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Recurrence plots analysis of the CNY exchange markets based on phase space reconstruction

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ABSTRACT

The employment of autocorrelation-based transformation to study the dynamics of the exchange rate system is meaningful because it benefits for chaotic prediction on the basis that the transformation from an exchange rate sequence to its associated autocorrelation sequence is reversible. This paper examines the influence of autocorrelation-based transformation on the systemic dynamics using exchange rates of CNY against different currencies among USD, EUR, JPY, GBP, MYR and RUB. First, we construct recurrence plots of exchange rate return sequences and autocorrelation sequences with a fixed sliding window length of 20. The recurrence quantification analysis (RQA) shows that the exchange rate return sequences exhibit lower degrees of determinism than the autocorrelation sequences. Further, by analyzing the RQA measures with bootstrap techniques and box plots, we reveal that the RQA measures of the exchange rate return systems and the autocorrelation sequence systems are mostly significant, and the vertical structures of recurrence plots of autocorrelation sequences are more sensitive to the shuffles of bootstrap techniques. Finally, we investigate the evolution of RQA measures with the changes of sliding window lengths. The analysis shows that appropriately adjusting the sliding window length can increase the systemic determinism.

1. Introduction

Since the first introduction of chaos theories by Lorenz (1965) in meteorology, researches on laws of random-looking processes which used to be regarded as noise have been highlighted. Chaos is a random-looking nonlinear deterministic phenomenon with characteristics of irregular periodicity and randomness. The application of chaos theories in economic spheres derived from the discovery of the chaos phenomenon by Stutzer (1980) in the Haavelmo macro model. Nonlinear deterministic process analysis is always one of the hottest issues for economic and financial experts to understand how the economy works. Just as its name implies, the detection of a nonlinear deterministic process can be divided into the nonlinear detection and the deterministic detection. While the nonlinear detection can be carried out with Hurst exponents by R/S analysis (Alvarezramirez, Echeverria, & Rodriguez, 2008; Cajueiro & Tabak, 2004; Yao, Lin, Liu, & Zheng, 2014) and BDS tests (Broock, Scheinkman, Dechert, & Lebaron, 1995; Brooks & Heravi, 1999; Kočenda, 2001), the determinism of nonlinear systems can be intuitively estimated with recurrence plots (Casdagli, 1997; Eckmann, Kamphorst, & Ruelle, 1987).

In descriptive statistics and chaos theories, a recurrence plot is a plot showing that measures the complexity and the determinism of a dynamical system. The texture of a recurrence plot contains the information of the systemic trajectory

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repetition which describes the structural determinism and the randomness of the reconstructed phase space. Thus a recurrence plot can reflect the autocorrelation of all the possible time scales generated in a system and is a representation of the global correlation of the system (Marwan, Romano, Thiel, & Kurths, 2007). The convenience of recurrence plot analysis makes it widely available for determinism analysis in bioscience (Carrubba, Frilot, Chesson, & Marino, 2008; Goshvarpour, 2012; Wu, 2004), climatology (Trauth, Bookhagen, Marwan, & Strecker, 2003; Yang, Wang, Bian, & Zhou, 2010) and financial fields (Addo, Billio, & Guégan, 2013; Bastos & Caiado, 2011).

Phase space reconstruction (Ma & Han, 2006) is the precondition of recurrence plot construction. The phase space reconstruction theory holds two viewpoints: first, all the dynamic information to determine the state of a system is included in the sequence evolution of the system; second, the state trajectories of univariate time series embedded in a new coordinate can preserve the most important characteristics of the original state and characterize some motion laws that cannot be described by the traditional coordinate. Abundant researches were carried out based on phase space reconstruction. Sivakumar, Jayawardena, and Fernando (2002) used phase space reconstruction approaches and artificial neural networks to understand river flow dynamics and found that not only were the major trends well captured but also the minor fluctuations reasonably preserved as well. Gao and Jin (2009) constructed a complex network from time series with nodes represented by phase space trajectories and edges determined by the trajectory distances, and found that the constructed network inherited main properties of the time series in its dynamic structure. Furthermore, some experimental results showed that the dynamic characteristics of systems can be controlled by different embedding dimensions or embedding delays. Hołyst, Żebrowska, and Urbanowicz (2001) investigated several economical exchange rates like USD/GBP, USA Treasury bond rates and the Warsaw stock index by recurrence plots, and showed how the appropriate time-delayed feedback influenced the simple chaotic economic model. Iwanski and Bradley (1998) represented that one can improve upon recurrence plot analysis by exploiting and incorporating structural characteristics of a system with different embedding dimensions.

More and more studies showed that the fluctuation modes of exchange rates are nonlinear, and the exchange rate systems possess the characteristics of complex nonlinear dynamic systems by both traditional regression models (Chinn, 1991; López-Suárez & Rodríguez-López, 2009; Priestley & Ødegaard, 2007) and nonlinear tests (Adrangi, Chatrath, & Raffiee, 2010; Pavlidis, Paya, & Peel, 2015; Stillwagon, 2016). Exchange rates from different markets can appear different dynamic characteristics. For instances, Bask (2002) showed the existence of chaos in the studies of Swedish against Deutsche Mark, ECU, USD, and Yen exchange rates while Dechert and Gencay (1992) showed no evidence of chaos in the studies of Canadian, German, Italian and Japanese monthly average spot exchange rates. Moreover, some further studies showed that exchange rates and their returns exhibited different dynamic characteristics. Lahmiri (2017) showed the existence of chaos in currency levels and the inexistence of chaos in return levels in the Moroccan exchange rate market.

The symbolic dynamics theories also provide us novel angels to understand the fluctuation modes of exchange rates. An, Gao, Fang, Huang, and Ding (2014) utilized an autocorrelation-based method to construct a symbol transfer network from a crude oil price sequence to study its fluctuation features, analyzing a one-dimensional sequence with network concepts. It led us to consider the autocorrelation of exchange rate systems which had been proved to be nonlinear.

Motivated by these previous studies, this paper applies recurrence plots to study the determinism of CNY against USD, EUR, JPY, GBP, MYR and RUB exchange rates based on phase space reconstruction. Unlike the previous studies, this paper will study the influence of autocorrelation-based transformation on the determinism of exchange rates in the CNY market as the supplement to the absent researches in this aspect and intuitively present in recurrence plot showing. Moreover, considering that the RQA (Marwan et al., 2007) provides quantitative indications for recurrence plots, this paper will briefly choose some measures to discuss the quantitative characteristics of recurrence plots of different exchange rate return sequences and the transformative sequences. Considering that the sliding window length is a key factor for autocorrelation-based transformation, this paper will investigate how the RQA measures evolve with different sliding window lengths.

