



Contents lists available at ScienceDirect

Human Movement Science

journal homepage: www.elsevier.com/locate/humov



Human movement variability, nonlinear dynamics, and pathology: Is there a connection?

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ARTICLE INFO

Article history:

Available online 29 July 2011

PsycINFO classification:

2330

2520

2260

Keywords:

Chaos

Stability

Anterior cruciate ligament

Cerebral concussion

Passive dynamic walker

Infant motor development

ABSTRACT

Fields studying movement generation, including robotics, psychology, cognitive science, and neuroscience utilize concepts and tools related to the pervasiveness of variability in biological systems. The concept of variability and the measures for nonlinear dynamics used to evaluate this concept open new vistas for research in movement dysfunction of many types. This review describes innovations in the exploration of variability and their potential importance in understanding human movement. Far from being a source of error, evidence supports the presence of an optimal state of variability for healthy and functional movement. This variability has a particular organization and is characterized by a chaotic structure. Deviations from this state can lead to biological systems that are either overly rigid and robotic or noisy and unstable. Both situations result in systems that are less adaptable to perturbations, such as those associated with unhealthy pathological states or absence of skillfulness.

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1. Introduction

One of the most common features of human movement is its variability. Human movement variability can be described as the normal variations that occur in motor performance across multiple repetitions of a task (Stergiou, Harbourne, & Cavanaugh, 2006). This variability is intrinsic in all biological systems and it can be observed quite easily. If a person tries to repeat the same movement twice, the

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two actions will never be identical. Bernstein (1967) used an expression “repetition without repetition” whereby each repetition of an act involved unique, non-repetitive neural and motor patterns. Recently, the role of movement variability in motor control has become an object of study in its own right (Bates, 1996; Newell & Corcos, 1993). Therefore, a number of questions have been raised (Stergiou et al., 2006). Examples of such questions are “How is variability controlled while learning a new skill?”, “Is variability associated with disease/health?”, and “What are the sources of variability, and how do they interact in the production of the observed variation in movement?”

In the past, variability in motor performance has been considered from a variety of theoretical perspectives (e.g., Newell & Corcos, 1993). A prominent theory is the Generalized Motor Program Theory (GMPT; Summers & Anson, 2009). This theory considers variation in a given movement pattern to be the result of error. This error in the ability to predict the necessary parameters for employing the underlying motor program results in variation in motor performance (Schmidt, 2003; Schmidt & Lee, 2005). With task-specific practice, prediction error is gradually eliminated or minimized, thereby optimizing the accuracy and efficiency of the movement pattern.

Another prominent theory is the uncontrolled manifold (UCM) hypothesis. Practically, motor variability has been associated with motor redundancy. Motor redundancy refers to having more elements than necessary to solve a task, resulting in the existence of multiple solutions to a given motor problem. Latash, Scholz, and Schöner (2002) described the UCM hypothesis to address this problem of motor redundancy. According to this hypothesis, when a multi-element system changes its state within a UCM computed for a particular performance variable (e.g., total force produced by a set of fingers), this variable is kept at a constant value. As long as the system does not leave the UCM, the hierarchically higher controller (e.g., central nervous system) does not need to interfere and, in that sense, the system of elemental variables does not need to be controlled within that manifold. If the system leaves the UCM and shows an acceptable error in the performance variable, the controller may have to interfere and introduce a correction (Latash, 2008). The UCM approach has been applied to several motor tasks such as maintaining quiet stance, finger force production, bimanual pointing, sit-to-stand, and pistol shooting (Domkin, Laczko, Jaric, Johansson, & Latash, 2002; Latash, Scholz, Danion, & Schöner, 2001; Scholz, Kang, Patterson, & Latash, 2003) to discover coordination strategies of apparently redundant motor systems and uncover the functional purposes that variability plays in those motor tasks.

A third theoretical perspective briefly presented here is the Dynamical Systems Theory (DST) which proposes that biological systems self-organize according to environmental, biomechanical, and morphological constraints to find the most stable solution for producing a given movement (Clark & Phillips, 1993; Hamill, van Emmerik, Heiderscheit, & Li, 1999; Kamm, Thelen, & Jensen, 1990; Kelso, 1995; Thelen, 1995; Thelen & Ulrich, 1991). Increased variability in a movement pattern generally indicates loss of stability, while decreased variability generally indicates a highly stable behavior. The GMPT, UCM, and DST perspectives are similar in that they all recognize that decreased variability results from the efficient execution of a given movement pattern. DST focuses more on behavioral transitions and provides tools to describe such phenomena. Specifically, DST suggests that, in certain dynamical systems and under certain conditions, when variability increases and reaches a specific critical point, the system becomes highly unstable and switches to a new, more stable movement pattern (with less variability). This proposition is a significant step forward because it explains transitions between behavioral states and implies that a persistent lack of movement variability may indicate rigid, inflexible motor behaviors with limited adaptability to changing task or environmental demands. However, a significant limitation of DST is that it does not account for the observation that some behaviors, which appear to be highly stable, paradoxically are performed in variable ways. This is especially evident when we observe elite sports players or musicians performing (e.g., Michael Jordan taking a jump shot or Yo-Yo Ma playing the cello). Not only is their performance more consistent than that of less capable individuals, but they also seem to have developed an infinite number of ways of performing. If we actually consider fundamental motor skills (i.e., gait) as activities when applied in “real life” contexts, we can actually say that every single one of us is a Michael Jordan in our abilities to walk through crowds or on diverse and challenging terrains. Therefore, it seems that in this sense, variability is closely related with a rich behavioral state.

The idea that variability decreases with skill acquisition in one context (motor learning paradigm) and increases with skill acquisition in another context (the development of a behavioral repertoire) is readily explained by the way in which variability is measured. Typical motor learning curves are constructed using traditional variability measures of skill performance to capture error in performance. Such linear statistical measures quantify the magnitude of variation in a set of values independently of their order in the distribution. The magnitude of variability continuously decreases and eventually plateaus as motor learning occurs. In contrast, variation in how a motor behavior emerges in time is best captured by measures where the temporal organization in distribution of values is the facet of interest. Temporal organization (or structure) of variability is quantified by the degree to which values emerge in an orderly manner, often across a range of time scales. Therefore, recent theoretical approaches have suggested that variability contains important information about movement (Amato, 1992; Cavanaugh, Guskiewicz, & Stergiou, 2005; Harbourne & Stergiou, 2009; Newell & Corcos, 1993). These approaches have now propagated in the human movement literature and lead the development of alternative theoretical frameworks and methodologies to study human movement related injuries and treatments.

Much of the controversy that exists in the literature with respect to human movement variability stems from the methodology used. Traditional linear measures, such as the standard deviation or the range, are measures of centrality and thus provide a description of the amount or magnitude of the variability around a central point (Fig. 1). From a human movement perspective, this approach in evaluating variability has led several practitioners and scientists to believe that the mean is the standard of performance and everything away from the mean is error. From a statistical standpoint, the valid usage of traditional linear measures to study variability assumes that variations between repetitions of a task are random and independent (of past and future repetitions) (Lomax, 2007). However, previous studies have shown that such variations are distinguishable from noise (Delignières & Torre, 2009; Dingwell & Cusumano, 2000; Dingwell & Kang, 2007; Stergiou, Buzzi, Kurz, & Heide, 2004a).

