

Differences Between Local and Orbital Dynamic Stability During Human Walking

Jonathan B. Dingwell¹
e-mail: jdingwell@mail.utexas.edu

Hyun Gu Kang

Department of Kinesiology & Health Education,
University of Texas,
1 University Station, D3700
Austin, TX 78712

Currently there is no commonly accepted way to define, much less quantify, locomotor stability. In engineering, “orbital stability” is defined using Floquet multipliers that quantify how purely periodic systems respond to perturbations discretely from one cycle to the next. For aperiodic systems, “local stability” is defined by local divergence exponents that quantify how the system responds to very small perturbations continuously in real time. Triaxial trunk accelerations and lower extremity sagittal plane joint angles were recorded from ten young healthy subjects as they walked for 10 min over level ground and on a motorized treadmill at the same speed. Maximum Floquet multipliers (Max FM) were computed at each percent of the gait cycle (from 0% to 100%) for each time series to quantify the orbital stability of these movements. Analyses of variance comparing Max FM values between walking conditions and correlations between Max FM values and previously published local divergence exponent results were computed. All subjects exhibited orbitally stable walking kinematics (i.e., magnitudes of Max FM < 1.0), even though these same kinematics were previously found to be locally unstable. Variations in orbital stability across the gait cycle were generally small and exhibited no systematic patterns. Walking on the treadmill led to small, but statistically significant improvements in the orbital stability of mediolateral ($p=0.040$) and vertical ($p=0.038$) trunk accelerations and ankle joint kinematics ($p=0.002$). However, these improvements were not exhibited by all subjects ($p \leq 0.012$ for subject \times condition interaction effects). Correlations between Max FM values and previously published local divergence exponents were inconsistent and 11 of the 12 comparisons made were not statistically significant ($r^2 \leq 19.8\%$; $p \geq 0.049$). Thus, the variability inherent in human walking, which manifests itself as local instability, does not substantially adversely affect the orbital stability of walking. The results of this study will allow future efforts to gain a better understanding of where the boundaries lie between locally unstable movements that remain orbitally stable and those that lead to global instability (i.e., falling).
[DOI: 10.1115/1.2746383]

Keywords: walking, falling, dynamic stability, Floquet multipliers, nonlinear dynamics

1 Introduction

Falls pose a significant and extremely costly [1] health problem for the elderly [2] and patients with gait disabilities [3–5]. Because most falls occur during whole-body movements like walking [6,7], addressing this momentous clinical problem effectively requires developing appropriate measures for quantifying dynamic stability during walking [8,9]. Increased variability during walking can predict increased risk of falls in the elderly [10,11]. However, statistical measures of variability do not quantify how the locomotor system responds to perturbations [9] and are not correlated with measures of local dynamic stability that do [8,9,12]. While many walking stability indices have been proposed [11,13–15], there is still no commonly accepted way to define, much less quantify, locomotor stability.

In theoretical mechanics, a system’s stability is defined by how its state variables respond to perturbations [16]. For limit cycle systems, defined as having a constant fixed period, “orbital stability” is defined using Floquet multipliers [16] that quantify, discretely from one cycle to the next, the tendency of the system’s states to return to the periodic limit cycle orbit after small perturbations. Floquet multipliers are the eigenvalues of the lineariza-

tion (or Jacobian) of the stride-to-stride function (or Poincaré map) [16]. They quantify the stability of periodic orbits by quantifying the local stability of the fixed point of the Poincaré map. Floquet multipliers are calculated at a single point during the cycle (e.g., heel strike) and quantify how much small perturbations grow or decay after one subsequent stride. Because this analysis assumes that walking is strictly periodic and because changes in the system states are evaluated only once per stride, temporal variations between strides and the specific trajectories the system follows through state space from one stride to the next are ignored. This approach has been used extensively to assess the orbital stability of walking robots (e.g., Refs. [17–19]), but has been applied only very limitedly to human experiments [20,21]. Human walking was found to be orbitally stable [20], but post-polio patients were less stable than healthy controls [21].

However, humans do not walk in a strictly periodic manner. Time series of human stride intervals (i.e., the time elapsed between subsequent ipsilateral heel strikes) exhibit aperiodic fractal-like fluctuations [22,23] that may have important implications for walking stability [24,25]. For aperiodic systems that exhibit no discernable periodic structure, “local stability” can be defined using local divergence exponents [26,27], which quantify how the system’s states respond to very small (i.e., “local”) perturbations continuously in real time. In contrast to Floquet analyses, the full temporal and spatial structure of all trajectories in the state space are used in these analyses. Previous studies have demonstrated

¹Corresponding author.

Contributed by the Bioengineering Division of ASME for publication in the JOURNAL OF BIOMECHANICAL ENGINEERING. Manuscript received January 28, 2006; final manuscript received December 6, 2006. Review conducted by Philip V. Bayly.

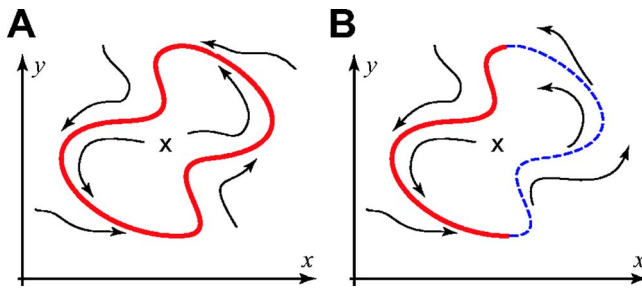


Fig. 1 (A) Schematic representation of an orbitally stable limit cycle that is also locally stable everywhere along the limit cycle. Being locally stable everywhere guarantees that the limit cycle must also be orbitally stable. (B) Representation of an orbitally stable limit cycle composed of both locally stable (solid line) and locally unstable (dashed line) regions. Trajectories that veer away from the limit cycle in the locally unstable regions are then “drawn back” toward it again in the locally stable region (adapted from Ref. [36]).

that patients with diabetic neuropathy improve their local stability by slowing down [28]; that walking on a motorized treadmill constrains local stability [12]; that slower walking speeds in general lead to improved local stability [8]; and that measures of gait variability [13,29] do not predict local stability during walking [8,9,12].

However, one perplexing finding from these studies was that all subjects tested (young healthy, older healthy, and older neuro-pathic) always exhibited a significant degree of local instability during walking, even though no subject ever fell or even stumbled while they were being tested. This is likely due in part to inherent biological noise (such as the signal-dependent noise that is an intrinsic property of muscle contraction [30–32]), which produces the same kind of local instability that local divergence exponents quantify [9,33]. This local instability may also result in part from small corrections made by the neuromuscular control system to maintain balance and smooth unintended irregularities during gait [28,34,35].