The remainder of this paper is organized as follows. Section 2 discusses the methodology used in this paper, including the methods to reconstruct phase space, the choices of recurrence thresholds, and the methods to construct recurrence plots. Section 3 is the part about data description including the exchange return sequences and autocorrelation sequences formed by the original exchange sequences. Section 4 provides the result exhibitions of recurrence plots and the analysis of how the RQA measures are influenced by sliding window lengths. Finally the concluding remarks are given in Section 5.

2. Methodology

2.1. Recurrence plot construction

A recurrence plot (RP) is a tool introduced to visualize the recurrent state of multidimensional phase space of a dynamical system. Referring to Guhathakurta, Bhattacharya, and Chowdhury (2010), given one-dimensional time series $\{v_i\}_{i=1}^{N}$, we reconstruct a phase space trajectory by time delay embedding:

$$x_i = \{v_i, v_{i+\tau}, \ldots, v_{i+\tau(m-1)}\} \tag{1}$$

where m is the embedding dimension and τ is the delay.

The recurrences can be expressed by the recurrence matrix

$$R_{ij} = H(\varepsilon - \|x_i - x_j\|), \quad i,j = 1 \ldots N, \tag{2}$$

with H being the Heaviside function and ε being a threshold distance for proximity. $\| \bullet \|$ is norm, a spatial distance which is mostly considered in terms of maximum or Euclidean norm. Hence, the cut-off distance ε defines a sphere centered at x_i. If the state x_j falls within that sphere, the state will be close to x_i and thus $R_{ij} = 1$. These ε can be either constant for all x_i or can vary in such a way that the sphere contains a predefined number of close states (Marwan, 2011). In this paper a fixed ε and the Euclidean norm are used for each RP.

The main diagonal line represents the identity, consisting entirely of recurrence points and thus $R_{ii} = 1$. In addition, since $R_{ij} = R_{ji}$, the plots are symmetrical. Small-scale features in a RP can be observed in terms of diagonal and vertical lines. Diagonal lines in general indicate deterministic behavior; regular patterns of parallel lines reveal a periodicity. Diagonal lines parallel with the main diagonal line occur when segments of the trajectory visit the same region of the phase space at distinct times. The length of these lines is determined by the duration of these visits. Vertical or horizontal lines suggest stationary states in which the system persists in the same region for some time. On the other hand, isolated recurrence points may occur when states are rare, especially for stochastic systems. While deterministic systems tend to exhibit long diagonal lines and few isolated points, stochastic systems present mostly isolated points or very short diagonal lines.

Before reconstructing the phase space, we need another two important parameters namely the embedding dimension m and the time delay τ.

2.2. Embedding dimension, delay and threshold choices

The construction of a RP requires the specification of the embedding dimension m, and the embedding time delay τ. Besides, the RQA requires the specification of the threshold ε. A sufficiently large embedding dimension which contains the associated dynamics of the system and accounts for the effect of noise should be chosen. We utilize false nearest neighbor methods (Kennel, Brown, & Abarbanel, 1992) to estimate embedding dimensions of exchange rate sequences. In an embedding dimension that is too small to unfold systemic attractors, not all points close to each other will become neighbors. Some will actually be far from each other and simply appear as neighbors because the geometric structures of the attractors have been projected down onto smaller spaces. An embedding dimension $m = 12$ is appropriate for the exchange rate sequences and the autocorrelation sequences under study.

As for the embedding time delay τ, according to Zbilut (2005), a lag value $\tau = 1$ is usually appropriate for discrete data. In the RQA of the sequences under study, a criterion suggesting that the recurrence threshold should be chosen in such a way that the recurrence rate is approximately 1% is adopted (Zbilut, Zaldivar-Comenges, & Strozzi, 2002). Thus in the processes of RQA, the recurrence rate is retained 1% in this paper. In addition, the prerequisite of this criterion to choose ε according to the recurrence rate is the sequence stationarity. We find the data under study stationary by ADF-tests (Hamilton, 1994).

3. Data description

The data in this paper consists of the daily exchange rates of CNY against USD, EUR, JPY, GBP, MYR and RUB for the period between November 22, 2010 and September 5, 2016. Consequently we obtain 1409 observations except weekends and national statutory holidays.

Specifically, the original sequences, the daily exchange rates in the CNY market under study are the amount of CNY to exchange for every one hundred USD, EUR, JPY, GBP, MYR and RUB. The dataset includes the representative exchange rates from Asia, Europe and America so that it is broad enough to represent the main global market information. And the exchange rates of CNY against USD, EUR and GBP are always the focus of the world.

The exchange rate stability is the assurance of foreign trades. Under the current international monetary systems, most countries implement managed floating exchange rate systems. In most cases, most regions will allow exchange rates to fluctuate within certain ranges but intervene in currency markets when exchange rates are outside the ranges. But in some cases, fluctuations of exchange rates may be out of control. As shown in Fig. 1, the exchange rate of CNY/RUB experienced huge fluctuations after July 2014. For CNY/EUR and CNY/GBP, the exchange rates also experienced several obvious fluctuation periods. In this paper, the returns of exchange rates are employed and they are calculated as the difference between the logarithmic prices at t and $t - 1$, shown as the below:

$$R(t) = \ln v(t) - \ln v(t-1) \tag{3}$$

where $R(t)$ denotes the exchange rate return at time t, $v(t)$ denotes the exchange rate at time t, and $v(t-1)$ denotes the exchange rate at time $t - 1$.

