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To cite this article: Michael A. Riley & M. T. Turvey (2002) Variability and Determinism in Motor Behavior, Journal of Motor Behavior, 34:2, 99-125, DOI: [10.1080/00222890209601934](https://doi.org/10.1080/00222890209601934)

To link to this article: <https://doi.org/10.1080/00222890209601934>



Published online: 02 Apr 2010.



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Variability and Determinism in Motor Behavior

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ABSTRACT. In investigations into perception–action systems, variability of observable behavior may be considered to (a) interfere with inquiry, (b) be neither detrimental nor particularly useful to inquiry, or (c) play a crucial role in inquiry. The authors underscore recent suggestions that alternative (c) is a preferred strategy for the study of many motor behaviors. In tutorial fashion, the authors review the concepts of variability and determinism with respect to postural and rhythmic movements. Study of the variability of those behaviors has revealed crucial features suggestive of underlying mechanisms and control, such as particular blends of noise and determinism (piecewise determinism). It has also revealed general lessons (for example, more variable does not mean more random and more controllable does not mean more deterministic) that may extend to other classes of perceptual–motor behavior.

Key words: motor control, random processes, time series analysis, variability

For most students of movement, regardless of theoretical persuasion and preferred scale of analysis, observable behavior is the basis for inferring motor function and its neural and environmental support. Many factors complicate the task of drawing inferences from behavior. Prominent among those factors is the variability of motor behavior. How should that variability be addressed? Should it be ignored, should it be included ad hoc as random “correction” terms of a model, or should it be taken as integral to the inference?

Consider everyday patterns of coordinated movement such as standing and walking. Like most behaviors to which one can readily attach labels, those coordination patterns are simultaneously stable and variable. They are persistent in the face of perturbations, sustainable for relatively long periods, and reproducible with a high degree of accuracy. At the same time, they are variable—from moment to moment and instance to instance. The body during upright quiet (unper-

turbed) standing exhibits time variations in the spatial location of its approximate center of mass (e.g., Edwards, 1942), and contralateral limbs moving rhythmically at the same tempo exhibit time variations in relative phase (e.g., R. C. Schmidt, Beek, Treffner, & Turvey, 1991). Paraphrasing the earlier question: Are those behavioral fluctuations merely a bothersome overlay on the output of the mechanism that is responsible for the stable, reproducible character of the behavior, or are they an intrinsic, essential, and telling feature of that mechanism? With respect to such a question, Newell and Slifkin (1998, p. 157) have conjectured that “contrary to traditional wisdom, it may be that the variance of movement dynamics is as revealing as, or more revealing than, the invariance in terms of unpacking the nature of the system organization” (see also Newell & Corcos, 1993; Post, Daffertshofer, & Beek, 2000; Slifkin & Newell, 1999a).

The present article is a synthesis of recent findings made largely from within the dynamical systems perspective on perception–action (e.g., Kelso, 1995; Turvey, 1990). The findings are with respect to the two classes of behavior recognized earlier, namely, upright standing and rhythmic movement. Although the two classes constitute a minimal sampling of the human behavioral repertoire, their universality suggests that the lessons learned from them may have far-reaching applications. We should not expect those lessons to be straightforward, however, given the topic. Notions such as variable, random, predictable, deterministic, programmed, and controllable are unevenly interpreted and complexly intertwined. As will become apparent, the

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lessons to be learned will take the following form: More variable does not mean more random, and more controllable does not always mean more deterministic. Most particularly, the lessons will take the form of new questions, such as: Are there behavior-specific blends of deterministic and random processes? and Is it proper to maintain movement invariants and movement variability on unequal explanatory footings?

In the following sections, we discuss the issues of determinism, randomness (particularly, measuring the degree of randomness in behavior), and stochasticity, and how the concepts are instantiated in motor behavior. Then, we review empirical work (focusing on postural control, rhythmic movements, and quasi-rhythmic motor behavior) in which variability has been a primary focus. In those sections, we note how a focus on variability has had (or may have) an impact on models of motor behavior. After that, we discuss some additional theoretical constraints that logically precede the modeling of motor behavior.

Practical Definitions

To be clear on the nature of the aforementioned lessons, one needs definitions of the key notions of variability and randomness as they apply to motor behavior. Patently, it is essential for our purposes that the two notions be held distinct.

Variability

A survey of the contemporary motor control literature reveals a number of different measures that serve to define motor variability. Nonetheless, we think that for most investigators the simple statistical idea of variance about the mean (typically expressed as its square root, the standard deviation) expresses their intuitive understanding of the term variability and guides their application of the term when qualifying motor processes. Accordingly, in the present article, we identify motor variability with motor variance.

Randomness and Stochasticity

Defining the notion of randomness is more complicated. A (discrete) random variable is defined in relation to a *probability function*, which assigns a probability to each possible value or outcome of the variable (Doucet & Sloep, 1992). The probability function completely describes the behavior of the variable. For a random variable, each possible value (e.g., the outcome of the throw of a die) is equally likely to occur. For continuous random variables (on which we will focus henceforth), the probability function is replaced with the *probability density function*, which gives the probability of observing a value of the variable within a certain range of possible values (the probability of a particular value falling within a given area of the curve defined by the function). Random variables are appropriately assessed in terms of the properties of the probability density function, such as variance.

A concept related to randomness is the notion of stochasticity. A *stochastic process* is one that evolves over time and for which the evolution at each time step is governed, at

least in part, by probability.¹ One can use the term *stochastic* to refer to a behavior that is random (e.g., ordinary Brownian motion, described later) or to a behavior that is influenced, to some degree, by both deterministic and random processes.

Difficulties with the concepts of randomness and stochasticity are magnified when one wishes to characterize a behavior by its degree of randomness. Frequently, questions arise in motor behavior research, such as "How noisy is this time series?" and "How random is this behavior?" To answer those questions, one needs an appropriate metric of randomness. As Morrison (1991, p. 275) remarked, however, "Random is a qualitative or heuristic term that describes a broad category of behavior or properties. There is no one test or measure of randomness." Thus, strictly speaking, degree of randomness is something of a misnomer—a process is either random or it is not. Generally, when one speaks of degree of randomness, one is speaking of the degree to which an observed stochastic behavior is influenced by a random process.

One practical solution to defining randomness in a manner that satisfies the need for a convenient metric—a solution that applies in some, but not all, cases—is provided by the joint consideration of Brownian motion and the Hurst exponent. Ordinary Brownian motion is observed when each increment in a time series (each displacement in a given time interval) is independent of preceding and subsequent increments. As originally spelled out by Einstein (1905), the distances covered by a random (Brownian) particle undergoing random collisions from all sides are directly related to the square root of time. That is,

$$R = kT^{1/2}, \quad (1)$$

where R is the distance covered, k is some constant, and T is a time index.

Wiener (1923) clarified the random function for Brownian motion as follows (see Feder, 1988). Consider a normalized independent Gaussian random process $\{\xi\}$. Such a process would follow from adding the impulses from very many independent collisions with a Brownian particle. Most impulses would cancel each other out; some would occasionally add. The result would be a bell-shaped distribution of impulses, with a peak at or very close to zero. Now, let the increments in the position of the Brownian particle be given by

$$X(t) - X(t_0) \approx \xi |t - t_0|^H, \quad (2)$$

for any two times t and t_0 . For ordinary Brownian motion, $H = .5$. The preceding relation is a random function that applies at an instant t regardless of whether the earlier values of $X(t)$ (for $t \leq t_0$) are known. One obtains the position $X(t)$ given $X(t_0)$ by choosing a random number ξ from a Gaussian distribution, multiplying it by the magnitude of the time increment $|t - t_0|$, and summing the result with the given position $X(t_0)$.

On generalization, a time series characterized by Equa-

tion 2 is referred to as a *Wiener process*. A student of movement would know such a time series more commonly as *white noise*. The time series is white because all frequencies contribute equally to the dynamics (just as all wavelengths contribute equally to white light); that is, its energy or amplitude (A) is the same constant value at all frequencies, $A \approx f^0$. The time series is “noise” because a value of the time series at one point in time is not influenced by past or future values of the time series. Consider the Fourier series

$$X(t) = \sum_{n=1}^N A_n \cos(\omega_n t + \Phi_n), \quad (3)$$

where A_n is the amplitude of a periodic term of frequency ω_n that has a phase Φ_n at time $t = 0$. $X(t)$ would be white noise if the right-hand side of Equation 3 summed a large number of random variables with common statistical properties. One can produce the latter by defining (a) $A_n = N^{-1/2}$ for all n , (b) $\{\Phi_n\}$ as a set of phases uniformly distributed on the interval $[0, 2\pi]$, and (c) ω_n as the harmonics of a fundamental frequency ω_0 , that is, $\omega_n = n\omega_0$ (B. West & Deering, 1995).

White noise cannot occur in a pure form. It is a mathematical model, not a physical phenomenon (Morrison, 1991). Consider that in its pure form white noise is a constant frequency-independent energy signal. That is, it is a signal of infinite energy up to infinite frequencies. Furthermore, by the mathematics, white noise jumps instantaneously between amplitudes, residing at any specific amplitude, or in the vicinity of any specific amplitude, for 0 s. In reality, noise can be white (can be of equal amplitude) over only a limited number of frequencies. Such misgivings should not blind us to the fact, however, that pure white noise or, equivalently, ordinary Brownian motion, has proven invaluable in modeling physical and biological systems. In the present article, it will be taken as the definition of randomness.

An immediate advantage of equating randomness with ordinary Brownian motion is that the definition of degree of randomness follows from generalizing Brownian motion to time series that lack an underlying Gaussian distribution. That generalization originated with and was inspired by Hurst (see Feder, 1988; Mandelbrot, 1983). His rescaled range analysis involves the dimensionless quantity R/S , where R is the range of $X(t)$ and S is the standard deviation:

$$R/S = kT^H. \quad (4)$$

In Equation 4, T is a time index and k is some constant, as in Equation 1. H is the Hurst exponent. Roughly speaking, Equation 4 is a generalization of the T^{-5} law of Equation 1 to a T^H law. For many natural processes, $H \neq .5$, implying that increments are not fully independent, that is, that the correlation of past and future increments is not null for all T . Such a process is said to exhibit fractional Brownian motion (Mandelbrot & van Ness, 1968). An H exponent in Equations 4 and 2 that exceeds .5 indicates that increases and decreases in the past imply increases and decreases,

respectively, in the future. Conversely, an H exponent that is less than .5 indicates negatively correlated past and future increments. Correlations between increments may extend well into the past (Beran, 1994). There is a continuum of degrees of serial correlation that can be quantified by the H exponent, that is, a continuum of deviations from white noise or a pure Wiener process (see Figure 1). The degree of randomness, therefore, may be given by the difference between an observed H exponent and the H exponent defining ordinary Brownian motion ($H = .5$).

The H metric applies to many, but not all, behaviors. For instance, it is not valid for non-Markov processes. Thus, H holds no privileged status—it is a convenient, yet arbitrary, measure of degree of randomness. As indicated previously, there is no single measure of randomness. Other measures could be used, based on estimates of scaling exponents obtained from the power spectrum (see the following paragraph), on scaling exponents obtained through techniques such as detrended fluctuation analysis (Hausdorff, Peng, Ladin, Wei, & Goldberger, 1995), or based on recurrence quantification analysis (RQA; Webber & Zbilut, 1994, 1996; Zbilut & Webber, 1992), which is described in a later section. Any measure will be somewhat arbitrary, and one should take great care to ensure that a given measure is an appropriate one for the perceptual-motor behavior of interest. For our present purposes, discussion of H is useful in two regards. First, H does serve as an appropriate measure of degree of randomness for spontaneous (i.e., nonperturbed) postural sway, which we discuss in a later section. Second, H is relatively easily appreciated as a measure of degree of randomness, given the relation between Equations 2 and 4. Therefore, H serves as a useful example for gaining intuitions about the degree of randomness of an observed behavior.

With respect to the Fourier transform defined in Equation 3, the variations in H correspond to variations in the spectral density function—specifically, how A scales with f . For $H \neq .5$, the power spectra scale as f^α , where $0 < \alpha \leq 3$. The noise represented by those scaling relations is termed *colored* (Schroeder, 1991) as opposed to white because the amplitude varies with frequency, indicating that, as is the case for colored light, the frequencies do not contribute equally to the observed behavior. When $H > .5$, low frequency noise generates large A compared with that generated by high frequency noise (Feder, 1988). The issue of degree of randomness can be addressed, therefore, in an additional and related way through the value of α . The questions become: Is the noise colored? How different is α from 0?

