AE 352: Homework 5

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October 1, 2016

1. **a.** Find $\omega^{'}$

$$\begin{aligned}(H_O)_1 &= (H_O)_2 \\ (0.4)(0.05)(250\cos(30^0)) - 3(6)(.2)^2 - 3(6)(.4)^2 &= (3+0.05)(0.4)^2\omega^{'} + 3(0.2)^2\omega^{'} \\ \omega^{'} &= 1.20087\frac{\text{rad}}{\text{s}}\end{aligned}$$

b. Find the maximum angular deflection θ_{max}

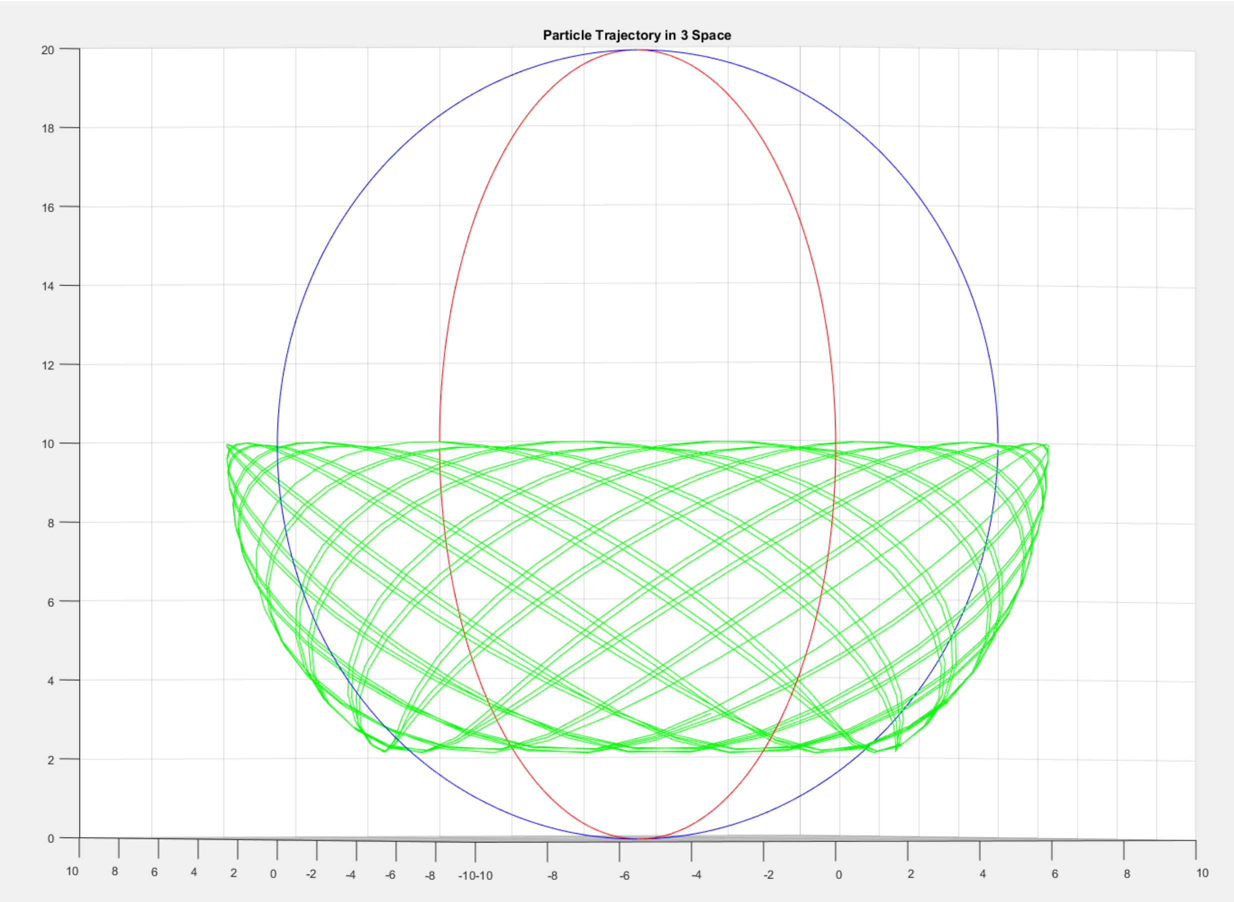
$$\begin{aligned}T+V &= T^{'}+V^{'} \\ 0+(3\cdot 0.2-(3+0.05)\cdot 0.4)g\cos(\theta) &= \frac{1}{2}(3+0.05)(0.4\cdot 1.20087)^2 + \frac{1}{2}(3)(0.2\cdot 1.20087)^2 \\ &\quad +(3\cdot 0.2-(3+0.05)\cdot 0.4)g \\ -.60822\cos(\theta) &= 0.35187+0.086525+-.60822 \\ \cos(\theta) &= 0.927922 \\ \theta_{max} &= \arccos(0.927922) \\ \theta_{max} &= 21.89^\circ\end{aligned}$$

