Optimizing Marketing Strategies in Social Networks: A Survey of Influence and Revenue Maximization

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Abstract

With the advent of social media platforms, marketing strategies have become increasingly dependent on the social network relationships between customers. Inherent to this shift was the observation that a customer's purchasing decisions are influenced not only by their own values, but also by those of their neighbors. These consumer dependencies raise the natural question of how best to develop marketing strategies that account for customers' influences on one another. This literature review aims to map the key developments in the fields of influence and revenue maximization in social networks. We first discuss the seminal papers that frame marketing as an influence maximization problem. We then analyze papers that build upon this work to instead frame this question as a revenue maximization problem by considering pricing strategies in addition to identifying influential nodes. Motivated by the limitations of the aforementioned works, we then consider various potential future directions of the field. Throughout the paper, we evaluate the works of this series of interconnected topics to share what is currently understood and identify important future directions as catalysts for further research.

1 Introduction

In the past three decades, the research field of marketing mechanisms has grown significantly with the newfound interest in social networks and their impact on marketing. In the past, marketing models used individual consumer data, such as demographic information and past buying behavior, to determine their responsiveness to marketing. However, the key insight that propelled the growth of this field was that consumers are influenced not only by their own valuations, but also those of their neighbors. Consequently, the question of how to develop optimal marketing strategies given these consumer dependencies emerged as an essential problem. These questions lie at the intersection of several prevailing fields: traditional market research and design, studies of influence maximization and information propagation in social networks, data analysis for empirical verification of theory and for the design of relevant recommendation systems, and algorithm development and analysis for market design, to list a few.

This literature review surveys a series of papers that capture key developments in the field of revenue maximization in social networks. In Section 2, we discuss early works to provide context for the emergence of social network considerations in direct marketing. These papers set the stage for Section 3, in which we analyze three seminal papers: two by Domingos and Richardson [1, 2], and one by Kempe et al. [3]. Published from 2001 to 2003, these papers lay the groundwork for modeling marketing strategies and influence maximization based on customer networks. Domingos and Richardson introduced the idea of viewing marketing as an influence maximization problem, and were among the first to consider the network value of a consumer. They also began a trend of empirically testing models with simulations and/or datasets. Kempe et al. focused less on the marketing component and instead framed the influence maximization question as a discrete optimization problem, developing approximation algorithms to find the optimal such set of nodes.

A fundamental limitation of the influence maximization framework is that it does not consider customers' *adoption on price*. A customer's adoption of a product depends not only on the purchases of their neighbors, but also on the offered price of the product. Thus, in Section 4, we introduce the notion of pricing strategies. The seminal 2008 paper by Hartline et al. [4] posed marketing on social networks as a problem of revenue maximization rather than influence maximization. This framework allows for the analysis of pricing strategies, which accounts for positive externalities, in addition to the selection of influential nodes in a network. We then consider extensions of this model that account for varying discounts, by Babaei et al. [5], and pricing trajectories, by Anari et al. [6].

The aforementioned papers focused strictly on influencing individual, myopic behavior in monopolistic settings (other than Anari et al.). Motivated by these limited scopes, in Section 5, we discuss four considerations for future directions to extend the current models: non-myopic buyers, non-monopolistic settings, collective buying decisions, and empirical applications.

2 Background: Growing Interest in Direct Marketing and Social Networks

2.1 Direct Marketing

With the advent of social media platforms, there has been a gradual shift in interest from mass marketing to direct marketing [7]. Mass marketing uses media such as television and radio to market to all potential customers indiscriminately. On the other hand, direct marketing promotes a product or service to a select group of customers. A fundamental question in direct marketing is how to determine which customers to market to given a finite amount of resources to maximize profit.

A growing body of research began applying data mining on customer data to predict a consumer's responsiveness to direct marketing. Ling and Li [7] used data mining to construct models based on customer data, such as demographic information and past buying behavior, to predict their response to targeted ads. Piatetsky-Shapiro and Masand [8] use measures such as *lift* to show that modeling this predictive behavior can yield a significant increase in profits.

However, an underlying limitation of this approach is that it assumes that customers make purchasing decisions independently of other customers. In practice, one might expect a customer's decision to purchase a product to be heavily influenced by that of the people around them. This shortcoming motivates the consideration of social networks in marketing.

2.2 Social Networks

Stanley Milgram's (1969) trailblazing Small-World experiment estimated that every person in the world is within six degrees of separation from any other person [9]. This work sparked interest in the study of the interconnectedness of people, even in large societies. A person's connections in a network could be used to learn information about the person's interests, and thus, responsiveness to marketing. Schwartz and Wood [10] developed an algorithm that deduces shared-interest relationships between people in a network based on their email communication history. They used this approach to identify people in the network that would be responsive to marketing.