Determinism

Obviously, to say that an observed time series of motor behavior is variable is not to say that it is random. Purely sinusoidal oscillation is a variable process (i.e., for a time series of a given sine wave, there exists variance about a mean), yet it is also a *deterministic* process—there is a law-

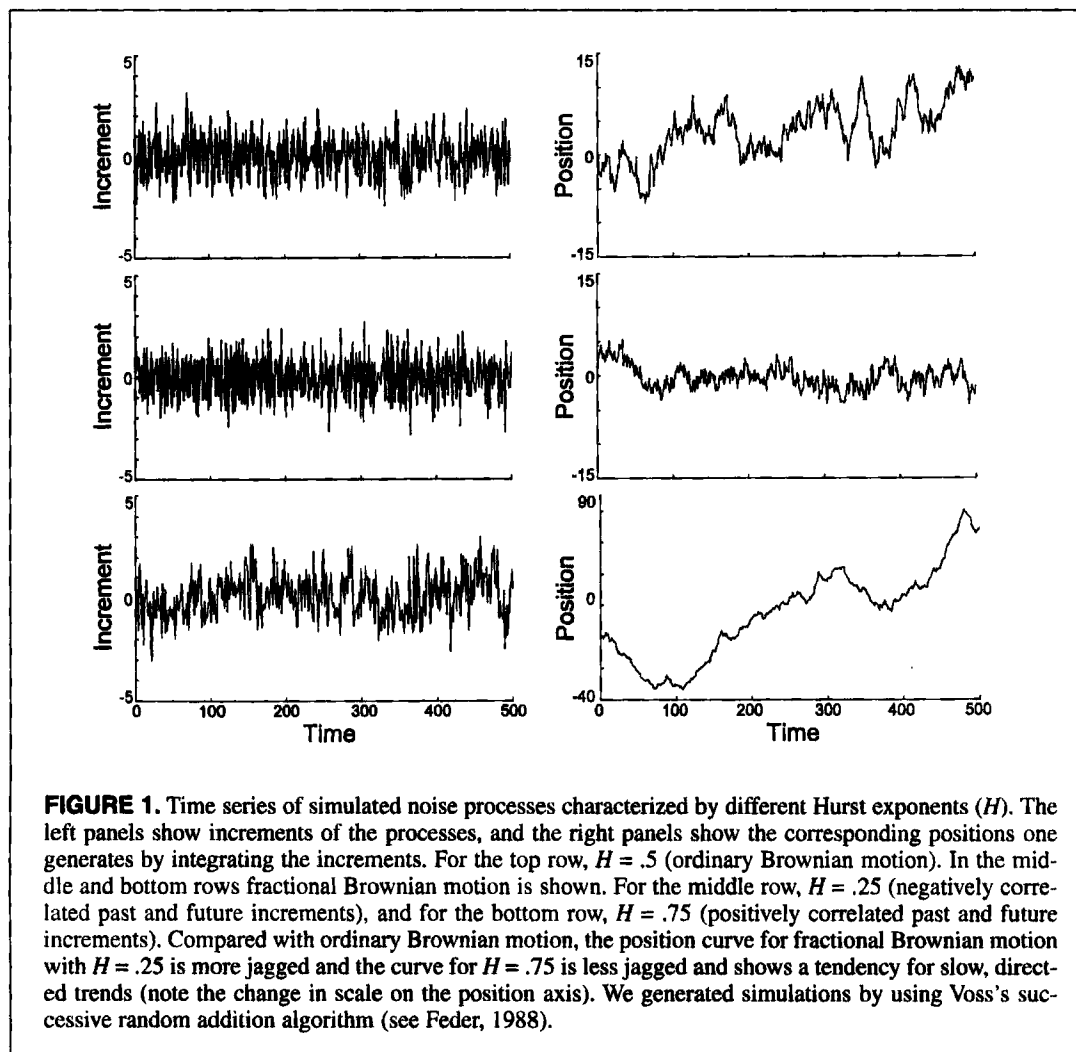


FIGURE 1. Time series of simulated noise processes characterized by different Hurst exponents (H). The left panels show increments of the processes, and the right panels show the corresponding positions one generates by integrating the increments. For the top row, $H = .5$ (ordinary Brownian motion). In the middle and bottom rows fractional Brownian motion is shown. For the middle row, $H = .25$ (negatively correlated past and future increments), and for the bottom row, $H = .75$ (positively correlated past and future increments). Compared with ordinary Brownian motion, the position curve for fractional Brownian motion with $H = .25$ is more jagged and the curve for $H = .75$ is less jagged and shows a tendency for slow, directed trends (note the change in scale on the position axis). We generated simulations by using Voss's successive random addition algorithm (see Feder, 1988).

ful generating mechanism, and prior states completely determine what future states will be.² For a purely random process, future states and prior states are independent of one another. Thus, given a sequence of data points from a random noise process, one would be unable to make precise predictions about future data points. Only gross features of a random noise process (e.g., mean values) are predictable. That characteristic differs in a deterministic process: Given knowledge of prior states of a deterministic process, future states are predictable with perfect precision. Randomness and determinism are opposite concepts.

Some stochastic processes, such as ordinary Brownian motion, are random. However, stochastic processes may involve deterministic elements. Whereas randomness and determinism are opposite concepts that are, in principle, easily demarcated, stochasticity and determinism are not. Rather, there is a continuum of stochastic processes, ranging from random to largely deterministic.

How Is Motor Variability Composed?

A deceptively simple equation allows us to collect the aforementioned deliberations and introduce what may well

be the key issue. From a commonly held point of view, a time series $X(t)$ of any given motor behavior, with mean value μ and standard deviation σ , can be understood as satisfying

$$X(t) = M(t) + N(t). \quad (5)$$

In that equation, $M(t)$ is the deterministic part (e.g., motor commands, motor program) and $N(t)$ is the random part—the signal and the noise, respectively. The usual interpretation of Equation 5 is that the more closely $N(t)$ approaches zero, the better is the fit of $X(t)$ to the intended or ideal movement, embodied by $M(t)$ (Newell & Corcos, 1993). But Equation 5 is actually open to many interpretations, reflecting, in part, variety in the composition of $N(t)$ and in the relation of $X(t)$ to $N(t)$. For example, a finding that $X(t)$ is correlated noise could be interpreted as evidence that (a) $N(t)$ is correlated noise but of a different kind (e.g., Chow & Collins, 1995; Peterka, 2000), (b) $N(t)$ is random noise appended to a particular form of $M(t)$ (Schöner, Haken, & Kelso, 1986), or (c) $N(t) \approx 0$ and $M(t)$ is a low-dimensional chaotic (e.g., Mitra, Riley, & Turvey, 1997; Yamada, 1995) or a piecewise-deterministic (Zak, Zbilut, & Meyers, 1997) process.

The aforementioned key issue is that the measure of degree of randomness in a motor behavior $X(t)$ is not, at the same time, a measure of the degree of randomness in the noise source $N(t)$ acting on the human motor control system. In brief, circumspection is required in drawing inferences about $N(t)$ from $X(t)$. Indeed, a possible implication of finding that $X(t)$ is correlated noise is that $M(t)$ is random noise. Within the familiar Wing-Kristofferson formulation for repetitive timing tasks (e.g., tapping), colored $X(t)$ is attributed to a (minimally) two-tiered random noise structure composed of a recurring central clock or time-keeper process that triggers a peripheral motor process (Pressing, 1999; Vorberg & Wing, 1996; Wing & Kristofferson, 1973). In terms of Equation 5, $M(t)$ and $N(t)$ would represent those random clock and motor processes, respectively.

How Could Both Determinism and Variability Inhere in $M(t)$?

The possibility of the previously mentioned alternative (c)—that the systematic and nonsystematic aspects of $X(t)$ are both caused by $M(t)$ —requires elaboration.

We have distinguished randomness and determinism on the basis that random processes are not predictable, whereas deterministic processes are. Certain types of processes, however, possess both random and deterministic characteristics, in that precise predictions of future states are difficult or impossible to achieve despite the fact that trajectories are governed by deterministic equations.

A *chaotic dynamical system* exhibits behavior that appears random but that, in fact, is generated from a deterministic process. For instance, the system of differential equations

$$\dot{x} = a(y - x), \quad (6)$$

$$\dot{y} = rx - xz - y, \quad (7)$$

$$\dot{z} = xy - bz, \quad (8)$$

known as the Lorenz equations, produced the chaotic behavior depicted in Figure 2. The figure shows a time series of the first variable, x (in the simulation, $a = 10$, $r = 28$, and $b = 8/3$). The variations in x appear random and are unpredictable in the long run, even though we generated the time series by using a purely deterministic set of equations (Equations 6–8).

If the initial conditions of a chaotic dynamical system, such as the Lorenz system, are known precisely, then precise predictions of the trajectories are possible. Such knowledge of initial conditions is typically not available, however. The defining characteristic of chaotic dynamics is that two trajectories with arbitrarily close initial conditions diverge from one another exponentially in time. In consequence, precise long-term predictions of chaotic dynamics are not possible in the absence of perfect knowledge of the initial conditions.

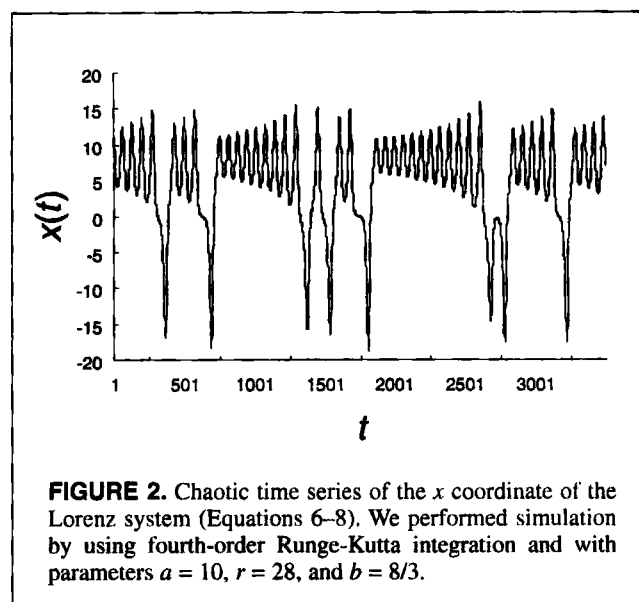


FIGURE 2. Chaotic time series of the x coordinate of the Lorenz system (Equations 6–8). We performed simulation by using fourth-order Runge-Kutta integration and with parameters $a = 10$, $r = 28$, and $b = 8/3$.

Dynamics that are piecewise deterministic are found at the intersection of random and deterministic processes (see Giuliani, Lo Giudice, et al., 1996; Giuliani & Manetti, 1996; Giuliani, Piccirillo, Marigliano, & Colosimo, 1998; Webber & Zbilut, 1996; Zak, 1993; Zak et al., 1997; Zbilut, Hübner, & Webber, 1996; Zbilut, Zak, & Meyers, 1996). Mathematically, piecewise determinism is captured by ordinary differential equations that violate the Lipschitz condition, which expresses the notion that all derivatives must be bounded (non-Lipschitz dynamics; Zak, 1993). When the Lipschitz conditions for ordinary differential equations are upheld, systems have unique solutions. The uniqueness criterion means that trajectories in the phase space are never allowed to intersect. Violation of the Lipschitz condition has two pathological (Strogatz, 1994) consequences. First, all trajectories reach singular solutions in a finite time period (rather than asymptotically). Second, because all trajectories reach the singular solution, solutions are not unique. When the Lipschitz restrictions are violated, trajectories intersect at the solution, thereby violating the uniqueness criterion.

Consider the following example, a generalization of Newton's second law, involving a mass subject to a force:

$$m\ddot{x} = \alpha \dot{x}^k \cos \omega t, \quad k = \frac{p}{p+2} < 1, \quad (9)$$

where x is the displacement of the mass, the overdots represent differentiation with respect to time, and p is an arbitrarily chosen integer (for a detailed treatment of the model, see Zak et al., 1997). That differential equation has a regular solution that allows two possible trajectories,

$$x = \pm \left[\frac{\alpha(1-k)}{m\omega} \sin \omega t \right]^{\frac{1}{1-k}}, \quad v \neq 0, \quad (10)$$

as well as a singular solution,

$$\dot{x} = 0 \quad (11)$$

(Zak et al., 1997). Within the interval $0 < t < \pi/2\omega$, the singular solution identified by Equation 11 is unstable (a repeller), and the system will follow one of the trajectories dictated by Equation 10, with a probability of .5 for each trajectory. Within the interval $\pi/2\omega < t < 3\pi/2\omega$, however, the system bifurcates—the singular point becomes stable and attracts all trajectories. Trajectories reach the singular point at $t = \pi/\omega$ and remain there until $t = 3\pi/2\omega$. The solution is not unique because all trajectories will intersect when they reach the solution. An infinitesimal amount of noise (or an infinitesimal perturbation from a deterministic source, such as a weakly coupled dynamical system) present during that interval will perturb the system away from the singular point. The attractor dynamics within $t = \pi/\omega$ and $t = 3\pi/2\omega$ are such, however, that they quickly dissipate that transient and pull trajectories back to the singular point so that the noise is not noticeable. The singular point during that regime possesses infinite stability. At $t = 3\pi/2\omega$, the system once again bifurcates, with the singular point returning to a repeller. During the latter regime, the presence of infinitesimal noise will perturb the system away from the singular solution point and the repeller dynamics will drive trajectories onto one of the regular solutions identified by Equation 10. The effects of noise during the regime beginning at $t = 3\pi/2\omega$ are noticeable, therefore, even though the noise's magnitude is infinitesimal, because the singular point is infinitely unstable.

The individual trajectories exhibited by that piecewise-deterministic dynamical system are governed purely by deterministic mechanisms (attractor and repeller dynamics). The particular trajectory that is observed, however, depends upon the infinitesimal destabilizing force. Different trajectories arise depending on the magnitude and direction of the perturbations. In consequence, the observed dynamics are unpredictable. Given the unpredictability of the magnitudes and the directions of the perturbations and the resulting trajectories, piecewise-deterministic dynamics are most appropriately conceived as stochastic processes.

In sum, returning to Equation 5, $X(t)$ could be variable in the absence of an independent $N(t)$ process in two distinct ways. The variability in $X(t)$ could arise from a deterministic chaotic $M(t)$ process or from a piecewise-deterministic $M(t)$ process. In both cases, there is structure (deterministic trajectories) that is variable. The variability in deterministic chaos arises from the instability that causes the divergence of trajectories with nearly identical initial conditions; the variability in piecewise determinism arises from the uniqueness of successive trajectories separated by an infinitesimally noisy singular point.

A First Summary and a Preview

In investigations of biological movement, variability has traditionally been considered to reflect random white noise of varying magnitude. Typically, noise, $N(t)$, is considered as superimposed on $M(t)$, an invariant deterministic signal

(Newell & Corcos, 1993). For instance, movement variability might be interpreted to be a result of white neuromotor noise (as a simplifying approximation) superimposed on the deterministic prescription of muscle activation provided by a motor program. Variability about the mean has frequently (though not always) been treated as random variation and is typically viewed as problematic, both to the theorist attempting to explain motor behavior and to the organism trying to produce goal-directed motor acts. Because movement variability is equated with randomness (noise), and because randomness and determinism are opposite concepts, movement variability and movement invariants (reflective of deterministic structure) are held as distinct and opposing facets of motor behavior.

Within that traditional perspective, a major challenge ascribed to the mechanisms implementing motor control is that of overcoming the undesirable effects of random noise (e.g., Meyer, Abrams, Kornblum, Wright, & Smith, 1988). How the challenge is met is presumably reflected in the invariants of movements (e.g., their mean states). Consequently, investigators focus upon the invariants, with the quantification of the amount of randomness a secondary concern, one that is usually addressed through gross measures such as standard deviation or root mean square variability. That perspective is reinforced by implicit assumptions of statistics as presented in the behavioral sciences—specifically, that moments of dependent measures exist, and, moreover, that the first and second moments (mean and variance, respectively) not only exist but are always meaningful descriptive quantities (Liebovitch & Todorov, 1996).

