Human Movement Science 30 (2011) 1144–1166

Contents lists available at ScienceDirect

Human Movement Science

journal homepage: www.elsevier.com/locate/humov

Common functional principal components analysis: A new approach to analyzing human movement data

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ARTICLE INFO

Article history:
Available online 2 May 2011

PsycINFO classification:
2240
2260

Keywords:
Functional data analysis
Common principal component analysis
Gait
Orthoses
Variability

ABSTRACT

In many human movement studies angle-time series data on several groups of individuals are measured. Current methods to compare groups include comparisons of the mean value in each group or use multivariate techniques such as principal components analysis and perform tests on the principal component scores. Such methods have been useful, though discard a large amount of information. Functional data analysis (FDA) is an emerging statistical analysis technique in human movement research which treats the angle-time series data as a function rather than a series of discrete measurements. This approach retains all of the information in the data. Functional principal components analysis (FPCA) is an extension of multivariate principal components analysis which examines the variability of a sample of curves and has been used to examine differences in movement patterns of several groups of individuals. Currently the functional principal components (FPCs) for each group are either determined separately (yielding components that are group-specific), or by combining the data for all groups and determining the FPCs of the combined data (yielding components that summarize the entire data set). The group-specific FPCs contain both within and between group variation and issues arise when comparing FPCs across groups when the order of the FPCs alter in each group. The FPCs of the combined data may not adequately describe all groups of individuals and comparisons between groups typically use *t*-tests of the mean FPC scores in each group. When these differences are statistically

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0167-9457/$ - see front matter
doi:10.1016/j.humov.2010.11.005

non-significant it can be difficult to determine how a particular intervention is affecting movement patterns or how injured subjects differ from controls. In this paper we aim to perform FPCA in a manner allowing sensible comparisons between groups of curves. A statistical technique called common functional principal components analysis (CFPCA) is implemented. CFPCA identifies the common sources of variation evident across groups but allows the order of each component to change for a particular group. This allows for the direct comparison of components across groups. We use our method to analyze a biomechanical data set examining the mechanisms of chronic Achilles tendon injury and the functional effects of orthoses.

1. Introduction

Human movement research studies often aim to determine differences in movement patterns between several groups of individuals, e.g., differences between injured and control subjects, differences between athletes grouped according to skill level, differences between children at various developmental stages, changes in movement patterns in response to some intervention, etc. Comparisons of group differences have typically involved the mean values in each group. Standard techniques such as *t*-tests require that the angle-time series data collected for each individual are summarized by single numbers (means, standard deviations, etc.). However such approaches lead to a severe reduction in the amount of data and much important information is discarded. This leads intuitively to the use of multivariate analysis methods such as principal components analysis, which summarizes the main modes of variation in a set of data. Such methods have been shown to be more useful than standard univariate techniques Deluzio, Wyss, Zee, Costigan, and Serbie (1997) and Daffertshofer, Lamoth, Meijer, and Beck (2004) and references therein but can also discard a large amount of information. In contrast, functional data analysis (FDA) (Ramsay & Silverman, 2005) is a statistical methodology that treats an entire sequence of measurements for an individual as a single functional entity rather than a set of discrete values. Treating the data as functions preserves all of the information contained in the data. In this paper, the term functional refers to the intrinsic structure of the data, i.e., the belief that the data are being generated by some underlying *function* and the discrete measurements collected are a snapshot of that function at various points in time. A central idea in FDA is smoothness, which implies that adjacent values in time are linked (i.e., are not independent) and it is unlikely that these values will differ largely (Ramsay & Silverman, 2005). The time-ordering of the data is vital since the value at one time point is dependent on the value at previous time point(s). This is a key difference between FDA and the multivariate techniques currently applied in human movement research which do not account for the time-ordering of the data. Treating human movement data as multivariate rather than functional has several drawbacks. First, using multivariate analyses requires that measurements for each individual are taken at exactly the same time points since each time point is treated as a separate variable. This is not the case in FDA where measurements can be taken at different time points. In addition, if the data were truly multivariate a measurement at one point in time could be exchanged with a measurement observed at any other point in time without altering the results since the time-ordering of the data is not accounted for in multivariate analyses. Therefore, we believe that FDA is more appropriate for analyzing human movement data.

Many multivariate techniques such as principal components analysis, regression analysis, etc. have been extended to analyze functional data. An important technique in FDA is functional principal components analysis (FPCA). Dynamical Systems Theory (DST) predicts that higher levels of kinematic or co-ordination variability may be a feature of good performance. The explanation of this is that higher levels of variability would indicate that the movement system (i.e., the body) is not being repeatedly loaded, whereas low variability would indicate that the system may be subject to repeated loading on

the same structures. If this repeated loading is present, for example, in running gait it could result over time in overuse injury. Therefore, examining variability and in particular how variability alters across treatment groups can provide real insight into the mechanism of injury or the effect of a particular treatment in kinematic data. Since FPCA provides a means of identifying and examining the main sources of variability of a set of curves, it is useful for analyzing human movement data where variability plays a key role. FPCA is described in full in Ramsay and Silverman (2005) and therefore the details are omitted here. When analyzing human movement data each FPC describes a particular movement pattern over the time interval being considered. Each subject receives a single FPC score on each component and a high score on FPC 1, for example, indicates that that subject is exhibiting the movement pattern described by FPC 1. An example is shown in Table 1 and Fig. 1, which examine the first FPC determined for the leg abduction angle of control subjects from the data set described in Section 2.1. Fig. 1(a) displays the mean leg ABD curve and the curves created by adding (+ signs) and subtracting (− signs) a multiple of FPC 1. This technique is used to aid the interpretation of the estimated FPCs (Ramsay & Silverman, 2005). The '+' signs represent the characteristic behavior of a high positive scorer on this component and the '−' signs represent the characteristic behavior of a high negative scorer on this component. In this case, FPC 1 represents variation around the mean and subjects with high positive (negative) scores (highlighted in bold in Table 1) tend to display leg ABD angles that are greater (less) than average throughout stance. Subjects 15, 17 and 22 have high positive scores on FPC 1 and therefore display increased leg ABD throughout stance as shown in Fig. 1(b). Subjects 27 and 28 have high negative scores on FPC 1 and therefore display decreased leg ABD throughout stance as can be seen in Fig. 1(b).

Table 1
FPC 1 scores of the leg ABD angle for the control group.

Subject	FPC 1 score	Subject	FPC 1 score
23	3.2374	24	−2.2537
17	**8.0822**	26	1.9603
28	**−13.39**	14	−0.5931
15	**5.1269**	6	−3.8642
21	4.3992	**27**	**−6.9527**
19	−3.1249	**22**	**7.3726**

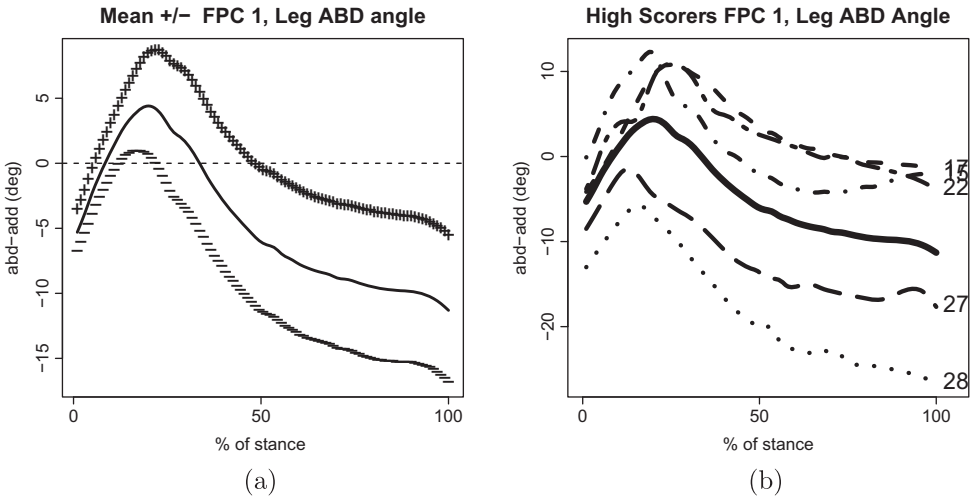

Fig. 1. (a) Depicts the mean curve of the leg ABD angle of control subjects plus (+) or minus (−) FPC 1, (b) depicts the mean curve of the leg ABD angle of control subjects and the leg ABD curves of high scorers on FPC 1.

The benefits of FPCA have been highlighted by a number of authors: Ormoneit, Black, Hastie, and Kjellström (2005) used FPCA to model human motion curves; Newell, McMillan, Grant, and McCabe (2006) studied the lactate curves of professional football players, Ryan, Harrison, and Hayes (2006) analyzed the dynamics underlying vertical jump performance of three groups of children and used the FPC scores to discriminate between the developmental stages; Harrison, Ryan, and Hayes (2007) examined the co-ordination of lower limb movements of children performing vertical jumps and used bivariate FPCA to determine the co-ordination patterns most effective at discriminating between developmental stages; Donoghue, Harrison, Coffey, and Hayes (2008) compared the rearfoot and lower limb kinematics of subjects with a history of Achilles Tendon injury and matched controls to determine movement patterns contributing to the mechanism of injury and used FPCA to examine the effect of orthoses on the movement patterns of subjects suffering from AT injuries and Doná, Preatoni, Cobelli, Rodano, and Harrison (2009) implemented the FPCA methodology to examine the technique of race-walkers. In each of these papers, FPCA was employed as a single sample technique, either by performing FPCA on the combined data set (where curves from all groups were aggregated into a single data set) or by performing FPCA on the data for each group individually, but there are problems associated with these approaches. Using the FPCs determined from the covariance function of the combined data set may lead to loss of information since the natural grouping structure evident in the data is ignored. For example if FPC 1 accounts for 85% of the variation in the combined data it is not clear how much variation is accounted for by this component in each individual group. The FPCs are not group-specific and the resulting components may not adequately describe the behavior in every group. In addition, the group with the highest variability will have the greatest influence on the resulting components. Comparisons between the groups are usually achieved through the analysis of the FPC scores (e.g., t-tests, discriminant analysis, etc.), but it can be difficult to determine significant differences between groups since such comparisons are based on the mean score in each group. Although the differences between the average scores in each group may be non-significant, the differences between the patterns of *variation* may contain more information as suggested by Donoghue et al. (2008). A basic approach to compare patterns of variation across treatment groups involves performing FPCA on the individual groups and requires extracting the FPCs of the covariance function of each group separately. This yields FPCs that are group-specific but the resulting components are disturbed by both within and between group variation (Fengler, Härdle, & Villa, 2003). This leads to difficulties in interpreting the FPCs. Similarly, since a reduced number of subjects is used to calculate the FPCs, the components can change their order from group to group. The order of the FPCs accounting for a small proportion of the variation can often change but can be ignored for the purposes of comparing groups since these components contain only a small amount of information. Problems arise when the order of the first few important FPCs is altered across groups (as occurs in the data set analyzed in this paper). In this case, it is not clear how to adequately compare FPCs across groups since each component describes a different aspect of motion. Therefore, while it may be clear that the same characteristic patterns of motion are evident in each group, the re-ordering of the group-specific FPCs means that it is not possible to compare the components and thus draw conclusions about how a particular movement pattern changes from group to group. There is clearly a need for a suitable method which allows for a comparison of the variance structure across treatment groups. The statistical method used here (common functional principal components analysis) determines the modes of variation that are *common* to each group but allows the ordering of the components to change across groups. This enables the direct comparison of variation patterns across groups and thus alleviates the limitations of standard FPCA as outlined above.