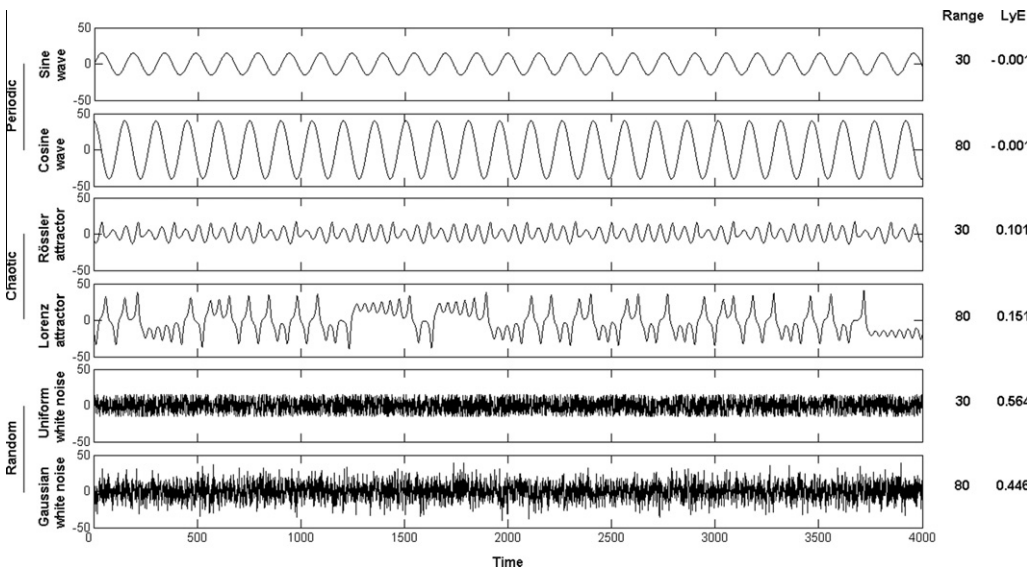


Fig. 1. Complementary linear and nonlinear measures from different signals; six signals are displayed, with the respective values for range and largest Lyapunov Exponent (LyE). The first two time series are periodic and have been generated using the sine function $15\sin(t/24)$ and the cosine function $40\cos(t/24)$. The following two time series are chaotic and have been generated using the Rössler and Lorenz systems, respectively. The final two time series are random and correspond to uniformly and Gaussian distributed white noise, respectively. All time series contain 4000 data points. The figure demonstrates that signals can have the same range but differ in terms of temporal structure (LyE) or they can have different ranges but the same LyE.

In addition, several studies have indicated that these variations have a deterministic origin (Dingwell & Cusumano, 2000; Dingwell & Kang, 2007; Harbourne & Stergiou, 2009; Miller, Stergiou, & Kurz, 2006). Thus, they are neither random nor independent. For instance, although variations between strides during walking appear to vary randomly, with no correlation between the present and future strides, the healthy adult locomotor system actually possesses “motor memory”, such that the fluctuations from one stride to the next display a subtle, “hidden” temporal structure. Mathematical tools, such as entropic or fractal measures or tools developed for the study of deterministic chaos have enabled the evaluation of this temporal structure of variability. From this approach, how human movement evolves over time becomes of importance. Therefore, the focus is not on the standard of performance represented by the average but rather on the exploratory nature of movement, which enhances practice and quality of performance.

From an evaluation perspective, these two approaches are complimentary since each explores different aspects of variability (Harbourne & Stergiou, 2009; Stergiou et al., 2004a). As mentioned above, conventional statistical tools quantify the magnitude of variation in a set of values independently of their order in the distribution; this works properly for linear systems. In contrast, variation in how a motor behavior emerges in time is best captured by tools developed for the study of nonlinear systems. These tools that have been used in the literature for this purpose include approximate entropy, sample entropy, correlation dimension, largest Lyapunov exponent, and detrended fluctuation analysis (Brujin, van Dieën, Meijer, & Beek, 2009; Cavanaugh, Kochi, & Stergiou, 2010; Delignières, Deschamps, Legros, & Caillou, 2003; Donker, Roerdink, Greven, & Beek, 2007; Gates & Dingwell, 2007, 2008; Hausdorff, 2009; Liao, Wang, & He, 2008; Kurz & Hou, 2010; Kurz, Markopoulou, & Stergiou, 2010; Sosnoff, Valentine, & Newell, 2006; Sosnoff & Voudrie, 2009; Stins, Michielsen, Roerdink, & Beek, 2009; Vaillancourt, Sosnoff, & Newell, 2004; Yang & Wu, 2010).

2. Further theoretical developments

There is a growing body of literature showing that the cycle-to-cycle variation seen in a wide variety of physiological systems is nontrivial and may offer insight into the control of these systems (Bassingthwaighte, Liebovitch, & West, 1994). This intrinsic movement variability is highly suggestive of a fundamental feature of the neural control of movement. Cai et al. (2006) provided some evidence with respect to this issue by studying the ability of spinal mice to learn to step. In their protocol, variation was permitted by applying an assist-as-needed mode of control of a robotic arm attached to the ankle of each hindlimb. The results showed that when the intrinsic variability was overridden (e.g., when a fixed pattern is imposed with no variability allowed), learning of a task was suboptimal relative to the condition when the training is assist-as-needed. The authors suggested that training with robotic control algorithms that provide a soft assist-as-needed control permits the intrinsic variability that characterized any neural controlled movements. This study provided strong evidence that a fundamental strategy of the neural control of a given motor task (stepping) is to incorporate a degree of variability in the sensorimotor pathways. Importantly, when the system is forced to adapt a rigid behavior, it produces suboptimal results. From a clinical point of view, these findings highlight the importance of variation of stepping kinematics as a feature of optimizing relearning to step.

Further evidence for the association of variability and health comes from research on higher neural functions and their association with gait. To better understand the underlying mechanisms of gait variability in community-dwelling older adults, Rosano, Brach, Studenski, Longstreth, and Newman (2007) investigated the relationships between the variability of different aspects of gait and subclinical brain vascular abnormalities in adults who are free of neurological diseases. Increased variability of step length was associated with greater prevalence of infarcts, including infarcts in the basal ganglia. It was also associated with greater white matter hyperintensities severity, independent of age, gender, cognitive function and cardiovascular disease. Importantly, these brain abnormalities were associated with increased movement variability in comparison with optimal healthy behavior. This is in the opposite direction of what was presented in the previous paragraph. Here we do not have rigidity and absence of movement variability, which is undesirable, but we have too much variability, which is also undesirable. Can it be possible then that healthy movement variability is associated with

an optimal state, which is in between too much and too little? Interestingly, Rocchi, Chiari, and Horak (2002) demonstrated that variability of postural sway was larger than normal in patients with Parkinson's disease without the effects of drugs and even larger with levodopa. However, with deep brain stimulation these patients exhibited smaller than normal variability of postural sway. Practically, the normal healthy controls were in between all these conditions suggesting that too much or too little were not optimal. Similarly, in Brach, Berlin, VanSwearingen, Newman, and Studenski (2005), elderly individuals with extreme step width variability (either low or high step width variability) were more likely to report a fall in the past year than those with moderate step width variability. Therefore, either too little or too much step width variability was associated with falls.