The emergence of nonlinear dynamics and fractals in mathematics and physics has led to an increasing appreciation of the fact that seemingly random behavior can emerge from nonrandom sources. There is also increasing recognition that what appears as white (unstructured, uncorrelated) noise in data collected from real, natural systems may possess structure in the form of correlation functions in the time and frequency domains—that is, the noise may be colored. The overall impact of those ideas within the biological sciences is being felt (see Basingthwaite, Liebovitch, & West, 1994; Goldberger, 1997; Liebovitch, 1998).

Proponents of the dynamical systems approach to perception-action have used those concepts to varying degrees to guide theory and research. Some advocates have noted the typical benefits that chaotic variability—but not white noise—would provide biological systems: namely, controllability stemming from a blend of flexibility and stability (Conrad, 1986; Kelso & Ding, 1993; Liebovitch, 1998; Mitra et al., 1997; Mpitsos & Soinila, 1993). Others have suggested that variability functions in facilitating transitions between behavioral modes (e.g., D. Collins, Park, & Turvey, 1998; Kelso, 1995) and that variability may have an exploratory, perceptual function (Riccio, 1993).

In the review of postural and rhythmic coordination research that follows, we endorse the arguments that func-

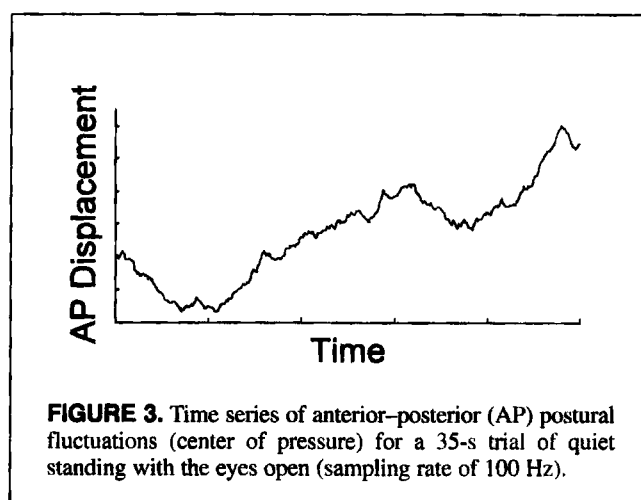
tional roles of movement variability exist and that variability is more constraining of theory than is usually assumed. Nontraditional analyses of variability are identified that appear capable of revealing structure (suggestive of underlying organizational principles and processes) undetected by more traditional measures of variability such as standard deviation (Liebovitch & Todorov, 1996; Post et al., 2000). Our goal in our review is two-fold: (a) to encourage researchers to use the nontraditional methods in their own data analyses to reveal properties of their data that might remain hidden if they use more traditional methods and (b) to encourage the development of models of motor control that explicitly incorporate the characteristics of motor behavior that the analyses identify.

Variability of Postural Motions

When standing quietly, in the absence of external perturbations, the body's center of gravity (COG) undergoes continuous fluctuations. Specifically, the intersection that the vertical vector from the center of mass makes with the horizontal plane varies from moment to moment in an apparently random fashion. In the laboratory, through the use of force plates, one measures fluctuations in the location of the vertical ground reaction vector, the so-called center of pressure (COP). Fluctuations of the COP are related to fluctuations of the COG but are by no means identical with them (Day, Steiger, Thompson, & Marsden, 1993). The COP is equal and opposite to a weighted average of all downward forces acting between the feet and the force plate, which arise from the muscular systems responsible for posture and balance (Winter, Prince, Frank, Powell, & Zabjek, 1996). The COP, therefore, is a collective variable in the literal sense that its value at any point in time reflects the activities of very many neuromuscular components acting in concert.

COP time series have complex, nonstationary, and extremely variable profiles (see Figure 3). There is face validity to an interpretation of COP fluctuations as stemming from white noise. Structure is revealed, however, by more careful inspection. Newell and Slifkin (1998) reviewed several methods by which the structure inherent in COP variability could be made evident (see also Newell, 1998). Those methods included estimates of dimensionality (Myklebust & Myklebust, 1989; Newell, van Emmerik, Lee, & Sprague, 1993; Yamada, 1995) and analyses and modeling in terms of stochastic processes such as fractional Brownian motion (stabilogram-diffusion analysis; J. Collins & De Luca, 1993, 1994, 1995; Riley, Balasubramaniam, Mitra, & Turvey, 1998; Riley, Mitra, Stoffregen, & Turvey, 1997; Riley, Wong, Mitra, & Turvey, 1997) and the Ornstein-Uhlenbeck process (Newell, Slobounov, Slobounova, & Molenaar, 1997).

Newell and Slifkin (1998) concluded that the irregularity of COP signals does not reflect white noise added to a deterministic signal. They also concluded that "[w]hat is less clear at this time is the veridical nature of the deterministic and stochastic profile of the [COP] dynamic" (p. 155). Although it is



now recognized that COP dynamics contain a blend of deterministic and random processes (Balasubramaniam, Riley, & Turvey, 2000; J. Collins & De Luca, 1994, 1995; Newell & Slifkin, 1998; Riley, Balasubramaniam, & Turvey, 1999), it is less clear how that blending is achieved. There is also contemporary recognition that a variety of factors related to the actor, the environment, perception, and suprapostural tasks shape or constrain COP dynamics (Riccio, 1993; Riccio & Stoffregen, 1988; Riley, Mitra, et al., 1997). Similarly opaque is the manner in which those constraints are played out in terms of deterministic and random processes in postural control. For instance, from the traditional perspective, vision has been taken to serve a noise-reducing feedback role—with vision available, the postural system is better able to eliminate random variation in body position by virtue of heightened perceptual sensitivity to postural noise. It seems, however, that vision plays a different role than that of a source of feedback corrections. Vision's influence seems to be packaged within posture and limb movements at a more fundamental level of their organization than a simple feedback-correction mechanism (Jeka, 1995; cf. Mitra, Riley, Schmidt, & Turvey, 1998). For instance, the data reviewed by Jeka (1995) suggested that postural sway is not passively driven by moving visual displays (e.g., "moving rooms" or computer graphics simulations thereof; Lee & Lishman, 1975; Schöner, 1991) but, rather, is actively adjusted, in an anticipatory fashion, so that it matches the time-varying optical structure.

Recurrence Quantification

Analyses of COP data provide clues about the blending of deterministic and random processes in postural control and about the influence of perceptual and task constraints on postural dynamics. Riley et al. (1999) used a technique termed *recurrence quantification analysis* on COP data. RQA is a multidimensional, nonlinear technique for uncovering subtle time correlations and repeating patterns in data that may be very irregular and nonstationary (Faure & Korn, 1998; Webber & Zbilut, 1994, 1996; Zbilut & Webber, 1992). Investigators originally introduced recurrence plots

to facilitate the detection of signals in complicated time series (Casdagli, 1997; Eckmann, Kamphorst, & Ruelle, 1987). The subsequent development of methods for quantifying recurrence has yielded an analytic tool that can distinguish signal from noise in hard cases such as the chaotic behavior of the logistic map limited to 1,000 points (Zbilut, Giuliani, & Webber, 1998).

RQA quantifies, among other factors, the degree of deterministic structure (%DET). %DET is computed as the percentage of recurrent data points (i.e., points that repeat [recur] in multidimensional, reconstructed phase space) that are part of a string of adjacent recurrent points. As we noted previously, determinism means, essentially, that a system's future states are governed (i.e., determined) by its previous and present states. Imagine we are given a sequence of data points x_1, x_2, x_3, x_4 , and x_5 from a deterministic system (the numerical subscripts denote discrete time steps). Because the system is purely deterministic, and because we know with perfect precision (in this contrived example) the system's previous states, we can then accurately predict the system's state at future time steps. We know what x_6 will be, because the system's state at time $t = 6$ is determined by the system's states at $t = 1, 2, \dots, 5$. One can therefore index determinism in observed data by isolating sequences of data points that, given previous values in the sequence, determine future values in the sequence. %DET is computed as the percentage of data points that recur as part of such a sequence of points (rather than as isolated points—data points that recur alone reflect data points that repeat themselves by chance). In Figure 4, a recurrence plot, both isolated recurrent points as well as deterministic strings of recurrent points (short line segments parallel to the main diagonal line of identity) can be seen.

%DET, like the Hurst exponent, H , can serve as a metric for quantifying the degree of randomness of an observed perceptual-motor process. It is important to note that, just as was the case with H , %DET is not a privileged metric, and it may not be appropriate for all behaviors. %DET does have the advantage over H that it can be applied to situations for which calculations of H fail—such as when postural sway is driven by a sinusoidal visual display (e.g., Dijkstra, Schöner, & Gielen, 1994). %DET is also an intuitively simple metric—its interpretation is similar, in some regards, to the common understanding of a signal-to-noise ratio. However, when applied to real-world data, it is not an absolute metric—an observed %DET = 90 does not indicate a behavior that may be simply interpreted as 90% deterministic and 10% random. The parameters derived by RQA are best used as relative descriptors of observed behaviors.

Using RQA, Riley et al. (1999) and Balasubramaniam et al. (2000) observed $0 < \%DET < 100$ for COP data (observed values of %DET were statistically different from 0 and 100). That observation indicates that deterministic structure was present in the COP data (%DET was statistically greater than 0) but that the data (specifically, recurrences) were not purely deterministic (%DET was statisti-

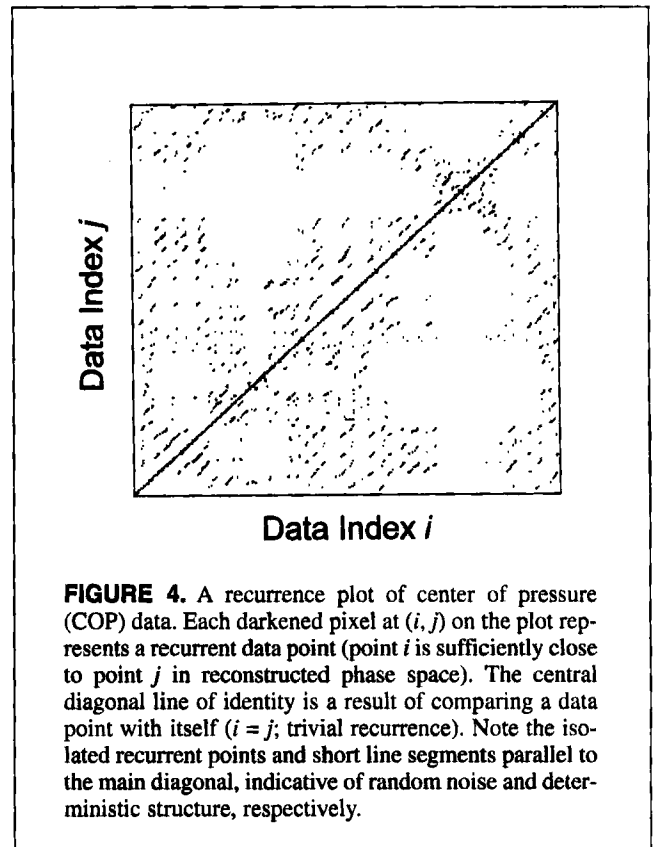


FIGURE 4. A recurrence plot of center of pressure (COP) data. Each darkened pixel at (i, j) on the plot represents a recurrent data point (point i is sufficiently close to point j in reconstructed phase space). The central diagonal line of identity is a result of comparing a data point with itself ($i = j$; trivial recurrence). Note the isolated recurrent points and short line segments parallel to the main diagonal, indicative of random noise and deterministic structure, respectively.

cally less than 100). There was a blend of determinism and noise. The aforementioned observation, therefore, confirms the basic result obtained from stabilogram-diffusion analysis: Time correlations (i.e., structure) in COP signals distinguish the signals from true random walk or Brownian motion; that is, postural fluctuations exhibit fractional Brownian motion (J. Collins & De Luca, 1993, 1994; Riley et al., 1998; Riley, Mitra, et al., 1997; Riley, Wong, et al., 1997). In terms of the issue of degree of randomness, the observation that $H \neq .5$ for postural fluctuations indicates that postural sway variability is not random. Stabilogram-diffusion analysis and RQA both reveal structure in COP signals and provide converging evidence that COP variability is not caused solely (if at all) by white noise. In terms of Equation 5, some of the variability in $X(t)$ originates in $M(t)$.

Perceptual Influences on Postural Dynamics

Riley et al. (1999) also examined the effects of vision on COP dynamics. The major result was that %DET was greater for eyes-closed than for eyes-open conditions. That result is somewhat paradoxical. A well-known fact (observed in the data in question) is that when people have their eyes closed, postural variability increases (e.g., Edwards, 1946; Paulus, Straube, & Brandt, 1984; Travis, 1945; Wapner & Witkin, 1950). The results of Riley et al. (1999) revealed that the increase in the amount of variability was accompanied by an increase in the degree of deterministic structure. That is, more variability with eyes closed was accompanied by less randomness (more determinism),

and less variability with eyes open was accompanied by more randomness (less determinism). That finding suggests that variability cannot be simply equated with randomness.

Those results also essentially replicate a result obtained numerous times under stabilogram-diffusion analysis: COP activity over both short and long time scales is less correlated (i.e., closer to random) with the eyes open than with the eyes closed (J. Collins & De Luca, 1995; Riley et al., 1998; Riley, Mitra, et al., 1997; Riley, Wong, et al., 1997). Riley, Wong, and their colleagues also obtained that result when they examined the perceptual role of light touch of an environmental surface (Jeka & Lackner, 1994). When participants lightly (nonsupportively) touched a surface, their COP variability decreased, suggesting that light touch may play a role similar to that of vision in the perceptual regulation of posture. They also found that COP correlations over both time scales were weaker (closer to random behavior) when participants touched an adjacent surface than when they did not touch the surface.

In the traditional view, postural control is identified as the task of overcoming postural noise (and large-amplitude perturbations) in order to maintain the COG as precisely centered above the base of support as possible. Postural noise is presumably unstructured (white) noise stemming from randomly variable force output from the involved skeletal muscles (De Luca, Le Feve, McCue, & Xenakis, 1982). The motor control system achieves perceptual regulation of the swaying body (e.g., by vision or touch) by either reducing the magnitude of postural noise or by countering the effects of random noise perturbations through activity of the postural musculature. If perceptual systems that are highly sensitive to postural fluctuations are used to regulate posture, then the noisy fluctuations may be more easily controlled or reduced, and the COG may be more accurately positioned so that biomechanical stability can be maximized.