Regardless of the underlying source, it remains unclear whether humans simultaneously exhibit significant local instability, and yet remain orbitally stable while walking. Some mathematical limit cycle attractors can simultaneously exhibit both locally stable and locally unstable regions and still remain orbitally stable (Fig. 1) [36]. This suggests that orbital and local dynamic stability may reflect different properties of system dynamics. Indeed, we recently examined how the local and orbital stability properties of the simplest mechanical model of bipedal walking [17] change when we added noise to the system in the form of making the model walk down an irregular (i.e., “bumpy”) slope. Our model simultaneously exhibited significant local instability, but remained orbitally stable. Moreover, while the local divergence exponents were very sensitive to changes in noise amplitude, the Floquet multipliers were not [37].

Furthermore, the orbital stability of a limit cycle should theoretically be the same at all points along the trajectory. For this reason, previous experiments have assessed the orbital stability of walking at only one or two points during the gait cycle [20,21]. However, perturbation experiments suggest that humans execute different responses to perturbations applied at different phases of the gait cycle [38–40]. This suggests that measures of walking stability may also be similarly phase dependent.

The locomotor control system must respond to perturbations both within the current stride and across subsequent strides. Therefore, to properly quantify locomotor stability it is necessary to collect data from multiple consecutive strides. Treadmills are useful in this regard because many consecutive strides can be sampled using existing gait analysis technologies. However, the validity of comparing treadmill to overground locomotion remains

Table 1 Basic characteristics (mean \pm SD), including gender ratio (M/F), age, height, and weight, for subjects tested in this study [12]. Average self-selected overground walking speeds, as recorded during the 10-min walking trials are also included.

Gender ratio (M/F)	5 / 5
Age (years)	27.1 \pm 3.2
Height (m)	1.71 \pm 0.09
Weight (kg)	64.8 \pm 12.5
Walking speed (m/s)	1.33 \pm 0.15

an issue of debate [12,41–44]. There are no theoretical mechanical differences between walking on an ideal treadmill (i.e., rigid surface and constant belt speed) and walking overground [45]. Experimentally, however, intrastride variations in instantaneous treadmill belt speed can alter locomotor kinematics [46]. More importantly, treadmills appear to artificially reduce the natural variability [12,43] and enhance the local dynamic stability [12] of locomotor kinematics. The effects of treadmills on the orbital stability of human walking have not been reported.

The present study analyzed the orbital stability of human walking kinematics during both overground and treadmill locomotion, using data from a previous experiment where the local stability of these walking patterns were analyzed in detail [9,12]. We hypothesized that human walking, while locally unstable, would simultaneously remain orbitally stable and that the degree of orbital stability exhibited would vary systematically across different phases of the gait cycle. We hypothesized that walking on a motorized treadmill at a constant externally imposed speed would lead to artificially more stable walking patterns compared to walking over level ground. Finally, we hypothesized that measures of orbital stability would not be correlated with previously quantified measures of local stability.

2 Methods

2.1 Subjects and Data. As reported previously [12], ten young healthy subjects (Table 1) gave informed consent before participating. Each subject underwent a screening exam (health history, height, weight, blood pressure, and lower extremity passive ranges of motion) to rule out any history of medications, illnesses, surgeries, or other injuries that may have affected their gait.

Subjects wore standardized walking shoes and completed a 10 min acclimatization period on the treadmill at their self-selected speed before testing began. Overground (OG) walking trials were performed first. Each subject walked around an approximately 7 m wide and 200 m long level indoor walking track at his/her own self-selected walking speed. Subjects were instructed “to walk in as consistent a manner as possible.” The total distance walked was measured with a rolling measuring wheel to calculate each subject’s average walking speed. Subjects rested 5 min before walking on the treadmill (TM) at their average overground speed.

Kinematic data were collected during continuous walking using a custom made data logger [12,28]. Strain gauge electrogoniometers (Biometrics Ltd., Ladysmith, VA) measured sagittal plane motions of the hip, knee, and ankle joints of the right leg. A triaxial accelerometer (Kistler Instrument Corp., Amherst, NY) was mounted at the base of the sternum to measure the three-dimensional accelerations of the trunk in the anterior-posterior (AP), mediolateral (ML), and vertical (VT) directions. All kinematic data were sampled at 66.67 Hz continuously for the entire duration of each 10 min trial.

2.2 State Space Reconstruction. All data were analyzed without filtering because the goal was to examine the spatio-temporal fluctuations within these signals, and because of the many complications associated with applying linear filters to non-

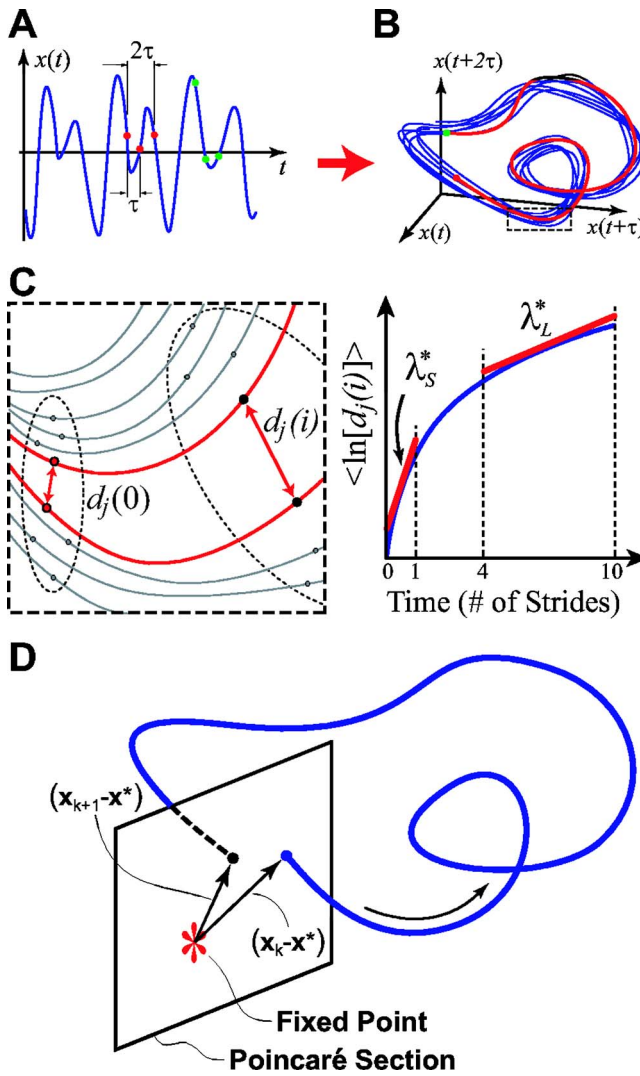


Fig. 2 Schematic representation of local and orbital dynamic stability analyses. (A) Original time series data, $x(t)$, plotted as a function of time (arbitrary units). (B) Reconstruction of a three-dimensional attractor for $x(t)$ such that $S(t)=[(t), x(t), x(t+\tau), x(t+2\tau)]$. The two triplets of points indicated in (A) and separated by time lags τ and 2τ each map onto a single point in the 3D state space. (C) Expanded view of a local section of the attractor shown in (B). An initial naturally occurring local perturbation, $d_j(0)$, diverges across i time steps as measured by $d_j(i)$. Local divergence exponents (λ^*) are calculated from the linear slopes of the average logarithmic divergence, $\langle \ln[d_j(i)] \rangle$ (Eq. (2)), of all pairs of initially neighboring trajectories versus time. (D) Representation of a Poincaré section transecting the state space perpendicular to the system trajectory. The system state, S_k , at stride k evolves to S_{k+1} one stride later. The Floquet multipliers quantify whether the distances between these states and the system fixed point, S^* , grow or decay on average across many cycles (Eq. (7)).

