# Chapter 7

# Conclusions and future work

# 7.1 Conclusions

In this thesis, nonlinear analysis methods have been explored from reconstructed state spaces (RSS) with uniform time-delay embedding (UTDE) to recurrence quantification analysis (RQA) with recurrence plots (RP). However, it is necessary to compute and then select appropriate embedding parameters before using any of the tools for nonlinear analysis (see Chapter 3). Iwanski and Bradley (1998) stated that patterns in recurrence plots and metrics for recurrence quantification analysis are independent of embedding dimension parameters. However, that is not the case for different recurrence thresholds. Hence, embedded parameters and recurrence thresholds were considered to create three dimensional surface plots of recurrence quantification analysis which was hypothesised to be a better approach to understand the impact of different characteristic of real-world time series data such as window size length, participants, sensors and levels of smoothness.

No scientific work has been reported regarding the use of nonlinear analysis (e.g. RSS with UTDE, RP and RQA) to quantify movement variability in the context of human-humanoid interaction. This thesis has explored the weaknesses and strengths

of RQA using 3D surfaces of the variation of embedding parameters and recurrence thresholds which lead me to conclude that this approach requires less parametrization than others used in this thesis (e.g. RSS with UTDE, RP and RQA). Additionally, it was found that the 3D surface plots for RQA ENTR metric can be used to model any of the effects of movement variability for different activities or different participants as well as the post processing of real-world time series data with different window size length, smoothness and structures (shape, amplitude and phase) of time series (see Sections of weaknesses and strengths of RQA in Chapters 5 and 6).

In the following sections, positives and negatives of this thesis are pointed out by answering the raised research questions posed in Chapter 1.

What are the effects on RSSs, RPs, and RQA metrics of different embedding parameters, different recurrence thresholds and different characteristics of time series (structure, smoothness and window length size)?

It is evident that time series from different sources of time series (e.g. participants, movements, axis type, window size lengths or levels of smoothness) present differences for not only embedding parameters but also for the patterns in RSS, RP, RQA and 3D surfaces of RQA metrics. With that in mind, it can be concluded that the selection of appropriate embedding parameters and recurrence threshold is crucial to get meaningful results from nonlinear analysis tools. However, in this thesis it has been found that the creation of 3D surface plots of RQA metrics is a new approach that is independent of the type of time series and the selection of embedding parameters. Specifically, it was found that 3D RQA ENTR is robust against different sources of time series data, which can led to insight into the quantification of movement variability.

# What are the weaknesses and strengths of RQA metrics when quantifying movement variability?

From the reported results in chapters 5 and 6, it can be stated that the weaknesses of RQA, investigated in this thesis, are three: (i) the requirement of an expert(s) to interpret and compute embedding parameters and recurrence thresholds, (ii) the implementation and computation of methods of nonlinear analysis is laborious and computation of the parameters for such methods is still an open problem, and (iii) the selection of particular parameters to apply methods of nonlinear analysis does not necessarily give the best representation of the dynamics of the time series.

Hence, by proposing a variation of embedded parameters and recurrence thresholds to create 3D surfaces of RQA, it can be stated two strengths of RQA metrics: (i) little set up of parametrisation for 3D RQA metrics is required and (ii) 3D RQA ENTR might be a suitable approach to give insight to the understanding of the dynamics of different characteristic of time series.

# How does the smoothing of raw time series affect methods of nonlinear analysis when quantifying movement variability?

The answer to this question depends on (i) what to quantify in movement variability and also (ii) which hardware is involved in the collection of time-series data. For instance, to avoid erratic changes in the metrics of nonlinear analysis, smoothing raw signals can both help to obtain well defined trajectories in RSS and patterns in RP as well as constant values in RQA's metrics. However, on one hand, it has been observed that the increase of smoothness of time-series data created more complex trajectories (i.e. not well defined) in the Reconstructed State Spaces and also added more black dots in Recurrence Plots (see RSSs and RPs sections in Chapters 5 and 6). On the

other hand, two metrics of RQA (e.g. DET and ENTR) are more robust against the effect of smoothness of time series.

