

Quantum Computing LaTeX Quick Reference

Quick Command Sheet for Quantum Computing Lecture Notes

1 Basic Quantum State Notation 4 Quantum Channels and Noise

1.1 State Vectors

Ket: $|\psi\rangle$ or $|\psi\rangle$ (1)

Bra: $\langle\psi|$ (2)

Bracket: $\langle\psi|\phi\rangle$ (3)

Outer product: $|\psi\rangle\langle\phi|$ (4)

Expectation: $\langle A\rangle = \langle\psi| A|\psi\rangle$ (5)

General channel: $\mathcal{E}(\rho)$ (22)

Depolarizing: $\mathcal{D}_p(\rho)$ (23)

Bit flip: $\mathcal{B}_p(\rho)$ (24)

Phase flip: $\mathcal{P}_p(\rho)$ (25)

5 Quantum Information Measures

1.2 Common States

Computational basis: $|0\rangle, |1\rangle$ (6)

Superposition: $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ (7)

$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ (8)

Bell states: $|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle, |\Psi^-\rangle$ (9)

Multi-qubit: $|\text{GHZ}\rangle, |\text{W}\rangle$ (10)

Von Neumann entropy: $S(\rho)$ (26)

Conditional entropy: $H(A|B)$ (27)

Mutual information: $I(A : B)$ (28)

Fidelity: $F(\rho, \sigma)$ (29)

Trace distance: $D(\rho, \sigma)$ (30)

6 Quantum Algorithms

6.1 Fourier Transform

QFT: $\text{QFT}|x\rangle = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi i x k / N} |k\rangle$ (31)

Inverse QFT: QFT^{-1} (32)

2 Quantum Gates and Operations

2.1 Single-Qubit Gates

Pauli matrices: $\sigma_X, \sigma_Y, \sigma_Z$ (11)

Hadamard: H (12)

Phase gates: $S, T, P(\theta)$ (13)

Rotations: $R_x(\theta), R_y(\theta), R_z(\theta)$ (14)

6.2 Oracles

Boolean oracle: $O_f|x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$ (33)

Phase oracle: $O_f|x\rangle = (-1)^{f(x)}|x\rangle$ (34)

2.2 Multi-Qubit Gates

CNOT: CNOT (15)

Toffoli: Toffoli or CCNOT (16)

Fredkin: Fredkin (17)

SWAP: SAWP (18)

7 Error Correction

Stabilizer group: $\mathcal{S} = \langle g_1, g_2, \dots, g_k \rangle$ (35)

Syndrome: $s = (s_1, s_2, \dots, s_k)$ (36)

Code space: \mathcal{C} (37)

Logical codeword: $|\bar{\psi}\rangle$ (38)

3 Quantum Measurements

Measurement operator: \mathcal{M} (19)

Projectors: $|0\rangle\langle 0|, |1\rangle\langle 1|, \P_i$ (20)

Probability: $\Pr[i] = \langle\psi|i\rangle\langle i|\psi\rangle$ (21)

8 Mathematical Utilities

8.1 Complex Numbers

Imaginary unit: i (39)

Real part: $\Re(z)$ (40)

Imaginary part: $\text{Im}(z)$ (41)

Complex conjugate: z^* (42)

8.2 Linear Algebra

8.3 Probability

Tensor product: $A \otimes B$ (43)

Trace: $\text{tr}(A)$ (44)

Partial trace: $\text{Tr}_B(\rho_{AB})$ (45)

Rank: $\text{rank}(A)$ (46)

Probability: $\Pr[\text{event}]$ (47)

Expectation: $\mathbb{E}[X]$ (48)

Variance: δX (49)

9 Circuit Notation (with quantikz)

Basic circuit elements:

```
\begin{quantikz}
\lstick{$\ket{0}$} & \gate{H} & \ctrl{1} & \meter{} \\
\lstick{$\ket{0}$} & \qw & \targ{} & \qw
\end{quantikz}
```

10 Complexity Classes

Bounded-error quantum polynomial: BQP (50)

Quantum Merlin-Arthur: QMA (51)

Quantum polynomial space: QPSPACE (52)

11 Usage Examples

11.1 Quantum Teleportation Protocol

The quantum teleportation protocol transfers the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ using the following steps:

- Protocol 11.1.**
1. Alice and Bob share the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
 2. Alice performs a Bell measurement on her qubit and the qubit to be teleported
 3. Alice sends the classical result to Bob
 4. Bob applies the appropriate correction based on Alice's measurement

11.2 Grover's Algorithm

Grover's algorithm amplifies the amplitude of marked states:

- Algorithm 11.1.**
1. Initialize: $|\psi_0\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$
 2. Apply $G = -H^{\otimes n} |0\rangle\langle 0|^{\otimes n} H^{\otimes n} O_f$ for $O(\sqrt{N})$ iterations
 3. Measure to obtain the marked state with high probability

11.3 Quantum Error Correction Example

The 3-qubit bit flip code protects against single bit flip errors:

Circuit 11.1. Encoding: $|0\rangle \rightarrow |000\rangle, |1\rangle \rightarrow |111\rangle$
Syndrome measurement: $s_1 = Z_1 Z_2, s_2 = Z_2 Z_3$

12 Common Quantum Gates

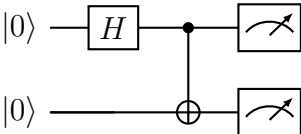


Figure 1: Quantum circuit for preparing and measuring a Bell state

Gate	Matrix	Gate	Matrix
Hadamard (H)	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	CNOT (CX)	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Identity (I)	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	Controlled-Z (CZ)	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
Phase (S)	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	Controlled- U (CU)	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & U_{00} & U_{01} \\ 0 & 0 & U_{10} & U_{11} \end{bmatrix}$
T-gate (T)	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$	SWAP	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
NOT (X)	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	Toffoli (CCNOT)	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$
Y-gate (Y)	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$		
Z-gate (Z)	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$		
Rotation $R(\theta)$ / Phase $P(\theta)$	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$		