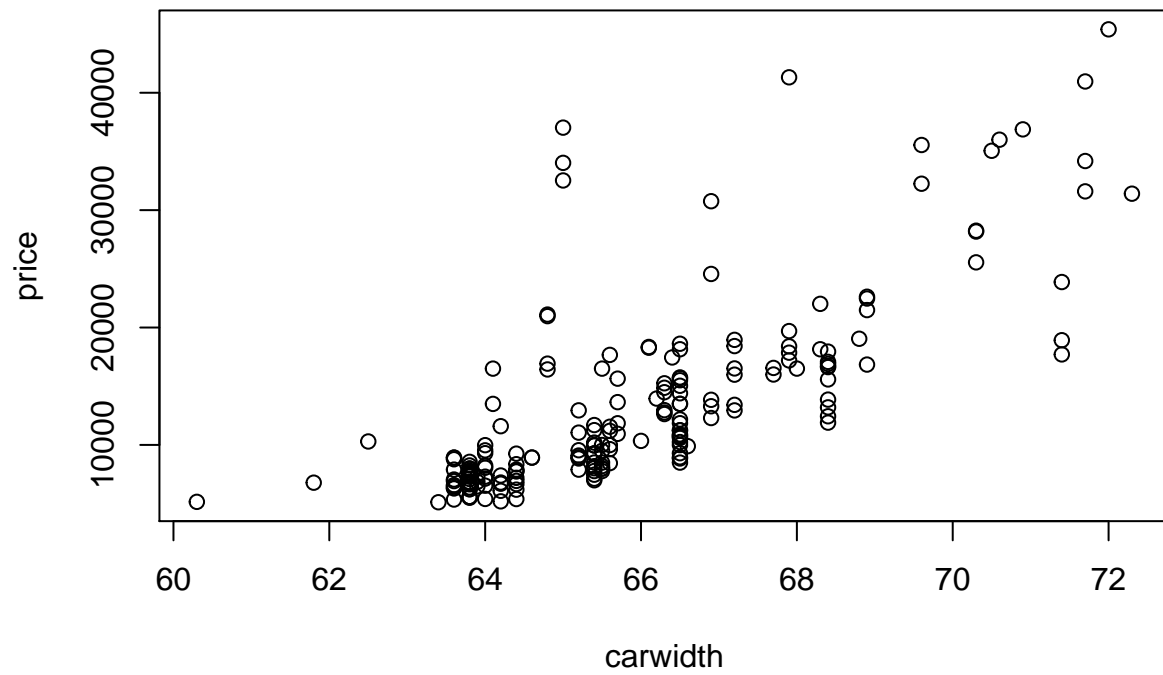
## Plot function from chapter 2

In this section using the plot option shown in chapter 2, I plot some the variables together to find the relationship between them and the output predictors. I used both categorical and continuous predictor to demonstrate the relationship between them.

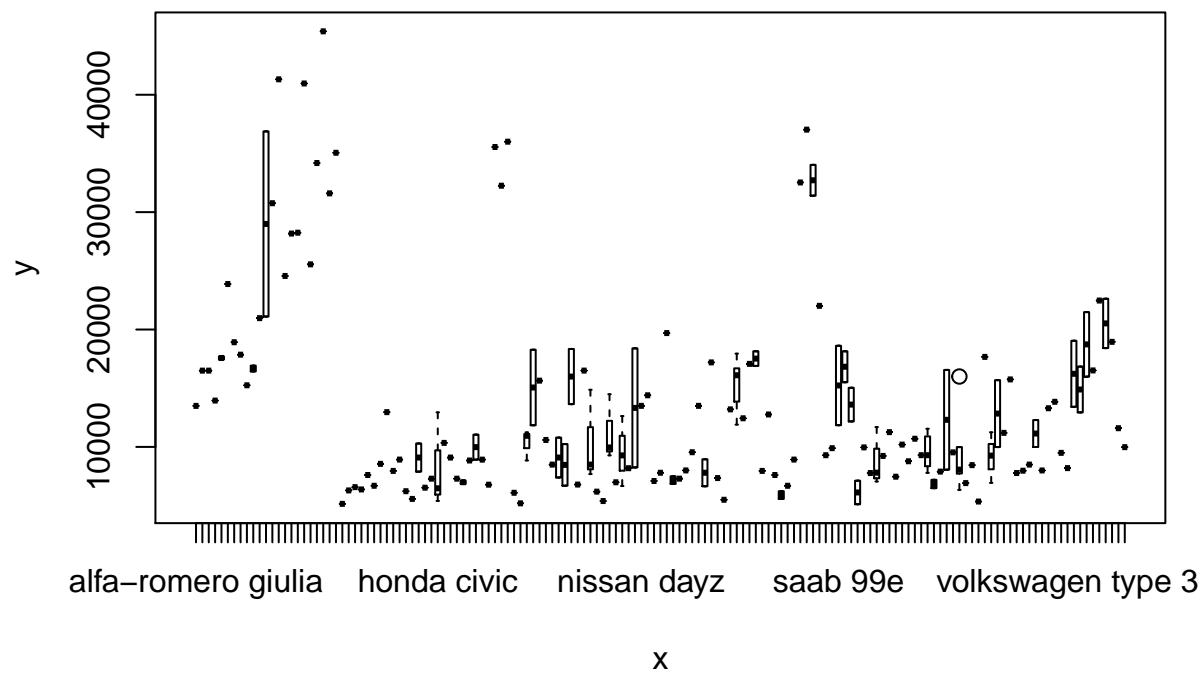
```
plot(engineSize, price)
```



```
plot(carwidth, price)
```

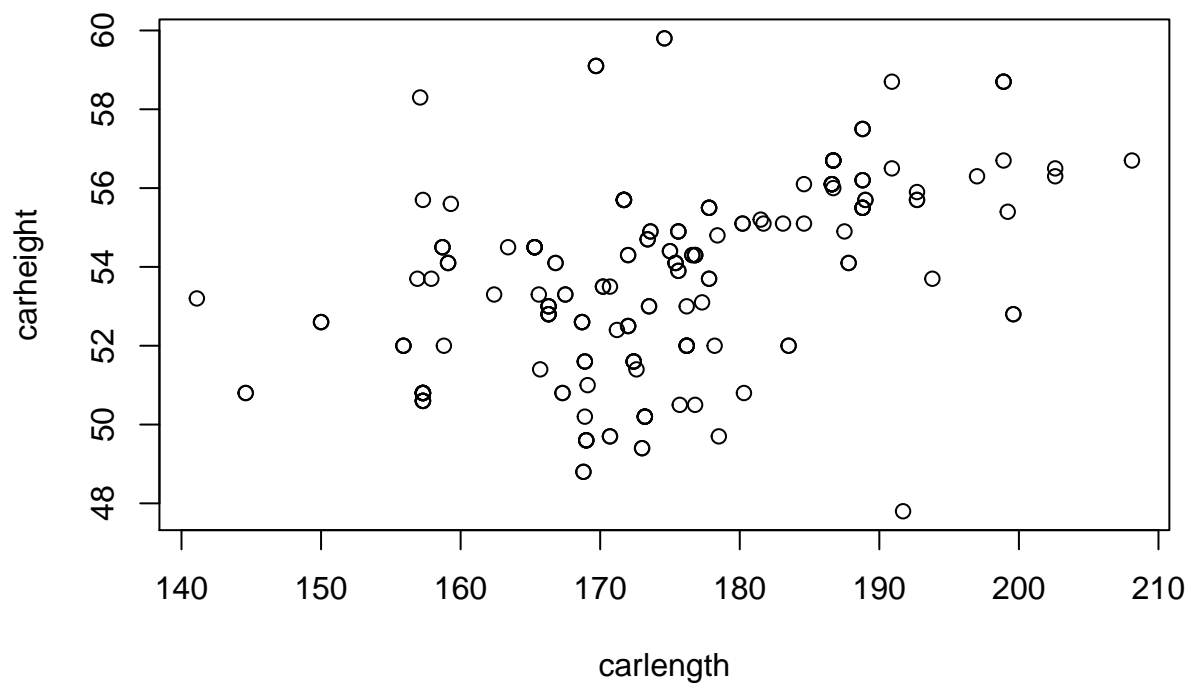


```
plot(CarName, price)
```

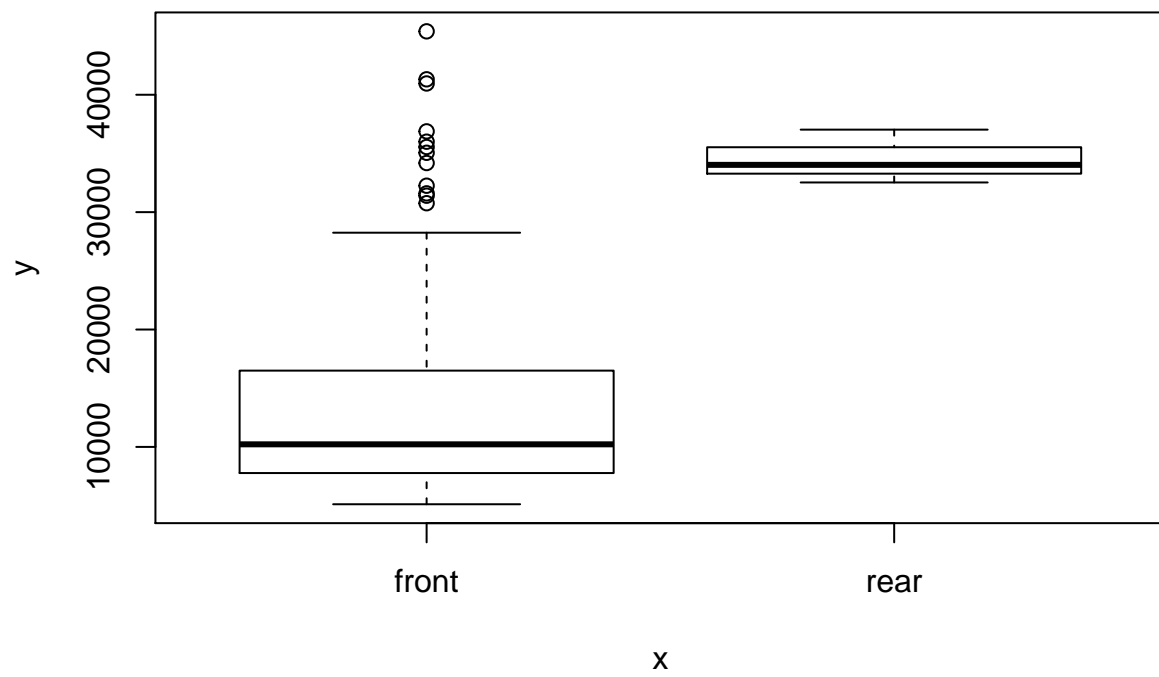


```
plot(carlength, carheight)
```



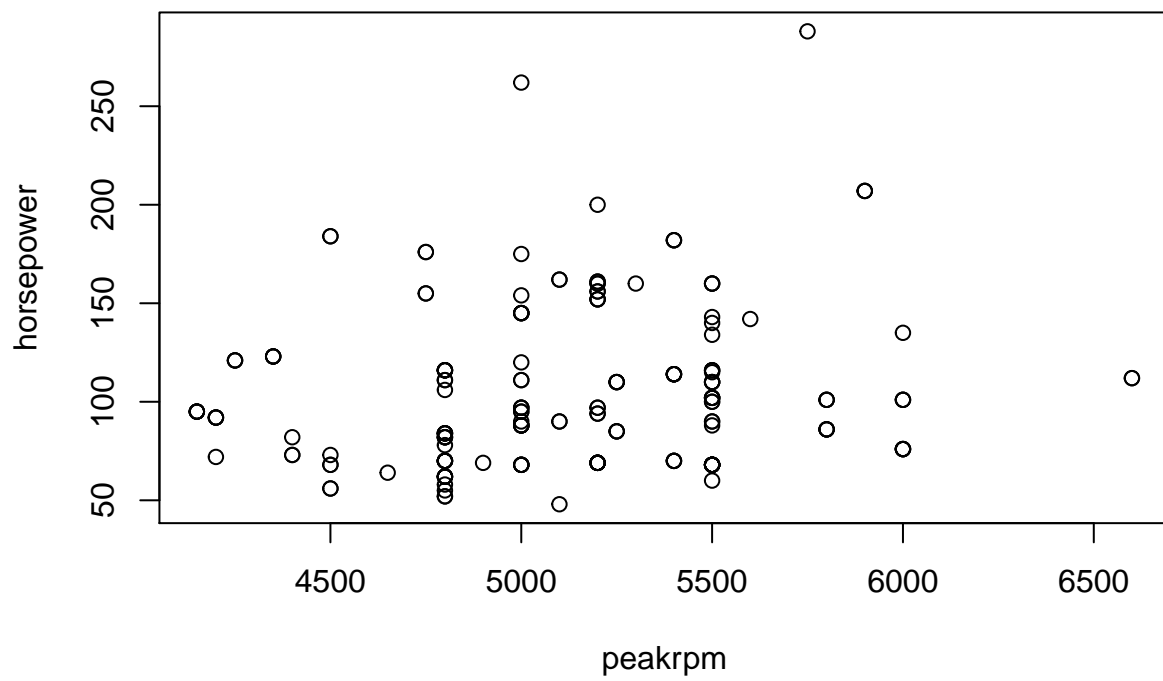


```
plot(engine.location, price)
```

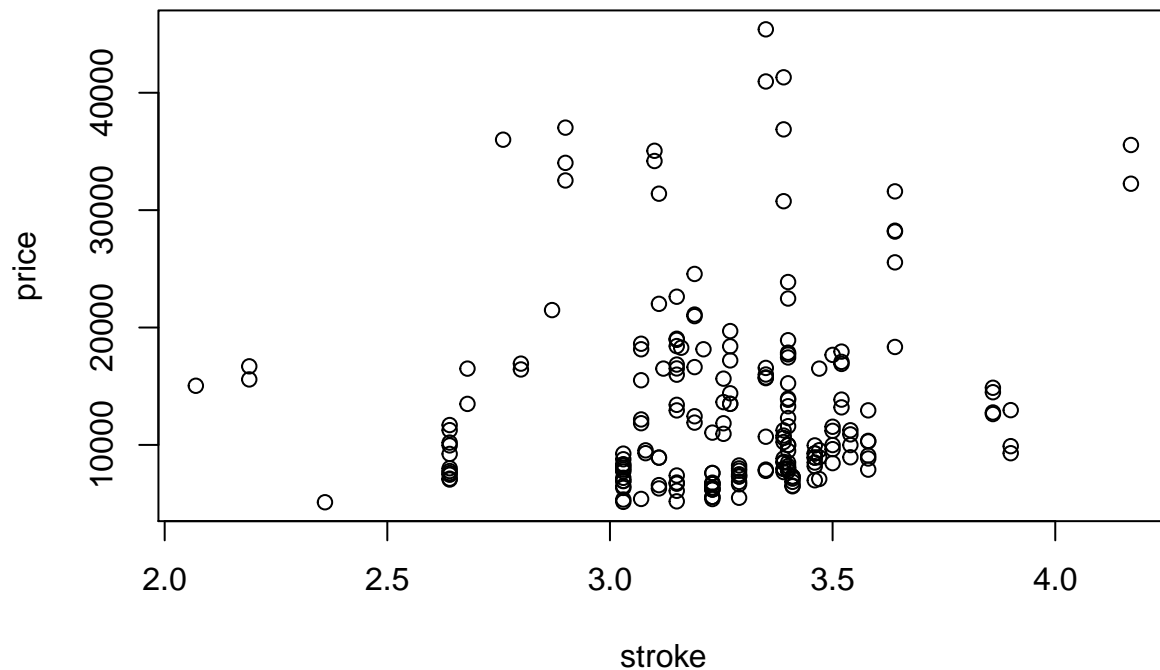


```
plot(engine type, price)
```





```
plot(stroke, price)
```



In those we can observe that some variables like enginesize, carwidth are closely related to predictor variables (Price). Some relations are not obvious from the plots like stroke and price.

## Chapter 3

In this section, I will be implementing codes of linear regression in my dataset. Firstly, I will apply linear regression using all the predictor features. Then I will narrow the features for better understanding and explain the code and methods.

```
attach(card)
```

```
## The following objects are masked from card (pos = 4):
##
## aspiration, boreratio, car_ID, carbody, carheight, carlength,
## CarName, carwidth, citympg, compressionratio, curbweight,
## cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
## enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
## price, stroke, symboling, wheelbase
```

```
attach(card)
```

```
## The following objects are masked from card (pos = 3):
##
## aspiration, boreratio, car_ID, carbody, carheight, carlength,
```

```
##      CarName, carwidth, citympg, compressionratio, curbweight,
##      cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##      enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
##      price, stroke, symboling, wheelbase
```

```
## The following objects are masked from card (pos = 5):
```

```
##
##      aspiration, boreratio, car_ID, carbody, carheight, carlength,
##      CarName, carwidth, citympg, compressionratio, curbweight,
##      cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##      enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
##      price, stroke, symboling, wheelbase
```

```
lm.fit = lm(price~., data = card)
```

```
#lm.fit = lm(price~fuelsystem+peakrpm+citympg+CarName+enginesize+enginetype+carwidth+curbweight+carlength+stroke+symboling+wheelbase, data = card)
summary(lm.fit)
```

```
##
```

```
## Call:
```

```
## lm(formula = price ~ ., data = card)
```

```
##
```

```
## Residuals:
```

```
##      Min      1Q  Median      3Q      Max
## -1576      0         0         0    1421
```

```
##
```

```
## Coefficients: (12 not defined because of singularities)
```

	Estimate	Std. Error	t value	Pr(> t )
## (Intercept)	1464.060	24067.028	0.061	0.951958
## car_ID	153.076	64.684	2.367	0.025687
## symboling	725.668	323.690	2.242	0.033723
## CarNamealfa-romero Quadrifoglio	5339.290	4141.961	1.289	0.208724
## CarNamealfa-romero stelvio	2851.924	1556.345	1.832	0.078362
## CarNameaudi 100 ls	3980.044	3341.799	1.191	0.244418
## CarNameaudi 100ls	5440.965	3939.323	1.381	0.178976
## CarNameaudi 4000	9453.414	4893.730	1.932	0.064359
## CarNameaudi 5000	7315.676	4148.741	1.763	0.089593
## CarNameaudi 5000s (diesel)	5048.734	4806.719	1.050	0.303223
## CarNameaudi fox	8004.584	3850.889	2.079	0.047659
## CarNamebmw 320i	3665.224	2932.400	1.250	0.222468
## CarNamebmw x1	16398.848	4064.364	4.035	0.000427
## CarNamebmw x3	15669.006	3870.528	4.048	0.000412
## CarNamebmw x4	16597.885	4245.075	3.910	0.000591
## CarNamebmw x5	26105.388	4116.060	6.342	1.02e-06
## CarNamebmw z4	15092.926	4104.480	3.677	0.001079
## CarNamebuick century	3394.510	6114.139	0.555	0.583510
## CarNamebuick century luxus (sw)	2830.130	6144.487	0.461	0.648917
## CarNamebuick century special	315.888	5317.826	0.059	0.953086
## CarNamebuick electra 225 custom	-238.538	5517.392	-0.043	0.965845
## CarNamebuick opel isuzu deluxe	-2746.797	4648.634	-0.591	0.559701
## CarNamebuick regal sport coupe (turbo)	8247.321	5282.930	1.561	0.130585
## CarNamebuick skyhawk	2498.152	5625.634	0.444	0.660669
## CarNamebuick skylark	-2256.734	5472.035	-0.412	0.683418
## CarNamechevrolet impala	745.695	4422.267	0.169	0.867398

## CarNamechevrolet monte carlo	-5791.923	3932.695	-1.473	0.152819
## CarNamechevrolet vega 2300	-6942.823	3806.475	-1.824	0.079675
## CarNamedodge challenger se	-5644.893	3638.855	-1.551	0.132922
## CarNamedodge colt (sw)	-8616.639	3757.617	-2.293	0.030177
## CarNamedodge colt hardtop	-9380.564	3719.607	-2.522	0.018133
## CarNamedodge coronet custom	-8832.211	3661.983	-2.412	0.023229
## CarNamedodge coronet custom (sw)	-5385.044	4548.548	-1.184	0.247162
## CarNamedodge d200	-7061.034	3456.794	-2.043	0.051348
## CarNamedodge dart custom	-3481.303	3974.749	-0.876	0.389129
## CarNamedodge monaco (sw)	-8467.722	3715.083	-2.279	0.031099
## CarNamedodge rampage	-7760.845	3894.157	-1.993	0.056861
## CarNamehonda accord	-3275.628	5032.000	-0.651	0.520785
## CarNamehonda accord cvcc	-6814.723	4655.065	-1.464	0.155198
## CarNamehonda accord lx	-5594.750	4749.667	-1.178	0.249496
## CarNamehonda civic	-4621.260	4100.458	-1.127	0.270032
## CarNamehonda civic (auto)	-8117.608	4535.662	-1.790	0.085153
## CarNamehonda civic 1300	-3924.742	4903.031	-0.800	0.430691
## CarNamehonda civic 1500 gl	-3391.219	5103.479	-0.664	0.512227
## CarNamehonda civic cvcc	-5930.849	4596.581	-1.290	0.208313
## CarNamehonda prelude	-4625.080	4938.351	-0.937	0.357602
## CarNameisuzu D-Max	-7981.442	4919.273	-1.622	0.116765
## CarNameisuzu D-Max V-Cross	-9077.711	4862.983	-1.867	0.073261
## CarNameisuzu MU-X	-7911.400	3885.498	-2.036	0.052043
## CarNamejaguar xf	160.068	3672.109	0.044	0.965564
## CarNamejaguar xj	-2986.856	3661.432	-0.816	0.422047
## CarNamejaguar xk	4389.084	6753.443	0.650	0.521456
## CarNamemaxda glc deluxe	-7881.451	4806.507	-1.640	0.113103
## CarNamemaxda rx3	-9080.132	4847.291	-1.873	0.072319
## CarNamemazda 626	-8611.028	4624.289	-1.862	0.073924
## CarNamemazda glc	-6522.541	4440.118	-1.469	0.153829
## CarNamemazda glc 4	-6814.221	5650.857	-1.206	0.238726
## CarNamemazda glc custom	-8221.173	4826.224	-1.703	0.100412
## CarNamemazda glc custom l	-11155.639	4696.154	-2.375	0.025183
## CarNamemazda glc deluxe	-8189.991	4851.734	-1.688	0.103362
## CarNamemazda rx-4	-9168.168	4685.529	-1.957	0.061203
## CarNamemazda rx-7 gs	-5694.495	4599.115	-1.238	0.226716
## CarNamemazda rx2 coupe	-7388.763	4860.722	-1.520	0.140556
## CarNamemercury cougar	-8178.650	6009.231	-1.361	0.185190
## CarNamemitsubishi g4	-15717.851	6169.643	-2.548	0.017103
## CarNamemitsubishi lancer	-16251.139	6357.561	-2.556	0.016772
## CarNamemitsubishi mirage	-18099.163	6498.938	-2.785	0.009854
## CarNamemitsubishi mirage g4	-16070.951	6325.950	-2.540	0.017384
## CarNamemitsubishi montero	-18648.094	6152.657	-3.031	0.005459
## CarNamemitsubishi outlander	-17041.307	6287.017	-2.711	0.011737
## CarNamemitsubishi pajero	-18035.063	6236.337	-2.892	0.007638
## CarNamenissan clipper	-14461.607	7557.316	-1.914	0.066743
## CarNamenissan dayz	-15537.918	7605.331	-2.043	0.051309
## CarNamenissan fuga	-13801.149	8073.239	-1.709	0.099271
## CarNamenissan gt-r	-15601.158	6975.502	-2.237	0.034110
## CarNamenissan juke	-13042.895	7277.852	-1.792	0.084757
## CarNamenissan kicks	-14465.254	8196.035	-1.765	0.089325
## CarNamenissan latio	-14503.856	7078.671	-2.049	0.050685
## CarNamenissan leaf	-13051.687	7260.070	-1.798	0.083843
## CarNamenissan note	-12338.572	7463.771	-1.653	0.110331

## CarNamenissan nv200	-16025.649	7182.645	-2.231	0.034509
## CarNamenissan otti	-15060.861	7682.618	-1.960	0.060749
## CarNamenissan rogue	-14305.794	7032.968	-2.034	0.052261
## CarNamenissan teana	-15631.312	7879.622	-1.984	0.057934
## CarNamenissan titan	-12235.253	7220.538	-1.695	0.102116
## CarNameNissan versa	-13413.773	6942.545	-1.932	0.064311
## CarNamepeugeot 304	-26389.554	6262.505	-4.214	0.000267
## CarNamepeugeot 504	-23978.749	6402.633	-3.745	0.000906
## CarNamepeugeot 504 (sw)	-23709.629	6380.998	-3.716	0.000977
## CarNamepeugeot 505s turbo diesel	-22733.899	6714.965	-3.386	0.002266
## CarNamepeugeot 604sl	-24539.992	6577.093	-3.731	0.000939
## CarNameplymouth cricket	-21289.464	8681.516	-2.452	0.021222
## CarNameplymouth duster	-23294.031	8326.902	-2.797	0.009567
## CarNameplymouth fury gran sedan	-23615.334	8797.386	-2.684	0.012478
## CarNameplymouth fury iii	-23123.441	8818.842	-2.622	0.014418
## CarNameplymouth satellite custom (sw)	-21743.841	8711.085	-2.496	0.019226
## CarNameplymouth valiant	-17988.655	8785.531	-2.048	0.050834
## CarNameporcshce panamera	-4303.206	10526.456	-0.409	0.686035
## CarNameporsche boxter	393.465	11113.478	0.035	0.972028
## CarNameporsche cayenne	-2956.282	10465.289	-0.282	0.779809
## CarNameporsche macan	-12031.956	8866.189	-1.357	0.186427
## CarNamerenault 12tl	-17786.564	8646.623	-2.057	0.049843
## CarNamerenault 5 gtl	-20579.681	8556.385	-2.405	0.023577
## CarNamesaab 99e	-16370.556	9256.390	-1.769	0.088699
## CarNamesaab 99gle	-15269.026	9336.815	-1.635	0.114025
## CarNamesaab 99le	-18228.042	9231.742	-1.974	0.059036
## CarNamesubaru	-27076.490	9392.453	-2.883	0.007808
## CarNamesubaru baja	-25469.712	9011.549	-2.826	0.008933
## CarNamesubaru brz	-24260.657	9253.575	-2.622	0.014427
## CarNamesubaru dl	-24848.991	9321.737	-2.666	0.013031
## CarNamesubaru r1	-26140.178	9207.082	-2.839	0.008665
## CarNamesubaru r2	-25641.614	8827.539	-2.905	0.007408
## CarNamesubaru trezia	-25979.569	9701.227	-2.678	0.012665
## CarNamesubaru tribeca	-24222.721	9501.112	-2.549	0.017031
## CarNametoyota carina	-28409.459	11515.664	-2.467	0.020530
## CarNametoyota celica gt	-25805.686	11595.954	-2.225	0.034939
## CarNametoyota celica gt liftback	-31145.270	10832.939	-2.875	0.007953
## CarNametoyota corolla	-24934.252	10863.751	-2.295	0.030042
## CarNametoyota corolla 1200	-25121.115	10594.321	-2.371	0.025424
## CarNametoyota corolla 1600 (sw)	-20241.570	10685.135	-1.894	0.069351
## CarNametoyota corolla liftback	-25071.958	10936.948	-2.292	0.030223
## CarNametoyota corolla tercel	-30453.878	10869.368	-2.802	0.009468
## CarNametoyota corona	-25299.866	10843.154	-2.333	0.027640
## CarNametoyota corona hardtop	-21796.585	11027.174	-1.977	0.058781
## CarNametoyota corona liftback	-26119.358	10798.687	-2.419	0.022875
## CarNametoyota corona mark ii	-25795.352	10651.811	-2.422	0.022725
## CarNametoyota cressida	-20941.386	11637.854	-1.799	0.083570
## CarNametoyota mark ii	-24785.364	10883.275	-2.277	0.031228
## CarNametoyota starlet	-25091.653	10746.339	-2.335	0.027541
## CarNametoyota tercel	-25336.364	11071.265	-2.288	0.030483
## CarNametoyouta tercel	-22470.447	11167.292	-2.012	0.054670
## CarNamevolkswagen rabbit	-28146.900	12718.048	-2.213	0.035870
## CarNamevolkswagen 1131 deluxe sedan	-28350.471	12222.975	-2.319	0.028491
## CarNamevolkswagen 411 (sw)	-30294.826	12296.250	-2.464	0.020682



## CarNamevolkswagen dasher	-27412.373	12723.706	-2.154	0.040649
## CarNamevolkswagen model 111	-29554.798	12696.486	-2.328	0.027974
## CarNamevolkswagen rabbit	-23080.543	12503.588	-1.846	0.076321
## CarNamevolkswagen rabbit custom	-25478.948	13041.257	-1.954	0.061573
## CarNamevolkswagen super beetle	-28172.032	13054.263	-2.158	0.040337
## CarNamevolkswagen type 3	-29758.368	12203.867	-2.438	0.021890
## CarNamevolvo 144ea	-27427.953	12668.497	-2.165	0.039744
## CarNamevolvo 145e (sw)	-29127.275	12239.829	-2.380	0.024949
## CarNamevolvo 244dl	-26297.009	12404.001	-2.120	0.043706
## CarNamevolvo 245	-25251.993	12782.992	-1.975	0.058923
## CarNamevolvo 246	-21108.975	13339.878	-1.582	0.125649
## CarNamevolvo 264gl	-25037.924	12552.574	-1.995	0.056664
## CarNamevolvo diesel	-23910.405	13115.100	-1.823	0.079804
## CarNamevw dasher	-27150.785	13299.630	-2.041	0.051474
## CarNamevw rabbit	-30000.371	12770.604	-2.349	0.026690
## fueltypegas	-27422.306	9966.212	-2.752	0.010662
## aspirationturbo	236.211	986.002	0.240	0.812547
## doornumbertwo	-1289.204	726.157	-1.775	0.087544
## carbodyhardtop	980.106	1811.658	0.541	0.593113
## carbodyhatchback	1074.707	1218.575	0.882	0.385892
## carbodysedan	2058.832	1031.011	1.997	0.056403
## carbodywagon	NA	NA	NA	NA
## drivewheel fwd	220.466	1260.739	0.175	0.862536
## drivewheel rwd	697.180	1435.842	0.486	0.631350
## enginelocationrear	9158.974	3987.839	2.297	0.029940
## wheelbase	289.818	128.512	2.255	0.032766
## carlength	-223.963	75.521	-2.966	0.006398
## carwidth	707.880	306.717	2.308	0.029218
## carheight	-461.834	272.256	-1.696	0.101767
## curbweight	10.847	3.395	3.195	0.003650
## enginetype dohc v	NA	NA	NA	NA
## enginetype l	NA	NA	NA	NA
## enginetype ohc	-3823.290	1866.254	-2.049	0.050717
## enginetype ohc f	NA	NA	NA	NA
## enginetype ohc v	804.447	2282.580	0.352	0.727358
## enginetype rotor	6623.579	3942.157	1.680	0.104898
## cylindernumber five	NA	NA	NA	NA
## cylindernumber four	7150.023	2131.208	3.355	0.002447
## cylindernumber six	NA	NA	NA	NA
## cylindernumber three	NA	NA	NA	NA
## cylindernumber twelve	NA	NA	NA	NA
## cylindernumber two	NA	NA	NA	NA
## enginesize	55.532	50.444	1.101	0.281043
## fuelsystem 2bbl	3954.307	2726.010	1.451	0.158854
## fuelsystem 4bbl	NA	NA	NA	NA
## fuelsystem i	NA	NA	NA	NA
## fuelsystem mfi	NA	NA	NA	NA
## fuelsystem mpi	2993.555	2482.520	1.206	0.238733
## fuelsystem spdi	2526.711	3442.636	0.734	0.469549
## fuelsystem spfi	2068.375	4073.432	0.508	0.615894
## boreratio	-3484.606	1792.683	-1.944	0.062818
## stroke	-1314.657	1072.834	-1.225	0.231407
## compressionratio	-1910.210	772.424	-2.473	0.020255
## horsepower	-42.876	34.010	-1.261	0.218616

## peakrpm	3.286	1.008	3.261	0.003099
## citympg	309.298	223.210	1.386	0.177618
## highwaympg	-124.218	164.066	-0.757	0.455783
##				
## (Intercept)				
## car_ID	*			
## symboling	*			
## CarNamealfa-romero Quadrifoglio				
## CarNamealfa-romero stelvio	.			
## CarNameaudi 100 ls				
## CarNameaudi 100ls				
## CarNameaudi 4000	.			
## CarNameaudi 5000	.			
## CarNameaudi 5000s (diesel)				
## CarNameaudi fox	*			
## CarNamebmw 320i				
## CarNamebmw x1	***			
## CarNamebmw x3	***			
## CarNamebmw x4	***			
## CarNamebmw x5	***			
## CarNamebmw z4	**			
## CarNamebuick century				
## CarNamebuick century luxus (sw)				
## CarNamebuick century special				
## CarNamebuick electra 225 custom				
## CarNamebuick opel isuzu deluxe				
## CarNamebuick regal sport coupe (turbo)				
## CarNamebuick skyhawk				
## CarNamebuick skylark				
## CarNamechevrolet impala				
## CarNamechevrolet monte carlo				
## CarNamechevrolet vega 2300	.			
## CarNamedodge challenger se				
## CarNamedodge colt (sw)	*			
## CarNamedodge colt hardtop	*			
## CarNamedodge coronet custom	*			
## CarNamedodge coronet custom (sw)				
## CarNamedodge d200	.			
## CarNamedodge dart custom				
## CarNamedodge monaco (sw)	*			
## CarNamedodge rampage	.			
## CarNamehonda accord				
## CarNamehonda accord cvcc				
## CarNamehonda accord lx				
## CarNamehonda civic				
## CarNamehonda civic (auto)	.			
## CarNamehonda civic 1300				
## CarNamehonda civic 1500 gl				
## CarNamehonda civic cvcc				
## CarNamehonda prelude				
## CarNameisuzu D-Max				
## CarNameisuzu D-Max V-Cross	.			
## CarNameisuzu MU-X	.			
## CarNamejaguar xf				

```

## CarNamejaguar xj
## CarNamejaguar xk
## CarNamemaxda glc deluxe
## CarNamemaxda rx3 .
## CarNamemazda 626 .
## CarNamemazda glc
## CarNamemazda glc 4
## CarNamemazda glc custom
## CarNamemazda glc custom l *
## CarNamemazda glc deluxe
## CarNamemazda rx-4 .
## CarNamemazda rx-7 gs
## CarNamemazda rx2 coupe
## CarNamemercury cougar
## CarNamemitsubishi g4 *
## CarNamemitsubishi lancer *
## CarNamemitsubishi mirage **
## CarNamemitsubishi mirage g4 *
## CarNamemitsubishi montero **
## CarNamemitsubishi outlander *
## CarNamemitsubishi pajero **
## CarNamenissan clipper .
## CarNamenissan dayz .
## CarNamenissan fuga .
## CarNamenissan gt-r *
## CarNamenissan juke .
## CarNamenissan kicks .
## CarNamenissan latio .
## CarNamenissan leaf .
## CarNamenissan note
## CarNamenissan nv200 *
## CarNamenissan otti .
## CarNamenissan rogue .
## CarNamenissan teana .
## CarNamenissan titan
## CarNameNissan versa .
## CarNamepeugeot 304 ***
## CarNamepeugeot 504 ***
## CarNamepeugeot 504 (sw) ***
## CarNamepeugeot 505s turbo diesel **
## CarNamepeugeot 604sl ***
## CarNameplymouth cricket *
## CarNameplymouth duster **
## CarNameplymouth fury gran sedan *
## CarNameplymouth fury iii *
## CarNameplymouth satellite custom (sw) *
## CarNameplymouth valiant .
## CarNameporcshce panamera
## CarNameporsche boxter
## CarNameporsche cayenne
## CarNameporsche macan
## CarNamerenault 12tl *
## CarNamerenault 5 gtl *
## CarNamesaab 99e .

