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Applying the method of surrogate data to cyclic time series

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Abstract

The surrogate data methodology is used to test a given time series for membership of specific classes of dynamical systems. Currently, there are three algorithms that are widely applied in the literature. The most general of these tests the hypothesis of nonlinearly scaled linearly filtered noise. However, these tests and the many extensions of them that have been suggested are inappropriate for data exhibiting strong cyclic components. For such data it is more natural to ask if there exist any long term (of period longer than the data cycle length) determinism. In this paper we discuss existing techniques that attempt to address this hypothesis and introduce a new approach. This new approach generates surrogates that are constrained (i.e., they look like the data) and for cyclic time series tests the null hypothesis of a periodic orbit with uncorrelated noise. We examine various alternative implementations of this algorithm, applying it to a variety of known test systems and experimental time series with unknown dynamics. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The rationale of surrogate data hypothesis testing is to generate an ensemble of artificial *surrogate* time series that are both "like" the original data and consistent with some null hypothesis. One then applies some test statistic (or indeed a battery of test statistics) to both the surrogates and the original data. If the test statistic value for the data is different from the ensemble of values estimated for the surrogates, then one may reject the underlying null hypothesis as being a likely origin of the data. If the test statistic value for the data is not distinct from that for the surrogates, then one may not reject the null hypothesis.

This scheme was suggested and implemented by Theiler et al. [26] and has been widely applied in the literature. Initially, surrogate methods were intended as a method to check the results of dimensional analysis against the possibility of misdiagnosing a purely random signal as deterministic chaos. Surrogate techniques are now widely applied as a form of hypothesis testing. The essential feature of this methodology is that one must have some algorithm with which to generate the surrogate data and some means to ensure the surrogate data are sufficiently "like" the original (indeed we are yet to define what we mean by "like" [27]) while being consistent with a specified null hypothesis.

The three algorithms proposed in [26] address the three hypotheses of: (0) independent and identically distributed (i.i.d) noise, (1) linearly filtered i.i.d. noise and (2) a static monotonic nonlinear transformation

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of linearly filtered noise. These three algorithms are widely known in the literature as algorithms 0, 1 and 2. In Section 2 we briefly review these algorithms and the major modifications suggested since this original implementation. An extensive review of the basic developments in this field and a summary of their own work has been provided by Schreiber and Schmitz [16].

The three hypotheses addressed by surrogates generated by algorithms 0, 1 and 2 are all some form of linear noise process (albeit with a static nonlinear filter). Often, for experimental systems one observes data with obvious periodic features, and these hypotheses should all be trivially false. In this case it is natural to ask if there is any additional determinism in the system. To this end, Theiler [25] proposed an alternative hypothesis: one tests for long term determinism by shuffling the individual cycles within a time series. More recently, Small et al. [21] outlined an improved algorithm that preserves both stationarity and differentiability while testing a similar hypothesis. The pseudo-periodic surrogate (PPS) algorithm suggested in [21], and the testing and application of it to experimental data, are the major novel contributions of this paper. This algorithm and the hypothesis it addresses will be discussed in Section 3. In Section 4 we present some computational examples of the application of the PPS algorithm.

2. Surrogate analysis: the current state of technology

Before considering the algorithm proposed in [21] it is necessary to examine, in some depth, current surrogate data technologies. The main observations of this section are:

- The three most commonly applied algorithms only provide surrogates to test for linearly filtered noise.
- Two of these algorithms are hampered by technical issues related to the Fourier transformation.
- Solutions to these technical problems are available but require either great computational expense or a reduction in the amount of usable data.
- An alternative is to choose a test statistic that is insensitive to the possible flaws in these algorithms.
- Two alternative algorithms have been suggested that allow for the generic testing of more general null hypotheses [16,18].
- However, both these methods suffer one important flaw—the level of noise added to the surrogates is at the user's discretion.
- An alternative method has been suggested for data exhibiting cyclic behavior; however, its application is limited to data that exhibits sufficiently sharp extrema and sufficient stationarity in the mean [25].

When we come to discuss the PPS algorithm, we will demonstrate that it addresses each of these points and provides a useful test for time series with a cyclic component.

Section 2.1 provides a quick review of the current surrogate technologies. In Section 2.2 we discuss issues concerned with generating constrained realizations (i.e., surrogates that are sufficiently like the data) and pivotal statistics. In Section 2.3 we discuss current techniques that can be employed to test for null hypotheses more general than that of a monotonic nonlinear transformation of linearly filtered noise. In Section 2.4 we discuss the algorithm proposed by Theiler [25] specifically to test for long term determinism in periodic data.

2.1. The state of surrogate technology

The three linear algorithms described by Theiler et al. [26] generate surrogates that preserve certain properties of the data while destroying others. For example, algorithm 0 surrogates are generated by shuffling (randomizing the order of) the data in the original time series. In this way the surrogates have the same rank distribution as the data, but there is no temporal correlation and therefore the surrogates provide a test of the hypothesis of i.i.d. noise. Similarly, algorithm 1 surrogates are generated by shuffling the phases of the Fourier transform of the data—thereby preserving linear correlations (the power spectrum), but destroying any additional (nonlinear) structure. Finally, algorithm 2 surrogates are generated to preserve both the rank distribution and the power spectrum of the data. Representative realizations of each

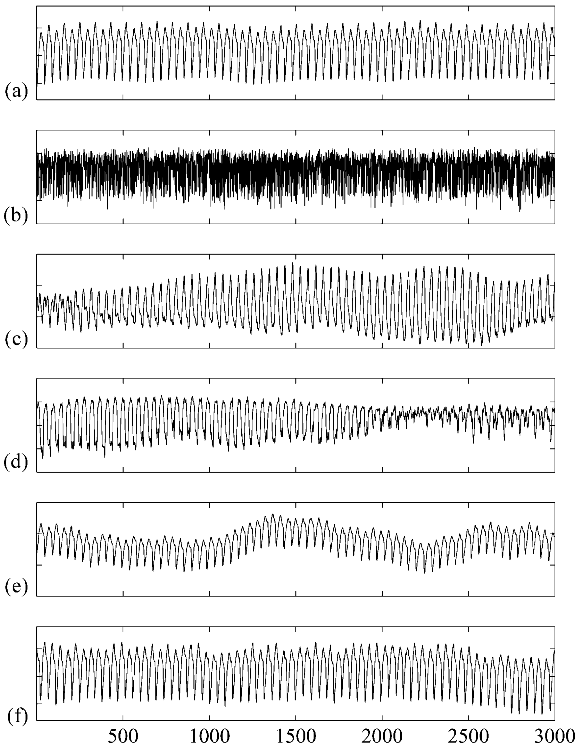

Fig. 1. Examples of typical surrogate data sets. Panel (a) is a human electroencephalogram (ECG) recording during ventricular tachycardia (VT). The purpose of the PPS algorithm is to provide a meaningful surrogate test for experimental data such as these. Panels (b), (c) and (d) are realizations of typical surrogates of that data set generated by algorithm 0, 1 and 2, respectively. These are all linear surrogates and clearly distinct from the data. Panel (e) is a cycle shuffled surrogate following the algorithm suggested by Theiler [25]. Although this surrogate appears qualitatively more like the data than the standard linear surrogates, there is a notable non-stationarity not present in the data. The non-stationarity is a result of the shuffling of individual cycles when peak values do not precisely coincide. Finally, panel (f) is a typical surrogate generated using the algorithm suggested in this paper. This surrogate is qualitatively very similar to the original data. In each panel the horizontal axis is datum number and is identical. The vertical axis units are arbitrary (proportional to surface ECG voltage in (a)).

of the surrogate generation algorithms for strongly periodic data are depicted in Fig. 1.

