Automatic Non-linear Analysis of the Variability of Human Activities

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Abstract—

Index Terms—Activity Recognition; On-Body Inertial Sensors

1 Introduction

HUMAN Activity Recognition (HAR) using body-worn sensors has received interest during the last 20 years [1], [2]. This is due to three factors: (i) technology advances in sensors, (ii) longer battery lifetimes and (iii) different application-oriented scenarios. In contrast to speech recognition and computer vision frameworks, HAR offers different challenges based on the complexity and diversity of human activities (e.g. ambulation, transportation, phone usage, daily activities, exercise, military); the selection of different sensors to use (e.g. inertial, light, temperature or audio sensors) [2] and different bodily locations for sensor placement (e.g. chest, wrist, lower back, hip, thigh, foot) [3] results in a large number of options for configurations [4].

According to Bulling *et al.* [1] the common challenges in HAR using body-worn sensors are: (i) *intraclass variability* which occurs when an activity is performed differently either by a single person or several people. For example, gait patterns may be more dynamic in the morning after sleep than in the evening after a day full of activities; (ii) *interclass similarity* occurs when the sensor data is very similar. For example, in recognising dietary activity, drinking water or coffee entails the same arm movements [5]; and (iii) *the NULL class problem* occurs when ambiguous activities are irrelevant for the recognition methods which leads to wrong classification of the activities [6].

For this PhD, the variability of dance activitities is a very rich case of study to explore and investigate the human movement variability. Variability is presented in either dance features (e.g. fluency of motion, coordination, steadiness of the rhythm, adding erratic or additional movements [7], [8]) or some biological and demographic features of dancers (e.g. gender, age, home country [7], [9]).

Hammerla *et al.* [10] have examined the effects of variability of motor performance using artificial signals so as to create motion structures (strategy of the motion activity) and motion noise (the precision of the motion) of human activities. To quantify the variability of motion activities, Hammerla *et al.* [10] proposed the use of PCA to compute the area underneath the cumulative energy curve which is used as a metric for motor skill assessment. The variability in human activities has therefore a relation with quantitative assessment of motion structures and motion noise of human activities. Velloso *et al.* [11], for example, assessed automatically the quality of weight-lifting activity. Similarly, Velloso *et al.* [12] quantify how *good* the repetition of weight-lifting activity is in terms of angles of each bone in relation to reference planes.

Similarly, concepts from non-linear analysis such as fractal dimensionality, the Lyapunov exponent or time-delay embedding have been applied to better understand the variability of human activities. For instance, Yamamoto *et al.* [13], [14] used the fractal dimensionality of the attractors to model repeated forehand and backhand tennis strokes. Gouwanda *et al.* [15] showed that the variability in walking speed has a linear relationship with the Lyapunov exponent. This exponent is therefore suitable for analysing the temporal variation in gait stability. Time-delay embedding has been used as a feature for general gait recognition [16] as well as for recognition of cycling, running, walking up stairs and downstairs activities [17]. Recently, Caballero *et al.* [18] reviewed further non-linear analysis tools (e.g. local dynamic stability, recurrent quantification analysis, entropy measurements, detrended fluctuation analysis) to measure human movement variability. However, the questions to ask, as pointed out by Caballero *et al.* [18], are: "...do these tools actually measure variability?" and "what kind of variability?". It should be noted that non-linear analysis offers a range of techniques for the study of human activity (see [19] for an overview of alternative techniques).

Given the case of the variability in dance activities, it is hypothesised that there are three possible reasons for variation: (i) inherent noise in body-worn sensors, (ii) inherent properties of the activity itself and (iii) discrepancies of biological features of people.

For this PhD, the following research questions will be addressed:

1) How can the time-delay embedding and PCA methods quantify the possible reasons of the variability of dance activities ?
2) In the light of limitations of time-delay embedding and PCA, which other non-linear analysis tools would be suitable to explore the variability in dancing activities and use them as a features for machine learning algorithms?

2 Recognising Dexterity In Dance

As Miura et al. [20] point out "... how the human motor system produces dance movements is still poorly understood." A key issue concerns the manner in which experienced dancers solve the 'degrees of freedom' problem in face of changing contextual demands. Miura et al. [21] measured muscle activation using electromyographic (EMG) data collected from muscles in the lower limb, for a task requiring participants to bounce up and down in time to a metronome beat. They demonstrated that experienced dancers show much better precision in synchronizing movements to beat than non-dancers, i.e., dancers maintained much lower standard deviation in temporal deviation against the beat than non-dancers. This result is consistent with work which shows that, compared with inexperienced- or non-dancers, trained ballet dancers exhibit superior postural stability [22], and show superior ability in position matching of upper limbs [23].