Common principal components analysis (CPCA) was originally developed as a multivariate method in a series of publications by Flury (1984, 1986, 1988). It has been extended to the functional case by Cizek, Härdle, and Weron (2005) who examined the use of common functional principal components analysis (CFPCA) in a financial application, modeling implied volatility data. To our knowledge neither CPCA nor CFPCA have been applied to human movement data. The method is based on the theory that the covariance functions of G independent groups have a common basic structure and that there exists a set of functional principal components that are common to all groups, i.e., that individuals in each group exhibit the same underlying characteristic behavior. CFPCA involves calculating the covariance functions of the individual groups and then determining the modes of variation that are common to

each. However the order (or relative importance) of the common functional principal components (CFPCs) can vary between groups. In essence, CFPCA assumes that while the same factors contribute to the variation in each group, the distribution of that variation across these factors can be very different for each group. This provides a meaningful way of comparing the covariance structure across groups based on an analysis of the CFPC eigenvalues (i.e., the variation of the CFPC scores). The eigenvalues determine the ranking of the CFPCs in each group and examining these rankings places the emphasis on comparing differences in the distribution of the variation across the factors in each group rather than comparing the means. While it may be possible to perform an F-test on the estimated eigenvalues for each CFPC in each group to determine if a particular CFPC accounts for significantly more variation in one group versus another, such a test has not been carried out in either the standard multivariate setting or the functional setting. In any case, such a test is not the main focus of CFPCA where the real interest is in how the influence of the CFPCs, and thus the distribution of the variation across the common factors, changes across groups. CFPCA is appropriate for analyzing human motion across treatment groups, since it is natural to expect that the same underlying factors influence the movement patterns of subjects in the individual groups though the influence of each factor may vary from group to group. The following paper aims to describe this novel approach to analyzing human movement data and show that it can provide real insight into the kinematics governing changes in movement patterns of individuals in independent groups.

CFPCA is suitable when groups are independent, e.g. when comparing treatment and control groups. It is not suitable when the groups are not independent, e.g. when measurements are taken on the same subjects before and after some treatment has been applied. CFPCA can be extended to incorporate this dependence across groups. This method, termed common functional principal components analysis for dependent groups (DCFPCA), is similar to CFPCA as it assumes that the covariance functions of the G groups have a common basic structure and that there exists a set of principal components that are common within groups. However DCFPCA also requires that the same common structure is evident between groups. DCFPCA involves calculating the covariance functions of each group (within-group covariance functions) and the cross-covariance functions (between-group covariance functions). The modes of variation that are common not only to each group but also across groups are then determined. This method provides a means of determining how movement patterns are changing across groups and thus establish how a particular intervention is influencing the movement patterns of individuals. DCFPCA allows us to determine the effect(s) of a treatment on the variability of movement patterns and identify the relative importance of these changes across groups. Again, such an approach is extremely useful in human movement studies since measurements are made on the same subjects before and after a particular treatment is applied. This paper outlines the considerable merits of this method and its capacity to ascertain further insight into how a particular treatment is influencing the movement patterns of an individual.

2. Methods

2.1. Data collection

A full description of how the data considered in this paper was collected can be found in Donoghue et al. (2008) and thus only a brief description is provided here. Written informed consent was obtained prior to commencement of testing and the study was approved by the local University Human Research Ethics Committee. Twenty-four subjects were initially classified into injured ($N_1 = 12$) and control ($N_2 = 12$) study groups. Eight retro-reflective markers were placed on the lower limbs of each subject. Subjects were observed running on a treadmill at self-selected speeds with eight Qualisys ProReflex MCU240 cameras obtaining three-dimensional coordinates of the markers. The injured subjects, or AT group, were observed wearing customized orthoses and not wearing customized orthoses; experimental conditions denoted hereafter as AT(O) and AT(NO) respectively. It was only possible to observe the control group (uninjured subjects) running without orthoses since it was not ethical to provide uninjured subjects with a treatment condition that was not required and may be harmful to them. During each footfall, angle-time series data were recorded on five angles of the lower limb, three of which

Table 2
Angles measured.

Angle	Description
Achilles tendon (EV) angle	Segment angle between rearfoot and lower leg; indicates eversion/inversion of rearfoot relative to the lower leg
Leg abduction angle (ABD) angle	Angle between the lower leg and the ground on the medial side as viewed from posterior; indicates level of varus/valus of lower leg
Ankle dorsiflexion (ADF) angle	Anatomical joint angle between the fibular head, ankle and 5th metatarsal; indicates level of dorsiflexion/plantarflexion

are listed in Table 2. Five replicate footfalls were obtained for each subject. The response variable data, i.e., the angle-time series, were curves and were functional in nature in the sense that for each kinematic variable they formed smooth real-valued functions. Data were padded with 10 additional frames at the start and end of each kinematic series to prevent end point distortion and so the time scale was normalized such that time $t = 0$ represented heel-strike and time $t = 1$ represented toe-off. We focused on the results for the leg abduction (leg ABD) angle, the ankle-dorsiflexion (ADF) angle and the Achilles Tendon (EV) angle since these angles were shown to be especially important in demonstrating the effects of orthoses on the kinematics of gait (Donoghue et al., 2008).

2.2. Smoothing

All of the following analysis was performed using the freely available R statistical software (http://www.cran.r-project.org). Functional data are usually measured at a discrete number of time points t_{ij}, $j = 1, \ldots, n_i$, where n_i is the number of observations for the ith individual and $t \in \mathscr{T}$. Since the records for individuals in a sample can have different lengths and measurements can be taken at different times for each individual, the index j is used to indicate the particular times at which a value was measured for an individual. Letting j range from 1 to n_i indicates that the total number of values measured can be different for each of the i individuals in the sample. Therefore if 9 values are measured for Subject 1, there are 9 distinct time points $(t_{11}, t_{12}, \ldots, t_{19})$ at which each value was taken, where t_{11} denotes the first time that a value was measured for Subject 1, t_{12} denotes the second time that a value was measured for Subject 1 and so on. The measurements taken typically contain some measurement error or noise, ε_{ij}. As a result we say that the observed (raw) values y_{ij} arise from a smooth function plus some noise, i.e.,

$$y_{ij} = y_i(t_j) + \varepsilon_{ij}. \tag{1}$$

A key preliminary step in FDA is to remove the measurement error (noise) from the raw data. This is called smoothing and provides an estimate of the smooth functions $y_i(t)$. Smoothing is achieved using a suitable smoothing method such as basis function expansions (Ramsay & Silverman, 2005). Basis function expansions involve representing $y_i(t)$ as a linear combination of K basis functions $\{\phi_1(t), \ldots, \phi_K(t)\}$ such that $y_i(t) = \sum_{k=1}^{K} c_{ik}\phi_k(t)$ and c_{ik} are coefficients to be chosen. There are many choices of possible basis functions including polynomial basis functions, Fourier basis functions, B-spline basis functions, wavelet basis functions, etc. The choice of basis function depends on the characteristic behavior of the data being analyzed and no single basis is suitable for all data types. In this paper, smoothing was carried out using cubic B-spline basis functions and a penalized least squares approach. This involved adding a penalty term to the least squares criterion, which penalized any roughness in the estimated curves. The coefficients c_{ik} were chosen to minimize the penalized criterion

$$\sum_{i=1}^{N}\sum_{j=1}^{n_i}\left[y_{ij} - \sum_{k=1}^{K} c_{ik}\phi_k(t_{ij})\right]^2 + \lambda \mathrm{PEN}_2\left(\sum_{k=1}^{K} c_{ik}\phi_k(t)\right), \tag{2}$$

where $\mathrm{PEN}_2()$ penalizes the curvature of the estimated functions. The trade-off between goodness of fit to the data and lack of smoothness was controlled by the smoothing parameter λ. The value of λ was chosen using generalized cross-validation (GCV); see Ramsay and Silverman (2005, Chapter 5) for

technical details and Ramsay, Hooker, and Graves (2009) for R and Matlab implementations. Once the functions $y_i(t)$ have been estimated from the raw data via smoothing, further analyses can be carried out.

2.3. Common principal components analysis

2.3.1. Independent Groups

Common principal components analysis was originally developed as a multivariate technique. Mathematical details on extracting the common principal components in a multivariate setting are given in Appendix A. This section generalizes those results to the functional case.

