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Chaotic dynamics applied to signal complexity in phase space and in time domain

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Abstract

In the past few years fractal analysis techniques have gained increasing attention in signal and image processing in Medicine. We concentrate on using fractal techniques for analysis of encephalographic data (EEG). Better understanding of general principles that govern discrete dynamics of these signals can help to reveal 'the signatures' of different physiological and pathological states. Fractal complexity of the signal in time domain, calculated using Higuchi's algorithm, seems to be the simplest method and may also be used in other biomedical applications.

1. Introduction

Processes that take place in a system often produce some 'imprints' – signals that can be registered. Analysis of complexity of such a signal helps to study the system itself, in particular dynamics of transitions between discrete system's states. For example, in Medicine signals produced by brain (EEG), heart (ECG), muscles (EMG) are used for diagnostic and assessment purposes. Better understanding of general principles that govern discrete dynamics of these signals can help to reveal 'the signatures' of different physiological and pathological states of body and mind. In the past few years fractal analysis techniques have gained increasing attention in medical signal and image processing, e.g., in pathology, neuro-psychiatry, cardiology.

1.1. Signal complexity

Signal complexity can be analyzed either in time domain, or in frequency domain, or in the system's phase space. Analysis in the frequency domain requires Fourier or wavelet transform of the signal, while analysis in the phase space requires embedding of the data in a multidimensional space. It is in the phase space where chaos meets fractals, since strange attractors have fractal dimension.

Complexity of the signal [30] may be characterized by its fractal dimension calculated directly in the time domain. *Fractal dimension*, D, calculated this way is a characteristics of the complexity of the curve representing the signal on a plane, e.g. that of the line traced out on a moving paper tape by a pen of a classical signal recorder (nowadays replaced by a computerized registration of signal's amplitude as a function of time) and similarly like that for the length of a

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coastline has its values always between 1 and 2. This fractal dimension *should not be confused with fractal dimension of the attractor of the system in the phase space* [19]. In time domain calculation of D is much quicker and so easier to be done in real time.

Woody Allen once said: “Brain is my second favorite organ”. We also are interested mainly in brain signals and concentrate our attention on fractal analysis of EEG in time domain.

1.2. Prediction vs. assessment

It is often assumed that the purpose of Non-linear Dynamics is to ‘predict and control’, that is to predict future states of the system (and so future events) possibly better than while using linear methods; this would also mean a possibility of controlling the system under consideration. Our point of view is different from the above, but the discrepancy is more of linguistic nature than really a substantial one.

Let us take an example. Suppose a doctor finds that a patient’s blood pressure is rather high and prescribes a drug to control the patient’s state; after some time the doctor measures the same patient’s blood pressure again and compares it with that from the previous test – if it is lower it means that the applied drug did work on this patient. But is the situation described above an example of dynamical prediction-control? In our opinion the answer is “NO”. Doctor’s decision is to prescribe a certain drug supposed to control patient’s blood pressure; but doctor’s decision is *not* based on prediction concerning individual patient’s case dynamics, but on medical statistics showing that high blood pressure means increased risk of heart failure and on pharmacological information that the given drug usually lowers blood pressure. The doctor compares patient’s blood pressure with that in the past. Comparing present value with previous value, is it *prediction?* Our answer is again “NO”.

To avoid misunderstandings we much prefer to use the term *assessment* for situation like this – the doctor *assesses* changes in patient’s state due to the prescribed drug. Facts that persons with high blood pressure have higher risk of death because of heart failure is not a prediction and facts that certain drugs lower blood pressure so decreasing the danger of heart failure is not a control, at least not in the sense of predicting and controlling the state of a system based on the system’s dynamics. But is assessment of patient’s state and its changes due to applied therapy important in Medicine? The answer is “YES”, it certainly is. Hence Non-linear Dynamics may be very useful in medical applications, helping doctors in *assessment* of patient’s state and its changes due to applied therapy as well as in *comparative assessment* of efficiency of different therapies.

2. Fractal dimension as a measure of complexity

2.1. Fractals and deterministic chaos

The problem with *fractal dimension* is that the same notion is used to denote many, often very different quantities (cf. [17]), e.g., a physicist while hearing the words ‘fractal dimension’ imagines almost automatically an attractor in a system’s phase space. It is in the phase space where Non-linear Dynamics and Deterministic Chaos theory are intimately related to Fractal Geometry, since systems that give rise to deterministic chaos have *chaotic* or *strange attractors* which are characterized by fractal (here meaning ‘not-integer’) dimension. Building up the phase space poses several problems – the data characterizing the system, like amplitudes of a signal produced by the system in consecutive moments of time, have to be embedded in the properly chosen multidimensional space.