linear signals [27]. First, appropriate state spaces for each individual time series were reconstructed from the original data and their time-delayed copies using standard techniques [27,47,48]

$$S(t) = [q(t), q(t+\tau), q(t+2\tau), \dots, q(t+(d_E-1)\tau)] \quad (1)$$

where $S(t)$ was the d_E -dimensional state vector; $q(t)$ was the original one-dimensional data; τ was the time delay; and d_E was the embedding dimension (Figs. 2(B) and 3(A)). Time delays were calculated from the first minimum of the average mutual information function [9,27,49]. An embedding dimension of $d_E=5$ was

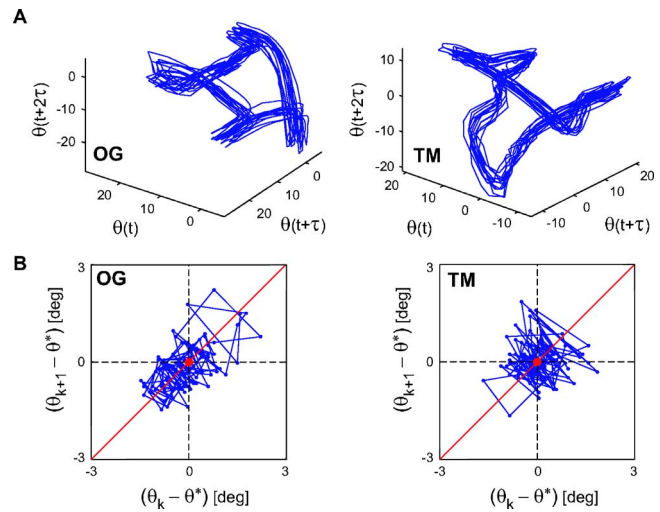


Fig. 3 (A) Example of reconstructed 3D state spaces from the ankle angle data recorded during overground (OG) and treadmill (TM) walking for a typical subject. While only three dimensions are shown; the data were analyzed in $d_E=5D$ state spaces. While the two plots have different shapes, as expected because different time lags (τ) were used in the reconstructions, they both qualitatively appear as noisy closed orbits (i.e., limit cycles). (B) First return maps for the ankle angle data shown in (A), plotting deviations away from the fixed point (θ^*) for each stride $k+1$, relative to the previous stride k . Each small dot represents the value recorded at the same phase (heel strike) of the gait cycle. Thus, the lines between the points serve only to indicate which strides occurred in what order and do not indicate trajectories between points. Fixed points are shown as large red (or gray) dots at $[0,0]$. These plots provide only a partial snapshot of only one of the five state variables used to define the full state space. Therefore, Floquet multipliers must be computed from the full state space data to determine if consecutive strides return, on average, to points closer to, or farther away from, the fixed point.

used for all data sets, as determined from a global false nearest neighbors analysis [9,27,50]. These reconstructed state spaces exhibit the same dynamical properties as state spaces constructed from the original state variables as long as τ is “reasonable” and d_E is “sufficiently large” [27,47,48]. Thus, the results of our subsequent stability analyses were insensitive to moderate changes in these parameters.

2.3 Estimating Local Stability. In the original study, local dynamic stability of walking was quantified [9] by estimating the average exponential rates of divergence of initially neighboring trajectories in state space as they evolved in real time. These local divergence exponents provided a direct measure of the sensitivity of the system to extremely small (i.e., local) perturbations [26,27]. Positive exponents indicate local instability, with larger exponents indicating greater sensitivity to local perturbations.

Nearest neighbor points on adjacent trajectories in the reconstructed state space represent the effects of small local perturbations to the system (Fig. 2(C)). A previously published algorithm ([26]; Totts Gap Medical Institute, Bangor, PA) provided estimates of the average exponential divergence for each embedded time series. Euclidean distances between neighboring trajectories in state space were computed as a function of time and averaged over all original pairs of initially nearest neighbors. Local divergence exponents (λ^*) were estimated from the slopes of linear fits to these exponential divergence curves

$$y(i) = \frac{1}{\Delta t} \langle \ln[d_j(i)] \rangle \quad (2)$$

where $d_j(i)$ was the Euclidean distance between the j th pair of initially nearest neighbors after i discrete time steps (i.e., $i\Delta t$ seconds) and $\langle \cdot \rangle$ denotes the average over all values of j [26]. Since the intrinsic time scales were different for each subject (i.e., different average stride times), the time axes of these curves were rescaled by multiplying by the average stride frequency for each subject. Short-term exponents (λ_S^*) were calculated from the slopes of linear fits to the divergence curve between 0 and 1 stride. Long-term exponents (λ_L^*) were calculated as the slope between 4 and 10 strides (Fig. 2(C)) [8,12,28].

These local divergence curves always exhibited positive divergence reflecting the natural variability and intrinsic biological noise in the system [9,28,51]. For “chaotic” systems, the local divergence would be *linear*, reflecting a *constant* exponential rate of divergence [26,52], and the slope of that line would approximate the maximum finite-time Lyapunov exponent for the system. Since the curves we obtained [9] were clearly *not* linear, we found no basis for defining a true Lyapunov exponent for human walking [51,52]. Nevertheless, these local divergence exponents still provided rigorously defined metrics for estimating the sensitivity of human walking to small intrinsic perturbations [9,53].

2.4 Estimating Orbital Stability. For the present study, orbital stability was quantified by calculating the Floquet multipliers (FMs) for the system [16] based on well-established techniques [18,20,21,54]. Because Floquet theory assumes the system is strictly periodic, the state space data for each stride were first time normalized to 101 samples (0–100%). This then allowed us to define 101 Poincaré maps (Figs. 2(D) and 3(B)) for the system as

$$\mathbf{S}_{k+1} = \mathbf{F}(\mathbf{S}_k) \quad (3)$$

where k was an index enumerating the individual strides and \mathbf{S}_k denoted the system state (Eq. (1)) for a single point in normalized time within each gait cycle. Limit cycle trajectories correspond to single fixed points in each Poincaré map

$$\mathbf{S}^* = \mathbf{F}(\mathbf{S}^*) \quad (4)$$

For our walking data, we defined the fixed points at each Poincaré section (i.e., at each % of the gait cycle) by the average trajectory across all strides within a trial. Orbital stability at each Poincaré section was estimated by computing the effects of small perturbations away from these fixed points, using a linearized approximation of Eq. (4)

$$[\mathbf{S}_{k+1} - \mathbf{S}^*] \approx \mathbf{J}(\mathbf{S}^*)[\mathbf{S}_k - \mathbf{S}^*] \quad (5)$$

where $\mathbf{J}(\mathbf{S}^*)$ defined the Jacobian matrix for the system at each Poincaré section. The Floquet multipliers are the eigenvalues of $\mathbf{J}(\mathbf{S}^*)$ [18,20,21,54]. Any deviation away from the fixed point is multiplied by FM by the subsequent cycle. Thus, for limit cycle to be orbitally stable, these complex-valued FM must have magnitude <1 (i.e., they must all lie *inside* the unit circle). If any of the FM has magnitude >1 , the limit cycle is orbitally unstable. We computed the magnitudes of the maximum FM for each % of the gait cycle to determine how the orbital stability varied across the gait cycle. For statistical analyses, we extracted the largest of these maximum FMs (“Max FM”) from across all Poincaré sections because this represented that instant during the gait cycle that was most unstable.