Let $y_{ig}(t)$ denote the functional data for the ith individual in the gth group, $i = 1, \ldots, N_g$, $g = 1, \ldots, G$. When analyzing functional data, covariance matrices become covariance functions and eigenvectors (principal components) become eigenfunctions (functional principal components). Let

$$\gamma(s,t) = N^{-1} \sum_{i=1}^{N} [y_i(s) - \bar{y}(s)][y_i(t) - \bar{y}(t)] \tag{3}$$

be the covariance function of the combined data set, where $\bar{y}(t)$ denotes the mean of the entire data set. Define

$$\gamma_g(s,t) = N_g^{-1} \sum_{i=1}^{N_g} [y_{ig}(s) - \bar{y}_g(s)][y_{ig}(t) - \bar{y}_g(t)] \tag{4}$$

to be the covariance function of the gth group, where $\bar{y}_g(t)$ denotes the mean of group g. In standard FPCA, when the data for each group are combined into a single data set, extracting the FPCs involves solving the eigenequation $\int_{\mathscr{T}} \gamma(s,t)\xi(t)dt = \rho\xi(s)$, where $\mathscr{T}$ denotes the time interval of interest. This essentially reduces to performing a singular value decomposition (SVD) of

$$\gamma(s,t) = \mathbf{\Delta}(s)\mathbf{\Lambda}\mathbf{\Delta}^T(t). \tag{5}$$

The columns of $\mathbf{\Delta}$ contain the eigenfunctions (FPCs) evaluated at time t and $\mathbf{\Lambda}$ contains the corresponding eigenvalues. The notation $\mathbf{\Delta}(t)$ is used to identify that we are dealing with functions. Similarly, when each group is considered separately, extracting the FPCs for the gth group involves solving

$$\gamma_g(s,t) = \mathbf{\Delta}_g(s)\mathbf{\Lambda}_g\mathbf{\Delta}_g^T(t). \tag{6}$$

The subscript g indicates that the FPCs and eigenvalues are now specific to the gth group. As outlined in Cizek et al. (2005) and Boente et al. (2010), extracting the common functional principal components (CFPCs) involves estimating the components such that

$$\gamma_g(s,t) = \mathbf{\Delta}^c(s)\mathbf{\Lambda}_g^c\mathbf{\Delta}^{cT}(t), \tag{7}$$

where the columns of $\mathbf{\Delta}^c$ now contain the common functional principal components (i.e., the components that are common to all groups) evaluated at time t and the matrix $\mathbf{\Lambda}_g^c$ contains the eigenvalues of the CFPCs in the gth group. The superscript c is used to denote the common functional principal components. Though the CFPCs are the same for each group (hence no subscript g on $\mathbf{\Delta}^c$), the eigenvalues (and therefore the order of the CFPCs) are different for each group (hence subscript g on $\mathbf{\Lambda}_g^c$). The CFPCs are determined using the equations presented in Appendix A and a modification of the F–G diagolization algorithm Flury and Gautschi (1986) and Clarkson (1988) for functional data. We have implemented this algorithm in R, adapting the code to perform CFPCA from the free statistical software package XploRe (http://fedc.wiwi.hu-berlin.de/xplore.php).

Due to the novelty of common functional principal components analysis, statistical tests for the method are currently under development (Boente, Rodriguez, & Sued, 2010; Benko, Härdle, & Kneip, 2009). To formally test the appropriateness of the CFPCA assumption, Benko et al. (2009) have developed several tests based on a resampling method called the bootstrap. One of these tests involves examining if the rth group-specific FPC (i.e., the rth FPC determined when using the data in each group alone) is identical across all groups. For example, when the data consists of two independent groups the following hypotheses are tested:

$$H_0 : \xi_r^{(1)}(t) = \xi_r^{(2)}(t), \quad r = 1, \ldots, R, \tag{8}$$

where $\xi_r^{(1)}(t)$ denotes the rth FPC determined using the data in group 1 only, $\xi_r^{(2)}(t)$ denotes the rth FPC determined using the data in group 2 only and R is the total number of components to be tested. If H_0 is not rejected for the first R FPCs, then the CFPCA assumption holds, the same factors are influencing the variance structure across groups and it is appropriate to extract the CFPCs. Utilizing the bootstrap method to test these hypotheses involves resampling with replacement from the original data in each group to form many "new" datasets. For example, let $\mathbf{y}^{(1)} = (y_1^{(1)}(t), y_2^{(1)}(t), y_3^{(1)}(t), y_4^{(1)}(t), y_5^{(1)}(t))$ denote the original data in group 1, where $y_1^{(1)}(t)$ denotes the data function for Subject 1 in group 1, $y_2^{(1)}(t)$ denotes the data function for Subject 2 in group 1, etc. Similarly, let $\mathbf{y}^{(2)} = (y_1^{(2)}(t), y_2^{(2)}(t), y_3^{(2)}(t), y_4^{(2)}(t), y_5^{(2)}(t))$ denote the original data in group 2, where $y_1^{(2)}(t)$ denotes the data function for Subject 1 in group 2, etc. Sampling with replacement from $\mathbf{y}^{(1)}$ and $\mathbf{y}^{(2)}$ forms a "bootstrap" sample for each group, e.g. $\mathbf{y}^{(1)^*} = (y_1^{(1)}(t), y_1^{(1)}(t), y_3^{(1)}(t), y_4^{(1)}(t), y_5^{(1)}(t))$ and $\mathbf{y}^{(2)^*} = (y_1^{(2)}(t), y_2^{(2)}(t), y_3^{(2)}(t), y_3^{(2)}(t), y_5^{(2)}(t))$. As can be seen, $\mathbf{y}^{(1)^*}$ no longer contains the data $y_2^{(1)}(t)$ but now has two copies of $y_1^{(1)}(t)$, while $\mathbf{y}^{(2)^*}$ no longer contains $y_4^{(2)}(t)$ but has two copies of $y_3^{(2)}(t)$. The group-specific FPCs for $\mathbf{y}^{(1)*}$ and $\mathbf{y}^{(2)*}$ are then calculated and the rth component in group 1 is compared with the rth component in group 2. The process of creating bootstrap samples, calculating and comparing the FPCs is then repeated hundreds of times to test the hypothesis. However, when the sample size is small (as is the case for the dataset described in this paper) the order of the FPCs in each group may change depending on the bootstrap sample drawn. This means that we can no longer compare FPCs across groups and leads to problems when testing the hypothesis outlined above. Rather than testing equality of each component individually, it may be more appropriate to test that the first R components span the same eigenspace. This is less restrictive than checking if individual components are equal and such a test may be especially of relevance when sample sizes are small. Concluding that the first R components span the same eigenspace in all groups is also evidence that the CFPCA assumption holds and that the CFPCs can be extracted. Full implementation details can be found in Benko et al. (2009).

However, for the purposes of this paper (which aims to introduce CFPCA and highlight its merits in a human movement context), a simpler approach, which considers the estimated eigenvalues, is used. If the hypothesis of common principal components is valid, it would be expected that the eigenvalues of the CFPCs should be close to the corresponding eigenvalues of the FPCs. To ensure that the CFPCA assumption was valid, the FPCs were extracted for each group individually as in Eq. (6), and then the CFPCs were extracted as in Eq. (7). The eigenvalues were then compared and if these values were suitably close, the analysis continued using CFPCA.

2.3.2. Dependent groups

Common principal components analysis for dependent groups was developed by Neuenschwander (1991) and Neuenschwander and Flury (2000) in the analysis of multivariate data. Mathematical details on extracting the common principal components for dependent groups in the multivariate case are provided in Appendix B. This section generalizes those results to the functional case. DCFPCA is similar to CFPCA as it assumes that some rotation simultaneously diagonalizes the within-group covariance functions and that all groups share the same components. However DCFPCA also requires that the same rotation diagonalizes the cross-covariance functions of measurements between groups.

Define the covariance functions by

$$\gamma_{g_1 g_2}(s,t) = N_{g_1}^{-1} N_{g_2}^{-1} \sum_{h=1}^{N_{g_1}} \sum_{m=1}^{N_{g_2}} [y_{hg_1}(s) - \bar{y}_{g_1}(s)][y_{mg_2}(t) - \bar{y}_{g_2}(t)], \quad g_1, g_2 = 1, \ldots, G, \tag{9}$$

where $\bar{y}_{g_1}(t)$ is the mean of group g_1 and $\bar{y}_{g_2}(t)$ is the mean of group g_2.

The within-group covariance functions are determined when $g_1 = g_2$. The between-group (cross) covariance functions are determined when $g_1 \neq g_2$. Extracting the common functional principal components for dependent groups involves estimating the components such that

$$\gamma_{g_1 g_2}(s,t) = \mathbf{\Delta}^c(s) \mathbf{\Lambda}^c_{g_1 g_2} \mathbf{\Delta}^{cT}(t), \tag{10}$$

where the columns of Δ^c now contain the principal components that are common within each group and across groups evaluated at time t, and the matrix $\Lambda^c_{g_1g_2}$ contains the corresponding eigenvalues. Again the $\Lambda^c_{g_1g_2}$ values determine the ranking of each component both within and between groups. This provides a means of examining how movement patterns vary across treatment conditions. The DCFPCs are determined using a modification of the F–G diagonlization algorithm and the equations provided in Appendix B.

3. Results

In order to compare movement patterns across groups, the data consisting of all five replicates for each subject in the AT(O), AT(NO) and control conditions were utilized. The data were initially smoothed as shown in Section 2.2 to remove measurement error and create smooth curves. For each subject in each condition, the five replicate angle-time series curves were aggregated into mean curves since the main aim of this analysis was to determine differences between the variation patterns of subjects across groups.

3.1. Injured versus control

Initially a comparison between injured subjects without orthoses (AT(NO)) and control subjects was carried out. Figs. 2(a), (b), 4(a), (b) and 6(a), (b) display the first three FPCs extracted for the leg ABD angle, ADF angle and EV angle of the AT(NO) and control groups considered separately. The overall shape of the first three FPCs is the same in all groups but the order of the second and third FPCs is altered across groups for both the leg ABD angle and EV angle. While it is clear that both groups have the same underlying variance structure and that the same factors have an effect on leg ABD, ADF and EV movement patterns of both injured and uninjured subjects, Donoghue et al. (2008) had difficulties performing a reasonable comparison of FPCs across groups because the order of the FPCs changes from group to group. To compare the groups, and since subjects in each group were different (i.e., the groups were independent), the common functional principal components (CFPCs) were extracted. The results for the leg ABD, ADF and EV angles are summarized below. The "CFPC" columns of Tables 3–5 display the eigenvalues and the percentage of variation accounted for by each CFPC in each group for the leg ABD, ADF and EV angles respectively. The eigenvalues for the group-specific FPCs (i.e., the FPCs extracted by estimating the components in each group separately) are displayed in the "FPC" columns. These were used to examine the appropriateness of the CFPCA assumption. Since the CFPC eigenvalues and the eigenvalues for the corresponding group-specific FPCs are very similar in the AT(NO) and control groups, the CFPCA assumption appears valid for each angle.

