AE 352: Final Project

Max A. Feinberger Steven P. Schlax Samuel T. Wagner

December 5, 2016

We will consider a 3-link planar robot arm with revolute joints:

The x and y positions of the end-effector can be defined as:

$$x = a_1 \cos(\theta_1) + a_2 \cos(\theta_1 + \theta_2) + a_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$y = a_1 \sin(\theta_1) + a_2 \sin(\theta_1 + \theta_2) + a_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$J = \begin{bmatrix} z_0^0 \times (O_3^0 - O_0^0) & z_1^0 \times (O_3^0 - O_1^0) & z_2^0 \times (O_3^0 - O_2^0) \\ z_0^0 & z_1^0 & z_2^0 \end{bmatrix}$$

$$T_1^0 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & a_1 \cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & a_1 \sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_2 \cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & a_2 \sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & a_3 \cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & a_3 \sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) & 0 & a_1 \cos(\theta_1) + a_2 \cos(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) & 0 & a_1 \sin(\theta_1) + a_2 \sin(\theta_1 + \theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} \cos(\theta_1 + \theta_2 + \theta_3) & -\sin(\theta_1 + \theta_2 + \theta_3) & 0 & a_1 \cos(\theta_1) + a_2 \cos(\theta_1 + \theta_2) + a_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ \sin(\theta_1 + \theta_2 + \theta_3) & \cos(\theta_1 + \theta_2 + \theta_3) & 0 & a_1 \sin(\theta_1) + a_2 \sin(\theta_1 + \theta_2) + a_3 \sin(\theta_1 + \theta_2 + \theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_2 c_{12} + a_3 c_{123} \\ a_2 s_{12} + a_3 s_{123} \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_3 c_{123} \\ a_3 s_{123} \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

$$T = \frac{1}{2} \begin{bmatrix} \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix} \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{12} & H_{22} & H_{23} \\ H_{13} & H_{23} & H_{33} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$H = \sum_{i=1}^{n} \left(m_i J_{v_i}^T J_{v_i} + J_{\omega_i}^T I_i J_{\omega_i} \right)$$

$$T = \sum_{i=1}^{n} \left(m_i \dot{q}^T J_{v_i}^T J_{v_i} \dot{q} + \dot{q}^T J_{\omega_i}^T I_i J_{\omega_i} \dot{q} \right)$$

$$U = \sum_{i=1}^{n} m_i g_i y_i$$

$$x_1 = \frac{a_1}{2} c_1$$

$$y_1 = \frac{a_1}{2} s_1$$

$$J_{v_1} = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_1} & \frac{\partial x_1}{\partial \theta_2} & \frac{\partial x_1}{\partial \theta_3} \\ \frac{\partial y_1}{\partial \theta_1} & \frac{\partial y_1}{\partial \theta_2} & \frac{\partial y_1}{\partial \theta_3} \\ \frac{\partial z_1}{\partial \theta_1} & \frac{\partial z_1}{\partial \theta_2} & \frac{\partial z_1}{\partial \theta_3} \end{bmatrix}$$

$$J_{v_1} = \begin{bmatrix} -\frac{a_1}{2} s_1 & 0 & 0 \\ \frac{a_1}{2} c_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_{\omega_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$U_1 = m \frac{a_1}{2} s_1$$

$$x_2 = a_1 c_1 + \frac{a_2}{2} c_{12}$$

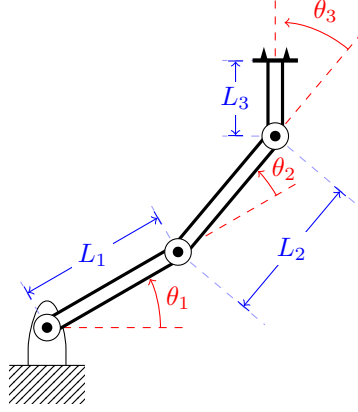$$y_2 = a_1 s_1 + \frac{a_2}{2} s_{12}$$

$$J_{v_2} = \begin{bmatrix} \frac{\partial x_2}{\partial \theta_1} & \frac{\partial x_2}{\partial \theta_2} & \frac{\partial x_2}{\partial \theta_3} \\ \frac{\partial y_2}{\partial \theta_1} & \frac{\partial y_2}{\partial \theta_2} & \frac{\partial y_2}{\partial \theta_3} \\ \frac{\partial z_2}{\partial \theta_1} & \frac{\partial z_2}{\partial \theta_2} & \frac{\partial z_2}{\partial \theta_3} \end{bmatrix}$$

$$J_{v_2} = \begin{bmatrix} -a_1 s_1 - \frac{a_2}{2} s_{12} & -\frac{a_2}{2} s_{12} & 0 \\ a_1 c_1 + \frac{a_2}{2} c_{12} & \frac{a_2}{2} c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_{\omega_2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$U_2 = m(a_1 s_1 + \frac{a_2}{2} s_{12})$$

Figure 1: A 3-Link Planar Robot Arm

$$x_3 = a_1c_1 + a_2c_{12} + \frac{a_3}{2}c_{123}$$

$$y_3 = a_1s_1 + a_2s_{12} + \frac{a_3}{2}s_{123}$$

$$J_{v_3} = \begin{bmatrix} \frac{\partial x_3}{\partial \theta_1} & \frac{\partial x_3}{\partial \theta_2} & \frac{\partial x_3}{\partial \theta_3} \\ \frac{\partial y_3}{\partial \theta_1} & \frac{\partial y_3}{\partial \theta_2} & \frac{\partial y_3}{\partial \theta_3} \\ \frac{\partial z_3}{\partial \theta_1} & \frac{\partial z_3}{\partial \theta_2} & \frac{\partial z_3}{\partial \theta_3} \end{bmatrix}$$

$$J_{v_3} = \begin{bmatrix} -a_1s_1 - \frac{a_2}{2}s_{12} - a_3s_{123} & -\frac{a_2}{2}s_{12} - \frac{a_3}{2}s_{123} & -\frac{a_3}{2}s_{123} \\ a_1c_1 + \frac{a_2}{2}c_{12} + \frac{a_3}{2}c_{123} & \frac{a_2}{2}c_{12} + \frac{a_3}{2}c_{123} & \frac{a_3}{2}c_{123} \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_{\omega_3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$U_3 = m(a_1s_1 + a_2s_{12} + \frac{a_3}{2}s_{123})$$

$$I_{\text{cuboid}} = \begin{bmatrix} \frac{1}{12}m(y^2 + z^2) & 0 & 0 \\ 0 & \frac{1}{12}m(x^2 + z^2) & 0 \\ 0 & 0 & \frac{1}{12}m(x^2 + y^2) \end{bmatrix} \tag{1}$$