Recently, it has been demonstrated that temporal variations in biological signals, even though they appear no different from random noise, exhibit deterministic patterns. These patterns have been defined as chaotic (Fig. 2: middle panel) and can have significant implications for medicine. For example, heart rhythms in which the variation in the time interval between subsequent QRS waves is either periodic or random (Fig. 2) have been associated with heart attacks (Denton, Diamond, Helfant, Khan, & Karagueuzian, 1990; Glass & Mackey, 1988). Conversely, chaotic heart rhythms are related to healthy states. Similar results have been found in other biological signals. These studies employed more advanced tools to describe conditions in which more conventional, linear techniques appeared inadequate, confounding scientific study and the development of meaningful therapeutic options. Research along these lines include investigations of heart rate irregularities, sudden cardiac death syndrome, blood pressure control, brain ischemia, epileptic seizures, and several other conditions (Amato, 1992; Buchman, Cobb, Lapedes, & Kepler, 2001; Faure & Korn, 2001; Garfinkel, Spano, Ditto, & Weiss, 1992; Goldberger, Rigney, Mietus, Antman, & Greenwald, 1988; Goldstein, Toweill, Lai, Sonnenthal, & Kimberly, 1998; Korn & Faure, 2003; Lanza et al., 1998; Orsucci, 2006; Sltzky, Cvitanovic, & Mogul, 2001; Toweill & Goldstein, 1998; Wagner, Nafz, & Persson, 1996), aiming to understand their effect on the human physiology and eventually develop prognostic and diagnostic tools. Based on such investigations, the presence of chaotic temporal variations in the steady state output of a healthy

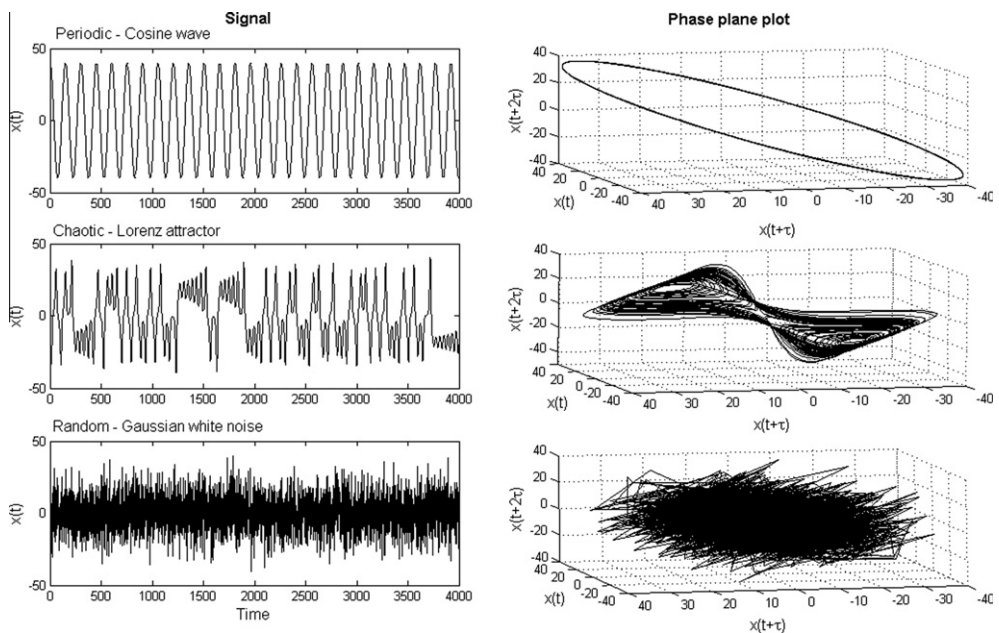


Fig. 2. Periodic, chaotic, and random time series and their corresponding three-dimensional phase space plots. The phase space plot is obtained by plotting the original time series and its time delayed copies. This figure provides an illustration of a chaotic signal and how it is different from other signals.

biological system can represent the underlying physiologic capability to make flexible adaptations to everyday stresses placed on the human body (Lipsitz, 2002; Lipsitz & Goldberger, 1992). Importantly, there are certain benefits for the nervous system for adopting chaotic regimes allowing a wide range of potential behaviors. This leads to healthy biological systems that are adaptable and flexible in an unpredictable and ever-changing environment (Faure & Korn, 2001; Korn & Faure, 2003). But what happens in diseased states? In which way are these deterministic properties of a healthy system and, in consequence, its behavior affected? Two main propositions have been developed recently to address these questions.

The first proposition has been stated by Lipsitz and Goldberger (1992) who proposed that healthy systems are characterized by the physiologic capability to make flexible adaptations to everyday stresses placed on the human body. In the case of human gait this is demonstrated as we observe variations over small time scales (i.e., a few strides) being statistically similar to those over larger and larger time scales (i.e., hundreds and even thousands of strides). The use of scaling analysis techniques (e.g., detrended fluctuation analysis) revealed that fluctuations in stride interval time series display long-range correlations (Hausdorff, Peng, Ladin, Wei, & Goldberger, 1995), and these correlation properties evolve during childhood (Hausdorff, Zeman, Peng, & Goldberger, 1999) and degrade both with physiologic aging and with certain degenerative neurological diseases (Hausdorff, 2009; Hausdorff et al., 1997). This breakdown in physiologic capability may be associated with the degradation of these properties (Peng, Hausdorff, & Goldberger, 2000). Thus, it is proposed that there is a positive correlation between this physiologic capability and healthy motor performance. On the other hand, it is proposed that there is a negative relation between physiologic capability and aging (Lipsitz & Goldberger, 1992).

However, not all studies are consistent with this proposition. It appears that the task goal plays a critical role in shaping the nature of the differences that arise with aging and disease (Vaillancourt & Newell, 2002, 2003). For instance, postural studies showed that the direction of change as a function of aging is opposite in the actions of posture and locomotion (Hausdorff et al., 1997; Newell, 1998). This contrasts with the above proposition. Rather it is the loss of adaptability of the intrinsic dynamics that is the key feature of change as a function of age. In an empirical examination of the “loss of adaptability hypothesis”, Vaillancourt and Newell (2003) examined whether age-related differences in the time and frequency structure of force output are dependent on task demands. They found that the specific direction of the change is dependent on the task demands and reflects the role of intentions and goals in organizing the dynamics of the motor output (Vaillancourt & Newell, 2002).

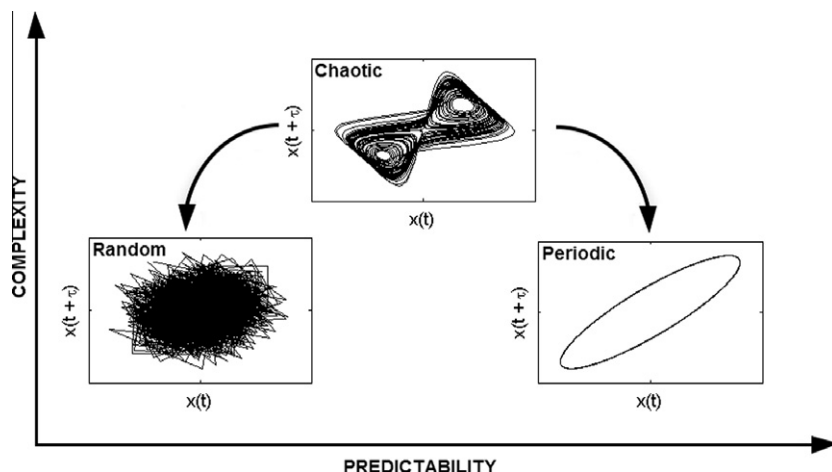


Fig. 3. Theoretical model of optimal movement variability illustrated using the signals from Fig. 2. For clarification purposes, the signals presented (“Periodic”, “Chaotic”, and “Random”) are not the only three possibilities. Behavior in terms of variability should be viewed in a continuum as being more or less predictable (on the X-axis) or exhibiting or not chaos (on the Y-axis). Thus, there are many other possibilities.