Fig. 3 displays the first three CFPCs extracted for the leg ABD angle of the AT(NO) and control groups and Figs. 5 and 7 display the first two CFPCs extracted for the ADF and EV angles respectively. These plots also display the mean leg ABD (ADF or EV) curve and the curves created by adding (+ signs) and subtracting (− signs) a multiple of each CFPC to aid interpretation. These components look very similar to those determined by Donoghue et al. (2008) and thus have essentially the same interpretations. CFPC 1 for the leg ABD angle represents an overall upward or downward shift from the mean curve, with reduced leg ABD ROM in the first 20% of stance. High positive (negative) scorers on this component tend to display leg ABD angle values that are lower (higher) than average throughout stance and move through a smaller (greater) range in the initial 20% of stance. CFPC 2 represents the leg ABD angle at heel-strike and changes in ROM that are particularly evident in the first 25% of stance. High positive (negative) scorers display decreased (increased) leg ABD in the first 25% of stance than average and reached their peak value later (earlier) in stance. CFPC 3 represents leg ABD range of motion (ROM), with particular emphasis on the first 50% of stance. High positive (negative) scorers display greater (less) leg ABD ROM than average. Though the analysis in Donoghue et al. (2008) based on the FPCs of the combined data revealed that on average injured subjects tended toward less leg ABD throughout stance and increased leg ABD at heel-strike than controls, the authors believed that differences in the variation of the scores may provide more in-depth information. In an attempt to examine these differences, they compared the group-specific FPCs 1 and suggested that a major

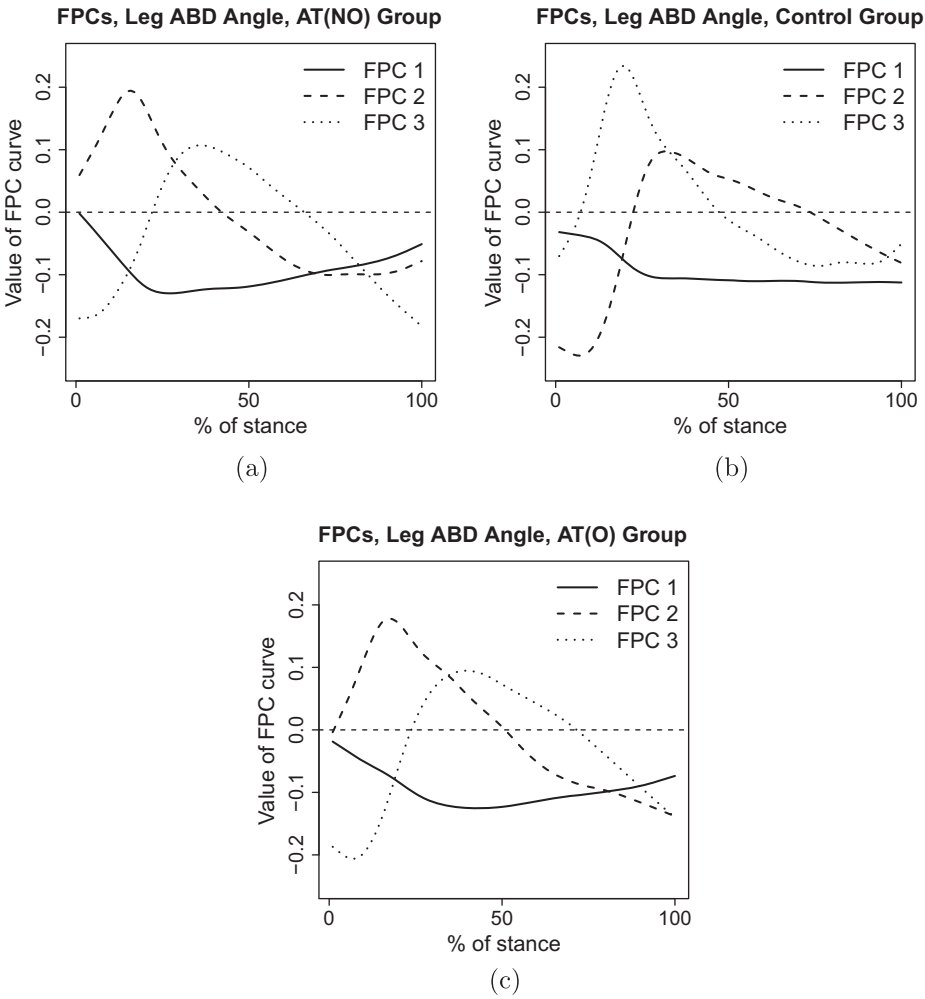

Fig. 2. Depicts the first three FPCs extracted for the leg ABD angle of (a) the AT(NO) group, (b) the control group and (c) the AT(O) group.

Table 3
Leg ABD eigenvalues for CFPCs. The table contains the eigenvalues for each group on each CFPC (CFPC columns) and the corresponding eigenvalues for the principal components extracted for each group separately (the value in the FPC columns). The value in brackets gives the percentage of variation accounted for by each CFPC in each group.

Component	AT(NO)		Control	
	CFPC	FPC	CFPC	FPC
1	10.46 (69.26%)	11.03	36.34 (90.42%)	36.40
2	1.62 (10.75%)	1.56	2.81 (7.00%)	2.86
3	2.64 (17.50%)	2.21	0.52 (1.30%)	0.56

Table 4
ADF eigenvalues for CFPCs. The table contains the eigenvalues for each group on each CFPC (CFPC columns) and the corresponding eigenvalues for the principal components extracted for each group separately (the value in the FPC columns). The value in brackets gives the percentage of variation accounted for by each CFPC in each group.

Component	AT(NO)		Control	
	CFPC	FPC	CFPC	FPC
1	14.74 (73.72%)	14.91	21.92 (82.47%)	21.96
2	3.21 (16.03%)	3.43	3.45 (12.97%)	3.50
3	1.23 (6.15%)	1.06	0.78 (2.92%)	0.77

Table 5
EV eigenvalues for CFPCs. The table contains the eigenvalues for each group on each CFPC (CFPC columns) and the corresponding eigenvalues for the principal components extracted for each group separately (the value in the FPC columns). The value in brackets gives the percentage of variation accounted for by each CFPC in each group.

Component	AT(NO)		Control	
	CFPC	FPC	CFPC	FPC
1	10.40 (42.04%)	10.53	15.39 (58.00%)	15.41
2	11.85 (47.89%)	11.98	8.39 (31.60%)	8.55
3	0.79 (3.18%)	1.09	1.06 (3.98%)	1.41

difference between injured and control subjects was the increased variation in leg ABD throughout stance for control subjects. However this difference could not be quantified and further comparisons of other group-specific FPCs was not carried out since the ordering of the components altered across groups. CFPCA provides a means of examining these differences in variation across groups.

CFPC 1 for the leg ABD angle explains over 90% of the variation in the control group and 69% of the variation in the injured group. This implies that over 90% of the variation in control subjects can be explained by changes in leg ABD throughout stance but only 69% of the variation in injured subjects can be explained by changes in leg ABD throughout stance. The leg ABD movement patterns of control subjects tended to vary almost 20% more around their mean curve than injured subjects. CFPC 2 accounted for 11% and 7% of the variation in the injured and control groups respectively, indicating that 11% of the variation in injured subjects can be attributed to changes in leg ABD at heel-strike and 7% of the variation in control subjects can be explained by changes at heel-strike. Finally, CFPC 3 accounted for a further 17% of the overall variation in injured subjects but only 1% of the variation in control subjects. Injured subjects had 16% more variation in leg ABD ROM than controls. These results show clear differences between the distribution of variation in leg ABD movement patterns of control subjects versus injured subjects. Over 90% of the variation in leg ABD of control subjects was explained by changes in movement patterns around the mean, with only a small percentage of changes in leg ABD being due to changes in leg ABD ROM (1%). In contrast, injured subjects had much lower variation in leg ABD around the mean and leg ABD ROM was a key source of variation in injured subjects accounting for approximately 16% more variation in this group than the control group. The leg ABD of injured subjects at heel-strike was also more variable than controls (11% versus 7%). These results are consistent with DST which predicts that uninjured subjects exhibit more variability than injured subjects. It is clear that control subjects display increased variability throughout stance than injured

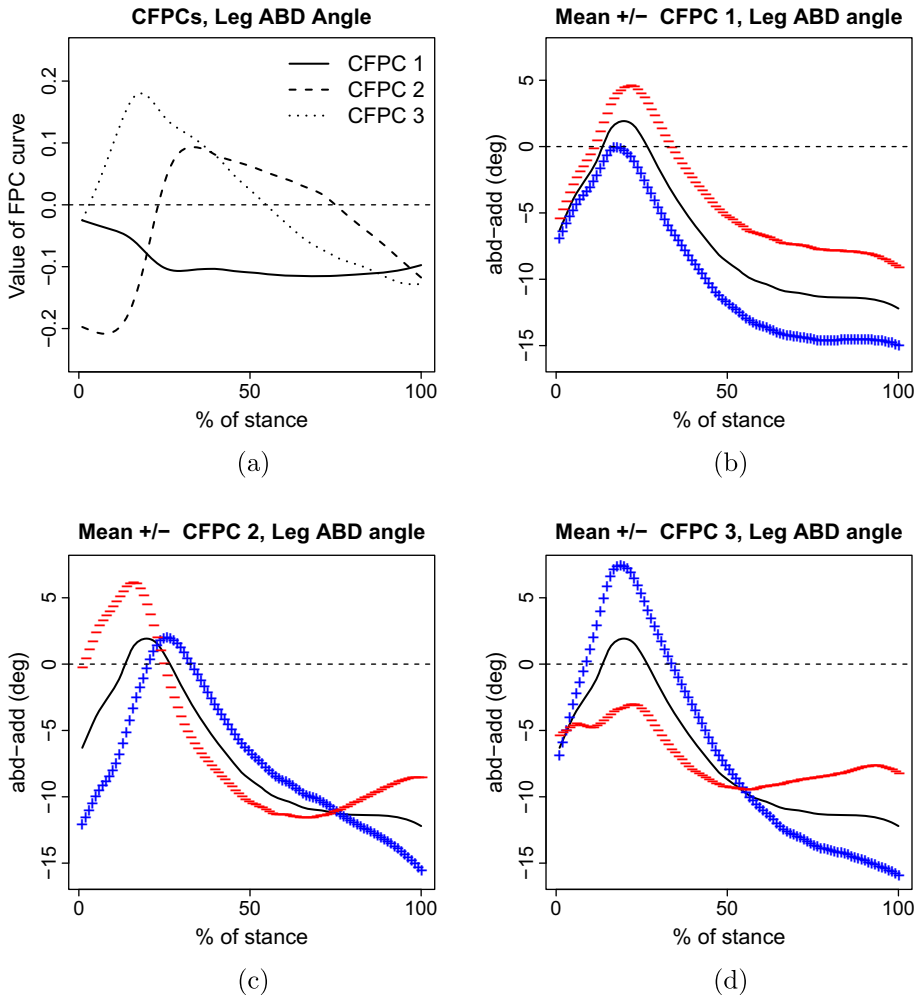

Fig. 3. (a) Depicts the first three CFPCs extracted for the leg ABD angle. (b–d) Depict the mean plus/minus each CFPC for the leg ABD angle.