```

```

## CarNamesaab 99gle
## CarNamesaab 99le .
## CarNamesubaru **
## CarNamesubaru baja **
## CarNamesubaru brz *
## CarNamesubaru dl *
## CarNamesubaru r1 **
## CarNamesubaru r2 **
## CarNamesubaru trezia *
## CarNamesubaru tribeca *
## CarNametoyota carina *
## CarNametoyota celica gt *
## CarNametoyota celica gt liftback **
## CarNametoyota corolla *
## CarNametoyota corolla 1200 *
## CarNametoyota corolla 1600 (sw) .
## CarNametoyota corolla liftback *
## CarNametoyota corolla tercel **
## CarNametoyota corona *
## CarNametoyota corona hardtop .
## CarNametoyota corona liftback *
## CarNametoyota corona mark ii *
## CarNametoyota cressida .
## CarNametoyota mark ii *
## CarNametoyota starlet *
## CarNametoyota tercel *
## CarNametoyouta tercel .
## CarNamevolkswagen rabbit *
## CarNamevolkswagen 1131 deluxe sedan *
## CarNamevolkswagen 411 (sw) *
## CarNamevolkswagen dasher *
## CarNamevolkswagen model 111 *
## CarNamevolkswagen rabbit .
## CarNamevolkswagen rabbit custom .
## CarNamevolkswagen super beetle *
## CarNamevolkswagen type 3 *
## CarNamevolvo 144ea *
## CarNamevolvo 145e (sw) *
## CarNamevolvo 244dl *
## CarNamevolvo 245 .
## CarNamevolvo 246
## CarNamevolvo 264gl .
## CarNamevolvo diesel .
## CarNamevw dasher .
## CarNamevw rabbit *
## fueltypegas *
## aspirationturbo
## doornumbertwo .
## carbodyhardtop
## carbodyhatchback
## carbodysedan .
## carbodywagon
## drivewheelfwd
## drivewheelrwd

```

```
## enginelocationrear          *
## wheelbase                   *
## carlength                   **
## carwidth                    *
## carheight                   *
## curbweight                  **
## enginetyperedohcv
## enginetype1
## enginetypeohc              .
## enginetypeohcf
## enginetypeohcv
## enginetyperotor
## cylindernumberfive
## cylindernumberfour         **
## cylindernumbersix
## cylindernumberthree
## cylindernumbertwelve
## cylindernumbertwo
## enginesize
## fuelsystem2bbl
## fuelsystem4bbl
## fuelsystemidi
## fuelsystemmfi
## fuelsystemmpfi
## fuelsystemspdi
## fuelsystemspfi
## boreratio                  .
## stroke
## compressionratio           *
## horsepower
## peakrpm                    **
## citympg
## highwaympg
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1100 on 26 degrees of freedom
## Multiple R-squared:  0.9976, Adjusted R-squared:  0.9811
## F-statistic: 60.35 on 178 and 26 DF, p-value: < 2.2e-16
```

By focusing on carname feaure only

```
lm.fit1 = lm(price~CarName)
summary(lm(price~horsepower+CarName))
```

```
##
## Call:
## lm(formula = price ~ horsepower + CarName)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -5274      0         0         0    5274
##
```

```

## Coefficients:
##
## (Intercept)
## horsepower
## CarNamealfa-romero Quadrifoglio
## CarNamealfa-romero stelvio
## CarNameaudi 100 ls
## CarNameaudi 100ls
## CarNameaudi 4000
## CarNameaudi 5000
## CarNameaudi 5000s (diesel)
## CarNameaudi fox
## CarNamebmw 320i
## CarNamebmw x1
## CarNamebmw x3
## CarNamebmw x4
## CarNamebmw x5
## CarNamebmw z4
## CarNamebuick century
## CarNamebuick century luxus (sw)
## CarNamebuick century special
## CarNamebuick electra 225 custom
## CarNamebuick opel isuzu deluxe
## CarNamebuick regal sport coupe (turbo)
## CarNamebuick skyhawk
## CarNamebuick skylark
## CarNamechevrolet impala
## CarNamechevrolet monte carlo
## CarNamechevrolet vega 2300
## CarNamedodge challenger se
## CarNamedodge colt (sw)
## CarNamedodge colt hardtop
## CarNamedodge coronet custom
## CarNamedodge coronet custom (sw)
## CarNamedodge d200
## CarNamedodge dart custom
## CarNamedodge monaco (sw)
## CarNamedodge rampage
## CarNamehonda accord
## CarNamehonda accord cvcc
## CarNamehonda accord lx
## CarNamehonda civic
## CarNamehonda civic (auto)
## CarNamehonda civic 1300
## CarNamehonda civic 1500 gl
## CarNamehonda civic cvcc
## CarNamehonda prelude
## CarNameisuzu D-Max
## CarNameisuzu D-Max V-Cross
## CarNameisuzu MU-X
## CarNamejaguar xf
## CarNamejaguar xj
## CarNamejaguar xk
## CarNamemaxda glc deluxe

```

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	3983.02	2577.88	1.545	0.127862	
horsepower	85.69	11.23	7.629	2.85e-10	***
CarNamealfa-romero Quadrifoglio	-679.82	3227.23	-0.211	0.833911	
CarNamealfa-romero stelvio	3005.00	3190.88	0.942	0.350297	
CarNameaudi 100 ls	1226.24	3192.49	0.384	0.702332	
CarNameaudi 100ls	3956.46	2763.44	1.432	0.157686	
CarNameaudi 4000	7894.89	3207.47	2.461	0.016889	*
CarNameaudi 5000	5510.69	3190.90	1.727	0.089584	.
CarNameaudi 5000s (diesel)	165.18	3238.01	0.051	0.959492	
CarNameaudi fox	1840.69	3190.90	0.577	0.566308	
CarNamebmw 320i	4039.44	2765.67	1.461	0.149626	
CarNamebmw x1	6618.06	3192.86	2.073	0.042726	*
CarNamebmw x3	12026.91	2800.58	4.294	6.89e-05	***
CarNamebmw x4	11180.76	3289.04	3.399	0.001239	**
CarNamebmw x5	21735.76	3289.04	6.609	1.42e-08	***
CarNamebmw z4	10213.06	3192.86	3.199	0.002255	**
CarNamebuick century	13652.68	3193.73	4.275	7.36e-05	***
CarNamebuick century luxus (sw)	13724.68	3193.73	4.297	6.82e-05	***
CarNamebuick century special	21209.37	3294.56	6.438	2.73e-08	***
CarNamebuick electra 225 custom	11028.68	3193.73	3.453	0.001051	**
CarNamebuick opel isuzu deluxe	16918.49	3228.93	5.240	2.43e-06	***
CarNamebuick regal sport coupe (turbo)	25649.37	3294.56	7.785	1.56e-10	***
CarNamebuick skyhawk	17076.68	3193.73	5.347	1.64e-06	***
CarNamebuick skylark	17790.49	3228.93	5.510	9.00e-07	***
CarNamechevrolet impala	-2945.31	3268.41	-0.901	0.371304	
CarNamechevrolet monte carlo	-3686.57	3223.95	-1.143	0.257613	
CarNamechevrolet vega 2300	-3406.57	3223.95	-1.057	0.295132	
CarNamedodge challenger se	-3433.18	3227.23	-1.064	0.291899	
CarNamedodge colt (sw)	-2201.18	3227.23	-0.682	0.497961	
CarNamedodge colt hardtop	-3118.18	3227.23	-0.966	0.338022	
CarNamedodge coronet custom	-4165.76	3192.49	-1.305	0.197183	
CarNamedodge coronet custom (sw)	-3444.58	3213.66	-1.072	0.288302	
CarNamedodge d200	-4766.76	3192.49	-1.493	0.140921	
CarNamedodge dart custom	-2603.05	3201.33	-0.813	0.419535	
CarNamedodge monaco (sw)	-3581.18	3227.23	-1.110	0.271799	
CarNamedodge rampage	-4238.18	3227.23	-1.313	0.194359	
CarNamehonda accord	-2257.66	2777.62	-0.813	0.419712	
CarNamehonda accord cvcc	-3966.73	3215.01	-1.234	0.222336	
CarNamehonda accord lx	-3200.73	3215.01	-0.996	0.323673	
CarNamehonda civic	-1964.31	2640.08	-0.744	0.459911	
CarNamehonda civic (auto)	-2207.37	3193.28	-0.691	0.492210	
CarNamehonda civic 1300	-2257.66	3203.22	-0.705	0.483798	
CarNamehonda civic 1500 gl	-3200.73	3215.01	-0.996	0.323673	
CarNamehonda civic cvcc	-3503.73	2791.21	-1.255	0.214503	
CarNamehonda prelude	-2507.66	3203.22	-0.783	0.436951	
CarNameisuzu D-Max	-856.25	2785.24	-0.307	0.759641	
CarNameisuzu D-Max V-Cross	-1065.07	3223.95	-0.330	0.742339	
CarNameisuzu MU-X	-3882.11	3212.34	-1.208	0.231845	
CarNamejaguar xf	16484.92	3273.35	5.036	5.09e-06	***
CarNamejaguar xj	13184.92	3273.35	4.028	0.000168	***
CarNamejaguar xk	9565.28	3613.67	2.647	0.010483	*
CarNamemaxda glc deluxe	-3715.18	3227.23	-1.151	0.254458	

## CarNamemaxda rx3	-4615.18	3227.23	-1.430	0.158155	
## CarNamemazda 626	-1321.87	2616.34	-0.505	0.615341	
## CarNamemazda glc	1610.35	2763.39	0.583	0.562365	
## CarNamemazda glc 4	93.36	3202.25	0.029	0.976844	
## CarNamemazda glc custom	-586.28	3205.26	-0.183	0.855518	
## CarNamemazda glc custom l	-2686.28	3205.26	-0.838	0.405485	
## CarNamemazda glc deluxe	-543.79	2809.24	-0.194	0.847199	
## CarNamemazda rx-4	-2025.73	2791.21	-0.726	0.470961	
## CarNamemazda rx-7 gs	4598.99	2777.06	1.656	0.103204	
## CarNamemazda rx2 coupe	-3015.18	3227.23	-0.934	0.354094	
## CarName mercury cougar	-2476.39	3270.86	-0.757	0.452106	
## CarName mitsubishi g4	-3199.80	2605.36	-1.228	0.224434	
## CarName mitsubishi lancer	-3621.18	3227.23	-1.122	0.266538	
## CarName mitsubishi mirage	-4421.18	3227.23	-1.370	0.176072	
## CarName mitsubishi mirage g4	-3509.50	2610.55	-1.344	0.184160	
## CarName mitsubishi montero	-4535.05	3201.33	-1.417	0.162038	
## CarName mitsubishi outlander	-3855.08	2605.39	-1.480	0.144472	
## CarName mitsubishi pajero	-3335.05	3201.33	-1.042	0.301916	
## CarName nissan clipper	-470.93	2763.67	-0.170	0.865299	
## CarName nissan dayz	-3509.43	3223.95	-1.089	0.280933	
## CarName nissan fuga	-2609.43	3223.95	-0.809	0.421656	
## CarName nissan gt-r	-1597.16	3252.30	-0.491	0.625250	
## CarName nissan juke	-2096.87	3225.57	-0.650	0.518254	
## CarName nissan kicks	-1422.72	3343.82	-0.425	0.672091	
## CarName nissan latio	-2721.87	2803.37	-0.971	0.335686	
## CarName nissan leaf	-2596.87	3225.57	-0.805	0.424114	
## CarName nissan note	-1896.87	3225.57	-0.588	0.558806	
## CarName nissan nv200	-2746.29	3194.76	-0.860	0.393597	
## CarName nissan otti	-3509.43	3223.95	-1.089	0.280933	
## CarName nissan rogue	-3296.58	2781.23	-1.185	0.240818	
## CarName nissan teana	-494.98	3238.01	-0.153	0.879044	
## CarName nissan titan	-2546.87	3225.57	-0.790	0.433041	
## CarName Nissan versa	-4396.87	3225.57	-1.363	0.178202	
## CarName peugeot 304	1076.10	3195.94	0.337	0.737574	
## CarName peugeot 504	3254.80	2443.16	1.332	0.188093	
## CarName peugeot 504 (sw)	144.71	3194.76	0.045	0.964030	
## CarName peugeot 505s turbo diesel	4951.10	3195.94	1.549	0.126873	
## CarName peugeot 604sl	3387.30	2764.67	1.225	0.225535	
## CarName plymouth cricket	-4766.76	3192.49	-1.493	0.140921	
## CarName plymouth duster	-3644.58	3213.66	-1.134	0.261504	
## CarName plymouth fury gran sedan	-2201.18	3227.23	-0.682	0.497961	
## CarName plymouth fury iii	-3909.68	2805.28	-1.394	0.168823	
## CarName plymouth satellite custom (sw)	-3118.18	3227.23	-0.966	0.338022	
## CarName plymouth valiant	-2603.05	3201.33	-0.813	0.419535	
## CarName porcshe panamera	10806.42	3368.17	3.208	0.002192	**
## CarName porcshe boxer	15306.42	3368.17	4.544	2.91e-05	***
## CarName porcshe cayenne	7522.08	3160.25	2.380	0.020670	*
## CarName porcshe macan	5780.81	3211.07	1.800	0.077109	.
## CarName renault 12tl	-2400.44	3199.59	-0.750	0.456201	
## CarName renault 5 gtl	-1800.44	3199.59	-0.563	0.575840	
## CarName saab 99e	-316.64	2776.51	-0.114	0.909603	
## CarName saab 99gle	1278.36	2776.51	0.460	0.646967	
## CarName saab 99le	195.69	2763.41	0.071	0.943792	
## CarName subaru	-4330.88	2792.01	-1.551	0.126396	

```

## CarNamesubaru baja -2078.21 3196.59 -0.650 0.518218
## CarNamesubaru brz -3234.89 3207.47 -1.009 0.317453
## CarNamesubaru dl -2654.80 2539.79 -1.045 0.300306
## CarNamesubaru r1 -1776.89 3207.47 -0.554 0.581756
## CarNamesubaru r2 -2236.00 3190.88 -0.701 0.486312
## CarNamesubaru trezia -3546.89 3207.47 -1.106 0.273449
## CarNamesubaru tribeca -1840.21 3196.59 -0.576 0.567098
## CarNametoyota carina -518.02 3238.01 -0.160 0.873462
## CarNametoyota celica gt 459.35 3219.31 0.143 0.887040
## CarNametoyota celica gt liftback -4282.69 3190.90 -1.342 0.184867
## CarNametoyota corolla -2194.00 2448.11 -0.896 0.373914
## CarNametoyota corolla 1200 -2795.79 2809.24 -0.995 0.323838
## CarNametoyota corolla 1600 (sw) -1398.02 3238.01 -0.432 0.667550
## CarNametoyota corolla liftback -1572.62 2763.85 -0.569 0.571594
## CarNametoyota corolla tercel -4042.69 3190.90 -1.267 0.210328
## CarNametoyota corona -1914.92 2454.29 -0.780 0.438482
## CarNametoyota corona hardtop -2378.02 3238.01 -0.734 0.465711
## CarNametoyota corona liftback -5474.47 3191.38 -1.715 0.091705
## CarNametoyota corona mark ii -3948.02 3238.01 -1.219 0.227762
## CarNametoyota cressida 3745.53 3191.38 1.174 0.245420
## CarNametoyota mark ii -1461.98 2632.65 -0.555 0.580843
## CarNametoyota starlet -2797.84 2777.62 -1.007 0.318058
## CarNametoyota tercel -2724.47 3191.38 -0.854 0.396847
## CarNametoyota tercel -1601.21 3230.67 -0.496 0.622063
## CarNamevolkswagen rabbit -664.08 3258.98 -0.204 0.839259
## CarNamevolkswagen 1131 deluxe sedan -3291.97 3204.22 -1.027 0.308579
## CarNamevolkswagen 411 (sw) -2771.97 3204.22 -0.865 0.390611
## CarNamevolkswagen dasher -895.71 2769.98 -0.323 0.747603
## CarNamevolkswagen model 111 -444.08 3258.98 -0.136 0.892093
## CarNamevolkswagen rabbit -114.31 3190.90 -0.036 0.971549
## CarNamevolkswagen rabbit custom 4034.82 3227.23 1.250 0.216320
## CarNamevolkswagen super beetle -315.18 3227.23 -0.098 0.922543
## CarNamevolkswagen type 3 -3071.97 3204.22 -0.959 0.341746
## CarNamevolvo 144ea 506.97 2778.78 0.182 0.855882
## CarNamevolvo 145e (sw) 1140.42 2763.59 0.413 0.681405
## CarNamevolvo 244dl 4125.98 2767.24 1.491 0.141473
## CarNamevolvo 245 2762.92 3191.06 0.866 0.390213
## CarNamevolvo 246 9403.47 3191.38 2.947 0.004648 **
## CarNamevolvo 264gl 4713.78 2779.98 1.696 0.095415
## CarNamevolvo diesel 1084.63 3241.90 0.335 0.739180
## CarNamevw dasher -100.44 3199.59 -0.031 0.975068
## CarNamevw rabbit -1715.44 3199.59 -0.536 0.593946
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2256 on 57 degrees of freedom
## Multiple R-squared: 0.9777, Adjusted R-squared: 0.9202
## F-statistic: 17.01 on 147 and 57 DF, p-value: < 2.2e-16

```

## Simple Linear Regression

From above result we can say that car name provides too much information regarding the car price. In this project we are more focused on getting car price from car inbuilt features like size, engine quality, top-speed.



We will be avoiding the carnames from now on as feature list. IN this pae we apply simple linear regression on the the car price dataset. As we are selecting only one feaures.

```
lm.fit = lm(price~enginesize)
lm.fit

##
## Call:
## lm(formula = price ~ enginesize)
##
## Coefficients:
## (Intercept)    enginesize
##      -8005.4         167.7
```

After fitting the model by calling names and coefficient the r will return model parameters and coefficients for considered variables.

```
names(lm.fit)

## [1] "coefficients" "residuals"      "effects"      "rank"
## [5] "fitted.values" "assign"         "qr"          "df.residual"
## [9] "xlevels"      "call"          "terms"       "model"

coef(lm.fit)
```

```
## (Intercept)    enginesize
##  -8005.4455      167.6984
```

```
confint(lm.fit)

##              2.5 %      97.5 %
## (Intercept) -9727.1913 -6283.6997
## enginesize   154.8047   180.5922
```

After fitting the model we can see the the performance of model fit. By summarizing the different statistical parameters; F-score, significance for the model we can understand the model fitness. We can see the p-value low for the significance and reject the null-hypothesis

```
summary(lm.fit)

##
## Call:
## lm(formula = price ~ enginesize)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10664.2  -2225.0   -482.4   1588.0  14271.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -8005.446    873.221  -9.168  <2e-16 ***
```

```
## enginesize      167.698      6.539  25.645   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3889 on 203 degrees of freedom
## Multiple R-squared:  0.7641, Adjusted R-squared:  0.763
## F-statistic: 657.6 on 1 and 203 DF,  p-value: < 2.2e-16
```

As we have a trained model we can use the model to predict the car price from car features using the trained linear regression model. We can also plot the output and variables. In R we do this by as follows

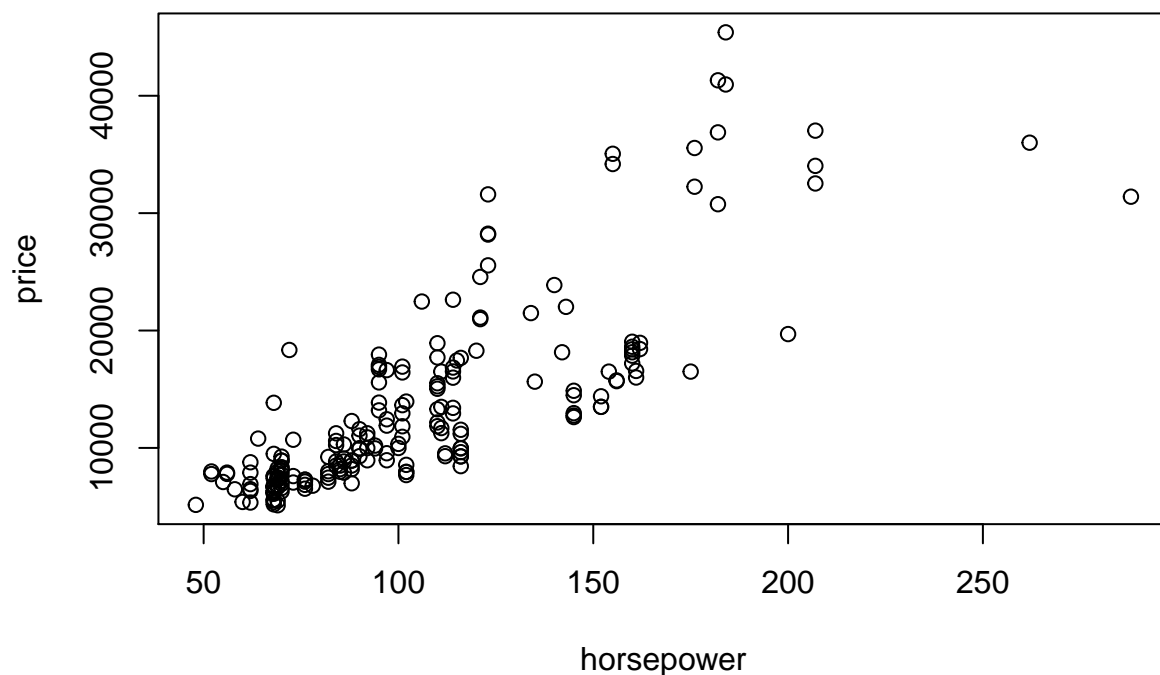
```
predict(lm.fit,data.frame(enginesize=(c(5,10,15))), interval="confidence")
```

```
##          fit          lwr          upr
## 1 -7166.953 -8827.550 -5506.356
## 2 -6328.461 -7928.170 -4728.752
## 3 -5489.969 -7029.082 -3950.857
```

```
predict(lm.fit,data.frame(enginesize=(c(5,10,15))), interval="prediction")
```

```
##          fit          lwr          upr
## 1 -7166.953 -15013.59  679.686
## 2 -6328.461 -14162.44 1505.518
## 3 -5489.969 -13311.80 2331.861
```

```
plot(horsepower, price)
```



Now considering adding more variables

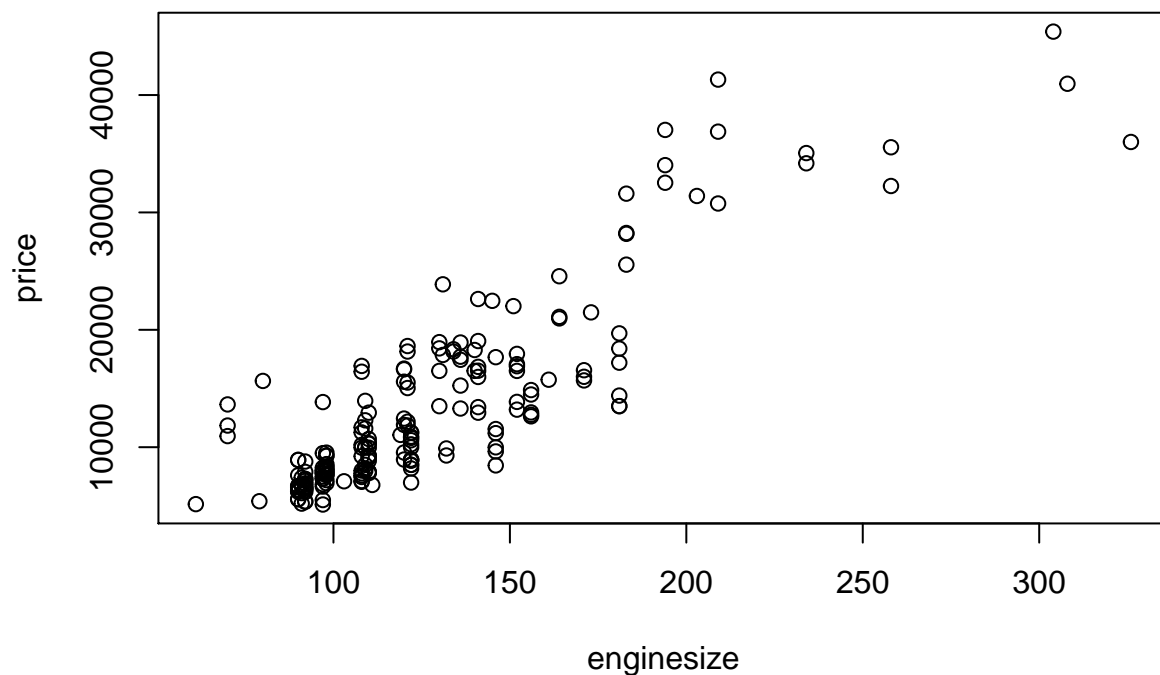
## Multiple Linear Regression

In the next section I will be using more than one variables and examine the summary of the linear regression model. For this we choose the car length and horsepower for discussing the results more clearly. We select the car length and horsepower by observing the summary of model from earlier analysis. The three stars shows their significance.

```
lm.fit = lm(price~carlength+horsepower)
lm.fit

##
## Call:
## lm(formula = price ~ carlength + horsepower)
##
## Coefficients:
## (Intercept)    carlength    horsepower
##    -38111.7         220.3         125.3

plot(engineSize, price)
```



In this experiment we conduct our analysis by incorporating more variables. We retrain the model using different features sets. We reevaluate the performance by considering the selected features set.

```
lm.fit = lm(price~enginelocation+enginesize+carlength+aspiration
            +curbweight+drivewheel, data = card)
summary(lm.fit)

##
## Call:
## lm(formula = price ~ enginelocation + enginesize + carlength +
##     aspiration + curbweight + drivewheel, data = card)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -7527.3 -1503.2  -100.9   1365.2 15175.9
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -14656.455    4933.580  -2.971  0.00334 **
## enginelocationrear 13029.009    2024.335   6.436 9.12e-10 ***
## enginesize       96.835      12.597   7.687 6.98e-13 ***
## carlength       19.853       42.969   0.462  0.64457
## aspirationturbo   438.735     694.900   0.631  0.52853
## curbweight       4.420       1.741   2.539  0.01189 *
## drivewheelfwd    -56.037    1208.333  -0.046  0.96306
## drivewheelrwd    1770.224    1196.704   1.479  0.14067
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3242 on 197 degrees of freedom
## Multiple R-squared:  0.841, Adjusted R-squared:  0.8354
## F-statistic: 148.9 on 7 and 197 DF, p-value: < 2.2e-16
```

In the following section, I will be implement the polymial regression by using polynomials of features and lm function in R.

```
summary(lm(price~poly(peakrpm,4)+aspiration+carlength+carheight
            +curbweight+fuelsystem+doornumber+wheelbase
            +enginetype, data =card))

##
## Call:
## lm(formula = price ~ poly(peakrpm, 4) + aspiration + carlength +
##     carheight + curbweight + fuelsystem + doornumber + wheelbase +
##     enginetype, data = card)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10540.7  -1989.4   -518.2   1636.9  14403.2
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -20524.628    9427.083  -2.177  0.030760 *
## poly(peakrpm, 4)1  21768.804    5075.402   4.289 2.92e-05 ***
## poly(peakrpm, 4)2  18325.242    4574.880   4.006 9.02e-05 ***
```

```
## poly(peakrpm, 4)3 -15038.206 4186.121 -3.592 0.000422 ***
## poly(peakrpm, 4)4 -23200.508 5173.456 -4.485 1.29e-05 ***
## aspirationturbo -1177.121 899.384 -1.309 0.192258
## carlength -25.794 66.235 -0.389 0.697409
## carheight -246.427 160.700 -1.533 0.126908
## curbweight 14.575 1.437 10.142 < 2e-16 ***
## fuelsystem2bbl 6464.969 1594.720 4.054 7.47e-05 ***
## fuelsystem4bbl 4375.865 4428.886 0.988 0.324458
## fuelsystemidi 5393.846 1865.420 2.891 0.004304 **
## fuelsystemmfi 2679.635 4111.384 0.652 0.515383
## fuelsystemmpfi 6151.123 1625.480 3.784 0.000209 ***
## fuelsystemspdi 3046.670 2149.272 1.418 0.158045
## fuelsystemspfi 981.934 4009.347 0.245 0.806803
## doornumbertwo 1773.610 654.866 2.708 0.007410 **
## wheelbase 71.127 107.366 0.662 0.508510
## enginetypeedohcv -133.867 3919.097 -0.034 0.972789
## enginetypeel -2586.584 1758.206 -1.471 0.142987
## enginetypeohc 1497.530 1334.909 1.122 0.263424
## enginetypeohcf 2270.551 1659.947 1.368 0.173056
## enginetypeohcv 1520.122 1546.661 0.983 0.326998
## enginetyperotor -7374.158 4055.943 -1.818 0.070700 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3569 on 181 degrees of freedom
## Multiple R-squared: 0.8229, Adjusted R-squared: 0.8004
## F-statistic: 36.58 on 23 and 181 DF, p-value: < 2.2e-16
```

From the F-statistics value 134 which is much higher than 1, it is evident that at least one features are related to the output variable car price.

## Chapter 4

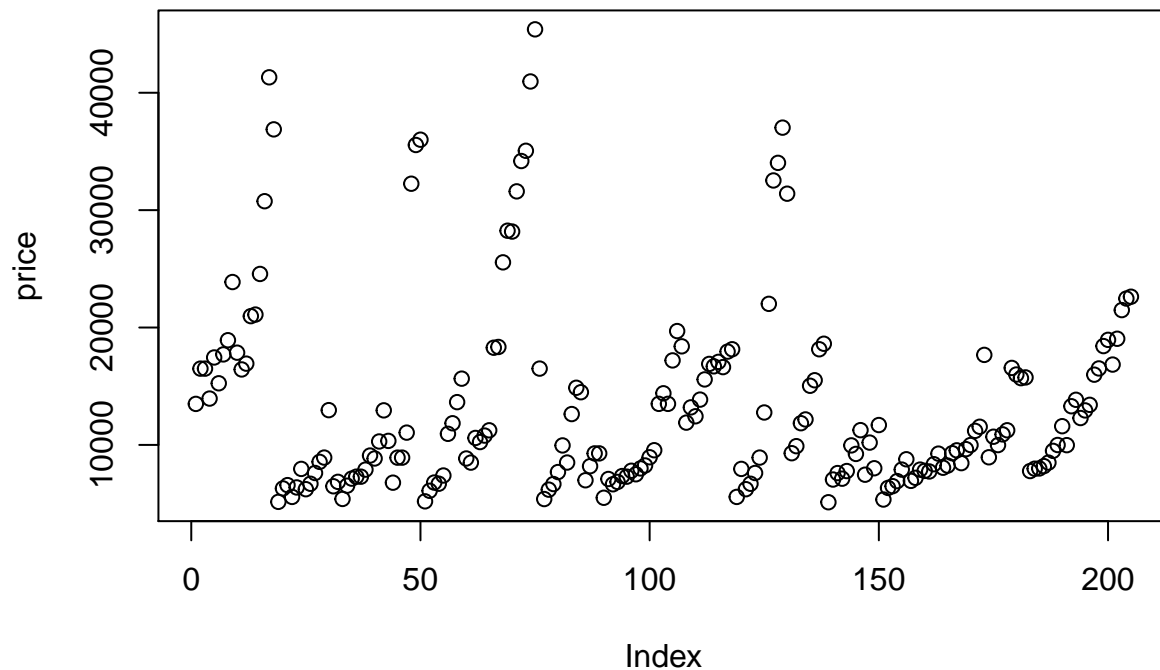
In the car dataset the output predictor is car price;a continuous variable. To apply classification techniques I reorganize my output predictor variable price as high and low based on median threshold. I have observe the price median as 10300. We label the price greater than threshold are high Yes and below No. We have created a classification problem of car price high or low in the car price dataset. We try to predict the car price label based on the data features. First we load the data.

```
library(ISLR)
```

```
dim(card)
```

```
## [1] 205 26
```

```
plot(price)
```



```
high = as.factor(ifelse(price<=10300, "No", "Yes"))
card = data.frame(card, high)
```

## Logistic Regression

We fit the model as targetting the created class labels high.

```
glm.fits=glm(as.numeric(high)~fuelsystem+peakrpm+citympg
+ enginesize+enginetype+carwidth+curbweight+carlength,
data = card)
```

After fitting the model we look into the fitted model by summary function.

```
summary(glm.fits)
```

```
##
## Call:
## glm(formula = as.numeric(high) ~ fuelsystem + peakrpm + citympg +
##     enginesize + enginetype + carwidth + curbweight + carlength,
##     data = card)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
```

```
## -0.65836 -0.14655 0.00762 0.13548 0.79154
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -4.709e-01  1.317e+00  -0.357  0.72119
## fuelsystem2bbl  3.976e-02  1.050e-01   0.379  0.70529
## fuelsystem4bbl  4.409e-01  3.507e-01   1.257  0.21025
## fuelsystemidi  3.301e-01  1.427e-01   2.313  0.02184 *
## fuelsystemmfi  5.407e-01  3.099e-01   1.745  0.08271 .
## fuelsystemmpfi  3.796e-01  1.112e-01   3.413  0.00079 ***
## fuelsystemspdi  1.173e-01  1.424e-01   0.824  0.41112
## fuelsystemspfi  6.309e-01  3.084e-01   2.046  0.04217 *
## peakrpm        9.905e-06  6.071e-05   0.163  0.87056
## citympg        -1.367e-02  8.064e-03  -1.695  0.09167 .
## enginesize     -1.174e-03  1.238e-03  -0.949  0.34396
## enginetyperedohcv -2.158e-01  3.321e-01  -0.650  0.51658
## enginetyperel   1.057e-01  1.324e-01   0.798  0.42579
## enginetyperohc  3.080e-02  9.804e-02   0.314  0.75374
## enginetyperohcf -3.959e-02  1.222e-01  -0.324  0.74642
## enginetyperohcv -4.262e-02  1.292e-01  -0.330  0.74195
## enginetyperotor  2.020e-01  3.153e-01   0.641  0.52258
## carwidth       2.712e-02  2.255e-02   1.202  0.23079
## curbweight     5.022e-04  1.622e-04   3.095  0.00227 **
## carlength      -5.292e-03  4.353e-03  -1.216  0.22557
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 0.08278914)
##
##      Null deviance: 51.249  on 204  degrees of freedom
## Residual deviance: 15.316  on 185  degrees of freedom
## AIC: 91.972
##
## Number of Fisher Scoring iterations: 2
```

```
coef(glm.fits)
```

```
##      (Intercept) fuelsystem2bbl fuelsystem4bbl fuelsystemidi fuelsystemmfi
## -4.708631e-01  3.975924e-02  4.409199e-01  3.301087e-01  5.406812e-01
## fuelsystemmpfi fuelsystemspdi fuelsystemspfi      peakrpm      citympg
##  3.795650e-01  1.173331e-01  6.309112e-01  9.905254e-06 -1.367271e-02
##      enginesize enginetyperedohcv enginetyperel enginetyperohc enginetyperohcf
## -1.174460e-03 -2.158418e-01  1.056797e-01  3.080150e-02 -3.958632e-02
## enginetyperohcv enginetyperotor      carwidth      curbweight      carlength
## -4.261800e-02  2.019611e-01  2.711590e-02  5.021979e-04 -5.292212e-03
```

```
summary(glm.fits)$coef
```

```
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -4.708631e-01  1.317430e+00 -0.3574102 0.7211921146
## fuelsystem2bbl  3.975924e-02  1.049685e-01  0.3787732 0.7052906342
## fuelsystem4bbl  4.409199e-01  3.507004e-01  1.2572554 0.2102460397
## fuelsystemidi  3.301087e-01  1.427342e-01  2.3127519 0.0218365932
```

```
## fuelsystemmfi      5.406812e-01 3.099105e-01 1.7446368 0.0827087800
## fuelsystemmpfi     3.795650e-01 1.112147e-01 3.4129028 0.0007895722
## fuelsystemspdi     1.173331e-01 1.424302e-01 0.8237940 0.4111173753
## fuelsystemspfi     6.309112e-01 3.083676e-01 2.0459709 0.0421744484
## peakrpm            9.905254e-06 6.070519e-05 0.1631698 0.8705628429
## citympg            -1.367271e-02 8.064297e-03 -1.6954627 0.0916699914
## enginesize         -1.174460e-03 1.237837e-03 -0.9487995 0.3439603704
## enginetyperedohcv -2.158418e-01 3.321260e-01 -0.6498793 0.5165763097
## enginetyperel      1.056797e-01 1.324017e-01 0.7981750 0.4257923143
## enginetypeohc      3.080150e-02 9.803978e-02 0.3141735 0.7537432676
## enginetypeohcf    -3.958632e-02 1.222364e-01 -0.3238506 0.7464169938
## enginetypeohcv    -4.261800e-02 1.292357e-01 -0.3297694 0.7419472714
## enginetyperotor    2.019611e-01 3.152725e-01 0.6405924 0.5225802287
## carwidth           2.711590e-02 2.255351e-02 1.2022916 0.2307871882
## curbweight         5.021979e-04 1.622369e-04 3.0954601 0.0022705322
## carlength         -5.292212e-03 4.352501e-03 -1.2159012 0.2255717778
```

```
summary(glm.fits)$coef[,4]
```

```
##      (Intercept) fuelsystem2bb1 fuelsystem4bb1 fuelsystemidi fuelsystemmfi
##      0.7211921146 0.7052906342 0.2102460397 0.0218365932 0.0827087800
## fuelsystemmpfi fuelsystemspdi fuelsystemspfi peakrpm citympg
##      0.0007895722 0.4111173753 0.0421744484 0.8705628429 0.0916699914
##      enginesize enginetyperedohcv enginetyperel enginetypeohc enginetypeohcf
##      0.3439603704 0.5165763097 0.4257923143 0.7537432676 0.7464169938
## enginetypeohcv enginetyperotor carwidth curbweight carlength
##      0.7419472714 0.5225802287 0.2307871882 0.0022705322 0.2255717778
```

From the summary above we see the significance of the fuelsystem and curbweight are highest based on their smaller p-value.

Now we check the classifiers performance based on its decision on the dataset. For that we create the confusion matrix for the classifier.

```
glm.probs=predict(glm.fits,type="response")
glm.probs[1:10]
```

```
##      1      2      3      4      5      6      7      8
## 1.642826 1.642826 1.765080 1.571926 1.872245 1.692959 1.918991 1.974233
##      9     10
## 2.073741 2.052672
```

```
contrasts(high)
```

```
##      Yes
## No      0
## Yes     1
```

```
glm.pred=rep("No", 205)
glm.pred[glm.probs>1.5]="Yes"
table(glm.pred, high)
```



```
##           high
## glm.pred No Yes
##           No  89   4
##           Yes  14  98
```

```
mean(glm.pred==high)
```

```
## [1] 0.9121951
```

From the result we can see that the model have correctly classified 89 no instances and 98 No instances. The model accuracy is 91.22% in the training instances.

Now in next case we only consider case where peakrpm is lower than 6000 we devide our dataset by taking the instances where peak values are smaller than 6000. We refit the model using the cropped dataset.

```
train=(peakrpm<6000)
ccard.6000=card[!train,]
dim(ccard.6000)
```

```
## [1] 11 27
```

```
high.6000=high[!train]
```

```
glm.fits=glm(as.numeric(high)~peakrpm+citympg
             + enginesize+carwidth+curbweight+carlength,
             data = card, subset = train)
```

```
glm.probs=predict(glm.fits,ccard.6000,type="response")
```

By selecting appropriate threshold we can get the prediction form the model for the 11 test dataset.

```
glm.pred=rep("No",11)
glm.pred[glm.probs>1.5]="Yes"
table(glm.pred,as.factor(high.6000))
```

```
##
## glm.pred No Yes
##           No   7   0
##           Yes  0   4
```

```
mean(glm.pred==high.6000)
```

```
## [1] 1
```

```
mean(glm.pred!=high.6000)
```

```
## [1] 0
```

Here the model sucessfully classified all the instances from the features; 7 no and 4 yes classes

We can consider only 3 variables to check the synergy. For that we take 3 variables to fit the high class.

```
glm.fits=glm(as.factor(high)~carwidth+curbweight+enginesize,
             data=card,family=binomial,subset=train)
glm.probs=predict(glm.fits,ccard.6000,type="response")
glm.pred=rep("No",11)
glm.pred[glm.probs>.2]="Yes"
table(glm.pred,high.6000)
```

```
##           high.6000
## glm.pred No Yes
##      No    7    0
##      Yes   0    4
```

```
mean(glm.pred==high.6000)
```

```
## [1] 1
```

After preparing the model, we can see that the model again perform well on the test dataset. It again classified all the test instance correctly. We can see the result in the above confusion matrix.

## Linear Discriminant Analysis

In this section we implement LDA on the high class for car data we created in earlier example.

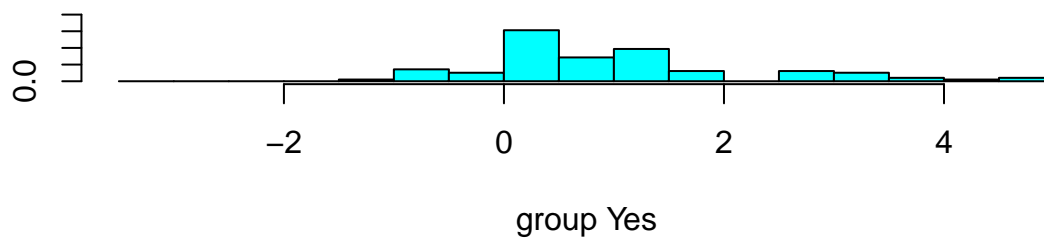
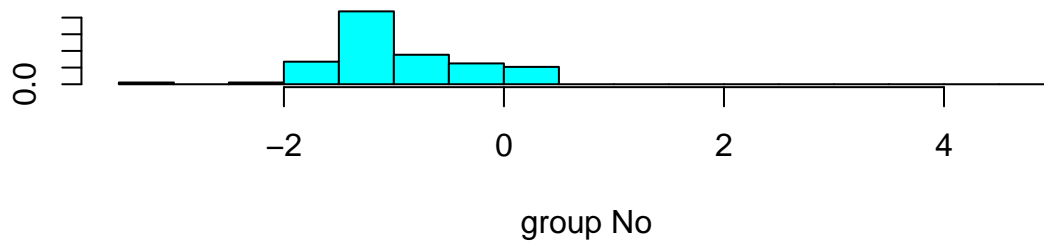
```
library(MASS)

lda.fit=lda(as.factor(high)~carwidth+enginesize,
            data=card,subset=train) # fitting model

lda.fit # provides summary
```

```
## Call:
## lda(as.factor(high) ~ carwidth + enginesize, data = card, subset = train)
##
## Prior probabilities of groups:
##      No      Yes
## 0.4948454 0.5051546
##
## Group means:
##      carwidth enginesize
## No   64.47917   103.2812
## Yes  67.45408   154.6429
##
## Coefficients of linear discriminants:
##              LD1
## carwidth    0.46165129
## enginesize  0.01191789
```

```
plot(lda.fit)
```



From above plot we see the difference in distribution between high and low class for the car price. We can use the previous model to predict new outcome

```
lda.pred=predict(lda.fit, ccard.6000)
names(lda.pred)
```

```
## [1] "class"      "posterior" "x"
```

```
lda.class=lda.pred$class
table(lda.class,high.6000)
```

```
##          high.6000
## lda.class No  Yes
##      No    7    4
##      Yes   0    0
```

```
mean(lda.class==high.6000)
```

```
## [1] 0.6363636
```

The model failed to correctly classify any yes instance in the previous section. It predicted all as NO. We can change the threshold to check result across the threshold.

```
# changing default threshold
sum(lda.pred$posterior[,1]>=.9)
```

```
## [1] 7
```

```
sum(lda.pred$posterior[,1]<.9)
```

```
## [1] 4
```

```
lda.pred$posterior[1:11,1]
```

```
##          32          34          35          36          37          56          57          58
## 0.9398168 0.9344186 0.9344186 0.9344186 0.9398168 0.8346551 0.8346551 0.8346551
##          59          166          167
## 0.7993691 0.9251628 0.9251628
```

```
lda.class[1:11]
```

```
## [1] No No No No No No No No No No No
## Levels: No Yes
```

```
sum(lda.pred$posterior[,1]>.9)
```

```
## [1] 7
```

In this case by changing the threshold we get better prediction for the same model.