The traditional view is subject to reexamination. First, the assumption that postural control is an automatic or reflexive process of maximizing biomechanical stability by maintaining the COG above the base of support is challenged by the idea that, for a broad variety of everyday tasks, maximal biomechanical stability is not optimal (Riccio, 1993; Riccio & Stoffregen, 1988; Riley, Mitra, et al., 1997). Later, we discuss the issue of suprapostural tasks (tasks people perform when standing that are above and beyond control of the COG) in the context of variability and determinism of postural motions. Second, the results just reviewed raise questions about the hypothesis that the role of perceptual information is that of reducing postural noise (random postural fluctuations). Although postural motions that occur during eyes-open and light-touch conditions are less variable than those performed when the eyes are closed or when a surface is not touched, they are also more random. Phrased differently, the more variable postural motions seen when visual regulation of postural motions is not allowed are at the same time more deterministic than those seen when vision is allowed. The degree of variability

and degree of randomness of COP motions are not directly related.

Suprapostural Task Influences on Postural Dynamics

In an examination of the role of task constraints on standing, Balasubramaniam et al. (2000) applied RQA to COP signals obtained from standing participants who performed a suprapostural aiming task. The task was to keep the projection of a laser pointer within the bounds of a small target. Participants stood facing the target, with the laser pointer held in the hand at hip level at the side of the body. The investigators manipulated task difficulty by placing the targets at different distances from the actor (for a given amount of laser pointer motion, the movement of the projected image is greater the more distant is the target surface).

The suprapostural aiming task placed strict demands on postural sway, but those demands depended upon the sway axis, either anterior-posterior (AP, front-back) sway or mediolateral (ML, side-to-side) sway. The task required reduced ML sway. Excessive ML sway would make keeping the laser pointer image on the target more difficult, especially when the target was placed farther away. In contrast, AP sway was of little consequence for the task. It simply made the projected image vary slightly in size. Thus, Balasubramaniam and colleagues expected that if the aiming task were to be completed successfully, ML sway variability would decrease as the task became more difficult. That was the observed result. It was expected that as the task was made more difficult, AP sway variability would perhaps decrease (i.e., postural sway would be reduced in general, without regard to sway axis) or remain constant (because AP sway did not substantially affect task performance, it would not be modulated). Instead, the investigators observed that AP sway variability increased as task difficulty increased.

RQA of those data revealed an inverse relation between variability and randomness, as had been observed in the study of visual influences on posture (Riley et al., 1999). For ML sway, %DET decreased with increasing task difficulty, and for AP sway, %DET increased with increasing task difficulty. Thus, ML sway became less variable but less deterministic (more random) as task difficulty increased, whereas AP sway became more variable, yet more deterministic. If increasing the demands of suprapostural aiming required increased precision of postural control (i.e., minimizing ML postural motions), then the paradoxical result was that increased precision in control was accompanied by more random COP fluctuations. Just as variability cannot be equated with randomness, controllability cannot be simply equated with determinism.

In a second experiment, Balasubramaniam et al. (2000) reversed the task's directional demands by having participants stand, with the body facing the side and with the head and laser pointer turned to face the target. When participants stood in that orientation with respect to the target, the task now required reduced AP sway, especially as difficulty

increased, and ML sway was now of little consequence. The pattern of variability and the %DET results obtained in the first experiment were still observed, but AP and ML sway results were reversed. Changing the directional demands of the task produced functional changes in sway in the two directions.

Three conclusions might be reached. First, the tasks that people perform while standing influence postural control both in the degree of variability and at the level of underlying deterministic structure of observed postural motions. The goals of suprapostural tasks constrain the organization of postural control. Second, one must again conclude that the degree of variability observed cannot be simply equated with the degree of randomness in the behavior itself and in the organization of the control underlying the behavior. Third, more random does not necessarily mean less controlled.

Summary

Analyses of postural variability have revealed that postural motions contain a blending of determinism and noise. Environmental (perceptual) and task constraints shape the organization of postural dynamics; that is, the nature of the structure of postural fluctuations depends upon factors such as the perceptual systems that are available and the particular tasks people perform when standing. The structure of postural fluctuations and the nature of postural dynamics might shed considerable light on how the control of upright posture—an elementary and overtly simple (but covertly complex) action—is achieved.

Analyses of postural fluctuations have constrained the formulation of new models of postural control. Chow and Collins (1995) proposed a pinned-polymer model of postural control. In their model formulation, they explicitly focused on the structure of postural variability as revealed by stabilogram-diffusion analysis (e.g., J. Collins & De Luca, 1993). In the pinned-polymer model, the upright body is treated as an elastically pinned polymer, under tension and subject to friction, that is driven by noise. Peterka (2000) developed an alternative model of postural control, also anchored in the results of stabilogram-diffusion analysis. Peterka's model is an inverted pendulum driven by a proportional, integral, and derivative (PID) neural controller and subject to random disturbance torques. The PID controller involves the generation of three types of corrective torques, each with a certain weight: (a) one torque in proportion to an error signal—the deviation of the body from the upright position (proportional term); (b) one torque proportional to the integral of the error signal (integral term); and (c) one torque proportional to the derivative of the error signal (derivative term). The Chow and Collins model and the Peterka model of postural control exemplify the strategy that we endorse in our present review: A focus on data variability and alternative analysis methods can constrain the development of new models of perceptual-motor behavior. In our review, we suggest continuations of those new directions in the development of postural control models.

For instance, one can use RQA to evaluate existing models of postural control—they must be able to produce the empirically observed patterns of %DET and the other RQA variables (see Riley et al., 1999). Beyond that advantage, RQA is informative with respect to the formulation of new models of postural control in that it suggests an integration of deterministic and random factors.

Variability of Coordinated Rhythmic Movement

Bernstein (1967) noted that when people perform a repetitive or cyclical movement the individual movement cycles are not exact duplicates of one another. There is always cycle-to-cycle variability in the rhythmic movements people produce, for example, when walking, drumming, hammering, or clapping. The apparent noisiness of motor (and other biological) rhythms is one of their most prominent characteristics. With respect to that apparent noisiness, we might ask the same question that we asked with respect to postural fluctuations: Does analysis of the variability suggest that it is additive white noise, or does the variability possess structure? If the variability possesses structure, then one must perform further investigation to identify the source of the noise, its nature, and the deterministic properties of the system of interest.

Degrees of Freedom and Variability

Kay (1988) addressed the organization of the control of coordinated rhythmic movements in the context of Bernstein's (1967) degrees of freedom (*df*) problem (see also Turvey, 1990). That problem, generally stated, is how relative simplicity in behavior emerges from the considerable complexity of the perception-action system that produces the behavior. Consider the task of producing a simple back-and-forth rhythm with the leg. A complete description of that behavior at a microscopic level entails measurement of very many interneurons, neurons, and motor units—a vast number of anatomical and physiological *df* support the behavior. At a more macroscopic scale, however, we need very few *df* to describe the leg (e.g., position, velocity).

In the dynamical systems perspective, the *df* problem is addressed from the following framework (see, e.g., Kay, 1988; Kugler & Turvey, 1987; Turvey, 1990). Microscopic *df* are constrained in the formation of a coordinative structure, defined as a group of muscles that might span several joints and that are constrained to act as a single functional unit (Turvey, 1977; Turvey, Shaw, & Mace, 1978). The *df* of a coordinative structure, by virtue of the constraints that operate in the assembly of the coordinative structure, are fewer than that of the microcomponents (constraints "freeze up," or reduce, *df*). A coordinative structure is assumed to operate as a dynamical system (Kugler, Kelso, & Turvey, 1980; Mitra, Amazeen, & Turvey, 1998). Because of the dissipative nature of stable dynamical systems, the behavior that results from the activity of a coordinative structure has fewer *df* than the coordinative structure itself (see Kay, 1986, 1988). Compression of *df* from the configuration

space to the solution manifold is a generic feature of dynamical systems (see Gershensfeld & Weigend, 1994). The *configuration space* or *phase space* is the space in which the system equations “live.” The *solution manifold* is where the solutions of the system equations “live.” The dimensionality of the latter subspace is the number of first-order differential equations one needs to fully describe the motions of the system—that is, the number of active degrees of freedom, or *adf*.

Understanding the organization of the control of movements, from that perspective, is tantamount to understanding (a) the formation of coordinative structures and (b) the constraints that operate on a coordinative structure to further reduce *df* in behavior (Kay, 1988; Post et al., 2000). That understanding requires one to specify the number of *df* of the microcomponents, the coordinative structure, and the behavior. It is here that coming to terms with variability will have important implications for understanding the nature of the organization of the movement system. The description of variability requires additional *df*. For idealized, perfectly rhythmic, nonvariable data, the minimal realistic model would be that of a limit-cycle oscillator. A limit-cycle oscillator (such as the hybrid model; Kay, Kelso, Saltzman, & Schöner, 1987) is expressed mathematically as a second-order differential equation and may alternatively be written as a set of two coupled, first-order, autonomous, ordinary differential equations. Because one needs two first-order equations to describe a limit-cycle oscillator, that system is said to have two *df*. Thus, two *adf* completely describe the motions of the idealized, perfectly regular motor rhythms.

As mentioned previously, however, actual data obtained from real rhythmic movements possess considerable cycle-to-cycle—as well as within-cycle—variability (see Figure 5). Realistic, variable motions are no longer fully captured by a system with two *adf* (e.g., Figure 5C). To account for movement variability, one must extrapolate beyond the dynamics of limit-cycle oscillators. Typically, the variability has been interpreted as local (i.e., fast time scale, small amplitude, or both) random physiological noise and is usually modeled as white noise added to global, deterministic, limit-cycle dynamics (e.g., Kay, 1988; Kelso, 1995). Here, as with postural fluctuations, the trend has been to attribute variability to additive randomness. Again, however, there are data that contradict that assumption.

Dynamics of Uncoupled Rhythmic Movements

The organization of the observed behavior of a dynamical system is represented by certain characteristics of the attractor underlying the behavior. An *attractor* is a closed set of points in phase space to which trajectories asymptotically converge (to which they are attracted). To learn about the dynamical organization of an observed behavior, one must uncover the characteristics of the underlying attractor. To do that directly would require that all of the active dynamical variables underlying the behavior are known and can be measured. For real data, that is very rarely ever the case (and if

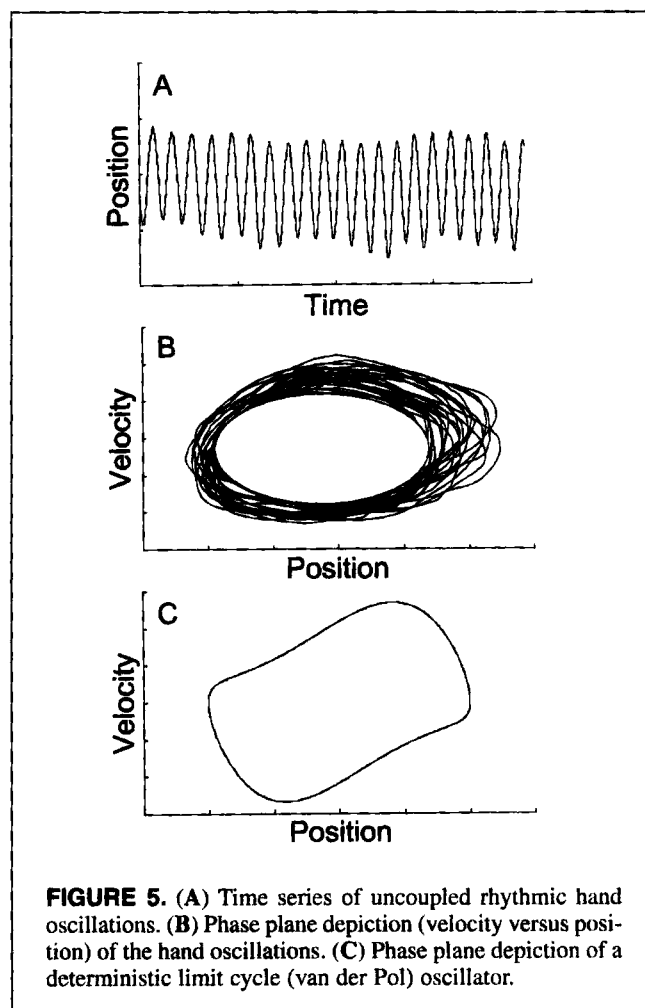


FIGURE 5. (A) Time series of uncoupled rhythmic hand oscillations. (B) Phase plane depiction (velocity versus position) of the hand oscillations. (C) Phase plane depiction of a deterministic limit cycle (van der Pol) oscillator.

those requirements were met, many of the important questions about the behavior would already be answered). However, in a mathematical theorem termed the *embedding theorem* (Takens, 1981), it is shown that for a nonlinear dynamical system, one needs to measure only one variable to gain information about the underlying dynamics. Using the measured signal and time-delayed copies of the signal, one can reconstruct the phase space of the dynamical system (for reviews of phase space or attractor reconstruction, see Abarbanel, 1996; Gershensfeld & Weigend, 1994; Sauer, Yorke, & Casdagli, 1991). Phase space reconstruction forms the basis of a variety of nonlinear time series analyses, including RQA (discussed earlier), various estimates of dimensionality, and computing the number of *adf* of a measured time series.

Kay (1986, 1988) first applied dimensionality measures to human rhythmic movement data (see also Ganz, Ehrenstein, & Cavonius, 1996; Kelso, 1995). Kay's efforts were directed toward an estimate of dimensionality of movement attractors termed the *correlation dimension* (Grassberger & Procaccia, 1983). Essentially, Kay (1986) attempted to determine if oscillations of the finger (uncoupled rhythmic movements) reflect the dynamics of a limit-cycle oscillator or of a higher dimensional system (e.g., a chaotic oscillator). The correlation dimension of an ideal (noise-free) limit-cycle oscillator

is 1.0. Kay computed the correlation dimension of rhythmic finger movements and observed an average correlation dimension of 1.17. Noninteger values for the correlation dimension indicate fractal dimensionality, a characteristic of chaotic dynamics. However, correlation dimension estimates from experimental data require large amounts of data for accuracy and are biased by even small amounts of measurement noise. Additionally, to obtain accurate results, one must obtain the data from a stationary dynamic. That is a difficult criterion to satisfy for biological (and especially cognitive and perceptual-motor) data. Because of those difficulties, Kay conservatively interpreted his result as suggesting that the variable finger oscillations he observed resulted from two processes, a fast time-scale noise process and global limit-cycle dynamics. The limit-cycle dynamics are responsible for the basic form of the motion (i.e., movement invariants), and the superimposed noise is responsible for the variability. Kay's interpretation is consistent with the standard interpretation of Equation 5— $M(t)$ is deterministic (an attractor), and $N(t)$ is random noise.