2. **a.** Find θ as a function of h

$$\begin{aligned}r_0\times mv_0 &= r\times mv \\ r_0mv_0 &= rmv\cos(\theta) \\ T+v &= T^{'}+v^{'} \\ \frac{1}{2}mv_0^2+mgh &= \frac{1}{2}mv^2 \\ v_0^2+2gh &= v^2 \\ v &= \sqrt{v_0^2+2gh} \\ r^2 &= r_0^2-h^2 \\ r_0mv_0 &= rmv\cos(\theta) \\ r_0v_0 &= \sqrt{r_0^2-h^2}\sqrt{v_0^2+2gh}\cos(\theta) \\ \frac{r_0v_0}{\sqrt{r_0^2-h^2}\sqrt{v_0^2+2gh}} &= \cos(\theta) \\ \frac{r_0v_0}{r_0v_0\sqrt{1-\frac{h^2}{r_0^2}}\sqrt{1+\frac{2gh}{v_0^2}}} &= \cos(\theta) \\ \theta &= \arccos\left(\frac{1}{\sqrt{1-\frac{h^2}{r_0^2}}\sqrt{1+\frac{2gh}{v_0^2}}}\right)\end{aligned}$$

b. As times progresses, θ will gradually increase and then begin decreasing after reaching a maximum value. θ will continue decreasing beyond the point where it changes signs and it will begin travelling up the hemisphere. Assuming that no damping occurs, as $t \to \infty$ the particle will continue circling the hemisphere as it oscillates up and down. In a real situation where damping occurs, the particle will eventually settle at the bottom of the hemisphere. Prior to it stopping, it will be circling the center of the hemisphere with $\theta \approx 0$.

c.



3. **a.**

$$
\begin{aligned}
2m \cdot x \cdot l_0 - m \cdot (1-x) \cdot l_0 &= 0 \\
x &= \frac{1}{3} \\
(H_O)_1 &= (H_O)_2 \\
2m\omega_0 \left(\frac{l_0}{3}\right)^2 + m\omega_0 \left(\frac{2l_0}{3}\right)^2 &= 2m\omega \left(\frac{l_0}{6}\right)^2 + m\omega \left(\frac{l_0}{3}\right)^2 \\
\frac{2}{9}\omega_0 l_0 + \frac{4}{9}\omega_0 l_0 &= \frac{1}{18}\omega l_0 + \omega l_0 \frac{1}{9} \\
\frac{6}{9}\omega_0 &= \frac{3}{18}\omega \\
\omega &= 4\omega_0
\end{aligned}
$$

b.

$$\begin{aligned}
\vec{F}_{str} &= \frac{mv^2}{r} \\
\vec{F}_{str} &= 2m\omega^2 r \\
\vec{F}_{str} &= 2m(4\omega_0)^2\frac{l_0}{6} \\
\vec{F}_{str} &= \frac{16}{3}m\omega_0^2 l_0 \text{ N}
\end{aligned}$$