2.5 Statistical Analyses. Detailed statistical analyses of the λ_S^* and λ_L^* measures were reported previously [9,12]. For this study, these analyses were replicated using the Max FMs as dependent measures. Differences between OG and TM walking were evaluated using a three-factor (subject \times condition \times interval) balanced analysis of variance (ANOVA) for randomized block design [12,55]. For each Max FM measure, p values for each

main effect and for subject \times condition and condition \times interval interaction effects were determined. Furthermore, to determine if trunk stability differed from leg stability, the Max FM data for each subject for each trial were averaged across the three trunk accelerations to obtain a single measure of “trunk stability” and across the three leg angle measures to obtain a single measure of “leg stability.” These data were analyzed using an additional three-factor (subject \times condition \times segment) balanced repeated measures ANOVA.

To determine if and how orbital stability was related to local stability, single averaged values of each measure of orbital stability (Max FM) and local stability (λ_S^* and λ_L^*) were computed for each subject for each condition. For each of the six time series measurements examined, correlation coefficients (r^2) were computed between each orbital and local dynamic stability measure to determine if these measures were linearly related. Because of the large number of comparisons made (12), statistical significance was judged based on a Bonferroni correction (i.e., $\alpha = 0.05 \div 12 = 0.004$).

3 Results

Visually, the embedded state space data for each time series exhibited a strongly periodic structure with the expected stride-to-stride variability that occurs naturally during normal walking (Fig. 3(A)). Because the data were analyzed in five-dimensional state spaces, which yielded four-dimensional Poincaré maps, these Poincaré maps themselves could not be represented graphically. Alternatively, stride-to-stride first return maps (Fig. 3(B)), which show how deviations away from the fixed point (i.e., the mean trajectory) for each individual state evolve across consecutive strides, did show that consecutive strides always returned close to the fixed point. However, because these plots only provide a partial snapshot of only one of the five state variables used to define the full state space, it is not immediately obvious from these plots whether or not consecutive strides return, on average, to points closer to, or farther away from, the fixed point. To determine this, the maximum Floquet multipliers must be computed from the full state space data.

All Floquet multipliers remained inside the unit circle (i.e., magnitude <1 ; Fig. 4) and their magnitudes were similar to those reported in previous human studies [20,21]. Thus, all of the subjects tested exhibited *orbitally stable* walking patterns, despite the fact that these same subjects all exhibited significant local *instability*, as previously reported [12]. This confirmed our hypothesis that subjects would be orbitally stable, despite being locally unstable. While the maximum FM values did fluctuate across the gait cycle, these fluctuations were generally small, did not follow any discernable pattern, and were not consistent across the different variables measured or conditions tested (Fig. 4). This contradicted our hypothesis that orbital stability would vary systematically across the gait cycle.

There was a trend for young healthy subjects to exhibit somewhat greater orbital stability when walking on the motorized treadmill than when walking over level ground at the same speed (Fig. 5(A)). These differences were statistically significant for three of the six variables tested. However, for all three of these variables, significant subject \times condition interaction effects were also found (Fig. 5(B)). For A_{ML} and A_{VT} , while most subjects exhibited slight decreases in Max FM on the TM compared to OG walking, others exhibited essentially no differences or even slight increases. At the ankle, although nearly all subjects exhibited decreases in Max FM on the TM, the magnitudes of these differences varied greatly across subjects. Thus, the decreases in Max FM values for TM versus OG walking were not consistent across subjects. Therefore, our hypothesis that treadmill walking would artificially enhance the orbital stability of walking compared to overground walking was only partially supported.

The average Max FM value for all three trunk acceleration directions combined, for all subjects and trials, was 0.707 ± 0.076 .

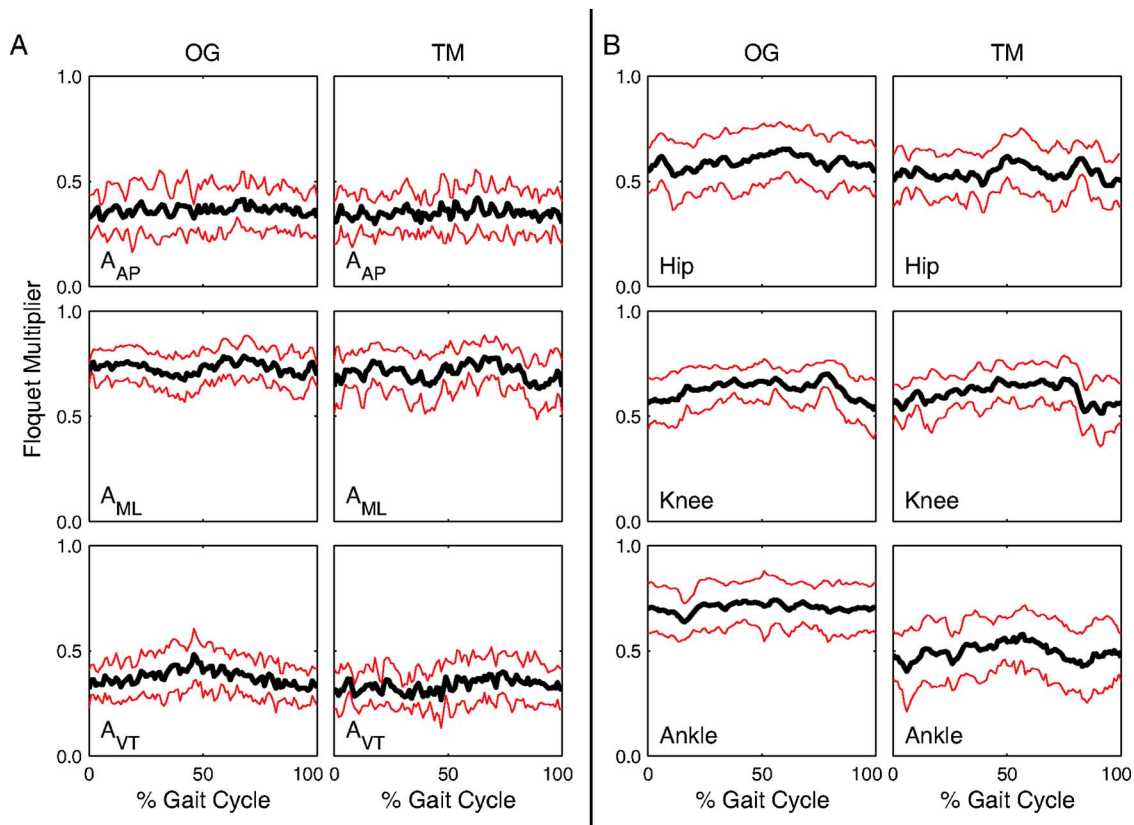


Fig. 4 Variations in the magnitudes of maximum Floquet multipliers (FM) across the gait cycle for (A) trunk acceleration data and (B) lower extremity sagittal plane joint angles. All FM remained inside the unit circle (magnitude <1) indicating that all subjects exhibited orbitally stable walking patterns. In general, although the FM did vary somewhat across the gait cycle, there were no obvious or consistent patterns to these variations.