Correlation dimension estimates, as noted, are very difficult to compute accurately. Moreover, it has been argued that although the correlation dimension is perhaps a useful geometric characterization of an attractor, a more useful quantity to obtain from data is the dimension of the solution manifold. The latter, it will be recalled, is the number of active dynamical variables (i.e., *adf*) underlying an observed time series (Abarbanel, 1996). That number is the number of first-order, autonomous, ordinary differential equations required to describe the time series. Methods for computing *adf* from data have been established (local false nearest neighbors analysis; see Abarbanel, 1996; Abarbanel & Kennel, 1993). The methods for computing *adf* are purportedly more robust against measurement noise contamination than are those used to estimate correlation dimension.

Mitra et al. (1997) computed *adf* for rhythmic hand movements. Mitra and colleagues asked essentially the same question that was asked by Kay (1986, 1988). Do oscillations of the hand reflect the dynamics of limit-cycle oscillators (two-*adf* systems) plus noise or of low-dimensional dynamical systems with more than two *adf*? In Mitra and his colleagues' experiment, participants held a pendular object with a shaft of one of two lengths. For a given trial, the length was either 36 or 66 cm (see Kugler & Turvey, 1987). Participants oscillated the hand-held pendulum, or "virtual limb," about the wrist at a comfortable tempo in the parasagittal plane (i.e., radial-ulnar flexion-extension about the wrist). Mitra et al. (1997) found more than two *adf*, suggesting that the motions were not describable as limit cycles. The number of *adf* depended upon the size of the virtual limb: For the small virtual limb, they found four *adf*, and for the large virtual limb, they found three *adf*. Most interesting, application of the same analysis to neurons in a well-studied central rhythm generator has yielded the same low-dimensional dynamics with three to four *adf* (e.g., Varona, Torres, Abarbanel, Rubinovich, & Elson, 2001).

Mitra et al. (1997) analyzed the stability of the trajectories in reconstructed phase space by estimating average local Lyapunov exponents (Abarbanel, 1996; Abarbanel, Brown, & Kennel, 1992) from the data. Lyapunov exponents index the behavior of the system in response to a small perturbation or to a change in initial conditions. Positive Lyapunov exponents indicate that trajectories diverge in such situations (i.e., chaotic dynamics), whereas negative Lyapunov exponents indicate that trajectories converge (i.e., dissipative dynamics). For every trial, and for both large and small virtual limbs, Mitra and colleagues observed a positive Lyapunov exponent, suggesting that the variability of rhythmic movements might stem from deterministic chaos rather than from noise. Although positive Lyapunov exponents were not observed in surrogate time series produced by shuffling the original time series, it is prudent to be circumspect given the difficulties of computing Lyapunov exponents from finite data contaminated by measurement noise (Theiler, Eubank, Longtin, Galdrikian, & Farmer, 1992). Nonetheless, Mitra and colleagues' results raise the possibility that the variability of rhythmic movements is not predominantly caused by random neuromotor noise but is of deterministic origin. That conclusion is echoed in research on the irregular spiking-bursting activity of neurons in central rhythm generators (e.g., Falcke et al., 2000). Chaotic variability would be an advantageous characteristic of rhythmic movement (Goodman, Riley, Mitra, & Turvey, 2000; Mitra et al., 1997) because it would simultaneously allow for behavioral flexibility (via extensive exploration of the system's phase space) and stability (trajectories are contained within a bounded region of phase space).

The possibility of chaotic variability has broad implications for theories of motor behavior. One way for motor behavior theorists to come to terms with the variability issue is by first searching for deterministic mechanisms that produce simple, predictable motor behavior (e.g., a motor program, or an underlying periodic attractor). The next step would be to identify sources of noise that could alter the deterministic mechanism's output so that variable motor behavior will occur. That approach—which is commonly used in theories of motor behavior—requires the assumption of two separate factors that contribute to an observed motor behavior (see Equation 5): deterministic structure, $M(t)$, and noise, $N(t)$. If motor variability is of deterministic origin, then the assumption of both factors is not needed. Deterministic models that generate realistically variable behavior (in the time series and phase plane) could be formulated.

For example, on the basis of observations by Beek, Turvey, and Schmidt (1992; see also Beek, 1989; Beek, & Beek, 1988; Beek, Schmidt, Morris, Sim, & Turvey, 1996), a model of uncoupled rhythmic movement with greater than two *adf* could be formulated. Beek and his colleagues found evidence that elastic and friction terms vary within a movement cycle, rather than being fixed parameters (see also Delignières, Nourrit, Deschamps, Lauriot, & Caillou, 1999; Mottet & Bootsma, 1999). Thus, instead of representing the elastic and friction terms as fixed parameters (e.g., instead of represent-

ing friction as a coefficient of velocity), a more general model might include elastic and friction terms that are dynamic variables (e.g., that are *adf*). A limit-cycle (two *adf*) model of oscillatory movement, which takes the form of

$$\ddot{x} = f(x, \dot{x}) \quad (12)$$

(the two *adf* are position, x , and velocity, \dot{x}), could be elaborated in the form of

$$\ddot{x} = f(x, \dot{x}, \theta), \quad (13)$$

where now the system's motion depends additionally on θ , the oscillator's phase angle. A dependence on θ could be caused by a phase-dependent energy injection that offsets energy losses resulting from friction (Beek et al., 1992). A system described by Equation 13 has three *adf* (x , \dot{x} , and θ), which meets the minimum required number of *adf* for generating chaotic variability. The possibility of phase-dependent escapement devices has not been explored since the initial work of Beek and his colleagues. It may be a fruitful avenue for developing new models of rhythmic movements that can account for variability caused by $M(t)$.

Another avenue for developing deterministic models that yield variable rhythmic movements was presented by Hatsopoulos (1996). He presented a model of the interaction between neural and physical (limb) limit-cycle dynamics, consisting of a van der Pol oscillator (representing neural pattern-generator dynamics) coupled with a hybrid spring-pendulum (representing the physical dynamics of a limb). The Hatsopoulos model has four *adf* and, under certain parameterizations, yielded simulated limb movement time series with variable profiles (see his Figure 4A).

The possibility that the variability of rhythmic movements is predominantly the result of deterministic instability (i.e., chaos) does not imply that noise in human rhythmic movements is entirely absent. In the analyses used by Mitra et al. (1997), assessment of the presence of noise in the hand oscillations was also allowed. At the scale of the attractor, high-dimensional noise was not prominent (for all but a few trials, no attractor-level noise was observed). At local scales of a fraction of the attractor size, though, noise was present, with the amount of noise dependent upon the size of the virtual limb (there was more noise for the small virtual limb).

Goodman et al. (2000) extended the investigations of Mitra et al. (1997) to manual oscillations at a hand-held pendulum's resonant frequency and at a higher and a lower frequency. The resonant frequency required three *adf* in comparison with the four *adf* required at each of the non-resonant frequencies. The resonant behavior also had the lowest residual high-dimensional noise. Whereas only 12% of the vectors in the phase spaces of the dynamics were unpredictable at resonance, 33% and 20%, respectively, were unpredictable at the frequencies higher and lower than resonance. Goodman and colleagues also found that the predictability of the evolving dynamics extended further into the future for oscillations at the resonant frequency. At

resonance, the prediction horizon was 5 times greater than the prediction horizon for the higher-than-resonance behavior and 2.5 times greater than that for the lower-than-resonance behavior. Goodman and colleagues interpreted the results as suggesting that attunement of the movement system to resonant frequencies minimizes the variables to be controlled and maximizes the predictability of a rhythmic movement's unstable and variable dynamics.

The prediction horizon measure used by Goodman et al. (2000) may be a useful measure of $N(t)$ in rhythmic movements. Prediction horizon is a functional measure of variability: Rather than quantifying the amount of variability or classifying it as deterministic or random, prediction horizon quantifies how predictable are the ongoing movement trajectories. Predictability is an important characteristic of movement because greater predictability of movements may reflect a greater ability to prospectively adjust movements to changing environmental conditions. Perceptual sensitivity to the predictability of movements would provide the movement system with a compact, abstract description of movement trajectories (including future states). That description would be beneficial because it could provide a basis for updating or adjusting a limb's motion. The further into the future the description extends, the easier, presumably, would be the anticipatory, adaptive control of the limb.

The number of *adf* and the amount of noise of a rhythmic movement also depend upon the degree to which the coordination task is practiced. Mitra et al. (1998) observed a decrease in *adf* and in attractor-level and local noises of coupled, bimanual rhythmic hand movements over the course of learning a 90° phase relation between the two oscillating hands. As their participants became increasingly fluent at the bimanual coordination task, the oscillations of the component oscillators (the left and right hands) became (a) governed by fewer *adf* and (b) increasingly less noisy. The decrease in local noise was observed even after changes in *adf* were no longer apparent. Thus, skill learning may involve processes of forming a coordinative structure (i.e., distilling a dynamical system of relatively fixed composition in terms of number of *adf*) and then building more determinism into movements by reducing the noise output of the coordinative structure.

Piecewise-Deterministic Rhythms

There is qualitative (graphical) and quantitative evidence for piecewise-deterministic dynamics in human rhythmic movements. Graphically, piecewise determinism is indicated by the presence of flattened "kink zones" (Zbilut et al., 1996) in a two-dimensional time-delay embedding plot (e.g., a plot of a measured trajectory versus a time-lagged copy of that trajectory; such a plot is related to the phase plane). In some instances of rhythmic movements produced by humans (e.g., rhythmic forearm pronation-supination, or rhythmic flexion-extension of the hand), there are similar flattened or compressed regions in the phase plane at oscillation extrema.

Those regions (see Figure 6) have previously been referred to as *anchor points*, and they have been hypothesized to play a role in the perceptual regulation of coordinated movements (Beek, 1989; Byblow, Carson, & Goodman, 1994; Byblow & Chua, 1998; Carson, Byblow, & Goodman, 1994; Chua, 1995; Fink, Foo, Kelso, & Jirsa, 2000; Goodman et al., 2000; Kay, Saltzman, & Kelso, 1991; Zak et al., 1997). If anchor points are the same as kink zones, then the flattening in the phase plane may represent the trajectories' convergence on and divergence from a fixed (singular) point solution in the phase plane. That is, the anchor points should contain singular points in the trajectory.

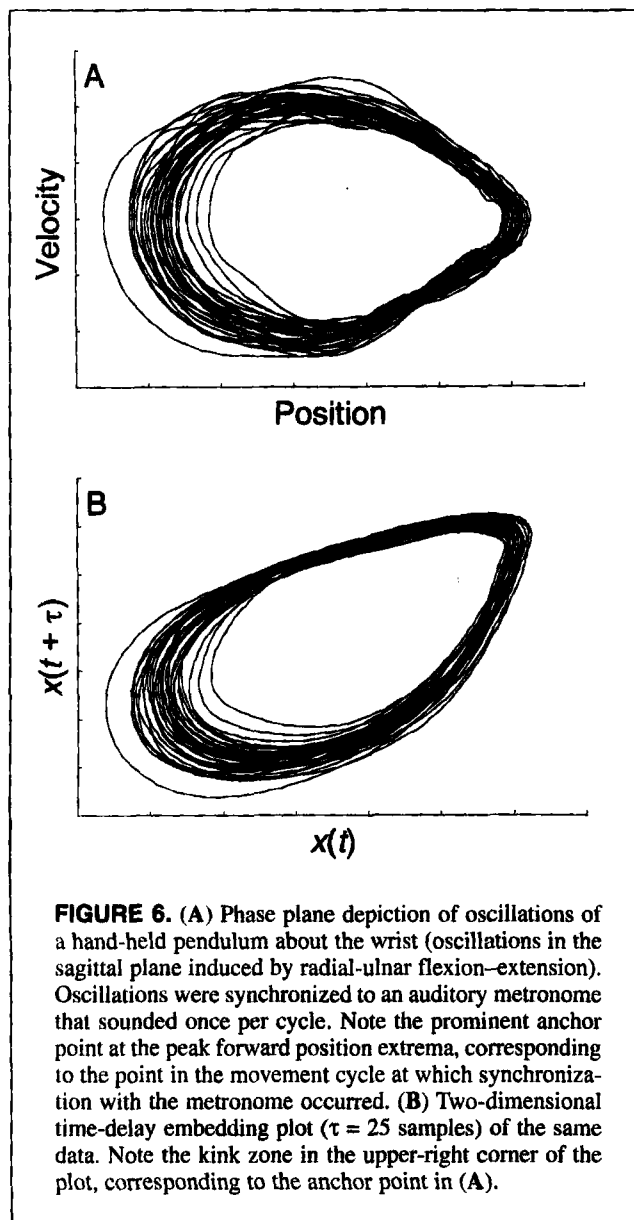
Several quantitative techniques for identifying piecewise determinism exist and have been applied to the analysis of human rhythmic movements (Zak et al., 1997). In one method, one uses wavelet decomposition to detect singularities (discontinuities) in trajectories (Mallat & Hwang,

1992). Another technique involves the previously discussed method of RQA. Detecting piecewise determinism with RQA involves a measure termed *MAXLINE*, which is defined as the length of the longest upward-diagonal (i.e., parallel to the main diagonal) line segment, excluding the main diagonal. *MAXLINE* is an index of dynamic stability and is inversely related to the Lyapunov exponent. Thus, the higher the value of *MAXLINE*, the more stable are the dynamics (conversely stated, the lower the *MAXLINE*, the less stable are the dynamics). If singular points indicative of piecewise determinism are present in a given trajectory, then measures of *MAXLINE* computed over a moving window placed at the singularity should drop sharply to zero. That should occur because high levels of instability are generated at the singular point when it alternates from an attractor to a repeller (technically, the Lyapunov exponent at that point diverges to infinity, so the singular point generates infinite instability).