Capturing dance activity through sensors has tended to rely on motion capture [24] or sensors mounted on the person [25] or in their shoes [26] or data recorded from their smartphones [27]. Much of this work has been concerned with using the dancers' motion to work with multimedia presentations that augment and complement the dance [28], [29] or as interfacing with a game [30] or commercial games, such as Dance Dance Revolution. While the range of sensing technology used in these papers is diverse and the results of the activity recognition are varied, it is fair to say that few of the papers have considered variability or dexterity in how a dance is performed. In their work, Aristidou et al. [8] have considered the manner in which dance steps conform to a set of defined templates that describe steps in terms of a three-dimensional rotation (described using quaternions). The implication is that a goodness-of-fit can be ascertained to determine how good a dancer performs a step, and how any deviation from good can be modified and improved through practice.

For this report, we are interested in the question of how time-delay embedding techniques can provide insight into the variability and dexterity of dancers. To this end, we consider the performance of a set of steps from Salsa dance as well as other dance styles in future and compare untrained, inexperienced or non-dancers in one cohort with experienced dancers in another.

Before explaining how the activity recognition chain, the next section outlines the approach to time-series time-delay embedding and the resulting phase space representation used in this report.

3 Time-delay Embedding

The aim of time-delay embedding, also known as Takens's Theorem, is to reconstruct a $k-$dimensional manifold M of an unknown dynamical system $s(t)$ from a time series $x(t)$ with discrete observations at given timepoints t. Time-delay embedding assumes that the time series is a sequence $x(t) = h[s(t)]$, where $h : M \to \mathbb{R}$ is a measurement function in the unknown dynamical system, being $x(t)$ measurable.

Thus, the time delay reconstruction is defined as: $\overline{x}(t) = (x(t), x(t-\tau), ..., x(t-(m-1)\tau))$ where m is the embedding dimension and τ is the embedding time-delay. $\overline{x}(t)$ defines a map $\Phi : M \to \mathbb{R}^m$ such that $\overline{x}(t) = \Phi(s)$. Similarly, $y(t) = \Psi[\overline{x}(t)]$ is a n-dimensional vector where $\Psi : \mathbb{R}^m \to \mathbb{R}^n$ is a further transformation (e.g., PCA [31], Nonlinear PCA [32], Locally Linear Embedding [33]). Figure 1 illustrate the time delay reconstruction process. For details, see the work of Uzal *et al.* [34].

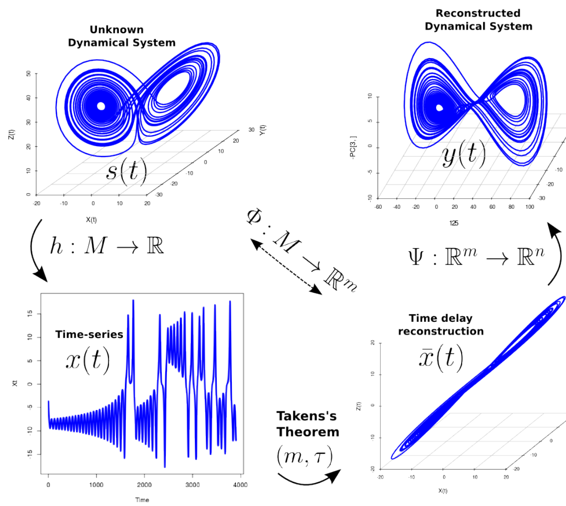

Fig. 1. The reconstruction problem. The figure is based on the work of Uzal *et al.* [34].

3.1 Embedding Parameters m and τ

Given any time series $x(t)$, the time delay reconstruction system, $\overline{x}(t)$, is easy to implement. For this work, Cao's method [35], a modification of the False Nearest Neighbours (FNN) algorithm, and mutual information algorithm by Fraser *et al.* [36] have been used to calculate minimum embedding parameters (m_{min}, τ_{min}).

3.1.1 *Minimum Embedding Dimension* m_{min}

Cao's method [35] for computing the minimal embedding dimension is based on the mean values $E1(d)$ and $E2(d)$ in which d is a given embedding dimension value.

$E1(d)$ is used to obtain the minimal dimension m_{min} and stops changing when the time series comes from an attractor (Figure 2 B). We computed $E1(d)$ values for $1 \leq \tau \leq 10$ to exemplify the minor dependency of τ given periodic, chaotic and random time series (Figures 2 (A,B,C)).