For all three lower extremity Max FM values combined, this average was 0.643 ± 0.056 . The repeated measures ANOVA confirmed that in general, overall trunk movements were significantly more stable (i.e., smaller Max FM; $p < 0.001$) than overall leg movements.

In general, correlations between local stability (λ_S^* and λ_L^*) and orbital stability (Max FM) were not particularly strong and most comparisons were not statistically significant (Fig. 6). Seven of the 12 correlations computed were positive and five were negative. Only one of the 12 correlations (hip) was statistically significant ($r^2 = 38.6\%$; $p = 0.003$). The remaining 11 correlations were all generally weak ($r^2 \leq 19.8\%$) and were not statistically significant ($p \geq 0.049$). Thus, our hypothesis that orbital stability would not be directly correlated with local dynamic stability was supported.

4 Discussion

To understand the mechanisms humans use to maintain locomotor stability, we first need to properly define and quantify locomotor stability. Engineering definitions of local and orbital stability define how systems respond to small perturbations either in real time or from one cycle to the next. While these measures are rigorously defined, they are based on fundamentally different assumptions about the underlying nature (i.e., aperiodic versus strictly periodic) of the system being studied. Because human walking is neither strictly periodic nor strongly aperiodic, it is not clear which of these approaches is most appropriate for studying locomotor stability. While some studies have shown that human walking is orbitally stable [20,21], others have demonstrated significant local instability in walking kinematics [8,9,12,28]. The purposes of this study were to determine if and how orbital sta-

bility was related to local stability during human walking; to determine how orbital stability varied across the gait cycle during human walking; and to determine how walking on a motorized treadmill affects the orbital stability of human walking.

All subjects exhibited orbitally stable walking kinematics during both overground and treadmill walking (Fig. 3), despite the fact that these same kinematics were previously shown to be locally unstable [9,12]. This demonstrates that the variability inherent in human walking, which manifests itself as local instability, does not significantly adversely affect the *orbital* stability of walking, at least under “normal” walking conditions where subjects are not subjected to additional large-scale external perturbations. These differences between orbital and local stability were further confirmed by the general lack of significant correlations between the Max FM and λ^* variables (Fig. 6). Together, these findings support the idea that these measures of orbital and local dynamic stability quantify different properties of system dynamics. This is due, at least in part, to how these measures are computed and to what they are quantifying. When computing Max FM, each trajectory in state space is compared to a single reference trajectory, $S^*(t)$, defined from the average of all trajectories (Eqs. (4) and (5)). Conversely, when computing λ_S^* and λ_L^* , each trajectory in state space is compared to its own nearest neighbor, regardless of how far those two trajectories are from the mean reference trajectory. The positive λ^* findings demonstrate that, on average, each trajectory diverges away from its own nearest neighbor. The finding of Max FM values with magnitude <1 demonstrates that all of the trajectories, on average, still generally converge toward the mean reference trajectory. Additional research is needed to determine just how much local instability locomotor systems can tolerate and still remain orbitally stable and to determine what me-

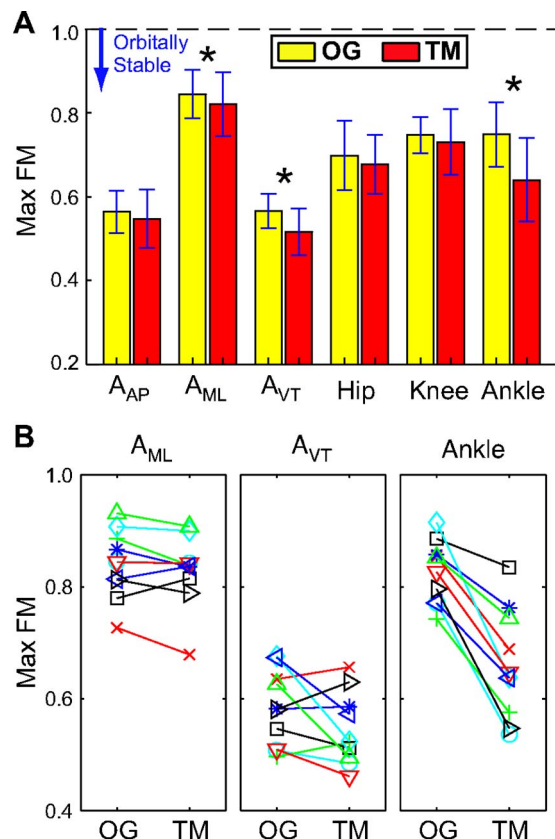


Fig. 5 Orbital stability results for overground (OG) versus treadmill (TM) walking. (A) Average Max FM values for both conditions for all six time series measures examined. Error bars indicate between-subject pooled standard deviations. Smaller Max FM values indicate greater orbital stability. Max FM values tended to be slightly lower (i.e., more orbitally stable) when walking on the treadmill. Differences were not statistically significant for anterior-posterior trunk accelerations (A_{AP} ; $p=0.434$), hip ($p=0.286$), or knee joint movements ($p=0.447$). Differences were statistically significant for mediolateral (A_{ML} ; $p=0.040$) and vertical (A_{VT} ; $p=0.038$) trunk accelerations and for ankle joint movements ($p=0.002$). However, the condition \times subject interaction effects for all three of these comparisons were also significant ($p \leq 0.012$). (B) Condition \times subject interaction plots for the three variables where the ANOVA revealed significant differences between OG and TM walking. Each line type represents the average results for one subject. The differences between OG and TM walking were clearly different for different subjects, with some subjects even showing slight increases in Max FM when walking on the TM.

chanical and neural mechanisms are used to regulate these processes.

Walking on the treadmill generally yielded small improvements in orbital stability for each of the six time series variables measured (Fig. 5(A)). While these differences were statistically significant for three of these six variables, the significant subject \times condition interaction effects meant that these differences between OG and TM walking were different for different subjects (Fig. 5(B)). Treadmill walking was previously shown to significantly reduce the natural variability [12,43] and enhance the local stability [12] of locomotor patterns in these same subjects. While there were somewhat similar effects on the *orbital* stability of walking, these effects were less pronounced. The Max FM variables were thus less sensitive to this experimental manipulation than previously reported findings for local divergence exponents or measures of variability. This finding suggests that the use of motorized treadmills is generally justified for studying orbital stability of

locomotion over multiple consecutive strides. The largest decreases in Max FM when walking on the treadmill were found at the ankle (Fig. 5). Likewise, trunk movements were, on average, somewhat more stable than leg movement during both walking conditions. This further supports the idea that a primary goal of the neuromuscular control system is to maintain the dynamic stability of superior segments (i.e., the head and trunk) more so than inferior segments (i.e., the legs) [8,28].