Additionally, smoothing time-series data can preserve the structure of the dynamics of NAO's arm movements when applying nonlinear analysis, as sometimes NAO produces jerky arm movements due to (i) its 14 degrees of freedom (DOF) for arms and head, (ii) the range of joint movement, (iii) joint torques and velocities, (iv) control of dynamic response (fast acceleration/deceleration), (v) stiffness of gear mechanics, or (vi) the number of degrees of freedom (see Gouaillier et al. (2009) for more references on NAO's mechanical and dynamic capabilities).

# 7.2 Future work

#### Inertial sensors

To have fundamental understating of the nature of signals collected through inertial sensor in the context of human-robot interaction, future experiments can be conducted considering the application of derivates of the accelerometer data. With that in mind, the following points can be explored (i) both the jerkiness of movements and the nature of arm movements which typically have minimum jerk (Flash and Hogan, 1985), (ii) the relationship of movement between different body parts, for instance, how rapidly or slowly a person performs arm and leg movements (de Vries et al., 1982; Mori and Kuniyoshi, 2012) or (iii) the application, to real-world time series data, of higher derivatives of displacement with respect time such as jounce, snap, crackle and pop (Eager et al., 2016).

## Smoothing time-series data

It has been hypothesised that one might create a closer representation of the nature of movement variability when using raw data from sensors. However, the quality of raw time-series data from inertial sensors can be affected by changes in sample rate, drift effect of long time-series data or changes of external variables such as temperature and magnetic fields to inertial sensors. Additionally, humanoid robots can sometimes produce jerky movements due its mechanical and dynamic capabilities. That said, further investigation is required to be done regarding the search of the appropriate balance between and the raw data and the degree of smoothness that can get closer to the quantification of the nature of movement variability in the context of human-humanoid interaction.

# Surrogate data analysis

Non-stationarity and non-linearity of experimental time-series data were assumed in this thesis (see Chapter 1). Such assumption was made based on the ambiguity of nonlinear analysis methods to quantify movement variability and the not yet fully explored area of application of nonlinear analysis methods in human-humanoid interaction (see Chapters 1 and 2). From the examiners of the PhD viva, one recommendation to avoid such prejudice of the type of data is to test the non-linearity and non-stationarity of the experimental time series data before nonlinear analysis methods are applied. Hence, a possible avenue to tackle such caveat is to apply surrogate data analysis to test that data have not been generated by "a stationary Gaussian linear stochastic process that is observed through an invertible, static, but possible linear stochastic function" (Schreiber and Schmitz, 2000, p. 2). However, applying surrogate data analysis to time series data that show strong periodicity or quasi-periodicity might create misleading results and perhaps provide unfair conclusion (see Figures in

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that illustrate how different realisations of the same periodic sinusoidal signal show to be sometimes stationarity and others non-stationarity). That said, further research require to be done, perhaps consider the works of Stam et al. (1998) and Small and Tse (2002) to test weak non-stationarity of periodic and quasi-periodic time series data. Also, for future work, it can be considered other time series data from activities that involve more than one joint in order to test the robustness of not only nonlinear analysis methods but also surrogate data analysis.

## Nonlinear analysis

#### Optimal embedding parameters

The method of False Nearest Neighbour (Cao, 1997) states that values of  $E_1(m)$  become insensitive to the increase of dimension, for which, in this thesis, a threshold has been defined in order to obtain the minimum embedding dimension  $m_0$ . However, a further investigation is required to be done for the selection of the threshold in the  $E_1(m)$  plots, as there were no particular method but visual inspection of the  $E_1(m)$  curves to set such a threshold (see Section 3.4.1 in Chapter 3). Similarly, further research is required to be done with regards to the selection of the minimum delay embedding because it is not clear: (i) why the choice of the first minimum of the AMI is the minimum delay embedding parameter (Kantz and Schreiber, 2003) or (ii) why the probability distribution of the AMI function is computed with the use of histograms which depend on a heuristic selection of number of bins for the AMI partitioning (Garcia and Almeida, 2005). Additionally, "the AMI method is proposed for two dimensional reconstructions and extended to be used in a multidimensional case which is not necessarily held in higher dimensions" (Gómez-García et al., 2014, p. 156).