More recently, and in an effort to bring together the above propositions, Stergiou et al. (2006) and refined in Harbourne and Stergiou (2009), proposed a new theoretical model to explain movement variability as it relates to motor learning and health. This model is based on the idea that mature motor skills and healthy states are associated with optimal movement variability that reflects the adaptability of the underlying control system. The principle of optimality in movement variability is pioneering in the sense that it relates in an inverted U-shape relationship the presence of chaotic temporal variations in the steady state output of a healthy biological system with the concept of predictability (see Fig. 3). Practically at this optimal state of movement variability the biological system is in a healthy state and is characterized by exhibiting chaotic temporal variations in the steady state output (i.e., the uppermost point along the inverted U-shaped function), attaining high values only in the intermediate region between excessive order (i.e., maximum predictability) and excessive disorder (i.e., no predictability). Thus, this variability has deterministic structure and reflects the adaptability of the system to environmental stimuli and stresses. Decrease or loss of this optimal state of variability renders the system more predictable, rigid and with a robotic type of motor behavior. This is fairly similar to the Lipsitz and Goldberger hypothesis. However, our research group also added that increases beyond optimal variability render the system more noisy and unpredictable, similar to what is observed for example in a very frail elder or a drunken sailor walking. Both situations result in decreased adaptability to perturbations and are associated with lack of health (see Fig. 3).

Recent empirical research in motor control supports the theoretical model of optimal movement variability. Among the most recent investigations, the study of Cignetti, Schena, and Rouard (2009) gives an illustration of the flexibility capabilities of the neuromuscular system to counteract the fatigue induced by a cross-country skiing effort. The study exemplifies the model developed by Stergiou et al. (2006). In this investigation, both the inter-cycle variability in cross-country skiing gait and its evolution with fatigue were examined to understand the flexibility capabilities of the neuromuscular system. The fluctuations of the limb movements of the skiers were not random but displayed a chaotic behavior, reflecting flexibility to adapt for possible perturbations present during skiing. This behavior degraded with fatigue through increased and more random fluctuations.

In the theoretical model of optimal movement variability, it is also proposed that motor development and learning processes obey these principles. In other words, the development of healthy and highly adaptable systems relies on the achievement of the optimal state of variability. Alternatively, abnormal development may be characterized by a narrow range of behaviors, some of which may be rigid, inflexible and highly predictable or, on the contrary, random, unfocused and unpredictable. Motor disabilities many times are described as such. In accordance with this proposition the authors also suggest that the goal of neurologic physical therapy and performance in sport activities should be to enhance the development of this optimal state movement variability by incorporating a rich repertoire of movement strategies, which can be achieved by implementing a multitude of experiences. Several such examples are given later in the review.

3. Variability does not equate with stability

Before we will continue with the presentation of our experimental work, which is based on the above proposition, we would like to address an issue where we believe that there is confusion in the literature. As mentioned above, variability was interpreted traditionally as noise superimposed upon a signal, where the signal is the intended movement and the variability is random noise about this intended movement (Newell & Corcos, 1993). The focus of this approach was to quantify the amount of variability associated with the movement of interest. Typically, the amount of variability was assessed by the standard deviation. Increased amount of variability found in postural sway as well as in gait has been linked to an increased risk of falling in the elderly (Demura, Kitabayashi, & Aoki, 2008; Maki, 1997). As increased amount of variability has been reported as a predictor of risk of falling, it has been assumed that variability and stability are negatively correlated, where increases in the amount of variability were assumed to equate with increases in instability. However, evidence shows that a moving system (e.g., a swaying body during posture or a moving body during gait) with large variability implies neither a highly stable system nor poor stability (Cavanaugh, Guskiewicz, &

Stergiou, 2005; Cavanaugh et al., 2005, 2006). For instance, a trained athlete can balance without falling while standing one-legged on a fully inflated soccer ball. Clearly, this demonstrates exceptional stabilizing capacity despite the fact that center-of-pressure measurements under the ball will demonstrate large movement variability. This simple example illustrates that variability does not necessarily predict instability.

As recently stated by Granata and England (Reply to the Letter to the Editor from Beauchet, Allali, Berrut, & Dubost, 2007), “it is incorrect to assume that variability can be equated to the biomechanics of stability”. According to their view, “variability” refers to the ability of the motor system to reliably perform in a variety of different environmental and task constraints, while “stability” refers to the dynamic ability to offset an external perturbation. Thus, variability and stability represent different properties within the motor control process. For these authors and others, variability is quantified using measures derived from linear statistics, such as the standard deviation of the mean ensemble curve (Li, Haddad, & Hamill, 2005), whereas stability is quantified using measures derived from nonlinear dynamics. Specifically, local stability is commonly defined as the “inverse of the rate of divergence from the intended trajectory after a small perturbation”, as quantified by the use of the largest Lyapunov exponent (Buzzi, Stergiou, Kurz, Hageman, & Heidel, 2003; Dingwell & Cusumano, 2000; Dingwell & Kang, 2007; Hurmuzlu & Basdogan, 1994; Hurmuzlu, Basdogan, & Stoianovici, 1996; Kang & Dingwell, 2006a, 2006b; Stergiou et al., 2004a). By the same token, stability can be inferred via the “presence of long-range, fractal correlations”, as quantified by the use of fractal analysis (e.g., detrended fluctuation analysis) (Hausdorff, 2009; Hausdorff et al., 1995, 1996, 1997, 1999, 2000; Jordan, Challis, & Newell, 2006). Thus, stability covers different aspects, that of deviations from deterministic orbits quantified through trajectory divergence (local stability) and that of temporal statistics quantified through correlations and entropies (self-similarity and regularity; see also below).

Presently and after significant work in this area, we humbly believe that the terminology used by human movement scientists to describe their findings should be more specific and straightforward. This is necessitated by our interactions with clinicians and practitioners where simplicity is important in order to establish proper communication and efficient collaboration. To date, there is general agreement that measures for linear systems (indexed by either the standard deviation in absolute terms, or the coefficient of variation in relative terms) quantify the amount or magnitude of the variations present in a time series (e.g., center of pressure oscillations or gait fluctuations), whereas the measures for nonlinear systems (e.g., approximate entropy, sample entropy, correlation dimension, largest Lyapunov exponent, and detrended fluctuation analysis) quantify the structure or organization of the variations present in a time series (i.e., changes observed in gait fluctuations or postural sway oscillations over time). But, there is no reason to infer that stability is uniquely related to any measure, since each of those measures quantifies different aspects of the time-dependent structural characteristics embedded in a given time series. For instance, the largest Lyapunov exponent quantifies the rate at which nearby trajectories from a time series in state space diverge over time; this equals the so-called local stability, i.e., deviation from a certain orbit (Abarbanel, 1996; Rosenstein, Collins, & De Luca, 1993; Wolf, Swift, Swinney, & Vastano, 1985); the approximate entropy quantifies the regularity of a time series (Pincus, 1991; Pincus, Gladstone, & Ehrenkranz, 1991); the detrended fluctuation analysis quantifies the presence of long-range correlations in a time series (Hausdorff et al., 1995). In conclusion, we suggest that interpretation of findings derived from nonlinear dynamics should not be made beyond what the nonlinear measures actually quantify, unless correlated with other measurements (e.g., electromyographic analysis for quantification of muscle fatigue, magnetoencephalographic analysis for direct quantification of cortical activity, etc.).