But an object in Euclidean space, like a coastline of an island, may also be characterized by a fractal. In a rough sense, it measures ‘how many points’ belong to a given set. A plane is ‘larger’ than a line, while Sierpinski triangle sits somewhere in between these two sets. On the other hand, all three of these sets have the same number of points in the sense that each set is uncountable. Somehow, though, fractal dimension captures the notion of ‘how large a set is’ while its fractional part captures the notion of *how complex a set is*. Similarly, fractal dimension, or rather its fractional part, characterizes *signal’s complexity* in time domain – the greater it is the more complex is the signal under consideration. In this paper we concentrate on signal complexity in time domain.

To the degree that the global fractal dimension is a statistical measure of the whole object, it represents a measure of its global complexity. On the other hand, the local fractal dimension represents the complexity and the fractal properties of different *loci* within the object. In a sense, this is the essence of multifractals – namely that objects can have global and different local fractal dimensions and, hence, local differences in complexity. Multifractals may possess an infinite number of fractal dimensions and the spectrum of fractal dimensions leads to the definition of quantities that are analogous to the thermodynamic properties [6,21,29].

2.2. Self-similarity and statistical self-similarity

The first formal definition of a fractal said that it is *a set for which the Hausdorff–Besicovitch dimension strictly exceeds the topological dimension* [22]. But this definition is too restrictive. An alternative definition uses the concept of self-similarity – *a fractal is a shape made of parts similar to the whole in some way* [6]. A mathematical fractal has an infinite amount of details. This means that magnifying it adds additional details, so increasing the overall size. In non-fractals, however, the size always stays the same, no matter of applied magnification. If one makes a graph log(*fractal's size*) against log(*magnification factor*) one gets a straight line. For non-fractals this line is horizontal since the size (e.g., length of a segment, area of a triangle, volume of a cylinder) does not change; for fractal object the line is no longer horizontal since the size increases with magnification and fractal dimension can be calculated from the slope of this line [21].

So, fractal object has a property that more fine structure is revealed as the object is magnified, similarly like morphological complexity means that more fine structure (increased resolution and detail) is revealed with increasing magnification. Fractal dimension measures the rate of addition of structural detail with increasing magnification, scale or resolution. The fractal dimension, therefore, serves as a *quantifier of complexity*.

Natural objects like coastlines or roots do not show exactly the same shape but look quite similar when they are scaled down. Due to their statistical scaling invariance they are called *statistically self-similar*. The miniature copy of a structure may be distorted, e.g. skewed; for this case there is the notion of *self-affinity*; within a self-affine structure the scaling factor is not constant [17,21]. Statistical self-similarity and self-affinity is often called *Brownian self-similarity*. An object showing this property may be decomposed into parts which looks much alike the whole object but are made up of randomly located (instead of being fixed) smaller parts, which in turn may be decomposed into still smaller randomly located parts etc. The path of a particle exhibiting Brownian motion, observed in shorter and shorter time intervals, is the canonical example of this type of fractal [24].

One familiar example of naturally occurring fractal curves is a coastline. Since all of the curve's features that are smaller than the size of the measuring tool will be missed whatever is the size of the measuring tool selected; therefore the result obtained depends not only on the coastline itself but also on the length of the measuring tool. Fractal dimension is the unique fractional power that provides the correct adjustments for all those details smaller than the measuring device.

The fractal dimension of a signal provides insight into how elaborate the process that generated the signal might have been [2], since the larger the dimension the larger the number of degrees of freedom likely have been involved in that process [24]. There are several methods of calculating fractal dimension of a signal or an object [17].

2.3. Fractal analysis as statistical analysis

Even in the apparent totally disordered systems such as glasses or polymer networks we may observe statistical self-similarity (repetition of characteristic local structures and certain typical correlations between them) if we use probabilistic description of the network [9–11]. This is the most characteristic feature of the self-similarity: the fundamental information about the structure of a complicated system is contained already in quite small samples, and we can reproduce all the essential features by adding up and repeating similar subsets ad infinitum even if they are not strictly identical like in a crystalline lattice, but just very much alike like in quasi crystals or network glasses [8].