## Quadratic Discriminant Analysis

```
qda.fit=qda(as.factor(high)~carwidth+enginesize,data=Smarket,subset=train)
qda.fit
```

```
## Call:
## qda(as.factor(high) ~ carwidth + enginesize, data = Smarket,
##     subset = train)
##
## Prior probabilities of groups:
##      No      Yes
## 0.4948454 0.5051546
##
## Group means:
##      carwidth enginesize
## No   64.47917   103.2812
## Yes  67.45408   154.6429
```

```
qda.class=predict(qda.fit, ccard.6000)$class
table(qda.class, high.6000)
```

```
##           high.6000
## qda.class No Yes
##      No    7    0
##      Yes   0    4
```

```
mean(qda.class==high.6000)
```

```
## [1] 1
```

From the output we can see that quadratic discriminant analysis provide accurate result on classifying test data. It predicted all 7 no and 4 yes class correctly.

## K-Nearest Neighbors

In this section, we implement k-nearest Neighbors for the car data to classify the car price as high or no.

```
# need class library
library(class)
train.X=cbind(carwidth,enginesize, fuelsystem, curbweight)[train,]
test.X=cbind(carwidth,enginesize, fuelsystem, curbweight)[!train,]
train.high=high[train]

set.seed(1) # reproducible result
# 4 arguments
knn.pred=knn(train.X,test.X,train.high,k=5)

table(knn.pred,high.6000)
```

```
##           high.6000
## knn.pred No Yes
##      No    7    3
##      Yes   0    1
```

```
# increasing the number of K (3 in this case)
knn.pred=knn(train.X,test.X,train.high,k=3)

table(knn.pred, high.6000)
```

```
##           high.6000
## knn.pred No Yes
##      No    7    1
##      Yes   0    3
```

```
mean(knn.pred==high.6000)
```

```
## [1] 0.9090909
```

The KNN failed to classify the car price high class. I have conducted experiment for different k values (5,10,15,25). In the previous result, the classifier correctly classified 10 instance out of 11 when KNN k-value is 3. By changing k to 5, the model loose it performance and detect 8 correctly out of 11 test instances.

## Chapter 5

In this section, I implement some of the resampling method on the car data from the textbook. Firstly, the validation set approach has been discussed.

### Validation Set Approach

The car dataset contains 205 instances. For this experiment, I take 195 for training and rest 10 for validation set. The random seed is here to remove the selection bias.

```
# seed 1
set.seed(1)
#attach(card)
train = sample(205, 195)
```

The train dataset consists of 195 instances randomly taken from the 205 instances. The model is trained using this 195 instances.

```
lm.fit = lm(price~ curbweight+carwidth + peakrpm+horsepower+
            +carlength+fueltype +carbody +enginesize+carheight,
            data = card, subset= train)

summary(lm.fit)
```

```
##
## Call:
## lm(formula = price ~ curbweight + carwidth + peakrpm + horsepower +
##      +carlength + fueltype + carbody + enginesize + carheight,
##      data = card, subset = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -8837.4 -1973.8    58.9  1463.0 14134.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -6.719e+04  1.391e+04  -4.832 2.87e-06 ***
## curbweight    2.845e+00  1.665e+00   1.708 0.089283 .
## carwidth      6.619e+02  2.379e+02   2.782 0.005972 **
## peakrpm       2.501e+00  6.851e-01   3.651 0.000341 ***
## horsepower    4.003e+01  1.403e+01   2.853 0.004833 **
```

```
## carlength      -5.428e+01  5.369e+01  -1.011  0.313434
## fueltypegas    -2.149e+03  9.956e+02  -2.158  0.032211 *
## carbodyhardtop -2.586e+03  1.790e+03  -1.445  0.150165
## carbodyhatchback -5.627e+03  1.417e+03  -3.972  0.000103 ***
## carbodysedan   -4.640e+03  1.464e+03  -3.170  0.001791 **
## carbodywagon   -6.559e+03  1.631e+03  -4.022  8.46e-05 ***
## enginesize      8.996e+01  1.374e+01   6.546  5.85e-10 ***
## carheight      3.263e+02  1.418e+02   2.301  0.022537 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3183 on 182 degrees of freedom
## Multiple R-squared:  0.8559, Adjusted R-squared:  0.8464
## F-statistic: 90.07 on 12 and 182 DF,  p-value: < 2.2e-16
```

```
mean((card$price-predict(lm.fit,card))[-train]^2)
```

```
## [1] 3626699
```

From above, we can see that the carbody, enginesize and incept has the smallest p-value. The F-statistic value is higher than 1. And R-squared value is close to one, meaning covering the data variance by the features.

We get the error of 3626699 for training a linear model. We will compare polynomial of two and three model for the same features on the same test dataset.

In the next section, I will use quadratic regression to experiment with the validation set approach. I will implement different model and check their performance based on the validation set result.

```
# preparing quadratic regression
lm.fit2 = lm(price~poly(curbweight,2)+poly(carwidth,2)+peakrpm+horsepower+
             +carlength+fueltype +carbody +enginesize+carheight,
             data = card, subset= train)

summary(lm.fit2)
```

```
##
## Call:
## lm(formula = price ~ poly(curbweight, 2) + poly(carwidth, 2) +
##     peakrpm + horsepower + +carlength + fueltype + carbody +
##     enginesize + carheight, data = card, subset = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -7388.6 -1605.6  -146.8   1244.0 14254.8
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -2.562e+04  1.108e+04  -2.312  0.021926 *
## poly(curbweight, 2)1  1.834e+04  1.177e+04   1.558  0.120889
## poly(curbweight, 2)2  9.579e+03  4.704e+03   2.036  0.043190 *
## poly(carwidth, 2)1   1.181e+04  7.100e+03   1.663  0.098084 .
## poly(carwidth, 2)2   1.110e+04  4.043e+03   2.746  0.006640 **
## peakrpm         1.990e+00  6.554e-01   3.036  0.002750 **
```

```
## horsepower          6.421e+01  1.471e+01   4.365 2.14e-05 ***
## carlength           5.409e+01  5.571e+01   0.971 0.332938
## fueltypepegas      -2.496e+03  9.424e+02  -2.649 0.008798 **
## carbodyhardtop     -2.833e+03  1.691e+03  -1.675 0.095634 .
## carbodyhatchback   -6.143e+03  1.370e+03  -4.484 1.30e-05 ***
## carbodysedan       -5.378e+03  1.419e+03  -3.790 0.000205 ***
## carbodywagon       -7.345e+03  1.575e+03  -4.663 6.04e-06 ***
## enginesize          5.748e+01  1.532e+01   3.753 0.000236 ***
## carheight          2.445e+02  1.355e+02   1.804 0.072903 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3004 on 180 degrees of freedom
## Multiple R-squared:  0.873, Adjusted R-squared:  0.8631
## F-statistic: 88.39 on 14 and 180 DF,  p-value: < 2.2e-16
```

```
# Prediction with rest
mean((card$price-predict(lm.fit2,card))[-train]^2)
```

```
## [1] 4236347
```

After training order 2 polynomials the prediction error in the model are 4236347 in the test dataset.

```
# preparing cubic regression
lm.fit3 = lm(price~poly(curbweight,3)+poly(carwidth,3)+peakrpm+horsepower+
              +carlength+fueltype +carbody +enginesize+carheight,
              data = card, subset= train)

mean((card$price-predict(lm.fit3,card))[-train]^2)
```

```
## [1] 4489315
```

By using the, the third order polynomial on curbwidth, we see the new errors are 4489315, slightly higher than the quadratic polynomial (4236347) on the first two features and also smaller than the linear regression model whose error was 3626699 To summarize the result, the linear worked best on the linear regression model.

By changing seed and re-evaluating the same model we can expect a slight different result. The seed changes the 10 test data samples randomly.

```
set.seed(4)
#attach(card)

train = sample(205, 195)

lm.fit = lm(price~ curbweight+carwidth+peakrpm+horsepower+
              +carlength+fueltype +carbody +enginesize+carheight,
              data = card, subset= train)

summary(lm.fit)
```

```
##
```



```
## Call:
## lm(formula = price ~ curbweight + carwidth + peakrpm + horsepower +
##      +carlength + fueltype + carbody + enginesize + carheight,
##      data = card, subset = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -8932.5 -1892.7    -8.5   1435.4  14157.1
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -6.413e+04  1.399e+04  -4.584 8.46e-06 ***
## curbweight     3.052e+00  1.677e+00   1.819 0.070499 .
## carwidth       6.598e+02  2.393e+02   2.757 0.006419 **
## peakrpm        2.183e+00  6.559e-01   3.328 0.001059 **
## horsepower     4.318e+01  1.403e+01   3.077 0.002414 **
## carlength     -6.422e+01  5.328e+01  -1.205 0.229645
## fueltypegas   -2.006e+03  9.846e+02  -2.037 0.043115 *
## carbodyhardtop -2.510e+03  1.790e+03  -1.402 0.162724
## carbodyhatchback -5.542e+03  1.421e+03  -3.901 0.000135 ***
## carbodysedan   -4.450e+03  1.462e+03  -3.043 0.002692 **
## carbodywagon   -6.446e+03  1.640e+03  -3.931 0.000120 ***
## enginesize      8.675e+01  1.386e+01   6.260 2.69e-09 ***
## carheight      3.221e+02  1.394e+02   2.310 0.021985 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3185 on 182 degrees of freedom
## Multiple R-squared:  0.8568, Adjusted R-squared:  0.8473
## F-statistic: 90.73 on 12 and 182 DF,  p-value: < 2.2e-16
```

```
mean((card$price-predict(lm.fit,card))[-train]^2)
```

```
## [1] 3290316
```

From the newdata sampling, the new error on the linear regression model is 3290316, slightly smaller than earlier experiment with seed 1. Next we train the quadratic model.

```
# preparing quadratic regression
lm.fit2 = lm(price~poly(curbweight,2)+poly(carwidth,2)+peakrpm+horsepower+
              +carlength+fueltype +carbody +enginesize+carheight,
              data = card, subset= train)

# Prediction with rest
mean((card$price-predict(lm.fit2,card))[-train]^2)
```

```
## [1] 2497317
```

In the quadratic model the new error on test set become the error is 2497317, almost half of the earlier example. In this case it seems quadratic model is better than linear model.

```
# preparing cubic regression
lm.fit3 = lm(price~poly(curbweight,3)+poly(carwidth,3)+peakrpm+horsepower+
             +carlength+fueltype +carbody +enginesize+carheight,
             data = card, subset= train)
summary(lm.fit3)
```

```
##
## Call:
## lm(formula = price ~ poly(curbweight, 3) + poly(carwidth, 3) +
##     peakrpm + horsepower + +carlength + fueltype + carbody +
##     enginesize + carheight, data = card, subset = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -7676.9 -1586.7   -92.4   1164.9 14391.7
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -2.234e+04  1.118e+04  -1.998  0.047231 *
## poly(curbweight, 3)1  2.475e+04  1.256e+04   1.971  0.050292 .
## poly(curbweight, 3)2  9.458e+03  4.793e+03   1.973  0.050002 .
## poly(curbweight, 3)3 -1.365e+03  3.953e+03  -0.345  0.730326
## poly(carwidth, 3)1   8.686e+03  7.848e+03   1.107  0.269878
## poly(carwidth, 3)2   1.129e+04  4.126e+03   2.735  0.006870 **
## poly(carwidth, 3)3   4.102e+03  3.926e+03   1.045  0.297496
## peakrpm          1.581e+00  6.375e-01   2.480  0.014055 *
## horsepower        6.601e+01  1.513e+01   4.364  2.16e-05 ***
## carlength         4.430e+01  5.711e+01   0.776  0.438974
## fueltypegas       -2.230e+03  9.371e+02  -2.379  0.018413 *
## carbodyhardtop    -2.705e+03  1.703e+03  -1.588  0.114105
## carbodyhatchback  -5.831e+03  1.388e+03  -4.200  4.20e-05 ***
## carbodysedan      -5.007e+03  1.430e+03  -3.501  0.000587 ***
## carbodywagon      -7.349e+03  1.598e+03  -4.598  8.05e-06 ***
## enginesize         5.214e+01  1.621e+01   3.216  0.001546 **
## carheight         2.544e+02  1.381e+02   1.842  0.067082 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3025 on 178 degrees of freedom
## Multiple R-squared:  0.8737, Adjusted R-squared:  0.8623
## F-statistic: 76.93 on 16 and 178 DF,  p-value: < 2.2e-16
```

```
mean((card$price-predict(lm.fit3,card))[-train]^2)
```

```
## [1] 2714493
```

In this case, the quadratic model gets the error of 2714493, a little higher than quadratic model. In this test case, the quadratic model performed best.

## Leave one-out-cross validation

In this resampling method, we put one training instance as the test example to check the model performances.

```
# used all continuous value predictor
```

```
glm.fit = glm(price~ curbweight+carwidth+peakrpm+horsepower+
               +carlength+fueltype +carbody +enginesize+carheight ,
               data = card)
coef(glm.fit)
```

```
##      (Intercept)      curbweight      carwidth      peakrpm
## -65287.278532      2.950661      660.153937      2.257717
##      horsepower      carlength      fueltypegas      carbodyhardtop
##      41.718643      -62.625543      -2012.332996      -2546.677919
## carbodyhatchback      carbodysedan      carbodywagon      enginesize
##      -5555.412413      -4573.041023      -6553.308168      88.441393
##      carheight
##      335.571161
```

```
lm.fit = lm(price~ curbweight+carwidth+peakrpm+horsepower+
               +carlength+fueltype +carbody +enginesize+carheight ,
               data = card)
coef(lm.fit)
```

```
##      (Intercept)      curbweight      carwidth      peakrpm
## -65287.278532      2.950661      660.153937      2.257717
##      horsepower      carlength      fueltypegas      carbodyhardtop
##      41.718643      -62.625543      -2012.332996      -2546.677919
## carbodyhatchback      carbodysedan      carbodywagon      enginesize
##      -5555.412413      -4573.041023      -6553.308168      88.441393
##      carheight
##      335.571161
```

The glm and lm provided the same result, which is evident by previous result of th coefficient values

```
#Library
```

```
library(boot)
set.seed(1)
glm.fit= glm(price~ curbweight+carwidth+peakrpm+horsepower+
               +carlength+fueltype +carbody +enginesize+carheight ,
               data = card)

cv.err = cv.glm(card, glm.fit)
cv.err$delta
```

```
## [1] 11009074 11004207
```

We get the error value of 11M.

```
# Polynomial
```

```
cv.error = rep(0,5)

for (i in 1:5){
  glm.fit = glm(price~ poly(curbweight, i)+carwidth+peakrpm+horsepower+
```

```

      +carlength+fueltype +carbody +poly(engine size,i)
      +carheight , data = card)
  cv.error[i] = cv.glm(card, glm.fit)$delta[1]
}
cv.error

```

```
## [1] 11009074 11145974 8861496 8826774 12322018
```

Now, applying polynomials upto 5, we see that the average error decreases in the third and forth polynomial about 8M on the test left data instance.

## k fold cross-validation

In k-fold we partition data in k sections and use k-1 as the training instance and test on the rest partition.

```

set.seed(17)

cv.error.10 = rep(0,10)

for (i in 1:10){
  glm.fit = glm(price~ poly(curbweight,i)+carwidth
    +peakrpm+horsepower+carlength+fueltype +carbody
    +poly(engine size,i)+carheight , data = card)
  cv.error.10[i] = cv.glm(card, glm.fit, K = 10)$delta[1]
}
cv.error.10

```

```
## [1] 10947388 11347044 9216694 9141835 12369026 18336440
## [7] 1819380000 2173132692 17382271800 2972794081
```

In the previous result, we see error decreases initially with the model degree and again rises showing overfit and huge training error on test dataset. The high polynomial model suffers from high variance problem.

## Bootstrap

To implement bootstrap we will use boot function.

```

alpha.fn=function(data,index){
  X=data$carlength[index]
  Y=data$price[index]
  return((var(Y)-cov(X,Y))/(var(X)+var(Y)-2*cov(X,Y)))
}

alpha.fn(card,1:100)

```

```
## [1] 1.001215
```

This provide the alpha value of 1.001215, now we select seed to recompute the alpha value for the car dataset. We take 100 samples in consideration.

```
set.seed(1)
alpha.fn(card,sample(100,100,replace=T))
```

```
## [1] 1.001173
```

Recomputing we get the value of 1.001173, very similar to the earlier sampling dataset.

```
boot(card,alpha.fn,R=1000)
```

```
##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = card, statistic = alpha.fn, R = 1000)
##
##
## Bootstrap Statistics :
##      original      bias      std. error
## t1* 1.001054 8.410011e-06 0.0001056349
```

The previous result shows the statistical distribution of alpha. The original value is about 1, with very low bias and standard deviation.

We will use the bootstrap model to analyse the performance of the linear model fit using the car dataset.

```
boot.fn=function(data,index)
  return(coef(lm(price~ curbweight+carwidth+peakrpm+horsepower+
    +carlength+fueltype +carbody +enginesize+carheight
    ,data=card,subset=index)))

boot.fn(card, 1:203)
```

```
##      (Intercept)      curbweight      carwidth      peakrpm
##      -63530.972604      2.747903      638.131288      2.175832
##      horsepower      carlength      fueltypegas      carbodyhardtop
##      43.344359      -60.676505      -1972.137018      -2573.963804
## carbodyhatchback      carbodysedan      carbodywagon      enginesize
##      -5550.370868      -4605.758455      -6477.290774      89.381308
##      carheight
##      334.490382
```

```
set.seed(1)

boot.fn(card,sample(205,205,replace=T))
```

```
##      (Intercept)      curbweight      carwidth      peakrpm
##      -5.437980e+04      1.323636e+00      5.404937e+02      9.925484e-01
##      horsepower      carlength      fueltypegas      carbodyhardtop
##      3.918690e+01      -3.556285e+01      -1.498810e+03      -3.917827e+03
## carbodyhatchback      carbodysedan      carbodywagon      enginesize
##      -9.299400e+03      -8.348023e+03      -1.171707e+04      1.031566e+02
##      carheight
##      4.167942e+02
```

```
boot.fn(card,sample(205, 205,replace=T))
```

```
##      (Intercept)      curbweight      carwidth      peakrpm
##      -73593.750256      2.725820      978.588449      1.279732
##      horsepower      carlength      fueltypegas      carbodyhardtop
##      40.618748      -84.377984      -917.918748      -716.341183
## carbodyhatchback      carbodysedan      carbodywagon      enginesize
##      -5859.570439      -4577.366174      -6303.931150      81.165508
##      carheight
##      272.748726
```

```
boot(Auto,boot.fn,100)
```

```
##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
## Call:
## boot(data = Auto, statistic = boot.fn, R = 100)
##
##
## Bootstrap Statistics :
##      original      bias      std. error
## t1*  -65287.278532 1218.37610669 1.307132e+04
## t2*      2.950661   0.08560465 1.340050e+00
## t3*   660.153937  14.77530690 2.554868e+02
## t4*    2.257717  -0.15537835 6.575903e-01
## t5*   41.718643  -1.07212993 1.271143e+01
## t6*   -62.625543  -1.60625643 4.727537e+01
## t7*  -2012.332996 195.83349433 9.875831e+02
## t8*  -2546.677919 -47.19188736 3.346254e+03
## t9*  -5555.412413  31.13727022 2.002547e+03
## t10* -4573.041023 145.08996152 2.051623e+03
## t11* -6553.308168  62.69901666 2.184819e+03
## t12*   88.441393  -2.60482640 1.930561e+01
## t13*  335.571161 -22.22436521 1.274739e+02
```

Here the above result gives the features value and their bias variances. We can see that carbody has the highest standard deviation of 3346.

```
summary(lm(price~carlength,data=card))$coef
```

```
##      Estimate Std. Error  t value    Pr(>|t|)
## (Intercept) -63690.6716  5792.7934 -10.99481 2.313319e-22
## carlength    442.2161    33.1996  13.31992 1.678707e-29
```

```
summary(lm(price~carlength+carwidth+peakrpm,data=card))$coef
```

```
##      Estimate Std. Error  t value    Pr(>|t|)
## (Intercept) -1.702890e+05 1.442212e+04 -11.807488 8.804821e-25
## carlength    1.219443e+02 5.487483e+01  2.222226 2.738293e-02
## carwidth     2.324874e+03 3.098843e+02  7.502395 1.980929e-12
## peakrpm      1.778318e+00 7.869361e-01  2.259799 2.490620e-02
```

```
boot.fn=function(data,index)
  coefficients(lm(price~carwidth+I(carwidth^2),data=card,subset=index))
set.seed(1)
boot(card,boot.fn,1000)
```

```
##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = card, statistic = boot.fn, R = 1000)
##
##
## Bootstrap Statistics :
##      original      bias      std. error
## t1* 671322.7695 -15668.054071 284011.07836
## t2* -22459.6189   471.898286   8633.88946
## t3*   189.0844   -3.549942    65.57634
```

```
summary(lm(price~carwidth+I(carwidth^2),data=card))$coef
```

```
##              Estimate Std. Error  t value    Pr(>|t|)
## (Intercept)  671322.7695 254902.47186   2.633646 0.009101236
## carwidth    -22459.6189   7628.20546  -2.944286 0.003616364
## I(carwidth^2)    189.0844    57.02578   3.315771 0.001083457
```

```
boot.fn=function(data,index)
  coefficients(lm(price~carlength+I(carlength^2),data=card,subset=index))
set.seed(1)
boot(card,boot.fn,1000)
```

```
##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = card, statistic = boot.fn, R = 1000)
##
##
## Bootstrap Statistics :
##      original      bias      std. error
## t1* 185969.293744 -1.952034e+03 60313.037599
## t2* -2425.276416  2.422859e+01  719.556222
## t3*    8.192746 -7.496051e-02   2.136189
```

```
summary(lm(price~carlength+I(carlength^2),data=card))$coef
```

```
##              Estimate Std. Error  t value    Pr(>|t|)
## (Intercept)  185969.293744 57979.22038   3.207516 1.556900e-03
## carlength    -2425.276416   663.62587  -3.654584 3.283737e-04
## I(carlength^2)    8.192746    1.89387   4.325929 2.386447e-05
```

In earlier result we see the comparison of applying the feature and the square of the features. The carlength both the parameter and square have similar bootstrapping error.

## Chapter 6

In this section, the model selection methods like best subset selection and dimensionality reduction techniques are applied on the car dataset.

### Best Subset Selection

Firstly we remove the missing data instances from the dataset.

```
### Lab 1 best subset selection
dim(card)
```

```
## [1] 205 27
```

```
card = na.omit(card)
dim(card)
```

```
## [1] 205 27
```

```
sum(is.na(card))
```

```
## [1] 0
```

From the sum result of 0 we know that there are no missing data points in the dataset instances.

```
## Choosing the best feature set by BIC, Cp , AIC ...
library(leaps)
attach(card)
```

```
## The following object is masked _by_ .GlobalEnv:
##
##     high
```

```
## The following objects are masked from card (pos = 7):
##
##     aspiration, boreratio, car_ID, carbody, carheight, carlength,
##     CarName, carwidth, citympg, compressionratio, curbweight,
##     cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##     enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
##     price, stroke, symboling, wheelbase
```

```
## The following objects are masked from card (pos = 8):
##
##     aspiration, boreratio, car_ID, carbody, carheight, carlength,
##     CarName, carwidth, citympg, compressionratio, curbweight,
##     cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##     enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
##     price, stroke, symboling, wheelbase
```



```
## The following objects are masked from card (pos = 10):
##
##   aspiration, boreratio, car_ID, carbody, carheight, carlength,
##   CarName, carwidth, citympg, compressionratio, curbweight,
##   cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##   enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
##   price, stroke, symboling, wheelbase

regfit.full = regsubsets(price~fuelsystem+peakrpm+citympg
                        + enginesize+enginetype+carwidth+curbweight+carlength
                        + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
                        + enginelocation+ aspiration+ doornumber
                        + horsepower+ compressionratio,
                        data = card)

summary(regfit.full)

## Subset selection object
## Call: regsubsets.formula(price ~ fuelsystem + peakrpm + citympg + enginesize +
##   enginetype + carwidth + curbweight + carlength + highwaympg +
##   boreratio + stroke + wheelbase + drivewheel + enginelocation +
##   aspiration + doornumber + horsepower + compressionratio,
##   data = card)
## 30 Variables (and intercept)
##               Forced in Forced out
## fuelsystem2bbl      FALSE      FALSE
## fuelsystem4bbl      FALSE      FALSE
## fuelsystemidi       FALSE      FALSE
## fuelsystemmfi       FALSE      FALSE
## fuelsystemmpfi      FALSE      FALSE
## fuelsystemspdi      FALSE      FALSE
## fuelsystemspfi      FALSE      FALSE
## peakrpm             FALSE      FALSE
## citympg             FALSE      FALSE
## enginesize          FALSE      FALSE
## enginetyopedohcv    FALSE      FALSE
## enginetyepel        FALSE      FALSE
## enginetyeohc        FALSE      FALSE
## enginetyeohcf       FALSE      FALSE
## enginetyeohcv       FALSE      FALSE
## enginetyeperotor    FALSE      FALSE
## carwidth            FALSE      FALSE
## curbweight          FALSE      FALSE
## carlength           FALSE      FALSE
## highwaympg          FALSE      FALSE
## boreratio           FALSE      FALSE
## stroke              FALSE      FALSE
## wheelbase           FALSE      FALSE
## drivewheel fwd      FALSE      FALSE
## drivewheel rwd      FALSE      FALSE
## enginelocation rear  FALSE      FALSE
## aspiration turbo     FALSE      FALSE
## doornumber two       FALSE      FALSE
## horsepower          FALSE      FALSE
```

```

## compressionratio      FALSE      FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
##      fuelsystem2bbl fuelsystem4bbl fuelsystemmidi fuelsystemmfi
## 1 ( 1 ) " "      " "      " "      " "
## 2 ( 1 ) " "      " "      " "      " "
## 3 ( 1 ) " "      " "      " "      " "
## 4 ( 1 ) " "      " "      " "      " "
## 5 ( 1 ) " "      " "      " "      " "
## 6 ( 1 ) " "      " "      " "      " "
## 7 ( 1 ) " "      " "      " "      " "
## 8 ( 1 ) " "      " "      " "      " "
##      fuelsystemmpfi fuelsystemspdi fuelsystemspfi peakrpm citympg
## 1 ( 1 ) " "      " "      " "      " "      " "
## 2 ( 1 ) " "      " "      " "      " "      " "
## 3 ( 1 ) " "      " "      " "      " "      " "
## 4 ( 1 ) " "      " "      " "      " "      " "
## 5 ( 1 ) " "      " "      " "      "*"      " "
## 6 ( 1 ) " "      " "      " "      " "      " "
## 7 ( 1 ) " "      " "      " "      " "      " "
## 8 ( 1 ) " "      " "      " "      " "      " "
##      enginesize enginetyperedohcv enginetyrel enginetypeohc enginetypeohcf
## 1 ( 1 ) "*"      " "      " "      " "      " "
## 2 ( 1 ) "*"      " "      " "      " "      " "
## 3 ( 1 ) "*"      " "      " "      " "      " "
## 4 ( 1 ) "*"      " "      " "      " "      " "
## 5 ( 1 ) "*"      " "      " "      " "      " "
## 6 ( 1 ) "*"      " "      " "      "*"      " "
## 7 ( 1 ) "*"      " "      " "      " "      "*"
## 8 ( 1 ) "*"      " "      " "      " "      " "
##      enginetypeohcv enginetyperotor carwidth curbweight carlength
## 1 ( 1 ) " "      " "      " "      " "      " "
## 2 ( 1 ) " "      " "      " "      " "      " "
## 3 ( 1 ) " "      " "      "*"      " "      " "
## 4 ( 1 ) " "      " "      "*"      " "      " "
## 5 ( 1 ) " "      " "      "*"      " "      " "
## 6 ( 1 ) " "      "*"      "*"      " "      " "
## 7 ( 1 ) "*"      "*"      "*"      " "      " "
## 8 ( 1 ) "*"      "*"      "*"      " "      " "
##      highwaympg boreratio stroke wheelbase drivewheel fwd drivewheelrwd
## 1 ( 1 ) " "      " "      " "      " "      " "      " "
## 2 ( 1 ) " "      " "      " "      " "      " "      "*"
## 3 ( 1 ) " "      " "      " "      " "      " "      " "
## 4 ( 1 ) " "      " "      " "      " "      "*"      " "
## 5 ( 1 ) " "      " "      " "      " "      "*"      " "
## 6 ( 1 ) " "      " "      "*"      " "      " "      " "
## 7 ( 1 ) " "      " "      "*"      " "      " "      " "
## 8 ( 1 ) " "      "*"      "*"      " "      "*"      " "
##      enginelocationrear aspirationturbo doornumbertwo horsepower
## 1 ( 1 ) " "      " "      " "      " "
## 2 ( 1 ) " "      " "      " "      " "
## 3 ( 1 ) "*"      " "      " "      " "
## 4 ( 1 ) "*"      " "      " "      " "
## 5 ( 1 ) "*"      " "      " "      " "

```

```
## 6 ( 1 ) "*" " " " "
## 7 ( 1 ) "*" " " " " "
## 8 ( 1 ) "*" " " " "
##      compressionratio
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
```

In above the model selected the best model based on the Residual sum of squared error. The \* locations in the model shows that the best model takes the feature of engine size and the second model considers the engine size and drivewheel position. This experiment showed top 8 models we can extend that by providing nvmax parameters as follows.

```
regfit.full = regsubsets(price~fuelsystem+peakrpm+citympg
                        + enginesize+engine+carwidth+curbweight+carlength
                        + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
                        + enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
                        data = card, nvmax = 19)

summary(regfit.full)
```

```
## Subset selection object
## Call: regsubsets.formula(price ~ fuelsystem + peakrpm + citympg + enginesize +
##      enginetype + carwidth + curbweight + carlength + highwaympg +
##      boreratio + stroke + wheelbase + drivewheel + enginelocation +
##      aspiration + doornumber + horsepower + compressionratio,
##      data = card, nvmax = 19)
## 30 Variables (and intercept)
##      Forced in Forced out
## fuelsystem2bbl      FALSE      FALSE
## fuelsystem4bbl      FALSE      FALSE
## fuelsystemidi        FALSE      FALSE
## fuelsystemmfi        FALSE      FALSE
## fuelsystemmpfi       FALSE      FALSE
## fuelsystemspdi       FALSE      FALSE
## fuelsystemspfi       FALSE      FALSE
## peakrpm              FALSE      FALSE
## citympg              FALSE      FALSE
## enginesize           FALSE      FALSE
## enginetyopedohcv     FALSE      FALSE
## enginetyepel         FALSE      FALSE
## enginetyeohc         FALSE      FALSE
## enginetyeohcf        FALSE      FALSE
## enginetyeohcv        FALSE      FALSE
## enginetyerotor       FALSE      FALSE
## carwidth             FALSE      FALSE
## curbweight           FALSE      FALSE
## carlength            FALSE      FALSE
```

```

## highwaympg          FALSE      FALSE
## boreratio           FALSE      FALSE
## stroke              FALSE      FALSE
## wheelbase           FALSE      FALSE
## drivewheel fwd      FALSE      FALSE
## drivewheel rwd      FALSE      FALSE
## enginelocation rear  FALSE      FALSE
## aspiration turbo     FALSE      FALSE
## doornumbertwo        FALSE      FALSE
## horsepower          FALSE      FALSE
## compressionratio     FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: exhaustive
##      fuelsystem2bbl fuelsystem4bbl fuelsystemidi fuelsystemmfi
## 1 ( 1 ) " " " " " "
## 2 ( 1 ) " " " " " "
## 3 ( 1 ) " " " " " "
## 4 ( 1 ) " " " " " "
## 5 ( 1 ) " " " " " "
## 6 ( 1 ) " " " " " "
## 7 ( 1 ) " " " " " "
## 8 ( 1 ) " " " " " "
## 9 ( 1 ) " " " " " "
## 10 ( 1 ) " " " " " "
## 11 ( 1 ) " " " " " "
## 12 ( 1 ) " " "*" " "
## 13 ( 1 ) " " "*" " "
## 14 ( 1 ) " " "*" " "
## 15 ( 1 ) " " "*" "*"
## 16 ( 1 ) " " "*" " "
## 17 ( 1 ) " " "*" " "
## 18 ( 1 ) " " "*" "*"
## 19 ( 1 ) " " "*" "*"
##      fuelsystemmpfi fuelsystemspdi fuelsystemspfi peakrpm citympg
## 1 ( 1 ) " " " " " " " "
## 2 ( 1 ) " " " " " " " "
## 3 ( 1 ) " " " " " " " "
## 4 ( 1 ) " " " " " " " "
## 5 ( 1 ) " " " " "*" " "
## 6 ( 1 ) " " " " " " " "
## 7 ( 1 ) " " " " " " " "
## 8 ( 1 ) " " " " " " " "
## 9 ( 1 ) " " " " " " " "
## 10 ( 1 ) " " " " "*" " "
## 11 ( 1 ) " " " " "*" " "
## 12 ( 1 ) " " " " "*" " "
## 13 ( 1 ) " " "*" " " "*" " "
## 14 ( 1 ) " " "*" " " "*" " "
## 15 ( 1 ) " " "*" " " "*" " "
## 16 ( 1 ) " " "*" " " "*" " "
## 17 ( 1 ) " " "*" " " "*" " "
## 18 ( 1 ) " " "*" " " "*" " "
## 19 ( 1 ) "*" "*" " " "*" " "
##      enginesize enginetype dohc v enginetype l enginetype ohc enginetype ohcf

```

## 1	( 1 )	"*"	" "	" "	" "	" "
## 2	( 1 )	"*"	" "	" "	" "	" "
## 3	( 1 )	"*"	" "	" "	" "	" "
## 4	( 1 )	"*"	" "	" "	" "	" "
## 5	( 1 )	"*"	" "	" "	" "	" "
## 6	( 1 )	"*"	" "	" "	"*"	" "
## 7	( 1 )	"*"	" "	" "	" "	"*"
## 8	( 1 )	"*"	" "	" "	" "	" "
## 9	( 1 )	"*"	" "	" "	"*"	" "
## 10	( 1 )	"*"	" "	" "	"*"	" "
## 11	( 1 )	"*"	" "	" "	"*"	" "
## 12	( 1 )	"*"	" "	" "	"*"	" "
## 13	( 1 )	"*"	" "	" "	"*"	" "
## 14	( 1 )	"*"	" "	" "	"*"	" "
## 15	( 1 )	"*"	" "	" "	"*"	" "
## 16	( 1 )	"*"	" "	"*"	"*"	" "
## 17	( 1 )	"*"	" "	"*"	"*"	" "
## 18	( 1 )	"*"	" "	"*"	"*"	" "
## 19	( 1 )	"*"	" "	"*"	"*"	" "
##		enginetypeohcv	enginetyperotor	carwidth	curbweight	carlength
## 1	( 1 )	" "	" "	" "	" "	" "
## 2	( 1 )	" "	" "	" "	" "	" "
## 3	( 1 )	" "	" "	"*"	" "	" "
## 4	( 1 )	" "	" "	"*"	" "	" "
## 5	( 1 )	" "	" "	"*"	" "	" "
## 6	( 1 )	" "	"*"	"*"	" "	" "
## 7	( 1 )	"*"	"*"	"*"	" "	" "
## 8	( 1 )	"*"	"*"	"*"	" "	" "
## 9	( 1 )	"*"	"*"	"*"	" "	" "
## 10	( 1 )	"*"	"*"	"*"	" "	" "
## 11	( 1 )	"*"	"*"	"*"	" "	" "
## 12	( 1 )	"*"	"*"	"*"	" "	" "
## 13	( 1 )	"*"	"*"	"*"	" "	" "
## 14	( 1 )	"*"	"*"	"*"	" "	" "
## 15	( 1 )	"*"	"*"	"*"	" "	" "
## 16	( 1 )	"*"	"*"	"*"	"*"	" "
## 17	( 1 )	"*"	"*"	"*"	"*"	" "
## 18	( 1 )	"*"	"*"	"*"	"*"	" "
## 19	( 1 )	"*"	"*"	"*"	"*"	" "
##		highwaympg	boreratio	stroke	wheelbase	drivewheelfwd drivewheelrwd
## 1	( 1 )	" "	" "	" "	" "	" "
## 2	( 1 )	" "	" "	" "	" "	"*"
## 3	( 1 )	" "	" "	" "	" "	" "
## 4	( 1 )	" "	" "	" "	" "	" "
## 5	( 1 )	" "	" "	" "	" "	" "
## 6	( 1 )	" "	" "	"*"	" "	" "
## 7	( 1 )	" "	" "	"*"	" "	" "
## 8	( 1 )	" "	"*"	"*"	" "	" "
## 9	( 1 )	" "	"*"	"*"	" "	" "
## 10	( 1 )	" "	"*"	"*"	" "	" "
## 11	( 1 )	" "	"*"	"*"	" "	" "
## 12	( 1 )	" "	"*"	"*"	" "	" "
## 13	( 1 )	" "	"*"	"*"	" "	"*"
## 14	( 1 )	" "	"*"	"*"	" "	"*"

```
## 15 ( 1 ) " "      "*"      "*"      " "      " "      "*"
## 16 ( 1 ) "*"      "*"      "*"      " "      " "      "*"
## 17 ( 1 ) "*"      "*"      "*"      " "      " "      "*"
## 18 ( 1 ) "*"      "*"      "*"      " "      " "      "*"
## 19 ( 1 ) "*"      "*"      "*"      " "      " "      "*"
##      enginelocationrear aspirationturbo doornumbertwo horsepower
## 1 ( 1 ) " "      " "      " "      " "
## 2 ( 1 ) " "      " "      " "      " "
## 3 ( 1 ) "*"      " "      " "      " "
## 4 ( 1 ) "*"      " "      " "      " "
## 5 ( 1 ) "*"      " "      " "      " "
## 6 ( 1 ) "*"      " "      " "      " "
## 7 ( 1 ) "*"      " "      " "      " "
## 8 ( 1 ) "*"      " "      " "      " "
## 9 ( 1 ) "*"      " "      " "      " "
## 10 ( 1 ) "*"      " "      " "      " "
## 11 ( 1 ) "*"      "*"      " "      " "
## 12 ( 1 ) "*"      " "      " "      " "
## 13 ( 1 ) "*"      " "      " "      " "
## 14 ( 1 ) "*"      "*"      " "      " "
## 15 ( 1 ) "*"      "*"      " "      " "
## 16 ( 1 ) "*"      " "      " "      " "
## 17 ( 1 ) "*"      "*"      " "      " "
## 18 ( 1 ) "*"      "*"      " "      " "
## 19 ( 1 ) "*"      "*"      " "      " "
##      compressionratio
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
## 9 ( 1 ) " "
## 10 ( 1 ) " "
## 11 ( 1 ) " "
## 12 ( 1 ) "*"
## 13 ( 1 ) "*"
## 14 ( 1 ) "*"
## 15 ( 1 ) "*"
## 16 ( 1 ) "*"
## 17 ( 1 ) "*"
## 18 ( 1 ) "*"
## 19 ( 1 ) "*"

```

```
reg.summary = summary(regfit.full)
names(reg.summary)
```

```
## [1] "which" "rsq" "rss" "adjr2" "cp" "bic" "outmat" "obj"
```

```
reg.summary$rsq
```

```
## [1] 0.7641291 0.7948774 0.8405232 0.8567114 0.8654310 0.8695393 0.8781805
## [8] 0.8849685 0.8902002 0.8946820 0.9000161 0.9030713 0.9062538 0.9080701
## [15] 0.9095691 0.9105845 0.9119274 0.9132864 0.9137280
```

Now the result shows top 17 models with the r squared values of the model with different top features.