#### Other methodologies for state space reconstruction.

In addition to the method of Uniform Time-Delay Embedding to reconstruct state spaces, other methods have been stated a better dynamic representations of time series in the reconstructed state spaces such as: (i) the nonuniform time-delay embedding methodology where the consecutive delayed copies of  $\{x_n\}$  are not equidistant (Pecora et al., 2007; Quintana-Duque and Saupe, 2013; Quintana-Duque, 2012, 2016; Uzal et al., 2011), or (ii) the uniform 2 time-delay embedding method which takes advantage of finding an embedding window instead of the traditional method of finding the embedding parameters separately (Gómez-García et al., 2014). As a future work, it might be worthwhile to apply (i) and (ii) methods to the current problem.

#### RP and RQA parameters

There are different avenues that can be investigated with regard to the computation of RP and RQA parameters. However from this thesis, it is suggested that the work of Marwan et al. (2007) and Marwan and Webber (2015) can be the starting point for further research with regards to different criteria for (i) neighbours, (ii) different norms ( $L_{1-norm}$ ,  $L_{2-norm}$ , or  $L_{\infty-norm}$ ) or (iii) different methods to select the recurrence thresholds such as: using only certain percentage of the signal ( $\sqrt{m_0} \times 10\%$  of the fluctuations of the time series) (Letellier, 2006), and selecting a determined amount of noise, and using a factor based on the standard deviation of the observational noise (Marwan et al., 2007).

#### Robustness of Entropy measures with RQA

Further investigation is required to be done with regards to the application of Shannon entropy with recurrence plots. Letellier (2006), for example, investigated the robustest of the Shannon entropy based on line segments distributions of recurrence plots  $S_{RP}$ 

against the Shannon entropy based on system dynamics  $S_{SD}$ . With that, Letellier (2006) pointed out that Shannon entropy based on recurrence plots has strong dependency with the choice of observable (i.e. variable of the dynamical system) while Shannon entropy based on system dynamics is more robust to noise-contaminated signals. Recently, with the introduction of the use of microstates, Corso et al. (2017) tackled the problem of Shannon entropy with RQA where ENTR values decrease despite the increase of non-linearity in a logistic map (Marwan et al., 2007). Additionally, Corso et al. (2017) presented the robustness of their method with changes to recurrence thresholds.

#### Advanced RQA quantifications

In addition to the application of RQA metrics (REC, RATIO, DET and ENTR) for recurrence quantification, advanced RQA metrics can be applied to the context of human-humanoid interaction. For example, RP based on complex networks statics, calculation of dynamic invariants, study of the intermittency in the systems, application of different windowing techniques, or the study of bivariate recurrence analysis for correlations, coupling directions or synchronisation between dynamical systems (Marwan et al., 2007; Marwan and Webber, 2015).

# Variability in perception of velocity

While conducting the experiments where participants performed arm movements with different velocities (e.g. normal and faster), it has been noted that participants perceive velocity differently. Particularly, some participants considered a normal velocity movement as being performed in slow velocity and others participants considered a slow velocity movement as being performed in normal velocity. With that in mind, it has been hypothesised that the differences in perception of velocities are related to different factors of a person such as (i) the background, (ii) personality traits or

(iii) even their movement experience (in music or sports) that make them more aware of their body movements. That said, further research require to be done to have better understanding on why each participant perceive the velocity of body movement differently, how such variability of perception of movement can be quantified, and what impact such differences might have for the control of movement or for the ability to recognise decrease in control ability.

### A richer dataset of real-world time series

It should be highlighted that the experiments for this thesis are limited to twenty three healthy right-handed participants of a range age of mean 19.8 and SD=1.39. Hence, participants of different ages, state of health and anthropomorphic features would create a richer dataset of real-world time series data to apply nonlinear analysis tools in the context of human-humanoid interaction.

# **Applications**

The application of the literature in human movement variability in the context of human-humanoid interaction can present different avenues. For instance, implement nonlinear analysis algorithms in humanoid robots in order to (i) evaluate the improvement of movement performances (Müller and Sternad, 2004), (ii) quantify and provide feedback of level skillfulness as a function of movement variability (Seifert et al., 2011) or (iii) quantify movement adaptations, pathologies and skill learning (Preatoni, 2007; Preatoni et al., 2010, 2013). Also applications in human-humanoid rehabilitation (Görer et al., 2013; Guneysu et al., 2015), where the use of nonlinear analysis can provide adequate metrics to quantify and provide feedback for movement variability.