In theory, the orbital stability of a deterministic limit cycle process should be the same, regardless of where along the trajectory the Poincaré section is made. This notion was generally supported by our results (Fig. 3). However, human walking is not strictly periodic [22–25] and people respond to perturbations differently during different phases of the gait cycle [38–40]. This suggests that the orbital stability of walking should be phase dependent in the same way that the local stability of certain theoretical limit cycles sometimes is ([36]; Fig. 1). We did not find any evidence of such phase-dependent variations in Max FM in our data (Fig. 3). However, this might have been due in part to how we constructed our state spaces. In the delay reconstruction (Eq. (1)), the additional “states” created were not aligned during the same “phase” of the gait cycle (i.e., the time delays, τ , were less than a complete gait cycle). It is possible this may have led to an “averaging out” of differences at individual phases of the gait cycle. To fully resolve this question, we would need to conduct the Floquet analyses using state spaces comprised of the full set of original states for the system. Therefore, the orbital stability of human walking might still be found to be phase dependent, if complete state space data were analyzed.

The present study was limited to experimentally quantifying the response of the locomotor system to those small perturbations that occur naturally during normal walking. These results may or may not extend to *global* stability [16], where the response of the system to much larger perturbations, such as tripping or slipping (e.g., Refs. [56,57]), would be assessed. Global stability can be quantified by mapping the complete basins of attraction of a system [58,59], or similarly by determining the largest singular perturbations to each system state that do not cause the system to fall over [60]. However, these techniques require applying a very large number of very large perturbations to the system to observe its response. So while they are very useful for examining the stability of simulations or robots, it would be very difficult to implement such analyses experimentally in humans. One important line of future research will be to determine if the Max FM measures adopted here can predict the resilience of humans to such larger perturbations.

There is clearly a limit to the magnitude of perturbations that humans can accommodate and we do not know how much inherent local instability the human locomotor system can tolerate while remaining orbitally stable. Recent modeling work from our lab using a passive dynamic walker suggests that orbital stability is quite robust to substantial perturbations: i.e., the Max FM measures remained nearly constant over the range of perturbations the model could tolerate without falling over. This suggests that Max FM measures may be relatively insensitive to changes in task demands that significantly challenge walking stability [37]. The results of the present study support those findings and suggest that local divergence exponents (λ^*) [12] may be more sensitive to experimental manipulations that are relevant to walking stability. The methods used and developed in the present study will allow future efforts to gain a better understanding of where the boundaries lie between locally unstable movements that remain orbitally and globally stable and those that lead to global instability (i.e., falling).

Acknowledgment

Partial funding for this project was provided by a Biomedical Engineering Research Grant (Grant No. RG-02-0354) from the Whitaker Foundation. The authors gratefully thank Dr. Peter R.

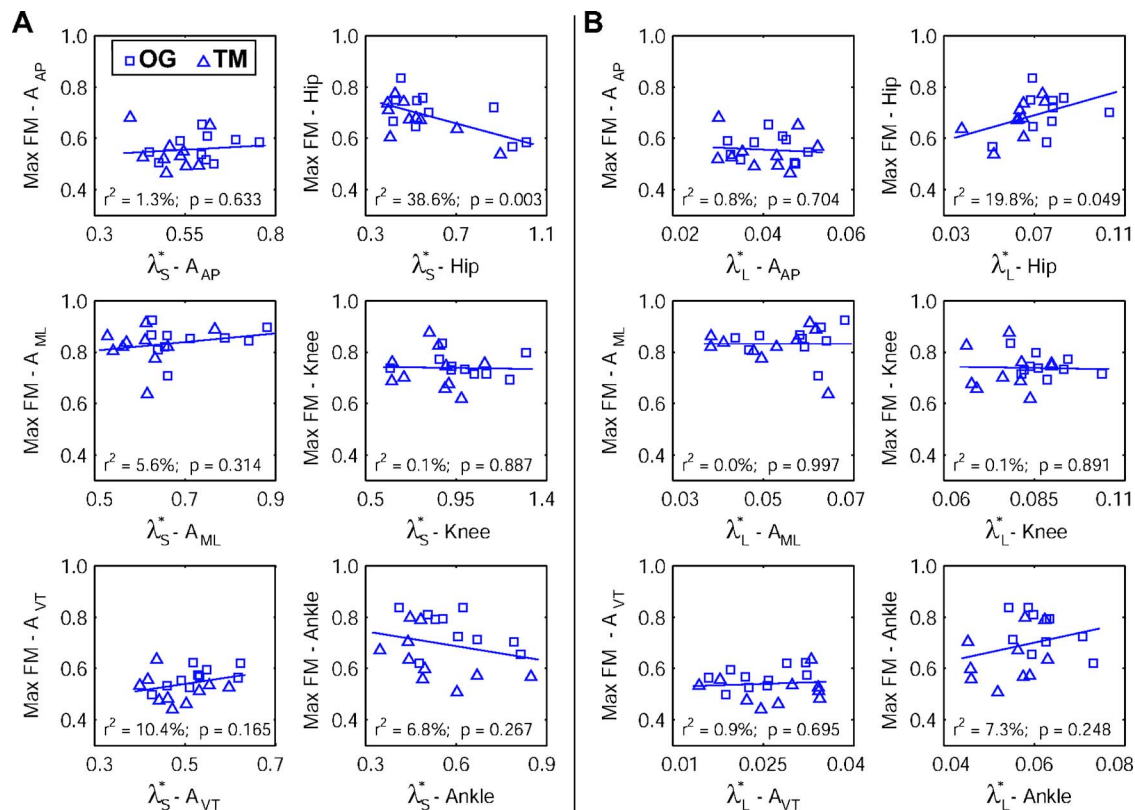


Fig. 6 Correlations between local (λ_S^* and λ_L^*) and orbital (Max FM) stability. (A) Correlations between short-term (λ_S^*) local divergence exponents and Max FM for all six time series examined. (B) Correlations between long-term (λ_L^*) local divergence exponents and Max FM. Only the comparison between λ_S^* and Max FM at the hip was statistically significant after Bonferroni correction ($p < 0.004$) and this relationship was negative: i.e., lesser local stability (larger λ_S^*) predicted greater orbital stability (smaller Max FM). None of the remaining 11 comparisons were statistically significant ($0.0\% \leq p \leq 19.8\%$; $0.997 \geq p \geq 0.049$). In general, no consistent relationships between local and orbital stability were found.

Cavanagh for his help and support in designing and conducting the original overground-treadmill experiment and Mrs. Mary B. Becker, R.N. for her assistance in collecting the data.