Zak et al. (1997) applied both the wavelet decomposition and the RQA methods to time series of rhythmic forearm pronation-supination produced at a rate of approximately 1 Hz (forearm rotation was paced by an auditory metronome). Both methods yielded evidence for singularities corresponding to the location of the kink zones in time-delay embedding plots. Those data are consistent with the notion that human rhythmic movements, at least those synchronized to an extrinsic metronome signal, possess characteristics of piecewise determinism.

It should be noted that those quantitative techniques require much higher sampling rates than are typically used in experiments in the movement sciences (typically, kinematic limb movement data are sampled at about 25–200 Hz). To detect the singular points in trajectories, one needs much higher sampling rates. In their analysis of rhythmic forearm movements, for instance, Zak et al. (1997) used a rate of 1 kHz. That requirement essentially precludes reanalysis of most existing data sets, because they are undersampled. Besides creating pragmatic problems related to equipment and technology, the requirement is additionally problematic because such high sampling rates allow more high-frequency noise to be introduced in the measured signal. Noise tends to obscure any singular points that might be present in a trajectory. One cannot obviate that problem by filtering, because filtering will annihilate singular points.

More research on that issue is needed. One critical comparison that needs to be made is between unpaced and metronome-paced movements of a single limb segment moving in isolation (i.e., not coordinated with other moving limb segments). It could be that the singular points detected by Zak et al. (1997) are not general features of rhythmic movements but instead reflect the actor's efforts to coordinate with a discrete environmental event (the metronome signal; Fink et al., 2000). If the latter were true, then unpaced movements would not exhibit singularities. As noted earlier, one cannot reanalyze previous data to perform



that comparison because of the undersampling issue. Therefore, new data must be collected, and experimental designs must be used that specifically address the piecewise-determinism hypothesis. If the new data support a claim for piecewise determinism in rhythmic movements, then piecewise-deterministic models of rhythmic movement should be developed and empirically evaluated.

Intersegmental Coordination: Relative Phase Fluctuations

The coordination of rhythmically moving body segments is essential for many everyday activities, such as locomotion. Rhythmic bimanual and intersegmental coordination has been extensively studied from the dynamical systems perspective, largely within the context of the Haken, Kelso, and Bunz (1985) model. A primary focus in that work has been variability (e.g., D. Collins & Turvey, 1997; Fuchs & Kelso, 1994; Kelso & Ding, 1993; Kelso, Scholz, & Schöner, 1986; Park, Collins, & Turvey, 2001; R. C. Schmidt et al., 1991; R. C. Schmidt, Shaw, & Turvey, 1993; R. C. Schmidt & Turvey, 1995; Schöner et al., 1986; Turvey, Schmidt, & Beek, 1993).

The definitive characteristic of 1:1 frequency-locked rhythmic intersegmental coordination is the requirement for invariant spatiotemporal relations between the two oscillating segments. For the synchronous, in-phase mode of coordination, the two segments should move together to satisfy a relative phasing requirement of $\phi = \theta_l - \theta_r = 0$ rad (relative phase ϕ , the difference of, respectively, the left and right phase angles θ_i , should be 0; that is, the segments should always be at the same positions in their respective movement cycles). For the antiphase mode of coordination (for which the limbs are at opposite points in their movement cycles at any given time), $\phi = \pi$ rad. The in-phase and antiphase coordination modes are spontaneously stable coordination modes. However, they are never executed without variability. If one observes ϕ over a bout of intersegmental coordination, the ϕ time series will exhibit fluctuations (see Figure 7, top).

Variability has played a pivotal role in the dynamical systems perspective on rhythmic intersegmental coordination, especially with respect to efforts to model both the steady-state behavior and the transitions that occur from antiphase to in-phase coordination modes as the coupled movement frequency of the coordination is increased (Kelso, 1995; R. C. Schmidt & Turvey, 1995). In initial treatments of the variability of intersegmental coordination, the variability has been attributed to additive stochastic (Gaussian, white) noise of constant magnitude (Schöner et al., 1986). The constant noise interacts with the strength of the attractive coordinative modes such that as the coordination weakens (e.g., as coupled movement frequency is scaled up), the effects of noise become more pronounced. As the coordination weakens, variability in ϕ increases, and, if the coordination is initially in the antiphase mode and if a critical coupled movement frequency is reached, the noise may cause a transition to in-phase coordination.

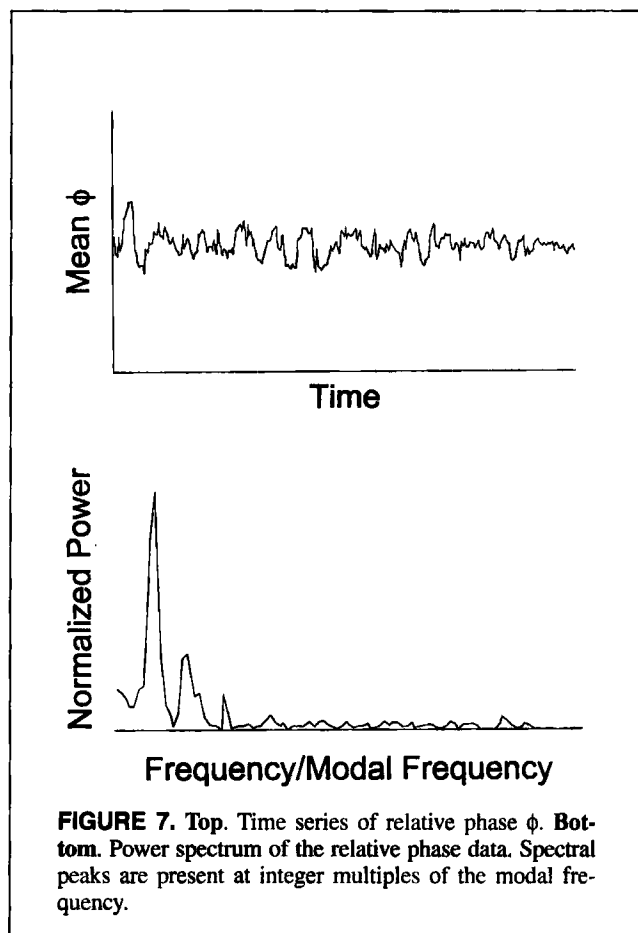


FIGURE 7. Top. Time series of relative phase ϕ . Bottom. Power spectrum of the relative phase data. Spectral peaks are present at integer multiples of the modal frequency.

Again, we examine the question of the nature of those fluctuations. R. C. Schmidt and his colleagues (R. C. Schmidt et al., 1991; R. C. Schmidt et al., 1993; R. C. Schmidt & Turvey, 1995) have argued that fluctuations in relative phase contain deterministic as well as random influences. R. C. Schmidt et al. (1993) performed spectral analyses on time series of relative phase. If relative phase fluctuations are caused by white noise, the power spectra should show an equal concentration of power across all frequencies (there would be no peaks in relative phase, and the spectrum would have a flat slope). Instead, R. C. Schmidt et al. (1993) observed that power tended to be concentrated at integer multiples of the coupled movement frequency (see Figure 7, bottom) and that the spectra showed a nonzero, negative slope (indicating $1/f^\alpha$ -type scaling of power with frequency; see, e.g., B. West & Shlesinger, 1990). The presence of structure in relative phase variability indicates that deterministic processes shape relative phase variability. R. C. Schmidt et al. (1993) proposed that the deterministic processes could stem either from higher-order coupling dynamics that link the two oscillating limbs (see R. C. Schmidt & Turvey, 1995) or from differences in the component oscillators that are coupled so that the coordination can be achieved (see Fuchs & Kelso, 1994; R. C. Schmidt & Turvey, 1995).

Riley, Santana, and Turvey (2001) confirmed through

modeling and data analysis that differences in the component oscillator dynamics (differences that cause the oscillators to have differently shaped phase plane orbits) can contribute to variability in ϕ . Their results revealed that although those differences were a prominent source of relative phase variability, they were not the only source. With respect to Equation 5, then, variations in ϕ seem to be caused partly by a deterministic source, $M(t)$, and partly by a random source, $N(t)$.

Riley et al. (2001) identified differences in component oscillator dynamics as a component of ϕ variability caused by $M(t)$. What is the nature of the $N(t)$ component of ϕ variability? In addition to the possibility that fluctuations in ϕ contain deterministic elements, it is possible that relative phase fluctuations are describable by stochastic processes other than additive white noise. Colored noise processes (processes for which the power spectra scale as f^α , with $0 < \alpha \leq 3$) or parametric noise processes (either white or colored) could be responsible for relative phase variability (R. C. Schmidt et al., 1993).

The focus on variability in the relative phasing of interlimb movements has thus far been rewarding. The focus has raised the important issue of the nature of the coupling function that links the cooperating body segments and has also shed light on the dynamics of the component oscillators that contribute to the coordination (thereby working downward to inform the research at a more fine-grained size of observation).

Summary

Variability in uncoupled and coupled rhythmic movements possesses structure and seems to stem from a blending of deterministic and random elements. Uncoupled rhythmic movements have usually been interpreted as limit-cycle oscillators with additive random noise (Kay, 1988; Kelso, 1995). Although more liberal interpretations of Kay's (1986, 1988) correlation dimension results have been offered (Mitra et al., 1997), an interpretation of rhythmic movement variability as caused by deterministic chaos could be premature. Regardless, it is clear that modeling variability in rhythmic movements as additive white noise is a strategy in which potentially relevant clues about the organization of actions will most likely be ignored. Rhythmic movement variability should be an explicit focus in efforts to disclose the principles of coordination, particularly with regard to the *df* problem and the formation of coordinative structures (see the following).

Fractal Structure of Fluctuations in Quasi-Rhythmic and Rhythmic Actions

Fractal processes have the characteristic of *self-similarity*—that is, across different temporal or spatial scales, similar features or behaviors are observed. Self-similarity is quantified by self-similarity parameters that index correlations in fluctuations (i.e., variability) across scales. In Equation 4, the scaling exponent H is a self-similarity parameter that indexes correlations across temporal scales. Equation 4

identifies a self-similar (fractal) stochastic process. Both Brownian motion ($H = .5$) and fractional Brownian motion ($H \neq .5$) are self-similar processes (the difference between the two, as stated previously, is that fractional Brownian motion represents a departure from complete randomness). The index α describing how A scales with f in the power spectral density function (Equation 3) is also a self-similarity parameter and is related to H .

Examples of fractal processes abound in the biological sciences. Fractal physiological rhythms that have been identified include heart rate and breathing variability (see Goldberger, 1997, for a review). Fractal variability in such processes is recognized as an indication of normal, healthy function, whereas regularity or periodicity may indicate pathology. Fractal structure has also been identified in a variety of quasi-rhythmic and rhythmic motor behaviors. We previously discussed analyses of postural fluctuations that revealed the presence of correlation functions over different time scales of observation (J. Collins & De Luca, 1993, 1994, 1995; Riley, Mitra, et al., 1997; Riley, Wong, et al., 1997; Riley et al., 1998). Postural fluctuations over different time scales are related by a scaling exponent (i.e., the Hurst exponent, H) and so reflect a correlated self-similar process. Postural fluctuations exhibit fractional Brownian motion.

Treffner and Kelso (1999) observed fractal structure in the variability of the quasi-rhythmic task of balancing an inverted pendulum-like object through motions of the hand. Correlations among fluctuations in hand displacements were observed for time scales of up to 3 min. Treffner and Kelso also observed changes in the correlation function over short and long time scales that resembled the changes in correlation functions for fluctuations of the body during upright standing (e.g., J. Collins & De Luca, 1993). Because the quasi-rhythmic task of Treffner and Kelso (1999) and maintaining an upright body posture both require balancing or stabilizing an intrinsically unstable object in the upright position, the similarity of the results is perhaps not surprising.

Fractal Rhythms

Hausdorff and colleagues (Hausdorff et al., 1995; Hausdorff et al., 1997; Hausdorff et al., 1996) identified fractal structure in stride-interval variations during human locomotion. Plots of stride-interval variation magnitude as a function of number of intervals revealed that variation magnitude scaled with interval size. Interval variations between strides were correlated with each other (i.e., statistically similar), even when the strides were separated by up to 1,000 strides (1 hr of walking). When walking rate was constrained by a metronome, however, the power-law correlations dropped out (Hausdorff et al., 1996). Moreover, the correlations were weaker for elderly participants and for participants with Huntington's disease (Hausdorff et al., 1997). The fractal structure of stride-interval variations reflects the intrinsic timing characteristics of the normal, healthy human movement system.

Hausdorff et al. (1996) showed that when the rhythms people produced with a segment were subject to an external, periodic timing constraint, stride-interval variability no longer possessed fractal structure. Chen, Ding, and Kelso (1997) noted, however, that this might have resulted from the analysis of an inappropriate measure. Chen and colleagues had participants tap the index finger at a pace prescribed by a metronome. Instead of analyzing the tapping fluctuations themselves (cf. Musha, Katsurai, & Teremachi, 1985), Chen and his group analyzed a time series of errors: the difference between the timing of the metronome pulse and that of the tapping. They found that the error signal exhibited long-range power-law correlations. In consequence, they hypothesized that the correlations might have emerged because of motor delays (i.e., the time lag between when a given command to the musculature is sent and when it is actually executed) and the numerous interactions among the very large number of microscopic *df*.

Summary

Fractal structure (long-range power-law correlations) is present in apparently unstructured noise for a variety of quasi-rhythmic and rhythmic coordinated actions, such as standing, balancing an object in the hand, walking, and tapping in synchrony with a metronome. Recognition of the fractal dynamics of coordinated actions is important because the presence of fractal dynamics means that such behaviors are best characterized not by gross statistical measures calculated at a particular scale of observation but instead by how statistical measures such as variance depend upon the scale of observation (Liebovitch & Todorov, 1996). Appropriate characterization of the behavior is necessary before we can hope to comprehend the underlying mechanisms and processes.

Additional Theoretical Considerations

In previous sections of this article, we have discussed empirical findings regarding movement variability and the implications of those findings for models of motor behavior. In the final section, we turn to broader theoretical considerations that underlie specific efforts to model motor behavior.