By the late 1990s, the use of customer networks for modeling the impact of direct marketing grew, but many of these studies had limited quantitative analyses. For instance, Krackhardt [11] constructed a model that determines how to select customers from a network to offer a free sample of a product to. The model was quite limited as it was only applied to a single rudimentary network of seven nodes. Moreover, it only accounted for the impact of a customer's immediate friends (i.e. not friends of friends) and assumed that each customer has the same probability of purchasing a product upon being given a free sample.

The limitations of these studies motivate the use of data mining to derive more specific information on the preferences and connections of individuals within a network, which is indicative of the structure of networks that are worth considering. This data can also be used to empirically test the performance of more complex models. These limitations also suggest the need for well-developed quantitative models of marketing in social networks.

3 Marketing and Influence Optimization

3.1 Overview

We now examine three fundamental papers in the field: Mining the Network Value of Customers [1], Mining Knowledge Sharing Sites for Viral Marketing [2], and Maximizing the Spread of Influence through a Social Network [3]. The first two papers, both authored by Domingos and Richardson, provided the first insights into quantitative modeling of markets of customers within a network. The third paper was published in the same conference the following year. Kempe et al. focused less on marketing but established the groundwork for influence maximization. We now present a brief overview of these papers.

In their seminal 2001 paper, Domingos and Richardson [1] were the first to present the question of marketing as an maximization problem with a *descriptive* model. This approach viewed purchases of certain customers as random variables and sought to model their joint distribution. They were also the first to empirically test their model on a large dataset. Their work expanded the concept of direct marketing to viral marketing, a marketing technique that involves the spread of information through a social network.

Expanding on [1], in their 2002 paper, Domingos and Richardson [2] then consider a variation of the same descriptive model, but with linear estimates of certain quantities within [1] that were less computationally tractable. This allowed for the maximization problem of [2] to be feasibly solved using analytical techniques. They also tested different marketing schemes on a dataset to verify the accuracy of their proposed marketing scheme.

In contrast, Kempe et al. [3] considers an *operational* model, which details the step-by-step dynamics of influence within a social network. This paper considers a few different variants of the same general question: how should we select an initial group of people within a network to influence to maximize the total influence? Using mathematical techniques, Kempe et al. analyzes the complexity of this problem and presents approximation methods to achieve within constant factor of the optimal value.

In this section, we describe the technical formulations of each of the models with their respective objectives. We briefly detail the proposed approaches to highlight the objective in each of the papers, and finally, summarize their contributions and limitations. We include an illustrative visual of each of the models in Appendix A.

3.2 Models: Domingos and Richardson, 2001 & 2002

Domingos and Richardson's [1, 2] model assumes there are n people in the market. For all i, X_i is the Boolean variable for whether person i buys the singular product being marketed, so the set of all such variables is $\mathbf{X} = \{X_1, \ldots, X_n\}$. The individuals are implicitly connected through a directed graph. This information is captured by considering the set $\mathbf{N}_i$ of *neighbors* of X_i, i.e. the set of Boolean variables X_j of the neighbors of each person i. The model assumes that an individual's purchasing decisions are not influenced by those of anyone outside their set of neighbors. We consider $\mathbf{X}$ to be partitioned into $\mathbf{X} = \mathbf{X}^k \cup \mathbf{X}^u$, where $\mathbf{X}^k$ is the set of X_i values known to the seller, whereas $\mathbf{X}^u$ is the set of unknown values. We can treat X_i values in $\mathbf{X}^u$ as random variables and examine probabilities relating to them.

The seller's singular product is denoted by Y. In the paper, Y is represented as a vector $\mathbf{Y} = \{Y_1, \ldots, Y_m\}$ for the sake of differentiating the effects of shared features among different products within the later data analysis, but we will omit this since it is not crucial to the model. Finally, M_i is the *marketing action* taken on customer i. In particular, [1] assumes M_i is either 1 or 0, which represents whether a customer is or is not marketed to (i.e. offered a discount), respectively. To extend this idea, [2] considers continuous values of M_i (e.g. to represent the varying sizes of discounts offered). Both papers set $\mathbf{M} = \{M_1, \ldots, M_n\}$ as the set of all marketing actions.

Finally, [1] defines the cost of marketing to any individual as c, and the revenues obtained as r_1 or r_0, depending on whether the individual who purchases the product was or was not marketed to, respectively. [2] expands upon this idea by considering c and r to be functions of the marketing actions performed.

We now discuss the quantities of interest in this descriptive model.

