Unified optimal transport for universal DA

labeled dataset Source $\mathcal{D}^s = \{x_i^s, y_i^s\}_{i=1}^{n_s} \in C_s$

Unlabeled target Data $\mathcal{D}^t = \{x_i\}_{i=1}^{n_t} \in C_t$

Common classes $C = C_s \cap C_t$

$\bar{C}_s = C_s \setminus C$ // source private class

$\bar{C}_t = C_t \setminus C$ // target private class

This paper ~~target~~

(i) Common classes Detection

(ii) Private class Discovery

preliminaries:

OT problem: transport one distribution → others.

$\Sigma_n := \{x \in \mathbb{R}_+^n \mid x^T 1_n = 1\}$ // simplex vector.

$$\left.\begin{array}{l}\alpha \in \Sigma_r \\ \beta \in \Sigma_C\end{array}\right\}$$ two simplex vector.

Transport prototype of α & β:

$$U(\alpha, \beta) = \{ \underline{Q} \in \mathbb{R}_+^{r \times C} \mid Q 1_c = \alpha, \; Q^T 1_r = \beta \}$$

$U(\alpha, \beta)$ is interpreted as possible joint prob of (x, y) with their marginal α, β

given a similarity matrix $M \in \mathbb{R}^{r \times C}$

Q^* maps α to β by following optim..

$$OT^{\epsilon}(M, \alpha, \beta) = \underset{Q \in U(\alpha, \beta)}{\arg\max} \; \mathrm{Tr}(Q^T M) + \epsilon \underline{H(Q)}$$

entropy regularisr

$$\underbrace{\text{Optimal } Q^* = \mathrm{Diag}(u) \exp(M/\epsilon) \, \mathrm{Diag}(v)}$$

Solved by Sinkhorn - algorithm

generalized form for unbalanced

$$\mathrm{UOT}^{\epsilon,\kappa}(M,\alpha,\beta) = \underset{Q\in\mathbb{R}^{+}}{\arg\max}\ \mathrm{Tr}(Q^{T}M) + \epsilon H(Q)$$

$$- \kappa\Big(\underbrace{D_{KL}(Q\mathbb{1}_c \,\|\, \alpha) + D_{KL}(Q^{T}\mathbb{1}_{p} \,\|\, \beta)}_{\text{Distribution matching}}\Big)$$

solved by generalized Sinkhorn algorithm.

Back to paper :)

Inter-domain partial Alignment of C Detection:

$(C \subset D)$

source prototype: $\underline{C_s} = [c_1^{s}, \dots c_{|C_s|}^{s}]^{T}$

Problem formulation: mapping target feature to $\underline{C_s}$

Target assignment matrix:

$$Q^{st} = \mathrm{UOT}^{\epsilon,\kappa}(S^{st}, \tfrac{1}{\cdot}\mathbb{1}, \tfrac{1}{\cdot}\mathbb{1})$$

$$Q = \mathrm{OT}\left(\cdot , \ _{B} \ _{+B}, \ |C_s| \ ^{-1}|C_s| \right)$$

classical formulation

where, $S^{st} = Z^{t} C_s^{T}$ // cosine similarity.

$\in \mathbb{R}^{B \times |C_s|}$

$$Z^{t} = \left[z_1^{t}, \ \ldots \ z_B^{t} \right]^{T}$$

$$z_i^{t} = \frac{f(x_i^{t})}{\| f(x_i^{t}) \|_2}$$ // normalized.

Network f

Assignment matrix $\bar{Q}^{st} = Q^{st} / \sum Q^{st}$

↓

find statistical properties

↓

each row vaninal prediction

Sample confidence :

$$w_i^{t} = \max \left(\left\{ \bar{Q}_{i,1}^{st}, \ \bar{Q}_{i,2}^{st} \ \ldots \ \bar{Q}_{i,|C_s|}^{st} \right\} \right)$$

Higher score means ~~+~~ close to common sample class.

Source prototype confidence Score:

$$w_j^s = \sum_{i=1}^{B} \bar{Q}_{i,j}^{st}$$

Higher score means common prototype.

$$\delta_i = \begin{cases} 1, & w_i^t \geq \frac{1}{B}, \text{ and } \underbrace{w_{\hat{y}_i^t}^s \geq \frac{1}{|C_s|}}_{\text{why ??}} \\ 0, & \text{otherwise} \end{cases}$$

top sample with top confidence.

$$\mathcal{L}_{CCD} = \frac{\sum_{i=1}^{B} \delta_i \, \mathcal{L}_{CE}(e_i^t, \hat{y}_i^t)}{\sum_{i=1}^{B} \delta_i}$$

[requires high samples in [illegible]

Adaptive filling for unbalance portion of +ve, -ve.

$$z_i^n = \frac{1}{2}\left(z_i^t + \arg\min_{c_k^s} \left(z_i^t {c_k^s}^T \right) \right)$$

Adaptive update for marginal prob vector:

initial: $\beta = \frac{1}{|C_s|} \mathbb{1}_{|C_s|}$

$$\beta^{(t+1)} = \mu \beta^{(t)} + (1-\mu) \bar{\beta}^{(t)}$$

sum of column in $\bar{Q}^{s,t}$ solve in $\beta^{(t)}$ case.

$$\beta^{(0)} = \frac{1}{|C_s|} \mathbb{1}_{|C_s|}$$

Intra-Domain Representation learning for Private class

predefine k learnable target prototype: C_t

initialize randomly

Assign target feature z^t

Assign target features t with prototype C_t by solving OT problem.

$$Q^{tt} = OT^{\epsilon}\left(S^{tt}, \frac{1}{K}1_K, \frac{1}{2B}1_{2B}\right)$$

where, $S^{tt} = \bar{z}^t C_t^T$

C_t^T → initially random prototype

$\bar{z}^t$ → minibatch target features.

Solution Q^{tt} satisfy the constraint strictly.

sum of each row $1/K$

$$\tilde{Q}^{tt} = Q^{tt} \times K$$

$$\mathcal{L}_{global} = \frac{1}{B}\sum_{i=1}^{B} \ell(\bar{q}^{tt}, z_i^t)$$

$$\ell(q_i, z_i) = -\sum_{k=1}^{K} q_{i,k} \log p_{i,k}$$

$$p_{i,k} = \frac{\exp(z_i C_k^T/\tau)}{\sum^{K} \exp(z_i C_k^T/\tau)}$$

$k=1$

Local Consistency loss:

Anchor feature

$$\mathcal{L}_{local} = \frac{1}{2B} \sum_{i=1}^{B} \Big[\, l\big(\hat{q}^{t+}_{B+i}, z_i^{t}\big) + l\big(\tilde{q}^{t+}_{i}, \tilde{z}_i^{t}\big) \Big]$$

nearest feature from memory queue

$$\mathcal{L}_{PCD} = \frac{1}{2} \left\{ \mathcal{L}_{global} + \mathcal{L}_{local} \right\}$$

$$\boxed{\mathcal{L}_{overall} = \mathcal{L}_{cls} + \lambda \left\{ \mathcal{L}_{CCD} + \mathcal{L}_{PCD} \right\}}$$