However, when applying the Fourier transform algorithm, end mismatch can be a problem. For a time series x_t of N measurements, the Fourier transform assumes that x_t is periodic with period N. So if the endpoints are not equal (or nearly equal) then this introduces a discontinuity that manifests as additional high frequency structure in the surrogate (see, for example [16]). This problem may be circumvented by ensuring that the end points do match (by reducing the length of the data appropriately) or with the use of a computationally much more expensive scheme suggested by Schreiber [15] (see also [16]).

The former approach is suggested by Stam et al. [22], who observed that for data exhibiting strong periodic components, the most significant of these algorithms (algorithm 2) does not perform well. The problem is that the length of the time series in unlikely to be an integer multiple of the dominant period, and one therefore observes precisely the end mismatch problem described above. In this case Stam et al. [22] advocate re-sampling or truncating the data so that its length is precisely a multiple of the dominant period.

Even forewarned with the above precautions, algorithm 2 surrogates have been shown to perform poorly. These surrogates are generated by shuffling the phases of the Fourier transform (to preserve linear but not nonlinear correlations) and then rescaling the data (to act as a static nonlinear transformation and preserve the rank distribution of the data). However, the rescaling of the time series does not preserve the power spectrum (nor vice versa), and therefore surrogates only have *approximately* the same power spectrum. While in many cases this may be sufficient for the cautious application of this algorithm [18], one has no guarantee that false results will not be obtained as a result of this approximation. In such cases Schreiber [15] provides a heuristic argument for an iterative version of algorithm 2, which they assert works in a wide variety of test systems. However, there is no guarantee that this iterative procedure will necessarily converge or do so within a feasible time.

2.2. Generating surrogates "like" the data

The effort that has gone into producing surrogates which exactly preserve certain properties of the data is to ensure the surrogates are sufficiently "like" the data. By "like", we mean that the surrogate time series are *constrained realizations* of the hypothesis under consideration. That is, if one were to estimate the specific process (i.e., model parameters) that is most likely to have generated the data and the specific process that is most likely to have generated the surrogate, then these processes (the specific model parameters) would be identical.

The alternative to constrained realizations are *typical realizations*. Typical realizations are simply realizations of any process consistent with the null hypothesis. However, typical realizations are generally perceived as undesirable because there is no guarantee that the distribution of statistic values will be the same for constrained realizations (the processes consistent with the hypothesis that are most like the data) [27]. Hence, although with typical realizations one is able to estimate a distribution of statistic values, one is unable to ensure that the distribution is the one expected *given* the test data.

However, if one chooses a test statistic for which the distribution is independent of the particular realization, then the problem of generating constrained realizations (and many of the technical issues described in the previous section) is immaterial. Such a statistic is said to be *pivotal*. Small and Judd [18] have shown that an unbiased estimate of correlation dimension is a pivotal statistic and demonstrated the application of correlation dimension as a test statistic numerically. Therefore, the effort spent achieving constrained realizations may be better expended trying to find a pivotal test statistic.

For the surrogate algorithm discussed in this paper we show that the realizations are indeed constrained. Furthermore, we apply a host of test statistics, including an estimate of correlation dimension, which we argue is pivotal.

2.3. A null hypothesis of not noise

For data exhibiting cyclic behavior the situation is particularly bad, not only are the algorithms prone to failure and the remedies computationally expensive, but also one expects data that exhibits periodic structure to be inconsistent with the hypothesis of a static monotonic nonlinear transformation of linearly filtered noise (i.e., algorithm 2 surrogates). Therefore, an alternative surrogate test and a more appropriate hypothesis are required. Currently two approaches have been suggested that extend to arbitrary classes of systems and hypothesis tests.

Schreiber [15] has proposed a computationally expensive scheme to generate surrogates consistent with a set of equality constraints. The scheme employs simulated annealing to solve the resultant nonlinear optimization problem (where N observations of the time series are the optimization parameters). Unfortunately, the authors suggest that this approach is rather demanding on computational resources, requiring many hours of computational time on a modern PC to generate a single surrogate [16]. This method also introduces many operator specifiable parameters that not only affect the speed but also the accuracy of the algorithm [16].

Finally, Small and Judd [18] proposed using noise driven iterated predictions of a nonlinear model of the original data as surrogates. Provided one employs a pivotal test statistic, it is not necessary to be concerned about achieving constrained realizations. This modeling process also necessarily introduces many additional parameters, and in this case the hypothesis being tested cannot be specified a priori, but rather is a result of the modeling process.

Both these methods suffer from one significant drawback: the noise level added to the surrogates needs to be specified. In the approach of Small and Judd [18], this noise level is an explicit parameter of the iterated prediction scheme. An appropriate value of noise may be suggested by the modeling procedure, but either too much or too little noise is likely to lead to misleading results. [1] In the approach suggested by Schreiber [15], the speed of the "cooling" during the simulated annealing affects the accuracy of the solution, and therefore the level of noise in the surrogate.

In both cases, a user specifiable noise level is undesirable and distinctly at odds with the simpler and more intuitive algorithms suggested by Theiler et al. [26]. Such flexibility opens the way for unscrupulous or unwise investigators to manipulate the noise level

[1] It is possible to re-frame the null hypothesis to include the specific noise level. But here we move from the realm of hypothesis testing to that of model validation [11].

to achieve desired results. However, with careful use these methods may still be employed fairly and accurately.

2.4. Testing pseudo-periodic data

If the time series under consideration contains obvious periodic components, then so too must the surrogates (see Fig. 1). This can be achieved by either building nonlinear models with these features and employing a pivotal test statistic [18] or by constraining surrogates to have these features and enlisting a simulated annealing approach [16]. However, both these methods are computationally expensive and employed for this specific purpose can be somewhat clumsy.

Theiler [25] suggested an alternative approach analogous to algorithm 0. Instead of shuffling the individual data points in a time series, one shuffles the individual cycles. This shuffling of cycles should destroy any structure with a period longer than the cycle length. However, this method relies on the data having a convenient place at which to break the cycles. For the data Theiler [25] considered, epileptic encephalograms, this was not a problem—the data was mean stationary and exhibited sudden strong peaks. However, if the data is smooth and one is forced to identify suitable breakpoints, then the surrogates necessarily have either spurious discontinuities (at the points where the cycles have been reassembled) or suffer from non-stationarity (due to intra-cycle dependence, but not necessarily of period longer than the pseudo-period). Fig. 2 illustrates this. Even with convenient breakpoints between cycles, Theiler and Rapp [28] observed spurious long term correlation in the autocorrelation plot for cycle shuffled surrogates.