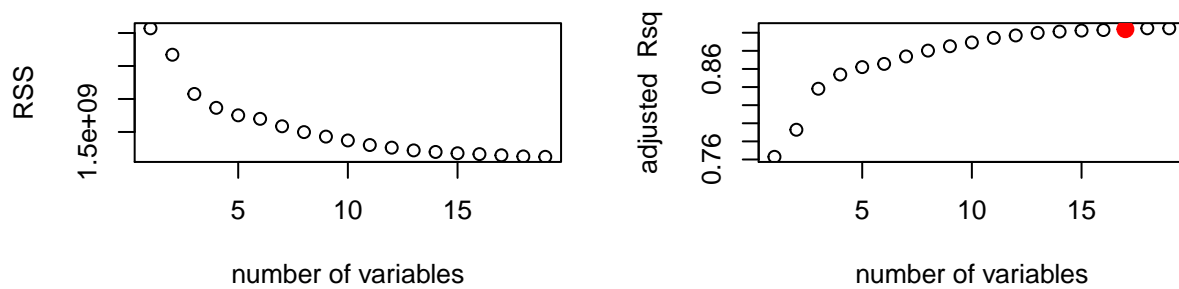
```
par(mfrow = c(2,2))

plot(reg.summary$rss, xlab= "number of variables", ylab = "RSS")

plot(reg.summary$adjr2, xlab= "number of variables", ylab = "adjusted Rsq")
which.max(reg.summary$adjr2) # return 17
```

```
## [1] 18
```

```
points(17, reg.summary$adjr2[17], col ="red", cex = 2, pch =20)
```



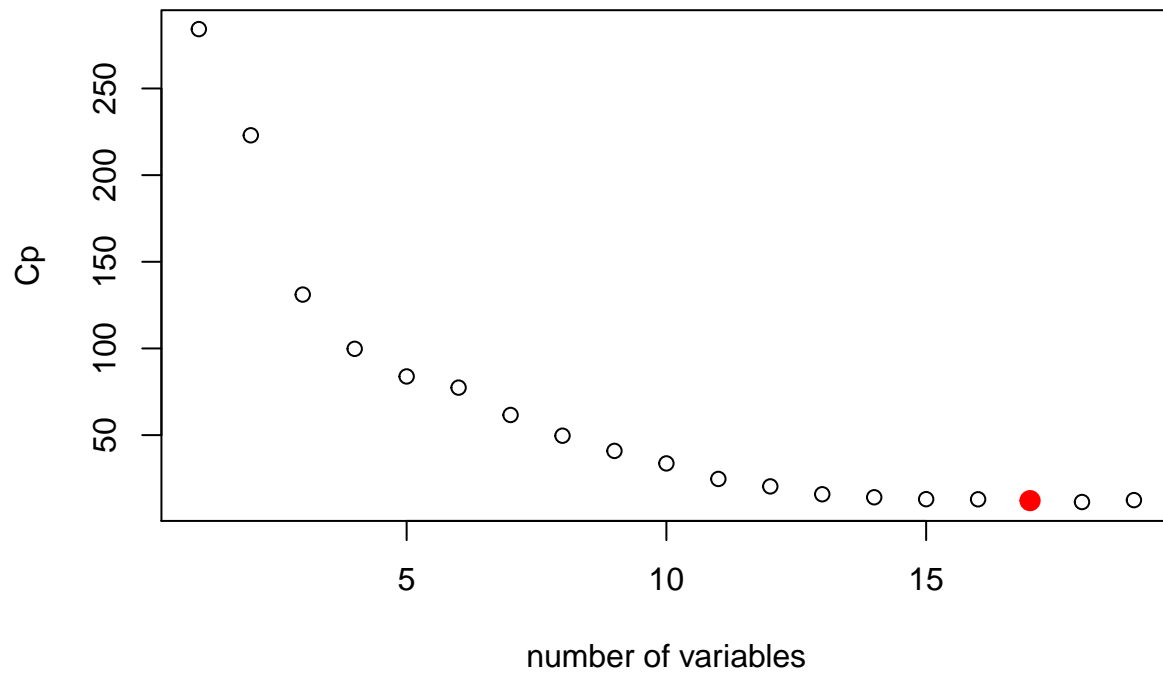
The plot shows the model error decrease with increasing variable numbers.

The previous value returns 17. We will use this to plot the cp and BIC statistics.

```
plot(reg.summary$cp, xlab= "number of variables", ylab = "Cp")
which.min(reg.summary$cp) #18
```

```
## [1] 18
```

```
points(17, reg.summary$cp[17], col = "red", cex = 2, pch = 20)
```



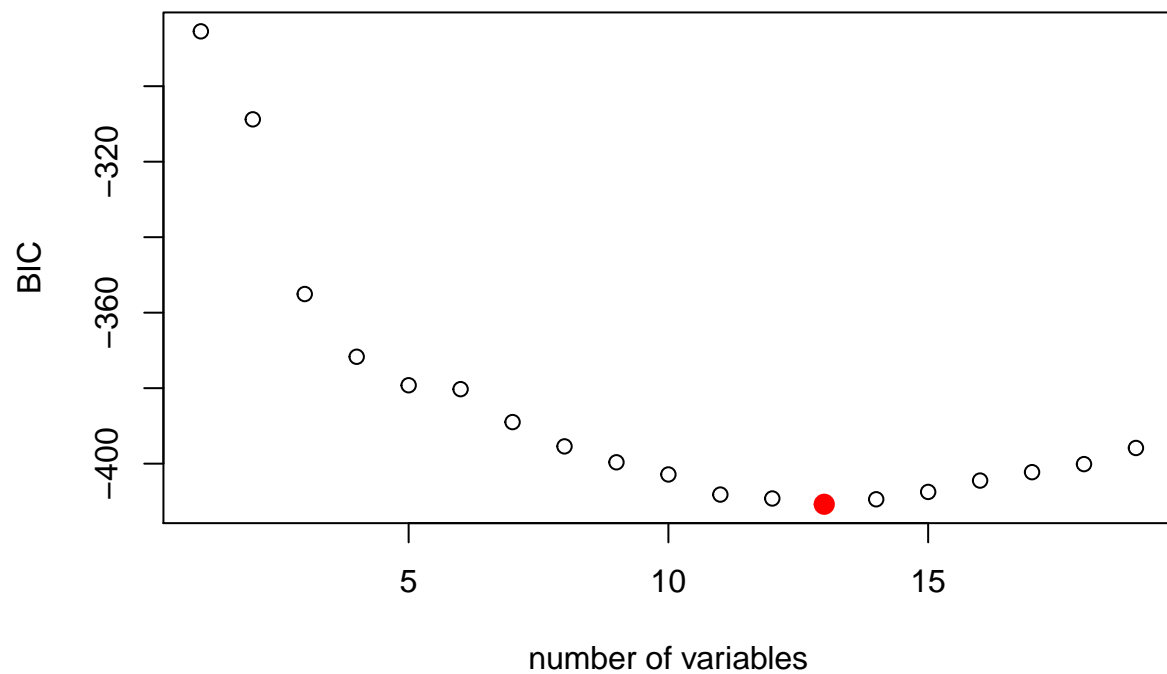
```
which.min(reg.summary$bic) #13
```

```
## [1] 13
```

```
plot(reg.summary$bic, xlab= "number of variables", ylab = "BIC")
```

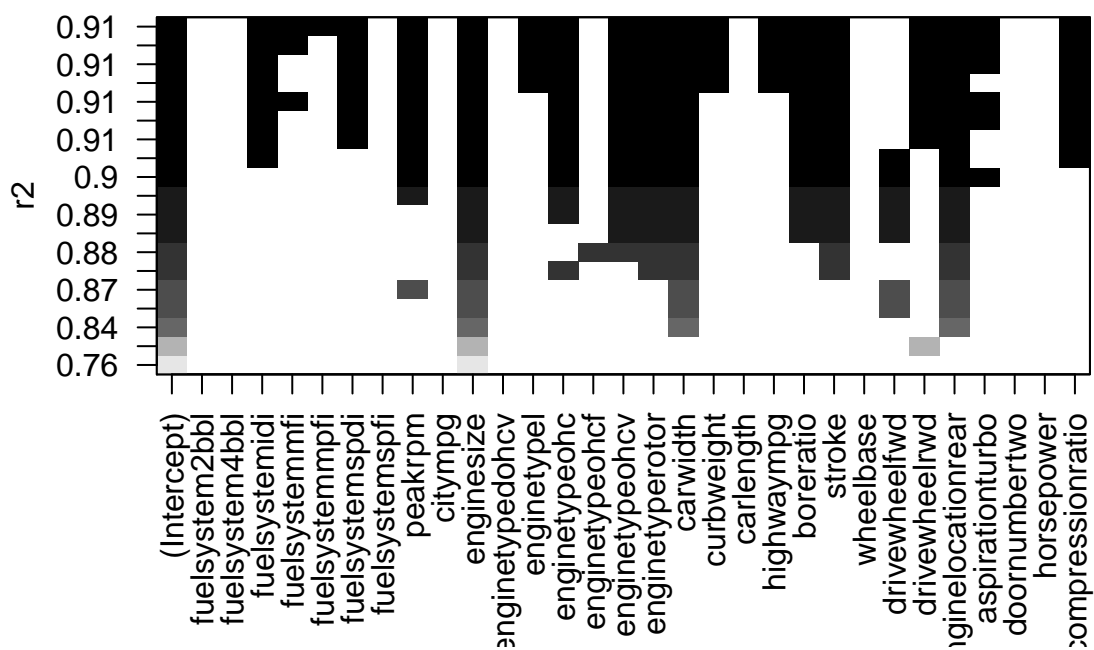
```
points(13, reg.summary$bic[13], col = "red", cex = 2, pch = 20)
```



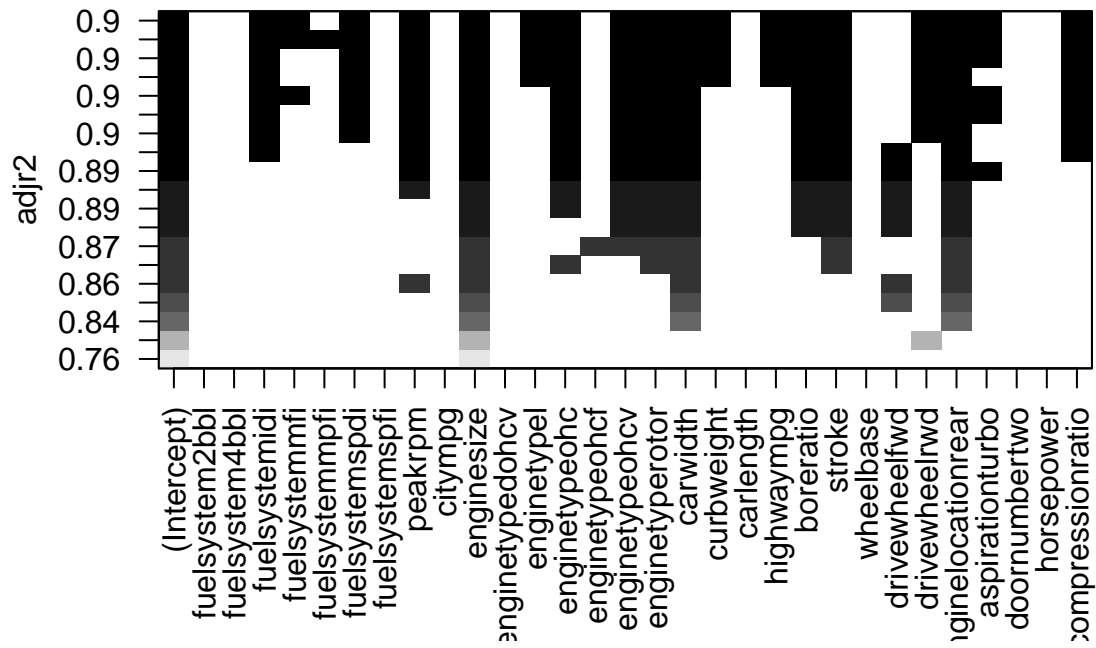


The Cp value decrease with new features, now we do the same for the BIC criterion for too in the car dataset. We observe from the above model that BIC selected the 13 feature model as the best candidate while Cp selected 17 feature/predictor variables models.

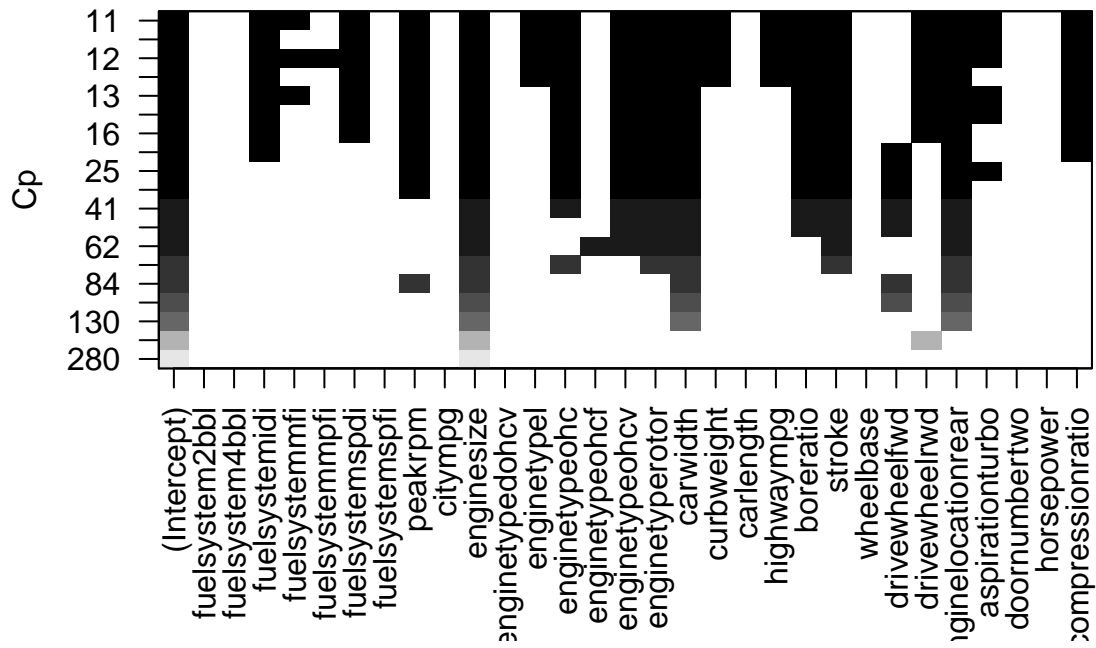
```
plot(regfit.full, scale = "r2")
```



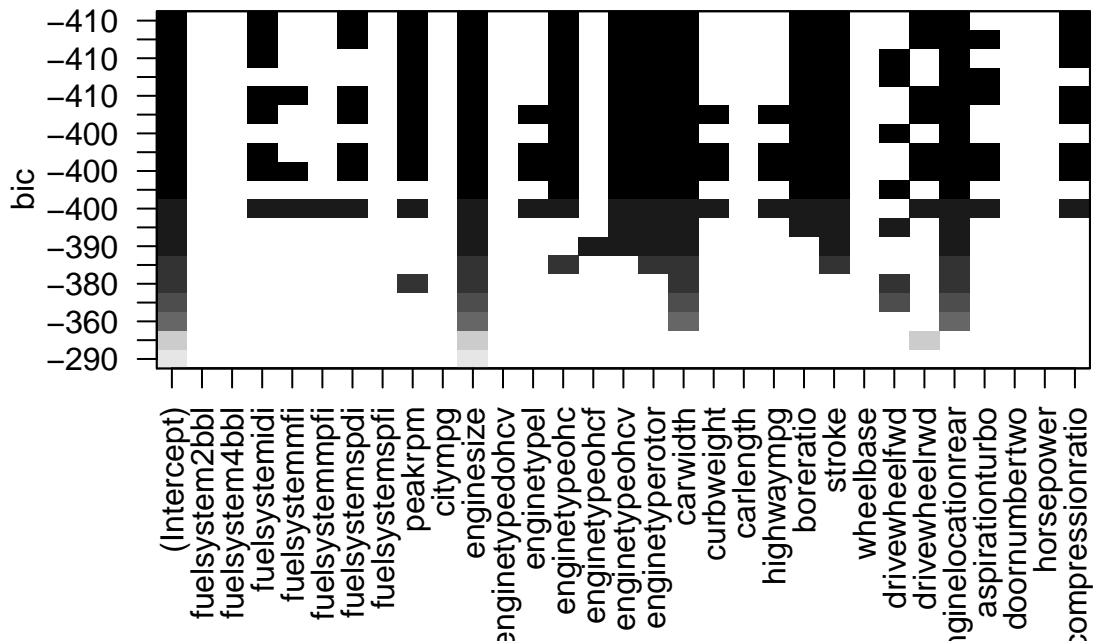
```
plot(regfit.full, scale = "adjr2")
```



```
plot(regfit.full, scale = "Cp")
```



```
plot(regfit.full, scale = "bic")
```



```
coef(regfit.full, 7)
```

```
##      (Intercept)      enginesize      enginetypeohcf      enginetypeohcv
##      -57712.5552      148.4221      -3804.7916      -4852.3248
##      enginetypeotor      carwidth      stroke      enginelocationrear
##      7800.8444      1007.4850      -4307.4211      14256.4615
```

Here the above plot show different model with different criterion BIC, Cp as they select different feature set.

## Foward and Backward stepwise selection

We use the parameter method to select backward or forward selection

In this method, the model start with smallest variables and add new variable in the next iteration.

```
regfit.fwd = regsubsets(price~fuelsystem+peakrpm+citympg
+ enginesize+enginetype+carwidth+curbweight+carlength
+ highwaympg+ boreratio+ stroke + wheelbase + drivewheel
+ enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
data= card, nvmax =8, method = "forward")

summary(regfit.fwd)
```

```
## Subset selection object
```

```

## Call: regsubsets.formula(price ~ fuelsystem + peakrpm + citympg + enginesize +
##     enginetype + carwidth + curbweight + carlength + highwaympg +
##     boreratio + stroke + wheelbase + drivewheel + enginelocation +
##     aspiration + doornumber + horsepower + compressionratio,
##     data = card, nvmax = 8, method = "forward")
## 30 Variables (and intercept)
##
##           Forced in Forced out
## fuelsystem2bbl      FALSE      FALSE
## fuelsystem4bbl      FALSE      FALSE
## fuelsystemmidi      FALSE      FALSE
## fuelsystemmfi       FALSE      FALSE
## fuelsystemmpfi      FALSE      FALSE
## fuelsystemspdi      FALSE      FALSE
## fuelsystemspfi      FALSE      FALSE
## peakrpm             FALSE      FALSE
## citympg             FALSE      FALSE
## enginesize          FALSE      FALSE
## enginetyopedohcv    FALSE      FALSE
## enginetyepel        FALSE      FALSE
## enginetyeohc        FALSE      FALSE
## enginetyeohcf       FALSE      FALSE
## enginetyeohcv       FALSE      FALSE
## enginetyerotor      FALSE      FALSE
## carwidth            FALSE      FALSE
## curbweight          FALSE      FALSE
## carlength           FALSE      FALSE
## highwaympg          FALSE      FALSE
## boreratio           FALSE      FALSE
## stroke              FALSE      FALSE
## wheelbase           FALSE      FALSE
## drivewheel fwd      FALSE      FALSE
## drivewheel rwd      FALSE      FALSE
## enginelocationrear  FALSE      FALSE
## aspirationturbo      FALSE      FALSE
## doornumbertwo        FALSE      FALSE
## horsepower          FALSE      FALSE
## compressionratio    FALSE      FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: forward
##           fuelsystem2bbl fuelsystem4bbl fuelsystemmidi fuelsystemmfi
## 1  ( 1 ) " "           " "             " "             " "
## 2  ( 1 ) " "           " "             " "             " "
## 3  ( 1 ) " "           " "             " "             " "
## 4  ( 1 ) " "           " "             " "             " "
## 5  ( 1 ) " "           " "             " "             " "
## 6  ( 1 ) " "           " "             " "             " "
## 7  ( 1 ) " "           " "             " "             " "
## 8  ( 1 ) " "           " "             " "             " "
##           fuelsystemmpfi fuelsystemspdi fuelsystemspfi peakrpm citympg
## 1  ( 1 ) " "           " "             " "             " "      " "
## 2  ( 1 ) " "           " "             " "             " "      " "
## 3  ( 1 ) " "           " "             " "             " "      " "
## 4  ( 1 ) " "           " "             " "             " "      " "
## 5  ( 1 ) " "           " "             " "             " *      " "

```

```

## 6 ( 1 ) " " " " " " "*" " "
## 7 ( 1 ) " " " " " " "*" " "
## 8 ( 1 ) " " " " " " "*" " "
##      enginesize enginetyperedohcv enginetyrel enginetypeohc enginetypeohcf
## 1 ( 1 ) "*" " " " " " " " "
## 2 ( 1 ) "*" " " " " " " " "
## 3 ( 1 ) "*" " " " " " " " "
## 4 ( 1 ) "*" " " " " " " " "
## 5 ( 1 ) "*" " " " " " " " "
## 6 ( 1 ) "*" " " " " " " " "
## 7 ( 1 ) "*" " " " " " " " "
## 8 ( 1 ) "*" " " " " " " " "
##      enginetypeohcv enginetyperotor carwidth curbweight carlength
## 1 ( 1 ) " " " " " " " " " "
## 2 ( 1 ) " " " " " " " " " "
## 3 ( 1 ) " " " " " " " " " "
## 4 ( 1 ) " " " " "*" " " " " "
## 5 ( 1 ) " " " " "*" " " " " "
## 6 ( 1 ) "*" " " "*" " " " " "
## 7 ( 1 ) "*" " " "*" " " " " "
## 8 ( 1 ) "*" " " "*" " " " " "
##      highwaympg boreratio stroke wheelbase drivewheel fwd drivewheelrwd
## 1 ( 1 ) " " " " " " " " " "
## 2 ( 1 ) " " " " " " " " "*"
## 3 ( 1 ) " " " " " " " " "*"
## 4 ( 1 ) " " " " " " " " "*"
## 5 ( 1 ) " " " " " " " " "*"
## 6 ( 1 ) " " " " " " " " "*"
## 7 ( 1 ) " " " " "*" " " " "*"
## 8 ( 1 ) " " "*" "*" " " " " "*"
##      enginelocationrear aspirationturbo doornumbertwo horsepower
## 1 ( 1 ) " " " " " " " "
## 2 ( 1 ) " " " " " " " "
## 3 ( 1 ) "*" " " " " " "
## 4 ( 1 ) "*" " " " " " "
## 5 ( 1 ) "*" " " " " " "
## 6 ( 1 ) "*" " " " " " "
## 7 ( 1 ) "*" " " " " " "
## 8 ( 1 ) "*" " " " " " "
##      compressionratio
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "

```

The forward selection method selects enginesize at first and then drivewheel and never drops the features. While in earlier method, in subset selection based on BIC/Cp some features were dropped later on. The backward traces in the reverse way it started with all the variables. In backward selection once the feature is dropped it is not recovered later.

We see the following result of backward in the following results

```
regfit.bwd = regsubsets(price~fuelsystem+peakrpm+citympg
                        + enginesize+engine+carwidth+curbweight+carlength
                        + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
                        + enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
                        data= card, nvmax =8, method = "backward")
```

```
summary(regfit.bwd)
```

```
## Subset selection object
## Call: regsubsets.formula(price ~ fuelsystem + peakrpm + citympg + enginesize +
##      enginetype + carwidth + curbweight + carlength + highwaympg +
##      boreratio + stroke + wheelbase + drivewheel + enginelocation +
##      aspiration + doornumber + horsepower + compressionratio,
##      data = card, nvmax = 8, method = "backward")
## 30 Variables (and intercept)
##              Forced in Forced out
## fuelsystem2bbl      FALSE      FALSE
## fuelsystem4bbl      FALSE      FALSE
## fuelsystemmidi      FALSE      FALSE
## fuelsystemmfi       FALSE      FALSE
## fuelsystemmpfi      FALSE      FALSE
## fuelsystemspdi      FALSE      FALSE
## fuelsystemspfi      FALSE      FALSE
## peakrpm             FALSE      FALSE
## citympg             FALSE      FALSE
## enginesize          FALSE      FALSE
## enginetyopedohcv    FALSE      FALSE
## enginetyepel        FALSE      FALSE
## enginetyeohc        FALSE      FALSE
## enginetyeohcf       FALSE      FALSE
## enginetyeohcv       FALSE      FALSE
## enginetyeperotor    FALSE      FALSE
## carwidth            FALSE      FALSE
## curbweight          FALSE      FALSE
## carlength           FALSE      FALSE
## highwaympg          FALSE      FALSE
## boreratio           FALSE      FALSE
## stroke              FALSE      FALSE
## wheelbase           FALSE      FALSE
## drivewheel fwd      FALSE      FALSE
## drivewheel rwd      FALSE      FALSE
## enginelocationrear  FALSE      FALSE
## aspirationturbo      FALSE      FALSE
## doornumbertwo        FALSE      FALSE
## horsepower          FALSE      FALSE
## compressionratio    FALSE      FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: backward
##              fuelsystem2bbl fuelsystem4bbl fuelsystemmidi fuelsystemmfi
## 1  ( 1 ) " "              " "              " "              " "
## 2  ( 1 ) " "              " "              " "              " "
## 3  ( 1 ) " "              " "              " "              " "
## 4  ( 1 ) " "              " "              " "              " "
```



```

## 5 ( 1 ) " " " " " " " "
## 6 ( 1 ) " " " " " " " "
## 7 ( 1 ) " " " " " " " "
## 8 ( 1 ) " " " " " " " "
##      fuelsystemmpfi fuelsystemspdi fuelsystemspfi peakrpm citympg
## 1 ( 1 ) " " " " " " " "
## 2 ( 1 ) " " " " " " " "
## 3 ( 1 ) " " " " " " " "
## 4 ( 1 ) " " " " " " " "
## 5 ( 1 ) " " " " " " " "
## 6 ( 1 ) " " " " " " " "
## 7 ( 1 ) " " " " " " "*" " "
## 8 ( 1 ) " " " " " " "*" " "
##      enginesize enginetyperedohcv enginetyrel enginetyrehc enginetyrehcf
## 1 ( 1 ) "*" " " " " " " " "
## 2 ( 1 ) "*" " " " " " " " "
## 3 ( 1 ) "*" " " " " " " " "
## 4 ( 1 ) "*" " " " " " " " "
## 5 ( 1 ) "*" " " " " " " " "
## 6 ( 1 ) "*" " " " " " " " "
## 7 ( 1 ) "*" " " " " " " " "
## 8 ( 1 ) "*" " " " " "*" " "
##      enginetyrehcv enginetyperotor carwidth curbweight carlength
## 1 ( 1 ) " " " " " " " "
## 2 ( 1 ) " " " " "*" " " "
## 3 ( 1 ) " " " " "*" " " "
## 4 ( 1 ) " " "*" "*" "*" " " "
## 5 ( 1 ) "*" "*" "*" " " "
## 6 ( 1 ) "*" "*" "*" " " "
## 7 ( 1 ) "*" "*" "*" " " "
## 8 ( 1 ) "*" "*" "*" " " "
##      highwaympg boreratio stroke wheelbase drivewheelfwd drivewheelrwd
## 1 ( 1 ) " " " " " " " "
## 2 ( 1 ) " " " " " " " "
## 3 ( 1 ) " " " " " " " "
## 4 ( 1 ) " " " " " " " "
## 5 ( 1 ) " " " " " " " "
## 6 ( 1 ) " " " " "*" " " "
## 7 ( 1 ) " " " " "*" " " "
## 8 ( 1 ) " " " " "*" " " "
##      enginelocationrear aspirationturbo doornumbertwo horsepower
## 1 ( 1 ) " " " " " " " "
## 2 ( 1 ) " " " " " " " "
## 3 ( 1 ) "*" " " " " " "
## 4 ( 1 ) "*" " " " " " "
## 5 ( 1 ) "*" " " " " " "
## 6 ( 1 ) "*" " " " " " "
## 7 ( 1 ) "*" " " " " " "
## 8 ( 1 ) "*" " " " " " "
##      compressionratio
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "

```

```
## 5 ( 1 ) " "
## 6 ( 1 ) " "
## 7 ( 1 ) " "
## 8 ( 1 ) " "
```

Here we see the difference in features for the backward model compared to forward model. For example the second best model for backward and forward are different as shown in above figure.

```
coef(regfit.full, 7)
```

```
##      (Intercept)      enginesize      enginetypeohcf      enginetypeohcv
##      -57712.5552      148.4221      -3804.7916      -4852.3248
##      enginetyperotor      carwidth      stroke      enginelocationrear
##      7800.8444      1007.4850      -4307.4211      14256.4615
```

```
coef(regfit.fwd, 7)
```

```
##      (Intercept)      peakrpm      enginesize      enginetypeohcv
##      -78543.314337      1.837289      127.381826      -3538.595514
##      carwidth      stroke      drivewheelrwd      enginelocationrear
##      1099.363038      -2110.755789      1962.714917      10219.114749
```

```
coef(regfit.bwd, 7)
```

```
##      (Intercept)      peakrpm      enginesize      enginetypeohcv
##      -73678.385495      1.555476      150.900289      -4741.677505
##      enginetyperotor      carwidth      stroke      enginelocationrear
##      6784.909984      1048.461782      -2839.813241      9839.862024
```

From above result we see that the coefficient and features are different three approaches of subset selection, forward and backward selection.

## Validation Approach

Firstly, we divide the car data instances in test and train set.

```
set.seed(1)

train = sample(c(TRUE, FALSE), nrow(card), rep= TRUE)

test = (!train)
```

Now we apply the subset selection method.

```
regfit.best = regsubsets(price~peakrpm+citympg
+ enginesize+enginetype+carwidth+ curbweight+carlength
+ highwaympg+ boreratio+ stroke + wheelbase + drivewheel
+ enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
data= card[train,], nvmax =19)
```

Now we test it on the separated set as we trained only using the training examples.

```
test.mat = model.matrix(price~peakrpm+citympg
                        + enginesize+engine+carwidth+ curbweight+carlength
                        + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
                        + enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio, data = card)

val.errors = rep(NA, 19)

for (i in 1:19){
  coefi = coef(regfit.best, id = i)
  pred = test.mat[, names(coefi)]%*%coefi
  val.errors[i] = mean((price[test]-pred)^2)
}
```

In previous we created test and cross validation set. I will use it to check model performance.

```
val.errors
```

```
## [1] 19984230 17389803 12842020 11551530 11345284 11966102 10394542 11589571
## [9] 11851994 11101306 10412482 13537134 12214163 13447630 12119045 11461321
## [17] 11548419 11466866 11931765
```

```
which.min(val.errors) # output 7
```

```
## [1] 7
```

```
coef(regfit.best, 7)
```

```
##      (Intercept)      enginesize      enginetypeohc      enginetyperotor
##      -63677.139       123.365       3133.558       6514.236
##      carwidth      stroke      drivewheelrwd      enginelocationrear
##      1074.321       -4063.746       2116.979       12109.372
```

In the output the model shows the 7 variables for the best model. Then we carry out our analysis by taking the best model.

```
predict.regsbsets = function(object, newdata, id, ...){
  form = as.formula(object$call[[2]])
  mat = model.matrix(form, newdata)
  coefi = coef(object, id =id)
  xvars = names(coefi)
  mat[, xvars]%*%coefi
}
```

The above function is an user defined prediction method.

```
regfit.best = regsubsets(price~peakrpm+citympg
                        + enginesize+engine+carwidth+ curbweight+carlength
                        + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
                        + enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
                        data= card, nvmax = 19)

coef(regfit.best, 7)
```

```
##      (Intercept)      enginesize      enginetypeohcf      enginetypeohcv
##      -57712.5552      148.4221      -3804.7916      -4852.3248
##      enginetyperotor      carwidth      stroke      enginelocationrear
##      7800.8444      1007.4850      -4307.4211      14256.4615
```

Now we take 10 training set.

```
k = 10
set.seed(1)
folds = sample(1:k, nrow(card), replace = TRUE)

cv.errors = matrix(NA, k, 10, dimnames = list(NULL, paste(1:10)))

for (j in 1:k){
  best.fit = regsubsets(price~peakrpm+citympg+ enginesize+fueltype
    +carwidth+curbweight+carlength
    + highwaympg+ boreratio+ stroke + wheelbase,
    data= card[folds!=j,], nvmax = 10)

  for (i in 1:10){
    pred = predict(best.fit, card[folds==j,], id = i)
    cv.errors[j, i]= mean((price[folds==j]-pred)^2)
  }
}
```

Now we have 10x10 matrix which (i,j) corresponds to the MSE of ith validation for the best j-variable model

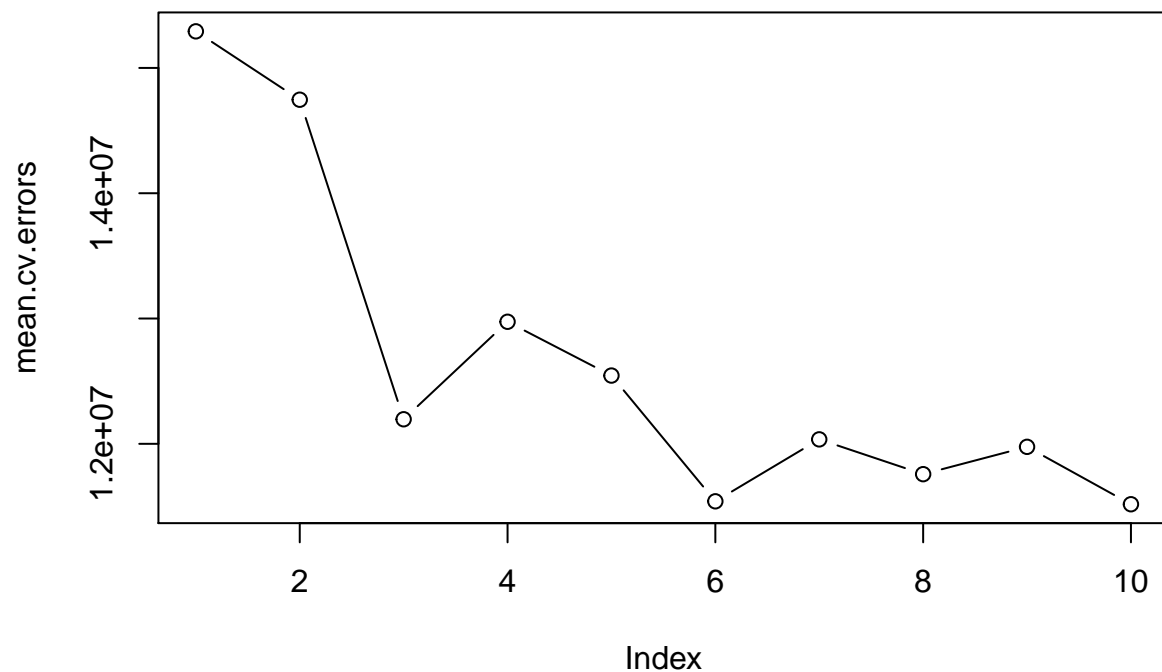
```
mean.cv.errors = apply(cv.errors, 2, mean)

mean.cv.errors
```

```
##      1      2      3      4      5      6      7      8
## 15291913 14746398 12195925 12974205 12544768 11541076 12035643 11757833
##      9      10
## 11975901 11517819
```

```
par(mfrow = c(1,1))

plot(mean.cv.errors, type = "b")
```



The plot shows mean error with different features using the cross validation approach.

```
reg.best = regsubsets(price~fuelsystem+peakrpm+citympg
+ enginesize+engine+type+carwidth+curbweight+carlength
+ highwaympg+ boreratio+ stroke + wheelbase + drivewheel
+ enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
data= card, nvmax = 19)

coef(reg.best, 11)
```

```
##      (Intercept)          peakrpm      enginesize      enginetypeohc
##      -53108.326745          1.416018      163.509438      1951.327128
##      enginetypeohcv      enginetyperotor      carwidth      boreratio
##      -4329.980717          8254.922067      993.205807      -4051.216381
##      stroke      drivewheel fwd      enginelocationrear      aspirationturbo
##      -4374.872812      -1906.461545      10841.290144      1722.404716
```

## Ridge Regression and Lasso

### Ridge Regression

In this experiment the glmnet r package will be used for carry out lasso and ridge regression. In the function we select alpha as 0 for ridge regression.

```
x = model.matrix(price~fuelsystem+peakrpm+citympg
+ enginesize+engine+type+carwidth+curbweight+carlength
+ highwaympg+ boreratio+ stroke + wheelbase + drivewheel
+ enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
data= card)[-1]

y = price
```

We have renamed our variables to conduct the next experiments.

```
library(glmnet)
```

```
## Loading required package: Matrix
```

```
## Loaded glmnet 3.0-2
```

```
grid = 10^seq(10,-2,length=100)
ridge.mod = glmnet(x,y, alpha = 0, lambda = grid)
```

```
dim(coef(ridge.mod))
```

```
## [1] 31 100
```

We get the size of 31x100, where 31 are for 9 predictors and intercept. We have total 30 variables under consideration. The 100 is for 100 different value of lambda.

```
ridge.mod$lambda[40]
```

```
## [1] 187381.7
```

```
coef(ridge.mod)[-1,40]
```

```
##      (Intercept)      fuelsystem2bbl      fuelsystem4bbl      fuelsystemidi
##      317.91583110      -255.71343575      -54.41856690      96.51008402
##      fuelsystemmfi      fuelsystemmpfi      fuelsystemspdi      fuelsystemspfi
##      -38.63725175      247.69181319      -92.64081048      -97.98608462
##      peakrpm           citympg           enginesize           enginetypeohcv
##      -0.03673657      -25.46071261      5.48887688      600.07601062
##      enginetypeel      enginetypeohc      enginetypeohcf      enginetypeohcv
##      -5.49532973      -174.39748241      18.46413682      397.08019754
##      enginetyperotor      carwidth           curbweight           carlength
##      -23.33075986      88.97961050      0.40035969      13.42014128
##      highwaympg        boreratio           stroke           wheelbase
##      -24.52574014      482.90216899      64.03113340      23.25517338
##      drivewheelrwd      drivewheelrwd      enginelocationrear      aspirationturbo
##      -297.38476312      326.17609539      804.93710933      102.08537890
##      doornumbertwo      horsepower      compressionratio
##      -5.15241216      5.24228034      5.12026915
```

```
sqrt(sum(coef(ridge.mod)[-1, 40]^2))
```

```
## [1] 1344.713
```

In the above result, we find the l2 norm of 1344 for lambda of 187381

```
ridge.mod$lambda[60]
```

```
## [1] 705.4802
```

```
coef(ridge.mod)[,60]
```

```
##      (Intercept)      fuelsystem2bbl      fuelsystem4bbl      fuelsystemidi
##      -4.867311e+04      -1.254438e+02      -6.842027e+01      1.490822e+03
##      fuelsystemmmfi      fuelsystemmpfi      fuelsystemspdi      fuelsystemspfi
##      -2.549469e+03      -1.007556e+02      -1.918467e+03      -1.923453e+03
##      peakrpm              citympg              enginesize      enginetyperedohcv
##      7.665295e-01        -7.194260e+00      8.228998e+01      4.227470e+02
##      enginetypeel        enginetypeohc      enginetypeohcf      enginetypeohcv
##      -1.923172e+03        1.352813e+03        1.706284e+02      -1.244039e+03
##      enginetyperotor      carwidth          curbweight          carlength
##      3.791835e+03          6.409885e+02          2.703191e+00      -1.099005e+01
##      highwaympg          boreratio          stroke              wheelbase
##      -5.437710e+00        -2.206767e+03        -2.274208e+03      1.151903e+02
##      drivewheel fwd      drivewheelrwd      enginelocationrear      aspirationturbo
##      -6.192992e+02        1.112299e+03        1.179112e+04        4.576378e+02
##      doornumbertwo        horsepower          compressionratio
##      1.734074e+02          3.074386e+01          -3.418842e+01
```

```
sqrt(sum(coef(ridge.mod)[-1, 60]^2))
```

```
## [1] 13753.58
```

In the above result, we find the l2 norm of 13753 for lambda of 705. So, we conclude in the car data that for smaller lambda we get smaller l2 error.

```
predict(ridge.mod, s = 50, type= "coefficients")[1:20,]
```

```
##      (Intercept)      fuelsystem2bbl      fuelsystem4bbl      fuelsystemidi      fuelsystemmmfi
##      -38060.213941      111.114417      -829.752265      9315.356962      -3040.814172
##      fuelsystemmpfi      fuelsystemspdi      fuelsystemspfi      peakrpm              citympg
##      678.706065      -2460.961617      -892.175511      1.931968      -20.656992
##      enginesize      enginetyperedohcv      enginetypeel      enginetypeohc      enginetypeohcf
##      151.111619      2598.204782      -1630.423702      1945.435424      333.657362
##      enginetypeohcv      enginetyperotor      carwidth          curbweight          carlength
##      -3760.346346      9501.808424      669.201875      3.265289      -50.702701
```

In above we get new prediction for a new lambda of 50.

