

QCI402 - Mathematical Foundations for Quantum Computing

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Fall Semester 2025

The underpinnings of all scientific advancements is the ability to express natural phenomena with the art of Mathematics; this is no different for the subject of Quantum Computing. While the boundaries of quantum computing have been pushed beyond limits in theoretical terms on university blackboards, it has become of great interest to realize the theoretical computational power with the advances of hardware and technology.

However, these notes mainly concerns itself with the mathematical underpinnings of quantum computing that the course surrounds itself with. Mathematical Foundations for Quantum Computing takes a scaffolding approach designed to efficiently convey the required theoretical understanding of mathematics in order to able to learn quantum computing. As of writing, we are basing the notes on verison one of the textbook published in March 2025. In this text, we will primarily be using dirac notation for the expression of vectors, operators, and their interactions.

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1 Summation and Product Notations

This section primarily focuses on the common notations applied across mathematics to denote and shorten addition and product notation.

1.1 Summation over a single Variable

The sigma notation is defined as follows

$$\sum_{i=1}^n f(i)$$

where we use sigma \sum to represent the sum of a series. For example, the sum of all numbers in a series beginning with m and ending at index n is written as:

$$\sum_{i=m}^n a_i = a_m + a_{m+1} + a_{m+2} + \dots + a_{n-1} + a_n$$

Sums can also be infinite, commonly seen when Sigma looks as follows: $\sum_{i=m}^{\infty}$. Infinite sums are either convergent or divergent. A few of the most common converging infinite sums are as follows:

$$\sum_{i=0}^{\infty} \frac{1}{2^i} = 1 + \frac{1}{2} + \frac{1}{4} + \dots = 2$$

$$\sum_{i=0}^{\infty} \frac{1}{i^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

The first example is an infinite geometric series, and the sum of the first n terms is given by:

$$S_n = \sum_{i=0}^n \frac{1}{2^i} = \frac{1 - \frac{1}{2^{n+1}}}{1 - \frac{1}{2}}$$

As $n \rightarrow \infty$, $\frac{1}{2^n} \rightarrow 0$. Consequently, $S_n \rightarrow \frac{1}{1 - \frac{1}{2}} = 2$. A rigorous proof of the second example requires extensive calculus and is not immediately obvious. While any mathematical symbol can be used for the index of a summation, it is more practical to use something other than i as in the context of complex numbers, i commonly denotes the complex number $\sqrt{-1}$. moreover, sume can also be specified using descriptions. For example,

$$\sum_{p \in P} f(p) \quad P \in \mathbb{N}'$$

where \mathbb{N}' is the set of all prime numbers. Summations can also contain parameters other than the index, which results in functions of those parameters. For example the discrete Fourier transform (DFT) is given by

$$\tilde{x}_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} kn}, \quad k = 0, 1, \dots, N-1$$

where x_n represents the N values index by n and \tilde{x}_k are the Fourier coefficients. Here, i is the imaginary numebr and N is a positive integer representing the dimension fo the DFT, of which we will cover in greater depth in Chapter 3. The following are some useful summation forumae commonly encountered in quantum computing:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2} \right)^2$$

$$\sum_{i=0}^n (a_0 + id) = (n+1) \left(a_0 + \frac{nd}{2} \right) \quad (\text{arithmetic series})$$

$$\sum_{i=0}^n a^i = \frac{1 - a^{n+1}}{1 - a} \quad (\text{geometric series})$$

$$(a+b)^n = \sum_{i=0}^n \binom{n}{i} a^{n-i} b^i \quad (\text{binomial theorem})$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \quad (|x| < 1)$$

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} nx^{n-1} = 1 + 2x + 3x^2 + 4x^3 + \cdots \quad (|x| < 1)$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \quad (|x| < 1)$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$