Furthermore, the degree to which this method actually randomizes the surrogates is dependent on the number of cycles present in the test time series. If, for example, one was to employ a dynamic measure (such as correlation dimension) as a test statistic and the embedding window was d_w, then the embedded points would only differ from the true points if the embedding window crossed the cycle breakpoint. If a time series of N points has p cycles, then each cycle is approximately of length N/p, and $N/p - d_w + 1$ embedded points will be identical for each cycle of data and surrogate (if $N/p > d_w$). One must therefore ensure the proportion of each embedded surrogate

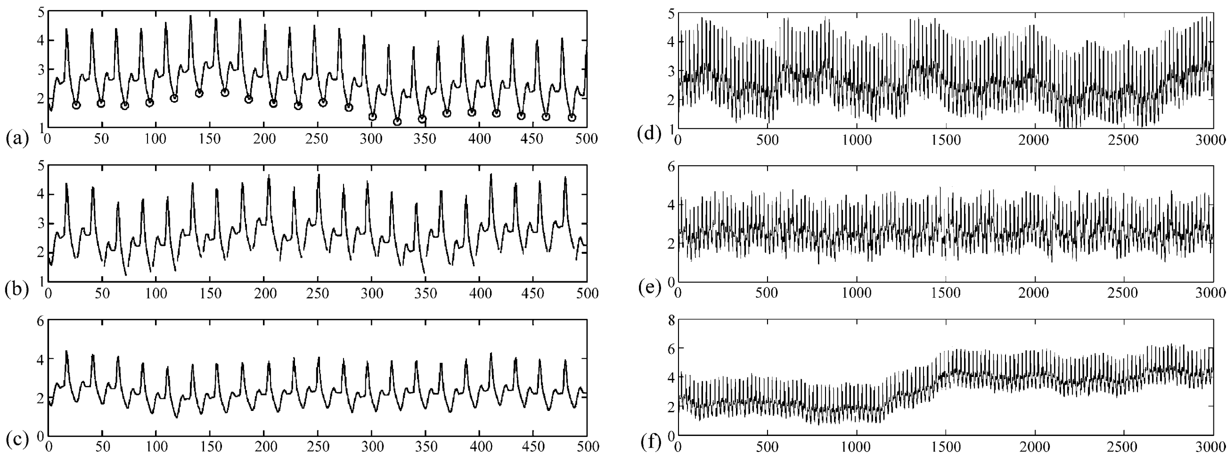

Fig. 2. Cycle shuffled surrogates. Generation of cycle shuffled surrogates according to the method describe by Theiler [25] is depicted here. The individual cycles are identified (panel (a)), separated (panel (b)) and reassembled in a random order (panel (c)). Panel (d) is a representative time series and panels (e) and (f) are cycle shuffle surrogates generated by two alternative methods. Note that, if the peak and trough values do not at all occur at exactly the same position, then the surrogate data is unable to preserve both stationarity and continuity. By re-aligning individual cycles vertically, one is able to preserve continuity but not stationarity (panel (f)). Conversely, preserving stationarity introduces spurious discontinuities or points of nondifferentiability in the surrogates (panel (e)). For the epileptic encephalogram data described in [25] and for infant respiratory data in [19], this method performed well. However, the technique lacks generality. In each panel the horizontal axis is datum number and the vertical axis is surface ECG voltage (in millivolts).

that is identical to the true data, $1-(p/N)(d_{\mathrm{w}}-1)$, is small.

In this paper we describe a new method, outlined in [21], that overcomes these problems. This method is the subject of the next section.

3. PPSs

In this section we will introduce the new surrogate generation algorithm in three parts. In Section 3.1 we describe the new PPS algorithm. Section 3.2 provides a mathematical argument that surrogates generated by this method do indeed mimic the periodic dynamics of the system. Finally, in Section 3.3 we address the issue of selecting parameter values for this algorithm.

3.1. The algorithm

Surrogate data suitable to test for pseudo-periodic determinism in a time series must address the null hypothesis of no determinism *other than the periodic behavior*. In what follows we employ a time delay embedding [24] of the data to extract the topological features of the underlying dynamics. We wish to generate surrogates that preserve the large scale behavior of the data (the periodic structure), but destroy any additional small scale (chaotic, linear or nonlinear deterministic) structure.

The PPS algorithm we employ here is inspired by the application of local modeling techniques to generate simulated time series. The various implementations of local modeling techniques are local linear models [4], local constant simplices [23] and triangulation and tesselations [14]. We have studied the application of these various methods for surrogate generation. However, we find that the simplest approach actually performs the best. Therefore, the implementation we employ here is a local constant model over spatial neighbors. This method avoids the added complications of these alternative schemes, and technically detailed improvements described more recently (e.g., [13]). Surrogate time series are generated by inferring the underlying dynamics from such a local model and contaminating a trajectory on the attractor with dynamic noise. With an appropriate choice of noise level, intra-cycle dynamics are preserved but inter-cycle dynamics are not. The null hypothesis these surrogates address is a periodic orbit with uncorrelated noise.

Our algorithm may be stated as follows:

(1) Define the time delay embedding $\{z_t\}_{t=1}^{N-d_{\mathrm{w}}}$ of the scalar time series $\{x_t\}_{t=1}^{N}$ as $z_t=(x_t, x_{t+\tau}, x_{t+2\tau}, \ldots, x_{t+d_{\mathrm{e}}\tau})$. For simplicity of notation we define the *embedding window* $d_{\mathrm{w}}=(d_{\mathrm{e}}-1)\tau$, where d_{e} and τ are the *embedding dimension* and *embedding lag*, respectively.
(2) Choose an initial condition $s_1 \in \{z_t | t=1,\ldots, N-d_{\mathrm{w}}\}$.
(3) Let $i=1$.
(4) Select one of the neighbors of s_i from $\{z_t | t = 1,\ldots, N-d_{\mathrm{w}}\}$ at random, say z_j.
(5) Set $s_{i+1}=z_{j+1}$ and increment i.
(6) Repeat this procedure from step 4 until $i=N$.
(7) Take as the surrogate time series $\{(s_t)_1 : t=1, 2,\ldots,N\}$ (where $(\cdot)_1$ denotes the scalar first coordinate of the vector).

Applying this algorithm, the vector time series $\{s_t\}_t$ is a stochastic trajectory on the attractor approximated by $\{z_t\}_t$. That is, it is a random walk on the attractor. It therefore has the same underlying dynamics, albeit contaminated by noise, as the original system. Also note that $\{s_t | t=1,2,\ldots,N\}$ and $\{z_t | t=1,2,\ldots,N-d_{\mathrm{w}}\}$ approximate the same attractor, but the reconstruction achieved by a delay embedding of $\{(s_t)_1\}_{t=1}^{N}$ does not.