The assumption that movement variability is a product of unstructured (additive, Gaussian, white) neuromotor noise superimposed on a deterministic signal is usually accompanied by the assumption that movement variability provides minimal, if any, insight into the nature of coordination, that is, the organization of the control of the motor apparatus (cf. Bernstein, 1967). However, variability is often structured rather than random. Structured variability appears to be the rule rather than the exception. Ignoring the structure and continuing to assume that movement variability is an unimportant factor is, perhaps, a misguided strategy.

Variability and Randomness

Motor variability cannot be simply equated with randomness. Although all of the summarized results support the lat-

ter assertion in the broad sense, the results obtained from the application of RQA to postural fluctuations are particularly compelling. Riley et al. (1999) and Balasubramaniam et al. (2000) all observed an inverse relation between degree of variability and degree of randomness of postural fluctuations. Riley and his colleagues observed that an increase in variability found when participants' eyes were closed (relative to eyes open) was accompanied by a decrease in randomness (i.e., by an increase in determinism, as indexed by %DET). As postural fluctuations became more variable (noisier), they became at the same time more deterministic (less random). Balasubramaniam and colleagues found essentially the same result in their analysis of the role of suprapostural tasks. As postural variability decreased with increasing task difficulty, so did %DET (less variable motions became more random). Both data sets revealed an inverse, rather than a direct, relation between degree of observed variability and degree of observed randomness.

Slifkin and Newell (1999b) also observed such a relation in their data. They had participants perform an isometric force-production task at different levels of required force and measured the time series of force production. They found that variability at the task was a U-shaped function of required force. Variability was highest for low and for high force levels and dropped for middle levels of force. They also observed—using spectral analysis and a measure of time series regularity termed *approximate entropy* (Pincus, 1991, 1998)—that the structure of the variability changed as a function of required force. The data were more regular for low and for high levels of force and more irregular for middle ranges (an inverted-U function). Thus, as the data became more variable (at low and high force requirements), they also became more regular, and as the data became less variable (middle levels of required force), the data also became more irregular. That is the same inverse variability–randomness trend observed by Riley et al. (1999) and by Balasubramaniam et al. (2000).

Dingwell and Cusumano (2000) observed a similar relation between kinematic variability and measures of deterministic structure. Patients suffering from peripheral neuropathy (a loss of afferentation from the feet and other extremities that is a frequent complication associated with diabetes) show greater variability in stride-to-stride kinematic measures than do normal participants. Dingwell and Cusumano showed that the increase in motor variability was accompanied by an increase in local dynamic stability (indexed by finite-time Lyapunov exponents). They also showed, using a surrogate data method, that stride-to-stride variability for both neuropathic and normal participants was not the product of a linear system driven by Gaussian noise.

The review of the preceding data shows that motor variability cannot be simply equated with randomness. Those data revealed inverse relations between motor variability, as indexed by measures such as standard deviation, and dynamic structure or stability. Put simply, data that appear to be more variable are often in fact less random.

Movement Organization and the Structure of Variability

To Bernstein (1967), a key to understanding perceptual-motor control was the *df* problem. One take on that problem (Turvey, 1990) can be expressed in the following form: How are the *df* of the high-dimensional movement system compressed and assembled into a low-dimensional, controllable coordinative structure? Kay (1988) proposed a framework for the solution of that question as a two-step process of constraint (see Kugler et al., 1980; Kugler & Turvey, 1987). First, microcomponents are constrained in the assembly of a coordinative structure, which is assumed to act as a dynamical system (Fowler & Turvey, 1978; Kugler et al., 1980; Mitra et al., 1997). Second, the coordinative structure is constrained so that it produces a particular movement or movement pattern available from a class of movements. Constraints operating at each step serve to compress the number of available *df* (see Turvey, Fitch, & Tuller, 1982). Figure 8 illustrates the process of compression of *df* at each of those levels and highlights various sources of constraint—those stemming from tasks, perceptual information, the environment, and other sources. Those sources of constraint mold or shape the process of compression and may do so at either step.

Mitra et al. (1998) addressed in detail vision's role in the shaping of coordinative structures and behavior. They speculated that the operation of constraints at various levels of movement system organization might be best expressed as processes of state-, parameter-, and graph-dynamics (Farmer, 1990; Saltzman & Munhall, 1992). *State-dynamics* refers to the evolution rules governing the motions of a system's state variables (e.g., position and velocity of a pendulum or of a limit-cycle oscillator). *Parameter-dynamics* refers to online changes in system parameters (such as damping or stiffness in a mass-spring system). *Graph-dynamics* refers to changes

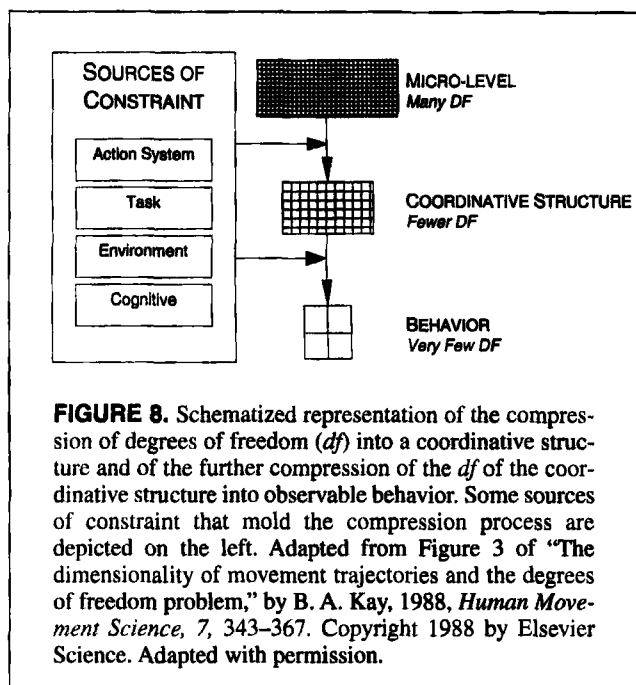
in the dynamical composition or "architecture" of a system (i.e., changes in the number and type of state variables and parameters). Mitra and colleagues showed vision's influence at each level of dynamics. Of particular relevance to the present discussion was the demonstration that the availability of vision influenced the number of *adf* (i.e., the number of state variables—a graph-dynamic effect) observed when participants performed a difficult rhythmic aiming task. The availability of vision influenced the compression of *df* into a coordinative structure and changed the nature of the dynamical system that described participants' motions. The reorganization of the underlying structure of movements appeared to be related to adaptive performance changes (Riley, Santana, & Mitra, 1998).

The preceding approaches to the *df* problem and to how constraints shape the movement system's solution of the *df* problem require an appreciation of what movement variability reveals about the organization of movement control (Kay, 1988; Post et al., 2000). The nature of movement variability provides important clues regarding the question of compression of *df* into a controllable, low-dimensional coordinative structure. To understand that process, and to understand how the behavior of the coordinative structure is further constrained so that it produces the observed behavior, one must first uncover the appropriate descriptions of the coordinative structure and the observed behavior (Kay, 1988). We hope to have shown in the present article that the structure in variability is highly informative of the nature of those appropriate descriptions.

Noise and the Nature of the "Motor Program"

With respect to the elementary perceptual-motor behaviors discussed in the present article, we have shown that although not all variability may be attributable to random noise (i.e., structure is present and might contribute to the apparent variability of behavior), in general it seems that $M(t)$ coexists with $N(t)$. In this section, we pose and attempt to answer the following question: Is there functional significance attached to the co-existence of determinism and noise in coordinated behavior? In particular, we address the implications of the co-existence of determinism and noise with respect to the concept of the motor program.

The notion of the motor program (e.g., Keele, 1968) is, broadly speaking, an account of the organization or order (invariance) of observed motor behaviors. A motor program, like a computer program, is often defined as a symbol string (but see Heuer, 1993). The symbols are commands that, when executed by the neuromuscular apparatus, dictate particular patterns of muscular activity that will produce a desired movement. Programs are deterministic—they are simply lists of commands (strings of symbols). Modifications to the concept of the motor program have included the addition of noise to parameters. Some authors (e.g., Meyer et al., 1988; Meyer, Smith, & Wright, 1982; R. A. Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979) have added noise (described as *neuromo-*



tor noise) to model parameters in order to explain spatial variability in rapid aimed movements, an idea traceable to Fitts (1954).

The coordination dynamics approach (e.g., Haken, 1996; Kelso, 1995), which was developed to enable researchers to account for behaviors such as those found in the experiments of Kelso (1984), and the dynamical systems perspective on coordinate movement, in general (e.g., Turvey, 1990), are also accounts of the order of observed behaviors. Therefore, the dynamical systems perspective and the motor programming perspective are both intended to account for the same thing, although in very different terms (cf. Amazeen, 1996). In the former perspective, the investigator seeks to replace the notion of a motor program with a notion such as a coordination dynamic. Besides the difference in how the two approaches account for movement invariants (dynamics versus symbol strings or prestructured muscle commands), they are also distinguishable in their respective emphases on the variance of behavior. Whereas advocates of the motor programming perspective use the concept of neuromotor noise, for the most part, to explain performance variability (e.g., Meyer et al., 1988), proponents of the dynamical systems perspective have treated variability as a fundamental, functional feature of perceptual-motor behavior.

Allometric Control

As noted, step-to-step fluctuations in human gait exhibit long-range correlations (e.g., Hausdorff et al., 1995; Hausdorff et al., 1996). An implication of that finding is that human locomotion is governed by an overall regulatory mechanism that incorporates scaling (B. West & Griffin, 1998). The hypothesized control mechanism, referred to as *allometric control* (B. West & Griffin), is the natural companion to *allometric laws* (Calder, 1984; Peters, 1983). Animals are to be found at multiple length and mass scales. What allometric laws reveal is that, despite the obvious variations in animals across scales, many biological functions are scale invariant. Behavioral (e.g., limb frequency in walking) and physiological (e.g., metabolic rate) measures scale as power functions of size, expressed either as a linear dimension (e.g., limb length) or as body mass. Not only are species that differ little in size dynamically similar, species that differ greatly in size are likewise dynamically similar. An allometric law reflects a general fractal (or scale-invariant) ordering principle that correlates species nearby and species far apart on the dimension of body size (B. West & Deering, 1995).

There are good reasons to suppose that the fractal ordering principle applies more widely (G. West, Brown, & Enquist, 1997). In particular, one might expect it to apply to the multiple allometric laws that span the spectrum of processes from those operating at short time scales (e.g., skeletal muscle contraction) to those operating at long time scales (life duration). The scaling of biological times (Lindstedt & Calder, 1981) suggests that the allometric laws governing biological functions of different durations exhibit

interrelated scaling exponents. The implication is that scaling is a mechanism that ensures the integrated function of the component biological structures and functions (Brown, West, & Enquist, 2000; B. West & Griffin, 1998; G. West et al., 1997).

That the variability of stride intervals is long-range correlation rather than uncorrelated random error (e.g., B. West & Griffin, 1998; Hausdorff et al., 1995) invites a rethinking of the mechanism of locomotion control along allometric lines. The inverse power-law behavior of stride-interval variance (plotted against number of strides in an observation window) may be the resultant of possibly many processes interacting over a multiplicity of interdependent scales. In locomotion, a variety of proprioceptive loops act concurrently and interactively with a variety of optical flow and vestibular functions that are, in turn, concurrent and interactive with muscular, respiratory, and metabolic functions. According to the allometric control hypothesis, an underlying fractal (scale-invariant) process ties together those component functions at short and long time scales (B. West & Griffin, 1998). Responding to the allometric hypothesis, and the experimental facts behind it, will challenge contemporary computational, neural, and dynamical models of rhythmic motor behavior (Hausdorff et al., 1996; Liebovitch & Todorov, 1996).

Linking Perspectives

As was mentioned earlier, in Wing-Kristofferson (W-K) theory (Wing & Kristofferson, 1973), timing variance is partitioned into that caused by a central clock and that caused by motor implementation. Application of W-K theory to the cycle-time variations of each hand in 1:1 coordination has shown that (a) the clock variances of the two hands are more strongly related than are the motor variances (Turvey, Rosenblum, & Schmidt, 1989; Turvey, Schmidt, Rosenblum, & Kugler, 1986) and (b) clock variances but not motor variances are affected by phase symmetry (clock variance is greater for antiphase; Turvey et al., 1986). Those outcomes suggest an organization of the limbs according to a single attractor in task space (Saltzman & Kelso, 1987) that acts implicitly as a clock. In principle, any functional division of the nervous system can be a clock (e.g., the cerebellum; Keele & Ivry, 1987) as long as its states abide a strict partial order, namely, asymmetric and transitive, so that no two states are concurrent (Bunge, 1977).³ The separation of timing and motor implementation in W-K theory challenges the *strong* dynamical perspective on rhythmic movement. The aim in the latter perspective is to account for the dynamics of observed repetitive behavior in terms of the quantitative dynamics of one autonomous oscillator (e.g., Kay et al., 1987; Kugler et al., 1980; Turvey, 1977). The aim in the *weak* dynamical perspective is to account for most of the behavior through the dynamics of an autonomous nonlinear oscillator but with an additional external forcing tailored to the autonomous dynamics (Beek & Beek, 1988; Beek et al., 1992).

Recent efforts to derive the basic property of W-K theory (negative lag-1 autocorrelations) from one autonomous dynamical system with nonspecific random forces have not succeeded (Daffertshofer, 1998). What would be needed, apparently, is a coupling of nonautonomous (deterministic) dynamics with autonomous dynamics—for example, a periodically forced oscillator (Daffertshofer, 1998). That conclusion is consistent with both the weak dynamical perspective and the W-K separation of timing and implementation. The hypothesized advantage of added nonautonomous dynamics is the openings they create for the influence of informational constraints on intention. As suggested by Beek et al. (1992), fully autonomous dynamics may be realizable, but only at the level of the perception-action cycle.

