ⓟ EqCo

EqCo (Equivalent Contrastive)

$$\mathcal{L}_{INCE} = \mathop{\mathbb{E}}_{\substack{q \sim D,\ k_i \sim \bar{D} \\ k_0 \sim D(q)}} \left[-\log \frac{e^{(q^T k_0 - m)/\tau}}{e^{(q^T k_0 - m)/\tau} + \sum_{i=1}^{N} e^{q^T k_i / \tau}} \right]$$

(margin, temperature)

$q \rightarrow$ Query Sample Representation.

$k \rightarrow$ keys. $0 \rightarrow$ positive; $k_i \rightarrow$ Negative.

$D \rightarrow$ Data Distribution.

Equivalent rule: $m = \tau \log \frac{N}{K}$ (A constant !!; No of Negatives.; Temperature.; margin)

EqCo: **Batch Size**: Number of queries in 'N' per batch.

Negative Samples: K per query

Query Embedding 'q'

Key Embedding $x = \{x_i\}_{i=0,\dots K}$

Conditional $P(x_i | q)$

independent dist $P(x_i)$

$$P_{H_i}(x) = P(x_i|q) \prod_{j \neq i} P(x_j)$$

↳ H_i (k+1) candidate distribution for x.

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$$\Pr\left[x \sim H_0 \mid q, x\right] = \frac{p^+ P_{H_0}(x)}{p^+ P_{H_0}(x) + p^- \sum_{i=1}^{K} P_{H_i}(x)}$$

$$= \frac{\frac{p^+}{p^-} \frac{p(k_0|q)}{p(k_0)}}{\frac{p^+}{p^-} \frac{p(k_0|q)}{p(k_0)} + \sum_{i=1}^{K} \frac{p(k_i|q)}{p(k_i)}}$$

Let, $\frac{p(k_i|q)}{p(k_i)} \propto e^{q^T k_i / \tau} \quad (i = 0, \cdots K)$

$$\frac{p^+}{p^-} = e^{-m/\tau}$$

then, $\mathcal{L}_{opt} \triangleq \mathbb{E}_{q,x} - \log \Pr\left[x \sim H_0 \mid q, x\right]$

$$\geq \log\left[1 + K e^{m/\tau}\right] - I(k_0, q)$$

$$I(k_0, q) \geq f_{bound}(m, K)$$

$$\triangleq \log\left(1 + K e^{m/\tau}\right) - \mathcal{L}_{opt}$$

$$\approx \log\left(1 + K e^{m/\tau}\right) - \mathop{\mathbb{E}}_{q \sim D, k_0 \sim D(q)} \log\left(1 + K e^{m/\tau} \frac{p(k_0)}{p(k_0|q)}\right)$$

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