4. Experimental work from our laboratory exploring the above theoretical frameworks

Armed with the above tools a great number of investigators have explored important questions on variability and sought to provide support for or against the above-mentioned theoretical frameworks. Here we will present some of our work including posture and gait from healthy and pathological populations at different stages of the lifespan.

4.1. Infant motor development

Harbourne and Stergiou (2003) explored the development of independent postural control in sitting. Specifically, they investigated whether developing postural control in sitting has deterministic origins, and if so, how this can be characterized using measures for nonlinear dynamics. Normal infants were examined longitudinally. Postural sway data were collected for ten seconds while the child attempted to maintain sitting postural control on a force platform. The resulting center of pressure (COP) time series during sitting were analyzed which revealed largest Lyapunov exponent values that were significantly different from their surrogate counterparts. This result indicated that the fluctuations observed in the COP time series were not randomly derived, and reflect deterministic processes by the neuromuscular system. The fluctuations in the time series were not noise, but had a structure or order that needed further investigation and description. Results further indicated differences in the largest Lyapunov exponent (LyE) and approximate entropy (ApEn) across the three stages of sitting development, reflecting changes in the dynamics of sitting postural control. The LyE values decreased as the sitting behavior emerged, indicating less divergence in the movement trajectories of the COP. Therefore, as the infants had more experience exploring the sitting position, they increasingly occupied trajectories that were closer together within the state space. The ApEn values also decreased, indicating that the child develops more repeatable movement patterns of the COP, which are most successful to maintain sitting postural control. Based on the findings the authors suggested that a centrally determined program of specific muscle responses is unlikely to provide successful postural control within the changing context of a growing infant. The findings of this study add to the evidence that infants dynamically assemble the sitting posture by originally organizing movement strategies that are more regular and repeatable, thus first allowing control of the degrees of freedom to approximate the skill, and then to explore adaptations to function in the environment.

Toward the goal of better understanding the control strategies that are involved in evaluating postural control during sitting in infants, Harbourne, Deffeyes, Kyvelidou, and Stergiou (2009) performed a principal component analysis as a data reduction method. Four factors were identified: the postural sway area and the amount of variability of that area (Factor 1: range and root-mean-square in the anterior-posterior and medial-lateral directions); the divergence of the sway movement trajectories and the regularity of the sway front-to-back (Factor 2: LyE and ApEn in the anterior-posterior direction); the speed and coordination of the postural sway (Factor 3: postural sway velocity and frequency dispersion); and the divergence of the sway movement trajectories and regularity of the sway side-to-side (Factor 4: LyE and ApEn in the medial-lateral direction). Thus, Factors 1 and 3 included measures of the amount of variation in the postural sway, and Factors 2 and 4 included measures of the temporal organization of these variations. The isolation of the two types of measures into separate factors indicated possible separate features of postural control during sitting in typically developing infants. For example, the measures of the amount of variation did not exhibit loading into different factors by direction (anterior-posterior vs. medial-lateral) suggesting that during development infants may concentrate in a specific direction for exploration. Furthermore, the authors stressed that different measures taken together offer a more comprehensive description of postural control, with the ability to understand specific characteristics in the system. Problems in the system may occur when one or more of several components are compromised. The authors also stressed the importance of this approach in determining the health of the developing postural control system in infants as well as for early diagnosis of postural disorders.

The above two studies suggested that learning of the sitting skill appears to be in line with theoretical suggestions and empirical results obtained when examining the acquisition of motor skills (e.g., Mitra, Riley, & Turvey, 1997; Newell & Vaillancourt, 2001). Specifically, the maintenance of equilibrium in the sitting skill is based on a problem of compressing a high-dimensional system composed of many components (e.g., neural, muscular and segmental components) into a low-dimensional system (such as in chaos which arises specifically in very low-dimensional nonlinear systems that are deterministic; Strogatz, 1994) with only few macroscopic or collective variables that need to be controlled. This could allow the attainment of the postural performance since these variables govern the coordination of the different system components. In this way, it is then possible to assume that the

rationale of decreasing the degrees of freedom and then the number of controlled collective variables makes the achievement of the postural control easier for the infants.

Our methodological approach was further used by [Deffeyes, Harbourne, Kyvelidou, Stuberger, and Stergiou \(2009\)](#) to investigate how sitting postural sway in typically developing infants differs from developmentally delayed infants. Infants in the developmentally delayed group were diagnosed with cerebral palsy, or else were developmentally delayed and at risk for cerebral palsy. Motor development in infants with cerebral palsy is delayed, meaning that developmental milestones such as sitting, standing, or walking may occur later than in infants with typical development, and in severe cases these milestones may never be met ([Fedrizzi et al., 2000](#); [Wu, Day, Strauss, & Shavelle, 2004](#)). The results from [Deffeyes, Harbourne et al. \(2009\)](#) showed that the LyE was the only parameter of COP time series that revealed significant differences ($p < .000$) between infants with typical versus delayed development. The authors suggested that the infants with delayed development appear to further minimize the fluctuations that are present in their postural sway patterns indicating more rigid control than infants with typical development. If it is assumed that the infants with typical development have better motor control, then it can be suggested that these infants are exploring a wider variety of solutions to postural control. It can also be assumed that infants with delayed development are further freezing degrees of freedom to have fewer control parameters to manipulate as they maintain upright posture. These results may seem contradictory with the results presented by the study of [Harbourne and Stergiou \(2003\)](#). However, this is not the case if we consider them under the prism of the theoretical framework of optimal state of variability. The infants with delayed development behave in a more robotic and periodic fashion ([Fig. 3](#)) than healthy typically developing infants. Furthermore, the healthy infants seem to “live” or move between randomness and optimal variability as they explore effective strategies for postural control. Importantly, the nonlinear measure of LyE has the potential to add the specificity of diagnosis in the early months of life, when most standardized tests of infant development have little predictive value.

Most recently, [Deffeyes et al. \(2009\)](#) examined the utility of different entropy algorithms to further explore if different control strategies exist between typically developing infants and developmentally delayed infants. Postural sway data were acquired while infants were sitting on a force platform. Two types of entropy measures were used: (1) symbolic entropy (SymEn), (2) a new asymmetric entropy (SymEn) measure, and (3) ApEn. For each method of analysis, parameters were adjusted to optimize the separation of the results between the infants with delayed development and the infants with typical development. The method that gave the widest separation between the two groups was the asymmetric SymEn method, which [Deffeyes et al. \(2009\)](#) developed by modification of the SymEn algorithm. The ApEn algorithm also performed well, using parameters optimized for the infant sitting postural sway data. As in the previous study the infants with delayed development were found to have more regular patterns of postural sway, while the infants with typical development are seen to have more information entropy in their movement. This further supported the conclusions from the previous study and suggested that the development of a postural control strategy involves an exploration of many possible solutions to arrive at a control strategy with an optimal state of variability. The authors further suggested that infants with typical development appear to be exploring more motor strategies, giving rise to the development of chaotic temporal variations in their postural sway.

Critical to the above studies was to establish the reproducibility of these measures for infant sitting posture. Therefore, [Kyvelidou, Harbourne, Stuberger, Sun, and Stergiou \(2009\)](#) and [Kyvelidou, Harbourne, Shostrom and Stergiou \(2010\)](#) investigated the intra-session and inter-session reliability of linear and nonlinear measures when used to analyze COP time series during the development of infant sitting postural control in both typically developing and developmental delayed infants. Overall, the results showed that the evaluation of COP time series using linear and nonlinear measures is a reliable method for quantifying incremental change across the development of sitting postural control in both typically developing infants and in infants with or at risk for cerebral palsy, and therefore the efficacy of therapeutic interventions directed at improving the sitting postural abilities in infants with motor developmental delays.