We split the training instances for estimating test error in ridge and lasso.

```
set.seed(1)
train = sample(1:nrow(x), nrow(x)/2)
test = (-train)
y.test = y[test]
```

Now we conduct experiment on the segmented data using glm.

```
ridge.mod = glmnet(x[train,], y[train], alpha = 0, lambda = grid, thresh = 1e-12)
ridge.pred = predict(ridge.mod, s = 4, newx = x[test,])
mean((ridge.pred - y.test)^2)
```

```
## [1] 9866127
```

Using features we find the error of 9866127

```
mean((mean(y[train]) - y.test)^2)
```

```
## [1] 65445936
```

If we use only the mean to predict the result we get higher error of 65445936.

We can also check same performance using very high lambda.

```
ridge.pred = predict(ridge.mod, s = 1e10, newx = x[test,])
mean((ridge.pred - y.test)^2)
```

```
## [1] 65445316
```

In above we get very similar value of 65M of the earlier mean only model.

```
ridge.pred = predict(ridge.mod, s = 0, newx = x[test,])
mean((ridge.pred - y.test)^2)
```

```
## [1] 9903438
```

```
lm(y~x, subset = train)
```

```
##
## Call:
## lm(formula = y ~ x, subset = train)
##
## Coefficients:
##      (Intercept)      xfuelsystem2bbl      xfuelsystem4bbl
##      -46019.584        -122.079        -1115.572
##      xfuelsystemidi      xfuelsystemmfi      xfuelsystemmpfi
##      583.330              NA              400.914
##      xfuelsystemspdi      xfuelsystemspfi      xpeakrpm
##      -2668.129              NA              1.754
##      xcitympg              xengineysize      xenginetyperedohcv
##      4.886                  200.276          4519.275
```



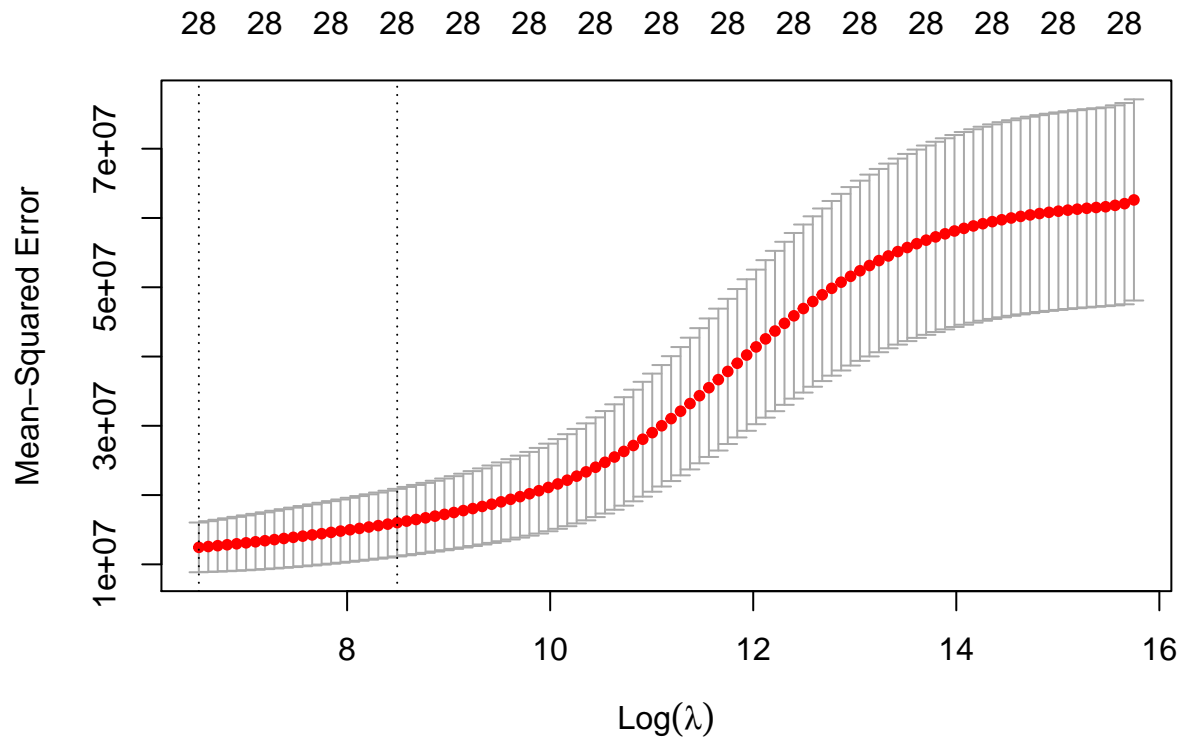
```
##      xenginetype1      xenginetypeohc      xenginetypeohcf
##      -2204.640        808.099        -2616.709
##      xenginetypeohcv  xenginetyperotor      xcarwidth
##      -5835.691        12691.241        1070.224
##      xcurbweight      xcarlength      xhighwaympg
##      1.886            -40.143         34.513
##      xboreratio      xstroke          xwheelbase
##      -2282.749        -5743.011        -134.104
##      xdrivewheel fwd  xdrivewheelrwd  xengine location rear
##      583.449          1486.237        15943.692
##      xaspiration turbo  xdoornumbertwo      xhorsepower
##      4833.875         -947.648        -50.918
##      xcompressionratio
##      -178.472
```

```
predict(ridge.mod, s=0, newx = x[test,], type="coefficients")[1:20,]
```

```
##      (Intercept)  fuelsystem2bb1  fuelsystem4bb1  fuelsystemidi  fuelsystemmfi
##      -46019.459092  -122.097895  -1115.355871  583.284935  0.000000
##      fuelsystemmpfi  fuelsystemspdi  fuelsystemspfi  peakrpm  citympg
##      400.769613  -2668.197866  0.000000  1.753604  4.893521
##      enginesize  enginety pedohcv  enginetype1  enginetypeohc  enginetypeohcf
##      200.267096  4518.078384  -2204.513145  808.249332  -2616.449885
##      enginetypeohcv  enginetyperotor  carwidth  curbweight  carlength
##      -5835.474487  12690.751953  1070.208519  1.885694  -40.150598
```

The mean for lambda 0 is similar to 9M of the prediction error of lambda 4.

```
set.seed(1)
cv.out = cv.glmnet(x[train, ], y[train], alpha = 0)
plot(cv.out)
```



```
bestlam = cv.out$lambda.min
bestlam
```

```
## [1] 691.9598
```

We find the value of best lambda is around 692 using the validation set approach. Now we predict using the best lambda and check the mse value on test dataset.

```
ridge.pred = predict(ridge.mod, s = bestlam, newx = x[test,])
mean((ridge.pred - y.test)^2)
```

```
## [1] 9602655
```

We also find the best model coefficient using the bestlambda.

```
out = glmnet(x,y, alpha = 0)
predict(out, type = "coefficients", s = bestlam)[1:20,]
```

```
##      (Intercept) fuelsystem2bbl fuelsystem4bbl fuelsystemmidi fuelsystemmfi
## -4.846268e+04 -1.307088e+02 -5.707612e+01 1.505479e+03 -2.550307e+03
## fuelsystemmpfi fuelsystemspdi fuelsystemspfi peakrpm citympg
## -9.830366e+01 -1.922857e+03 -1.907042e+03 7.611355e-01 -7.971368e+00
## enginesize enginetypeohcv enginetypeel enginetypeohc enginetypeohcf
## 8.259412e+01 3.796631e+02 -1.906869e+03 1.364857e+03 1.739892e+02
## enginetypeohcv enginetypeotor carwidth curbweight carlength
## -1.258224e+03 3.810482e+03 6.398897e+02 2.670809e+00 -1.220085e+01
```

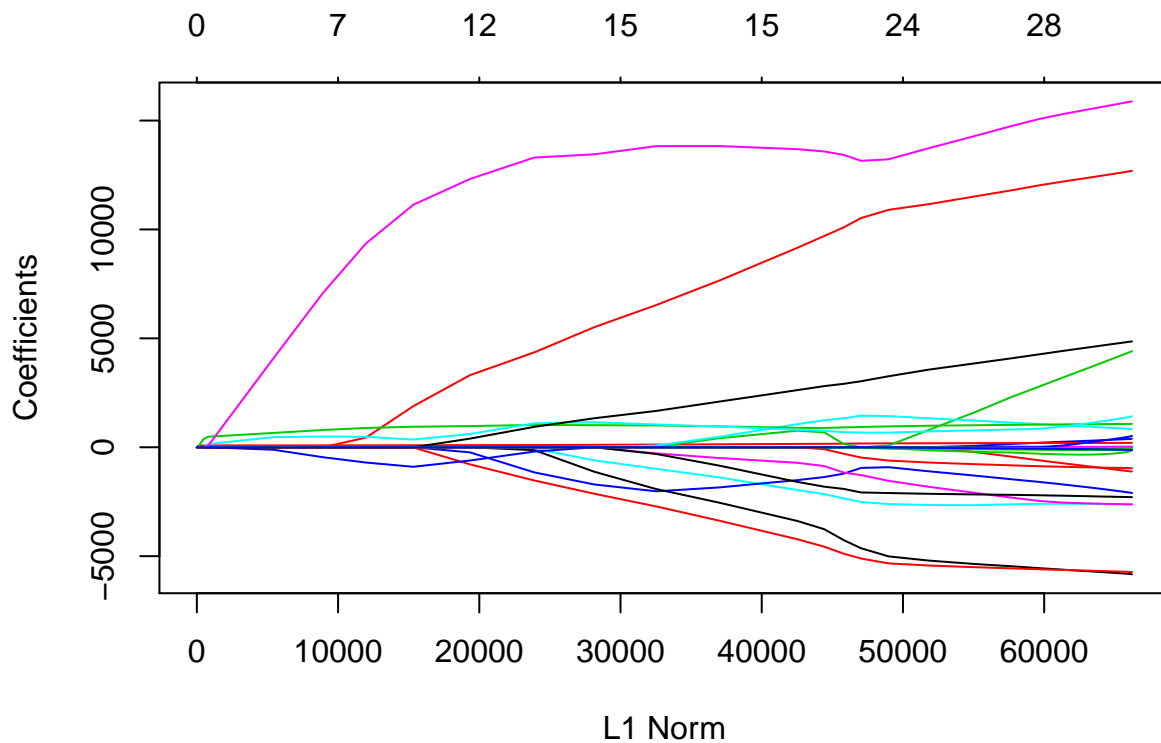
We see the coefficients are not zero that often.

## Lasso

We use the alpha of 1 to implement lasso using the similar method used for ridge regression. We also plot the model for visualization.

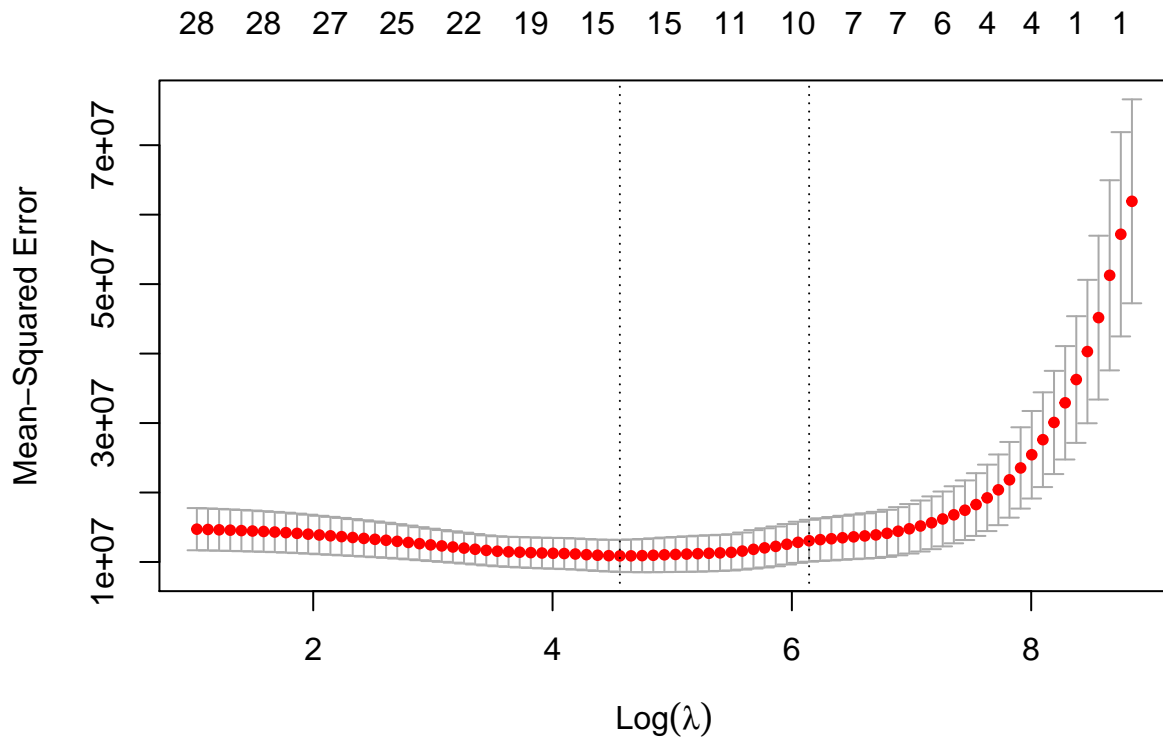
```
lasso.mod = glmnet(x[train,], y[train], alpha = 1, lambda = grid)
plot(lasso.mod)
```

```
## Warning in regularize.values(x, y, ties, missing(ties)): collapsing to unique
## 'x' values
```



I use the random train test split for the lasso and find the best lambda value.

```
set.seed(1)
cv.out = cv.glmnet(x[train,], y[train], alpha=1)
plot(cv.out)
```



```
bestlam = cv.out$lambda.min
lasso.pred = predict(lasso.mod, s =bestlam, newx = x[test,])
mean((lasso.pred-y.test)^2)
```

```
## [1] 9456810
```

For best lambda the lasso also provides the error of about 9M like the ridge model.

```
out = glmnet(x,y, alpha =1, lambda =grid)
lasso.coef = predict(out, type ="coefficients", s= bestlam)[1:20,]
lasso.coef
```

```
##      (Intercept)  fuelsystem2bbl  fuelsystem4bbl  fuelsystemidi  fuelsystemmfi
## -48252.335050    -2.068202      0.000000      479.481926    -1806.104148
## fuelsystemmpfi  fuelsystemspdi  fuelsystemspfi      peakrpm      citympg
##  56.647436     -1604.347645    -102.546469      1.332641      0.000000
##      enginesize  enginetyopedohcv  enginetyepel  enginetyeohc  enginetyeohcf
##  132.917423     331.801144     -543.830839     1396.374341      0.000000
## enginetyeohcv  enginetyeperotor      carwidth      curbweight      carlength
## -2633.789111     6013.736600      705.197259      1.674569      0.000000
```

```
lasso.coef[lasso.coef != 0]
```

```
##      (Intercept)  fuelsystem2bbl  fuelsystemidi  fuelsystemmfi  fuelsystemmpfi
```

```
##      -48252.335050      -2.068202      479.481926      -1806.104148      56.647436
## fuelsystemspdi fuelsystemspfi      peakrpm      enginesize enginetypedohcv
##      -1604.347645      -102.546469      1.332641      132.917423      331.801144
##      enginetypel      enginetypeohc enginetypeohcv enginetyperotor      carwidth
##      -543.830839      1396.374341      -2633.789111      6013.736600      705.197259
##      curbweight
##      1.674569
```

In lasso coefficient we find many values are zeros unlike the ridge regression.

## PCR and PLS

### Principle Components Regression (PCR)

For the experiment the pls library of r will be used.

```
library(pls)
```

```
##
## Attaching package: 'pls'

## The following object is masked from 'package:stats':
##
##      loadings
```

```
set.seed(2)
pcr.fit = pcr(price~peakrpm+citympg+ enginesize
              +carwidth+curbweight+carlength
              + highwaympg+ horsepower+engineLocation,
              data = card, scale = TRUE, validation = "CV")
```

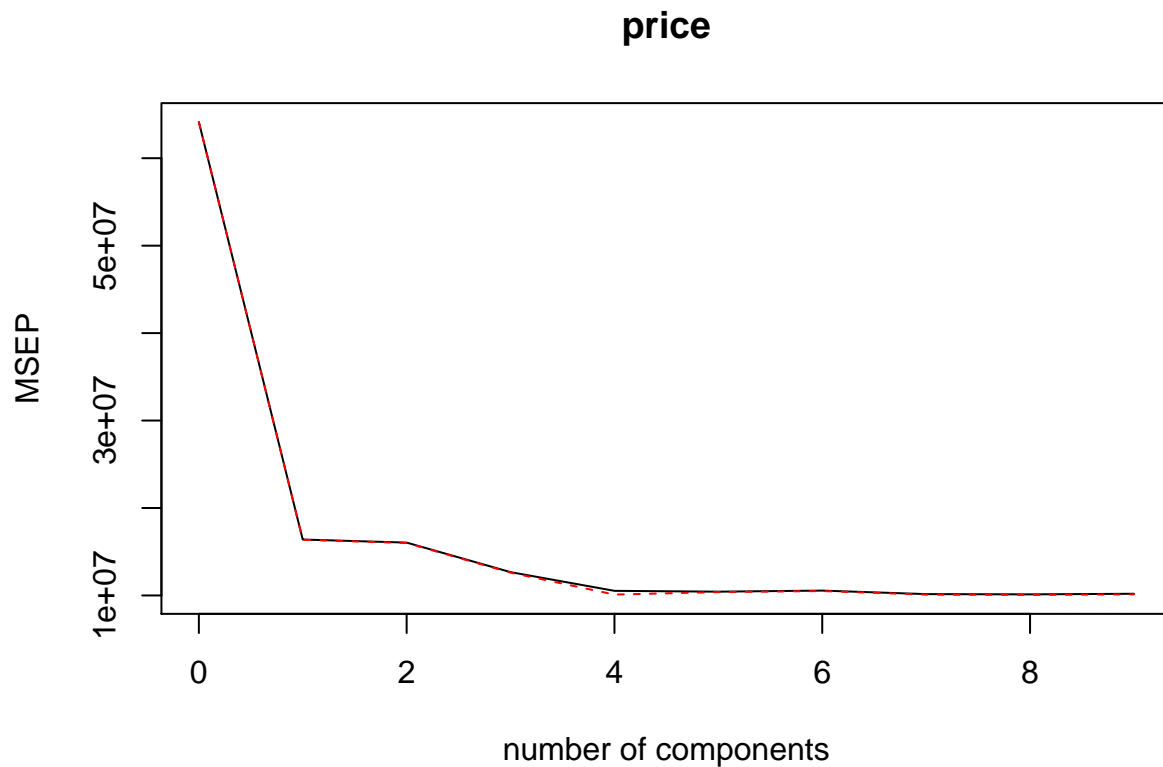
```
summary(pcr.fit)
```

```
## Data:      X dimension: 205 9
## Y dimension: 205 1
## Fit method: svdpc
## Number of components considered: 9
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##      (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV              8008    4049    4006    3559    3245    3230    3249
## adjCV           8008    4046    4000    3550    3178    3222    3241
##      7 comps  8 comps  9 comps
## CV          3187    3182    3193
## adjCV        3177    3172    3182
##
## TRAINING: % variance explained
##      1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
```

```
## X      61.48    78.30    88.21    92.44    96.46      98    99.05    99.77
## price   74.98    76.18    81.35    84.88    84.99      85    85.78    85.90
##      9 comps
## X      100.00
## price   85.92
```

We find the the PCR model performed bes in case of 8 components when error is 3182 the lower than anyother. The 5 componest also covered 97% of the total variance.

```
validationplot(pcr.fit, val.type = "MSEP")
```

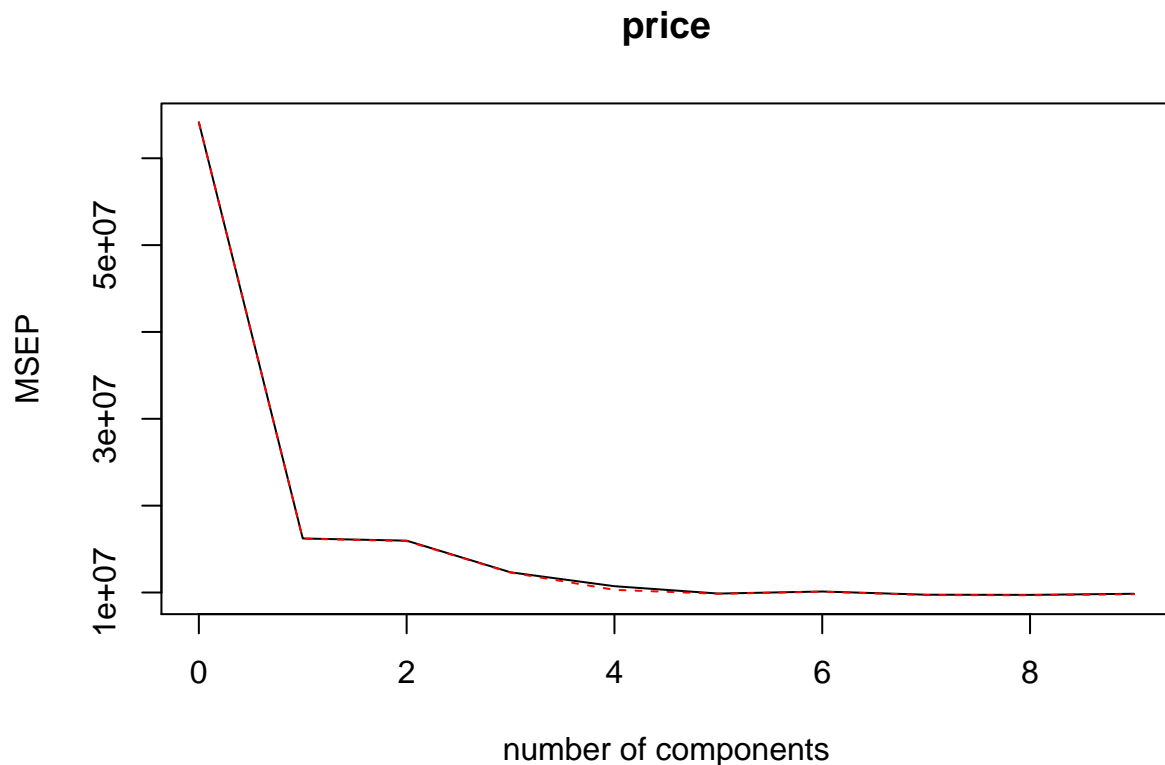


We see the result in the plot with the error and components added.

```
set.seed(1)

pcr.fit = pcr(price~peakrpm+citympg+ enginesize
              +carwidth+curbweight+carlength
              + highwaympg+ horsepower+engine.location,
              data = card, scale = TRUE, validation = "CV")

validationplot(pcr.fit, val.type = "MSEP")
```



The above plot shows the model performance on the test data. We also observe the lowest error around the 8th components. At next we use the 8 components to predict the error on test instances.

```
x = model.matrix(price~peakrpm+citympg+ enginesize
                  +carwidth+curbweight+carlength
                  + highwaympg+ horsepower+enginelocation,
                  data= card)[,-1]

y = price
pcr.pred = predict(pcr.fit, x[test,], ncomp =8)
mean((pcr.pred - y.test)^2)
```

```
## [1] 9278896
```

The test error is 9278896 using the best components. We then retrain the model using the 8 component as found for smallest error.

```
pcr.fit = pcr(y~x, scale= TRUE, ncomp = 8)

summary(pcr.fit)
```

```
## Data:      X dimension: 205 9
## Y dimension: 205 1
## Fit method: svdpc
```

```
## Number of components considered: 8
## TRAINING: % variance explained
##      1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
## X      61.48    78.30    88.21    92.44    96.46     98    99.05    99.77
## y      74.98    76.18    81.35    84.88    84.99     85    85.78    85.90
```

## Partial Least Squares

```
set.seed(1)

#partial least square

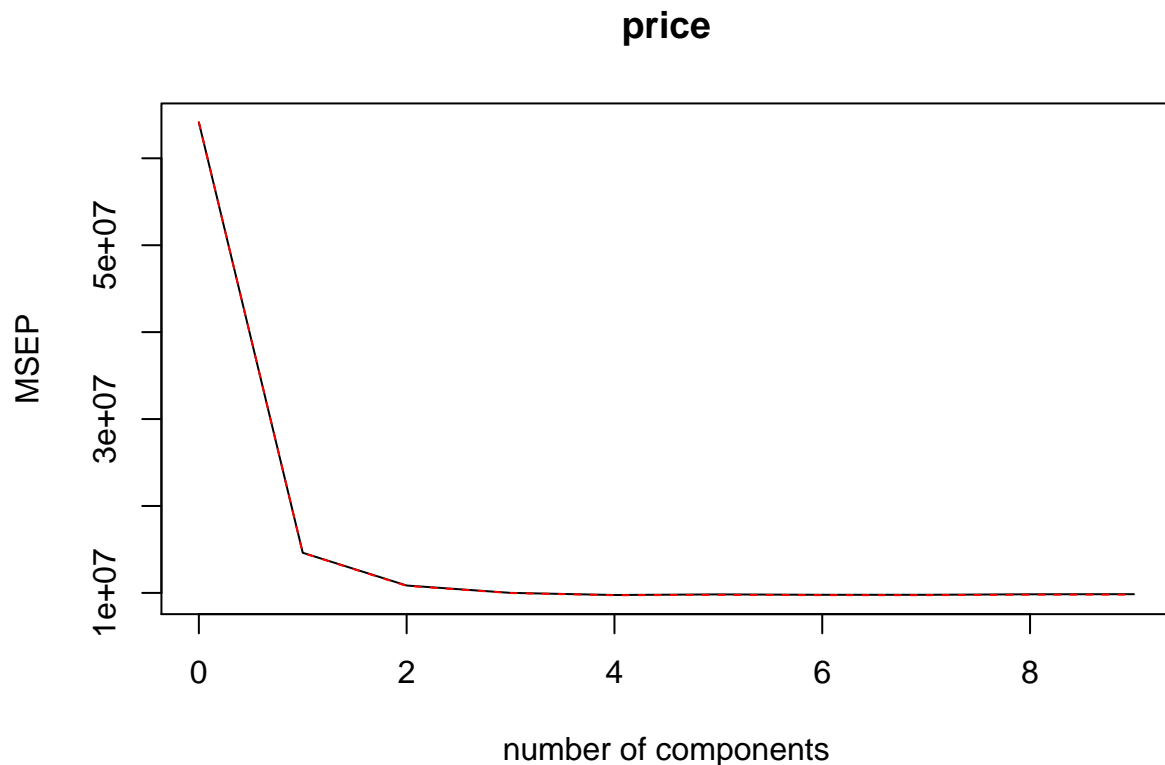
pls.fit = plsr(price~peakrpm+citympg+ enginesize
               +carwidth+curbweight+carlength
               + highwaympg+ horsepower+engine.location,
               data = card, scale = TRUE, validation = "CV")

summary(pls.fit)
```

```
## Data:      X dimension: 205 9
## Y dimension: 205 1
## Fit method: kernelpls
## Number of components considered: 9
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##      (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV              8008    3823    3294    3162    3124    3135    3128
## adjCV           8008    3821    3287    3160    3119    3126    3121
##      7 comps  8 comps  9 comps
## CV          3127    3137    3139
## adjCV        3120    3129    3131
##
## TRAINING: % variance explained
##      1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
## X          61.32    72.93    84.85    92.20    93.46    96.13    98.30    99.48
## price       77.61    83.84    85.03    85.56    85.90    85.91    85.91    85.91
##      9 comps
## X          100.00
## price       85.92
```

```
validationplot(pls.fit, val.type = "MSEP")
```





From above result we see the lowest error occurs for the 4th component the value of 3124. In test case we use 4 components as found here.

```
pls.pred = predict(pls.fit, x[test,], ncomp = 4)
```

```
mean((pls.pred - y.test)^2)
```

```
## [1] 9449577
```

```
pls.fit = plsr(price~peakrpm+citympg+ enginesize
               +carwidth+curbweight+carlength
               + highwaympg+ horsepower+engine.location,
               data = card, scale = TRUE, ncomp =4)
```

```
summary(pls.fit)
```

```
## Data:      X dimension: 205 9
## Y dimension: 205 1
## Fit method: kernelpls
## Number of components considered: 4
## TRAINING: % variance explained
##          1 comps  2 comps  3 comps  4 comps
## X          61.32   72.93   84.85   92.20
## price      77.61   83.84   85.03   85.56
```

Using 4 components we find error of 9446577, very similar to PCR but with smaller number of components. PCR chose more components.

## Chapter 7

### Polynomial Regression and Step function

In this section I will implement different nonlinear estimation method to experiment on car dataset. Firstly we will use poly function with lm function to implement the polynomial regression. In this experiment, I will use only one feature of enginesize for better explanation. We have seen the significance of enginesize feature in last chapter.

```
library(ISLR)
attach(card)

## The following object is masked _by_ .GlobalEnv:
##
##      high

## The following objects are masked from card (pos = 6):
##
##      aspiration, boreratio, car_ID, carbody, carheight, carlength,
##      CarName, carwidth, citympg, compressionratio, curbweight,
##      cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##      enginetype, fuelsystem, fueltype, high, highwaympg, horsepower,
##      peakrpm, price, stroke, symboling, wheelbase

## The following objects are masked from card (pos = 11):
##
##      aspiration, boreratio, car_ID, carbody, carheight, carlength,
##      CarName, carwidth, citympg, compressionratio, curbweight,
##      cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##      enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
##      price, stroke, symboling, wheelbase

## The following objects are masked from card (pos = 12):
##
##      aspiration, boreratio, car_ID, carbody, carheight, carlength,
##      CarName, carwidth, citympg, compressionratio, curbweight,
##      cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##      enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
##      price, stroke, symboling, wheelbase

## The following objects are masked from card (pos = 14):
##
##      aspiration, boreratio, car_ID, carbody, carheight, carlength,
##      CarName, carwidth, citympg, compressionratio, curbweight,
##      cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##      enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
##      price, stroke, symboling, wheelbase

fit = lm(price~poly(enginesize, 4), data = card)
coef(summary(fit))
```

```
##               Estimate Std. Error    t value    Pr(>|t|)
## (Intercept)      13276.7106    255.8457  51.89341988  5.439147e-118
## poly(engine size, 4)1  99743.0989  3663.1536  27.22875167  3.369131e-69
## poly(engine size, 4)2 -1165.2811  3663.1536  -0.31810873  7.507344e-01
## poly(engine size, 4)3 -19642.6343  3663.1536  -5.36221972  2.257410e-07
## poly(engine size, 4)4  -154.3066  3663.1536  -0.04212397  9.664419e-01
```

There are also alternative ways to implement the polynomial regression over the dataset. We can use `I` or use the `cbind` function for concision.

```
fit2 = lm(price~poly(engine size, 4, raw=T), data = card)
coef(summary(fit))
```

```
##               Estimate Std. Error    t value    Pr(>|t|)
## (Intercept)      13276.7106    255.8457  51.89341988  5.439147e-118
## poly(engine size, 4)1  99743.0989  3663.1536  27.22875167  3.369131e-69
## poly(engine size, 4)2 -1165.2811  3663.1536  -0.31810873  7.507344e-01
## poly(engine size, 4)3 -19642.6343  3663.1536  -5.36221972  2.257410e-07
## poly(engine size, 4)4  -154.3066  3663.1536  -0.04212397  9.664419e-01
```

```
fit2a = lm(price~engine size+I(engine size^2)+I(engine size^3) + I(engine size^4), data=card)
coef(fit2a)
```

```
##      (Intercept)      engine size I(engine size^2) I(engine size^3) I(engine size^4)
##      2.171021e+04    -4.221207e+02    3.530566e+00    -6.129592e-03    -7.917883e-07
```

```
fit2b = lm(price~cbind(engine size, engine size^2, engine size^3, engine size^4), data=card)
coef(fit2b)
```

```
##                                     (Intercept)
##                                     2.171021e+04
## cbind(engine size, engine size^2, engine size^3, engine size^4)engine size
##                                     -4.221207e+02
##      cbind(engine size, engine size^2, engine size^3, engine size^4)
##                                     3.530566e+00
##      cbind(engine size, engine size^2, engine size^3, engine size^4)
##                                     -6.129592e-03
##      cbind(engine size, engine size^2, engine size^3, engine size^4)
##                                     -7.917883e-07
```

In the previous two sections, we find similar coefficients for all the `poly`, `I` and `cbind` function methods. This shows the equivalence of the implementations.

Here we specify the range of engine size for prediction.

```
engsrange = range(engine size)

engs.grid = seq(from=engsrange[1], to = engsrange[2])

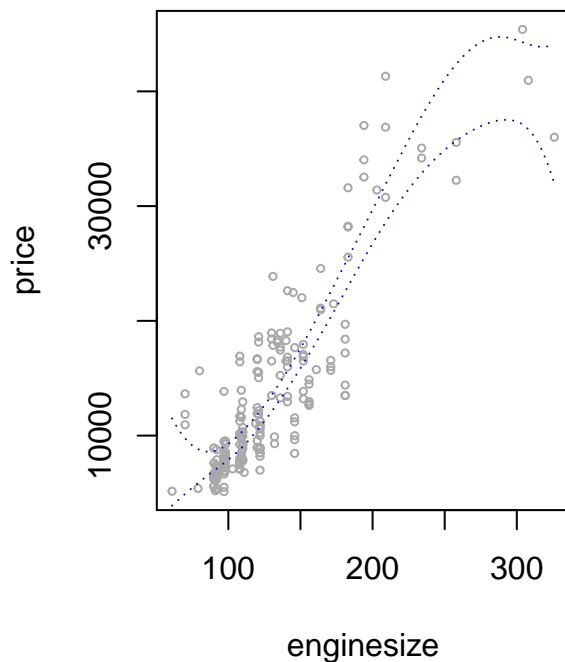
preds= predict(fit, newdata = list(engine size=engs.grid), se=TRUE)

se.bands = cbind(preds$fit+2*preds$se.fit, preds$fit-2*preds$se)
```

Now we can plot the result of previous section.

```
#plot
par(mfrow=c(1,2), mar = c(4.5, 4.5,1,1), oma=c(0,0,4,0))
plot(engineSize, price, xlim= engsrange, cex=0.5, col="darkgrey")
title("Degree 4 polynomial", outer=T)
matlines(engs.grid, se.bands, lwd=1, col="blue", lty=3)
```

## Degree 4 polynomial



Next, we re-evaluate the equivalence between the `poly()` and `I()` method by check the prediction differences.

```
preds2= predict(fit2, newdata=list(engineSize=engs.grid), se=TRUE)
max(abs(preds$fit- preds2$fit))
```

```
## [1] 2.582965e-10
```

The prediction are almost same.

```
fit.1 = lm(price~engineSize, data=card)
fit.2 = lm(price~poly(engineSize,2), data=card)
fit.3 = lm(price~poly(engineSize,3), data=card)
fit.4 = lm(price~poly(engineSize,4), data=card)
fit.5 = lm(price~poly(engineSize,5), data=card)
anova(fit.1, fit.2, fit.3, fit.4, fit.5)
```

```
## Analysis of Variance Table
```

```
##
## Model 1: price ~ enginesize
## Model 2: price ~ poly(enginesize, 2)
## Model 3: price ~ poly(enginesize, 3)
## Model 4: price ~ poly(enginesize, 4)
## Model 5: price ~ poly(enginesize, 5)
##   Res.Df      RSS Df Sum of Sq      F      Pr(>F)
## 1     203 3070953588
## 2     202 3069595708  1   1357880  0.1009    0.7511
## 3     201 2683762625  1 385833083 28.6606 2.365e-07 ***
## 4     200 2683738814  1    23811  0.0018    0.9665
## 5     199 2678969866  1   4768948  0.3542    0.5524
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Here, the linear polynomial seems fit. But changing from 1 to 2 is not significant.

```
coef(summary(fit.5))
```

```
##               Estimate Std. Error    t value      Pr(>|t|)
## (Intercept)    13276.7106    256.2598  51.80957650 1.758719e-117
## poly(enginesize, 5)1  99743.0989  3669.0816  27.18475861  6.373145e-69
## poly(enginesize, 5)2 -1165.2811  3669.0816  -0.31759477  7.511253e-01
## poly(enginesize, 5)3 -19642.6343  3669.0816  -5.35355607  2.364929e-07
## poly(enginesize, 5)4  -154.3066  3669.0816  -0.04205591  9.664963e-01
## poly(enginesize, 5)5 -2183.7922  3669.0816  -0.59518767  5.523942e-01
```

The result is also evident from the anova test above. We see the relation of t value and p value from the above result

```
(-5.3535)^2
```

```
## [1] 28.65996
```

The same as 28.6606 of the avona result earlier.

In next section, we add another feaure carwidth for the analysis

```
fit.1 = lm(price~carwidth+enginesize, data=card)
fit.2 = lm(price~poly(enginesize,2)+carwidth, data=card)
fit.3 = lm(price~poly(enginesize,3)+carwidth, data=card)
anova(fit.1, fit.2, fit.3)
```

```
## Analysis of Variance Table
##
## Model 1: price ~ carwidth + enginesize
## Model 2: price ~ poly(enginesize, 2) + carwidth
## Model 3: price ~ poly(enginesize, 3) + carwidth
##   Res.Df      RSS Df Sum of Sq      F      Pr(>F)
## 1     202 2686419306
## 2     201 2671194631  1  15224675  1.3799    0.2415
## 3     200 2206634677  1 464559954 42.1057 6.661e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We find the model 2 to model 3 is insignificant.