subjects (90% versus 69%), which suggests that the leg ABD angle of injured subjects is subjected to repeated loading on the same structures, thus contributing to injury. In addition, injured subjects have much more variable leg ABD ROM than control subjects which also contributes to injury. Such results were not evident in the original analysis of Donoghue et al. (2008).

The first three CFPCs extracted for the ADF angle are plotted in Fig. 5(a). Since CFPC 3 accounts for only 6% and 3% of the total variation, we focus on the first two CFPCs. CFPC 1 describes an overall upward or downward shift compared with the mean curve. Positive (negative) scorers displayed decreased (increased) ADF values throughout stance. CFPC 2 describes peak ADF and ADF ROM with subjects receiving high positive (negative) scores displaying less (greater) ADF ROM and reduced (increased) peak ADF during stance. Donoghue et al. (2008) report that there are no clear changes in average ADF movement patterns of injured and control subjects, though again they suggest that there may be some changes in the variation of movement patterns across groups. Our results clarify where these changes exist. 74% of the variation in ADF angle values of injured subjects can be explained by changes in ADF throughout stance but 82% of the variation in control subjects can be explained by these

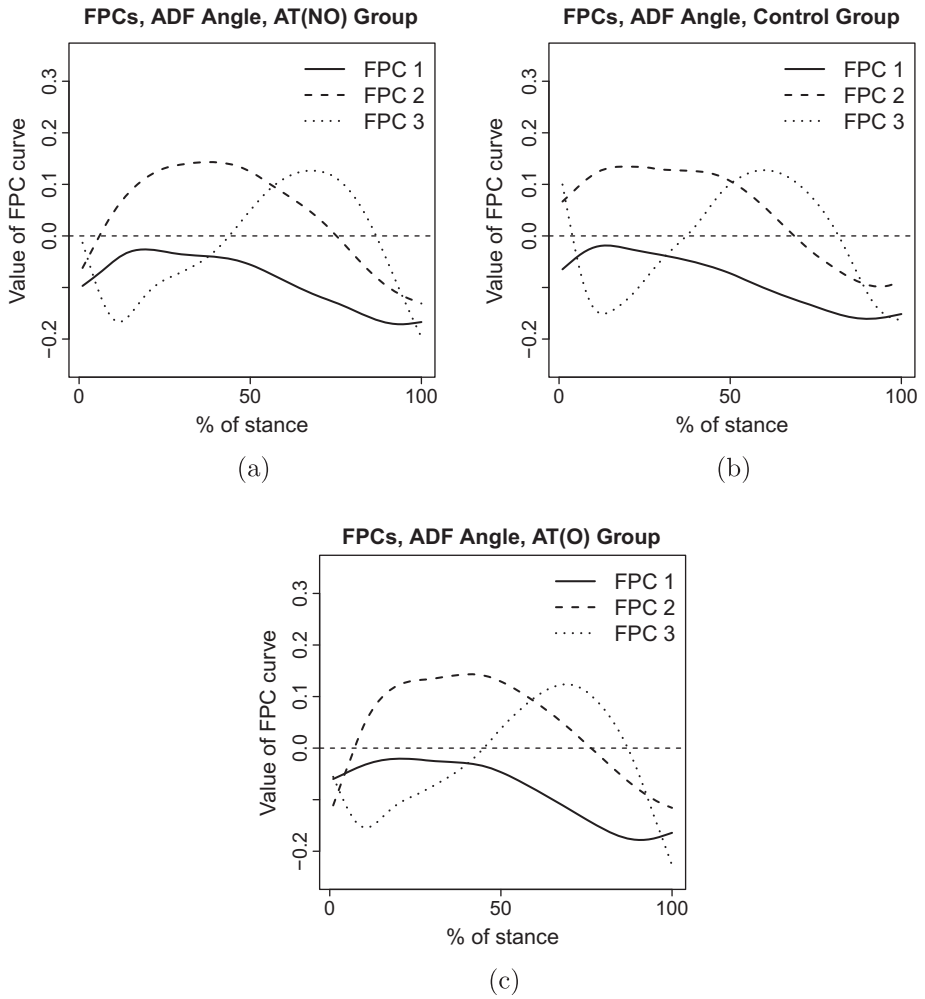

Fig. 4. Depicts the first three FPCs extracted for the ADF angle of (a) the AT(NO) group, (b) the control group and (c) the AT(O) group.

changes. The control group were again slightly more variable than injured subjects indicating that uninjured subjects had increased variability in ADF movement patterns throughout stance than injured subjects. CFPC 2 accounts for a further 16% and 13% of the total variation in the injured and control groups respectively. Subjects in the injured group tended toward increased variation in ADF ROM during stance compared with controls, as revealed by the additional 4% variation accounted for by this component. This is consistent with current theory that injured subjects exhibit excessive ROM contributing to injury.

Fig. 7(a) displays the first three CFPCs for the EV angle. Since CFPC 3 accounts for approximately 3% of the variation and may be group-specific, the analysis focused on the first two components. CFPC 1 represented variation around the mean and subjects receiving high positive (negative) scores displayed increased (decreased) EV throughout stance. CFPC 2 represented EV ROM and peak EV. Subjects receiving high positive (negative) scores displayed increased (decreased) EV ROM and reached peak EV earlier (later) in stance. Donoghue et al. (2008) showed that on average there is a tendency toward increased EV throughout stance for injured subjects and increased EV ROM. The authors also showed

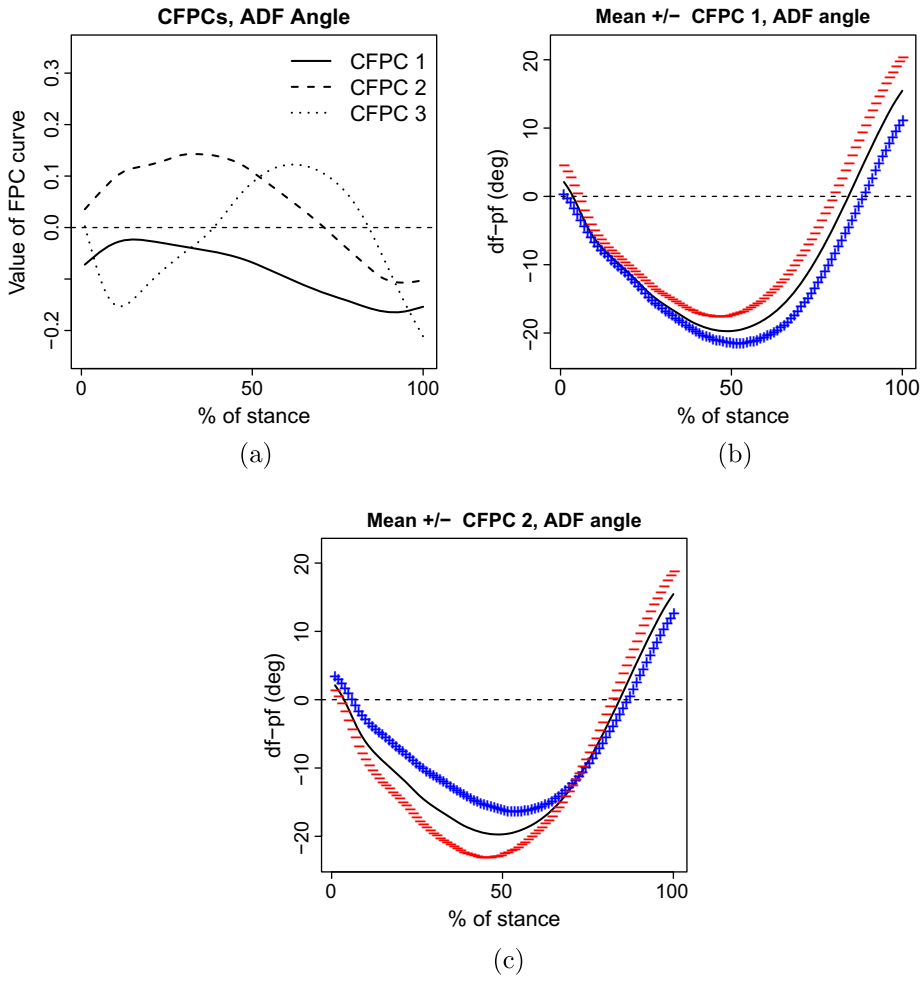

Fig. 5. (a) Depicts the first three CFPCs extracted for the ADF angle. (b–c) Depict the mean plus/minus each CFPC for the ADF angle.

that control subjects had increased variation in EV movement patterns throughout stance versus injured subjects. Other changes in variation between groups were unclear. More in-depth analysis of the changes in variation was provided by the CFPCA analysis. 42% of the total variation in the injured group was explained by changes in EV throughout stance with 58% of the variation in control subjects explained by this component. Control subjects exhibited EV movement patterns that were 16% more variable than injured subjects. CFPC 2 accounted for 48% of the variation in the injured group (making it the most important mode of variation in that group) and 32% of the variation in control subjects. This component represented EV ROM and peak EV. The re-ordering of the CFPCs in the injured group imply that EV ROM was the key factor influencing movement patterns in injured subjects. The EV ROM of injured subjects was 16% more variable than EV ROM in control subjects. Our results show obvious differences between the variation in EV movement patterns of injured and control subjects. EV ROM is the main source of variation in injured subjects and these subjects also exhibit less variable EV movement throughout stance than controls. Again, the repeated loading on the EV angle of injured subjects as predicted by DST in conjunction with excessive EV ROM are believed to be influential in the mechanism of Achilles Tendon injury.