In order to study the influence of autocorrelation-based transformation on the RP, we divide the original exchange rate sequence into several terms with a sliding window of w days and thus we get a new sequence with $T = 1409 - w$ terms. The autocorrelation between t-th and time $(t - 1)$-th terms is defined as the following formula:

$$\rho_t = \frac{\mathrm{cov}(term_t, term_{t-1})}{\sqrt{\mathrm{var}(term_t)}\sqrt{\mathrm{var}(term_{t-1})}} \tag{4}$$

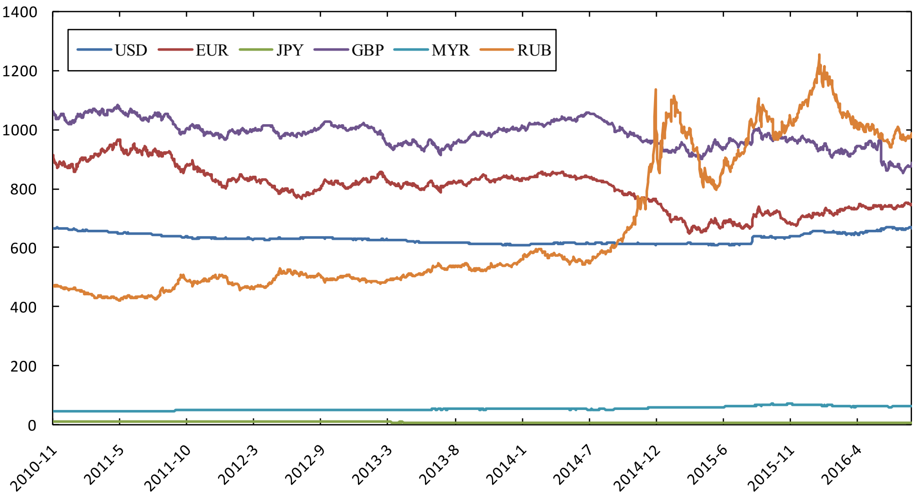

Fig. 1. Exchange rates of CNY against different currencies.

Table 1
The processes to construct an autocorrelation sequence.

Original sequence	term	ρ
v_t v_{t-1} v_{t-2} … $v_{t-(w-1)}$	t	ρ_t
v_{t-w}	t-1	ρ_{t-1}
$v_{t-(w+1)}$	t-2	ρ_{t-2}
…	…	…

Table 2
Statistical properties of exchange rate return sequences and autocorrelation-based sequences.

Type	Mean	STD	Skewness	Kurtosis	JB-Statistic	Prob.
R_{USD}	5.16E−06	0.001332	3.845409	49.78262	131868.7	0.000000
R_{EUR}	−0.000143	0.005687	0.147466	5.099915	263.8022	0.000000
R_{JPY}	−0.000150	0.005894	−0.362381	10.82299	3621.172	0.000000
R_{GBP}	−0.000126	0.005216	−1.734510	30.13819	43912.90	0.000000
R_{MYR}	0.000187	0.005505	−0.866573	17.28964	12155.59	0.000000
R_{RUB}	0.000523	0.012249	0.129951	20.94332	18892.44	0.000000
ρ_{USD}	0.781034	0.159643	−1.353523	5.379332	751.7568	0.000000
ρ_{EUR}	0.776128	0.156374	−1.518207	5.998354	1053.900	0.000000
ρ_{JPY}	0.779143	0.160098	−1.371575	5.429985	777.2439	0.000000
ρ_{GBP}	0.768586	0.162595	−1.260572	5.170347	640.4781	0.000000
ρ_{MYR}	0.734701	0.199951	−1.461735	5.809178	951.3584	0.000000
ρ_{RUB}	0.773616	0.160132	−1.370459	6.200626	1027.666	0.000000

Notes: the null hypothesis that the sequence obeys normal distribution is rejected if the prob. is less than 5%.

where var($term_t$) denotes the variance and cov($term_t$, $term_{t-1}$) denotes the autocovariance. In this paper, the length of the sliding window w is initially set to 20. As shown in Table 1, we finally get a new sequence $\boldsymbol{\rho_t}$ (Table 2).

4. Recurrence quantification analysis of the CNY exchange market

4.1. Recurrence plots

4.1.1. Recurrence plots of exchange rate returns

A RP colorized according to the phase space distances provides a more detailed overview of a system which can reveal the dynamics of the system in a cleaner way than a conventional black-and-white RP whose recurrence dots are determined according to whether the phase distances are shorter than a specified threshold. In the RPs of this paper, The dot at coordi-