In next section we create class label for the price in car dataset by selecting the modality.

```
fit = glm(I(price>10300)~poly(engine size, 4), data=card, family = binomial)

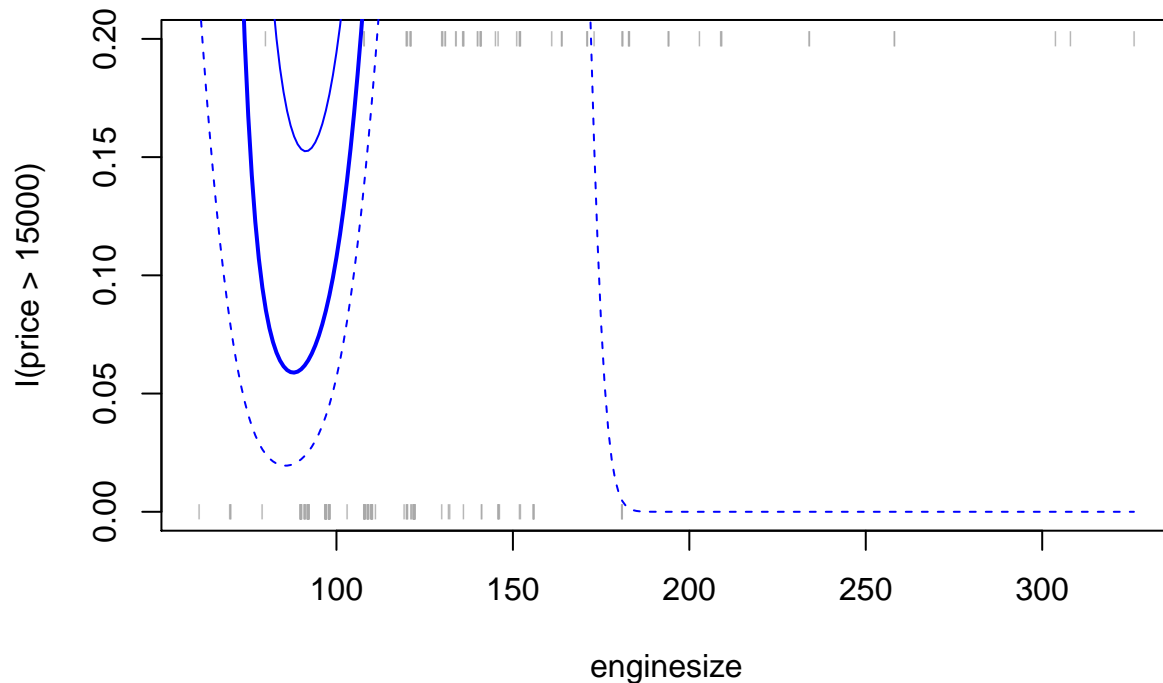
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

preds = predict(fit, newdata = list(engine size=engs.grid), se=T)

pfit = exp(preds$fit)/(1+exp(preds$fit))
se.bands.logit = cbind(preds$fit+2*preds$se.fit, preds$fit-2*preds$se.fit)
se.bands = exp(se.bands.logit)/(1+exp(se.bands.logit))

preds = predict(fit, newdata = list(engine size=engs.grid), type="response", se=T)

plot(engine size, I(price>15000), xlim=engs.range, type="n", ylim= c(0,0.2))
points(jitter(engine size), I((price>15000)/5), cex=0.5, pch="|", col="darkgrey")
lines(engs.grid, pfit, lwd=2, col="blue")
matlines(engs.grid, se.bands, lwd = 1, col="blue", lty=3)
```



```
table(cut(engine size, 4))

##
## (60.7,127] (127,194] (194,260] (260,326]
##          130         61          11          3
```

```
fit =lm(price~cut(engine size, 4), data=card)
coef(summary(fit))
```

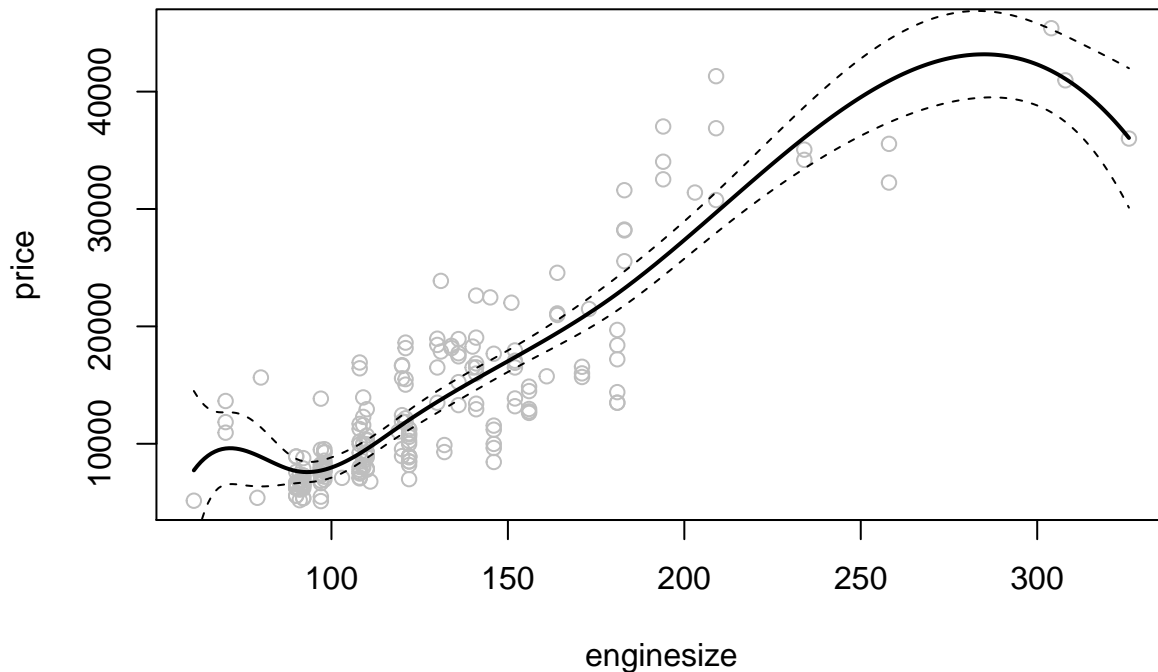
```
##              Estimate Std. Error  t value    Pr(>|t|)
## (Intercept)      9078.862    314.6253  28.85611 2.098258e-73
## cut(engine size, 4)(127,194]   7939.731    556.7310  14.26134 2.427508e-32
## cut(engine size, 4)(194,260] 25555.638   1126.4369  22.68715 2.459773e-57
## cut(engine size, 4)(260,326] 31707.805   2094.8786  15.13587 4.816748e-35
```

The cut method selected the point 127, 194 and 260 engine size.

## spline

```
library(splines)

fit = lm(price~bs(engine size, knots = c(90,120,180)), data=card)
pred = predict(fit, newdata = list(engine size=engs.grid), se=T)
plot(engine size, price, col="grey")
lines(engs.grid, pred$fit, lwd=2)
lines(engs.grid, pred$fit+2*pred$se, lty="dashed")
lines(engs.grid, pred$fit-2*pred$se, lty="dashed")
```



In the implementation we specified 90, 120 and 180 as knots to create spline of 6 basis functions.

```
dim(bs(engineSize, knots=c(90,120,180)))
```

```
## [1] 205 6
```

```
dim(bs(engineSize, df=6))
```

```
## [1] 205 6
```

```
attr(bs(engineSize, df=6), "knots")
```

```
## 25% 50% 75%
```

```
## 97 120 141
```

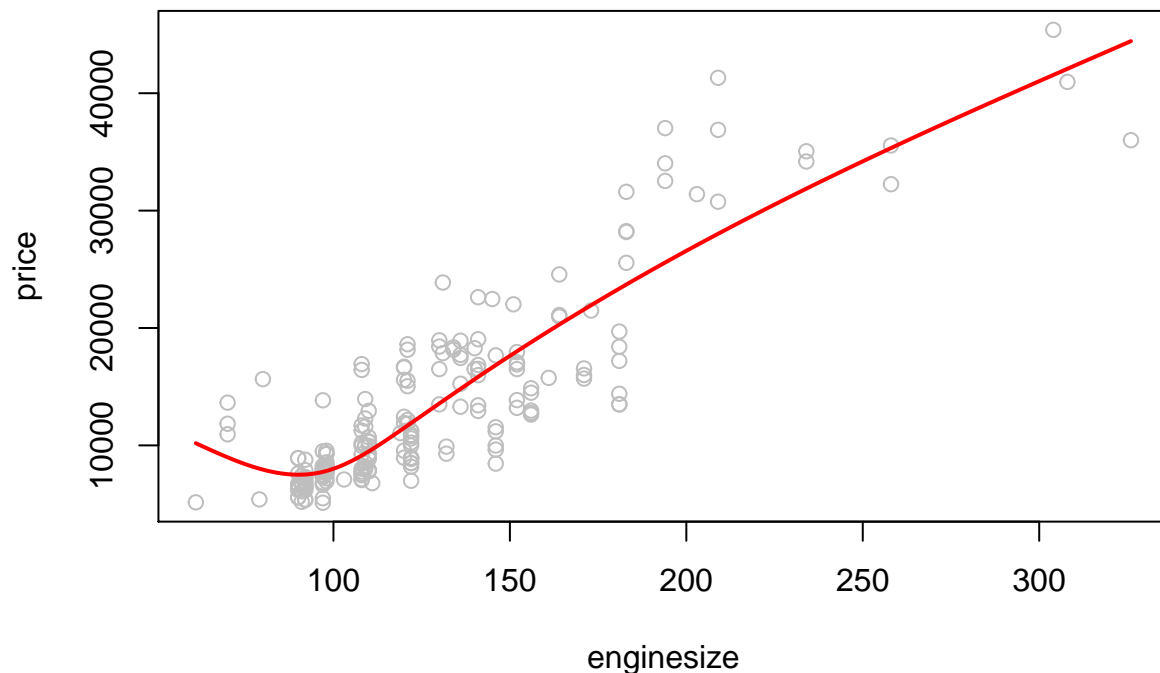
We see that the r select knots in 97, 120 and 141, near the points we selected in earlier methods.

```
fit2 = lm(price~ns(engineSize, df=4), data=card)
```

```
pred2= predict(fit2, newdata = list(engineSize=engs.grid), se=T)
```

```
plot(engineSize, price, col="grey")
```

```
lines(engs.grid, pred2$fit, col="red", lwd=2)
```



We fit the previous model using 4 degree of freedom.

In next experiment we use the smooth spline method.



```

plot(engineSize, price, xlim= engsrange, cex=0.5, col="darkgrey")
title("smoothing Spline")
fit = smooth.spline(engineSize, price, df=16)
fit2 = smooth.spline(engineSize, price, cv=TRUE)

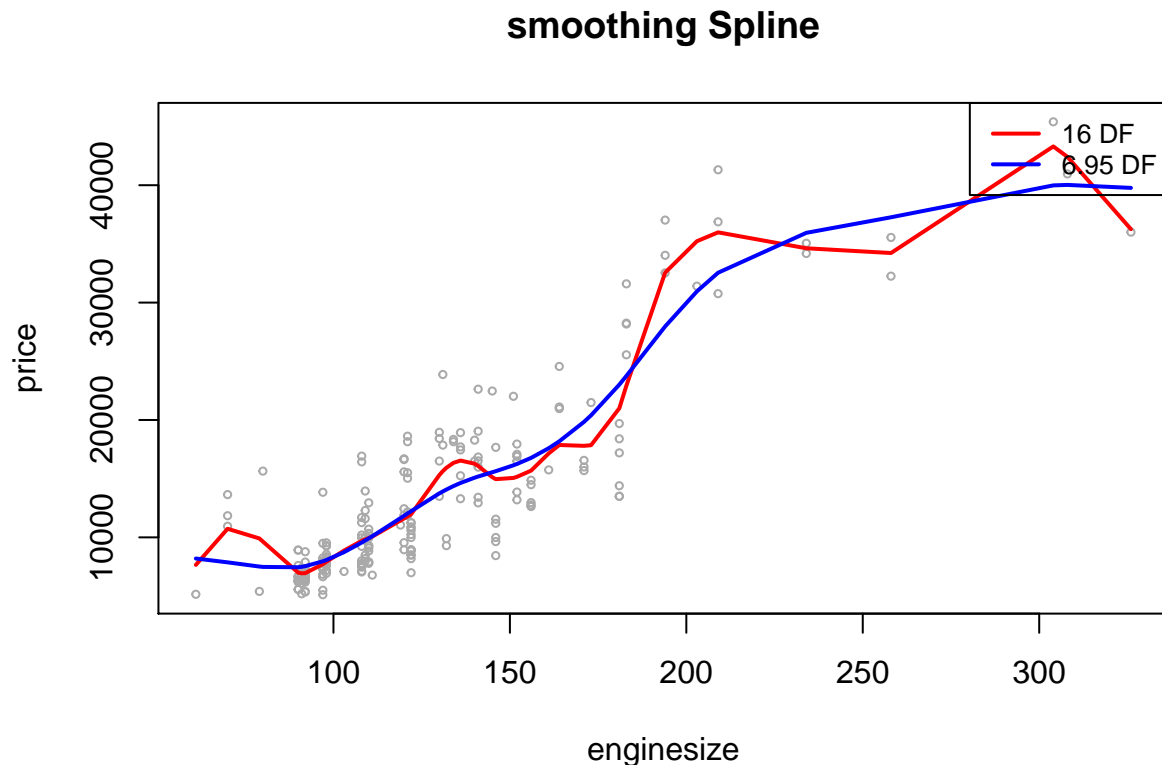
## Warning in smooth.spline(engineSize, price, cv = TRUE): cross-validation with
## non-unique 'x' values seems doubtful

fit2$df

## [1] 6.953457

lines(fit,col="red", lwd=2)
lines(fit2,col="blue", lwd=2)
legend("topright", legend = c("16 DF", "6.95 DF"), col=c("red", "blue"), lty=1, lwd=2, cex=0.8)

```



We see the comparison between 16 and 6.5 degree of freedom. 16 DF model fit the data with high accuracy by taking more wibble form.

```

plot(engineSize, price, xlim = engsrange, cex=.5, col="darkgrey")
title("local regerssion")
fit = loess(price~engineSize, span=.2, data=card)

```

```

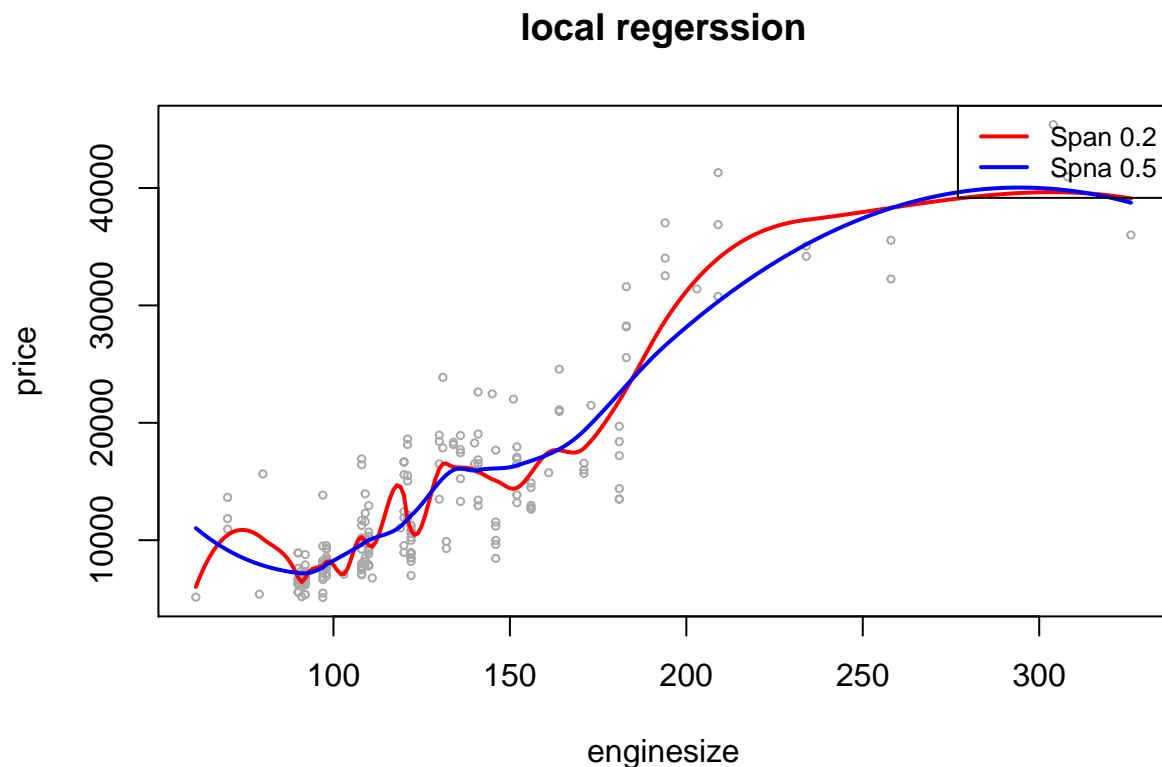
## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : pseudoinverse used at 97

```

```
## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : neighborhood radius 5

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : reciprocal condition number 5.0063e-017

fit2 = loess(price~enginesize, span=.5, data=card)
lines(engs.grid, predict(fit, data.frame(enginesize=engs.grid)), col="red", lwd=2)
lines(engs.grid, predict(fit2, data.frame(enginesize=engs.grid)), col="blue", lwd=2)
legend("topright", legend = c("Span 0.2", "Span 0.5"), col=c("red", "blue"), lty=1, lwd=2, cex=0.8)
```



We can also select the span parameters to control the model fitness over the training instances

## General additive model (GAM)

Using general additive model we cab combine different methods together.

```
gam1 = lm(price~ns(enginesize, 4)+ns(carwidth,3), data=card)
library(gam)
```

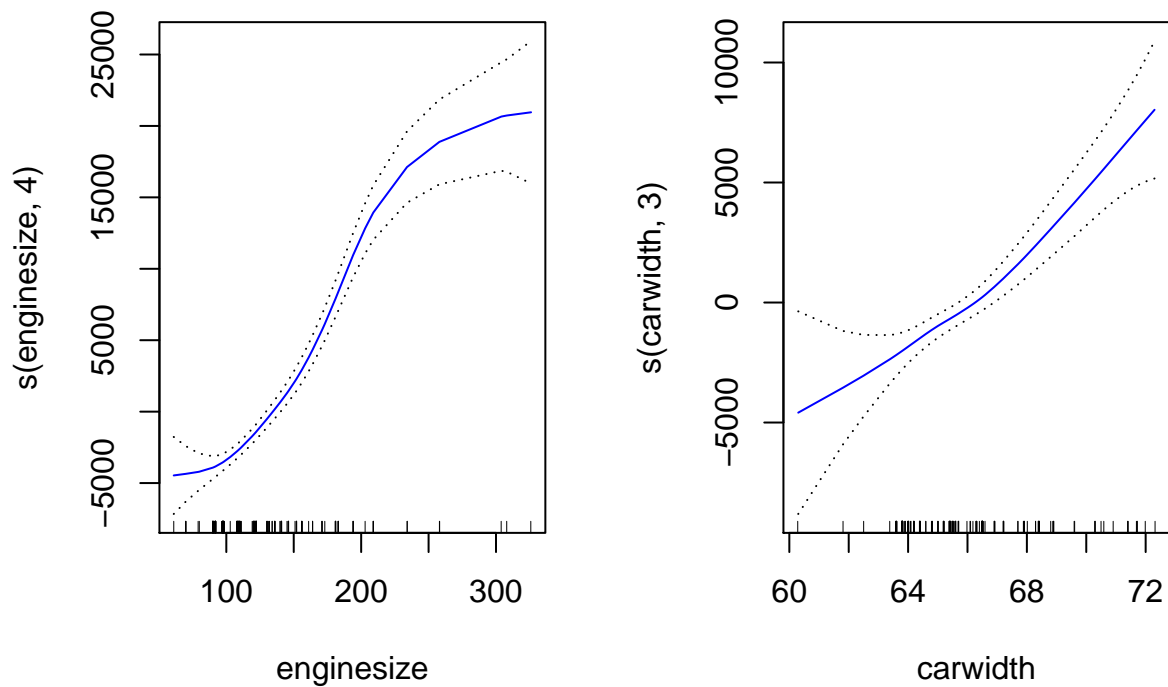
```
## Loading required package: foreach
```

```
## Loaded gam 1.16.1
```

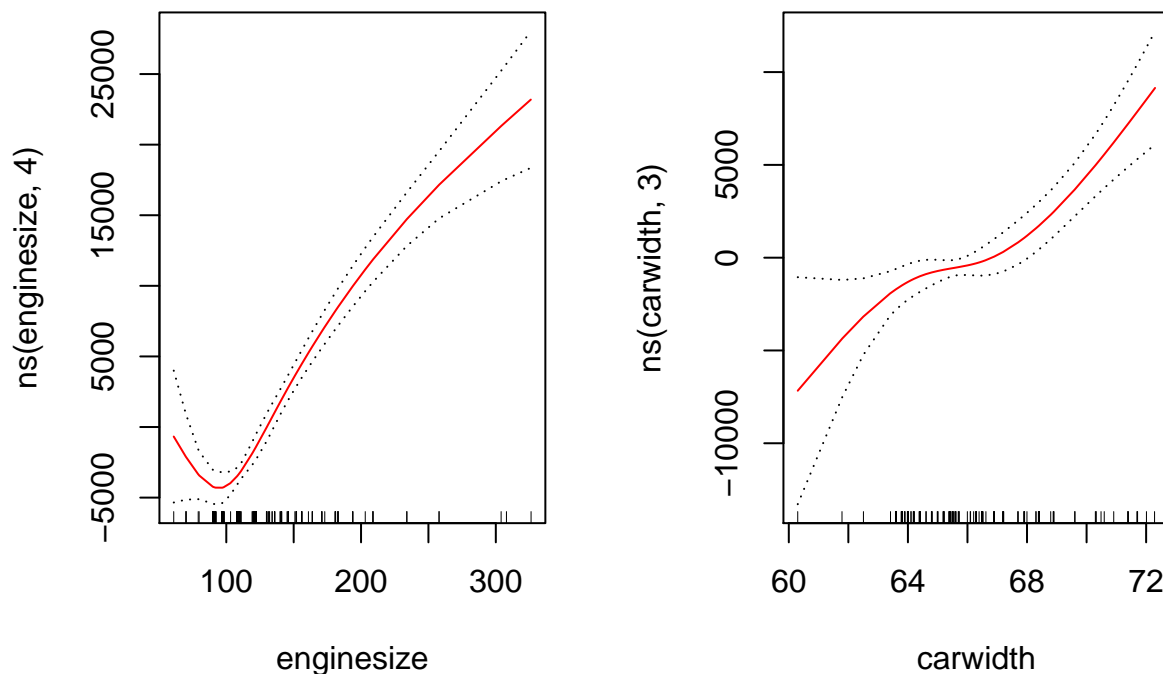
```
gam.m3 =gam(price~s(engineSize,4)+s(carwidth,3), data=card)
```

```
## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts argument
## ignored
```

```
par(mfrow = c(1,2))
plot(gam.m3, se=TRUE, col='blue')
```



```
par(mfrow = c(1,2))
plot.Gam(gam1, se=TRUE, col='red')
```



The previous two section we implement spline and smooth spline as additive model. We find the fitness and difference in the plot in the boundary regions.

```
gam.m1= gam(price~s(engine size,4), data=card)
```

```
## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts argument
## ignored
```

```
gam.m2= gam(price~s(engine size,4)+car width, data=card)
```

```
## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts argument
## ignored
```

```
anova(gam.m1, gam.m2, gam.m3, test="F")
```

```
## Analysis of Deviance Table
```

```
##
```

```
## Model 1: price ~ s(engine size, 4)
```

```
## Model 2: price ~ s(engine size, 4) + car width
```

```
## Model 3: price ~ s(engine size, 4) + s(car width, 3)
```

```
##   Resid. Df Resid. Dev    Df Deviance      F      Pr(>F)
```

```
## 1      200 2576573634
```

```
## 2      199 2132406763 1.0000 444166871 41.4787 8.919e-10 ***
```

```
## 3      197 2109535663 2.0002 22871101  1.0678  0.3457
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

In previous section we define three models (1. Linear and smooth spline 2. add features, linear and smooth spline 3. Smooth spline for two features and linear for the other) and compare the significance of going from one model to another.

The model summary of model are are given below

```
summary(gam.m3)

##
## Call: gam(formula = price ~ s(engineSize, 4) + s(carwidth, 3), data = card)
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -7913.5 -1767.3  -656.6  1447.1 13783.1
##
## (Dispersion Parameter for gaussian family taken to be 10708319)
##
##      Null Deviance: 13019639362 on 204 degrees of freedom
## Residual Deviance: 2109535663 on 196.9997 degrees of freedom
## AIC: 3909.844
##
## Number of Local Scoring Iterations: 2
##
## Anova for Parametric Effects
##              Df      Sum Sq    Mean Sq F value    Pr(>F)
## s(engineSize, 4)   1 9870407416 9870407416  921.75 < 2.2e-16 ***
## s(carwidth, 3)     1  492363471  492363471   45.98 1.359e-10 ***
## Residuals         197 2109535663   10708319
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##              Npar Df  Npar F      Pr(F)
## (Intercept)
## s(engineSize, 4)      3 15.1189 6.791e-09 ***
## s(carwidth, 3)       2  1.8634  0.1579
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We find the significance of the polynomial features and spline models.

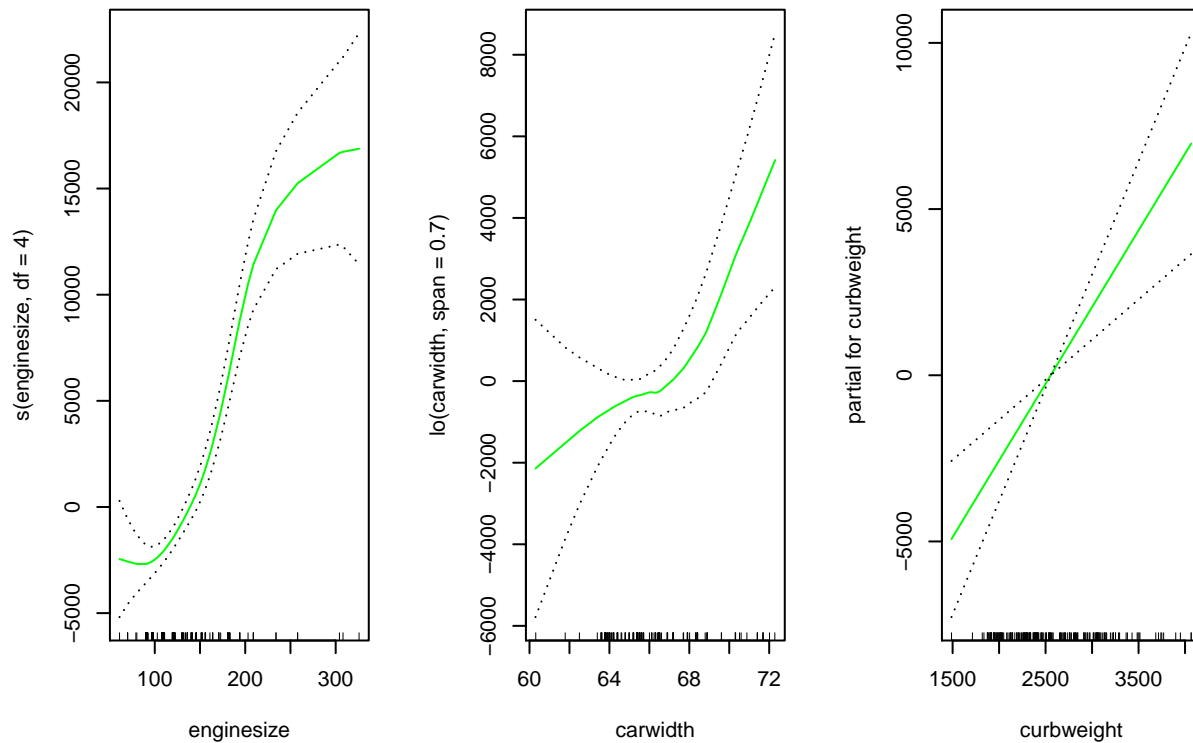
We use the predict and gam library to plot the prediction of the models in the next section with different additive models.

```
preds =predict(gam.m2, newdata=card)

par(mfrow = c(1,3))
gam.lo= gam(price~s(engineSize, df=4)+lo(carwidth, span = 0.7)
          +curbweight, data=card)

## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts argument
## ignored
```

```
plot.Gam(gam.lo, se=TRUE, col="green")
```

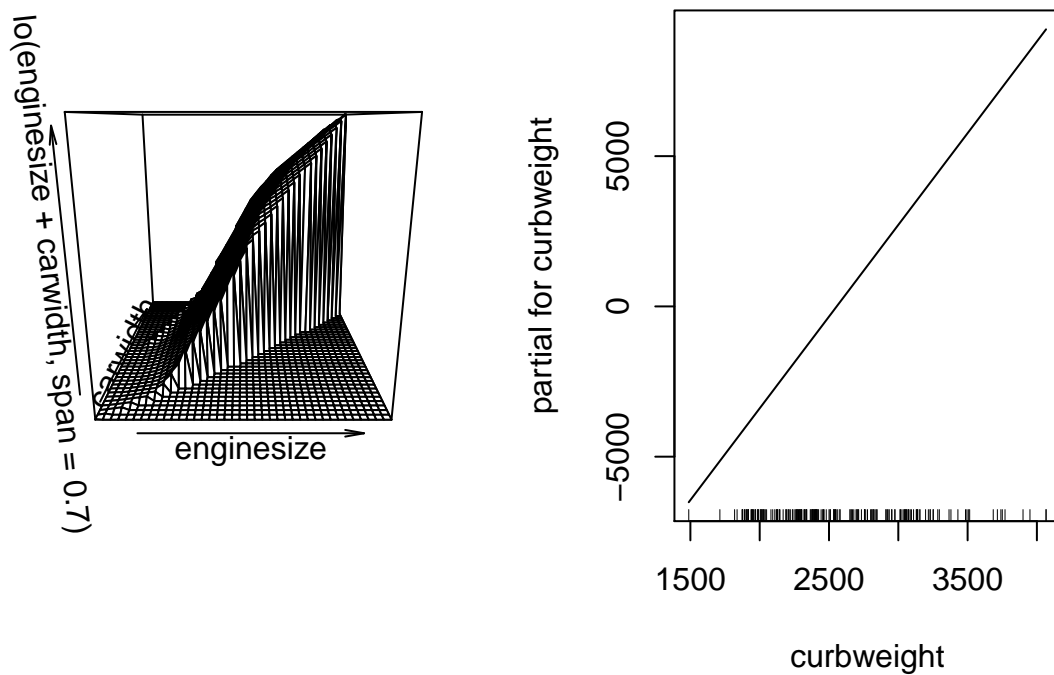


```
gam.lo.i= gam(price~lo(engine size+carwidth, span = 0.7)
              +curbweight, data=card)
```

```
## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts argument
## ignored
```

We can use akima to see the two dimension of plot for two variable for the car price dataset against the carwidth and engine size variable.

```
library(akima)
par(mfrow = c(1,2))
plot(gam.lo.i)
```

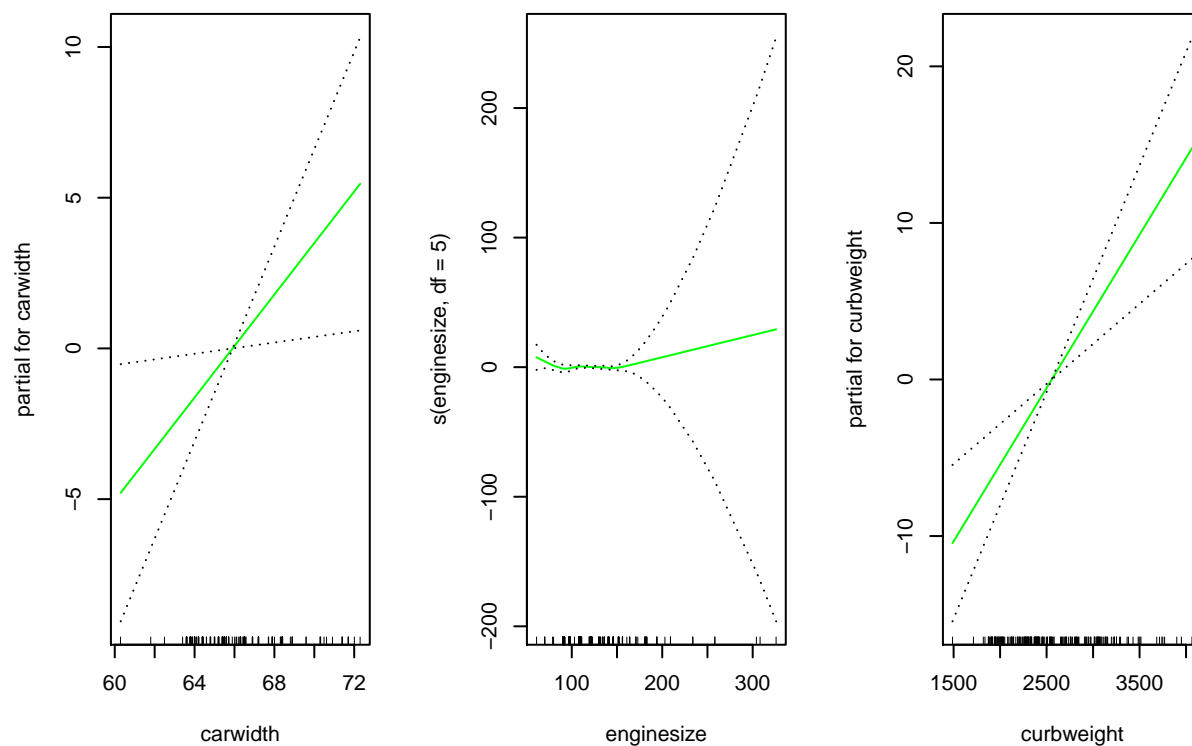


```
gam.lr = gam(I(price>10400)~carwidth+s(engine size, df=5)
             +curbweight, family = binomial, data=card)
```

```
## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts argument
## ignored
```

```
par(mfrow = c(1,3))
```

```
plot(gam.lr, se=T, col="green")
```



```
table(curbweight, I(price>15000))
```

```
##
##  curbweight  FALSE  TRUE
##      1488      1    0
##      1713      1    0
##      1819      1    0
##      1837      1    0
##      1874      2    0
##      1876      2    0
##      1889      1    0
##      1890      1    0
##      1900      1    0
##      1905      1    0
##      1909      2    0
##      1918      3    0
##      1938      1    0
##      1940      1    0
##      1944      1    0
##      1945      1    0
##      1950      1    0
##      1951      1    0
##      1956      1    0
##      1967      2    0
##      1971      1    0
```



##	1985	1	0
##	1989	3	0
##	2004	1	0
##	2008	1	0
##	2010	1	0
##	2015	1	0
##	2017	1	0
##	2024	2	0
##	2028	1	0
##	2037	1	0
##	2040	1	0
##	2050	1	0
##	2081	1	0
##	2094	1	0
##	2109	1	0
##	2120	1	0
##	2122	1	0
##	2128	2	0
##	2140	1	0
##	2145	2	0
##	2169	1	0
##	2190	1	0
##	2191	2	0
##	2204	1	0
##	2209	1	0
##	2212	1	0
##	2221	1	0
##	2236	1	0
##	2240	1	0
##	2254	1	0
##	2261	1	0
##	2264	1	0
##	2265	1	0
##	2275	3	0
##	2280	1	0
##	2289	1	0
##	2290	2	0
##	2293	1	0
##	2300	2	0
##	2302	1	0
##	2304	1	0
##	2319	1	0
##	2324	1	0
##	2326	1	0
##	2328	1	0
##	2337	2	0
##	2340	1	0
##	2365	1	0
##	2370	1	0
##	2372	1	0
##	2380	2	0
##	2385	4	0
##	2395	0	2
##	2403	2	0

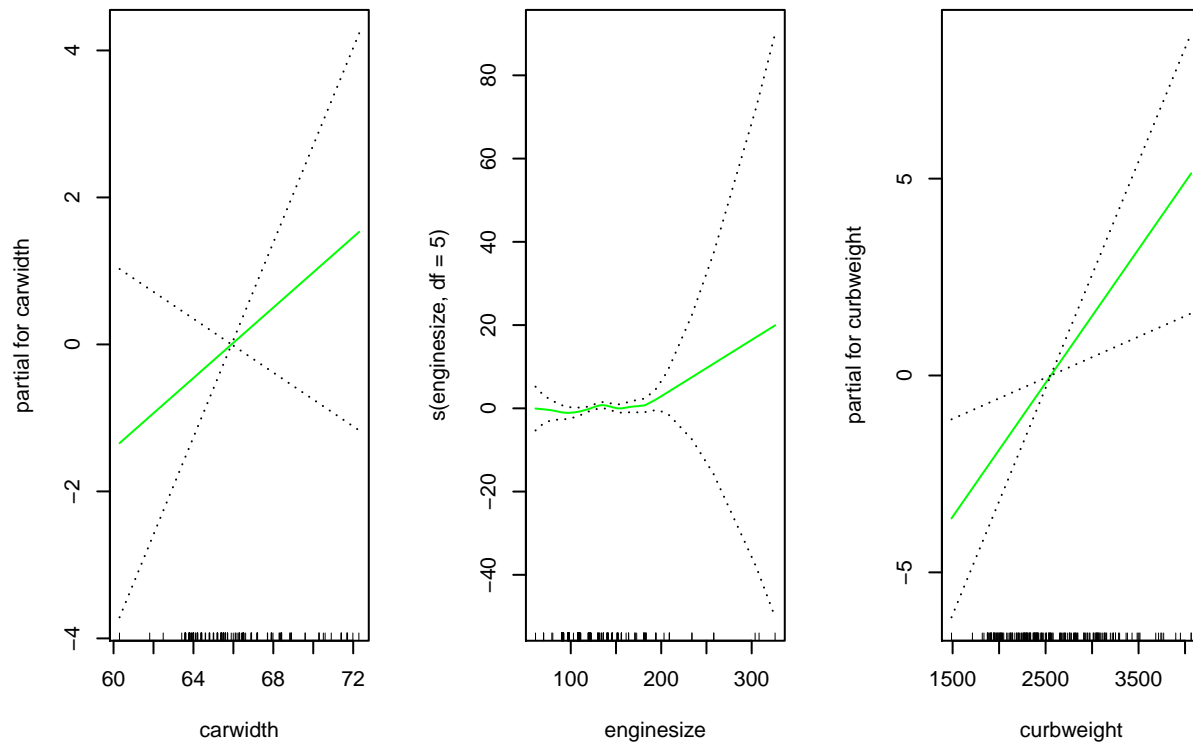
##	2405	1	0
##	2410	2	0
##	2414	2	0
##	2420	1	0
##	2425	1	0
##	2443	1	0
##	2455	1	0
##	2458	1	0
##	2460	1	0
##	2465	1	0
##	2480	1	0
##	2500	0	1
##	2507	0	1
##	2510	1	0
##	2535	2	0
##	2536	1	0
##	2540	1	0
##	2548	1	1
##	2551	1	0
##	2563	1	0
##	2579	2	0
##	2650	1	0
##	2658	1	0
##	2661	1	0
##	2670	0	1
##	2679	1	0
##	2695	1	0
##	2700	0	1
##	2707	0	1
##	2710	0	1
##	2714	1	0
##	2734	1	0
##	2756	0	2
##	2758	0	1
##	2765	0	1
##	2778	0	1
##	2800	0	1
##	2808	0	1
##	2811	1	0
##	2818	1	0
##	2823	0	1
##	2824	0	1
##	2833	1	0
##	2844	0	1
##	2847	0	1
##	2910	0	1
##	2912	1	0
##	2921	1	0
##	2926	1	0
##	2935	0	1
##	2952	0	1
##	2954	0	1
##	2975	0	1
##	2976	0	1

```
##      3012      0      1
##      3016      0      1
##      3020      1      0
##      3034      1      0
##      3042      0      1
##      3045      0      1
##      3049      0      1
##      3053      0      1
##      3055      0      1
##      3060      1      0
##      3062      0      1
##      3071      0      1
##      3075      0      2
##      3086      0      1
##      3095      1      0
##      3110      1      0
##      3130      0      1
##      3131      0      1
##      3139      0      2
##      3151      0      1
##      3157      0      1
##      3197      1      0
##      3217      0      1
##      3230      1      1
##      3252      0      2
##      3285      0      1
##      3296      1      0
##      3366      0      1
##      3380      0      1
##      3430      1      0
##      3485      0      1
##      3495      0      1
##      3505      0      1
##      3515      0      1
##      3685      0      1
##      3715      0      1
##      3740      0      1
##      3750      0      1
##      3770      0      1
##      3900      0      1
##      3950      0      1
##      4066      0      2
```

```
gam.lr.s = gam(I(price>15000)~carwidth+s(engine size, df=5)
               +curbweight, family = binomial, data=card)
```

```
## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts argument
## ignored
```

```
plot(gam.lr.s, se =T, col ="green")
```



## Chapter 8

In this section, I will implement different tree based methods on the car dataset.