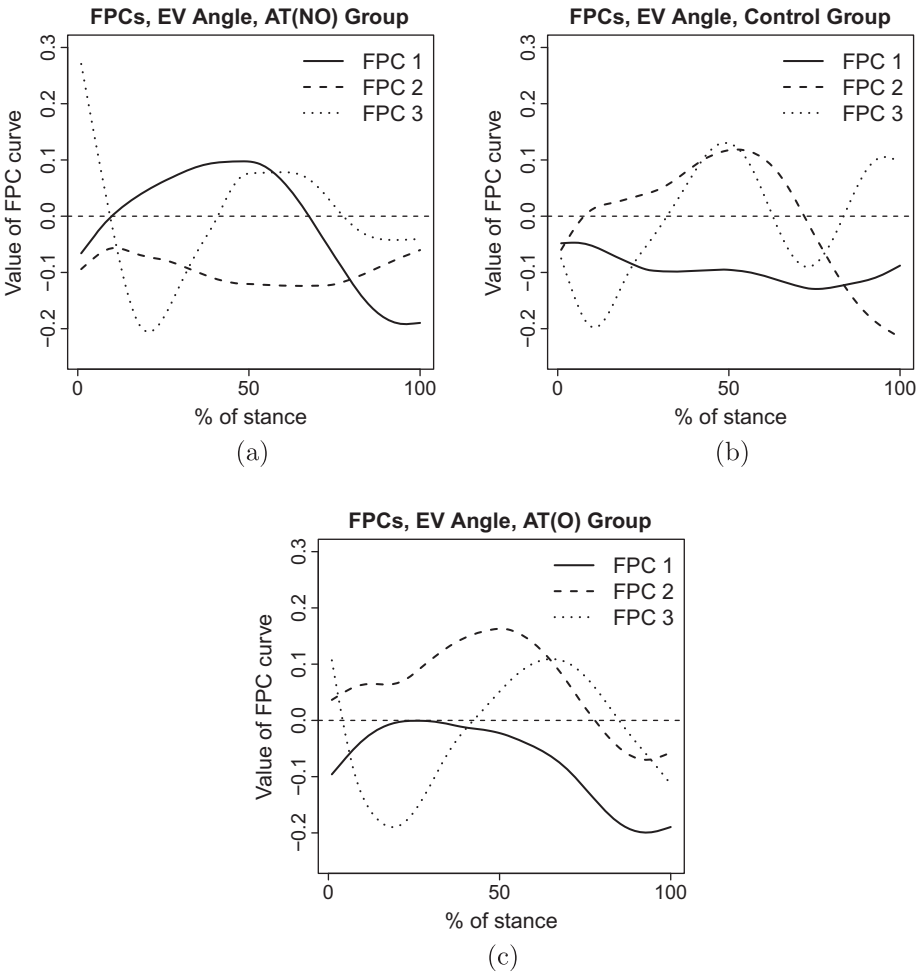

Fig. 6. Depicts the first three FPCs extracted for the EV angle of (a) the AT(NO) group, (b) the control group and (c) the AT(O) group.

The above results indicate that decreased variability of leg ABD, ADF and EV movement patterns of injured subjects coupled with the more variable ROM exhibited by these subjects play a key role in the mechanism of injury. Placing the emphasis on the variance of the scores (and hence the ordering of the CFPCs in each group) has revealed changes that were not obvious from an analysis of the average scores.

Though the latter results provided insight into the mechanism of injury by examining how injured and control subjects differed, it did not examine how wearing orthoses altered the movement patterns of injured subjects between treatment conditions and thus relieved symptoms of Achilles tendonitis. Common functional principal components analysis for dependent groups was applied to determine how the leg ABD, ADF and EV angle values varied across treatment conditions in the injured group.

3.2. AT(O) versus AT(NO)

To determine the effect of orthoses, the data consisting of the mean of the five replicate curves for each individual in the AT(O) and AT(NO) conditions was utilized. The subjects in the AT(O) group are

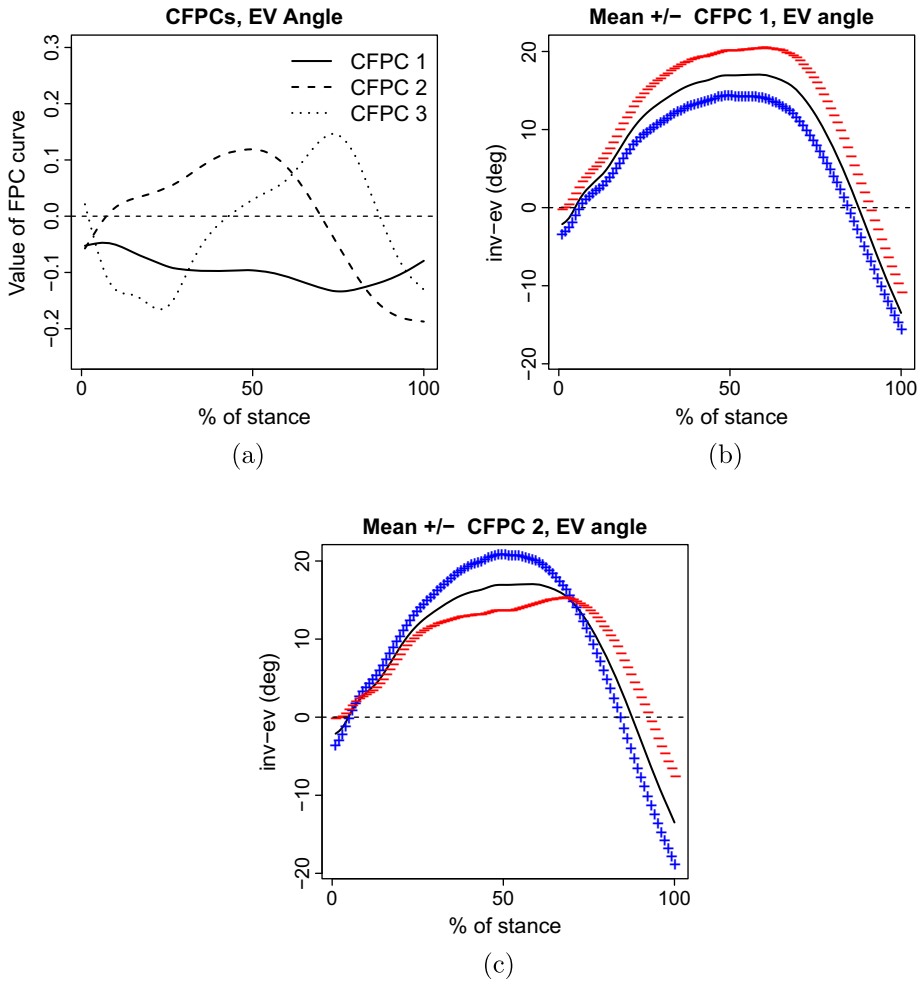

Fig. 7. (a) Depicts the first three CFPCs extracted for the EV angle. (b–c) Depict the mean plus/minus each CFPC for the EV angle.

the same as those in the AT(NO) group and so the groups were dependent. Hence, the common functional principal components for dependent groups (DCFPCs) were extracted. The results for the leg ABD, ADF and EV angles are given below.

Examining Figs. 2(a), (c), 4(a), (c) and 6(a), (c) it is clear that the same characteristic behavior is evident between the AT(O) and AT(NO) groups and it is not particularly clear how the presence of orthoses alters movement patterns of injured subjects. Figs. 8, 9 and 10 display the first three common functional principal components for dependent groups (DCFPCs) extracted for the leg ABD angle, ADF angle and EV angle respectively. These components are similar to the CFPCs extracted in the previous section and their overall shape remains the unchanged. As a result, the interpretations provided for the CFPCs can be applied to the DCFPCs. Tables 6–8 display the eigenvalues and percentage of variation accounted for by each DCFPC within each condition (AT(O) or AT(NO)) and across conditions (AT). The eigenvalues in the AT column show us how the presence of orthoses alters the movement patterns of injured subjects.

DCFPC 1 for the leg ABD angle represents overall variation around the mean curve with reduced leg ABD ROM in the first 20% of stance, DCFPC 2 represents leg ABD ROM particularly in the first 50% of

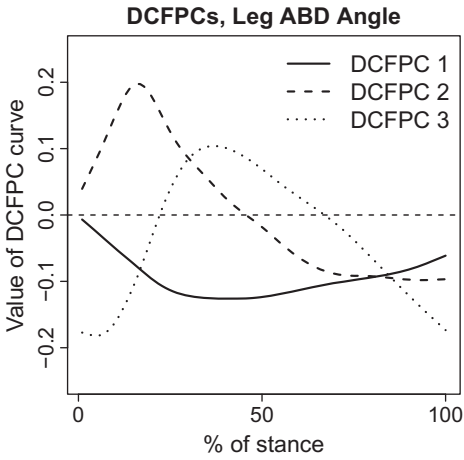

Fig. 8. Depicts the first three DCFPCs extracted for the leg ABD angle.