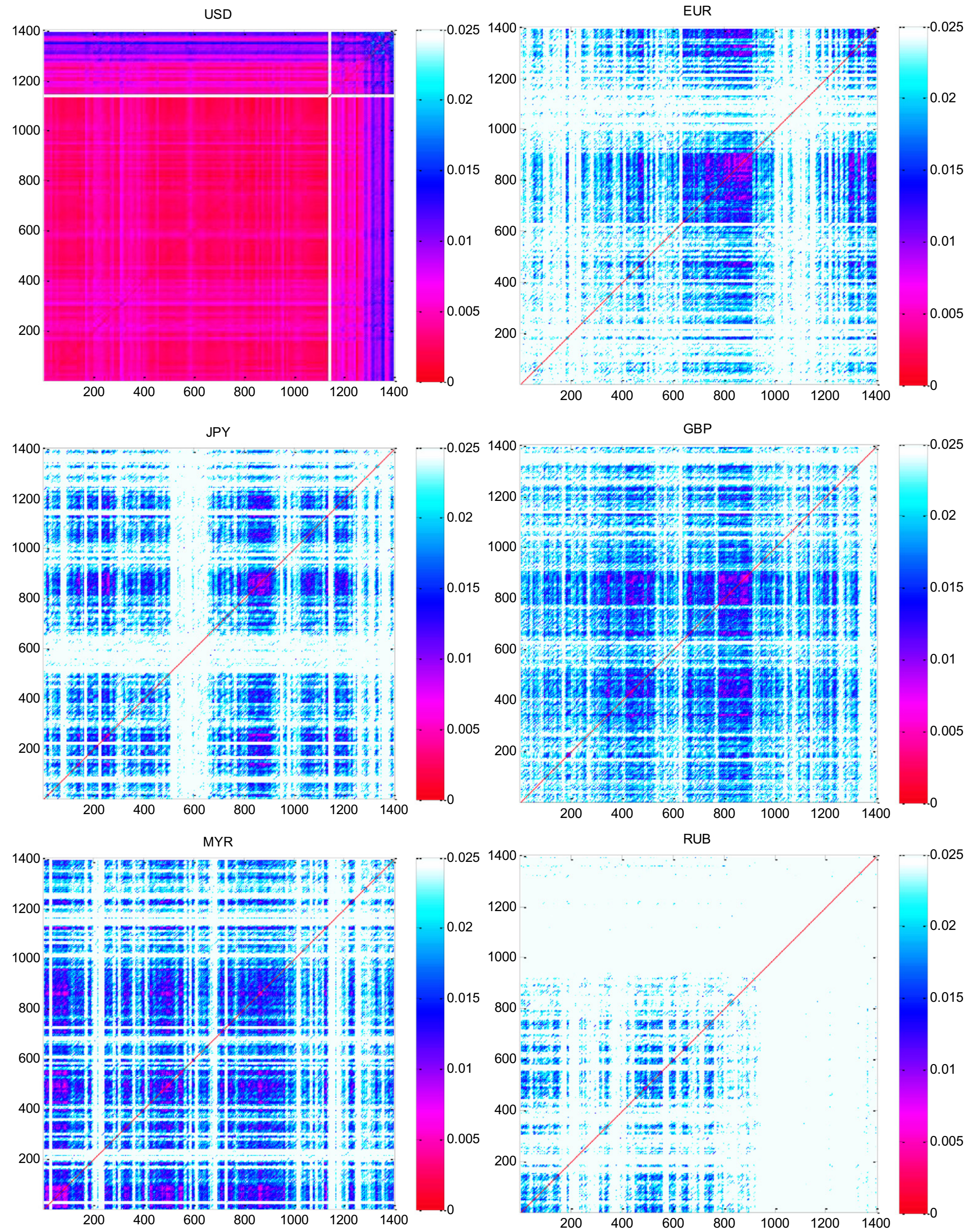

Fig. 2. The RPs of exchange return sequences. Parameters: the embedding dimension $m = 12$, the delay $\tau = 1$.

nate (i, j) is colorized according to the value of the distance $\|x_i - x_j\|$. The RPs of CNY against different currencies are shown in Fig. 2. In Fig. 2, the colors in color bars ranging from red, purple, blue and cyan to white correspond to the distances ranging from 0 to 0.025.

For time series from periodic systems, the recurrence plots consist of some diagonal lines parallel to main diagonal lines. For time series from nonlinear deterministic systems, the structures of recurrence plots are more complex and appear with some analogous periodicity that the diagonal lines are parallel to the main diagonal line and the number of the diagonal lines parallel to the main diagonal line is greater than the stochastic system. The massive structures in RPs potentially reveal the congregation of phase space trajectories, suggesting the sequence stationarity. Thus, by observing the diagonal lines parallel to the main diagonal and the massive structures in the RP, one can qualitatively estimate the systemic determinism and the fluctuation mode.

The main diagonal lines entirely consist of recurrence points with the highest levels of recursiveness in RPs, and therefore they are colored in red corresponding to the color bars. As shown in Fig. 2, the RPs of return sequences include few red and purple dots, except USD. In addition, the red and purple dots are relatively dispersive, generally sticking onto the central segments of massive structures which are the aggregation of recurrence points in RPs. The RP of the CNY/USD return sequence consists mainly of red and purple dots. Another two interesting structures found in the RP of CNY/USD are the roughly blue vertical pattern in the rightmost and the blue horizontal pattern in the uppermost. These features are corresponding to the fluctuation mode of the CNY/USD exchange rate: quite stable and slightly fluctuant before and after August 2015. In contrast, the RP of the CNY/RUB exchange rate return consists mainly of white points, and overall few massive structures, potentially exhibiting a higher degree of instability and unpredictability compared with the others. The economically interdependent bodies, the United Kingdom and the European Union, display some common features in RPs in term of massive structure positions, whereas their structures in the rightmost and the uppermost are quite different, potentially revealing their deviation in fluctuation modes of exchange rates during those periods. As for the CNY/MYR exchange return sequence, the lower left part of the RP whose coordinate axis is within [1, 1000] includes nine roughly similar massive structures. This feature shows the structural periodicity of the CNY/MYR exchange system.

In conclusion, the RPs of exchange rate returns under study have many small strips parallel to the main diagonals, indicating that the exchange rate returns have certain degrees of determinism. Besides, the presences of massive structures in the RPs indicate the structural stationarity of the exchange rate systems.

4.1.2. Recurrence plots of autocorrelation sequences

Fig. 3 shows the RPs of the autocorrelation sequences with embedding dimensions being 12, embedding time delays being 1 and sliding window lengths being 20:

In Fig. 3, the different colors in color bars correspond to the distances within [0, 0.35]. From Figs. 2 and 3, we observe two phenomena: first, the massive structures become much smaller and the diagonals are more clearly visible, which may be an exhibition of a lager degree of determinism than the return sequences; second, the RPs tend to feature periodic trajectories consisting of profuse discontinuous diagonals. This consequence is more in conformity with characteristics of chaotic determinism.

As shown in Fig. 3, the percentages of colored dots of the six RPs are of less difference in numerical values, compared with the RPs in Fig. 2. Even though few dots in the USD plot and a large proportion of dots in the RUB plot are colored in white in Fig. 2, the RPs of the autocorrelation sequences are constructed with much more similar features. In addition, the colored dots in Fig. 3 distribute more uniformly, and the diagonals as well as the small massive structures are clearer to see, which may be interpreted as a signature of determinism. The distributions of colored dots may be contributed from the self-correlation of exchange rate sequences. Yet, the RP of MYR is intuitively much simpler with a lower degree of recursiveness. There are many purple or even red dots in the lower right corner and the upper left corner in spite of the majority of the dots colored in blue and cyan. The diagonal lines and the small massive structures also distribute in the lower right corner and the upper left corner. It means that the phase space trajectories are relatively concentrated and the autocorrelation sequence systems are of higher stability compared with the exchange rate return systems.

4.2. Recurrence quantification analysis of the CNY exchange market

4.2.1. Recurrence quantification analysis and reliability measures

4.2.1.1. Recurrence quantification analysis. Referring to the research of Marwan et al. (2007) which shows the processes of RQA, this paper utilizes some RQA measures to study the similarity and the difference between exchange rate return sequences and autocorrelation sequences transformed from original exchange rate sequences.

RR: recurrence rate, the percentage of recurrence points in the RP. This measure provides the density of recurrence points in the RP.