### Classification Tree

I create class label for the car price by selecting the median as threshold value. I use the tree library to implement the classification tree.

```
library(tree)
high = ifelse(price <= 10300, "No", "Yes")

card = data.frame(card, high)

attach(card)
```

```
## The following object is masked _by_ .GlobalEnv:
```

```
##
```

```
##   high
```

```
## The following objects are masked from card (pos = 8):
```

```
##
```

```
##   aspiration, boreratio, car_ID, carbody, carheight, carlength,
```

```

##      CarName, carwidth, citympg, compressionratio, curbweight,
##      cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##      enginetype, fuelsystem, fueltype, high, highwaympg, horsepower,
##      peakrpm, price, stroke, symboling, wheelbase

## The following objects are masked from card (pos = 12):
##
##      aspiration, boreratio, car_ID, carbody, carheight, carlength,
##      CarName, carwidth, citympg, compressionratio, curbweight,
##      cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##      enginetype, fuelsystem, fueltype, high, highwaympg, horsepower,
##      peakrpm, price, stroke, symboling, wheelbase

## The following objects are masked from card (pos = 17):
##
##      aspiration, boreratio, car_ID, carbody, carheight, carlength,
##      CarName, carwidth, citympg, compressionratio, curbweight,
##      cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##      enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
##      price, stroke, symboling, wheelbase

## The following objects are masked from card (pos = 18):
##
##      aspiration, boreratio, car_ID, carbody, carheight, carlength,
##      CarName, carwidth, citympg, compressionratio, curbweight,
##      cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##      enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
##      price, stroke, symboling, wheelbase

## The following objects are masked from card (pos = 20):
##
##      aspiration, boreratio, car_ID, carbody, carheight, carlength,
##      CarName, carwidth, citympg, compressionratio, curbweight,
##      cylindernumber, doornumber, drivewheel, enginelocation, enginesize,
##      enginetype, fuelsystem, fueltype, highwaympg, horsepower, peakrpm,
##      price, stroke, symboling, wheelbase

tree.card = tree(high~fuelsystem+peakrpm+citympg
                  + enginesize+enginetype+carwidth+curbweight+carlength
                  + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
                  + enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
                  data = card)

summary(tree.card)

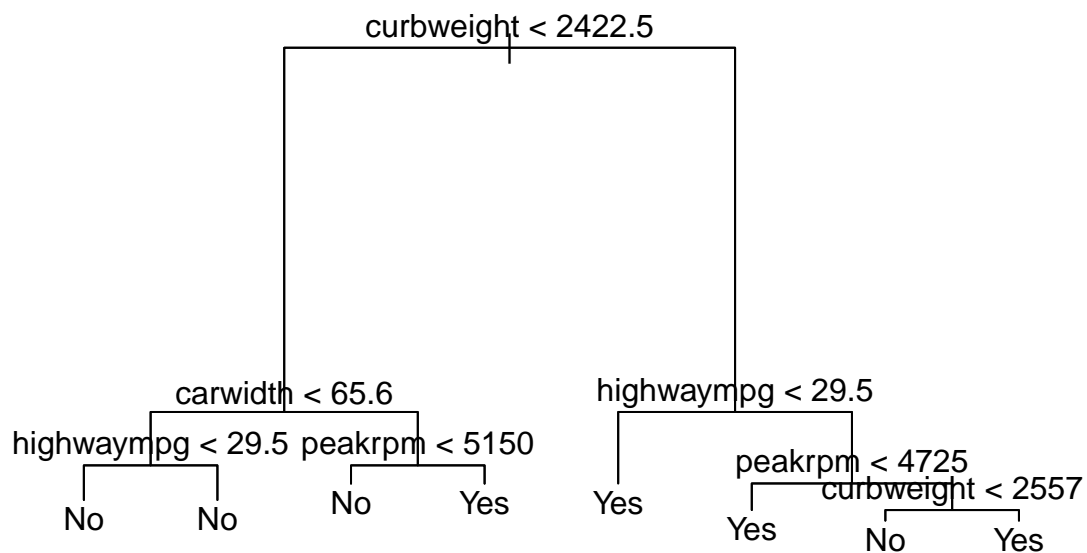
##
## Classification tree:
## tree(formula = high ~ fuelsystem + peakrpm + citympg + enginesize +
##      enginetype + carwidth + curbweight + carlength + highwaympg +
##      boreratio + stroke + wheelbase + drivewheel + enginelocation +
##      aspiration + doornumber + horsepower + compressionratio,

```

```
##      data = card)
## Variables actually used in tree construction:
## [1] "curbweight" "carwidth"  "highwaympg" "peakrpm"
## Number of terminal nodes:  8
## Residual mean deviance:  0.17 = 33.5 / 197
## Misclassification error rate: 0.03902 = 8 / 205
```

From the summary, we see the performance of the tree, it misclassified 8 instances in the training examples. We observe the graphical representation of the tree in the following section.

```
plot(tree.card)
text(tree.card, pretty= 0)
```



We see the classified tree via the classifier method. It selected curbweight as 1st label features. We can also get the description by following code

```
tree.card
```

```
## node), split, n, deviance, yval, (yprob)
##      * denotes terminal node
##
##  1) root 205 284.200 No ( 0.50244 0.49756 )
##    2) curbweight < 2422.5 104 65.840 No ( 0.90385 0.09615 )
##      4) carwidth < 65.6 92 26.440 No ( 0.96739 0.03261 )
##        8) highwaympg < 29.5 9 11.460 No ( 0.66667 0.33333 ) *
```

```
##      9) highwaympg > 29.5 83    0.000 No ( 1.00000 0.00000 ) *
##      5) carwidth > 65.6 12   16.300 Yes ( 0.41667 0.58333 )
##     10) peakrpm < 5150 7     8.376 No ( 0.71429 0.28571 ) *
##     11) peakrpm > 5150 5     0.000 Yes ( 0.00000 1.00000 ) *
##     3) curbweight > 2422.5 101 60.700 Yes ( 0.08911 0.91089 )
##     6) highwaympg < 29.5 79    0.000 Yes ( 0.00000 1.00000 ) *
##     7) highwaympg > 29.5 22   29.770 Yes ( 0.40909 0.59091 )
##    14) peakrpm < 4725 8     0.000 Yes ( 0.00000 1.00000 ) *
##    15) peakrpm > 4725 14   18.250 No ( 0.64286 0.35714 )
##    30) curbweight < 2557 8     6.028 No ( 0.87500 0.12500 ) *
##    31) curbweight > 2557 6     7.638 Yes ( 0.33333 0.66667 ) *
```

The above result shows the rule of tree classifier for classifying car price as high or low.

Now I use the validation set to test the model performance. The model is trained only on the training instances.

```
set.seed(2)

train = sample(1:nrow(card), 150)
card.test = card[-train,]
high.test = high[-train]
tree.card = tree(high~fuelsystem+peakrpm+citympg
                  + enginesize+engine+carwidth+curbweight+carlength
                  + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
                  + enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
                  data = card, subset=train)

tree.pred = predict(tree.card, card.test, type = "class")

table(tree.pred, high.test)

##      high.test
## tree.pred No Yes
##      No  25  4
##      Yes  1 25
```

From the table above we find that the classifier was able to correctly classify 50 instances out of 54 test instances. It was trained on 150 training examples.

```
set.seed(3)

cv.card = cv.tree(tree.card, FUN=prune.misclass)
names(cv.card)

## [1] "size" "dev" "k" "method"

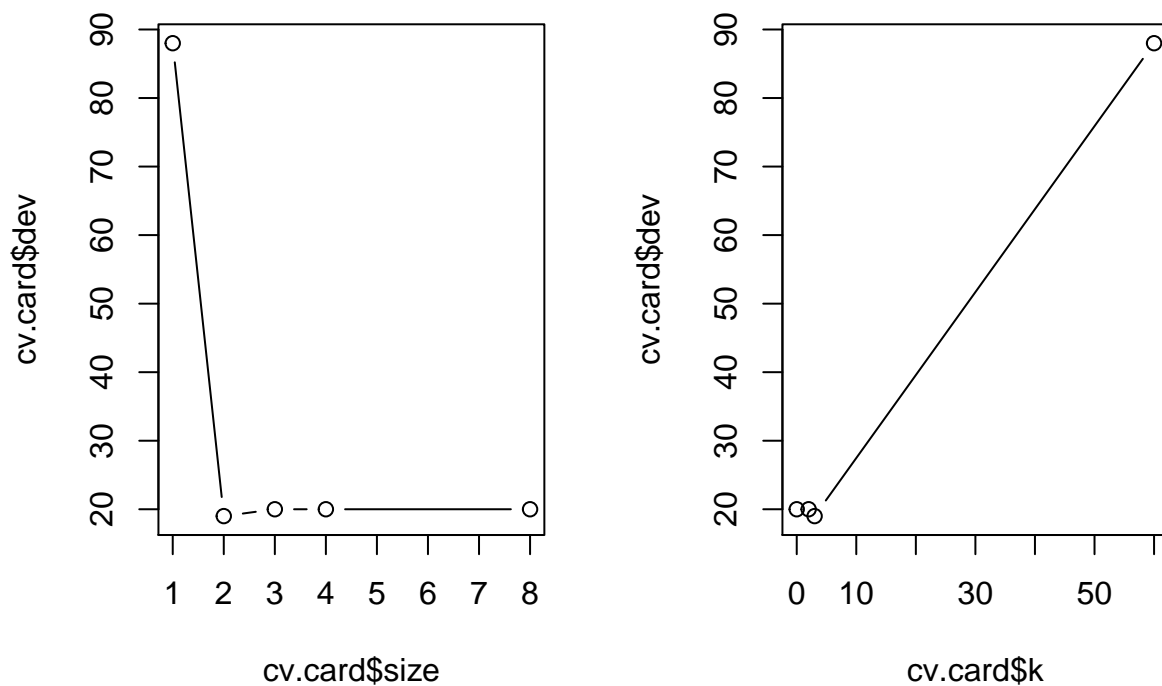
cv.card

## $size
## [1] 8 4 3 2 1
##
```

```
## $dev
## [1] 20 20 20 19 88
##
## $k
## [1] -Inf    0     2     3    60
##
## $method
## [1] "misclass"
##
## attr("class")
## [1] "prune"          "tree.sequence"
```

The dev corresponds to misclassification. For 2 the misclassification rate is minimum for the tree method.

```
par(mfrow= c(1,2))
plot(cv.card$size, cv.card$dev, type="b")
plot(cv.card$k, cv.card$dev, type="b")
```

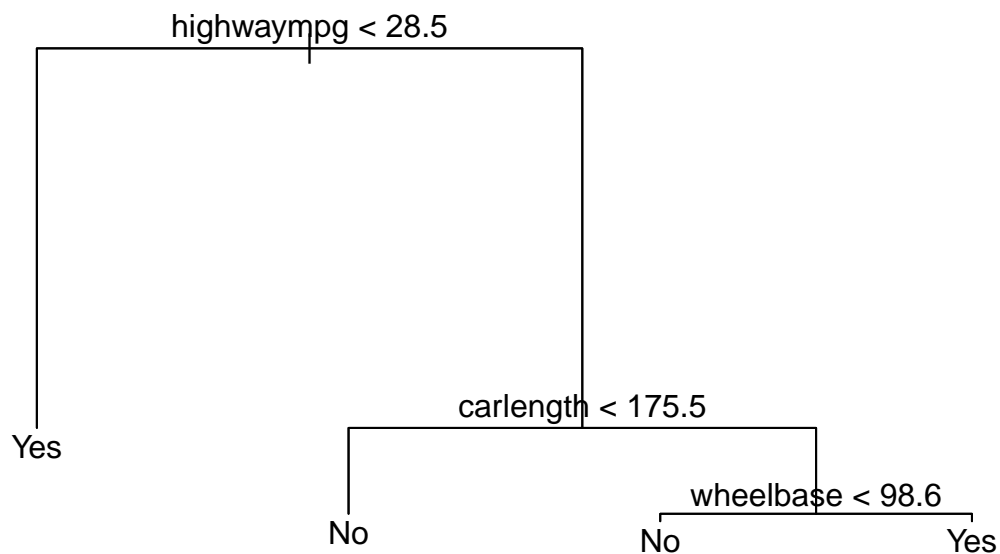


The result shows that with tree size the misclassification decreases.

We now create a 4 node tree using prune missclassification.

```
prune.card = prune.misclass(tree.card, best=4)
plot(prune.card)
text(prune.card, pretty = 0)
```





```
tree.pred = predict(prune.card, card.test, type = "class")
table(tree.pred, high.test)
```

```
##           high.test
## tree.pred No  Yes
##      No   24   4
##      Yes   2  25
```

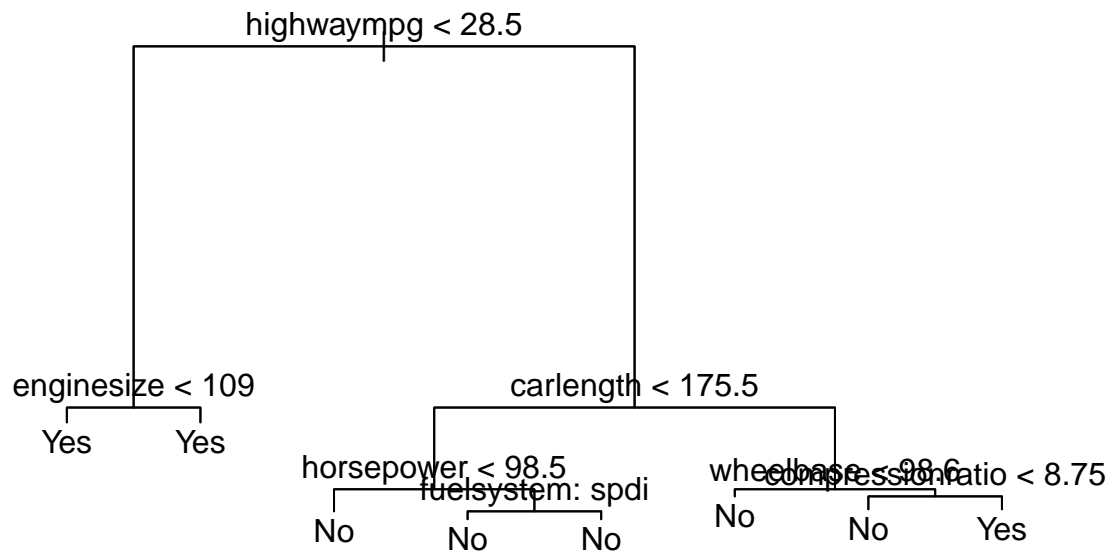
The previous result shows 6 misclassification result on the validation set. The accuracy is 89%.

```
(24+25)/(26+29)
```

```
## [1] 0.8909091
```

Now we fit the tree model for different tree size and check model performance. I use more tree label than earlier example.

```
prune.card = prune.misclass(tree.card, best=6)
plot(prune.card)
text(prune.card, pretty = 0)
```



```
tree.pred = predict(prune.card, card.test, type = "class")
table(tree.pred, high.test)
```

```
##           high.test
## tree.pred No  Yes
##      No   26   5
##      Yes   0  24
```

We see than the error has decrease in the result as we have used bigger tree.

## Fitting regression trees

Similar to classifier, we first fit the model,

```
set.seed(1)

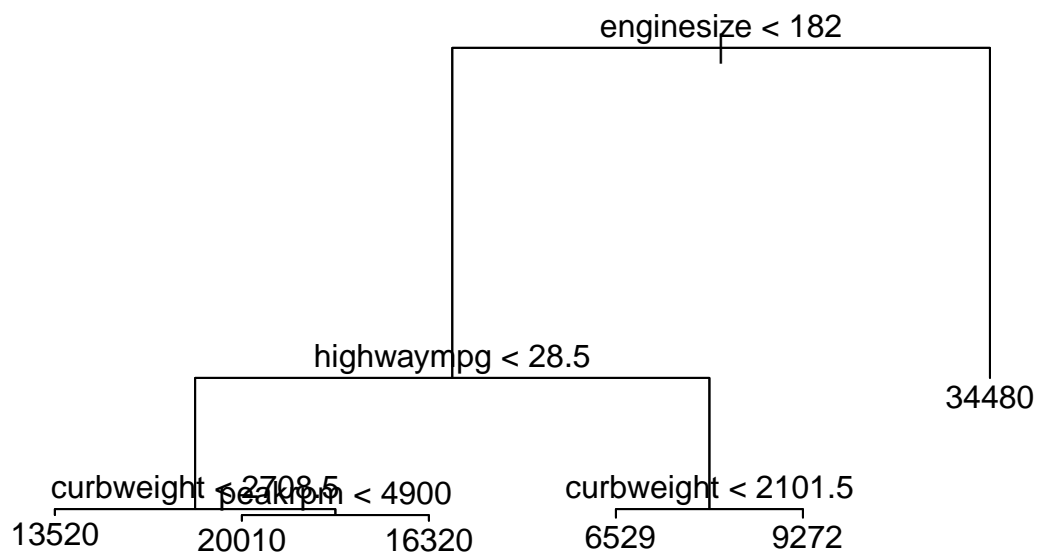
train = sample(1:nrow(card), nrow(card)/2)
tree.card = tree(price~fuelsystem+peakrpm+citympg
  + engine size+engine type+carwidth+curbweight+carlength
  + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
  + enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
  data = card, subset=train)

summary(tree.card)
```

```
##
## Regression tree:
## tree(formula = price ~ fuelsystem + peakrpm + citympg + enginesize +
##       enginetype + carwidth + curbweight + carlength + highwaympg +
##       boreratio + stroke + wheelbase + drivewheel + enginelocation +
##       aspiration + doornumber + horsepower + compressionratio,
##       data = card, subset = train)
## Variables actually used in tree construction:
## [1] "enginesize" "highwaympg" "curbweight" "peakrpm"
## Number of terminal nodes: 6
## Residual mean deviance: 6695000 = 642700000 / 96
## Distribution of residuals:
##      Min.   1st Qu.   Median     Mean   3rd Qu.     Max.
## -8926.00 -1193.00   -19.28     0.00  1227.00 10920.00
```

In summary the model finds four variables to decide the car price. It created 6 nodes tree. The graphical structure is as follow;

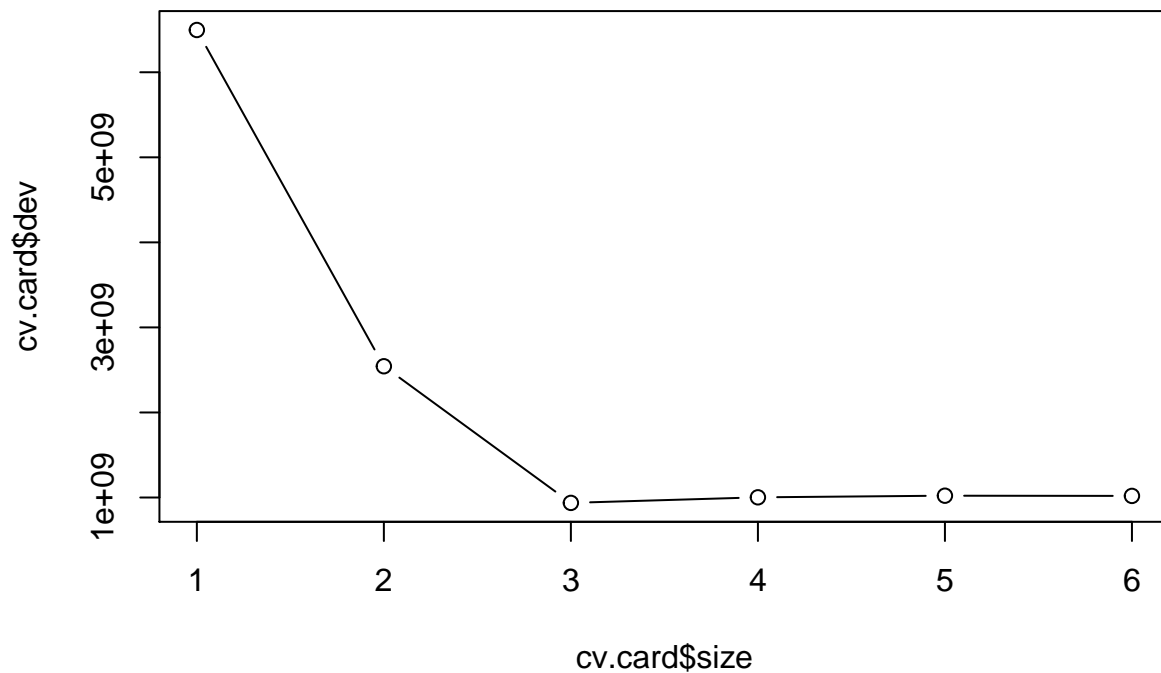
```
plot(tree.card)
text(tree.card, pretty = 0)
```



In above figure, we see the 6 node tree found by the model.

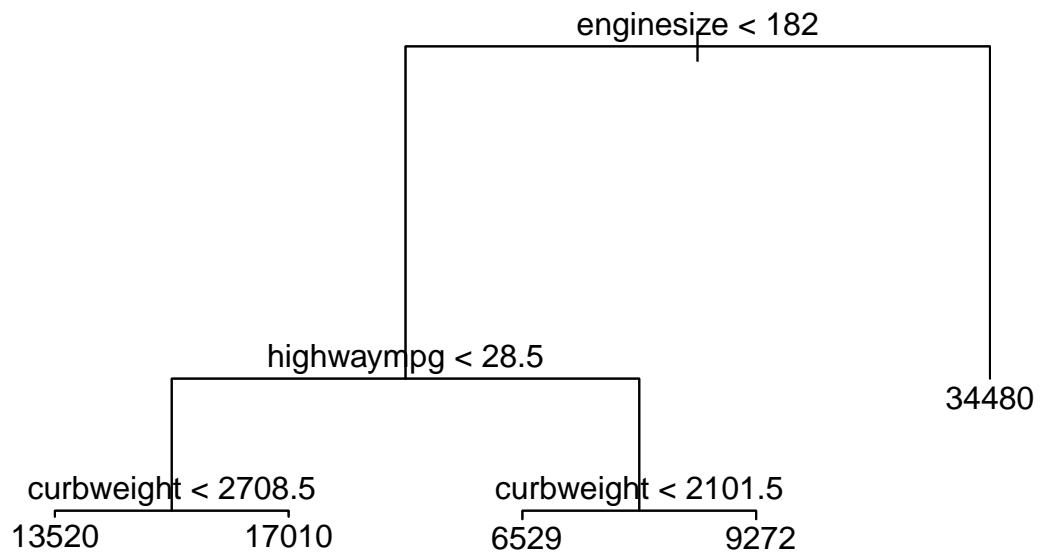
We apply pruning to check model performance across the tree size.

```
cv.card = cv.tree(tree.card)
plot(cv.card$size, cv.card$dev, type = "b")
```



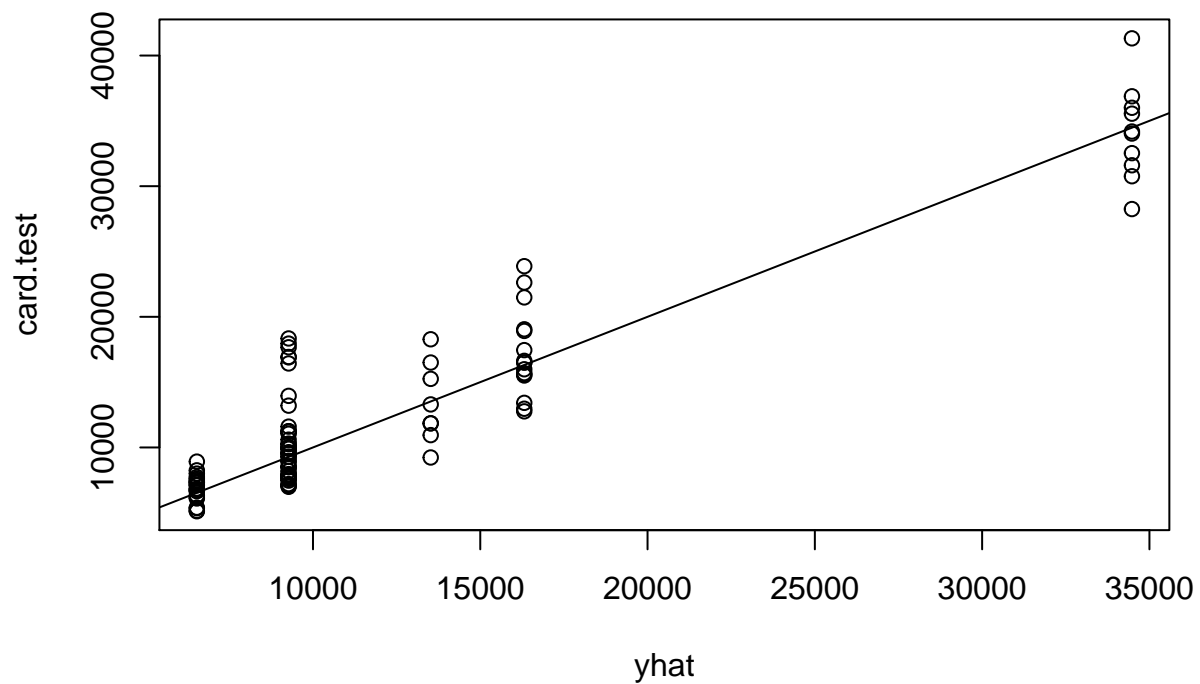
The above graph shows tree size vs error graph for the car price using regression tree.

```
prune.card = prune.tree(tree.card, best=5)
plot(prune.card)
text(prune.card, pretty = 0)
```



We controlled the tree size by specifying 5. The plot shows the estimated 5 node tree for the car price dataset.

```
yhat = predict(tree.card, newdata= card[-train,])
card.test = card[-train, "price"]
plot(yhat, card.test)
abline(0,1)
```



```
mean((yhat-card.test)^2)
```

```
## [1] 8637294
```

In last example, we see the tree performance using the cross-validation approach. We find the final error on test set is 8637294.

In next we will see the bagging and random forest and compare the error result.

## Bagging and Random forest

In this part we will apply bagging and random forest by randomforest function.

### Bagging

The bagging is special case of random forest with considering all the features at a time.

```
library(randomForest)
```

```
## randomForest 4.6-14
```

```
## Type rfNews() to see new features/changes/bug fixes.
```

```

set.seed(1)
bag.card = randomForest(price~fuelsystem+peakrpm+citympg
                        + enginesize+enginetype+carwidth+curbweight+carlength
                        + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
                        + enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
                        data = card, subset=train, mtry =18, importance =TRUE)

bag.card

##
## Call:
## randomForest(formula = price ~ fuelsystem + peakrpm + citympg +      enginesize + enginetype + carw
##              Type of random forest: regression
##              Number of trees: 500
## No. of variables tried at each split: 18
##
##              Mean of squared residuals: 6074559
##              % Var explained: 90.13

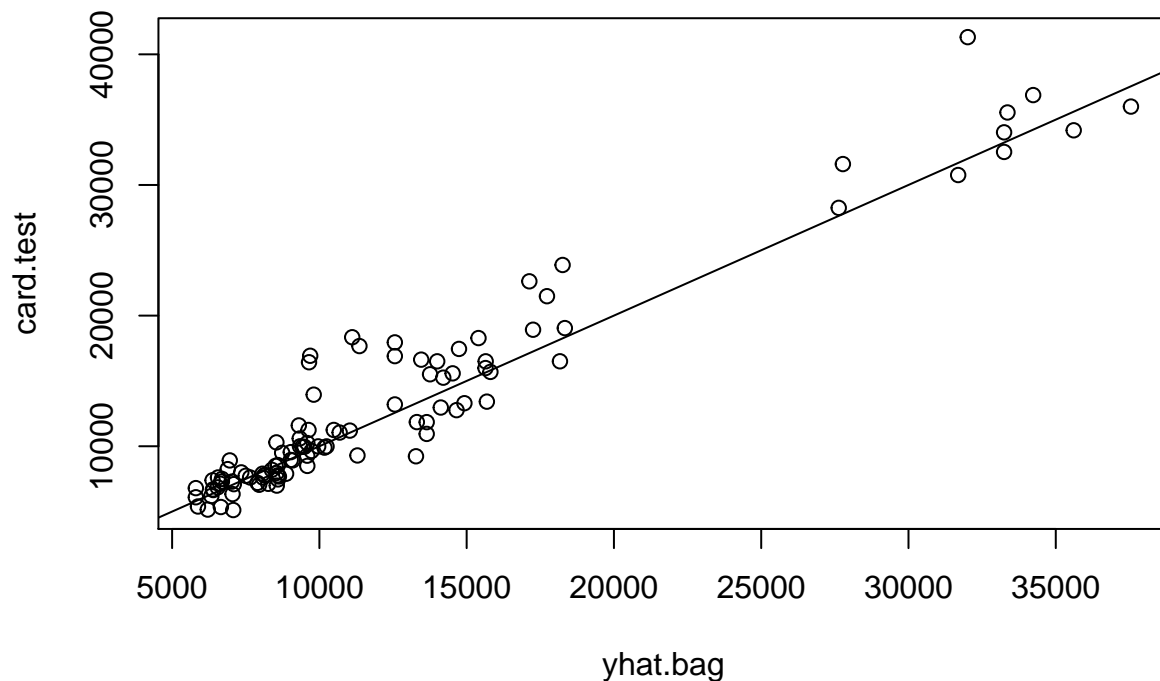
```

We specified `mtry = 18`, as we have 18 features for the car price dataset. The random forest follows bagging approach for estimation.

```

yhat.bag = predict(bag.card, newdata = card[-train,])
plot(yhat.bag, card.test)
abline(0,1)

```



```
mean((yhat.bag - card.test)^2)
```

```
## [1] 5694545
```

In the cross-validation approach we observe the error of 5694545. This is smaller than tree regression method since bagging combines result different regressor.

We can also control the number of tree in bagging and and check its performance on the test result.

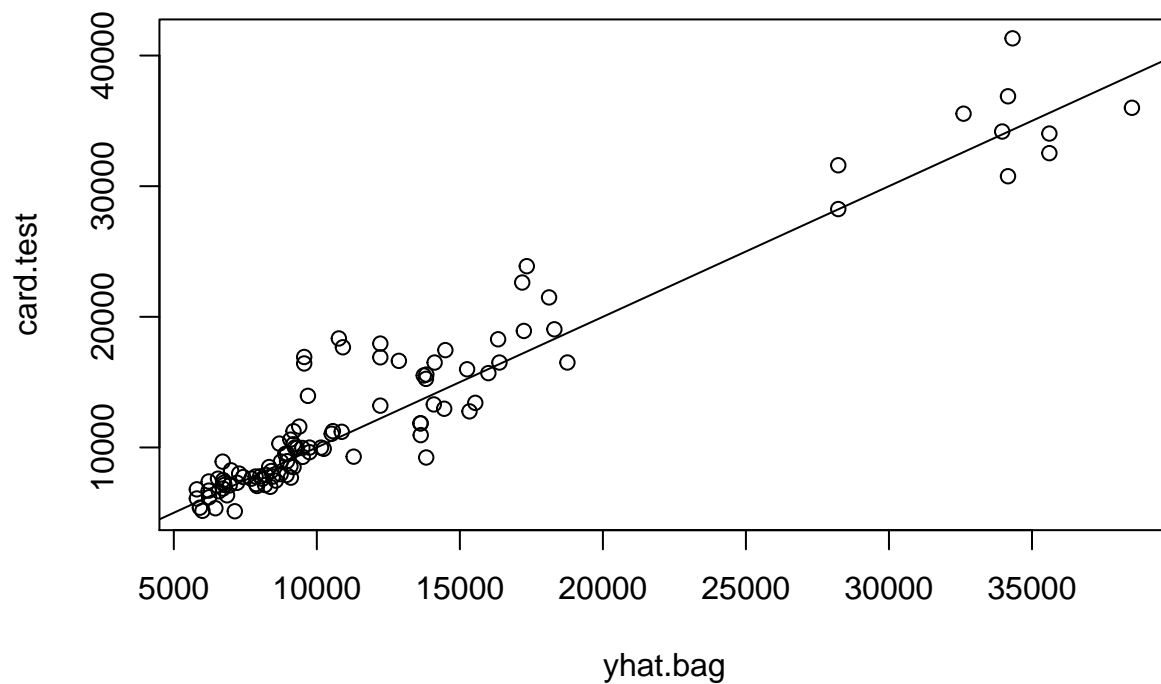
```
set.seed(1)
bag.card = randomForest(price~fuelsystem+peakrpm+citympg
                        + enginesize+enginetype+carwidth+curbweight+carlength
                        + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
                        + enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
                        data = card, subset=train, mtry =18, ntree = 25)
bag.card
```

```
##
## Call:
## randomForest(formula = price ~ fuelsystem + peakrpm + citympg +      enginesize + enginetype + carw
##               Type of random forest: regression
##               Number of trees: 25
## No. of variables tried at each split: 18
##
##               Mean of squared residuals: 5560265
##               % Var explained: 90.97
```

We evaluate the bagging model on the validation dataset.

```
yhat.bag = predict(bag.card, newdata = card[-train,])
plot(yhat.bag, card.test)
abline(0,1)
```





```
mean((yhat.bag - card.test)^2)
```

```
## [1] 6000532
```

We find the final error is 6000532, similar to earlier bagging method but smaller than decision tree regression method.

## Random forest

By controlling mtry parameters we implement random forest over the car dataset. I used  $18/3 = 6$  features for the car data.

```
set.seed(1)
rf.card = randomForest(price~fuelsystem+peakrpm+citympg
                        + enginesize+enginetype+carwidth+curbweight+carlength
                        + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
                        + enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
                        data = card, subset=train, mtry = 6, importance=TRUE)
yhat.rf = predict(rf.card, newdata = card[-train, ])
mean((yhat.rf - card.test)^2)
```

```
## [1] 5559902
```

Using random forest we find the error of 5559902 comparable to the bagging method by using 6 features at maximum each time. The random forest also better estimates than the decision tree method.

We can also observe the variable importance in the random forest in r.

```
importance(rf.card)
```

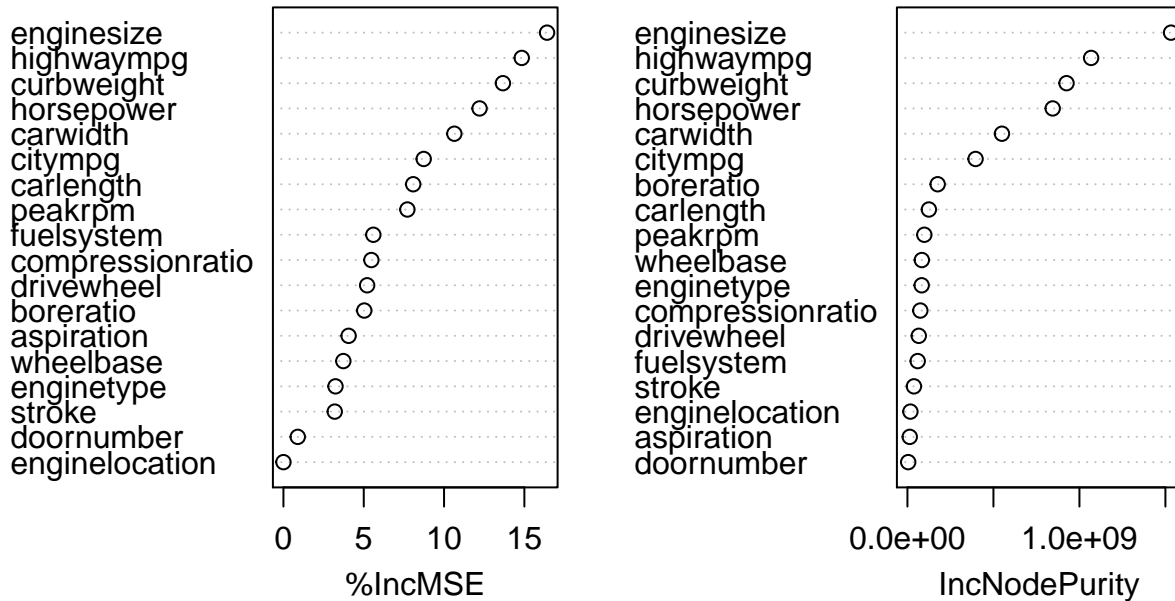
##	%IncMSE	IncNodePurity
## fuelsystem	5.5964884	59649716
## peakrpm	7.7166068	98176631
## citympg	8.7319301	396102906
## enginesize	16.4120100	1532751906
## enginetype	3.2414265	82863997
## carwidth	10.6398536	549513349
## curbweight	13.6639477	925056694
## carlength	8.0778921	124732748
## highwaympg	14.8362882	1067921018
## boreratio	5.0328482	175789746
## stroke	3.1947744	37816082
## wheelbase	3.7281764	83666728
## drivewheel	5.2116266	65821154
## enginelocation	0.0000000	17265212
## aspiration	4.0570564	13486013
## doornumber	0.8913836	4503901
## horsepower	12.2105869	844513965
## compressionratio	5.4739152	74924174

From the importance we see that, enginesize is the most important feature for the random forest. Since enginesize gets maximum value in the random forest approach.

We can also plot their respective importance.

```
varImpPlot(rf.card)
```

## rf.card



We can see the previous result in the plot in this section. The engine size gets chosen as the best important feature.