Taken together, the findings from the motor development investigations conducted by our research group identified control strategies that point towards a new approach with respect to therapy ([Harbourne & Stergiou, 2009](#)). In this approach, the therapist assumes that the general rule for

the patient is to optimize variability of movement for improving functional mobility and therefore health. This will include keeping the patient in a state of dynamic equilibrium during therapy sessions. Additionally, the therapist uses the strategy of providing only information for the patient on how to do a task if the patient does not have a way to get the information. The rationale is that variability is encouraged if the patient seeks information independently, and the patient is kept in a dynamic state. The therapist does not focus on a particular movement form or strategy, but rather allows the patient to discover that enhanced deterministic variability of various movements has an inherent value in promoting success during a task. Importantly, the findings from our motor development studies recommend that measures for studying nonlinear dynamics reveal that exhibiting chaotic temporal variations is probably inherent in normal variations, indicating features of motor control that are important for physical therapists to measure as they implement intervention. The application of principles based on our theoretical framework capitalize on concepts and measures of nonlinear dynamics to provide with innovative approaches to guide physical therapist practice and research in motor development.

4.2. Sports medicine

Over the past ten years, we have conducted several studies on musculoskeletal injuries such as anterior cruciate ligament injury or brain injury such as cerebral concussion.

4.2.1. Anterior cruciate ligament: A sport-related injury to the neuromuscular system

Anterior cruciate ligament (ACL) is the most commonly injured ligament in sports (Zarins & Adams, 1988) and its reconstruction is a common operation among orthopaedic surgeons who are involved in sports medicine. The purpose of ACL reconstruction and subsequent rehabilitation is to restore complete and normal functionality of the knee joint in terms of muscular strength and stability (Chmielewski, Rudolph, & Snyder-Mackler, 2002). The ability of the surgical procedure to achieve complete and normal functionality of the knee joint is assessed with either static measures (i.e., KT-1000, pivot-shift test) or questionnaires (i.e., Lysholm score) or combinations of both (i.e., International Knee Documentation Committee score). However, none of these measures is a true assessment of the dynamic functionality of the reconstructed knee under low demanding activities, such as walking, or higher demanding activities, such as those encountered in sports. To address this critical knowledge gap, our research group conducted explored variability during gait to assess the efficacy of anatomical ACL reconstruction for restoring normal knee mechanics and preserving long-term joint health.

In our first study, Stergiou, Moraiti, Giakas, Ristanis, and Georgoulis (2004b) investigated the effect of walking speed on the dynamic function of the ACL deficient knee using nonlinear measures. Dynamic function of the knee was assessed in terms of the structure of the variations that exist in the natural stride-to-stride movements of the knee. Individuals with unilateral deficiency walked on a treadmill at different speeds while kinematics was collected. The deficient knee was found to have significantly larger LyE values than the intact contralateral knee. Furthermore, increases in walking speed did not affect these differences in the LyE values. However, these results were limited because comparisons with healthy controls were not included. It is quite possible that the intact contralateral knee is not absolutely healthy and several compensations occur leading to contradictory results.

Therefore, Moraiti, Stergiou, Ristanis, and Georgoulis (2007) extended the above research by investigating the temporal structure of the variations present in the ACL deficient knee as compared to that of a healthy control knee during walking. Individuals with unilateral ACL deficiency and healthy controls walked at their self-selected speed on a treadmill, while lower extremity kinematics was collected for 80 consecutive strides. The ACL deficient knee exhibited smaller LyE values than a healthy control knee, indicating less divergence in the flexion-extension movement trajectories of the deficient knee. The results also verified the hypothesis made in the previous study that the intact contralateral knee is not absolutely healthy. The fact that the ACL deficient knee exhibited smaller LyE values than the healthy control is likely non-desirable since it may represent a decrease or loss of the optimal state of variability (which is exhibited by the healthy controls) rendering the system more predictable, periodic and with a rigid type of motor behavior. Neurologically this can be explained if we consider that the ACL plays an important role in knee function because of its mechanical properties

and the mechanoreceptors that exist in it (Johansson, Sjölander, & Sojka, 1991; Solomonow & Krogsgaard, 2001). For instance, it has been shown that activations of the ACL mechanoreceptors induce hamstring contraction resisting anterior tibial translation (ACL-hamstring reflex), in both animals and humans (Dyhre-Poulsen & Krogsgaard, 2000; Fujita, Nishikawa, Kambic, Andrich, & Grabiner, 2000; Tsuda, Okamura, Otsuka, Komatsu, & Tokuya, 2001). It has been proposed that the loss of proprioceptive input from the mechanoreceptors that exist in the ACL may lead to changes in the central nervous system which, in turn, leads to the development of altered muscle patterns and postural synergies (Courtney, Rine, & Kroll, 2005; Di Fabio, Graf, Badke, Breunig, & Jensen 1992; Valeriani et al., 1996). It has been further suggested that this kind of injury might be regarded as a neurophysiological dysfunction, not being a simple musculoskeletal injury (Kapreli & Athanasopoulos, 2006). Therefore, ACL deficiency can lead to altered somatosensory input, which results in decline in the system's flexibility and narrowed functional responsiveness reflected as rigidity.

Importantly, degeneration of the knee joint and eventually development of osteoarthritis has been associated with ACL deficiency. Longitudinal follow-up studies have shown that ACL deficiency leads to the development of chondral injuries (Mankin, 1982), meniscal tears, degeneration of the articular cartilage and eventually post-traumatic arthritis (Fithian, Paxton, & Goltz, 2002; McDaniel & Dameron, 1983; Noyes, Matthews, Mooar, & Grood, 1983; Noyes, Mooar, Matthews, & Butler, 1983). Therefore, it is possible that the increased behavioral rigidity found in these patients could lead to continuous systematic loading of the same areas on the articulating surfaces of the bones resulting over time in these pathological results. The absence of flexibility in the system does not practically allow for the loading to be more dispersed and over time the result is osteoarthritis. This hypothesis, however, needs to be further explored via additional research. Nonetheless, from this theoretical standpoint, initial experimental work has demonstrated the ability of nonlinear analysis to provide insight into specific causal physiological mechanisms of motor pathology.

Based on the above, it can then be asked if ACL reconstruction can restore the LyE values to normative levels. Thus, Moraiti, Stergiou, Vasiladias, Motsis, and Georgoulis (2010) investigated the functional outcome after ACL reconstruction using bone-patellar tendon-bone (BPTB) and quadrupled semitendinosus and gracilis tendon (ST/G) autografts by evaluating the stride-to-stride fluctuations present in the knee flexion-extension time series. Patients with BPTB and patients with ST/G ACL reconstruction, two years postoperatively, and healthy controls walked on a treadmill at their self-selected pace, while lower extremity kinematics was collected for 100 consecutive strides. Both the BPTB and the ST/G groups had significantly larger LyE values than the healthy controls, even though clinical outcomes indicated complete restoration. No differences were found between the BPTB and the ST/G LyE values. Practically, the ACL reconstruction using either BPTB or ST/G renders the system more noisy and unpredictable as compared to healthy controls. This is probably because ACL reconstruction cannot restore the proprioceptive pathways found in a healthy knee (Johansson, Sjölander, & Sojka, 1990; Solomonow et al., 1987). These results may indicate that the current reconstruction techniques or the grafts used are not sufficient in restoring knee kinematic variability to normal (i.e., absence of complete reinstatement of the actual anatomy of the ACL; Arnoczky, Tarvin, & Marshall, 1982).