3.2.1 Probability of Purchase

We first consider the probability that buyer i purchases the product given the known purchasing information of other buyers, the product's information, and the marketing action, denoted as $P(X_i|\mathbf{X}^k, Y, \mathbf{M})$. This probability will be essential to compute the Expected Lift in Profit defined in the next section. We assume that whether buyer i has purchased the product is unknown (i.e. $X_i \in \mathbf{X}^u$), so we can consider X_i to be a random variable. Domingos and Richardson [1] approximate this expression using maximum entropy estimates as follows:

$$
\begin{aligned}
P(X_i|\mathbf{X}^k, Y, \mathbf{M}) &= \sum_{C(\mathbf{N}_i^u)} P(X_i|\mathbf{N}_i, Y, \mathbf{M}) P(\mathbf{N}_i^u|\mathbf{X}^k, Y, \mathbf{M}) && (1) \\
&\approx \sum_{C(\mathbf{N}_i^u)} P(X_i|\mathbf{N}_i, Y, \mathbf{M}) \prod_{X_j \in \mathbf{N}_i^u} P(X_j|\mathbf{X}^k, Y, \mathbf{M}) && (2)
\end{aligned}
$$

where $C(\mathbf{N}_i^u)$ denotes the set of all configurations of the unknown neighbors of X_i. Equation 1 is based on the law of total probability, summing over all possible configurations of unknown neighbor values, which is implicitly based on the network. In the authors' data analysis, given quantities of the form $P(X_i|\mathbf{N}_i, Y, \mathbf{M})$, they are able to solve for $P(X_j|\mathbf{X}^k, Y, \mathbf{M})$ iteratively using Equation 2.

In [2], the authors further separate the probability of a customer into *internal* probabilities of purchasing the product, $P_0(X_i|Y, M_i)$, and *external* probabilities of purchasing the product, $P_N(X_i|\mathbf{N}_i, Y, \mathbf{M}_i)$ as follows:

$$
P(X_i|\mathbf{X} - \{X_i\}, Y, \mathbf{M}) = \beta_i P_0(X_i|Y, M_i) + (1 - \beta_i) P_N(X_i|\mathbf{N}_i, Y, \mathbf{M}) \tag{3}
$$

where β_i is a scalar that measures how self-reliant X_i is. Furthermore, they use a linear model to approximate the external probability:

$$
P_N(X_i = 1|\mathbf{N}_i, Y, \mathbf{M}) = \sum_{X_j \in \mathbf{N}_i} w_{ij} X_j, \tag{4}
$$

where w_{ij} represents how much buyer i is influenced by their neighbor j, with $w_{ij} \geq 0$ and $\sum_{X_j \in \mathbf{N}_i} w_{ij} = 1$.

3.2.2 Expected Lift in Profit and its Maximization

Domingos and Richardson define a central quantity, the global *expected lift in profit* (ELP), which they seek to maximize in both [1] and [2]. As a function of marketing actions, the ELP is intuitively the profit that a particular marketing scheme $\mathbf{M}$ attains relative to the null marketing scheme $\mathbf{M}_0$ (in which no marketing is done, i.e. $M_i = 0$ for all i). Using the Boolean framework of [1], this is defined as:

$$ELP(\mathbf{X}^k, Y, \mathbf{M}) = \sum_{i=1}^{n} r_i P(X_i = 1|\mathbf{X}^k, Y, \mathbf{M}) - r_0 P(X_i = 1|\mathbf{X}^k, Y, \mathbf{M}_0) - |\mathbf{M}|\mathbf{c} \tag{5}$$

where the revenues are given by $r_i = r_1$ if $M_i = 1$ and $r_i = r_0$ if $M_i = 0$, and $|\mathbf{M}|\mathbf{c}$ is the total cost of marketing. They define this quantity in [2] very similarly, but consider c and r to be functions of the marketing actions performed. The goal of both [1] and [2] is to maximize this ELP with respect to $\mathbf{M}$, the marketing strategy. Since the probabilities within the ELP expression depend on the network structure (i.e. people are influenced by their neighbors), this problem goes beyond that of simply maximizing profit by considering each person independently.

With the linear model defined in [2], the authors are able to solve directly for the optimal marketing strategy. However, in [1], a general solution may involve trying all possible $\mathbf{M}$ combinations, so the authors suggest three approximation methods for optimization: using a single pass, doing a greedy search, or using a variant of hill-climbing search. Empirically, [1] verifies that hill-climbing strategy generally finds the best marketing scheme, but at the cost of higher amounts of computational time.

3.3 Model: Kempe et al.

As opposed to [1] and [2], Kempe et al.'s [3] model does not consider the marketing aspect of networks – instead, it focuses exclusively on influence maximization. Kempe et al. defines an operational model that describes how influence in a network unfolds with time. Specifically, Kempe et al. considers a directed graph G with nodes that are *active*, e.g. willing to adopt a product in the marketing scenario, or *inactive* at any point in time. There is an initial set of active nodes A_0, and at each time step, inactive nodes may become active due to their active neighbors, while active nodes will stay active. Kempe et al. presents two models to capture such processes: the Linear Threshold Model and the Independent Cascade Model. The paper then describes more general models, but we focus on these two as they are at the heart of the paper's discussion.

