Recurrence structure analysis: revealing metastable attractor dynamics from time series

Axel Hutt

email: axel.hutt@dwd.de

in close collaboration with

Peter beim Graben

BTU Cottbus - Senftenberg

Outline

(I) metastable dynamics

(II) detection and models

(III) applications

(IV) extensions

(I) metastable dynamics

(II) detection and models

(III) applications

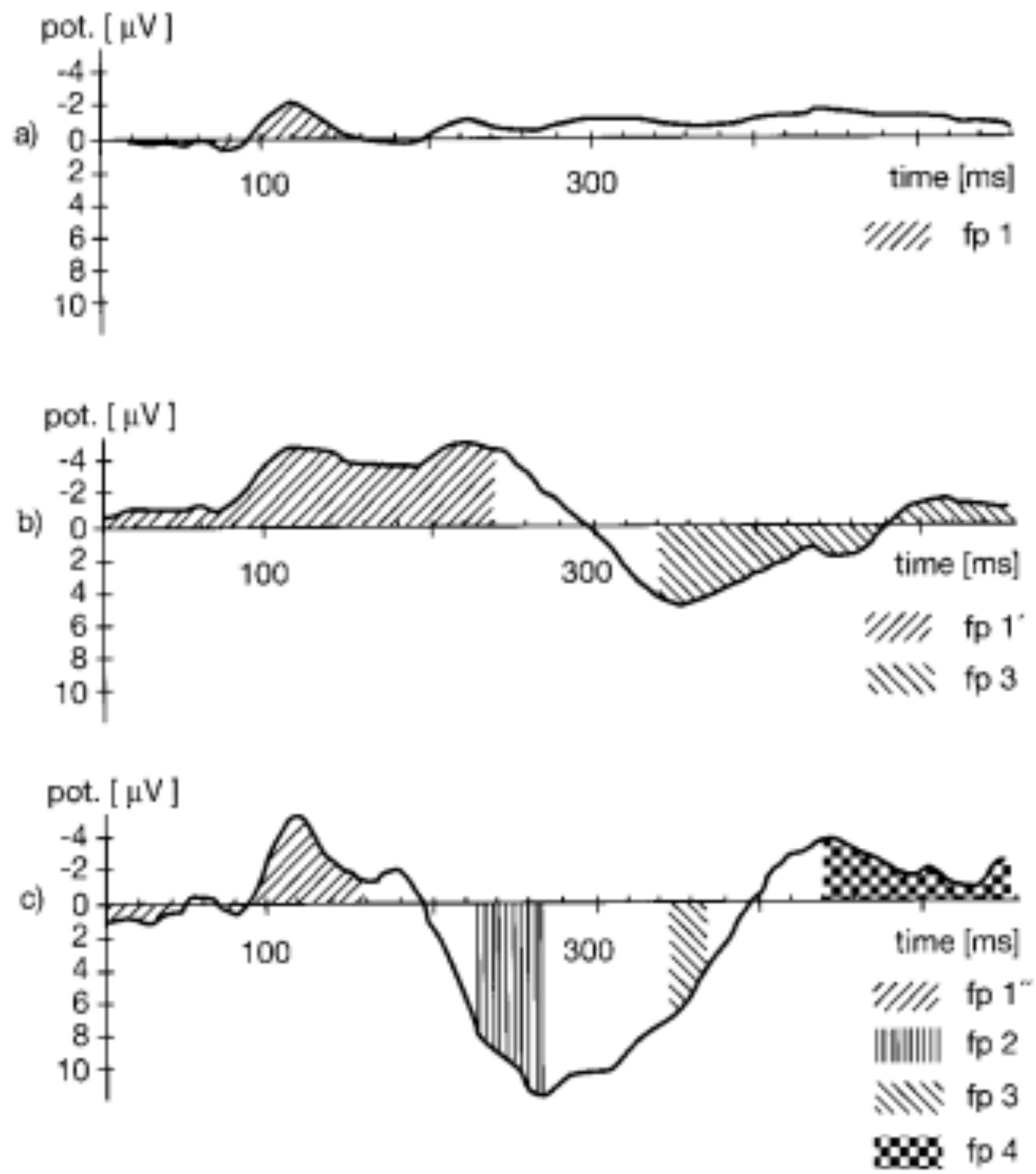
(IV) extensions

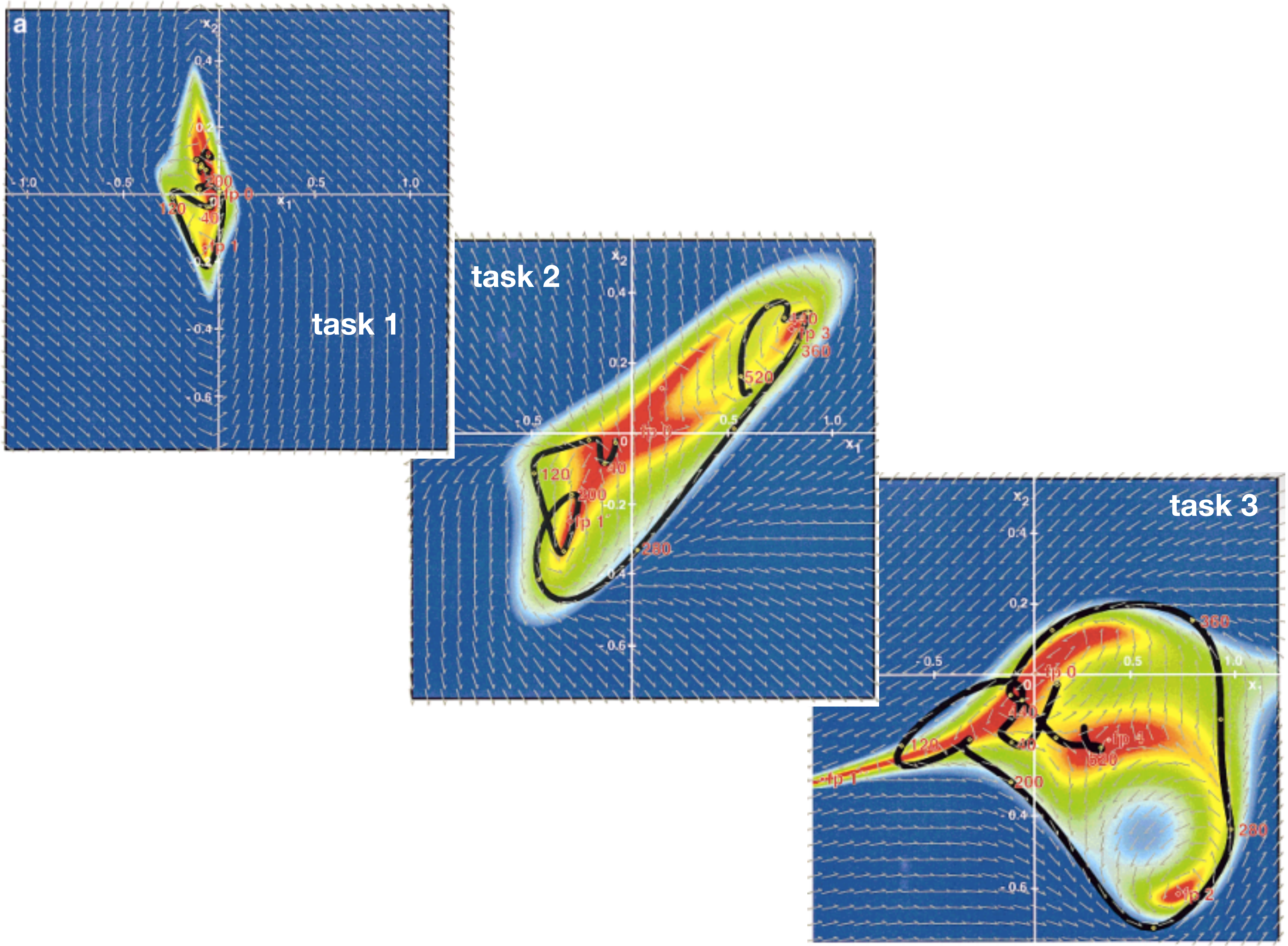
Event-related brain potentials (ERP)