3.2. Shadowing surrogates

Surrogates constructed according to this algorithm follow the same vector field as the original data but are contaminated with dynamic noise. The addition of dynamic noise obliterates fine scale dynamics, i.e., any deterministic inter-cycle dynamic behavior.

We can demonstrate this more formally. Let Δ be the underlying attractor and ϕ the underlying dynamical operator. Let $\mathcal{A}_N=\{x_i : i=1,\ldots,N\}$ and $\mathcal{A}=\lim_{N\to\infty}\mathcal{A}_N$. Takens' theorem says that a homeomorphism h exists between $\mathcal{A}$ and Δ. Let f denote the composition of that homeomorphism with ϕ (i.e., $f = h^{-1} \circ \phi \circ h$). Now we appeal to the shadowing lemma [5]. By construction, $\{s_i\}_{i=1}^N$ is an *α-pseudo orbit* [2] of f where $\alpha \geq \bar{\alpha}$ and

$$\bar{\alpha} := \max_{i=1,2,\ldots,N} \| f(s_i) - s_{i+1} \|.$$

(Note that $\bar{\alpha}$ is just the maximum perturbation applied in step 4 of the PPS algorithm.) The shadowing lemma implies that for all $\beta > 0$ there exists $\hat{\alpha} > 0$ such that if $\bar{\alpha} \leq \hat{\alpha}$ then $\{s_i\}_{i=1}^N$ is *β-shadowed* [3] by a point $y \in \mathcal{A}$.

Hence, if the perturbations applied in step 4 are sufficiently small, then the surrogates will look like a real trajectory of $\mathcal{A}$. Conversely, at each point s_i the distance to the true state $f^{(i)}(y)$ is bounded, $d(s_i, f^{(i)}(y)) < \beta$. Hence one requires that the randomization is sufficient to perturb $\{s_i\}$ from being β-shadowing a point in $\mathcal{A}$, but no more.

It is interesting to note the similarity between surrogates generated in this manner and the so called *attractor surrogates* described by Dolan et al. [3]. In [3] attractor surrogates are constructed (in two dimensions) as a coarse-grained, second-order Markov model. The surrogates therefore resemble the original two-dimensional attractor with the addition of noise. The technique we describe here does not aim to exactly reproduce the attractor in each surrogate, only the periodic structure of the data. Furthermore, the above shadowing arguments could not necessarily be applied to attractor surrogates.

3.3. The parameters of the algorithm

In Section 2.3 we criticized two alternative surrogate generation algorithms for user specifiable parameters and adjustable noise level. However, one immediately notices that the above algorithm has precisely three parameters: the embedding parameters d_e and τ and the noise level (or rather the method for selecting one of the near neighbors of s_i at random). Since the aim of embedding the original time series is to reproduce dynamics topologically equivalent to the true system dynamics, we are attempting to satisfy the necessary conditions of Takens' embedding theorem [24]. Selection of parameters d_e and τ for successful embedding is considered at great length in a wide body of literature (see [1] for just one review). We therefore do not consider that problem here; indeed, suitable selection of embedding parameters is likely to be dependent on the particular data set under consideration. [4]

Selection of appropriate noise level is an entirely different problem. Excessive noise will produce surrogates entirely unlike the data—in the limiting case the surrogates will be equivalent to algorithm 0 surrogates. [5] Insufficient noise will produce surrogates excessively like the data, leading to increased likelihood of false positive results.

First let us define the form of the noise. We select z_j from among the neighbors of s_i with probability

$$\mathrm{Prob}(z_j = z_t) \propto \exp \frac{-\|z_t - s_i\|}{\rho}. \quad (1)$$

Admittedly, the restriction to this particular form of randomization is arbitrary, but we feel that it is not entirely unreasonable. However, we have experimented with many alternative schemes, using, for example a fully local linear model [4] or constructing local simplices [23]. We concluded that (for our purposes) these methods performed no better, and in most cases worse, than with the current scheme. These methods generally included additional user specifiable parameters. In the absence of any evidence to the contrary, we prefer to employ the simplest model that will produce the required dynamics (see [12]).

With the specification of Eq. (1), we have now reduced the problem to selection of the *noise radius* ρ. If ρ is too small then the surrogate and original data will be identical (or largely identical). If ρ is too large

[2] That is, $d(s_{i+1}, f(s_i)) < \alpha$ for $i = 1, \ldots, N$.

[3] That is, $d(f^{(i)}(y), s_i) < \beta$ for $i = 1, \ldots, N$.

[4] It is important to observe that the embedding parameters used to construct the surrogates may be selected with reference to the data. However, any embedding parameters required for computation of the test statistic (say correlation dimension) must be selected independent of the test data. Otherwise one is effectively tuning the test statistic to the data, thereby increasing the likelihood of a false rejection of the null hypothesis, see [16].

[5] This is not exactly true: algorithm 0 surrogates are typically selected randomly *without* replacement. Under the influence of excessive noise this method would randomly select points on the attractor *with* replacement.

then the surrogate will be effectively i.i.d. noise. For a finite data set, this means that at either extreme the number of short (length ≥ 2) segments of the surrogate that are identical to the data will be small. At some intermediate value, the number of short segments that are identical will reach a maximum, and it is this maximum that we choose as the value of the noise radius to generate surrogates. In all cases (to be discussed in Section 4) we observe this characteristic behavior and selecting ρ in this way produces surrogates that appear visually identical to the data but lack any long term determinism. Further discussion of selection of ρ may be found in [21].

4. Examples

In this section we apply the PPS algorithm to several artificial and experimental systems. For each time series that exhibits pseudo-periodic behavior, we apply this algorithm to test the null hypothesis of a periodic orbit with uncorrelated noise. However, we must first determine a suitable test statistic.

4.1. The test statistics

Following the original motivation for surrogate data, we employ a variant of correlation dimension described by Judd [7,8]. The algorithm we employ estimates correlation dimension d_c as a function of observation scale ϵ_0, and hence the results presented here are curves of $d_c(\epsilon_0)$. When applying correlation dimension, one first needs to determine an embedding dimension m and embedding lag ℓ.[6] If we select optimal values of m and ℓ for the test data and then employ the same values of the surrogates we are effectively selecting a test statistic (from a host of such statistics) that is most likely to reject the given hypothesis. Therefore, we select ℓ as the first zero of the autocorrelation of the data (this will be the same for data and surrogates as periodic structure is preserved) and let $m = 3, 4, 5, \ldots, 15$.

As a further precaution we employ several additional test statistics. We choose to employ a variant of nonlinear prediction error as a test statistic, see [22].[7] We also estimate mutual information [1], Lyapunov exponent based statistics[8] described by Hegger et al. [6], and higher order moments (kurtosis and skewness).