In the Linear Threshold Model, each node v is influenced by each neighbor w according to weight $b_{v,w}$ (similar to the weights w_{ij} in [2]). Each node v has a threshold value drawn uniformly at random: $\theta_v \in [0, 1]$. A diffusion process then occurs where, at each time step t, active nodes stay active and, for any node v that was inactive in the previous time step $t - 1$ but satisfies the equation

$$\sum_{w \text{ active neighbor of } v} b_{v,w} \geq \theta_v$$

at time t, v becomes active in step t.

In the Independent Cascade Model, at the time step at which a node v becomes active, it has a single opportunity to activate any inactive neighbor w, and it succeeds with probability $p_{v,w}$. If it fails, it cannot attempt to activate w again in the future.

For both models, the influence of a set of nodes A is defined to be the expected number of active nodes at the end of the diffusion process starting with $A_0 = A$. The goal of Kempe et al. [3] is to find the set of k nodes with the maximum influence given the parameter k. A major result in the paper is proving that this problem is NP-hard. However, a hill climbing strategy similar to the one described in [1] is shown to be within a factor of at least 63% of the optimal solution.

3.4 Empirical Studies

Domingos and Richardson [1, 2] are among the first to apply a quantitative model to a large dataset. In [1], they use the EachMovie collaborative filtering systems to perform an analysis of different marketing schemes and test how well the model fits the data. For the first task, they implemented three marketing strategies on the data: mass-marketing (targeting all), direct marketing (targeting those who individually have positive expected profit), and the network based marketing. At a high level, they found that mass marketing always performed poorly, direct marketing did slightly better, and the network-based marketing approach found by maximizing ELP (described in the model) consistently did the best. To test the fit of their model, they measured the correlation between the actual values of X_i with the estimated probabilities $P(X_i|\mathbf{X}^k, Y, \mathbf{M})$ and obtained a weak correlation, but they attribute it to the low dimensional input data and expected the correlation to increase upon more informative input data.

In [2], Domingos and Richardson apply their model to Epinions, an extensive knowledge-sharing site at the time. Using the product reviews on Epinions, they perform a similar experiment to [1], testing different marketing strategies to find the optimal one. They find that the viral marketing scheme (i.e. the optimal one maximizing the version of Equation 5 in [2]) performs the best out of the chosen schemes. Moreover, they experiment with situations in which parts of the network itself are unknown and find that the viral marketing scheme is robust and does not require all of the network information to achieve large lifts in profit.

Finally, Kempe et al. [3] use the network data from the collaboration graph of co-authorships in the high-energy physics theory section of arXiv. Using this graph, they compare different influence maximization heuristics with the hill climbing (greedy) strategy which was proven to be within 63% of optimal, along with the crude baseline of a random choice of nodes. They find that, while heuristics of choosing nodes of high degrees or "central nodes" perform much better than the random baseline, the greedy strategy still performed the best among all those considered.

3.5 Contributions and Limitations

The key contributions of both [1] and [2] are the construction of a quantitative, descriptive model for marketing in a social network, as well as the consideration of different strategies for achieving an optimal marketing scheme. Moreover, they demonstrate how to apply their model to a relatively large dataset and are able to empirically verify the quality of their proposed optimal marketing scheme using data. However, these papers do not offer significant considerations for best pricing strategies, and certainly not for different types of markets. A few immediate future directions here include better search algorithms (as opposed to hill-climbing, which is costly computationally and only finds an approximation) for finding the best marketing strategies and searching for stronger and more nuanced causal relationships between customers within data.

The key contributions of Kempe et al. [3] are the consideration of the influence maximization through an operational approach and showing that, although the problem is NP-hard, there are approximation solutions that achieve within a constant factor of the optimal value. This paper's focus on influence suggests that there are many opportunities to extend the model to consider revenue maximization within the network more directly, for instance if active nodes are interpreted as the purchase of a product. This leads us to the next section, where we discuss pricing strategies within a network.

4 Pricing Strategies in Social Networks

4.1 Motivation: Limitations of Influence Maximization Models

Domingos and Richardson [1] and Kempe et al. [3] posed the question of marketing in social networks as an influence maximization problem. Although they studied the question of identifying a set of influential nodes in a network, their marketing strategies did not consider customers' *adoption on price*. Indeed, a customer's adoption of a product depends on the pricing strategy used by the seller. They also did not offer deep considerations for the problem of revenue maximization for broader models.

Motivated by these limitations, the seminal 2008 paper by Hartline et al. [4] posed marketing on social networks as a problem of *revenue maximization* rather than influence maximization. In their model, a buyer's decision is influenced not only by the set of other buyers who purchase an item, but also the *price* at which the item is offered. This framework allows for the analysis of pricing strategies in addition to the selection of influential nodes in a network. It also allows for considerations of how many people to market to.

In particular, Hartline et al. [4] study optimal marketing strategies in the scenario in which the seller first offers a product to a set of influential buyers for free. The product is then offered to the remaining customers in some sequence, at a price proportional to the influence that was exerted on them by previous buyers. This model accounts for *positive externalities*, which are sales to buyers that have a positive impact on the likelihood of other customers buying. Hartline et al. show that finding the optimal marketing strategy is NP-hard, and instead construct a family of approximation algorithms called *influence-and-exploit* strategies. We include an illustrative visual of the model in Appendix A.