Most fractals in nature is only statistically self-similar, i.e. a portion of the object 'looks' *qualitatively* like the whole, and more formally the log–log plots of length of the object vs. the length of measuring elements produces a 'good' straight line fit over several orders of magnitude (in general, instead of length one may use other characteristics e.g., equivalent mass) [26]. For example, supporting evidence that lungs are fractal comes from measurements of the alveolar area, which was found to be 80 m^2 with light microscopy and 140 m^2 at higher magnification with electron microscopy – the increase in size with magnification is one of the properties of fractals. Fractal structure of the bronchial tree is imposed by optimization of resource utilization requirements in the lung, such as efficient distribution of blood and air. Thus, the morphology of the lung is directly related to its function and changes in its structure can be linked to dysfunction. Since the bronchial tree is a fractal structure, its fractal dimension can be used as a tool for the detection of structural changes and quantification of lung diseases.

Biological fractal structures like lungs and the vascular bed represent probably optimal design for their particular functions, like air flow or blood flow, respectively. It may well be that the fractal, dendritic trees of neurons are also optimally designed, but in this case for the flow of the most important commodity: information [26]. The surface of human *brain* contains a large number of folds. Its fractal dimension is between 2.73 and 2.79, the highest in the animal kingdom.

It should be emphasized that fractal dimension, D, is a descriptive, quantitative measure; *it is a statistic, in the sense that it represents an attempt to estimate a single-valued number for a property (complexity) of an object with a sample of data from the object.* One can, for example, view D in much the same way that thermodynamics might view intense measures such as temperature, even if not much is known about the underlying mechanisms leading to this value. D is not a unique, sufficient measure; for example, two objects may appear visually very different from one another and yet have the same fractal dimension [26].

3. Signal complexity in time domain

3.1. Fractal dimension of EEG-signal

Until introduction of personal computers biosignals like EEG were registered as curves written by special pens on long and wide paper tapes, in such a way that pens' displacement perpendicular to the direction of paper movement was proportional to the amplitude of the signal registered on the given channel. A curve like this shows statistical self-similarity and may be treated as a fractal, similarly like a coastline despite the fact that EEG-signals, unlike coastlines, are noisy [31]. Now we understand that EEG traces (curves) corresponding to different physiopathological conditions can be characterized by their complexity as measured by fractal dimension D, but calculation of D was rather impossible before introduction of computers.

With introduction of computerized data-acquisition systems biosignals are registered numerically, in a form of electrophysiological time series. Fractal dimension calculated in the system's phase space was proposed as a useful measure for the characterization of such series. First the data were embedded in an appropriately chosen phase space by using Takens' delayed coordinate method [28]. Then a fractal dimension of the system's strange attractor (if any), e.g. pointwise correlation dimension (which should not be confused with the fractal dimension D of a curve representing the signal on a paper tape like in classical EEG apparatus), was evaluated. For this purpose, however, relatively long stationary signal intervals were needed and consequently only long-term events could be analyzed; also much calculation time was required. For example, a reliable estimate of the pointwise correlation dimension [25] of EEG-signal could be calculated from time series of at least few seconds duration, which for EEG-signals that are very non-stationary is still rather long. Moreover, our calculations show that such 'classical' chaotic quantifiers do not exhibit any consistent pattern of changes (due for example to administration of a drug like Diazepam, cf. Table 1) that may be applied for therapy assessment [13,27].

It is feasible to use fractal dimension as a tool to characterize the complexity of short electroencephalographic time series. But to analyze events of brief duration it is necessary to make calculations directly in the time domain, which is analogous to calculate D of a curve representing the signal on a paper tape. Such fractal analysis allows investigating relevant EEG 'events' shorter than those detectable by means of other linear and non-linear techniques [1].

Calculation of fractal dimension of EEG-signals does make sense if D is not treated as an absolute measure but as a *comparative measure.* It enables doctors to assess changes in EEG due for example to applied therapy, for example to assess the influence of magnetic field on the brain, to asses the progress of phototherapy on patients with SAD (Seasonal Affective Disorder, colloquially called Seasonal Depression), to compare influence of different drugs including alcohol on the same patient, etc.