(Uhl et al. (1998), Human Brain Mapping 6:137–149)

- 128 channel EEG
- three cognitive auditory tasks

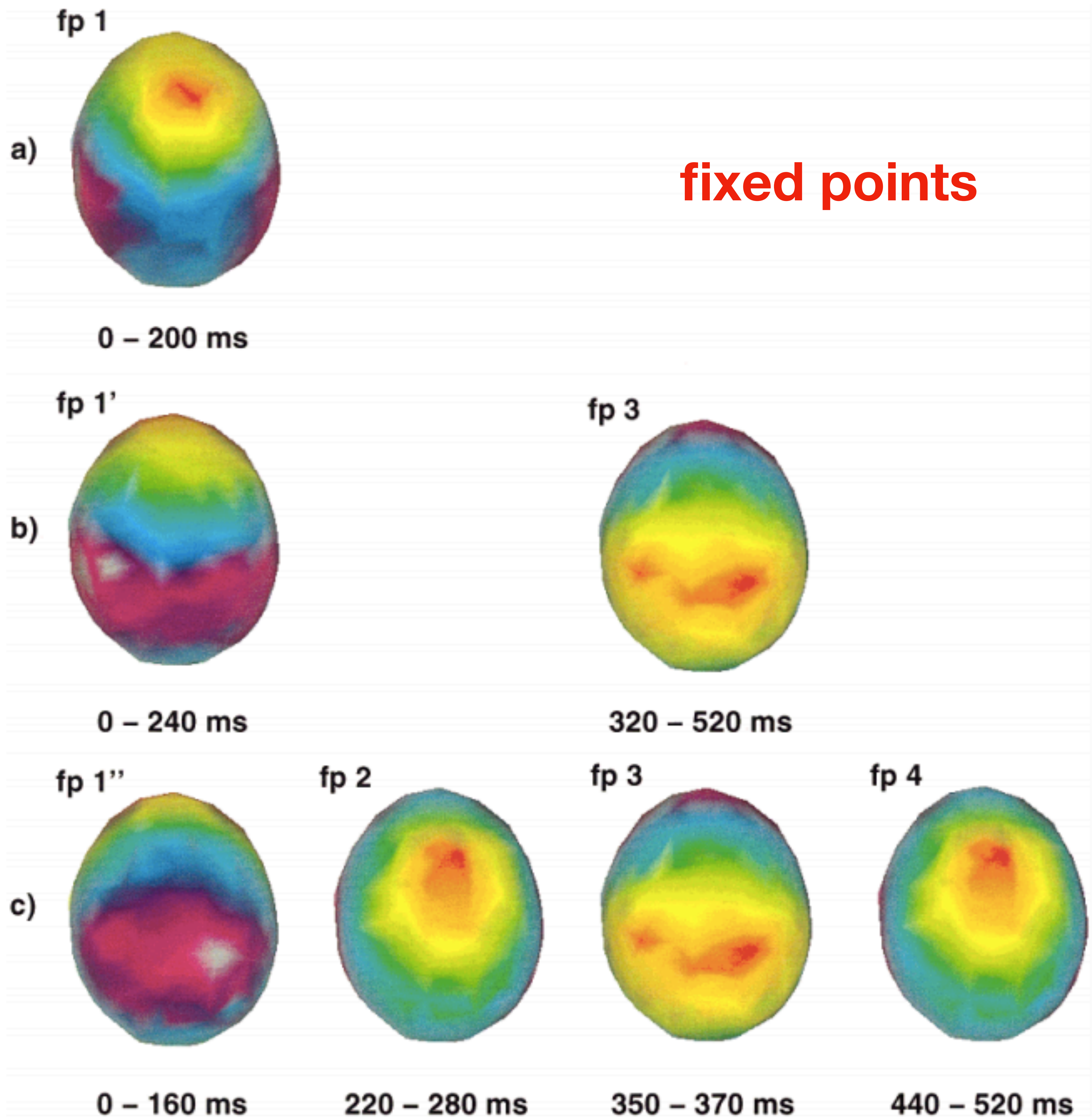
example: electrode Fz





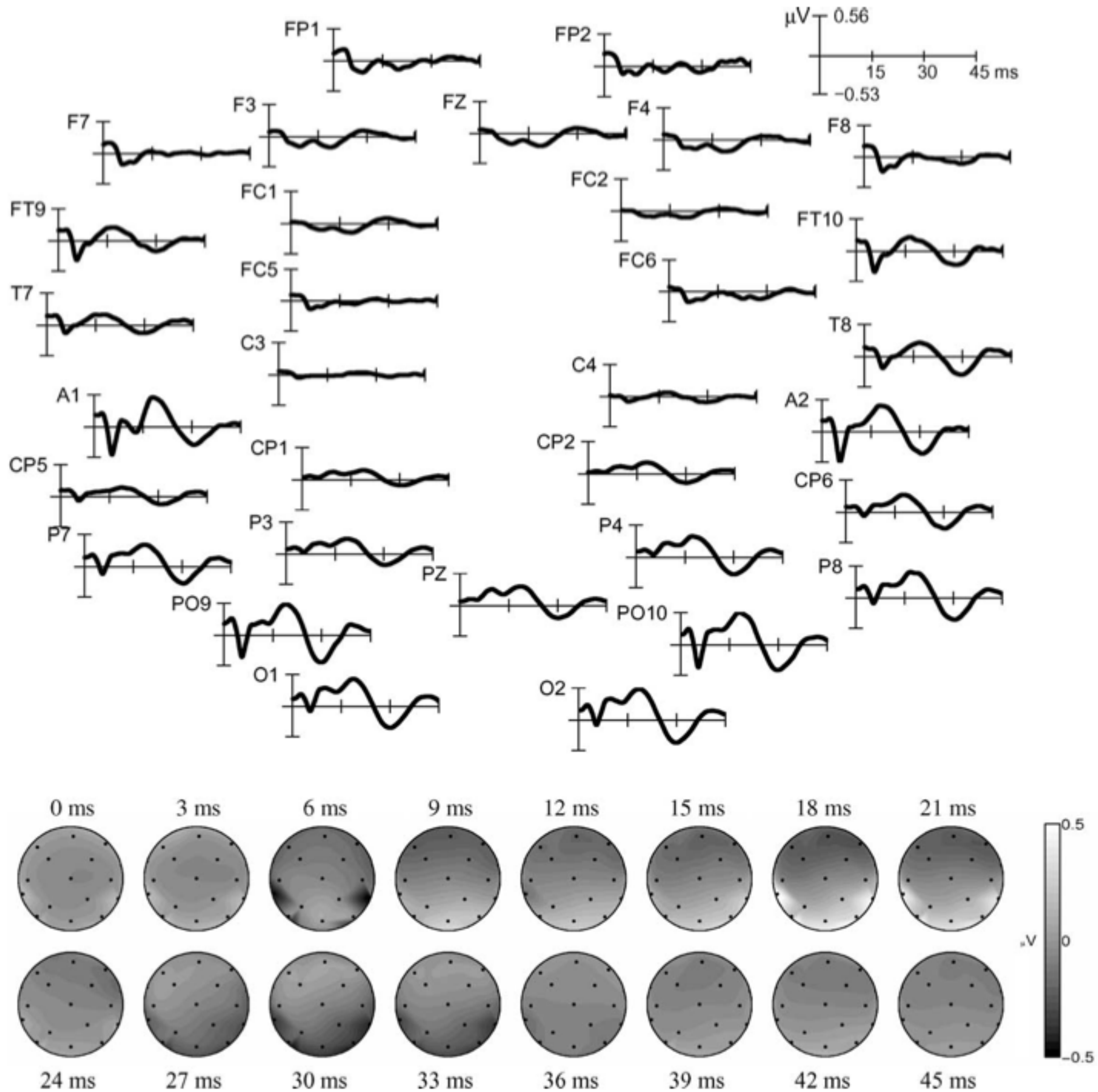