4.2 Model: Hartline et al.

The model assumes that there is a seller who wants to maximize their revenue from selling a good to a set X of potential buyers. The seller has an unlimited supply of the good, each unit of which costs them zero. Each buyer i's value, drawn from a known distribution F, is given by a function $v_i : 2^X \to \mathbb{R}^+$, so their value $v_i(S)$ is a function of the set of customers S that already own the good. The seller knows the distribution $F_{i,S}$ of the random variables $v_i(S)$ for all $S \subseteq X$, and the buyers' values are distributed independently. A *marketing strategy* has two components: the order in which the buyers are offered the good, and the price that each buyer is offered. The sequence and prices can be *adaptive*, which means they can be based on the purchases or rejections made by customers in the sequence so far. Buyers are assumed to be *myopic*, so their value depends only previous buying decisions.

4.3 Influence-and-Exploit Marketing

Hartline et al. prove that finding the optimal marketing strategy is NP-hard. Motivated by this result, they instead construct a simple strategy called *influence-and-exploit* (IE). The strategy has two steps.

1. Influence step: the seller gives the item to a selected set of buyers $A \subseteq X$ for free.

2. Exploit step: the seller visits the remaining buyers in $X \setminus A$ in an arbitrarily selected order. For each visited buyer i, given that a set $S \subseteq X \setminus \{i\}$ of buyers has already boughten the item, the seller offers i the good at the *optimal myopic price*, which is the price p that maximizes the expected revenue extracted from that buyer. In other words, this is the value p that maximizes $p \cdot (1 - F_{i,S}(p))$ (since the buyer accepts if and only if their value is at least p).

Hartline et al. show that IE strategies perform well even in relatively general settings. In the case that the revenue functions are submodular, monotone and non-negative, and the value distributions satisfy the monotone hazard rate condition, there exists a set A such that IE is a $\frac{e}{4e-2}$-approximation of the optimal marketing strategy. They present two approximation algorithms for generating the optimal such set A: a local search $\frac{1}{3}$-approximation algorithm, and a randomized local search 0.4-approximation algorithm. We describe the idea behind these algorithms below.

4.4 Connections to Kempe et al.

Hartline et al.'s [4] local search algorithm begins by selecting a singleton set $\{v\}$ for the node v that yields the maximum revenue. At a high level, the algorithm then iteratively adds any node to the set that increases the expected revenue and removes any node from the set that decreases the expected revenue. This algorithm is similar to the hill climbing algorithm proposed in Kempe et al. [3].

Furthermore, these papers have a more fundamental connection. Kempe et al. [3] sought to answer the question of how to pick the k most influential nodes in a network. This problem is deeply connected to the Influence step of the IE strategy, since the seller is aiming to pick the set of buyers A that will allow for the maximal possible exploitation in the Exploit step. While under the Kempe et al. model, the seller would only pick the initial set, the IE strategies then allow the seller to determine the sequence in which to offer the product to the buyers, as well as the price at which to offer them. Thus, Hartline et al.'s model can be seen as an extension of that of Kempe et al.

4.5 Contributions and Limitations

The key contribution of Hartline et al. [4] is the consideration of pricing strategies to frame marketing as a revenue maximization problem more directly. In particular, they construct the class of IE strategies, which are shown to perform relatively well in the general setting despite their simplicity. However, the simplicity of these models also suggest that there are many potential future directions of this work. For instance, the IE strategy picks a sequence of the remaining buyers at random. One can imagine that we could instead pick an ordering of the buyers to increase positive externalities, for instance by visiting the nodes with more neighbors earlier. Moreover, in the case in which the seller does not visit each of the buyers in $X \setminus A$ in sequence, but rather posts prices to all of them simultaneously, it is worth considering how to determine the optimal fixed price to maximize revenue in this scenario.

Moreover, we observe a fundamental limitation of the IE strategy: the pricing used in the Exploit step is the optimal *myopic* price. Indeed, the strategy assumes that the seller ignores buyer i's ability to influence other buyers given that it buys the good and simply wishes to optimize their revenue for the current buyer i. This assumption restricts the seller to a very near-sighted approach. In particular, the division of the strategy into two steps, i.e. influence and exploit, is quite limiting since the seller cannot consider the effect of positive externalities for any of the buyers not in the original set A. Babaei et al. [5] present a model that addresses this shortcoming.

4.6 Extension: Varying Discounts

Babaei et al. [5] extend Hartline et al.'s [4] model by offering varying discounts to buyers rather than simply providing the good for free to an initial set of buyers. Babaei et al.'s [5] model considers both the sequence in which to visit buyers and the optimal discount prices to offer them in order to maximize the seller's revenue. Unlike the Exploit step of the IE strategy, their pricing is not myopic: the model attempts to balance the tradeoff between maximizing the revenue from the current buyer and maximizing the influence of the current buyer on the remaining buyers.