Behaviorally, the findings from the above studies indicate that the ACL deficient individual exhibits a more predictable and rigid behavior with respect to their knee movement variability. On the other hand, after ACL reconstruction the knee demonstrates a more noisy and unpredictable behavior. Clinically, these results can be explained as follows. An individual that knows that the ACL is reconstructed feels "secure" to increase and add extra movement. However, since the proper proprioceptive channels are not there, the temporal structure of the stride-to-stride variations of the knee is not restored to normative levels. On the contrary, the rigidity found in the ACL deficient knee signifies that ACL deficient patients are more "careful" in the way they walk trying to eliminate any extra movements. These behavioral phenomena are well described by the theoretical proposition of the optimal movement variability. Specifically, healthy gait is characterized by an optimal state of movement variability. This state allows for flexibility, adaptability, and ability to respond to unpredictable stimuli and stresses. In our above experiments this is the state that is exhibited by our healthy controls. Decrease or loss of this optimal state is associated with a system that is more rigid and very repeatable, as in the ACL deficient knee. Increase beyond optimal variability is associated with a system, which is noisy and irregular, as in the ACL reconstructed knee (Fig. 3).

Furthermore, the impaired variability noted in the reconstructed knee using either graft could be the reason that ACL reconstruction is still linked to susceptibility to further sports injury and development of future pathology without alleviating the problems that were mentioned above for the ACL deficiency. Specifically, long-term follow-up studies have shown an increased incidence of osteoarthritis in ACL-reconstructed knees (Pinczewski et al., 2007). The studies described above used ACL reconstruction techniques representative of the standard of care for the last fifteen years (i.e., single graft bundle, typically transtibial drilling of femoral tunnel). Cadaver and in vivo studies have highlighted limitations of this approach for restoring normal knee anatomy and function, and led to a surge of interest in anatomical ACL reconstructions that attempt to better reproduce its actual two-bundle anatomy and insertion sites. The approach used here can provide similar insights for the efficacy of these new surgical techniques for ACL reconstruction for restoring normal knee movement patterns and preserving long-term joint health.

4.2.2. Cerebral concussion: An example of sport-related injury to the brain

Complete recovery of postural control after cerebral concussion is an important determinant of an athlete's readiness to return to competitive activity. Athletes who return to competitive activity too early after injury are potentially more vulnerable to injury recurrence, the consequences of which can be dramatic (Kelly et al., 1991). The assessment of postural control provides an indirect means of identifying concussion-related neurophysiological abnormality (Guskiewicz, Ross, & Marshall, 2001). Postural control traditionally has been characterized according to a biomechanical framework as postural balance. Changes in postural control in athletes after cerebral concussion previously have been measured with a metric known as the equilibrium score resulting from the Sensory Organization Test (SOT) (Guskiewicz, 2002; Guskiewicz, Riemann, Perrin, & Nashner, 1997; Guskiewicz et al., 2001). This score estimates the maximum anterior-posterior angular displacement of the whole body center of gravity based on the range of the anterior-posterior COP displacement. Higher equilibrium scores are derived from lower amplitude COP displacement, thereby assuming greater postural stability. Several studies have raised the possibility that traditional postural stability measures (such as the SOT equilibrium scores; Guskiewicz, 2002) may not be capable of detecting subtle changes in postural control.

In response to this concern, Cavanaugh et al. (2005) recently investigated whether ApEn could detect changes in postural control in athletes with normal postural stability after cerebral concussion. COP data were collected from NCAA Division I (USA) athletes prior to and within 48 hours after injury. After injury, athletes displayed normal postural stability equivalent to preseason levels. For comparison, COP data also were collected from healthy non-athletes on two occasions. Compared to healthy controls, COP oscillations among athletes generally became more regular (lower ApEn value) after injury despite the absence of postural instability. For anterior-posterior time series, declines in ApEn values were much larger in SOT conditions 1 (eyes open on a firm surface) and 2 (eyes closed on a firm surface) than for all other conditions. For medial-lateral time series, ApEn values declined after injury in all sensory conditions. Thus, if an investigator's goal is solely to assess changes in the variability of COP oscillations after cerebral concussion, standing quietly with eyes open and eyes closed on a stable platform may be the only sensory condition that would be needed to evaluate. Overall, these findings provided preliminary evidence that ApEn could be a sensitive indicator of change in postural control in the acute stage after concussion. However, the authors also wanted to identify why ApEn seems to be sensitive to these changes.

Therefore, Cavanaugh et al. (2006) examined the post-concussion recovery of postural control using ApEn. Collegiate athletes from whom COP and symptom data were collected at preseason, less than 48 hours after injury, and 48 to 96 hours after injury, were included in the analysis. Compared with the healthy preseason state, ApEn values for the anterior-posterior and medial-lateral time series declined immediately after injury in both steady and unsteady injured athletes. The decline in ApEn values after concussion reflects changes in the neurophysiological and mechanical constraints on postural control. Diffuse axonal injury reduces and distorts the interactions among neurons in the brain (McCrory, Johnston, Mohtadi, & Meeuwisse, 2001), thereby increasing the regularity of cortical oscillations (Pincus, 1995) that are subsequently manifested in more regular patterns of COP oscillation. Increased co-contraction of the lower extremity musculature is also generated by injured athletes

in an attempt to gain control over postural sway. This mechanism can also result in more regular COP oscillations. Above and beyond, the positive relationship between ApEn values and equilibrium scores indicated that larger amplitude COP oscillations (diminished postural control reflected in a lower equilibrium score) tended to be more regular (lower ApEn values), whereas lower amplitude COP oscillations (better postural control reflected in a higher equilibrium score) tended to be more noisy (higher ApEn values). It appears, therefore, that effective postural control is achieved through relatively unconstrained, more irregular patterns of motor output. The ApEn and the equilibrium score have distinct theoretical constructs. ApEn quantifies regularity in the system output to provide clues to underlying system organization (Pincus & Goldberger, 1994). The ApEn algorithm is a highly iterative process that analyzes the recurrent nature of short sequences of data points considered incrementally throughout a time series. In contrast, the equilibrium score provides little insight into the evolving patterns of variation in postural control performance during the course of a trial. Equilibrium scores are calculated using only two COP data points, the maximum and the minimum, regardless of when they occur. As a biomechanical measure, the resulting range of COP displacement reflects only the amount of variability in the system output.

Importantly, the ApEn provides a theoretically distinct and valuable measurement alternative that may prove effective for reducing uncertainty in the return-to-play decision. Another very interesting finding of the above study was that, at 48 to 96 hours after injury, ApEn values for the medial-lateral time series remained significantly depressed, even among athletes whose initial postural instability had resolved. In other words, the effects of cerebral concussion on postural control appeared to persist for longer than 3 to 4 days, even among athletes with no clinical signs of unsteadiness. This finding contrasts with the SOT equilibrium score data that demonstrated that postural instability generally resolves within that time frame allowing athletes to return to sports (Guskiewicz et al., 1997, 2001; McCrea et al., 2003; Riemann & Guskiewicz, 2000). It is then possible that the documented recurrence of cerebral concussions is due to undetected pathology that ApEn is more sensitive to identify (Cavanaugh, Guskiewicz, & Stergiou, 2005).

