Quantum Computing LaTeX Quick Reference

Quick Command Sheet for Quantum Computing Lecture Notes

1 Basic Quantum State Notation

1.1 State Vectors

$$\text{Ket:} \quad |\psi\rangle \text{ or } |\psi\rangle \tag{1}$$
$$\text{Bra:} \quad \langle\psi| \tag{2}$$
$$\text{Braket:} \quad \langle\psi|\phi\rangle \tag{3}$$
$$\text{Outer product:} \quad |\psi\rangle\langle\phi| \tag{4}$$
$$\text{Expectation:} \quad \langle A\rangle = \langle\psi\rangle\, A|\psi \tag{5}$$

1.2 Common States

$$\text{Computational basis:} \quad |0\rangle, |1\rangle \tag{6}$$
$$\text{Superposition:} \quad |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \tag{7}$$
$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \tag{8}$$
$$\text{Bell states:} \quad |\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle, |\Psi^-\rangle \tag{9}$$
$$\text{Multi-qubit:} \quad |\text{GHZ}\rangle, |\text{W}\rangle \tag{10}$$

2 Quantum Gates and Operations

2.1 Single-Qubit Gates

$$\text{Pauli matrices:} \quad \sigma_X, \sigma_Y, \sigma_Z \tag{11}$$
$$\text{Hadamard:} \quad H \tag{12}$$
$$\text{Phase gates:} \quad S, T, P(\theta) \tag{13}$$
$$\text{Rotations:} \quad R_x(\theta), R_y(\theta), R_z(\theta) \tag{14}$$

2.2 Multi-Qubit Gates

$$\text{CNOT:} \quad \text{CNOT} \tag{15}$$
$$\text{Toffoli:} \quad \text{Toffoli or CCNOT} \tag{16}$$
$$\text{Fredkin:} \quad \text{Fredkin} \tag{17}$$
$$\text{SWAP:} \quad \text{SAWP} \tag{18}$$

3 Quantum Measurements

$$\text{Measurement operator:} \quad \mathcal{M} \tag{19}$$
$$\text{Projectors:} \quad |0\rangle\langle 0|, |1\rangle\langle 1|, \P_i \tag{20}$$
$$\text{Probability:} \quad \Pr[i] = \langle\psi\rangle\, |i\rangle\langle i|\, |\psi \tag{21}$$

4 Quantum Channels and Noise

$$\text{General channel:} \quad \mathcal{E}(\rho) \tag{22}$$
$$\text{Depolarizing:} \quad \mathcal{D}_p(\rho) \tag{23}$$
$$\text{Bit flip:} \quad \mathcal{B}_p(\rho) \tag{24}$$
$$\text{Phase flip:} \quad \mathcal{P}_p(\rho) \tag{25}$$

5 Quantum Information Measures

$$\text{Von Neumann entropy:} \quad S(\rho) \tag{26}$$
$$\text{Conditional entropy:} \quad H(A|B) \tag{27}$$
$$\text{Mutual information:} \quad I(A:B) \tag{28}$$
$$\text{Fidelity:} \quad F(\rho, \sigma) \tag{29}$$
$$\text{Trace distance:} \quad D(\rho, \sigma) \tag{30}$$

6 Quantum Algorithms

6.1 Fourier Transform

$$\text{QFT:} \quad \text{QFT}\,|x\rangle = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi i x k/N}\,|k\rangle \tag{31}$$
$$\text{Inverse QFT:} \quad \text{QFT}^{-1} \tag{32}$$

6.2 Oracles

$$\text{Boolean oracle:} \quad O_f\,|x\rangle\,|y\rangle = |x\rangle\,|y \oplus f(x)\rangle \tag{33}$$
$$\text{Phase oracle:} \quad O_f\,|x\rangle = (-1)^{f(x)}\,|x\rangle \tag{34}$$

7 Error Correction

$$\text{Stabilizer group:} \quad \mathcal{S} = \langle g_1, g_2, \ldots, g_k\rangle \tag{35}$$
$$\text{Syndrome:} \quad s = (s_1, s_2, \ldots, s_k) \tag{36}$$
$$\text{Code space:} \quad \mathcal{C} \tag{37}$$
$$\text{Logical codeword:} \quad |\overline{\psi}\rangle \tag{38}$$