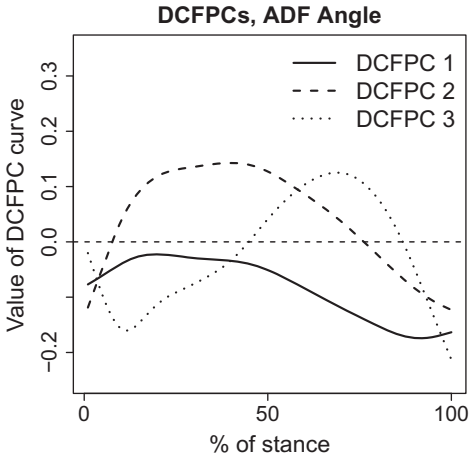

Fig. 9. Depicts the first three DCFPCs extracted for the ADF angle.

stance and DCFPC 3 represents the leg ABD angle at heel-strike and peak leg ABD. The analysis outlined in Donoghue et al. (2008) did not reveal differences between average leg ABD angle behavior in the presence and absence of orthoses and thus it was not obvious if orthoses had an effect on leg ABD movement patterns. Our results show that orthoses do have an effect on leg ABD by altering the variability of leg ABD movement patterns. DCFPC 1 was the main mode of variation within each condition and accounted for 75% and 72% of the variation in the AT(O) and AT(NO) groups respectively. This component was also the main mode of variation across conditions, accounting for approximately 74% of the total variation. Because this component is ranked first in the AT column (i.e., across conditions), it implies that the main way in which wearing orthoses alters leg ABD angle of subjects in the injured group is via an overall change in leg ABD throughout stance. DCFPC 2 was the second most important component across conditions accounting for a further 15% of the variation thus showing that the second most important way in which orthoses alters leg ABD values is through inducing changes in leg ABD ROM. DCFPC 3 accounted for approximately 9% of the variation across conditions implying that 9% of the differences in movement patterns exhibited between the AT(O) and AT(NO)

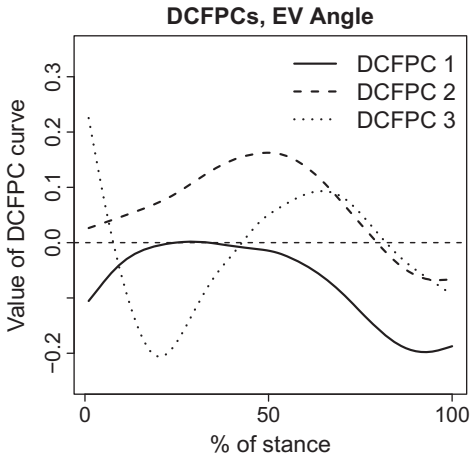

Fig. 10. Depicts the first three DCFPCs extracted for the EV angle.

Table 6
Leg ABD eigenvalues for DCFPCs. The table contains the eigenvalues for each DCFPC within-conditions (AT(O), AT(NO) columns) and across conditions (AT column). The values in brackets give the percentage of variation accounted for by each DCFPC both within each condition and across conditions. The values in the AT column reveal the main effects of orthoses.

Component	AT(O)	AT(NO)	AT
1	16.81 (77.24%)	10.95 (72.46%)	11.83 (74.02%)
2	2.48 (11.38%)	2.27 (15.05%)	2.35 (14.69%)
3	2.03 (9.33%)	1.56 (10.32%)	1.50 (9.37%)

Table 7
ADF eigenvalues for DCFPCs. The table contains the eigenvalues for each DCFPC within-conditions (AT(O), AT(NO) columns) and across conditions (AT column). The values in brackets give the percentage of variation accounted for by each DCFPC both within each condition and across conditions. The values in the AT column reveal the main effects of orthoses.

Component	AT(O)	AT(NO)	AT
1	21.58 (77.47%)	16.67 (74.21%)	16.67 (79.03%)
2	4.58 (16.43%)	3.39 (16.98%)	3.20 (15.18%)
3	1.19 (4.28%)	1.07 (5.36%)	1.01 (4.79%)

conditions can be explained by changes in leg ABD at heel-strike. These findings show that the most important change invoked by orthoses is increasing the variation in leg ABD movement throughout stance. Orthoses then invoke a change in leg ABD ROM, where wearing orthoses appears to reduce the excessively variable ROM of injured subjects. Finally, orthoses alter the leg ABD angle values at

Table 8
EV eigenvalues for DCFPCs. The table contains the eigenvalues for each DCFPC within-conditions (AT(O), AT(NO) columns) and across conditions (AT column). The values in brackets give the percentage of variation accounted for by each DCFPC both within each condition and across conditions. The values in the AT column reveal the main effects of orthoses.

Component	AT(O)	AT(NO)	AT
1	21.25 (62.48%)	11.30 (45.65%)	14.52 (57.40%)
2	8.93 (26.25%)	11.13 (44.98%)	8.46 (33.44%)
3	1.44 (4.24%)	1.06 (4.28%)	1.12 (4.42%)

heel-strike by slightly decreasing the amount of variation evident in leg ABD values when orthoses are worn. The results from DCFPCA show that orthoses do have an effect on leg ABD movement, by increasing the variation in leg ABD movement throughout stance and decreasing the variation in leg ABD ROM. There is also a slight effect of orthoses at heel-strike, where orthoses decrease the amount of variation exhibited at heel-strike.

Only the first two components are of interest for the ADF angle since the last component accounts for a small proportion of the overall variation. DCFPC 1 represents an overall upward or downward shift from the mean curve and DCFPC 2 represents peak ADF and ADF ROM. Donoghue et al. (2008) show that orthoses have a moderate effect on average ADF ROM, where wearing orthoses appear to decrease peak ADF in midstance resulting in less ADF ROM. However further information is lacking. Our approach shows that orthoses also have an effect on the variability of ADF movement patterns. CFPC 1 was the main source of variation both within and between conditions, accounting for approximately 77%, 74% and 79% of the total variation in the AT(O) group, AT(NO) group and between conditions respectively. This implies that the principal way in which the ADF angle of subjects in the injured group changes across conditions is via an overall increase or decrease in ADF angle values throughout stance. The presence of orthoses tended to slightly increase the amount of variation in ADF angle movement patterns of injured subjects as indicated by the higher percentage of variation accounted for by this component in the AT(O) group. CFPC 2 was the second most important source of variation within each condition [AT(O) – 16%, AT(NO) – 17%] and across conditions (15%). Approximately 15% of the differences exhibited between the AT(O) and AT(NO) conditions can be explained by changes in ADF ROM. In this case, orthoses appear to slightly decrease the variation in ADF ROM of injured subjects. From these findings, it appears that orthoses initially increase the variation in ADF movement throughout stance and then alter ADF ROM, where wearing orthoses appears to reduce the more variable ROM of injured subjects. This was not obvious from the original results.

Three DCFPCs were also extracted for the EV angle. DCFPC 1 again represents an overall shift around the mean, which increases in magnitude in the latter half of stance. DCFPC 2 describes peak EV and EV range of motion (ROM), while DCFPC 3 describes the rate of eversion. DCFPC 1 was the main mode of variation in the AT(O) group, accounting for 62% of the variation in that group. It was also the main mode of variation across conditions accounting for 57% of the variation; however this component had essentially the same ranking as DCFPC2 in the AT(NO) group, with DCFPC 1 and DCFPC 2 both accounting for approximately 45% of the variation in these subjects. This indicated that EV ROM was also a key source of variation in this group. It can be seen from Table 8 that the presence of orthoses increased the variation in EV movement patterns throughout stance (and particularly in the latter half of stance) by approximately 20%. This is the main effect of orthoses on the EV angle. DCFPC 2 accounted for 26% and 33% of the variation in the AT(O) group and across treatment conditions respectively. The second effect of orthoses is through changes in EV ROM. Orthoses reduced variability in EV ROM by almost 20% in injured subjects. Finally, it can be seen that subjects are also varying slightly in terms of rate of eversion in the presence of orthoses, though this component only accounts for 4% of the variation across conditions. Donoghue et al. (2008) show that on average orthoses increase EV

throughout stance and induce increases in peak EV and EV ROM for some subjects when orthoses are worn. These findings again only apply to average behavior and it is also not clear which effect is most important. The results given here show that the main effect of orthoses is to increase variation in EV movement throughout stance, followed by a decrease in variation of EV ROM.

4. Discussion

Functional data analysis and in particular functional principal components analysis has been shown to be effective in analyzing human movement and biomechanical data. This paper has highlighted some of the limitations of standard FPCA as implemented by Ramsay and Silverman (2005) in the analysis of the types of designs typically employed in human movement research. Donoghue et al. (2008) initially combined the leg ABD angle (or ADF or EV angle) data for all three groups and extracted the FPCs of the entire data set. This approach assumed that the curves for each subject were independent. Since injured subjects were measured twice, this is clearly not the case. The resulting FPCs described the main patterns of variation evident in the entire data set and did not give a measure of how much variation is accounted for by a particular component in each group. Comparisons between groups were made using ANOVA tests and boxplots of the scores in each group on each component. These tests focused on comparisons of the average behavior across groups and though many of these tests failed to reveal clear differences between the three groups, the authors were aware of the limitations of the methods they applied and hinted that examining the variation of the scores in each group may provide additional information. An attempt was then made to examine this within-group variation by extracting the FPCs for each group separately. These FPCs were group-specific but as outlined in Section 1 were disturbed by both within and between group variation. Only the first FPCs of the leg ABD and EV angles and the second FPCs of the ADF angle were compared across groups. This was because the order of the remaining FPCs altered across groups and no further comparisons could be made.

While FPCA (and comparisons of the average scores) examines changes in average behavior across groups, CFPCA and DCFPCA examine changes in variability across groups. The CFPCA methodology outlined in this paper places the emphasis on comparing the changes in patterns of variation across groups by extracting the common modes of variation and examining the group-specific eigenvalues. The method shows that although the same factors may influence the variation across groups, the distribution of the variation across these factors can be different. For example, the same three main factors (increased/decreased leg ABD throughout stance (as described by CFPC 1), leg ABD at heel-strike (as described by CFPC 2) and leg ABD ROM (as described by CFPC 3)) influenced the variation in movement patterns of the leg ABD angle in both AT(NO) and control subjects. The belief that these factors were *common* to both groups led to the extraction of the CFPCs. Examining the eigenvalues (and hence ordering of the CFPCs) revealed that although the factors influencing the variation were the same, the distribution of the variation across those three factors was very different in injured versus control subjects. The distribution of the variation across the first three factors for injured subjects was 69%, 11% and 18% while the distribution of the variation for control subjects was 90%, 7% and 1%. This implies that the movement pattern described by CFPC 3 (which represented leg ABD ROM) constituted 18% of the total variation in leg ABD movement of injured subjects but essentially contributed nothing to the variation of movement patterns of control subjects. Such differences in the distribution of the variation cannot be seen in standard analyses of the mean, cannot be examined when the FPCs of the combined data are determined since the proportions of variation accounted for by each component in each group are not calculated, and are not available when group-specific FPCs are determined since one cannot be sure that the components extracted represent the same movement patterns across groups. We found that placing the emphasis on differing variance percentages in each group provided real insight into the kinematics governing the mechanism of injury and the way in which the movement patterns of injured and control subjects differed. Examining the CFPCs provided a means of explaining how subjects in a particular group tended to behave and determined distinguishing patterns in each group. CFPCA revealed clear differences in the variation of movement patterns of injured versus control subjects that were not evident

in the previous analysis. Injured subjects exhibited reduced variation in leg ABD, ADF and EV movement patterns throughout stance than control subjects. They also exhibited more variable leg ABD, ADF and EV ROM than controls. CFPCA also overcame the problem of re-ordered components allowing for meaningful comparisons of the variability across conditions that was not possible in Donoghue et al. (2008).