In Section 4.2 we demonstrate the straightforward application of this method to differentiate between the chaotic and periodic behavior of the Rössler system contaminated with noise. Although the time series and embedded attractors appear exceedingly similar [21], the PPS algorithm is able to correctly identify the chaotic system. In Section 4.3 we take the other extreme situation and consider the simplest possible systems that could lead to rejection of the null hypothesis: a periodic oscillator driven by linear noise and a sinusoidal signal with chaotic forcing. In both the cases, the PPS algorithm correctly identifies the presence of long term determinism, despite the fact that the systems appear indistinguishable from a simple noisy periodic orbit.

The remainder of this section is devoted to the application of this algorithm to a variety of experimental time series. Sections 4.4–4.6 present the application of this algorithm to human ECG data, annual global climatic time series and the canonical annual sunspots time series, respectively. In each case the system under consideration exhibits some periodic structure. We apply the machinery of PPS analysis and test for evidence of additional long term determinism in these systems.

Finally, in Section 4.7 we consider a slightly more unusual application. We consider three financial data sets, none of which exhibit obvious periodic cycles. However, we find that application of the PPS algorithm can still be employed in conjunction with the standard algorithm 2 surrogates. Although algorithm 2 surrogates (and therefore the hypothesis

[6] These embedding parameters need not, and in general will not, be the same as those used in Section 3 to generate the surrogate. They may be the same, but it would be unwise to constrain them to be so.

[7] Note that nonlinear prediction error is derived from a local constant model of the same form as that which we employ to generate PPS data [23]. It is therefore unclear to us, a priori, that such a statistic would not be unduly biased [27].

[8] Lyapunov exponent estimates are also based on local modeling and may suffer the same weakness as nonlinear prediction error.

of nonlinearly transformed linearly filtered noise) is clearly rejected, PPS surrogates are not (in two cases). We therefore conclude that the financial data examined exhibits short term *nonlinear* deterministic dynamics but no long term determinism.

4.2. The Rössler system

In [21] it was shown that this algorithm can differentiate between chaotic and periodic systems in the presence of noise. These results are summarized in Fig. 3. The results presented in Fig. 3 and subsequent plots compare correlation dimension estimates [7,8] for the original time series and 50 PPS data sets. The correlation dimension estimates for the surrogates are presented as a contour plot. This contour plot is a pictorial representation of the probability distribution function (PDF) of $d_c(\epsilon_0)$ for fixed distinct values of ϵ_0. That is, each vertical slice is a different PDF computed using the kernel estimator described by [17]. The isocline lines are selected automatically and uniformly. Any value outside the lowest isocline corresponds to a value outside the distribution of surrogate values.

From the Rössler differential equations [29] in a chaotic and periodic regime (both period 2 and period 6), we generate a simulation of 5000 points (sampling rate 0.2 time units) with Gaussian variants added to each component. Typical time series exhibit an apparently periodic structure with large oscillatory motion that repeats irregularly. We found that the time series could not be easily differentiated by inspection, but application of the PPS algorithm with correlation dimension as a test statistic easily differentiated between the non-periodic and periodic dynamics [21]. Higher order moments and Shannon entropy as test statistics were unable to differentiate between data and surrogates. Mutual information provided significant separation between data and surrogates only for a small range of time lags.

4.3. Linear noise and chaos

Fig. 3 panel (b) shows the results of this algorithm applied to a periodic system with noise. In this section we tested the algorithm with periodic systems under the influence of various noise sources. In each case we examined a sine wave signal with additive noise. Fig. 4 summarizes the results for white and colored noise. In the case of white noise, the hypothesis test is unable to reject the null hypothesis, whereas for colored noise we conclude rejection is possible. This is precisely as expected—a periodic orbit with white (i.e., uncorrelated) noise is consistent with the null hypothesis; in the presence of colored (correlated) noise, the null hypothesis is rejected.

We also tested the algorithm in the presence of deterministic chaotic dynamics (the Ikeda map [29]) superimposed on a periodic orbit. Although the

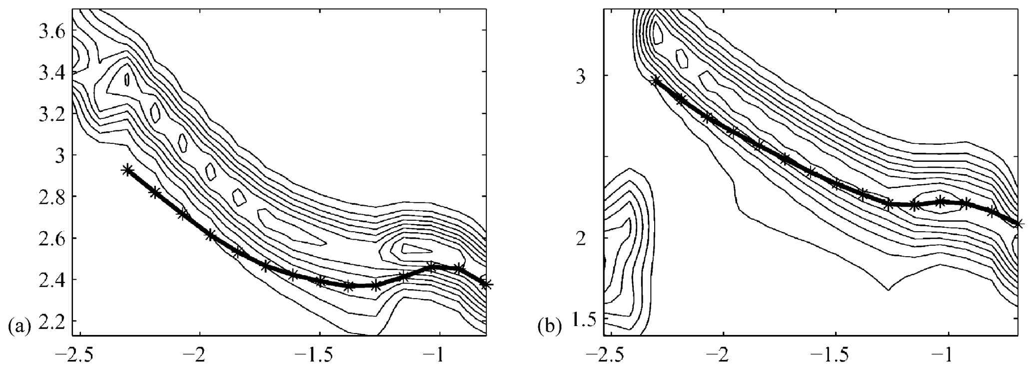

Fig. 3. Surrogate test results for the Rössler system. Each panel depicts a probability distribution of correlation dimension estimates (as a contour plot) from 50 PPS data sets and the corresponding correlation dimension estimate for the data. Correlation dimension is estimated using the method described by Judd [7,8] (thick solid line). For the chaotic Rössler data with dynamic noise (panel (a)), these results indicate rejection of the null hypothesis. For the noisy periodic Rössler (panel (b)), these results indicate the null hypothesis cannot be rejected. The horizontal and vertical axes are the dimensionless quantities $\log \epsilon_0$ and $d_c(\epsilon_0)$, respectively.