Collectively, the above findings support the theoretical model of optimal movement variability, indicating that effective postural control in quiet standing is achieved via relatively unconstrained patterns of motor output. As sensory information was withdrawn or degraded, COP oscillations became more regular. Pincus (1994) and Pincus and Keefe (1992) gave heuristic support for the idea that systems with a relatively limited number of viable interconnections among components may generate more regular output. Newell (1998) proposed a similar idea using a degrees of freedom theoretical framework. Accordingly, either fewer or more poorly organized degrees of freedom reduce the adaptive capability of the individual (Newell, van Emmerik, & Sprague, 1993). Together, these hypotheses suggest that a healthy postural control system, because of numerous interconnections among its components, is capable of adapting to a wide variety of task and environmental demands. Hence, when the system is allowed to operate with minimal constraints (e.g., at rest during quiet standing under normal sensory conditions), the system (i.e., COP oscillations) output appears to fluctuate in relatively random fashion, presumably reflecting the readiness of the system to rapidly respond to perturbation. In the presence of injury, however, normal interconnections among system components would be compromised, thereby reducing the motor flexibility and adaptability of the system. As a result, fluctuations in the system output at rest would be more constrained, appearing more regular. Consistent with the aforementioned hypotheses, the removal of accurate sensory feedback not only made it more difficult for individuals to precisely control body position, but also artificially constrained interactions among control system components, producing more predictable oscillations in system output (Cavanaugh et al., 2005; Cavanaugh, Guskiewicz, & Stergiou, 2005).

Based on the above presented studies, future investigations in the postural control after cerebral concussion could explore the following questions: “How long ApEn values remain depressed after injury?”, “Which factors correlate with the eventual return of ApEn values to pre-injury levels?”, and “Which specific neurophysiological or mechanical mechanisms explain the changes in regularity of postural sway after concussion?”. These investigations can lead to the determination of whether the changes in the ApEn values after injury are associated with an increased risk of recurrence of cerebral concussion.

4.3. Chaos in passive dynamic gait models

Full and Koditschek (1999) suggested that the multifactorial nature of locomotion can be approached by using simple models or templates that can be made to resolve the redundancy of multiple legs, joints and muscles by seeking synergies and symmetries. Using this approach, our group sought to identify a template that can exhibit chaos in its gait variability. Such a template can verify that chaos can be present in the fluctuations that are present from one step to the next during locomotion, and then can be used to investigate how chaos in gait can be controlled.

A relatively simple model that has been used as a template to address questions about the biomechanical requisites and energetics of bipedal human locomotion is the passive dynamic walking model that walks down a slightly sloped surface (Garcia, Chatterjee, Ruina, & Coleman, 1998; Goswami, Thuijlot, & Espiau, 1998; Kuo, 2001; Kuo, 2002; McGeer, 1990). Garcia et al. (1998) demonstrated that a simple passive dynamic walking model can exhibit a cascade of period doublings in the walking pattern. They noted that the distances between consecutive period doublings appear to converge to the Feigenbaum constant (4.669201...). This suggested that a passive dynamic walking model might exhibit a chaotic bipedal locomotive pattern (Alligood, Sauer, & Yorke, 1997). However, Garcia et al. (1998) did not examine or prove the presence of chaos *per se* in the model's locomotion. Nor did they identify which ramp angle is associated with the onset of a chaotic walking pattern. Kurz, Stergiou, Heidel, and Foster (2005), using simulations of the model, were the first to identify that as the ramp angle was increased, a cascade of bifurcations were present in the model's locomotive pattern that lead to a chaotic attractor from $0.01839 \text{ rad} < \text{ramp angle} < 0.0189 \text{ rad}$. These results provided evidence that such a model can be used as a template for exploring the biomechanical control parameters responsible for chaos in human locomotion.

Subsequently our group proceeded to investigate how the presence of chaos on our template can be controlled. In two subsequent studies, Kurz and Stergiou (2005, 2007a) demonstrated that implementing “muscles” in the model in the form of hip joint actuations during the swing phase can provide slight perturbations to the unstable manifolds of points in a chaotic system that will promote the transition to new stable behaviors embedded in the rich chaotic attractor. Stable behavior here is when the passive walker does not fall down. The simulations indicated that systematic alterations of the hip joint actuations resulted in rapid transitions to any stable locomotive pattern available in the chaotic locomotive attractor (Kurz & Stergiou, 2007a). Based on these findings, they investigated the benefits of having a chaotic gait with a biologically inspired artificial neural network (ANN) that employed this chaotic control scheme. The ANN was robust and capable of selecting a hip joint actuation that transitioned the passive dynamic model to a stable gait embedded in the chaotic attractor. Additionally, the ANN was capable of using hip joint actuations to accommodate environments that were previously unstable and to even overcome unforeseen perturbations. These simulations provided with an understanding of the advantages that exist when we have a locomotive system that exhibits chaos and provide insight as to how chaos can be used as an advantageous control scheme for the nervous system (Kurz & Stergiou, 2005).

Similar results as with the hip joint actuation were produced using toe-off impulses that assist the forward motion of the center of mass (Kurz & Stergiou, 2007b). Furthermore, results from human experiments supported the model's prediction that the control of the forward progression of the center of mass influences the gait dynamics. More recently, Kurz et al. (2010) and Kyvelidou, Kurz, Ehlers, and Stergiou (2008) used the passive bipedal walking model to relate attractor divergence and walking balance. Their simulations revealed that attractors that have a greater amount of divergence are more susceptible to falls from external perturbations. They supplemented these results with human experiments where they demonstrated that elderly and patients with Parkinson's disease have walking patterns that are more noisy with increased LyE values than their young healthy counterparts. These results suggested that elderly and patients with Parkinson's disease may have a higher likelihood of falling as predicted by the theoretical framework of the optimal movement variability.

Together, these studies demonstrated that chaos could be a powerful component of the locomotive system. As we mentioned earlier in this review, chaos is necessary for the control of locomotion by allowing the nervous system to rapidly transition to new gaits that are embedded within the chaotic attractor demonstrating healthy flexibility and adaptability. This is reflected in our optimal movement

variability model where deterioration of these properties results in lack of health. As demonstrated in the above studies, joint actuations and mechanical perturbations could be used to rapidly transition to any gait available in the bifurcation map of the passive dynamic walking model. In a similar fashion, humans demonstrate predictable scaling in the chaotic structure of the gait pattern as the dynamics of the locomotive system are assisted and the mechanics of the locomotive system are altered. The above-presented experimental results demonstrate that the presence and the way chaos is being exhibited could be controllable which is fundamentally important for the nervous system.

5. Concluding comments

In conclusion, using analysis for nonlinear dynamical systems to human behavior provides a better understanding of variability and how it relates to pathology. In this context, the theoretical model of optimal movement variability developed by our research group provides the framework for interpreting both simulated and empirical results. Fields studying movement generation, including robotics, psychology, and neuroscience have utilized concepts and tools related to the pervasiveness of variability in biological systems. The concepts of variability and chaotic variations in human movement along with the advanced tools used to measure these concepts open new vistas for research in movement dysfunction and pathology. In this review we described innovations in the exploration of variability and their potential importance in understanding human movement. Far from being a source of error, evidence supports the necessity of an optimal state of variability for health and functional movement. Concepts of and methods used for nonlinear dynamics offer significant application possibilities to guide rehabilitation practice and research in human movement.

Acknowledgments

This work is supported by the NIH (K25HD047194), the NIDRR (H133G040118 and H133G080023) and the Nebraska Research Initiative.

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