References

- [1] Englander, F., Hodson, T. J., and Terregrossa, R. A., 1996, "Economic Dimensions of Slip and Fall Injuries," *J. Forensic Sci.*, **41**, pp. 733–746.
- [2] Fuller, G. F., 2000, "Falls in the Elderly," *Am. Fam. Physician*, **61**, pp. 2173–2174.
- [3] Richardson, J. K., and Hurvitz, E. A., 1995, "Peripheral Neuropathy: A True Risk Factor for Falls," *J. Gerontol., Ser. A*, **50**, pp. M211–M215.
- [4] Forster, A., and Young, J., 1995, "Incidence and Consequences of Falls Due to Stroke: A Systematic Inquiry," *Br. Med. J. (Clin. Res. Ed.)*, **311**, pp. 83–86.
- [5] Hausdorff, J. M., Schaafsma, J. D., Balash, Y., Bartels, A. L., Gurevich, T., and Giladi, N., 2003, "Impaired Regulation of Stride Variability in Parkinson's Disease Subjects With Freezing of Gait," *Exp. Brain Res.*, **149**, pp. 187–194.
- [6] Tinetti, M. E., Doucette, J., Claus, E., and Marottoli, R., 1995, "The Contribution of Predisposing and Situational Risk Factors to Serious Fall Injuries," *J. Am. Geriatr. Soc.*, **43**, pp. 1207–1213.
- [7] Niino, N., Tsuzuku, S., Ando, F., and Shimokata, H., 2000, "Frequencies and Circumstances of Falls in the National Institute for Longevity Sciences, Longitudinal Study of Aging (NILS-LSA)," *J. Epidemiol.*, **10**, pp. S90–S94.
- [8] Dingwell, J. B., and Marin, L. C., 2006, "Kinematic Variability and Local Dynamic Stability of Upper Body Motions When Walking at Different Speeds," *J. Biomech.*, **39**, pp. 444–452.
- [9] Dingwell, J. B., and Cusumano, J. P., 2000, "Nonlinear Time Series Analysis of Normal and Pathological Human Walking," *Chaos*, **10**, pp. 848–863.
- [10] Maki, B. E., 1997, "Gait Changes in Older Adults: Predictors of Falls or Indicators of Fear?," *J. Am. Geriatr. Soc.*, **45**, pp. 313–320.
- [11] Hausdorff, J. M., Rios, D. A., and Edelberg, H. K., 2001, "Gait Variability and Fall Risk in Community-Living Older Adults: A 1-Year Prospective Study," *Arch. Phys. Med. Rehabil.*, **82**, pp. 1050–1056.
- [12] Dingwell, J. B., Cusumano, J. P., Sternad, D., and Cavanagh, P. R., 2001, "Local Dynamic Stability Versus Kinematic Variability of Continuous Overground and Treadmill Walking," *ASME J. Biomech. Eng.*, **123**, pp. 27–32.
- [13] Holt, K. G., Jeng, S. F., Ratcliffe, R., and Hamill, J., 1995, "Energetic Cost and Stability During Human Walking at the Preferred Stride Frequency," *J. Motor Behav.*, **27**, pp. 164–178.
- [14] Karcnik, T., 2004, "Stability in Legged Locomotion," *Biol. Cybern.*, **90**, pp. 51–58.
- [15] Hof, A. L., Gazendam, M. G. J., and Sinke, W. E., 2005, "The Condition for Dynamic Stability," *J. Biomech.*, **38**, pp. 1–8.
- [16] Nayfeh, A. H., and Balachandran, B., 1995, *Applied Nonlinear Dynamics: Analytical, Computational, and Experimental Methods*, Wiley, New York, NY.
- [17] Garcia, M., Chatterjee, A., Ruina, A., and Coleman, M., 1998, "The Simplest Walking Model: Stability, Complexity, and Scaling," *ASME J. Biomech. Eng.*, **120**, pp. 281–288.
- [18] Kuo, A. D., 1999, "Stabilization of Lateral Motion in Passive Dynamic Walking," *Int. J. Robot. Res.*, **18**, pp. 917–930.
- [19] Russell, S., Granata, K. P., and Sheth, P., 2005, "Virtual Slope Control of a Forward Dynamic Bipedal Walker," *ASME J. Biomech. Eng.*, **127**, pp. 114–122.
- [20] Hurmuzlu, Y., and Basdogan, C., 1994, "On the Measurement of Dynamic Stability of Human Locomotion," *ASME J. Biomech. Eng.*, **116**, pp. 30–36.
- [21] Hurmuzlu, Y., Basdogan, C., and Stoianovici, D., 1996, "Kinematics and Dynamic Stability of the Locomotion of Post-Polio Patients," *ASME J. Biomech. Eng.*, **118**, pp. 405–411.
- [22] Hausdorff, J. M., Peng, C. K., Ladin, Z., Wei, J. Y., and Goldberger, A. L., 1995, "Is Walking a Random Walk? Evidence for Long-Range Correlations in Stride Interval of Human Gait," *J. Appl. Physiol.*, **78**, pp. 349–358.
- [23] Ashkenazy, Y., Hausdorff, J. M., Ivanov, P. C., and Stanley, H. E., 2002, "A Stochastic Model of Human Gait Dynamics," *Physica A*, **316**, pp. 662–670.
- [24] Hausdorff, J. M., Mitchell, S. L., Firtion, R., Peng, C. K., Cudkowicz, M. E., Wei, J. Y., and Goldberger, A. L., 1997, "Altered Fractal Dynamics of Gait: Reduced Stride Interval Correlations With Aging and Huntington's Disease," *J. Appl. Physiol.*, **82**, pp. 262–269.
- [25] Herman, T., Giladi, N., Gurevich, T., and Hausdorff, J. M., 2005, "Gait Instability and Fractal Dynamics of Older Adults With a "Cautious" Gait: Why do Certain Older Adults Walk Fearfully?," *Gait and Posture*, **21**, pp. 178–185.
- [26] Rosenstein, M. T., Collins, J. J., and DeLuca, C. J., 1993, "A Practical Method for Calculating Largest Lyapunov Exponents From Small Data Sets," *Physica D*, **65**, pp. 117–134.