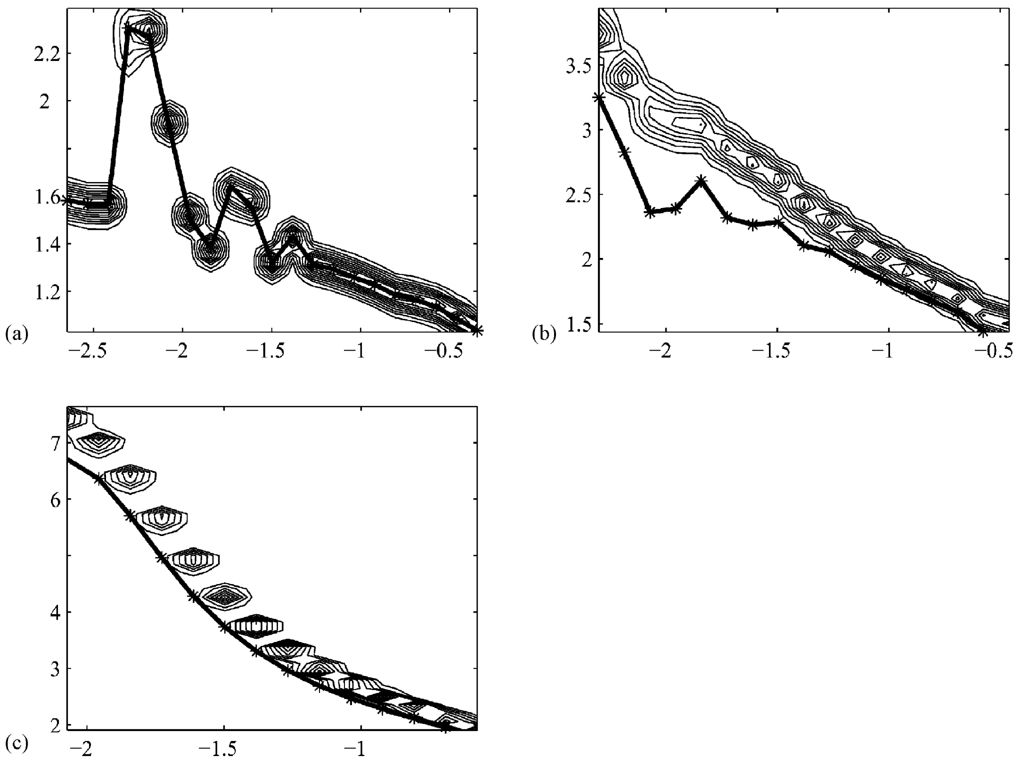

Fig. 4. Surrogate test results for periodic orbits with noise. Each panel depicts a probability distribution of correlation dimension estimates (as a contour plot) from 50 PPS data sets and the corresponding correlation dimension estimate for the data. Correlation dimension is estimated using the method described by Judd [7,8] (see text). For the periodic orbit with white noise (panel (a)), these results indicate the null hypothesis cannot be rejected. For the periodic orbit with colored noise (panel (b)), and for the periodic orbit with deterministic chaos (the Ikeda map, panel (c)), these results indicate rejection of the null hypothesis. The horizontal and vertical axes are the dimensionless quantities $\log \epsilon_0$ and $d_c(\epsilon_0)$, respectively.

non-periodic component in this case is temporally correlated, that correlation is only short term. Higher order iterates appear less correlated. Hence, the first return map of the periodic orbit appears to behave much like independent and identically distributed noise (i.e., high dimensional dynamics). The PPS algorithm generated surrogates that appeared qualitatively similar to the data; however, application of correlation dimension as a test statistic led to clear rejection of the associated null hypothesis.

4.4. Human electrocardiogram

Results of analysis of human electrocardiogram (ECG) data have been presented in [21]. The results of that work showed that human ECG during both sinus rhythm and ventricular tachycardia (VT) were not consistent with a periodic orbit with uncorrelated noise. This result is significant because ECG during both rhythms is regular and appears largely periodic. Representative data during VT is depicted in Fig. 1. Fig. 5 depicts typical PPS data generated for a recording of sinus rhythm. Qualitatively the ECG time series and PPS data are indistinguishable. However, surrogate analysis demonstrates that these time series are inconsistent with the null hypothesis of a periodic orbit with uncorrelated noise. The presence of long term correlations and possibly determinism indicates that analysis using techniques of nonlinear dynamics, and examining inter-beat dynamics (the so called *RR intervals*) should prove fruitful. See [2] for a compelling example of the control of cardiac

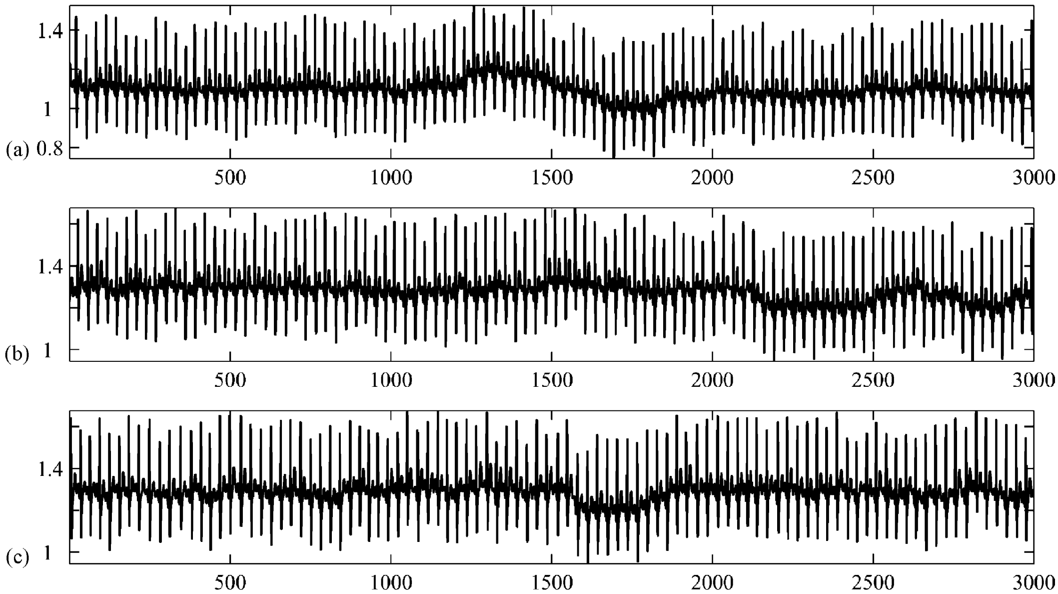

Fig. 5. Data and representative surrogates for human sinus rhythm ECG. Panel (a) depicts a representative recording of human sinus rhythm ECG. Panels (b) and (c) show representative PPS data sets. Qualitatively the data and surrogates appear indistinguishable. On each panel the horizontal axis is the datum number and the vertical axis is surface ECG voltage (in millivolts).

arrhythmia in human subjects using the methods of unstable periodic orbit detection and targeting.

4.5. Global climatic data

Time series data of monthly deviations from monthly mean global air temperature [9] are depicted in Fig. 6, along with representative PPS and algorithm 2 surrogates. This data appears extremely noisy and somewhat (mean) non-stationary. However, previous analysis has shown that this data exhibits long term periodic like behavior [20]. We do not differentiate here between a limit cycle and a stable focus. The data does not exhibit obvious periodicity, but the analysis [20] suggests the existence of periodic structure.

We apply the PPS generation method and hypothesis test to determine if this data may be adequately modeled as a noisy periodic orbit or limit cycle. The results of Fig. 6 show that this data is indistinguishable from the surrogates using correlation dimension and mutual information as test statistics. Results for all other statistics (kurtosis, skewness, Shannon entropy, local linear prediction and Lyapunov exponents) were identical. We therefore find no evidence that these data are inconsistent with the noisy linear model described in [20].

4.6. Sunspots

In this section we examine a much shorter time series than previously considered. We test 301 annual sunspot counts recorded since 1700. Unfortunately, we find that the data is insufficient to make any conclusion using this technique. Fig. 7 demonstrates that the PPS algorithm is behaving as expected—in this case shuffling the 11-year cycles. However, this randomization is insufficient to differentiate between data and surrogates using any of the test statistics discussed in this paper. Because of the limited length of the time series (and the expected complexity of the physical system) we find that the most likely explanation is simply a lack of data.