Check:

$$\begin{aligned}
\vec{F}_{str} &= \frac{mv^2}{r} \\
\vec{F}_{str} &= m\omega^2 r \\
\vec{F}_{str} &= m(4\omega_0)^2\frac{l_0}{3} \\
\vec{F}_{str} &= \frac{16}{3}m\omega_0^2 l_0 \text{ N}
\end{aligned}$$

4. **a.**

$$\begin{aligned}
\sum F &= 0 \\
\mu m\frac{L-x}{L}g &= m\frac{x}{L}g \\
\mu(L-x) &= x \\
\mu L &= x+\mu x \\
x &= \frac{\mu L}{1+\mu} \text{ m}
\end{aligned}$$

b.

$$\begin{aligned}
T+V &= T^{'}+v^{'} \\
0+m\frac{L-\frac{\mu L}{1+\mu}}{L}gL+m\frac{\frac{\mu L}{1+\mu}}{L}g(L-\frac{x}{2}) &= mg\frac{L}{2}+\frac{1}{2}v^2 \\
(gL-g\frac{\mu L}{1+\mu})+\frac{\frac{\mu L}{1+\mu}}{L}g(L-\frac{x}{2}) &= g\frac{L}{2}+\frac{1}{2}v^2 \\
gL-g\frac{\mu L}{1+\mu}+g\frac{\mu L}{1+\mu}-g\frac{x}{2}\frac{\frac{\mu L}{1+\mu}}{L} &= g\frac{L}{2}+\frac{1}{2}mv^2 \\
g\frac{L}{2}-g\frac{x}{2}\frac{\frac{\mu L}{1+\mu}}{L} &= \frac{1}{2}v^2 \\
gL-gx\frac{x}{L} &= v^2 \\
v &= \sqrt{g(L-\frac{x^2}{L}} \\
v &= \sqrt{g(L-\frac{(\frac{\mu L}{1+\mu})^2}{L}} \\
v &= \sqrt{gL\Big(1-\Big(\frac{\mu}{1+\mu}\Big)^2\Big)} \ \frac{\text{m}}{\text{s}}
\end{aligned}$$

MATLAB Code

```
%%
% AE 352
% HW 5
% 2C EC
close all; clear all;

R = 10; %define initial Radius
v_0 = 10; %define initial speed
g = -9.81; %gravity constant

theta(1) = 0; % starting parameters
omega(1) = v_0/R;

dt = 100/1000;% time discretization
t_Max = 100;

x(1) = 0;
y(1) = R;
z(1) = 10;

vx(1) = 0;
vy(1) = v_0;
vz(1) = 0;
az(1) = 0;


for t=1 : 1000

 %utilize a ratio of radii (r_0)^2/r^2 to determine omega,
 %then use omega^2*r to find centripetal acceleration
 az(t+1) = (omega(1)*(100/(R^2-(R-z(t))^2)))^2*(R-z(t));
 vz(t+1) = vz(t) + dt*((g)+az(t+1)); %alter gravity influence with centripetal
 z(t+1) = z(t) + dt*vz(t+1); %numerically integrate to find z

 omega(t+1) = omega(1)*(100/(R^2-(R-z(t+1))^2)); % find omega as before with radii ratio

 theta(t+1) = theta(t) +dt*omega(t+1); %numerically integrate to find theta

 x(t+1) = sin(theta(t+1))*sqrt(R^2-(R-z(t+1))^2); % numerically intrage to find new x and y
 y(t+1) = cos(theta(t+1))*sqrt(R^2-(R-z(t+1))^2);

end

%make circles for visualization purposes
x1(1)=0;
y1(1)=0;
z1(1)=0;
x2(1)=0;
y2(1)=0;
z2(1)=0;
for t = 1:360
    x1(t) = 10*cosd(t-1);
    x2(t) = 0;
    y1(t) = 0;
    y2(t) = 10*cosd(t-1);
    z1(t) = 10*sind(t-1)+10;
    z2(t) = 10*sind(t-1)+10;
end

figure(1)
title('Particle Trajectory in 3 Space');
hold on;
grid on;
plot3(x,y,z,'g-');
plot3(x1,y1,z1,'b-');
plot3(x2,y2,z2,'r-');
```