

**Divide and conquer:NCD**

Notations:

Base set (labeled) :  $\mathcal{D}^b = \{(x_i, y_i)\}_{i=1}^{N^b}$

Novel set (unlabeled) :  $\mathcal{D}^n = \{x_j\}_{j=N^b+1}^{N^b+N^c}$

↳ number of classes  $C^N$  (known)

Encoder,  $z = \phi(x)$

Objective:

mapping image space,  $\mathcal{X} = \{x_i\}_{i=1}^{N^b+N^c} \rightarrow \mathcal{Y} = \{y_k\}_{k=1}^{C^b+C^c}$

Compositional Experts:

Batchwise Experts: → linear classifier  $\phi^b(\cdot)$

Prediction output:  $\hat{y}_i^{\phi^b} = \phi^b(z_i)$  ;  $z_i \in \mathcal{D}^b$   
 ↳ normalized

Novel batch expert: Another prediction layer  $\phi^n(\cdot)$

↳  $z_j^{\phi^n}$  (normalized)

$$\boxed{\hat{y}_i^{\phi^*} = \phi^* (\phi^T(x_i))} \quad ; x_i \in \mathcal{D}^*$$

Cross entropy loss:

$$L_{CE}(\hat{y}^{\phi}, y) = -y \log \sigma(\hat{y}^{\phi}/c)$$

[utilize the labels & apply pseudo label  
for the rest]

Regularization to avoid class leakage:

$$L_{reg}(\hat{y}^{\phi}) = \left( \sum_{c \in \hat{y}} (\hat{y}_c^{\phi})^2 \right)^{1/2}$$

$\hookrightarrow \quad , - c^a + c^b$

(kinda entropy loss)

Encourage minimum entropy

classwise expert:

$$\hat{\phi}^b = \phi^b(x_i) \quad \text{[only base class]}$$

concat  $\left[ \begin{array}{c} d \times 1 \times \dots \times y \\ y^{\psi^n} = \psi^n(\psi'^n(z_i)) \end{array} \right] \text{ [only expert class]}$

$$\mathcal{L}_{CE}(\left[\hat{y}^{\psi^b}, \hat{y}^{\psi^n}\right], y)$$

Training target:

pseudo-labeling:  $\left\{ \begin{array}{l} \text{Equally partition the} \\ \text{novel clusters. } \psi^n, \phi^n \end{array} \right.$

Batch of novel samples:

$$Z = \left[ z_1^{\phi}, z_2^{\phi}, \dots, z_{B^n}^{\phi} \right] \in \mathbb{R}^{B^n \times d}$$

cluster center:  $W = [w_1, w_2, \dots, w_c] \in \mathbb{R}^{c \times d}$

Goal  $\Rightarrow \max_{y \in \mathcal{P}} \text{tr} \left( \overset{\text{labels}}{y} \overset{\text{embedding}}{W} z^T \right) + \epsilon \mathcal{H}(y)$   
 $\downarrow$  smoothness.

$$\left[ \overset{\downarrow}{y_1^{\phi^n}}, y_2^{\phi^n}, \dots, y_{s^n}^{\phi^n} \right] \in \mathbb{R}^{s^n \times c^n}$$

Solution by Sinkhorn-Knopp algorithm. mean of  $z^{\phi}, z^{\psi}$

local aggregation:

$$y^n = \alpha \frac{y^{\phi^n} + y^{\psi^n}}{2} + (1-\alpha) \sum_{k=1}^{n^q} s_k y_k^q$$

$\frac{\exp(\bar{z} \cdot z_k^q / c)}{\sum_{k=1}^{n^q} \exp(\bar{z} \cdot z_k^q / c)}$   
 $\hookrightarrow$  Attention mechanism  
[  $n^q$  novel sample in the dictionary ]