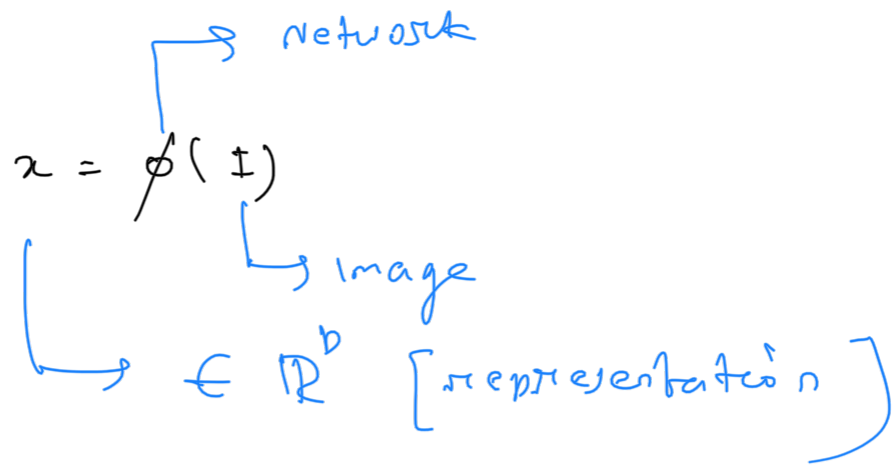


Self labelling via simultaneous clustering

methods:



n data points I_1, I_2, \dots, I_N

[k labels $y_1, \dots, y_N \in \{1, \dots, k\}$]

classification head $h: \mathbb{R}^D \rightarrow \mathbb{R}^k$

$$\Rightarrow p(y = \cdot | x_i) = \text{softmax}(h \circ \phi(x_i))$$

$$\Rightarrow \mathbb{E} \left(p | y_1, \dots, y_N \right) = - \frac{1}{N} \sum_{i=1}^N \log p(y_i | x)$$

[where is the label??]

Self-labeling

$n \quad k$ \nearrow Deterministic posterior

$$E(p, q) = -\frac{1}{N} \sum_{i=1}^N \sum_{y=1}^K q(y|x_i) \log p(y|x_i)$$

objective

Delta label

$$\min_{p, q} E(p, q) \quad \text{Subj. to} \quad \forall y: q(y|x_i) \in \{0, 1\} \quad \& \quad \sum_{i=1}^N q(y|x_i) = \frac{N}{K}$$

equipartition

[optimal transport problem]

Let, $p_{yi} = p(y|x_i) \frac{1}{N}$; $k \times n$ // joint pdf

$Q_{yi} = q(y|x_i) \frac{1}{N}$; $k \times n$ // assigned joint pdf

↳ relax the Q to be transport prototype.

$$U(\pi, c) := \{Q \in \mathbb{R}_+^{k \times n} \mid Q\mathbf{1} = \pi, Q^T\mathbf{1} = c\}$$

$\xrightarrow{\frac{1}{K} \cdot \mathbf{1}}$ $\xrightarrow{\frac{1}{n} \cdot \mathbf{1}}$
 $\xrightarrow{\mathbf{1} \times 1}$ $\xrightarrow{\mathbf{1} \times 1}$

rewriting

dot product

minimize

$$\underbrace{E(p, q) + \log N}_{\text{MI maximization}} = \langle Q, -\log P \rangle$$

Require to Solve.

$$\min_{Q \in U(\pi, c)} \langle Q, -\log P \rangle$$

Sin known - known:

$$\min_{Q \in U(\pi, c)} \langle Q, -\log P \rangle + \frac{1}{\lambda} \text{KL}(Q \| P e^T)$$

Element wise

$$\Rightarrow \text{then } Q = \text{diag}(\alpha) P^T \text{diag}(\beta)$$

$$\text{Learn: } \left[\begin{array}{c} \text{h o } \phi \\ \downarrow \end{array} \right] \begin{array}{l} \text{determines} \\ p_i = \text{softmax}(\text{h o } \phi(x_i)) \end{array}$$

\hookrightarrow a label assignment matrix Q

Two alternating approach

① Representation learning

h o ϕ : cross entropy

⑪ Self-labeling → given current h.o.p
compute P

Then find $Q = \text{diag}(\alpha) P^T \text{diag}(\beta)$

by iterative approach

$$\forall y : \alpha_y \leftarrow [P^T \beta]_y^{-1} ; \forall i : \beta_i \leftarrow [x_i^T P^T]^{-1}$$