

**Neighborhood contrastive learning.**

problem statement:

given: labeled set  $D^l$   
unlabeled  $D^u$

target: learn to cluster  $D^u$ , leveraging  $D^l$

feature extractor  $\mathcal{R}: x \rightarrow z \in \mathbb{R}^H$

$$\phi^l \rightarrow c^l$$

$$\phi^u \rightarrow c^u$$

Baseline:

$$(x, y) \in D^l$$

optimize cross-entropy loss:

$$\mathcal{L}_{CE} = - \frac{1}{C^l} \sum_{i=1}^{C^l} y_i \log \phi_i^l(\mathcal{R}(x))$$

pairwise binary CF loss:

$$x_i^u, x_j^u \in \mathcal{X}^u \mapsto z_i^u, z_j^u$$

$$\cos \text{Sim} : \mathcal{D}(z_i^u, z_j^u) = \frac{z_i^{u^T} \cdot z_j^u}{\|z_i^u\|_2 \|z_j^u\|_2}$$

$$\hat{y}_{ij} = \mathbb{1} [\mathcal{D}(z_i^u, z_j^u) \geq \gamma]$$

$$\mathcal{L}_{\text{bce E}} = \hat{y}_{ij} \log y_{ij} + (1 - \hat{y}_{ij}) \log (1 - y_{ij})$$

$$p_{ij} = \underline{\phi^u(z_i^u)^T \phi^u(z_j^u)}$$

Extra head compared to prev. work.

consistency loss:

$$\begin{aligned} \mathcal{L}_{\text{MSE}} &= \frac{1}{C^L} \sum_{i=1}^{C^L} (\phi_i^L(z^L) - \phi_i^L(\hat{z}^L))^2 \\ &+ \frac{1}{C^u} \sum_{i=1}^{C^u} (\phi_i^u(z^u) - \phi_i^u(\hat{z}^u))^2 \end{aligned}$$

1 . . . 1 + util

$$\mathcal{L}_{\text{NCE}} = \mathcal{L}_{\text{CE}} + \mathcal{L}_{\text{CE}} + \text{MSE}$$

Neighborhood Contrastive L:

Queue:  $M^u$  [regarded as negative]  
sample  $\bar{z}^u$

$$l(z^u, \hat{z}^u) = -\log \frac{e^{\delta(z^u, \hat{z}^u)/c}}{e^{\delta(z^u, \hat{z}^u)/c} + \underbrace{\sum_{m=1}^{|M^u|} e^{\delta(z^u, \bar{z}^m)/c}}_{\text{fake negative issue}}}$$

Retrieve top- $k$  similar features:

$$\rho_k = \underset{\bar{z}_i^u}{\text{argtop}_k} \left( \left\{ \delta(z^u, \bar{z}_i^u) \mid \forall i \in \{1 \dots |M^u|\} \right\} \right)$$

consider them as pseudo positive

$$l(z^u, \rho_k) = -\frac{1}{k} \sum_{\bar{z}_i^u \in \rho_k} \log \frac{e^{\delta(z^u, \bar{z}_i^u)/c}}{e^{\delta(z^u, \bar{z}_i^u)/c} + \sum_{m=1}^{|M^u|} e^{\delta(z^u, \bar{z}_m^u)/c}}$$

$$z_i^u \sim \mathcal{D}$$

$$m \in \mathcal{I}$$

$\therefore$  NCL:

$$l_{NCE} = \alpha l(z^u, \hat{z}^u) + (1-\alpha) l(z^u, z^+)$$

Supervised contrastive loss:

$$\mathcal{P} = \left\{ \frac{-1}{e_j} \in m^L : y_i = y_j \right\} \cup \hat{z}_j^+$$

$$l_{scr} = - \frac{1}{|\mathcal{P}|} \sum_{\frac{-1}{e_j^L} \in \mathcal{P}} \log \frac{\sum_i \exp(-\sqrt{d(z_i^L, z_j^L)}) / \kappa}{\sum_i \exp(-\sqrt{d(z_i^L, \hat{z}_i^L)}) + \sum_{m \in \mathcal{I}} \exp(-\sqrt{d(z_i^L, \hat{z}_m^L)}) / \kappa}$$

Hard negative generation:

Easy negatives:

$$e_k = \arg \min_{\hat{z}_i^u} \left( \left\{ -\sqrt{d(z^u, \hat{z}_i^u)} \mid \forall i \in 1 \dots |m^u| \right\} \right)$$

for samples in  $m^l$ , true negative to  $m^u$

$$\mathcal{S} = \mu \cdot \bar{\mathcal{E}}^u + (1 - \mu) \cdot \bar{\mathcal{E}}^l$$

$\nearrow \epsilon \in \epsilon_k$ 
 $\downarrow$

mixing coef.
randomly sampled

hard negative:

$$n_k = \arg \max_{\mathcal{S}_i} \left( \{ \mathcal{S}(z^u, \mathcal{S}_i) \mid \forall i \in \{1, \dots, n_{x2}\} \} \right)$$

new memory queue:

$$m^{u'} = m^u \cup n_k$$

Overall loss:

$$l_{\text{loss}} = l_{\text{base}} + l_{\text{sc1}} + l_{\text{sc2}}$$