We also used another comparative measure taken from Non-linear Dynamics – cumulative pattern entropy [16,18,19].

Table 1
Pointwise correlation dimension (PD2) for raw EEG-data, calculated before and after administration of diazepam

Subject#	655		642		638		623	
Channel	Before	After	Before	After	Before	After	Before	After
Fp1-F7	7.0	6.6	8.3	8.0	N.C.	6.3	9.2	9.5
Fp2-F8	3.7	6.2	7.8	8.4	7.4	6.1	5.1	8.7
C3-P3	9.5	9.2	8.7	7.6	8.3	9.3	9.0	7.5
C4-P4	9.1	9.1	8.3	8.4	8.7	9.4	N.C.	N.C.
T5-O1	9.3	8.7	7.6	7.8	8.3	9.0	8.9	6.6
T6-O2	9.2	8.9	8.0	9.0	7.0	7.6	N.C.	7.8

3.2. Higuchi's fractal dimension

We use quick and easy algorithm proposed by Higuchi [7,20]. A fractal curve may be subdivided into k curves ($k = k_1, k_2, \ldots, k_{max}$) that are similar; then the length of this curve may be expressed as proportional to k^{-D}, where fractal dimension, D, measures complexity of the curve – for a simple curve D is equal 1, for a curve which nearly fills out the plane D is close to 2.

For the jth time window containing only small number, say $N = 128$ data points (e.g., length of 0.25 to 1.0 s, depending on the sampling frequency), local fractal dimension, D_j, may be calculated using Higuchi's algorithm; then the window is moved (overlapping of the consecutive windows of about 10% is often used) and local fractal dimension D_{j+1} is calculated, etc. This way the continuous signal $x(t)$ (where $x(i)$ is the signal's amplitude in the ith discrete moment of time), is transformed into $D(t)$, where $D(j)$ is the signal's fractal dimension in the jth time window. Characteristics of $D(t)$ shows changes of EEG-signal complexity due e.g., to such 'events' as eyes opening or eyes closing (Fig. 1; [4]).

These characteristics (like the 'tooth' in Figs. 2–4) are also demonstration of EEG-signal chaocity since they are destroyed if raw data are transformed into surrogate data and then fractal dimension $D'(t)$ of the surrogate data is calculated in the same way (Fig. 2; [14]).

Random signal (white noise) has fractal dimension equal practically 2 (Fig. 3; [14]).

Transformation of original signal $x(t)$ into fractal dimension $D(t)$ is prone to noise – adding artificial noise to the signal $x(t)$ does not change characteristic qualitative features of the transformed signal, $D(t)$, and causes only small changes of fractal dimension (Fig. 4).

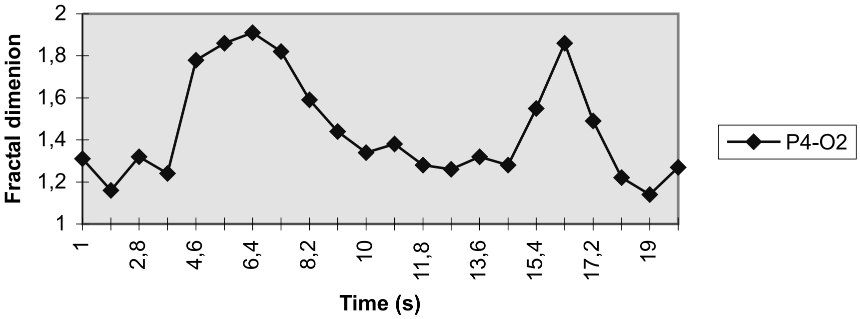

Fig. 1. Fractal dimension for 19 s epoch of raw EEG-data (channel P4-O2) with 'events' of eyes opening (in 4th s) and eyes closing (in 15th s).