[9] These data were obtained from the web site http://www.cru.uea.ac.uk/tiempo/floor2/data/gltemp.htm.

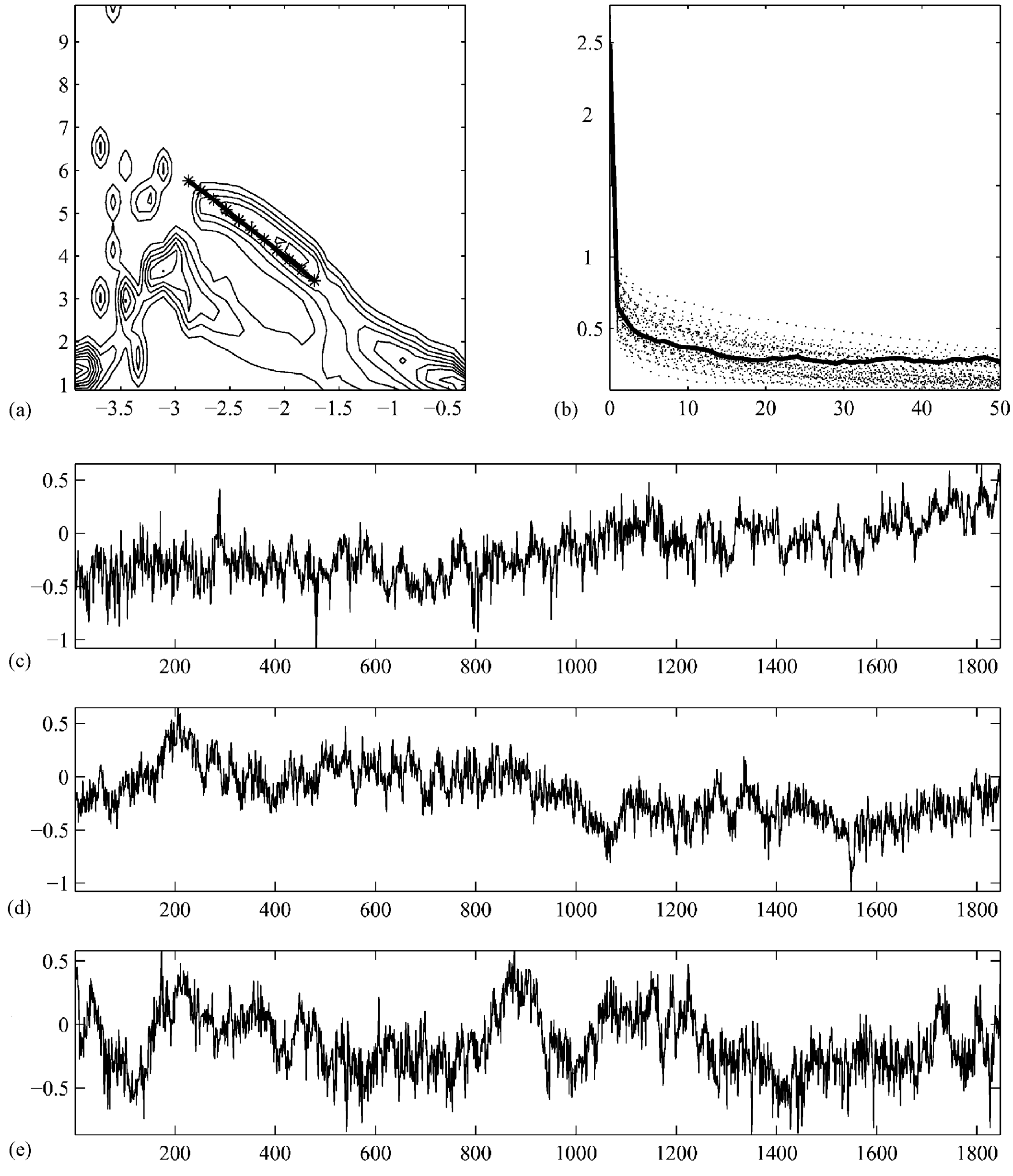

Fig. 6. Surrogate test results for global climatic data. Panel (a) depicts a probability distribution of correlation dimension estimates (as a contour plot) from 50 PPS data sets and the corresponding correlation dimension estimate for the data (thick line). In this panel $\log \epsilon_0$ is plotted against $d_c(\epsilon_0)$. Panel (b) is a plot of the mutual information (in bits) against time lag for 50 PPS data sets (dotted) and the original time series (thick line). Panel (c) shows the original data. Panel (d) is a representative algorithm 2 surrogate, and panel (e) is a representative PPS data set. Panels (a) and (b) show that the original time series and PPS data are indistinguishable; therefore, the null hypothesis cannot be rejected. For panels (c–e) the horizontal axis is datum number and the vertical axis is proportional to the monthly deviations from monthly mean global air temperature.

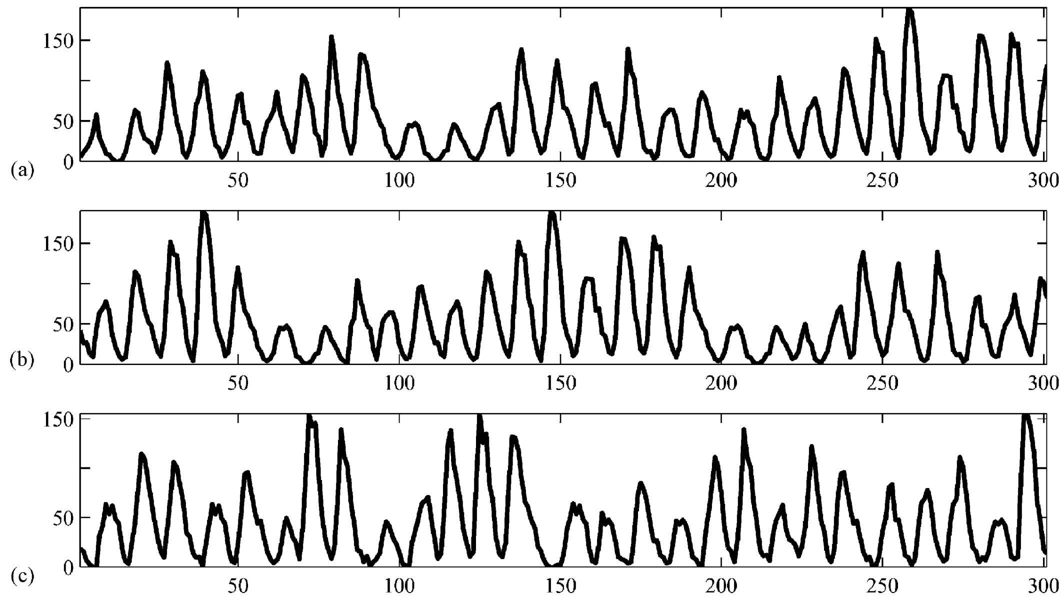

Fig. 7. The annual sunspot time series and representative surrogates. Panel (a) depicts the original data, panels (b) and (c) are representative PPS data sets. Analysis with PPS surrogates indicates the null hypothesis cannot be rejected. By examining the surrogates produced we believe this may be due to insufficient data. Each surrogate appears qualitatively similar to the true data; however, upon closer inspection one notices that entire orbits and indeed large sections of data are repeated. This is a result of insufficient data in the embedded space. For each panel the horizontal axes is datum number (year) and the vertical axes in the normalized sunspot index.