- [27] Kantz, H., and Schreiber, S., 2004, *Nonlinear Time Series Analysis*, 2nd ed., Cambridge University Press, Cambridge, UK.
- [28] Dingwell, J. B., Cusumano, J. P., Sternad, D., and Cavanagh, P. R., 2000, "Slower Speeds in Neuropathic Patients Lead to Improved Local Dynamic Stability of Continuous Overground Walking," *J. Biomech.*, **33**, pp. 1269–1277.
- [29] Hausdorff, J. M., Edelberg, H. K., Mitchell, S. L., Goldberger, A. L., and Wei, J. Y., 1997, "Increased Gait Unsteadiness in Community-Dwelling Elderly Fallers," *Arch. Phys. Med. Rehabil.*, **78**, pp. 278–283.
- [30] Harris, C. M., and Wolpert, D. M., 1998, "Signal-Dependent Noise Determines Motor Planning," *Nature (London)*, **394**, pp. 780–784.
- [31] Jones, K. E., Hamilton, A. F. d. C., and Wolpert, D. M., 2002, "Sources of Signal-Dependent Noise During Isometric Force Production," *J. Neurophysiol.*, **88**, pp. 1533–1544.
- [32] Hamilton, A. F. d. C., Jones, K. E., and Wolpert, D. M., 2004, "The Scaling of Motor Noise With Muscle Strength and Motor Unit Number in Humans," *Exp. Brain Res.*, **157**, pp. 417–430.
- [33] Theiler, J., Eubank, S., Longtin, A., Galdrikian, B., and Farmer, J. D., 1992, "Testing for Nonlinearity in Time Series: The Method of Surrogate Data," *Physica D*, **58**, pp. 77–94.
- [34] Nashner, L. M., 1980, "Balance Adjustments of Humans Perturbed While Walking," *J. Neurophysiol.*, **44**, pp. 650–664.
- [35] Haridas, C., Zehr, E. P., and Misiasek, J. E., 2005, "Postural Uncertainty Leads to Dynamic Control of Cutaneous Reflexes From the Foot During Human Walking," *Brain Res.*, **1062**, pp. 48–62.
- [36] Ali, F., and Menzinger, M., 1999, "On the Local Stability of Limit Cycles," *Chaos*, **9**, pp. 348–356.
- [37] Su, J. L., and Dingwell, J. B., 2006, "Dynamic Stability of Passive Dynamic Walking on an Irregular Surface," *ASME J. Biomech. Eng.*, in press.
- [38] Dietz, V., Colombo, G., and Müller, R., 2004, "Single Joint Perturbation During Gait: Neuronal Control of Movement Trajectory," *Exp. Brain Res.*, **158**, pp. 308–316.
- [39] Berger, W., Dietz, V., and Quintern, J., 1984, "Corrective Reactions to Stumbling in Man: Neuronal Co-ordination of Bilateral Leg Muscle Activity During Gait," *J. Physiol. (London)*, **357**, pp. 109–125.
- [40] Duysens, J., Tax, A. A., Trippel, M., and Dietz, V., 1992, "Phase-Dependent Reversal of Reflexly Induced Movements During Human Gait," *Exp. Brain Res.*, **90**, pp. 404–414.
- [41] Pearce, M. E., Cunningham, D. A., Donner, A. P., Rechnitzer, P. A., Fullerton, G. M., and Howard, J. H., 1983, "Energetics of Treadmill and Floor Walking at Self-Selected Paces," *Eur. J. Physiol.*, **52**, pp. 115–119.
- [42] Arsenaault, A. B., Winter, D. A., and Marteniuk, R. G., 1986, "Treadmill Versus Walkway Locomotion in Humans: An EMG Study," *Ergonomics*, **29**, pp. 665–676.
- [43] Wank, V., Frick, U., and Schmidtbleicher, D., 1998, "Kinematics and Electromyography of Lower Limb Muscles in Overground and Treadmill Running," *Int. J. Sports Med.*, **19**, pp. 455–461.
- [44] White, S. C., Yack, H. J., Tucker, C. A., and Lin, H. Y., 1998, "Comparison of Vertical Ground Reaction Forces During Overground and Treadmill Walking," *Med. Sci. Sports Exercise*, **30**, pp. 1537–1542.
- [45] van Ingen Schenau, G. J., 1980, "Some Fundamental Aspects of the Biomechanics of Overground Versus Treadmill Locomotion," *Med. Sci. Sports Exercise*, **12**, pp. 257–261.
- [46] Savelberg, H. H. C. M., Vortenbosch, M. A. T. M., Kamman, E. H., van de Weijer, J. G. W., and Schambardt, H. C., 1998, "Intra-Stride Belt Speed Variation Affects Treadmill Locomotion," *Gait and Posture*, **7**, pp. 26–34.
- [47] Takens, F., 1981, "Detecting Strange Attractors in Turbulence," *Dynamical Systems and Turbulence, Warwick 1980*, D. Rand and L. S. Young, eds., Springer-Verlag, Berlin, Germany, Vol. 898, pp. 366–381.
- [48] Sauer, T., Yorke, J. A., and Casdagli, M., 1991, "Embedology," *J. Stat. Phys.*, **65**, pp. 579–616.
- [49] Fraser, A. M., 1986, "Using Mutual Information to Estimate Metric Entropy," *Dimensions and Entropies in Chaotic Systems*, G. Mayer-Kress, ed., Springer-Verlag, Berlin, Germany, pp. 82–91.
- [50] Kennel, M. B., Brown, R., and Abarbanel, H. D. I., 1992, "Determining Embedding Dimension for Phase-Space Reconstruction Using a Geometrical Construction," *Phys. Rev. A*, **45**, pp. 3403–3411.
- [51] Timmer, J., Häussler, S., Lauk, M., and Lücking, C.-H., 2000, "Pathological Tremors: Deterministic Chaos or Nonlinear Stochastic Oscillators?," *Chaos*, **10**, pp. 278–288.
- [52] Kantz, H., and Schreiber, S., 1997, *Nonlinear Time Series Analysis*, Cambridge University Press, Cambridge, UK.
- [53] Kang, H. G., and Dingwell, J. B., 2006, "A Direct Comparison of Local Dynamic Stability During Standing and Walking," *Exp. Brain Res.*, **172**, pp. 35–48.
- [54] Donelan, J. M., Shipman, D. W., Kram, R., and Kuo, A. D., 2004, "Mechanical and Metabolic Requirements for Active Lateral Stabilization in Human Walking," *J. Biomech.*, **37**, pp. 827–835.
- [55] Ramsey, F. L., and Schafer, D. W., 1997, *The Statistical Sleuth: A Course in Methods of Data Analysis*, Duxbury Press, Wadsworth Publishing Company, Belmont, CA.
- [56] Pavol, M. J., Owings, T. M., Foley, K. T., and Grabiner, M. D., 2001, "Mechanisms Leading to a Fall From an Induced Trip in Healthy Older Adults," *J. Gerontol., Ser. A*, **56A**, pp. M428–M437.
- [57] Smeesters, C., Hayes, W. C., and McMahon, T. A., 2001, "Disturbance Type and Gait Speed Affect Fall Direction and Impact Location," *J. Biomech.*, **34**, pp. 309–317.
- [58] Schwab, A. L., and Wisse, M., 2001, "Basin of Attraction of the Simplest Walking Model," *Proceedings of the 2001 ASME Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society for Mechanical Engineering, Pittsburgh, PA, Paper No. DETC01/VIB-21363.
- [59] Wisse, M., Schwab, A. L., Linde, R. Q. v. d., and Helm, F. C. T. v. d., 2005, "How to Keep From Falling Forward: Elementary Swing Leg Action for Passive Dynamic Walkers," *IEEE Trans. Rob.*, **21**, pp. 393–401.
- [60] Mombaur, K. D., Longman, R. W., Bock, H. G., and Schlöder, J. P., 2005, "Open-Loop Stable Running," *Robotica*, **23**, pp. 21–33.