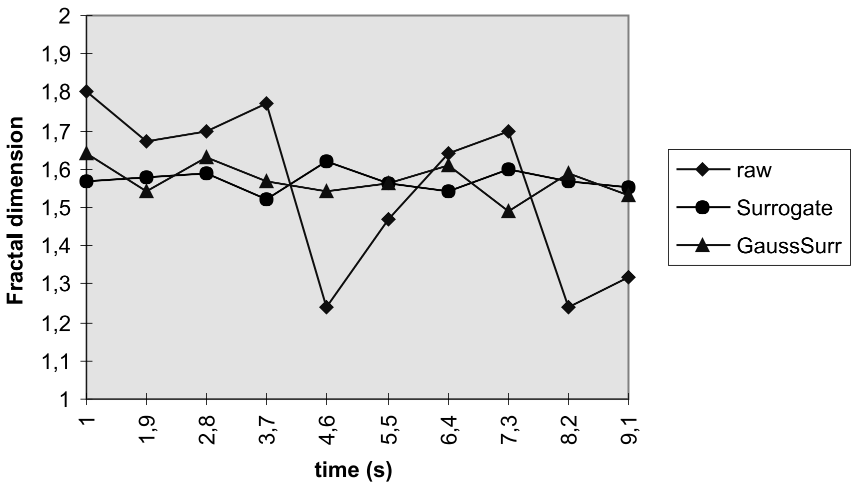

Fig. 2. Short epoch of EEG-data with the 'event' of eyes opening in 4.6th s – comparison of fractal dimension of raw data with fractal dimension calculated for surrogate data (for the same epoch); for surrogate data characteristic shape ('tooth') completely disappears.

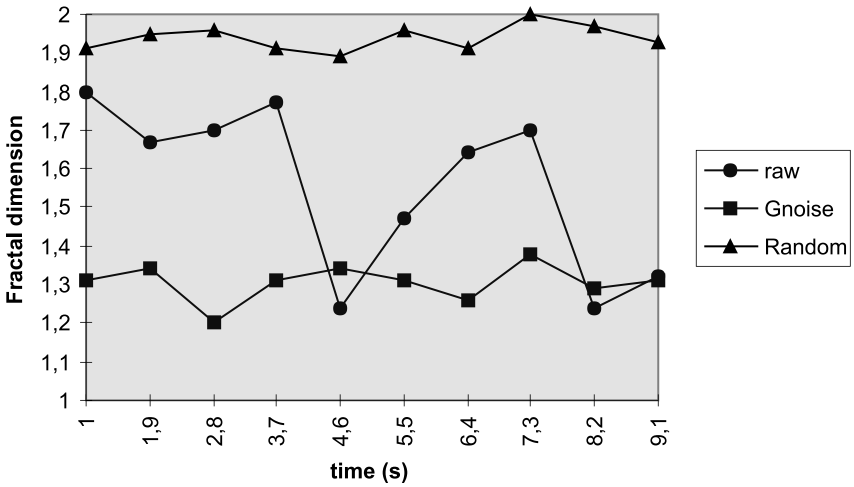

Fig. 3. Fractal dimension of the same short epoch of EEG-data as shown in Fig. 2 (the 'event' of eyes opening in 4.6th s) compared with fractal dimension calculated for Gaussian noise and random data (white noise); for noise on random data no characteristic shape ('tooth') is observed.

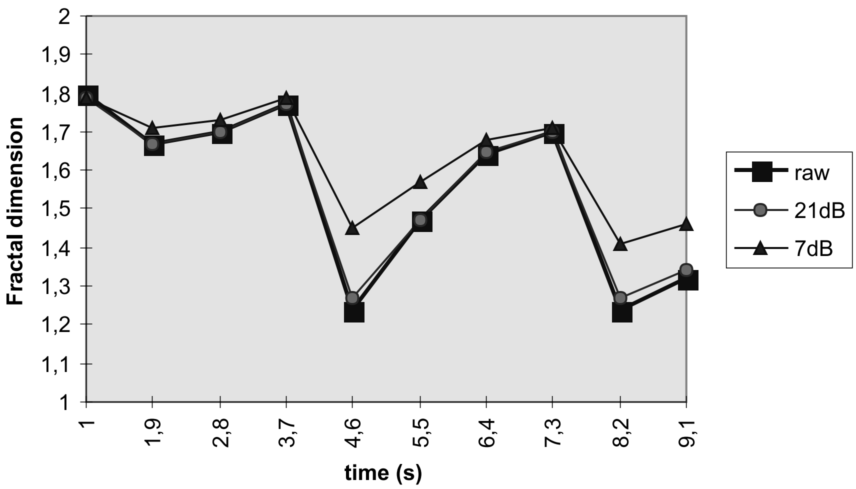

Fig. 4. Fractal dimension of the same short epoch of EEG-data as shown in Fig. 2 (the 'event' of eyes opening in 4.6th s) compared with fractal dimension calculated for data with artificially added noise; when signal-to-noise ratio is 21 dB there is practically no difference with the fractal dimension for raw data; when signal-to-noise ratio is 7 dB the differences are still small and 'tooth' is still preserved.