$$RR = \frac{1}{N^2}\sum_{i,j} R_{ij}(\varepsilon) \tag{5}$$

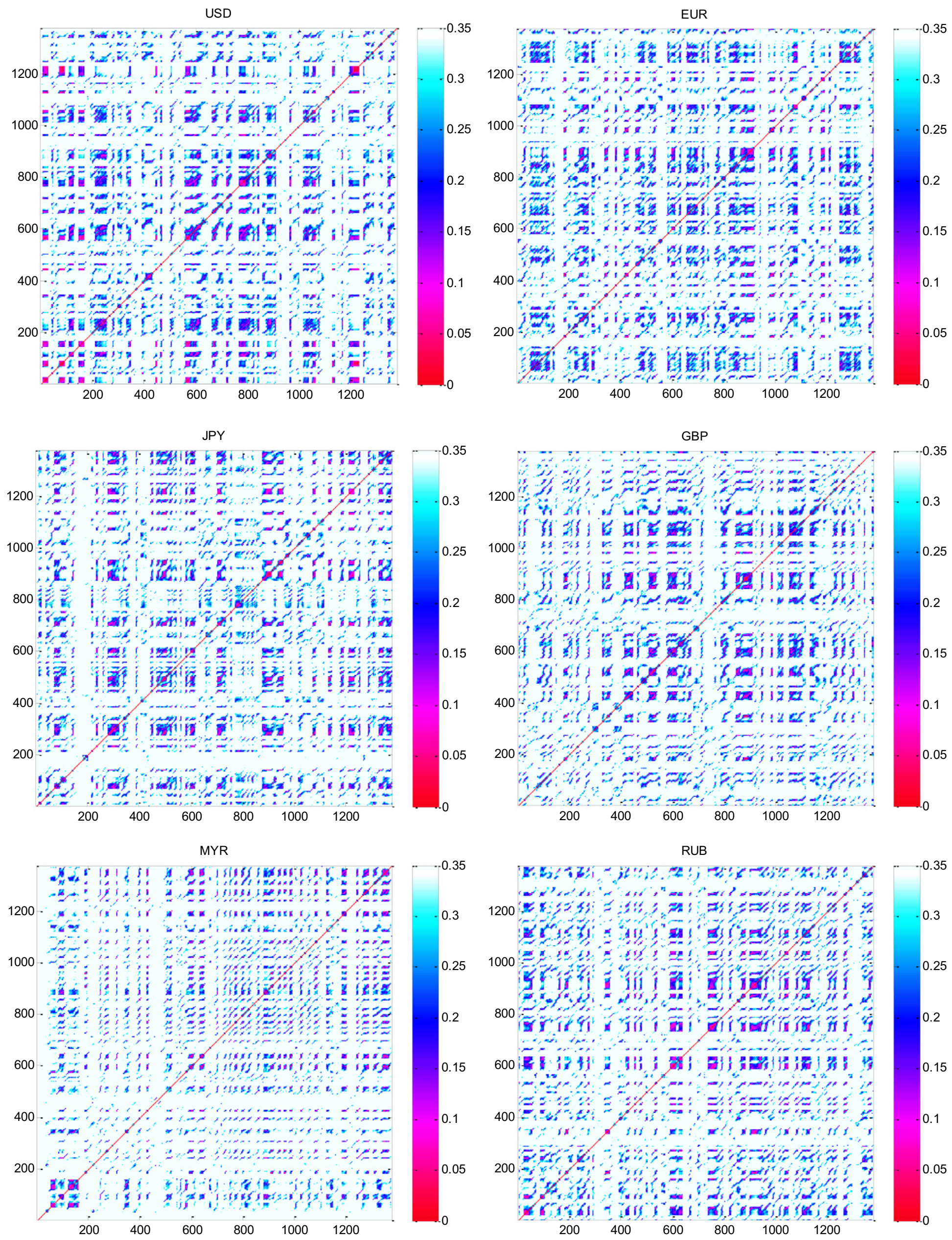

Fig. 3. The RPs of exchange rate autocorrelation sequences Parameters: the embedding dimension $m = 12$, the delay $\tau = 1$ and the sliding window length $w = 20$.

DET: determinism, the percentage of recurrence points forming diagonal structures in the RP:

$$\mathrm{DET} = \frac{\sum_{l \geqslant l_{\min}} lP(l)}{\sum_{l} lP(l)} \tag{6}$$

where l is the diagonal line length, $P(l)$ is the histogram of the diagonal line length, and l_{min} is the threshold to exclude the diagonal lines which are formed by the tangential motion of the phase space trajectory. In this paper l_{min} is set to 2. Deterministic processes have more and longer diagonals and less isolated points. Thus this measure provides an indication for determinism and predictability of the system.

L: the average diagonal line length:

$$L = \frac{\sum_{l \geqslant l_{\min}} l P(l)}{\sum_{l \geqslant l_{\min}} P(l)} \tag{7}$$

Similar to *DET*, this measure also provides an indication of systemic determinism, the greater the value, the smaller the systemic randomness.

L_{max}: the longest diagonal line found in the RP. This measure provides the information about system stability, the more the value, the more stability the system.

ENTR: Shannon entropy of distribution of diagonal line lengths:

$$ENTR = -\sum_{l \geqslant l_{\min}} p(l) \ln p(l) \tag{8}$$

where $p(l) = P(l)/\sum_{l \geqslant l_{\min}} P(l)$. This measure reflects the diversity of diagonal lines and the complexity of the system, the greater the entropy, the more complex the system.

LAM: laminarity, the percentage of the recurrence points forming the vertical or horizontal structures:

$$LAM = \frac{\sum_{v \geqslant v_{\min}} v P(v)}{\sum_{v} v P(v)} \tag{9}$$

where v is the vertical or horizontal line length, $P(v)$ is the histogram of the vertical or horizontal line length, and v_{min} is the threshold to exclude the vertical or horizontal lines. In this paper v_{min} is set to 2. It measures the probability that a state will not change for the next time step.

TT: trapping time, the average length of vertical or horizontal structures:

$$TT = \frac{\sum_{v \geqslant v_{\min}} v P(v)}{\sum_{v \geqslant v_{\min}} P(v)} \tag{10}$$

This measure provides the information about the number of the vertical or horizontal structures in the RP, reflecting the average time of the system in a particular state, the greater the value, the less complex the system.

4.2.1.2. Reliability measures. Our guideline for the RQA reliability measure is the bootstrap technique, a nonparametric method widely used for estimating statistical significance (see, e.g., Kónya, 2006; Yao, Lin, & Liu, 2016; Zharkikh & Li, 1995). The bootstrap technique requires us to construct a number of replicas. Specifically the steps of bootstrap techniques for the reliability measures of *DET* are elaborated as follows with the RQA bootstrap methods proposed by Schinkel, Marwan, Dimigen, and Kurths (2009):

① Generate a matrix $X = \{x_1, x_2, \ldots, x_N\}$ based on the phase space reconstruction, where $x_i = \{v_i, v_{i+\tau}, \ldots, v_{i+\tau(m-1)}\}$ (i = 1, 2,. . ., N) is a phase space trajectory.

② Generate a bootstrap sample $X* = \{x_{i_1}, x_{i_2}, \ldots, x_{i_N}\}$ by randomly sampling N phase space trajectories of X with replacement where i_k (k = 1, 2,. . ., N) denotes a random integer within [1, N].

③ Construct a RP and compute the *DET* based on the bootstrap sample $X*$ with a 1% *RR* threshold.