Babaei et al. present three potential approaches to select a discount offer sequence. They are built on the following intuition. As before, the seller can initially offer the item for free to the most influential buyers, which will increase the value of the remaining buyers. The seller can then gradually lower the discount and continue offering the item to the remaining buyers with the greatest influence. Specifically, they consider discounting based on average degree, a greedy discount approach, and discounting based on standard deviation of the degree distribution. They show that these strategies outperform the traditional algorithms, with the greedy discount strategy yielding a 15% increase in performance. They also study two marketing strategies based on deterministic local search and the greedy hill climbing strategy, which are practical methods that can be applied in online social network settings.

4.7 Extension: Price Trajectories

Hartline et al. [4] and Babaei et al. [5] considered positive externalities in social networks: when more buyers own a product, other buyers' valuations of the product may increase. In particular, they proposed pricing strategies to exploit this increase in a buyer's valuation of the good on a given day. Anari et al. [6] built upon this notion of positive historical externalities by introducing the notion of price *trajectories*.

While Hartline et al. [4] assumed that buyers act myopically, Anari et al. [6] assume that buyers have full information on the future states of the world. In particular, they require the seller to post a sequence of public posted prices. The buyers then make simultaneous decisions for which day they will buy the product (if ever) and commit to those after seeing the price trajectory. This model relaxes myopic assumptions and allows buyers to be fully rational. It also prevents the seller from exercising price discrimination.

Anari et al. [6] model this setting as a game and study optimal pricing strategies by considering the game's equilibria. In particular, they determine the necessary and sufficient conditions on the buyers' valuation functions for the existence and uniqueness of equilibria. They define the revenue maximization problem in settings in which the equilibrium exists and is unique, and in these scenarios, present nearly-optimal algorithms in the symmetric setting (when there is a single buyer type) and the linear setting (when there is a buyer an initial type-independent bias and a linear type-dependent influenceability coefficient).

5 Future Directions

The field of optimized marketing in social networks was founded on the works of Domingos and Richardson [1, 2] and Kempe et al. [3] discussed in Section 3, which optimized the spread of influence through identifying the optimal nodes in a social network to market to. This work provided limited consideration for how to monetize these nodes and their surrounding network through optimal pricing strategies. As discussed in Section 4, Hartline et al. [4] and Babaei et al. [5] filled this gap by framing the situation as a revenue maximization problem directly. These works provide a robust framework for the consideration of markets as social networks.

However, they also give rise to numerous future directions that are highly applicable to real world situations. Indeed, the aforementioned papers focused strictly on influencing individual, myopic behavior in monopolistic settings (other than Anari et al.). Motivated by these limited scopes, we present four key future directions in the field.

5.1 Non-myopic Buyers

One area of expansion is the consideration of *non-myopic buyers*. In much of the current literature, agents are assumed to act myopically: they make purchasing decisions solely based on past purchasing events, rather than potentially optimizing for utility in the long run. In particular, these models do not rationalize the possibility of *future* discount rates or price adjustments as a component of their model. Thus, most of the current approaches depend on Bayesian models to understand buyers' purchasing decisions. In practice, buyers may wait for the price of a product to fall before buying it. This direction of research may call upon the need to transition away from strictly Bayesian modelling to account for the consideration of future price adjustments.

One key instance of the consideration of non-myopic buyers is the model by Anari et al. [6] described in Section 4.7. In the model, buyers could select the future day on which they wished to buy given full information on the future states of the world and a sequence of public posted prices. This model is significant since it allows us to assume buyers are fully rational. However, one can imagine scenarios in which agents only have partial information about the future, with which they make (potentially incorrect) predictions on future prices to base their current decisions. In the real world, we may expect buyers to behave in a way that is partly myopic, in that they are skewed to make decisions based on past events due to their uncertainty of the future, but still have some foresight on whether they can benefit by purchasing the product later on. Thus, it may be worth considering a model that is in between the two extremes presented by Hartline et al. [4] (completely myopic) and Anari et al. [6] (non-myopic).

An additional direction is that some buyers may have more information about the future than others, which suggests the consideration of price discrimination for non-myopic buyers.

5.2 Non-monopolistic Settings

The majority of models of marketing in social networks assume monopolistic settings. In practice, however, the same product is often sold by multiple competing sellers. Indeed, while we have discussed the influence of the purchasing decisions of neighboring buyers, as well as the offered price itself, on a buyer, it seems equally important to rationalize the effects of competing prices and products as externalities. For instance, one can imagine that more sellers will be incentivized to market to buyers earlier on, since once a buyer buys from a particular seller, they may become more likely to buy from them again. Thus, an essential area for further research is the consideration of *oligopoly* settings, which are markets with multiple sellers.