The second of these values, $E2(d)$, is used to distinguish deterministic signals from random signals in which case the $E2(d)$ values will be approximately equal to 1 for any d (Figure 2 F). Similarly, we computed $E2(d)$ values for periodic, chaotic and random time series, to exemplify the no significative dependency on τ, where $1 \leq \tau \leq 10$ (Figures 2 (D,E,F)).

Cao's method is a modified version of the FNN method, and $E1(d)$ and $E2(d)$ values are only dependant on m and τ [35].

3.1.2 *Minimum Time-delay Embedding* τ_{min}

The method of choosing the minimum Time-delay embedding, τ_{min}, was proposed by Fraser *et al.* [36] in which the

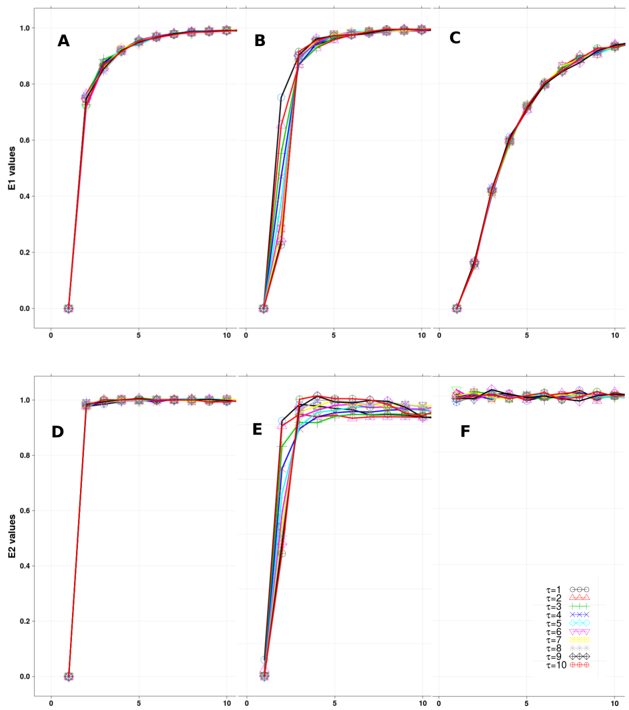

Fig. 2. The values of $E1(d)$ and $E2(d)$ with different time delay embedding parameters from periodic (A,D), chaotic (B,E) and random (C,F) time series.

first minimum of the mutual information graph is chosen to estimate the minimal time-delay embedding parameter. For instance, Figure 3 illustrates the mutual information from periodic, chaotic and random time series. The local minimum for the Chaotic series in Figure 3 is $\tau_{min} = 18$. On the other hand, for the periodic and random time series the mutual information plots have no local minimum and values are monotonically decreasing which means that $\tau_{min} = 1$ for both (Figure 3) [36].

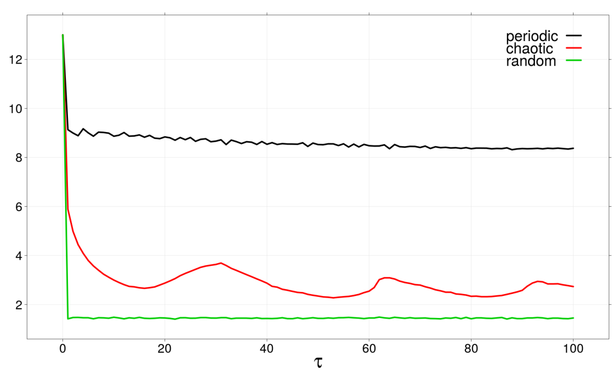

Fig. 3. Mutual information plots from periodic, chaotic and random time series.

3.1.3 *Embedding Parameters Setbacks*

Although the time-delay embedding method using inertial sensors has been used extensively in gait recognition [16], gait stability [15] and walking, running and cycling activities [17], some problems with the minimal embedding parameter estimation (m_{min} and τ_{min}) still remain to be solved.

Sama *et al.* [16] and Gouwanda *et al.* [15] estimated the minimal embedded dimension (m_{min}) with the False Nearest Neighbours (FNN) method. However, Cao [35] pointed out that the FNN algorithm introduces new parameters (R_{tol} and A_{tol}) that lead to different results and cannot differentiate random series from deterministic series. Frank *et al.* [17] proposed a grid search method to find the minimal embedded parameters, but there are no details about their approach.