8 Mathematical Utilities

8.1 Complex Numbers

$$\text{Imaginary unit:} \quad \mathrm{i} \tag{39}$$
$$\text{Real part:} \quad \Re(z) \tag{40}$$
$$\text{Imaginary part:} \quad \mathrm{Im}(z) \tag{41}$$
$$\text{Complex conjugate:} \quad z^* \tag{42}$$

8.2 Linear Algebra

$$\text{Tensor product:} \quad A \otimes B \tag{43}$$
$$\text{Trace:} \quad \mathrm{tr}(A) \tag{44}$$
$$\text{Partial trace:} \quad \mathrm{Tr}_B(\rho_{AB}) \tag{45}$$
$$\text{Rank:} \quad \mathrm{rank}(A) \tag{46}$$

8.3 Probability

$$\text{Probability:} \quad \Pr[\text{event}] \tag{47}$$
$$\text{Expectation:} \quad \mathbb{E}[X] \tag{48}$$
$$\text{Variance:} \quad \delta X \tag{49}$$

9 Circuit Notation (with quantikz)

Basic circuit elements:

```
\begin{quantikz}
\lstick{$\ket{0}$} & \gate{H} & \ctrl{1} & \meter{} \\
\lstick{$\ket{0}$} & \qw & \targ{} & \qw
\end{quantikz}
```

10 Complexity Classes

$$\text{Bounded-error quantum polynomial:} \quad \mathrm{BQP} \tag{50}$$
$$\text{Quantum Merlin-Arthur:} \quad \mathrm{QMA} \tag{51}$$
$$\text{Quantum polynomial space:} \quad \mathrm{QPSPACE} \tag{52}$$

11 Usage Examples

11.1 Quantum Teleportation Protocol

The quantum teleportation protocol transfers the state $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ using the following steps:

Protocol 11.1.
1. Alice and Bob share the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
2. Alice performs a Bell measurement on her qubit and the qubit to be teleported
3. Alice sends the classical result to Bob
4. Bob applies the appropriate correction based on Alice's measurement

11.2 Grover's Algorithm

Grover's algorithm amplifies the amplitude of marked states:

Algorithm 11.1.
1. Initialize: $|\psi_0\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$
2. Apply $G = -H^{\otimes n} |0\rangle\langle 0|^{\otimes n} H^{\otimes n} O_f$ for $O(\sqrt{N})$ iterations
3. Measure to obtain the marked state with high probability

11.3 Quantum Error Correction Example

The 3-qubit bit flip code protects against single bit flip errors:

Circuit 11.1. Encoding: $|0\rangle \to |000\rangle$, $|1\rangle \to |111\rangle$
Syndrome measurement: $s_1 = Z_1 Z_2$, $s_2 = Z_2 Z_3$

12 Common Quantum Gates

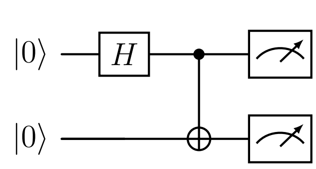

Figure 1: Quantum circuit for preparing and measuring a Bell state

Gate	**Matrix**
Hadamard (H)	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$
Identity (I)	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
Phase (S)	$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$
T-gate (T)	$\begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$
NOT (X)	$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$
Y-gate (Y)	$\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$
Z-gate (Z)	$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$
Rotation $R(\theta)$ / Phase $P(\theta)$	$\begin{bmatrix}1 & 0\\ 0 & e^{i\theta}\end{bmatrix}$

Gate	**Matrix**
CNOT (CX)	$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$
Controlled-Z (CZ)	$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1\end{bmatrix}$
Controlled-U (CU)	$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & U_{00} & U_{01}\\ 0 & 0 & U_{10} & U_{11}\end{bmatrix}$
SWAP	$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$
Toffoli (CCNOT)	$\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\end{bmatrix}$