④ Repeat steps (①–③) 1000 times and obtain a *DET* collection $DETC = \{DET_1, DET_2, \ldots, DET_{1000}\}$. Typically in the systemic analysis 1000 is thought of as an enough number of replicas.

⑤ The bootstrap processes of the other RQA measures are defined in similar ways.

4.2.2. The influence of the embedding dimension m

As a crucial parameter for a RP and its RQA, special attention on the embedding dimension is required. In this part, the influence of the embedding dimension on the RQA measure is investigated with its value evolving from 1 to 20. The results are shown in Fig. 4.

As shown in Fig. 4, the *MDET* of ERR remains stable with the embedding dimension greater than 12, while the *MDET* of AUT remains stable with the embedding dimension greater than 9. This exhibition may also provide an indication for the selection of the embedding dimension. The measures *ML* of ERR and AUT both exhibit upward tendencies and the AUT curve is "S" shaped. It is remarkable that the ML_{max} of AUT is more sensitive to the changes of the embedding dimension, and it tends to experience a sudden rise with the embedding dimension greater than 12. For *MENTR*, the tendencies of ERR and AUT are roughly consistent when their embedding dimensions are greater than 12, meaning the roughly consistent complexity of their systems in term of the diagonals. As for *MLAM* and *MTT*, the values of AUT are always greater than the values of ERR, suggesting the less complexity of the AUT systems.

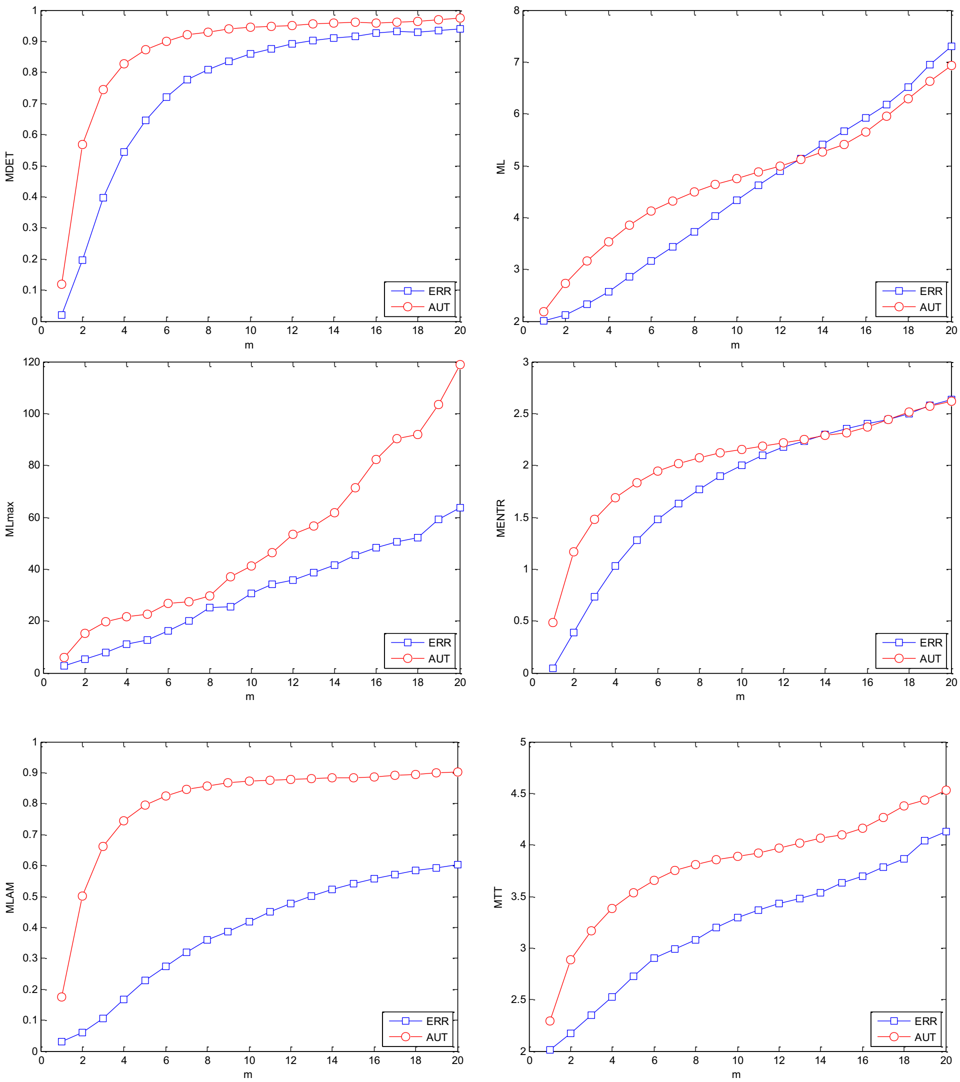

Fig. 4. The evolution of mean RQA measures with embedding dimensions. ERR denotes the exchange rate return, AUT denotes the autocorrelation sequence. A capital 'M' in front of the name of a RQA measure denotes the mean value of the RQA measures of different exchange sequences at each observation point. Parameters: ε corresponding to the 1% *RR*, the embedding delay $\tau = 1$, and the sliding window length $w = 20$.

4.2.3. The measure of reliability with the bootstrap technique

The box and whisker plots of bootstrap results of exchange return and autocorrelation sequences are shown in Fig. 5. The plots of AUT sequences are colored in blue, the plots of ERR sequences are colored in black, and the outliers are colored in red. Table 3 is about the comparative analysis of RQA measures and 95th percentiles of RQA measure collections based on bootstrap techniques.

According to the above-mentioned bootstrap technique, the RQA bootstrap samples with 1000 surrogate elements for each are produced by shuffling the phase space trajectories with replacement. This operation could preserve not only distributions but also dependencies compared with simple shuffles of observations of original sequences which could only preserve distributions. Therefore, while the simple shuffle of the observations provides the information about the linear stochasticity of the currency, the shuffle of the phase space trajectories provides the information about the systemic reliability.

As the box and whisker plots shown in Fig. 5, the median values of *DET* of AUT sequences are greater than ERR sequences for EUR, JPY, GBP and MYR, whereas it turns out the opposite results for USD and RUB. As for the measures *L*, L_{max} and *ENTR*, the median values are roughly equal. It is noteworthy that the distances between the left and right ends of the whiskers are much smaller for the AUT sequences. Furthermore for the measures *LAM* and *TT*, the median values of AUT sequences are smaller than the values of ERR sequences, especially for the measure *LAM*, in sharp contrast to the comparisons of *LAM* and *TT* between AUT and ERR sequences shown in Fig. 4.