In the case of the minimal time delay embedding value, τ_{min}, Fojt *et al.* [37] mentioned a method in which the chosen τ is made in function of filling optimally the reconstructed state space; however, Fojt *et al.* [37] mentioned that "it is a rough estimation based on a graphical procedure." Although, Sama *et al.* [16] computed τ_{min} using the method proposed by Fraser *et al.* [36], they pointed that the chosen τ_{min} largely depend on the application.

4 The Activity Recognition Chain

Bulling *et al.* [1] reviewed the state of the art of HAR using body-worn inertial sensors. Figure 4 illustrates the typical activity recognition chain (ARC) to identify activities with body-worn sensors.

The first stage of the ARC is the raw data collection from several sensors attached to different parts of the body. Sensors data over a given time, s_i, provide multiple values d^i, (e.g. d^1, d^2, d^3 for 3-D acceleration referred to as x, y and z direction)

$$s_i = (\mathbf{d}^1, \mathbf{d}^2, \dots \mathbf{d}^t), \text{for } i = 1, \dots, k \tag{1}$$

where k denotes the number of sensors.

In the preprocessing stage of the ARC, raw multivariable time series are transformed into a pre-processed time series $D' = (d'_1, \dots, d'_n)^T$, where d'_i is one dimension of the data for the preprocessed time series and n is the number of total data dimensions. Different methods for the preprocessing tasks may be applied to the raw data (e.g. synchronisation, calibration, unit conversion, normalisation, resampling, denoising or baseline drift removal [1]).

The stage of data segmentation identifies segments within the continuous data stream that are likely to have information about activities. The segmentation stage creates a set of segments w_m such that

$$W = \{w_1, \dots, w_m\}, \tag{2}$$

where m correspond to the number of segments. Since the segmentation of the data is a difficult problem, there are various methods in the literature to tackle this problem: sliding window, energy-based segmentation, rest-position segmentation, additional sensors and external context sources.

In the feature extraction stage, a feature extraction function F reduces the signals D' into segmented signals W. The total number of features X_i is the feature space.

$$X_i = F(D', w_i) \tag{3}$$

In the literature on activity recognition, different methods for feature extraction can be found including signal-based features, body model features, event-based features, multilevel features or automatic feature ranking and selection.

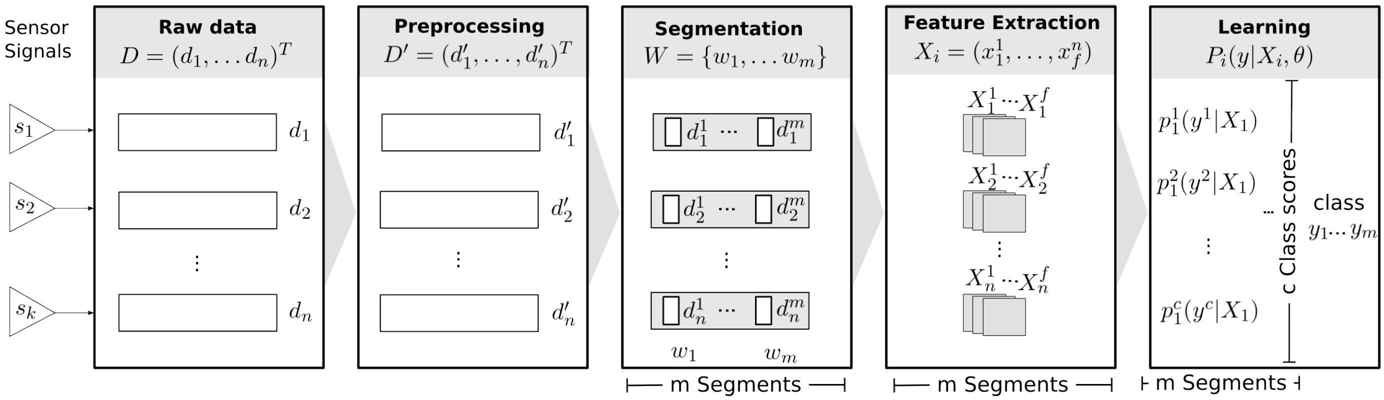

Fig. 4. Typical activity recognition chain (ARC) to identify activities or gestures from body-worn inertial sensors. Diagram is replicated from the work of Bulling *et al.* [1].