The W-K formulation contrasts markedly with the fractal dynamics of allometric control described earlier. It is presumed in that formulation that rhythmic behavior is characterized by a short-term correlation with a characteristic time scale. Whereas the autocorrelation function for W-K theory decays within a time lag of 1, a process with long-range correlations, such as human gait, has an autocorrelation function that scales as $\tau^{-\lambda}$, where τ is a time lag and $0 < \lambda < 1$. Most important, it is a fact that both repetitive tapping tasks (Wing, 1977) and continuous rhythmic tasks (Turvey et al., 1989) show correlations at larger lags. Furthermore, it is not uncommon to find that the time series of rhythmic behaviors are characterized by positive lag-1 autocorrelations; such a finding rules out the W-K analysis (but not necessarily W-K theory, given that positive lag-1 autocorrelations could arise for trivial reasons, for example, improper experimental procedures). Those observations suggest that repetitive behaviors may conform more to the allometric formulation than to the W-K formulation. If so, then timing variability will resist interpretation in terms of two or more interacting elements (e.g., a clock element and a motor element), and a major reconsideration of the “clock” in a typical repetitive movement experiment will be required. An alternative to the idea that memory for the required cycle time is contained in a central clocking component (e.g., Keele & Ivry, 1987) is the idea that the memory “evolves dynamically through activities at multiple time scales—a kind of dynamic memory” (Chen, Ding, & Kelso, 2001, p. 7).

The Positive Side of Noise

Within the dynamics perspective, the most often cited function of noise is that it allows for flexibility in behavior that cannot be provided by a purely deterministic system. For example, noise may facilitate transitions between different modes of intersegmental coordination (D. Collins et al., 1998; Haken, 1996; Kelso, 1995). Consider the task of coordinating the oscillations of the two hands as the frequency of oscillation is gradually increased (Kelso, 1984). If the initial intersegmental coordination state is $\phi = 0$ rad (in-phase coordination), then the primary effect of increasing the oscillation frequency is to slightly increase the vari-

ability of the coordination. If the initial state is $\phi = \pi$ rad (antiphase coordination), however, then at some critical frequency there is a spontaneous shift to in-phase coordination. Noise is critical in generating the shift to in-phase coordination in a timely fashion.

Certain combinations of noise and deterministic signals can have functional properties beyond their suggested role in behavioral flexibility. For instance, adding noise to a chaotic dynamical system may render the process controllable (Ott, Grebogi, & Yorke, 1990). Chaotic systems are much more flexible than simple deterministic systems are and are even more flexible than simple deterministic systems with added noise, but (in the absence of noise) they suffer with respect to the problem of controllability (i.e., chaotic dynamics are not predictable in the long run and are sensitive to initial conditions). Integrating chaotic dynamics with noise may represent a solution that biological systems have evolved to exploit. Noise, measured as the correlation of the system's time course with itself, is known to increase the coherence of a dynamical system. It can induce synchronization of coupled oscillators and regular synchronized oscillations in networks of coupled subsystems (*stochastic coherence*; Pradines, Osipov, & Collins, 1999). Another example of a functional combination of noise and determinism is that adding noise to a very weak and otherwise undetectable nonlinear deterministic signal may amplify the signal beyond threshold levels. That amplification is termed *stochastic resonance* (J. Collins, Imhoff, & Grigg, 1996; Wiesenfeld & Moss, 1995). Acknowledging the co-existence of determinism and noise in coordinated movement raises the possibility that those or related processes are important features of movement control. The implications for coordinated movement of the dynamics of chaos in noisy systems (e.g., Kapitaniak, 1988) and of the control of chaotic dynamics (e.g., Ott et al., 1990) remain to be explored (see Kelso & Ding, 1993).

Programs as Piecewise Deterministic

The co-existence of noise and determinism raises another possibility: Noise is perhaps much more integral to coordinated behavior than has been previously considered, even from within the dynamical systems framework. As summarized in Equation 5, in previous acknowledgments of the role of noise in coordinated behavior (made from both the motor programming and dynamical systems theories), the problem has always been approached from the perspective of noise added to determinism (variability superimposed on movement invariants, i.e., the standard interpretation of Equation 5). Noise plays an important yet somewhat secondary role: Variability provides some benefits (e.g., increased behavioral flexibility), but determinism is primary and does the bulk of the work. It is also usually assumed that the deterministic structure is in the form of a continuous dynamical system. An alternative to the noise-added-to-determinism approach is to consider noise and determinism

on more equal footing in the context of piecewise determinism (Zak et al., 1997).

Consideration of piecewise-deterministic dynamics raises new ideas regarding the nature of coordinated motor behavior. For instance, what is the fundamental nature of the invariance of movements that investigators have attempted to account for in such concepts as the motor program and coordination dynamics? Paradoxically, in the analysis presented here, we suggest that variability (stochasticity at singular points) may be as integral a feature of movement invariants as are deterministic trajectories. Moreover, the very distinction drawn between movement invariants and variability is blurry, at best, when viewed from the piecewise-deterministic dynamics perspective, because it is the deterministic components of the behavior that vary (the successive trajectories). The successive trajectories vary by virtue of the jumps (that occur because of stochasticity at the singular points) among possible trajectories.

Piecewise-deterministic dynamics possess characteristics that would be advantageous for the control of coordinated movement. The greatest advantage afforded by piecewise-deterministic dynamics is that that type of dynamics yields ultimate flexibility. Previously we noted that chaotic dynamical systems are typically considered to possess more flexibility than deterministic systems with added noise, which in turn possess more flexibility than purely deterministic systems. However, chaotic dynamical systems are sensitive to initial conditions. Therefore, initial conditions place fundamental constraints on the flexibility of the behavior of a chaotic dynamical system—the system is bound by its own history and cannot assume states not allowed by its initial conditions. Piecewise-deterministic dynamics are not bound by historical constraints: The dynamics are potentially adjustable each time a singular point is encountered.

The potential to adjust the ongoing dynamics each time a singular point is encountered does not necessarily require the assumption of a continuous, time-dependent clock mechanism to monitor the dynamics. The dynamics might, but do not necessarily have to, be intentionally adjusted at the singular points. Noise will perturb the system away from the singular point and onto a new trajectory; no monitoring or control mechanism is required for that to occur. Thus, although singular points might represent a convenient place for discrete control to be exerted, when needed, that notion does not imply that one must have a continuous control mechanism to monitor and to generate movement trajectories. The proposal just described is reminiscent of Pattee's (1977, 1979, 1983, 1986) principle of the complementarity of symbols and dynamics. According to Pattee, biological systems operate in two complementary (but not interchangeable) modes, the symbolic (discrete) and the dynamic (continuous). The symbolic mode is used sparingly, whereas use of the dynamic mode is maximized. Discrete (i.e., symbolic) control could be "used, now and then, to direct dynamical processes and to limit their complexity" (Kugler & Turvey, 1987, p. 5; see also

Carello, Turvey, Kugler, & Shaw, 1984). Such a scenario could be hypothesized with respect to piecewise-deterministic motor dynamics—discrete control could be exercised at the singular points but need not be exercised every time the system reaches a singular point. Most of the time, the interplay between noise and deterministic dynamics could simply be allowed to unfold.

However, intentional or perceptually guided motor control, singly or in combination, would seem to require more than random noise perturbations at the singular points. Some means of modeling intentional changes of dynamic regimes is necessary, but how can one formally account for the injection of control at singular points? One approach could be to model the control in terms of infinitesimal guiding forces that drive the system away from the singular point (Zak et al., 1997, p. 304). Perceptually guided control at the singular points could be modeled as a dynamical system that is coupled to the piecewise-deterministic process. If the amplitude of the coupled dynamical system's motion is very, very small relative to that of the piecewise-deterministic system, then the infinitesimally weak "guiding dynamic" will have a remarkable effect on the piecewise-deterministic system only during the regime of infinite instability. In principle, the guiding dynamic could be represented as an explicit function of time (i.e., a clock) that drives the piecewise-deterministic system and that would render the coupled dynamical system nonautonomous. Again, however, we argue that the assumption of a clock is not necessary. The guiding dynamic could instead be represented as an autonomous dynamical system (one that may or may not be piecewise deterministic) that is parametrically coupled to the piecewise-deterministic dynamic. The second strategy would avoid an explicit appeal to a clock but still allow perceptually guided control to be represented in a model. The distinction between those strategies reflects the distinctions discussed earlier regarding W-K theory, the weak dynamical perspective, and the strong dynamical perspective (Beek & Beek, 1988; Beek et al., 1992; Chen et al., 2001; Daffertshofer, 1998; Turvey et al., 1989). In the piecewise-deterministic hypothesis, the issue of clocking in motor behavior is not solved but the problem is cast in a different light.

Because piecewise-deterministic dynamics are not bound by historical constraints, they are free of initial conditions. With respect to coordinated movement, that means that the product of a movement (e.g., the placement of the hand in a particular spatial location) is potentially free of initial conditions (e.g., the starting location of the hand). That freedom is an extremely important feature, because the initial conditions for any given real-world movements are rarely identical, despite the fact that the endpoint requirements might be identical. For example, when hammering, one might begin the striking motion from a variety of initial positions, and the trajectories of successive strikes may differ from one another substantially (Bernstein, 1967). The endpoint requirements for accurate hammering are basically identical, however—for each movement, one must bring

the head of the hammer to a fixed location, the head of the nail. That relative insensitivity to initial conditions or exquisite sensitivity to final conditions (Shaw & Kinsella-Shaw, 1988) reflects an ability to adapt movements to novel conditions and is a prominent characteristic of coordinated movement. The ability to adapt to novel initial conditions and to produce unique and novel movement trajectories reflects the *productivity* or *generativity* of the movement system. In any model of the movement system, those features must be accounted for, and piecewise-deterministic dynamics may provide a basis for such a model. The flexibility of piecewise determinism might facilitate an organism's behavioral adjustments to environmental demands (Giuliani et al., 1998; Zak et al., 1997).

Currently, there is some evidence to suggest that coordinated rhythmic movements possess discontinuities consistent with the presence of piecewise-deterministic dynamics (Beek, 1989; Byblow & Chua, 1994; Byblow et al., 1998; Carson et al., 1994; Chua, 1995; Kay et al., 1991; Zak et al., 1997). New experiments with the explicit focus of quantitatively confirming or disconfirming these suggestions are needed. The appropriate tools for detecting the signatures of piecewise determinism should be brought to bear on rhythmic, as well as postural, movements.

Such analyses would help investigators to constrain theoretical models of motor behavior. The evidence for piecewise determinism in movement trajectories is inconsistent with the strategy of modeling rhythmic movements as continuous dynamical systems, which is widely practiced (e.g., Haken et al., 1985; Kay, 1986; Kay et al., 1987). The mathematical foundations for modeling piecewise-deterministic dynamics have been established by Zak (1993; Zak et al., 1997). If in new analyses it is suggested that rhythmic movements are piecewise deterministic, then a shift in modeling strategies (from continuous, ordinary differential equations to piecewise-deterministic models) may be necessary.

Broader Lessons

The most general lesson that the present review provides is straightforward. The strategy of first parceling movement into deterministic and random components (Equation 5), and then focusing on the deterministic component to the exclusion of the random component, does not tell the entire story of motor behavior. That lesson is not a new one, but its importance makes it worth repeating. Newell and Corcos (1993) and Newell and Slifkin (1998) suggested that motor variability might be more revealing than invariants in efforts to understand the coordination and control of movement. In the present article, we have summarized evidence in support of that suggestion and, ideally, provided further groundwork for what we hope will prove to be a useful strategy for inquiry into motor behavior. An explicit focus on variability brings to light important features that promise to constrain inferences regarding action and perception functions and their neural and environmental support (e.g., piecewise-

deterministic dynamics). In particular, an explicit focus on variability promises insights and lessons of potentially broad scope, for example, that more variable does not equal more random and that more controllable does not mean more deterministic.

Moreover, in the present review we suggest that the simple parceling of movement into deterministic and random components is not as clear-cut as it might seem. Motor behaviors might not be easily divisible into the two components identified in Equation 5, especially those describable by piecewise-deterministic dynamics. Distinguishing the random from the deterministic, or classifying a behavior along the random-deterministic continuum, is a difficult challenge. Addressing the challenge requires an appreciation of potential sources of variability, and an appreciation of the fact that the distinction between $M(t)$ and $N(t)$ is perhaps a blurry one. Addressing the challenge also requires the application of analytic tools (e.g., the use of RQA, prediction horizon, approximate entropy, local false nearest neighbors analysis to calculate *adf*, Hurst's rescaled range analysis, and relative dispersion) designed to address variability in the face of nonlinearity and noise. Because no single analysis is foolproof, such tools should be used in conjunction with one another on single data sets so that converging evidence can be obtained for the classification of a particular motor behavior as random, deterministic, or somewhere in between. Clearly, however, the blind application of analyses, no matter how sophisticated they are, will not lead to significant progress in theories of motor control. New models of perceptual-motor behavior—models in which the various possible forms of $M(t)$ and $N(t)$ reviewed in the present article are explicitly accounted for—might be required. Our aim in this review was to highlight the importance of observed motor variability in constraining models of perceptual-motor behavior.

ACKNOWLEDGMENT

This work was supported in part by National Science Foundation Research Grant SBR 97-28970 (M. T. Turvey). We thank J. Zbilut for discussions on piecewise-deterministic dynamics and an anonymous reviewer for many helpful, critical comments.

NOTES

1. It is not a requirement that stochastic processes be partitioned into discrete time steps—they may be either continuous or discrete.
2. The variability of a sine wave time series is a consequence of viewing the process in a space with too small an embedding dimension. When the process is viewed in the phase plane (velocity versus position), the variability vanishes—a single, closed orbit is seen.
3. The formal analysis of relations is the starting point for addressing the processes of sorting and ordering. The concept of a clock refers to a particular ordering of states. Bunge (1977, pp. 296–304) took advantage of the following properties of relations to express the notion of clock in its most abstract and general way. A relation on a set A is (a) reflexive if (x, x) belongs to the solution set of the relation for every $(x \in A)$; (b) symmetric if (y, x) belongs to the solution set whenever (x, y) belongs to the solution

set $(x, y \in A)$, where the term symmetric refers to the property that the solution set is unchanged if the order of the elements in every pair is changed (the relation \geq on the set of real numbers, for example, is not symmetric); (c) antisymmetric if, whenever (x, y) and (y, x) both belong to the solution set, then x and y are the same element (the relation \geq , for example, is antisymmetric); and (d) transitive if (x, z) belongs to the solution set whenever (x, y) and (y, z) both belong to the solution set. An equivalence relation is defined by (a), (b), and (d); an order relation is defined by (a), (c), and (d). The strict partial ordering that defines a clock is given by (c) and (d).

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Submitted December 8, 1999

Revised March 21, 2001

Second revision September 13, 2001

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