Fractal dimension correlates with power spectrum of the EEG-signal – for the intervals for which fractal dimension is below the average D calculated for the whole signal, the power spectrum is moved towards lower frequencies; for those intervals for which fractal dimension is above the average D, the power spectrum is moved towards higher frequencies (Fig. 5; [15]).

Since transformation of $x(t)$ into $D(t)$ gives 100-fold data compression it enables quick visual control of the signal properties and doctor may choose the most interesting intervals of the original signal for more detail inspection, whereas the detail inspection of the whole original signal would take much, much more time. *Fractal diagnoses method* is a computationally feasible way to achieve substantial reduction in the volume of EEG-data without undue loss of diagnostically important information in the primary signal [3].

Local (in time) changes of fractal dimension of EEG-signal due to different 'events', for example FD-ratio (eyes opening/eyes closing ratio) we have proposed, are even more suitable for assessment than average value of D and

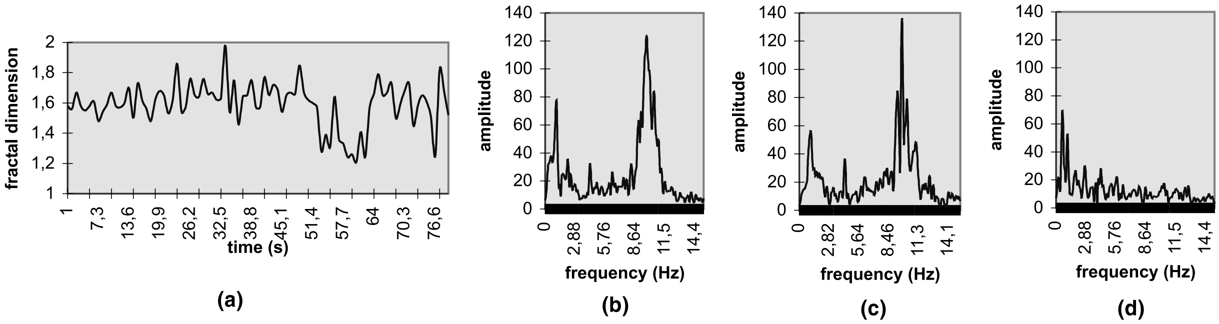

Fig. 5. Fractal dimension of EEG-signal correlates with the spectrum of the signal. (a) Fractal dimension for 80 s. EEG-epoch (channel O2-O1); (b), (c), (d) FFT-analysis for 10 s intervals of the same EEG-epoch: (b) for the interval 1 to 10 s, for which fractal dimension is around average; (c) for the interval 40 to 50 s, for which fractal dimension is above average; (d) for the interval 54–64 s, for which fractal dimension is below average; the amplitude is normalized – 350 corresponds to 50 μV.

correlate well with some psychological tests used by doctors to assess the advance of therapy (e.g., phototherapy in patients with SAD, cf. [4]).

4. Examples of application

We investigated possible influence of magnetic field on human brain by analyzing EEG-signal using Higuchi's method. No influence of magnetostimulation could be noticed by naked eye inspection of EEG-recordings. Also linear methods did not reveal any evident changes indicating possible influence of magnetostimulation whereas calculation of fractal dimension of EEG-signal clearly demonstrates an influence of magnetic field on brain (Fig. 6; [18]).

Fractal dimension was also used to assess influence of phototherapy on patients suffering with SAD; again, neither naked eye inspection of EEG even by a specialist nor analysis using linear methods revealed any evident changes, but calculation of D demonstrates influence of applied phototherapy (Fig. 7; [17,19]).

Fractal dimension may also be applied to two-dimensional patterns since it is possible to transform a pattern in such a way to obtain a one-dimensional signal containing the information about the pattern, called a *landscape*. For example, starting with some gray level images, first by proper segmentation binary images are produced; then one takes a strip of the binary image total length of N pixels and height of M pixels, with N several times greater than M. At each point of the long axis, denoted as $t \in [1, N]$, the fraction of 'white' pixels in the column orthogonal to the long axis is calculated:

$$x_1(t) = M_1(t)/M \in [0, 1],$$

where $M_1(t)$ denotes the number of white pixels in the tth column. The resulting series of N rational numbers $x_1(t)$ serves as input for the subsequent 'signal analysis'. Such approach was adapted for example by Mattfeld [23] for analysis of histological texture of tumors.