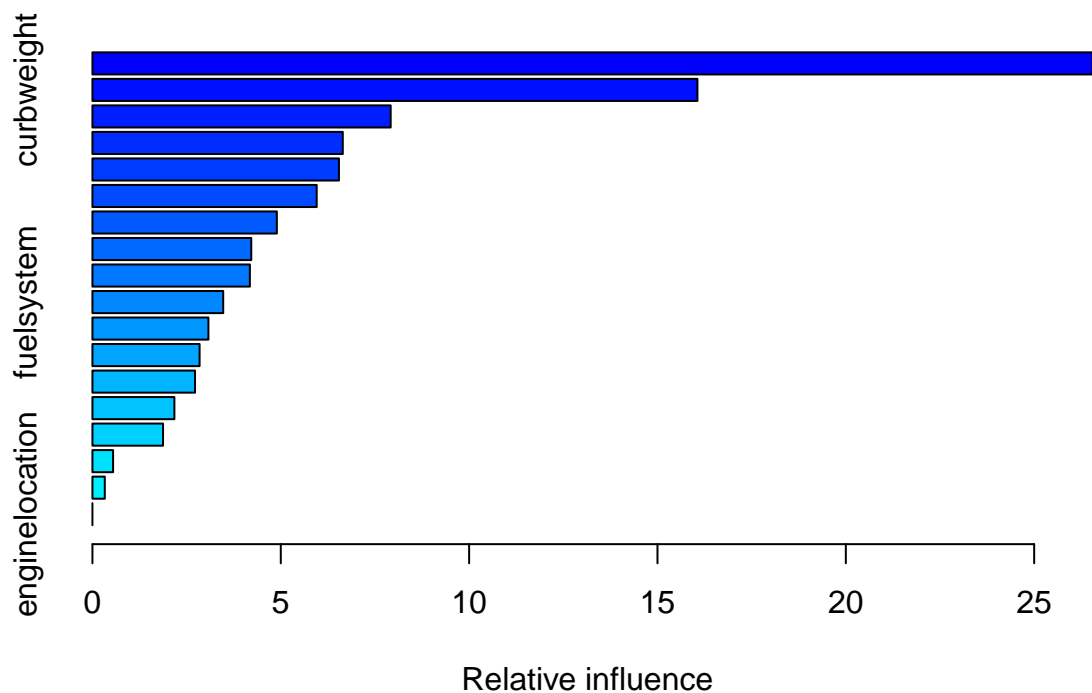
## boosting

We will use gbm package for applying boosting over the car price dataset. In the r we can selection the interaction option in boosting method. In my experiment, I have chosen 6.

```
library(gbm)
```

```
## Loaded gbm 2.1.5
```

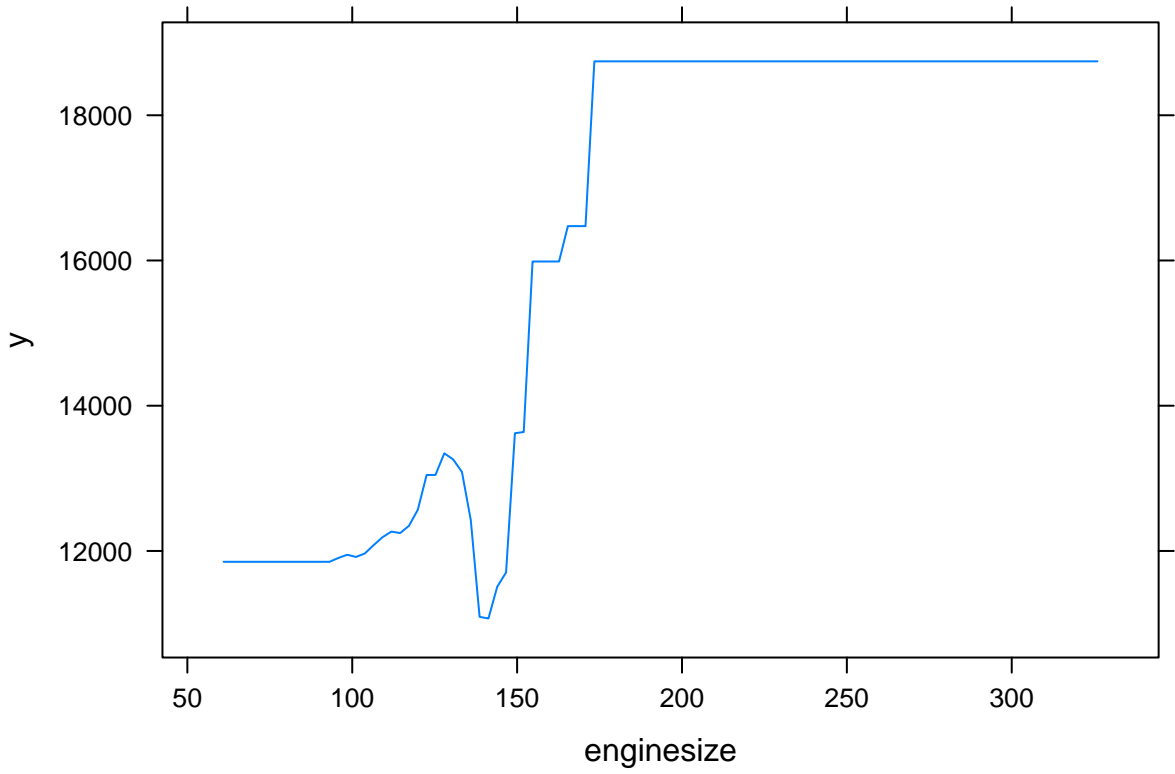
```
set.seed(1)
boost.card = gbm(price~fuelssystem+peakrpm+citympg
  + enginesize+engine type+carwidth+curbweight+carlength
  + highwaympg+ boreratio+ stroke + wheelbase + drivewheel
  + enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
  data = card[-train,], distribution = "gaussian", n.trees = 5000,
  interaction.depth = 6)
summary(boost.card)
```



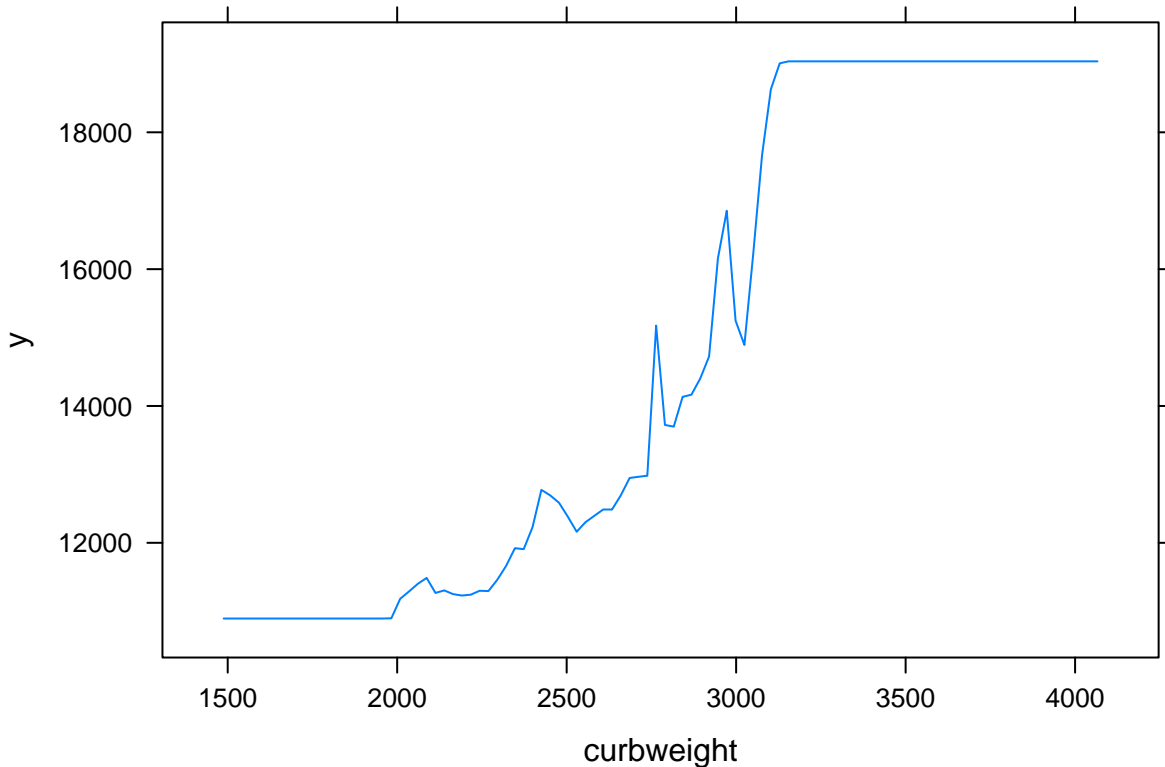
```
##           var      rel.inf
## enginesize  enginesize 26.5446690
## curbweight curbweight 16.0603711
## highwaympg highwaympg  7.9170659
## carwidth   carwidth   6.6460265
## carlength  carlength  6.5441080
## horsepower horsepower  5.9538873
## wheelbase  wheelbase  4.8950393
## boreratio  boreratio  4.2160147
## citympg    citympg    4.1796560
## fuelsystem fuelsystem  3.4713013
## enginetype enginetype  3.0789409
## compressionratio compressionratio 2.8443091
## stroke      stroke     2.7229956
## peakrpm     peakrpm    2.1756799
## drivewheel  drivewheel  1.8741826
## doornumber  doornumber  0.5477344
## aspiration  aspiration  0.3280184
## enginelocation enginelocation 0.0000000
```

We again see that the most important feature in boosting is enginesize as it gets most rel.inf parameter of 26.5447

```
par(mfrow = c(1,2))
plot(boost.card, i ="enginesize")
```



```
plot(boost.card, i= "curbweight")
```



In above plots we see the car price estimation based on the enginesize and curbweight predictors.

```
yhat.boost = predict(boost.card, newdata = card[-train,], n.trees = 5000)
mean((yhat.boost - card.test)^2)
```

```
## [1] 215913.8
```

We estimate the cross validation error for boosting method and found the value of 215913, smaller than both the random forest and decision trees. The tree performed better in the car price than the linear methods.

Finally we experiment with the shrinkage parameter lambda.

```
boost.card = gbm(price~fuelsystem+peakrpm+citympg
+ enginesize+engine+type+carwidth+curbweight+carlength
+ highwaympg+ boreratio+ stroke + wheelbase + drivewheel
+ enginelocation+ aspiration+ doornumber+ horsepower+ compressionratio,
data = card[-train,], distribution = "gaussian", n.trees = 5000,
interaction.depth = 4, shrinkage = 0.2, verbose = F)
yhat.boost = predict(boost.card, newdata = card[-train,], n.trees = 5000)
mean((yhat.boost - card.test)^2)
```

```
## [1] 209760
```

By tuning lambda, we get a little better result (209460) in the cross validation data compared to earlier boosting cross-validation result (215913.8).

## Chapter 9

To apply the support vector classifier and SVM we will use e1071 library in R.

### Support Vector Classifier

First I format the car dataset for training the support vector classifier. I have used engineize and curbweight. We also created class label for the target using the median value for the car price.

```
set.seed(1)

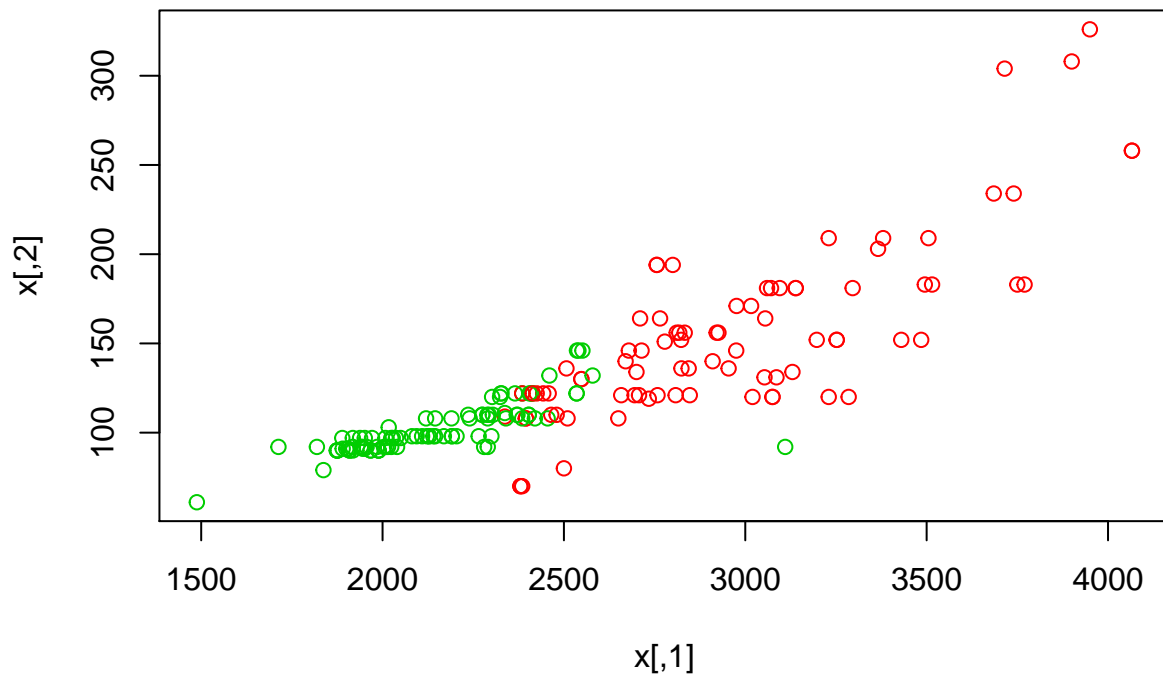
high = ifelse(price<=10400, 0, 1)
y = high
cutlen = 180 # upto 205

x = matrix( c(curbweight[1:cutlen], enginesize[1:cutlen]),ncol = 2, nrow = cutlen) #very important
y = high[1:cutlen]

cardshort = data.frame(x = x, y = as.factor(y))
attach(cardshort)

## The following object is masked _by_ .GlobalEnv:
##
##      y

plot(x, col = (3-y))
```



We see the distribution of the car price for the two variables. The car price red denotes high price.

I use the svm function from the library to implement car price classifier based on the two features I selected earlier.

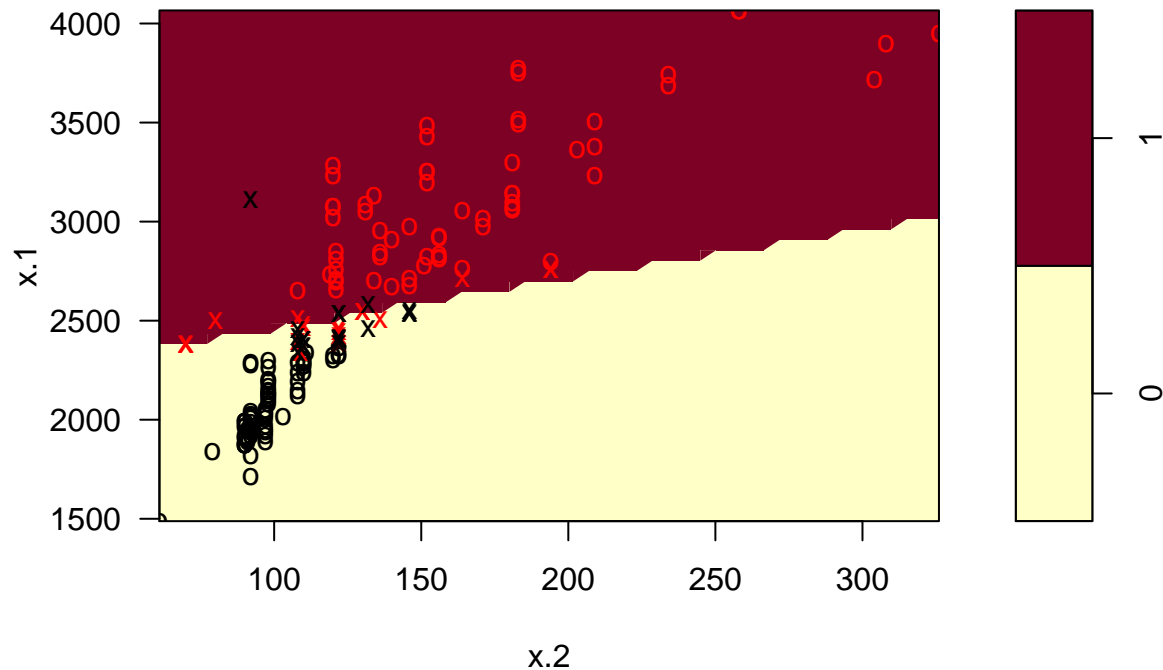
```
library(e1071)
svmfit = svm(y~., data = cardshort, kernel = "linear", cost = 20, scale = FALSE)
```

As we have already trained the SVM classifier we can plot the classifier by as follow

```
plot(svmfit, cardshort)
```



## SVM classification plot



We see that, the two region separated the car prices. The red region return 1 and the greyish area return 0 in the feature space.

```
svmfit$index
```

```
## [1] 1 2 4 6 11 12 13 42 56 57 58 59 62 64 65 127 128 146 175
## [20] 177 178 29 41 60 61 63 81 87 88 89 124 131 132 144 145 148 149 156
## [39] 168 169 170 176
```

```
summary(svmfit)
```

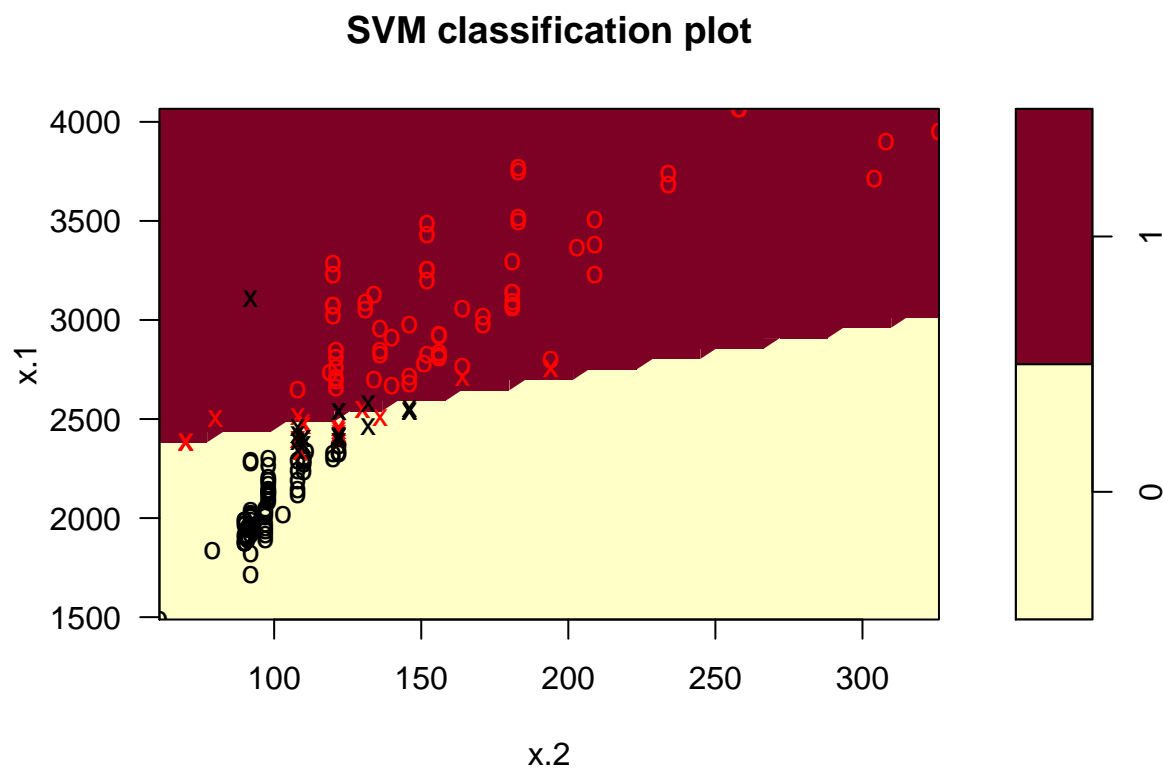
```
##
## Call:
## svm(formula = y ~ ., data = cardshort, kernel = "linear", cost = 20,
##      scale = FALSE)
##
##
## Parameters:
##   SVM-Type:  C-classification
##   SVM-Kernel: linear
##       cost:  20
##
## Number of Support Vectors:  42
##
## ( 21 21 )
##
```

```
##
## Number of Classes: 2
##
## Levels:
## 0 1
```

Here we see that linear kernel was used with cost 20.

We can change the cost parameters to smaller value and conduct experiment on the car price dataset.

```
svmfit = svm(y~., data = cardshort, kernel = "linear", cost = 0.01, scale = FALSE)
plot(svmfit, cardshort)
```



```
svmfit$index
```

```
## [1] 1 2 4 6 11 12 13 42 56 57 58 59 62 64 65 127 128 146 175
## [20] 177 178 29 41 60 61 63 81 87 88 89 124 131 132 144 145 148 149 156
## [39] 168 169 170 176
```

With the cost we can control the number of support vector. With smaller cost parameter we find higher number of support vectors.

Now we sweep the value of cost in implement svm for different cost values

```
set.seed(1)

tune.out = tune(svm,y~.,
                data = cardshort, ranges = list(cost =c(0.001, 0.01, 0.1, 1 ,5 ,10, 100)) )
```

We can observe the summary of the model as follows.

```
summary(tune.out)

##
## Parameter tuning of 'svm':
##
## - sampling method: 10-fold cross validation
##
## - best parameters:
##   cost
##   100
##
## - best performance: 0.08333333
##
## - Detailed performance results:
##   cost      error dispersion
## 1 1e-03 0.46666667 0.10861391
## 2 1e-02 0.46666667 0.10861391
## 3 1e-01 0.11111111 0.05237828
## 4 1e+00 0.11111111 0.06415003
## 5 5e+00 0.08888889 0.05367177
## 6 1e+01 0.08888889 0.05367177
## 7 1e+02 0.08333333 0.05399030
```

From summary value we can see the best cost for linear kernel is 0.0833.

We can also see the summary of the best model using R for the car data for the linear kernel.

```
bestmod = tune.out$best.model
summary(bestmod)

##
## Call:
## best.tune(method = svm, train.x = y ~ ., data = cardshort, ranges = list(cost = c(0.001,
##   0.01, 0.1, 1, 5, 10, 100)))
##
##
## Parameters:
##   SVM-Type:  C-classification
##   SVM-Kernel: radial
##   cost: 100
##
## Number of Support Vectors: 39
##
## ( 22 17 )
```

```
##
##
## Number of Classes: 2
##
## Levels:
## 0 1
```

In previous section, we see the summary for the best model. It has 39 support vectors.

We can also predict the model performance on the test data.

```
xtest = matrix(c(curbweight[(cutlen+1):205], enginesize[(cutlen+1):205] ),
               ncol = 2, nrow = 205 - cutlen)
ytest = high[(cutlen+1):205]

cardshorttest = data.frame(x = xtest, y = as.factor(ytest))

ypred = predict(bestmod, cardshorttest)

table(predict = ypred, truth = cardshorttest$y)
```

```
##      truth
## predict 0  1
##      0  8  1
##      1  0 16
```

From previous result, we see that the model performance on the validation set. The model correctly classified 24 instances and failed in estimating 1 instance.

```
svmfit = svm(y~., data = cardshort, kernel = "linear", cost = 1, scale = FALSE)
ypred = predict(svmfit, cardshorttest)

table(predict = ypred, truth = cardshorttest$y)
```

```
##      truth
## predict 0  1
##      0  8  1
##      1  0 16
```

In the car data the cost from 0.1 to 1 didn't impact the test performances.

Now we change the cost to a high values.

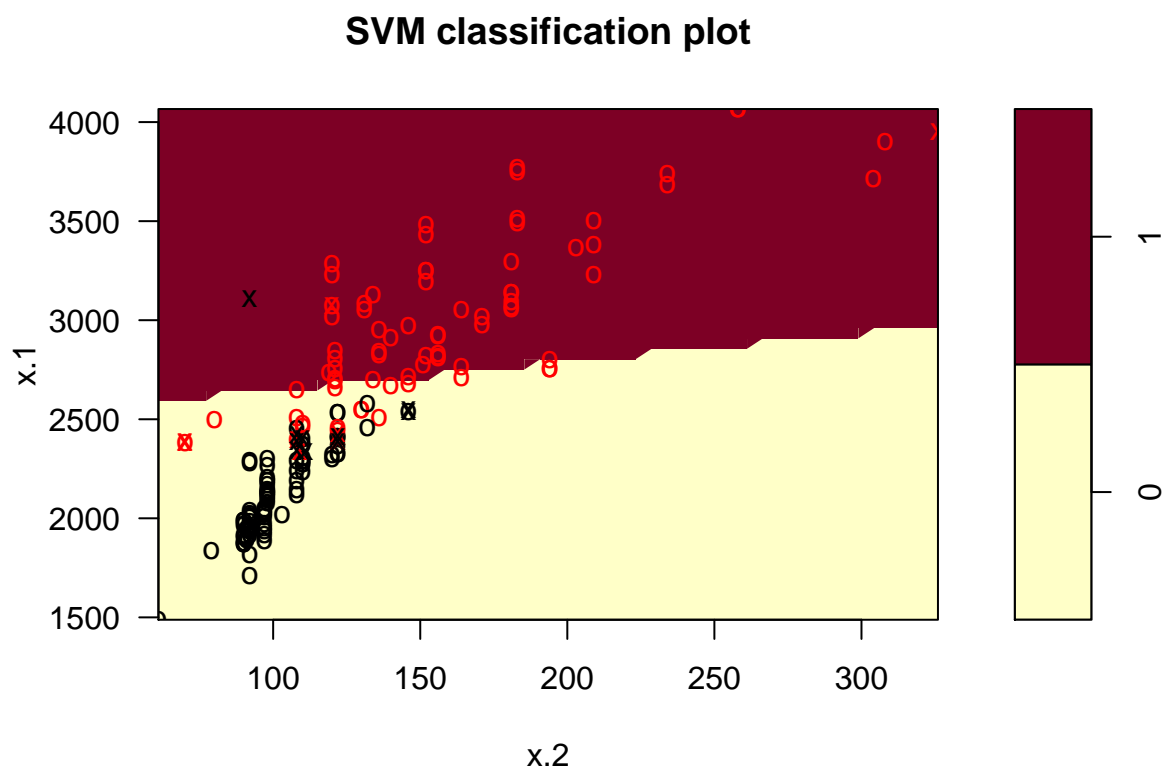
```
svmfit = svm(y~., data = cardshort, kernel = "linear", cost = 1e05, scale = FALSE)
summary(svmfit)
```

```
##
## Call:
## svm(formula = y ~ ., data = cardshort, kernel = "linear", cost = 1e+05,
##      scale = FALSE)
##
##
```

```
## Parameters:
##   SVM-Type:  C-classification
##   SVM-Kernel: linear
##       cost:  1e+05
##
## Number of Support Vectors:  19
##
## ( 9 10 )
##
##
## Number of Classes:  2
##
## Levels:
##  0 1
```

Now we see the support vectors number is 19, smaller than earlier cost = .1 (39).

```
plot(svmfit, cardshort)
```

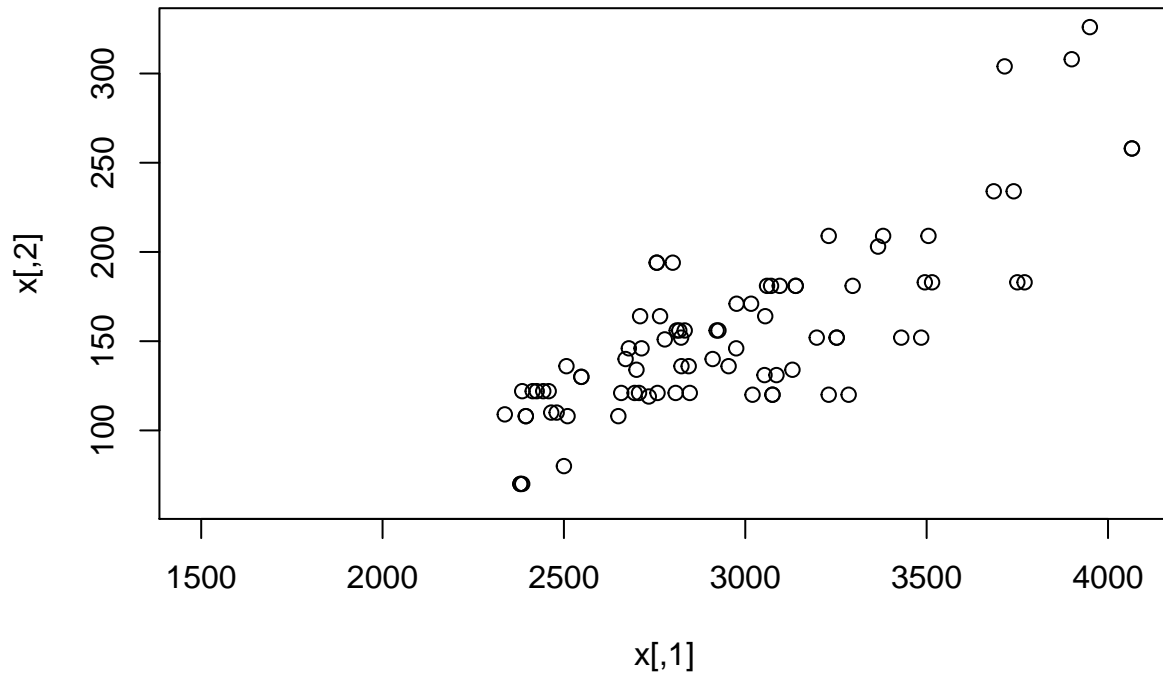


We plot the model and see the boundary lines are smooth compared to the earlier small cost value.

## SVM

In this section I experiment with the non-linear kernel for the car price dataset. We use previously defined class labels for this experiments.

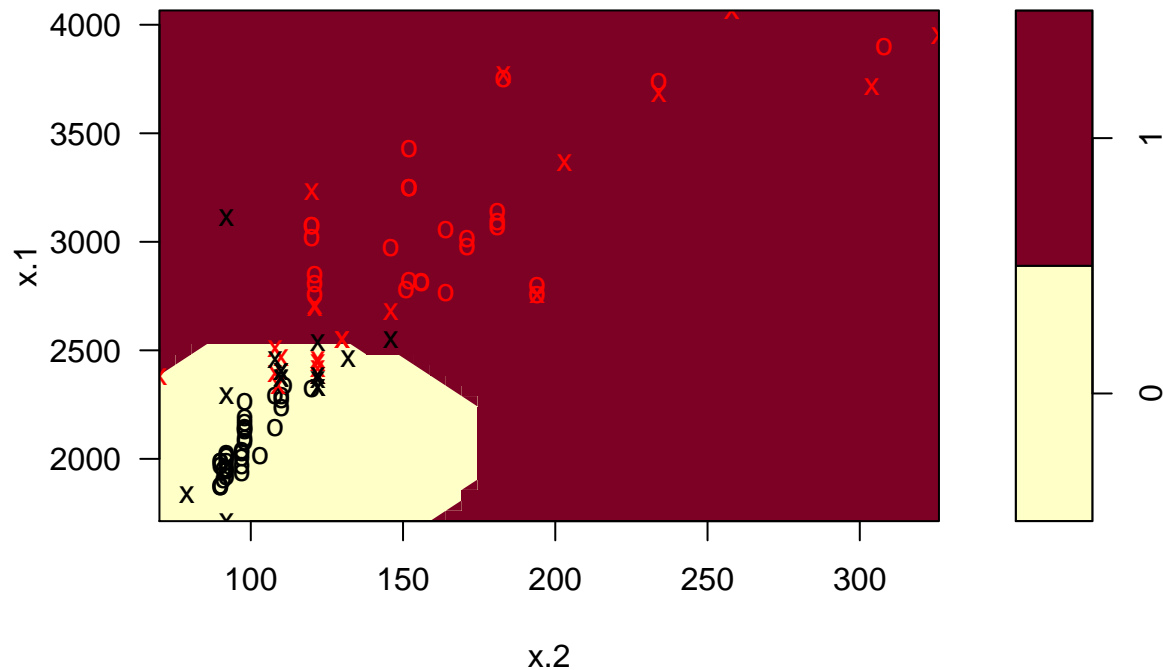
```
plot(x, col = y)
```



```
train = sample(180,100)
```

```
svmfit = svm(y~., data = cardshort[train,], kernel = "radial", gamma = 1, cost = 1)  
plot(svmfit, cardshort[train,])
```

## SVM classification plot



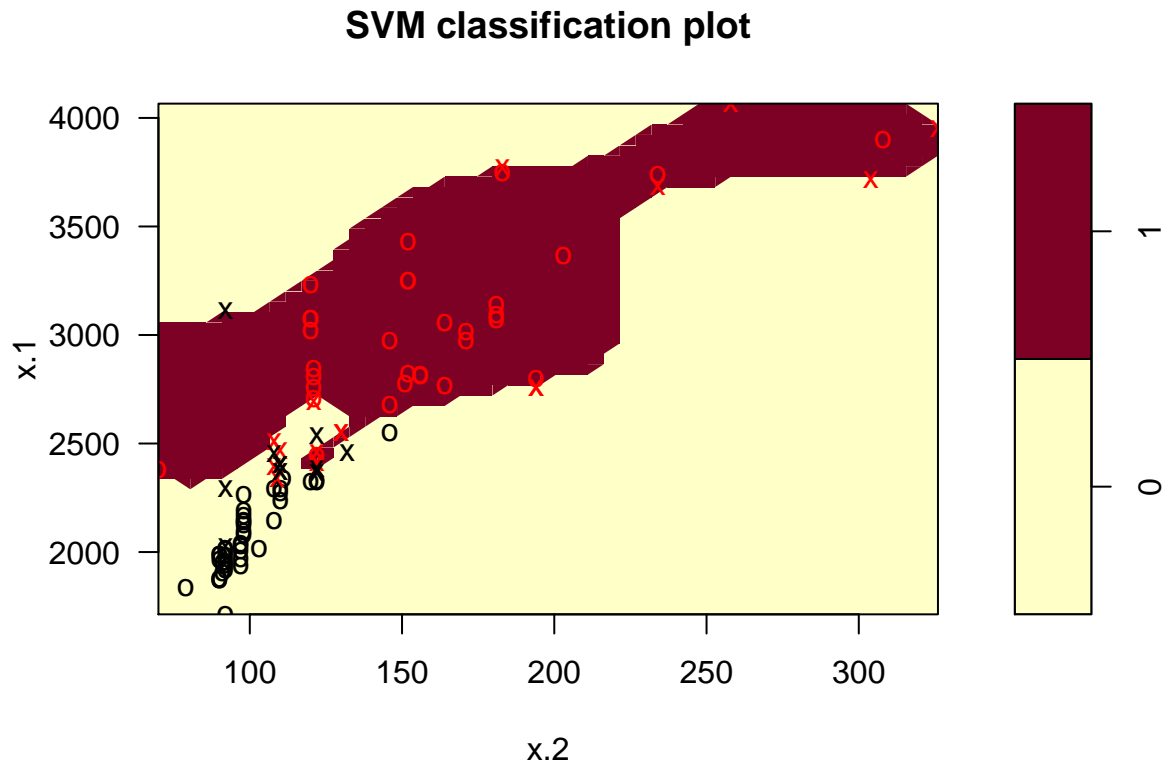
We see curved boundary for the decision classifier generated by radial basis classifiers.

```
summary(svmfit)
```

```
##
## Call:
## svm(formula = y ~ ., data = cardshort[train, ], kernel = "radial",
##      gamma = 1, cost = 1)
##
##
## Parameters:
##   SVM-Type:  C-classification
##   SVM-Kernel: radial
##       cost:  1
##
## Number of Support Vectors:  38
##
## ( 22 16 )
##
##
## Number of Classes:  2
##
## Levels:
##  0 1
```

The radial kernel selected 38 support vectors to draw the boundary between the class labels.

```
svmfit = svm(y~., data = cardshort[train,], kernel = "radial", gamma = 1, cost = 1e5)
plot(svmfit, cardshort[train,])
```



```
summary(svmfit)
```

```
##
## Call:
## svm(formula = y ~ ., data = cardshort[train, ], kernel = "radial",
##      gamma = 1, cost = 1e+05)
##
##
## Parameters:
##   SVM-Type:  C-classification
##   SVM-Kernel: radial
##      cost:  1e+05
##
## Number of Support Vectors:  29
##
##   ( 17 12 )
##
##
## Number of Classes:  2
##
## Levels:
##  0 1
```



We see complex boundary generated for the the decision by the svm with a high cost function with 29 support vectors.

```
set.seed(1)
tune.out = tune(svm, y~., data = cardshort[train,], kernel = "radial",
               ranges = list(cost = c(0.1, 1, 10, 100, 1000)) )
summary(tune.out)
```

```
##
## Parameter tuning of 'svm':
##
## - sampling method: 10-fold cross validation
##
## - best parameters:
##   cost
##   100
##
## - best performance: 0.1
##
## - Detailed performance results:
##   cost error dispersion
## 1 1e-01  0.13  0.1159502
## 2 1e+00  0.13  0.1159502
## 3 1e+01  0.13  0.1159502
## 4 1e+02  0.10  0.1247219
## 5 1e+03  0.11  0.1286684
```

We use the tune to find the best classifier by sweeping the cost value. We find that the model gets cost 0.1 as the best model. We evaluated the model on the test cases.

```
table(truc = cardshort[-train, "y"], pred = predict(tune.out$best.model,
                                                    newdata = cardshort[-train,]))
```

```
##      pred
## trunc 0  1
##      0 41  3
##      1  2 34
```

Now we see the model performed with high accuracy on large number of test case and trained upon small number of instance. The model accuracy is

```
(41+34)/(43+37)
```

```
## [1] 0.9375
```

## ROC curve

Firstly, we define a function for plotting the ROC curves

```
library(ROCR)
```

```
## Loading required package: gplots
```

```
##
```

```
## Attaching package: 'gplots'
```

```
## The following object is masked from 'package:stats':
```

```
##
```

```
##      lowess
```

```
rocplot = function(pred, truth, ...){  
  predob = prediction(pred, truth)  
  perf = performance(predob, "tpr", "fpr")  
  plot(perf,...)  
}
```

Now we implement the actual svm fit using the train car data portion.

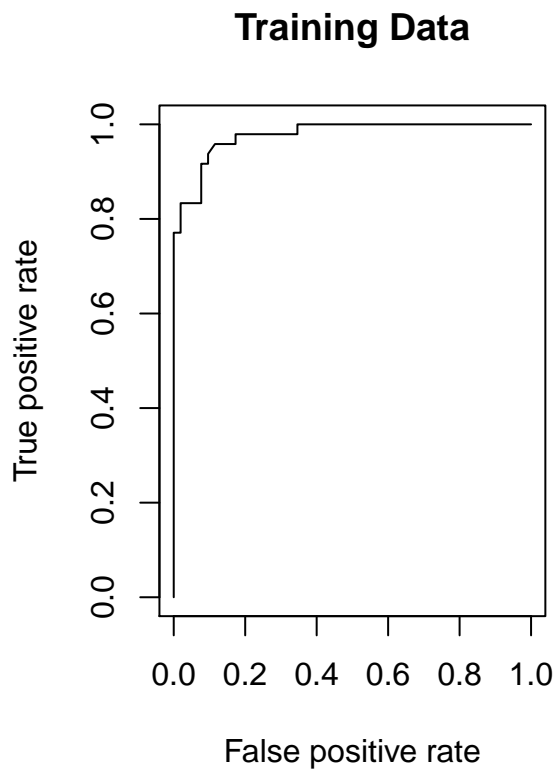
```
svmfit.opt = svm(y~., data = cardshort[train,],  
                 kernel = "radial", gamma = 2, cost = 1, decision.values = T)
```

```
fitted = attributes(predict(svmfit.opt, cardshort[train,], decision.value = T))$decision.values
```

Next, we use the fitted model to plot the ROC curve.

```
par(mfrow = c(1,2))
```

```
rocplot(fitted, cardshort[train, "y"], main = "Training Data")
```

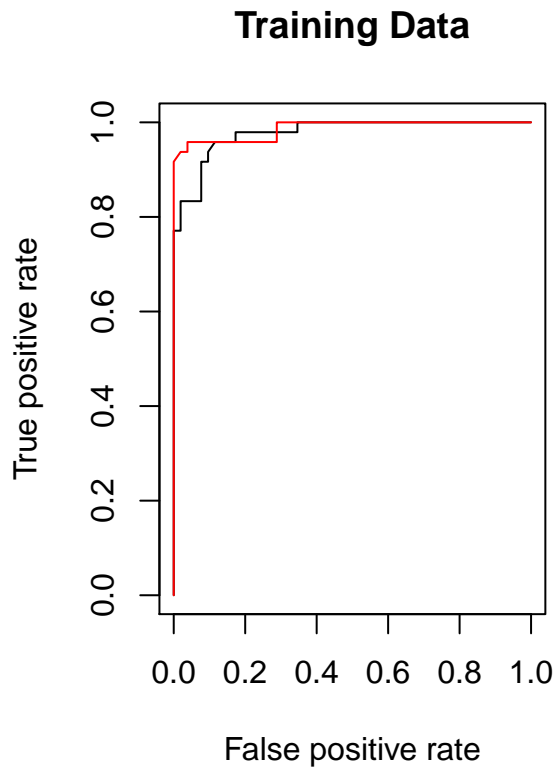


The previous plot shows the ROC curves. The classifier seems working great on training instances as we see the area under the ROC curve is close to 1.

```
par(mfrow = c(1,2))
rocplot(fitted, cardshort[train, "y"], main = "Training Data")

svmfit.flex = svm(y~., data = cardshort[train,],
                  kernel = "radial", gamma = 2, cost = 10000, decision.values = T)

fitted = attributes(predict(svmfit.flex, cardshort[train,], decision.value = T))$decision.values
rocplot(fitted, cardshort[train, "y"], add = T, col = "red")
```



We overlap the two roc curve for two different cost function. In the lower FPR the red model seems a little better and at the FPR 0.4, the black ROC curve performed better.

Now we finally plot the the test models ROC curves in the same plot for comparison.

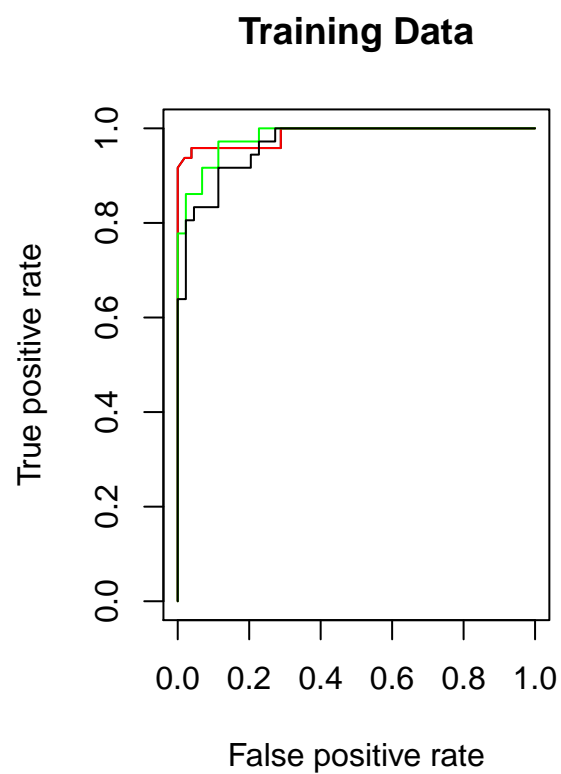
```
par(mfrow =c(1,2))
rocplot(fitted, cardshort[train, "y"], main = "Training Data")

svmfit.flex = svm(y~., data = cardshort[train,],
                  kernel = "radial", gamma = 2, cost = 10000, decision.values =T)

fitted = attributes(predict(svmfit.flex, cardshort[train,], decision.value = T))$decision.values
rocplot(fitted, cardshort[train, "y"], add =T, col ="red")

fitted = attributes(predict(svmfit.opt, cardshort[-train,], decision.value = T))$decision.values
rocplot(fitted, cardshort[-train, "y"], add =T, col ="green")

fitted = attributes(predict(svmfit.flex, cardshort[-train,], decision.value = T))$decision.values
rocplot(fitted, cardshort[-train, "y"], add =T, col ="black")
```



From the above figure, we see that the RED and green (optimal) model performed better than the black curve (gamma parameter 2) in terms of area under the curves.