5.3 Collective Buying Decisions

The aforementioned works assumed that individuals make purchasing decisions individually (though they may be influenced by others' decisions). However, one can imagine the scenario in which a group of buyers is deciding on a product together, and they may be influenced by the decisions of other individuals and/or groups. Examples of this phenomena include convincing employers to switch health insurance products for their collective workforce, marketing to school districts to adopt new software technologies for classrooms, and advertising medical record systems for hospital networks.

Marketing to social networks in the context of group buying decisions is less studied, in part because much of the past literature cites that the influence effects that occur in the case of group settings are similar to those in individual settings. However, group settings present a number of new considerations, since sellers may need to consider the influence of not only individuals, but also clusters of individuals, i.e. groups. For instance, one can imagine the scenario in which it is unprofitable to market to a buyer, even if they are the most influential node, if they do not market to anyone else in their group.

5.4 Empirical Applications

More broadly, a more general future direction is in the area of real-world data analysis using empirical results. In recent years, much of the literature in the field has converged onto the area of applications of social network marketing optimization for influence and revenue. This is likely a result of the reduced cost and increased speed of computing across extensively large datasets (such as social networks). Although it is beyond the scope of this literature review to discuss the current papers in this area in detail, it is worth highlighting a few of the notable ones. Namely, Gong et al. [12]'s work on wireless communications companies and telecom pricing, and Chen and Xie's [13] work on AirBNB's marketplace are two well-cited works in this domain. Other applications include influence maximization on Twitter data [14], as well as information spread to homeless shelters [15].

6 Conclusion

The past few decades gave rise to a number of key developments in the field of optimized marketing in social networks. Broadly, the field has evolved from considering mass marketing strategies before 2000 to growing an increased focus on direct marketing through leveraging the social networks of buyers. These networks raised the natural question of how to develop marketing strategies that account for customers' influences on one another. In their seminal 2001 and 2002 papers, Domingos and Richardson [1, 2] were the first to present a quantitative descriptive model for this problem. In 2003, Kempe et al. [3] built on [1, 2] by focusing on the influence maximization problem of picking nodes to market to in order to maximize influence. Each paper incorporated data analysis in order to test and empirically verify their proposed strategies.

A fundamental limitation of the influence maximization framework presented by the above papers was the lack of consideration of customers' adoption on price. The seminal 2008 paper by Hartline et al. [4] addressed this issue by considering pricing strategies to pose marketing as a revenue maximization problem more directly. This work was followed by various others that extended the pricing model [5, 6].

These works provide a robust framework for the consideration of markets as social networks. However, they are also still limited in scope: most of the foundational works in the field focus strictly on influencing individual, myopic buyers in monopolistic settings. Motivated by these limitations, we presented four key future directions: non-myopic buyers, non-monopolistic settings, collective buying decisions, and empirical applications. As the interconnectedness of digital and social networks steadily grows, we expect the literature to expand in each of these directions.

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A Appendix A: Figures of Models Discussed in the Core Papers

To visually aid the reader's understanding of the various models discussed in this paper, we developed the following figures for the three foundational papers in this review. These include Domingos and Richardson's [1] seminal 2001 paper, Kempe et al.'s [3] 2003 paper on influence maximization, and Hartline et al.'s 2008 paper on pricing strategies [4].

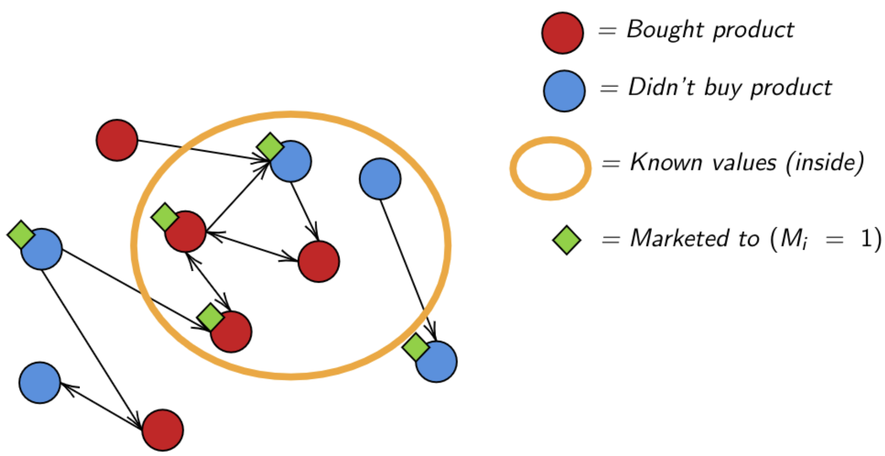

Figure 1: Domingos and Richardson [1] modelled a customer's network value based on known and unknown values for whether their neighbors bought a product.