DCFPCA was then used to determine how injured subjects varied between conditions. DCFPCA allows for the direct comparison of dependent groups of data which arise when the same individuals are measured under several treatment conditions. DCFPCA has the additional advantage of examining how the groups vary across conditions, which reveals the main effects of a particular treatment. This is not a consideration in standard FPCA analysis and as shown can provide real insight into the effect(s) of a particular treatment. Examining the eigenvalues of the cross-condition covariance function identified the largest sources of variation between conditions and established the main effects (and their order of importance) of orthoses on the movement patterns of injured subjects. Orthoses increased leg ABD variation, ADF variation and EV variation throughout stance. Wearing orthoses reduced the more variable leg ABD ROM evident in injured subjects while marginally reducing variation in ADF ROM. Orthoses also had a pronounced effect on the EV ROM, reducing the variation by approximately 20%. These results are sensible from a biomechanical perspective since orthoses are designed to control excessive ROM in injured subjects. Orthoses also appeared to slightly reduce variation in leg ABD at heel-strike. It is thought that the reduction in average ADF values in conjunction with the increase in variation of leg ABD movement patterns throughout stance, decrease in the amount of variability in leg ABD and EV ROM for injured subjects and decrease in leg ABD variability at heel-strike provided enough alteration to the kinematics to relieve symptoms of injury.

5. Conclusion

Previous research has emphasized the importance of analyzing the variation present in human movement data sets. We have presented an extension of functional principal components analysis (called common functional principal components analysis) which provides a means of determining the common modes of variation across groups and thus allows for meaningful comparisons between independent groups to be made. A novel application of our method to a highly structured data set has been discussed and has shown that CFPCA is useful when comparison of FPCs for several independent groups is required. Utilizing the CFPC model has several advantages. The method allowed us to achieve additional dimension reduction (since $R = 3$ components are examined rather than $R = 3 \times 2$ components when each group is considered separately). Benko et al. (2009) also point out that the CFPC model results in higher estimation precision, since the CFPCs are determined from the pooled sample of data. CFPCA alleviates the pooling effect of combining all of the data and extracting the FPCs of the combined data where the resulting components are affected by the group with the highest variability and may result in FPCs that do not adequately describe the patterns of variation in each group. In addition, it allows for the direct comparison of independent groups of individuals, even when the ordering of the components differs across groups. This is a problem in classic FPCA as it is not clear how to adequately compare FPCs across groups when the ordering of the FPCs is altered from one group to another. DCFPCA allows for the comparison of patterns of variability across treatment conditions when the groups are dependent, e.g., when measurements are made on the same individual before and after a particular intervention. Both of the methods proposed in this paper place the emphasis on changes in the variation of movement patterns, which can reveal differences between treatment conditions that may not become evident through the examination of the average scores in each group. Therefore we believe that the methods proposed are useful for analyzing human movement data. The experimental designs outlined in Section 1 are often encountered in human movement studies that wish to examine changes in movement patterns between groups. Therefore we believe that CFPCA and DCFPCA can provide more in-depth information about changes in the variation of human movement across groups than current methods such as classical FPCA.

Appendix A. Details for extracting the common principal components for independent groups

Let $\mathbf{\Gamma}_1, \ldots, \mathbf{\Gamma}_G$ denote the covariance matrices for each of G groups considered separately and let $\mathbf{\Gamma}$ denote the covariance matrix for the aggregated data set (where the data from all G groups is accumulated together and the covariance matrix determined). Extracting the principal components involves performing a singular value decomposition (SVD) of either $\mathbf{\Gamma}$ or a SVD of each of $\mathbf{\Gamma}_1, \ldots, \mathbf{\Gamma}_G$ such that:

$$\mathbf{\Gamma} = \mathbf{\Delta}\mathbf{\Lambda}\mathbf{\Delta}^T \tag{A.1}$$

or

$$\mathbf{\Gamma}_g = \mathbf{\Delta}_g\mathbf{\Lambda}_g\mathbf{\Delta}_g^T, g = 1, \ldots, G, \tag{A.2}$$

where $\mathbf{\Delta}$ is a matrix containing the principal components and $\mathbf{\Lambda}$ is a diagonal matrix containing the corresponding eigenvalues. In the first case, as given by Eq. (A.1), the principal components and eigenvalues are the same for all groups (hence no subscript on $\mathbf{\Delta}$ or $\mathbf{\Lambda}$). In the second case, as given by Eq. (A.2), both the principal components and eigenvalues are group-specific (hence the subscript g on $\mathbf{\Delta}$ and $\mathbf{\Lambda}$). In contrast, common principal components analysis assumes that some rotation simultaneously diagonalizes $\mathbf{\Gamma}_1, \ldots, \mathbf{\Gamma}_G$ in all G populations, i.e., that there exists some matrix $\mathbf{\Delta}^c$ such that

$$\mathbf{\Gamma}_g = \mathbf{\Delta}^c\mathbf{\Lambda}_g^c\mathbf{\Delta}^{cT}. \tag{A.3}$$

(Note the superscript c indicates that we are referring to the *common* principal components and their associated eigenvalues.) This implies that the eigenvectors are the same across all groups, but the eigenvalues differ (note there is no subscript on $\mathbf{\Delta}^c$ indicating the principal components are common to all groups but there is a subscript g on $\mathbf{\Lambda}^c$ indicating the eigenvalues are group-specific). Determining the common principal components (CPCs) involves solving a system of linear equations given by

$$\xi_r^T\left(\sum_{g=1}^{G} N_g \frac{\rho_{gr} - \rho_{gm}}{\rho_{gr}\rho_{gm}}\mathbf{\Gamma}_g\right)\xi_m = 0, \quad r, m = 1, \ldots, n;\ r \neq m, \tag{A.4}$$

where N_g is the total number of individuals in the gth group, n is the total number of common components extracted, ξ_r denotes the rth CPC and ρ_{gh} denotes the corresponding eigenvalue for the gth group on the rth CPC. The matrix $\mathbf{\Delta}^c$ is comprised of the estimated CPCs $(\xi_1, \xi_2, \ldots)$ while the matrices $\mathbf{\Lambda}_g^c$ are comprised of the group-specific eigenvalues $(\rho_{g1}, \rho_{g2}, \ldots)$. Eq. (A.4) is solved subject to $\mathbf{\Delta}^{cT}\mathbf{\Delta}^c = \mathbf{I}$ (i.e., the CPCs are orthogonal) and is achieved using an algorithm called the Flury–Gatuschi (F–G) diagolization algorithm as given in Flury and Gautschi (1986) and Clarkson (1988).

Appendix B. Details for extracting the common principal components for dependent groups

DCPCA is similar to CPCA as it assumes that some rotation simultaneously diagonalizes the within-group covariance matrices and that all groups share the same components. However, DCPCA also requires that the same rotation diagonalizes the cross-covariance matrices of measurements between groups. For example, in longitudinal studies where the same measurements are made on the same subjects at G different stages, the covariance matrix consists of G^2 blocks of the form

$$\mathbf{\Gamma} = \begin{pmatrix} \mathbf{\Gamma}_{11} & \mathbf{\Gamma}_{12} & \ldots & \mathbf{\Gamma}_{1G} \\ \mathbf{\Gamma}_{21} & \mathbf{\Gamma}_{22} & \ldots & \mathbf{\Gamma}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Gamma}_{G1} & \mathbf{\Gamma}_{G2} & \ldots & \mathbf{\Gamma}_{GG}, \end{pmatrix} \tag{B.1}$$

where the matrices on the diagonal of $\mathbf{\Gamma}$ denote the within-stage covariance matrices and the off-diagonal matrices denote the covariance matrices of measurements between stages for $g_1, g_2 = 1, \ldots, G$. CPCA extracts the CPCs for the matrices on the diagonal of $\mathbf{\Gamma}$. However, DCPCA also incorporates the off-diagonal matrices and renders them into diagonal form. The extracted dependent common

principal components (DCPCs) give a measure of the variation within stages and the variation across stages.

The DCPC model assumes that there exists an orthogonal matrix $\mathbf{\Delta}^c$ such that

$$\mathbf{\Gamma}_{g_1g_2} = \mathbf{\Delta}^c \mathbf{\Lambda}^c_{g_1g_2} \mathbf{\Delta}^{cT}, \quad \forall g_1, g_2 = 1, \ldots, G. \tag{B.2}$$

This reduces to solving the system of equations

$$\xi_r^T \left\{ \sum_{g_1=1}^{G} \sum_{g_2=1}^{G} (\rho_{g_1g_2,r} - \rho_{g_1g_2,m})(\mathbf{\Gamma}_{g_1g_2} + \mathbf{\Gamma}_{g_2g_1}) \right\} \xi_m = 0, \quad r, m = 1, \ldots, \ n, r \neq m, \tag{B.3}$$

such that $\mathbf{\Delta}^{cT}\mathbf{\Delta}^c = \mathbf{I}$ and $\rho_{g_1g_2,h} = \xi_h^T \mathbf{\Gamma}_{g_1g_2} \xi_h, g_1, g_2 = 1, \ldots, G, h = 1, \ldots, n$ and ξ_h is the hth column of $\mathbf{\Delta}^c$. Estimates of the DCPCs can again be determined using a modification of the F–G diagonalization algorithm.

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Further Reading

Phillips, P. C., & Arnold, S. J. (1999). Hierarchical comparison of genetic variance–covariance matrices using the Flury hierarchy. *Evolution, 53*(5), 1506–1515.