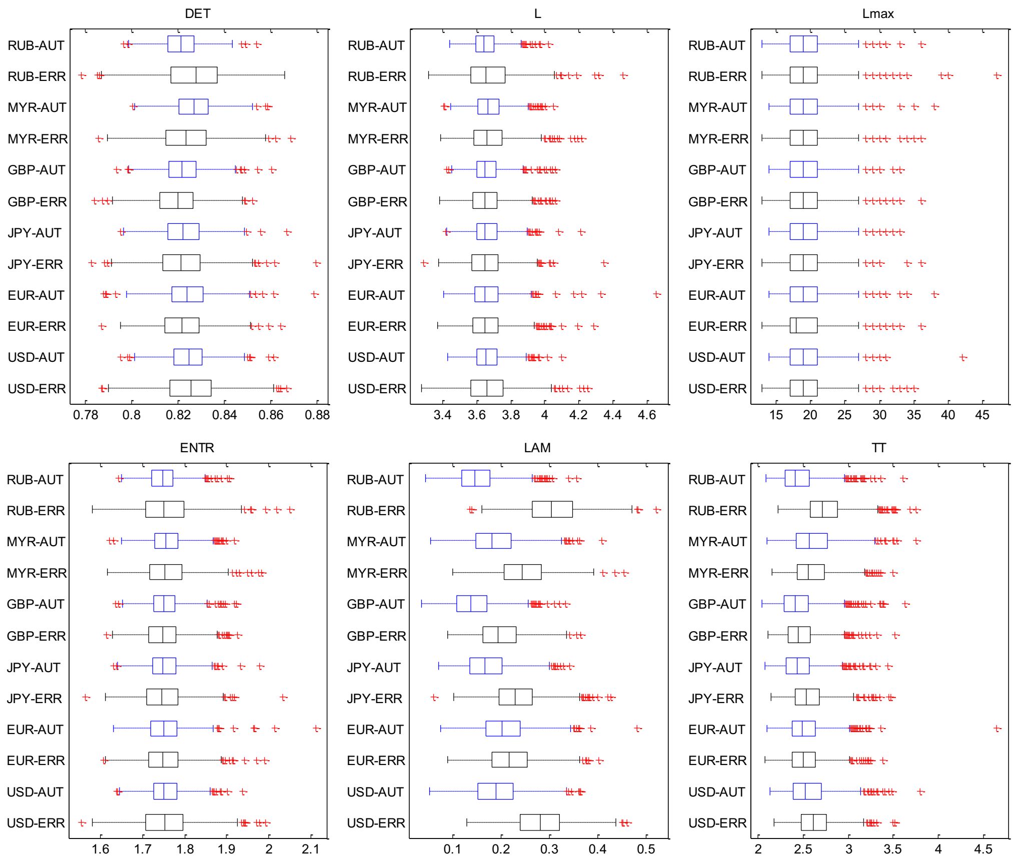

Fig. 5. Box and whisker plots of RQA measures based on bootstrap samples. AUT denotes the autocorrelation sequence; ERR denotes the exchange rate return sequence. Parameters: ε corresponding to the 1% *RR*, the embedding dimension $m = 12$, the embedding delay $\tau = 1$, and the sliding window length $w = 20$.

Table 3
RQA measures of exchange return and autocorrelation sequences.

Type	ε	*DET*	*L*	L_{max}	*ENTR*	*LAM*	*TT*
USD-ERR	0.0015	88.83 (84.88)	4.68 (3.92)	33 (26)	2.12 (1.87)	45.83 (38.51)	3.31 (3.01)
USD-AUT	0.0902	95.65 (83.97)	5.43 (3.84)	70 (24)	2.34 (1.83)	90.91 (28.59)	4.80 (2.97)
EUR-ERR	0.0104	90.66 (84.06)	5.69 (3.85)	56 (25)	2.35 (1.84)	54.85 (31.45)	3.28 (2.90)
EUR-AUT	0.0998	95.33 (84.07)	5.13 (3.84)	50 (25)	2.26 (1.83)	87.06 (29.73)	3.90 (2.91)
JPY-ERR	0.0105	89.63 (84.12)	4.70 (3.88)	35 (25)	2.12 (1.85)	53.66 (31.73)	3.47 (2.93)
JPY-AUT	0.0924	95.46 (83.85)	5.10 (3.83)	49 (25)	2.25 (1.83)	89.49 (25.87)	4.25 (2.85)
GBP-ERR	0.0103	88.52 (83.76)	4.97 (3.86)	36 (25)	2.19 (1.84)	50.09 (29.36)	3.81 (2.84)
GBP-AUT	0.1012	95.12 (83.60)	4.83 (3.81)	57 (26)	2.17 (1.82)	86.44 (22.53)	3.37 (2.87)
MYR-ERR	0.0090	88.33 (84.45)	4.79 (3.89)	28 (26)	2.17 (1.85)	46.25 (34.59)	4.05 (3.01)
MYR-AUT	0.1109	94.39 (84.37)	4.74 (3.83)	46 (25)	2.15 (1.83)	87.19 (28.55)	3.76 (3.11)
RUB-ERR	0.0151	89.29 (85.03)	4.58 (3.92)	27 (26)	2.11 (1.87)	**36.38** (41.13)	**2.69** (3.15)
RUB-AUT	0.0949	94.65 (83.60)	4.75 (3.80)	49 (25)	2.15 (1.82)	86.09 (23.28)	3.72 (2.87)

Notes: DET and LAM values are given in %. ERR denotes the exchange rate return sequence. AUT denotes the autocorrelation sequence. The values in parentheses denote the 95th percentiles of the corresponding RQA measure collections based on bootstrap techniques. The values of RQA measures less than 95th percentiles of RQA bootstrap collections are shown in bold font. Parameters: ε corresponding to the 1% *RR*, the embedding dimension $m = 12$, and the embedding delay $\tau = 1$.

Consequently we summarize three main points with Fig. 5 and Table 3:

(1) The vertical structures of RPs of AUT sequences are more sensitive to the shuffles of bootstrap techniques, revealing the vertical structures tend to be destroyed by the shuffling operations.
(2) For AUT sequences, the bootstrap collections of RQA measures exhibit peak distributions in the sense that the distances between the left and right ends of the whiskers are smaller.

(3) The comparative analysis of RQA measures and 95th percentiles of RQA bootstrap collections shows that the majority of the RQA measures are reliable at 5% critical levels in the sense that the values of RQA measures are greater than the 95th percentiles of RQA bootstrap collections.