This conclusion may appear disappointing, but it is not a negative result. The PPS algorithm is providing a demonstration that if we are to do more than characterize the periodic orbit of this time series then we do not (yet) have sufficient data. This is consistent with results reported elsewhere [9,10].

4.7. Financial time series

The previous examples considered in this paper have all had some "pseudo-periodic" structure. In this section we examine three financial time series, none of which have any periodic determinism. The data we present are (i) daily gold prices (from 2 June 1969 to 19 August 1999), (ii) daily Dow-Jones industrial average (DJIA) (from 5 August 1956 to 24 April 1998) and (iii) daily Japanese Yen/US Dollar (YEN/USD) exchange rates (from 2 January 1980 to 7 June 2001). Each time series was pre-processed by taking the difference between the logarithm of successive values, i.e.,

$$z_t = \frac{\log y_{t+1}}{\log y_t}.$$

Furthermore, the resulting time series were tested with standard algorithm 2 type surrogates and found to be clearly distinct. However, it is possible to reach this conclusion from simply inspecting the data, which has significant short term correlation (over successive days) and long term trends (variability in volatility).

Using only algorithm 2 surrogates, one may conclude that these systems are not consistent with a monotonic nonlinear transformation of linearly filtered noise. However, this does not mean that the appealing alternative hypothesis of long term determinism is true. To test this data against the null hypothesis of short term dynamics with uncorrelated noise, we apply the PPS algorithm and hypothesis testing as described in this paper. For each data set we employed the ensemble of test statistics described previously. In

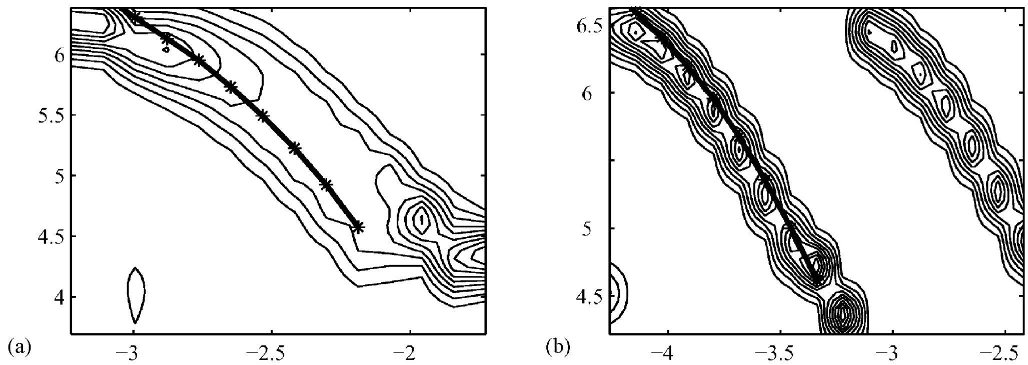

Fig. 8. PPS surrogate calculation for YEN/USD and DJIA data. Panel (a) depicts the results of PPS analysis for the YEN/USD exchange rate data, panel (b) is the same calculation for the DJIA data. Analysis with PPS surrogates indicate the null hypothesis cannot be rejected. Therefore, these systems are largely stochastic and exhibit no long term deterministic dynamics. The horizontal and vertical axes are the dimensionless quantities $\log \epsilon_0$ and $d_c(\epsilon_0)$, respectively.

the YEN/USD and DJIA we found no evidence to reject the null hypothesis. The likely conclusion is that these time series exhibit only short term stationary deterministic dynamics with uncorrelated noise (see Fig. 8). Coupled with the rejection of the hypothesis of a monotonic nonlinear transformation of linearly filtered noise (algorithm 2 surrogates), we are led to conclude that this data contains *dynamic nonlinearly* filtered noise.

Conversely, for the daily gold price data, we found that the time series and PPS data were significantly different (Fig. 9). Therefore, for this system we were able to reject the null hypothesis of short term stationary deterministic dynamics with uncorrelated noise. This may indicate that this system exhibits greater determinism and greater predictability. The simplest explanation for this result is that the time series exhibits correlated noise with a long correlation time. In either case this represents deterministic and (because of the rejection of the hypothesis associated with algorithm 2) nonlinear, information that may be exploited for prediction.

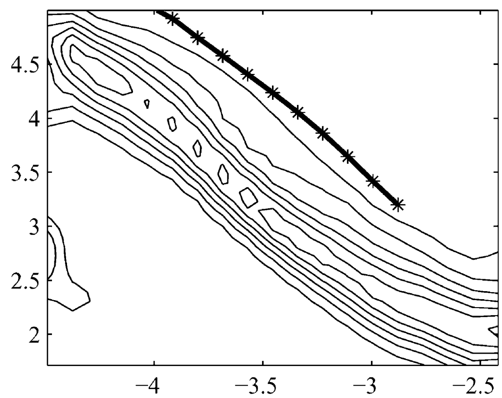

Fig. 9. PPS calculation for daily gold price data. Analysis with PPS surrogates indicate the null hypothesis can be rejected for this data. This may indicate additional long term determinism not found in the other two financial time series considered in this paper. The horizontal and vertical axes are the dimensionless quantities $\log \epsilon_0$ and $d_c(\epsilon_0)$, respectively.

5. Conclusion

The algorithm we have described in this paper provides a robust method to test pseudo-periodic time series data for deterministic non-periodic inter-cycle dynamics. We have shown that the PPS algorithm is capable of testing against the null hypothesis of a periodic orbit with uncorrelated noise in time series with pseudo-periodic structure. This provides a useful extension of existing techniques.

Of the alternative nonlinear surrogate tests that have previously been proposed [15,18], this algorithm is simpler and requires no user adjustable parameters. It is also computationally less expensive than the methods proposed by Schreiber [15], and Small and Judd [18]. We have found that this technique does not suffer from stationarity and continuity problems encountered with Theiler's cycle shuffled surrogates [25].

We have demonstrated the application of this algorithm to a variety of experimental systems. We found the technique to be robust to noisy and non-stationary data. In all cases higher order moments provided poor discrimination as test statistic. Conversely, correlation dimension performed the best. Significantly, we concluded that human ECG is not consistent with a noisy periodic orbit and the daily gold price data is inconsistent with a system exhibiting only short term determinism. These results suggest possible future treatment regimes (ECG) and improved prediction (gold). Practical application of nonlinear control techniques to human cardiac arrhythmia has already been demonstrated [2]. Successful application to financial prediction is unclear.

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