Machine learning tools have been used in HAR over the last 15 years so as to describe, analyse and predict human activities [1]. However, the chosen approach is subject to computational complexity, recognition performance or latency. Generally for the learning stage, a training data set $T = \{X_i, y_i\}_{i=1}^N$ is computed prior to the classification with N pairs of feature vectors X_i and ground truth labels y^i (possible activities to recognise). For this stage, model parameters θ can be learned to decrease the classification error on T. Then, with the trained model T, each feature vector X_i is mapped to a set of class labels $Y = \{y^1, \dots, y^c\}$ with scores $P_i = \{p_i^1, \dots, p_i^c\}$:

$$p_i(y \mid X_i, \theta) = I(X_i, \theta), \text{for } y \in Y \tag{4}$$

and inference method I. Finally, the classification output y_i is computed with the maximum score P_i

$$y_i = \underset{y \in Y, p \in P_i}{\operatorname{argmax}}\, p(y|X_i, \theta) \tag{5}$$

The most common classification algorithms are: decision trees, Bayesian models, domain transform, fuzzy logic, Markov models, support vector machines (SVM), artificial neural networks (ANN) and ensembles of classifiers [2].

Similarly, when the recognition of activities can miss, confuse or falsely recognise activities that did occur, several metrics can be used to optimise the classification. Some of the metrics are confusion matrices, accuracy, precision, recall, and F-scores, decision-independent Precision-Recall or receiver operating characteristic curves (ROC curves) [1].

5 Artificial Signals

Following the proposal of Hammerla *et al.* [10], artificial signals are created to examine the effects of variability in the precision of motion (additive noise) and in the strategy of motion (structural noise) of activities.

Additive noise is normalised noise with σ_a^2 added to the sinusoid signal S:

$$S^a = S + \mathbf{N}(0, \sigma_a^2) \tag{6}$$

Structural noise is a sinusoid signal distorted with different variance in frequency and amplitude σ_s^2 and window length w_s. Algorithm 1 describes the creation of structural noise. To make the data less redundant for possible variations of environmental conditions or body-worn sensor mobility in users, the data is whitened (i.e. data is normalised to have zero mean and unit variance)

Algorithm 1 Structural Noise

Input: time-series S^a, variance σ_s^2, window length w_s
Output: Structurally distorted signal S^s

```
for j = 1 to L, j = j + w_s do
    u' ← N(0, σ_s^2)
    S^a = sinusoid with frequency |u'| and variance σ_a^2 of length w_s
    S^s_{j→j+w_s} = S^s_{j→j+w_s} + S^a × σ_s^2
end for
S^s = whiten(S^s)
return S^s
```

By varying both σ_a^2 and σ_s^2 is possible to simulate and control the additive noise and the structural noise in the structure of the human activity. For example, low values of σ_a^2 are associated with precise movements while low values of σ_s^2 correspond to a well chosen strategy for a motion.

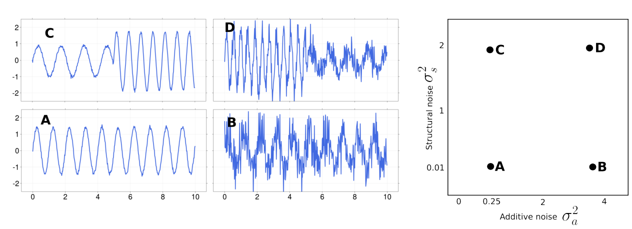

Fig. 5. Graphs (A, B, C and D) present the variability of additive noise (σ_a^2) and structural noise (σ_s^2) on the sinusoid signals with $w_s = 500$ according to the parameters indicated on the left plot.

For graphs in Figure 5, the base frequency of the sinusoid is 1 Hz sampled at 50Hz with an amplitude 1 and window length of 250.

6 WORKING ON HAMMERLA METHOD TO TAKENSPCA

7 WORKING ON RESULTS WITH NONDANCERS DATA

8 PUBLICATION PLAN

Publish my research's results in Measuring Behaviour 2016 and Augmented Human 2016 conferences so as to push myself to submit a journal in the 15th Month of the PhD.

9 CONCLUSION AND FUTURE WORK

Preliminary results were submitted to ISWC 2016; however, the submission was rejected because the reviewers argued that the method of using the reconstructed state space to quantify dexterity for salsa dancers is too specific and it is not transferible to other applications. Additionally, data analyis were collected from 11 novices, 1 intermediate and 1 expert dancers, and no classification framework was performed.

To raise the bar in the field of human activity recognition, we are planning to collect more data from experts dancers so as to capture as much of the variability as possible in different dancing tasks. Classify our data according to the better feature representation as well as the evaluation of the approach for different variation of activities such as tool usage, sport skills to mention but a few.

ACKNOWLEDGMENTS

Miguel Pérez-Xochicale gratefully acknowledges the studentship from the National Council for Science and Technology (CONACyT) Mexico to pursue his postgraduate studies at University of Birmingham.

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