4.2.4. The influence of the sliding window length *w*

Another important parameter for the RP of each autocorrelation sequence is the sliding window length. We investigate the evolution of the RQA measures with the changes of sliding window lengths in Fig. 6.

As shown in Fig. 6, for the measures *DET* and *LAM*, sudden increases are observed when *w* values are smaller than 40, which may be related to the fact that the autocorrelation sequences are normally more and more predictable with *w* values from 10 to 40. For the measure *L*, all the curves exhibit upward tendencies in fluctuation with *w* smaller than some specific critical values but downward tendencies with *w* greater than those critical values. For the measure L_{max}, the curve of GBP is of considerable volatility. The others overall experience the downward tendencies after the upward. For the measures *L*, *ENTR* and *TT*, it is remarkable to observe that the fluctuation of EUR is much larger than the others.

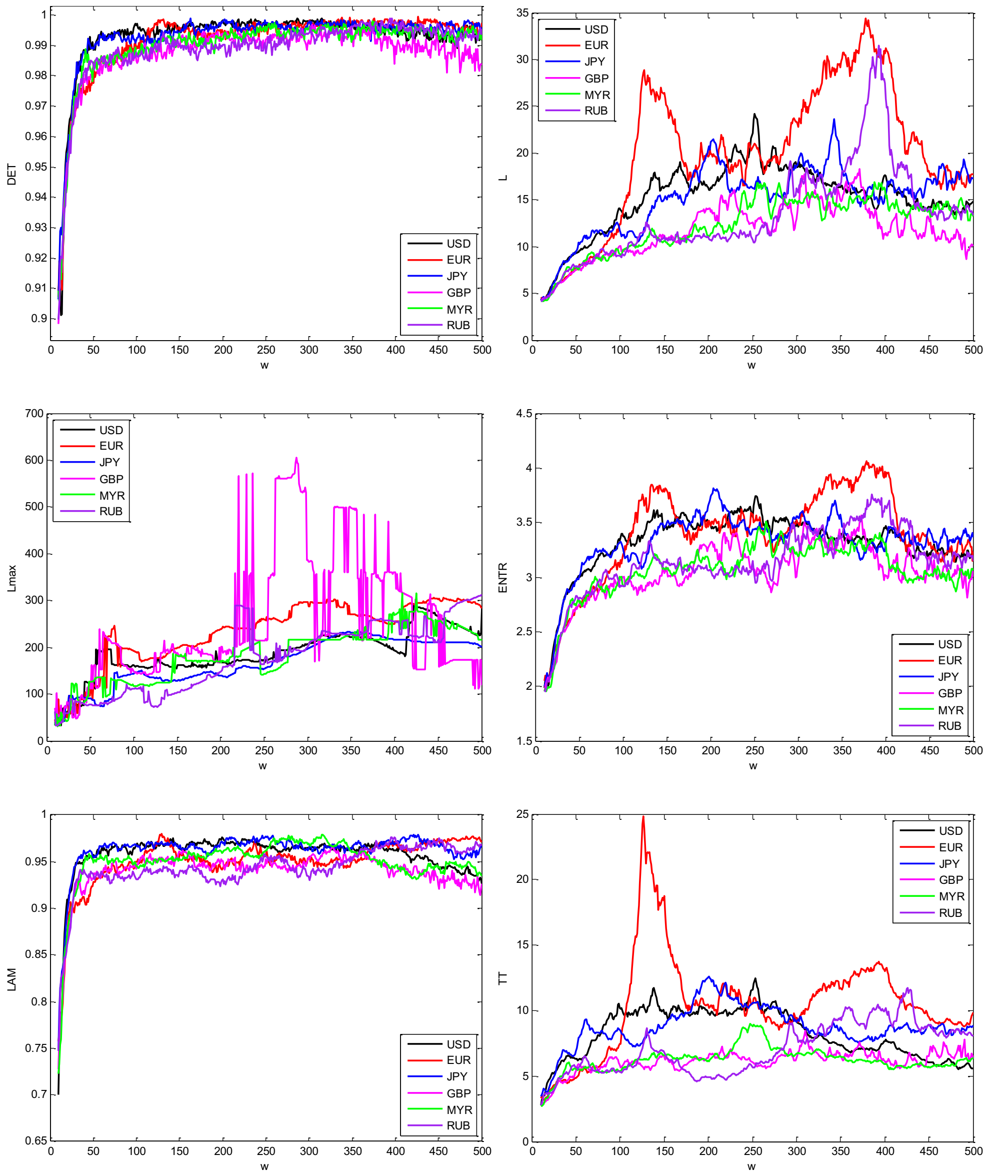

Fig. 6. The evolution of RQA measures with sliding window lengths. *w* denotes the sliding window length within [10, 500]. The *RRs* are retained 1%

In the case of choosing an optimal sliding window length, the smallest w value when the DET becomes stationary is suggested because it overall means a relatively larger degree of determinism of the system with the shortest period of autocorrelation. Another potential criterion may be the largest L, because it may mean that the determinism of the system may become large in some short periods of the overall observation period. In conclusion, those results reveal that one can change the recurrence characteristics and consequently improve the predictive accuracy of the system by appropriately adjusting the sliding window length.

5. Conclusions

Recently, the absent studies about the influence of autocorrelation transformation on the dynamics of exchange rates make the supplement in this field necessary. Based on the RP and RQA theories, this paper investigates the deterministic characteristics of exchange rates of CNY against different currencies among USD, EUR, JPY, GBP, MYR and RUB. Two main problems are focused: first, to exhibit the difference between exchange rate return and autocorrelation sequences in RPs; second, to study how the RQA measures evolve with different embedding dimensions and sliding window lengths.

The exhibitions of RPs include the several exchange return sequences and the autocorrelation sequences with an initial sliding window length of 20. It intuitively distinguishes the similarity and difference between these two types of sequences.

To quantitatively analyze the characteristics of the RPs, we choose several RQA measures to analyze the recurrence characteristics with the recurrence threshold corresponding to the 1% recurrent rate. We investigate the influence of embedding dimensions on the RQA measures by changing the embedding dimension from 1 to 20. Moreover, we show how the RQA measures evolve by changing the sliding window lengths from 10 to 500. The comparative analysis of RQA measures with bootstrap techniques could also show the reliability of the exchange systems. Our work not only helps to understand the deterministic characteristics of exchange rates in the CNY market, but also benefits for further studies about chaotic prediction because the transformation from an original exchange rate sequence to its associated autocorrelation sequence is reversible, given the sliding window length and the original exchange rate sequence.

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