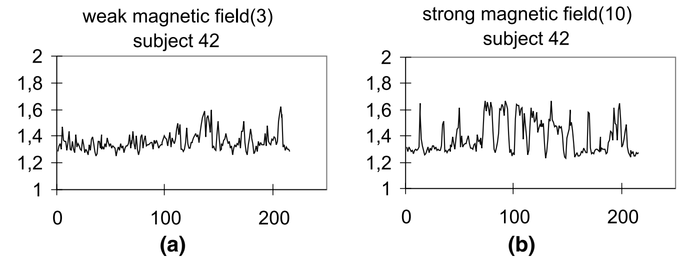

Fig. 6. Fractal dimension reveals influence of magnetic field on human brain. One unit on the horizontal axes denotes 5 s. For the first 7 min (84 units) magnetostimulator is "off"; next 7 min (85 to 168 units) magnetostimulator is "on"; and then it is "off" again. (a) In 'weak' magnetic field ('3' on the magnetostimulator scale) the subject shows no noticeable reaction. (b) In 'strong' magnetic field ('10' on the magnetostimulator scale) reaction of the subject is strongly marked. We used magnetostimulating device VIOFOR JPS MRS 2000® made by VitaLife Ltd. Poland that is attested for physiotherapeutic use by the Ministry of Health.

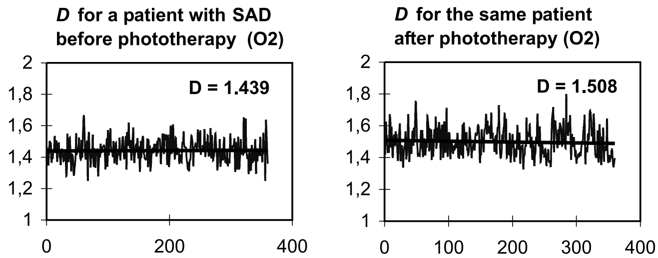

Fig. 7. Change of fractal dimension of EEG-signal due to phototherapy, calculated using Higuchi's algorithm for a subject with SAD (sampling frequency $v_{prob} = 128$ Hz, time in seconds, $k_{max} = 8$).

Fractal diagnoses became an important classificatory methodology for objective characterization of and discrimination between closely related conditions. The general approach of such *diagnostic scheme* is: 1st – characterize the case by several features, numerically describing some or all of the factors considered subjectively by pathologists, and 2nd – assign diagnosis to the case based on these features, in accordance with a prescribed classificatory approach determined and validated by a representative set of cases [5]. Fractal dimension is the parameter most often used in such diagnoses.

Non-linearity is very important in all living systems. Dose–response ratio are nearly always non-linear but this phenomenon is rarely taken into account in Medical Statistics [12]. The using of non-linear methods in medical applications is highly desirable.

5. Discussion and conclusions

Higuchi's fractal dimension is always between 1 and 2 since it characterizes complexity of the curve representing the signal under consideration on a two-dimensional plane. We need to stress once again that this fractal dimensions should not be mistaken with fractal dimension of an attractor, which is calculated in the system's phase space. Attractor dimension, e.g., pointwise correlation dimension, is usually fractal but it may be significantly greater than 2; it may provide some measure about how many relevant degree of freedom are involved in the dynamics of the system under consideration. Calculation of attractor's fractal dimension requires previous embedding of the data in phase space, using for example Takens' time delay method [16,28]. For Higuchi's method construction of phase space and data embedding are not needed, the algorithm works on raw data.

It was proven that fractal analysis of biosignals is a useful tool for medical applications. Processes taking place in a complex system like human organism often produce dynamical signals that can be registered. But System Theory and Non-linear Dynamics are very interdisciplinary. It is not only by accident that colloquially one speaks about 'social organisms'. The same methods that are used for analysis of complex systems in natural sciences may be adapted for socio-economic system analysis. These systems also produce some specific 'signals' and complexity analysis of these signals can help to reveal 'the signatures' of different 'physiological and pathological states' of society and government.

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