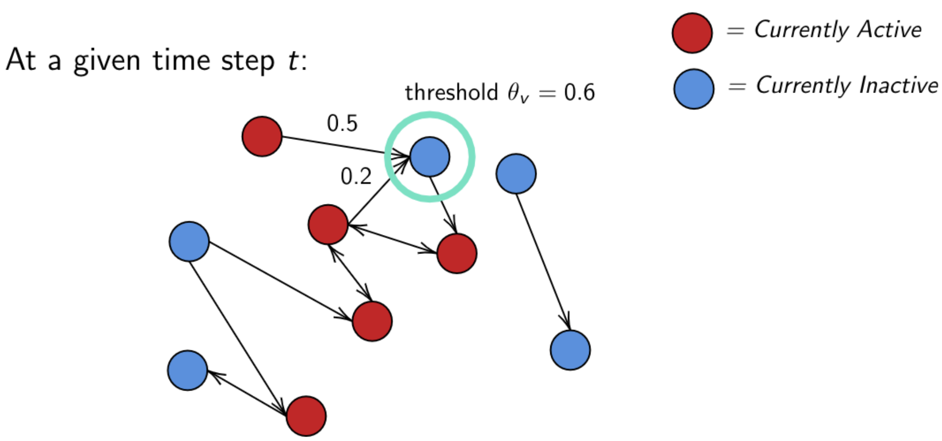

Figure 2: Kempe et al.'s [3] work in 2003 aimed to identify the most influential nodes within a network to market a product to, in turn optimizing the overall spread of information to increase adoption, i.e. active nodes. The figure illustrates the Linear Threshold Model at time step t, when the circled node is currently inactive.

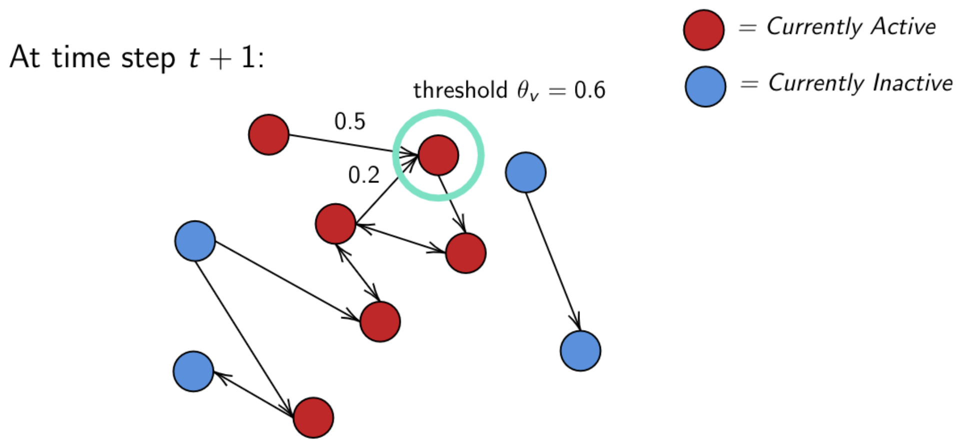

Figure 3: Kempe et al.'s [3] work in 2003 aimed to identify the most influential nodes within a network to market a product to, in turn optimizing the overall spread of information to increase adoption, i.e. active nodes. The figure illustrates the Linear Threshold Model at time step $t + 1$, when the circled node is now active.

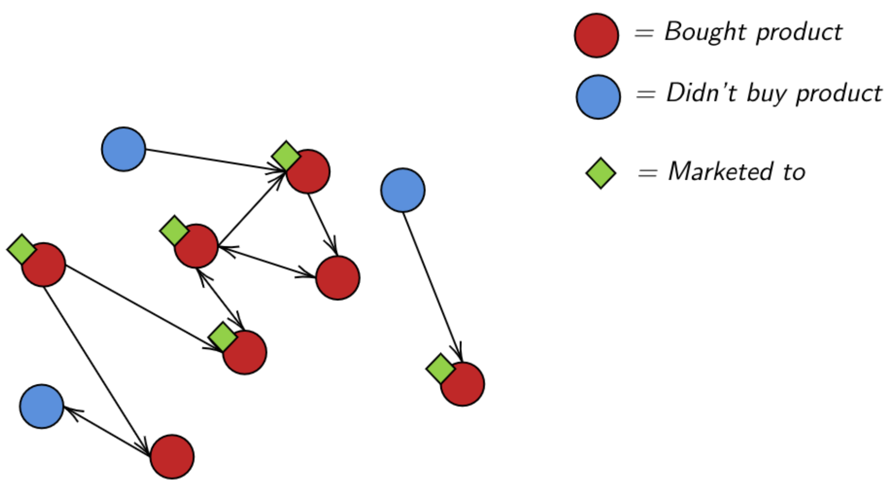

Figure 4: Hartline et al. [4] extended the above models to focus on revenue maximization directly. They identified influential nodes to first offer the product for free to (i.e. market to), and then sequentially to the remaining buyers at strategic prices.

B Appendix B: Contributions and Acknowledgments

Researchers: Max Guo, Haneul Shin, and Zeel Patel

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