a
task 1
task 2
task 3

fixed points

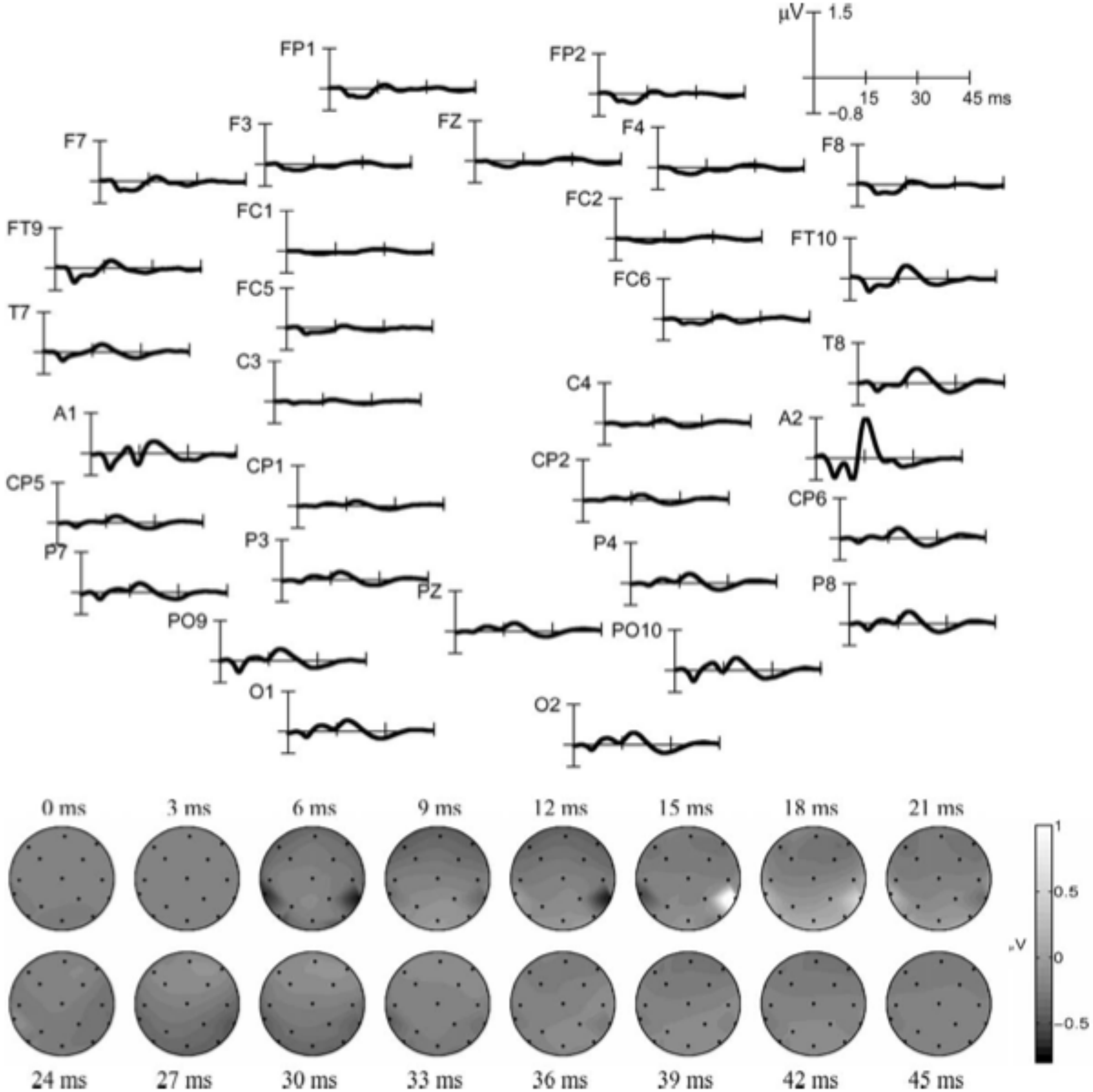


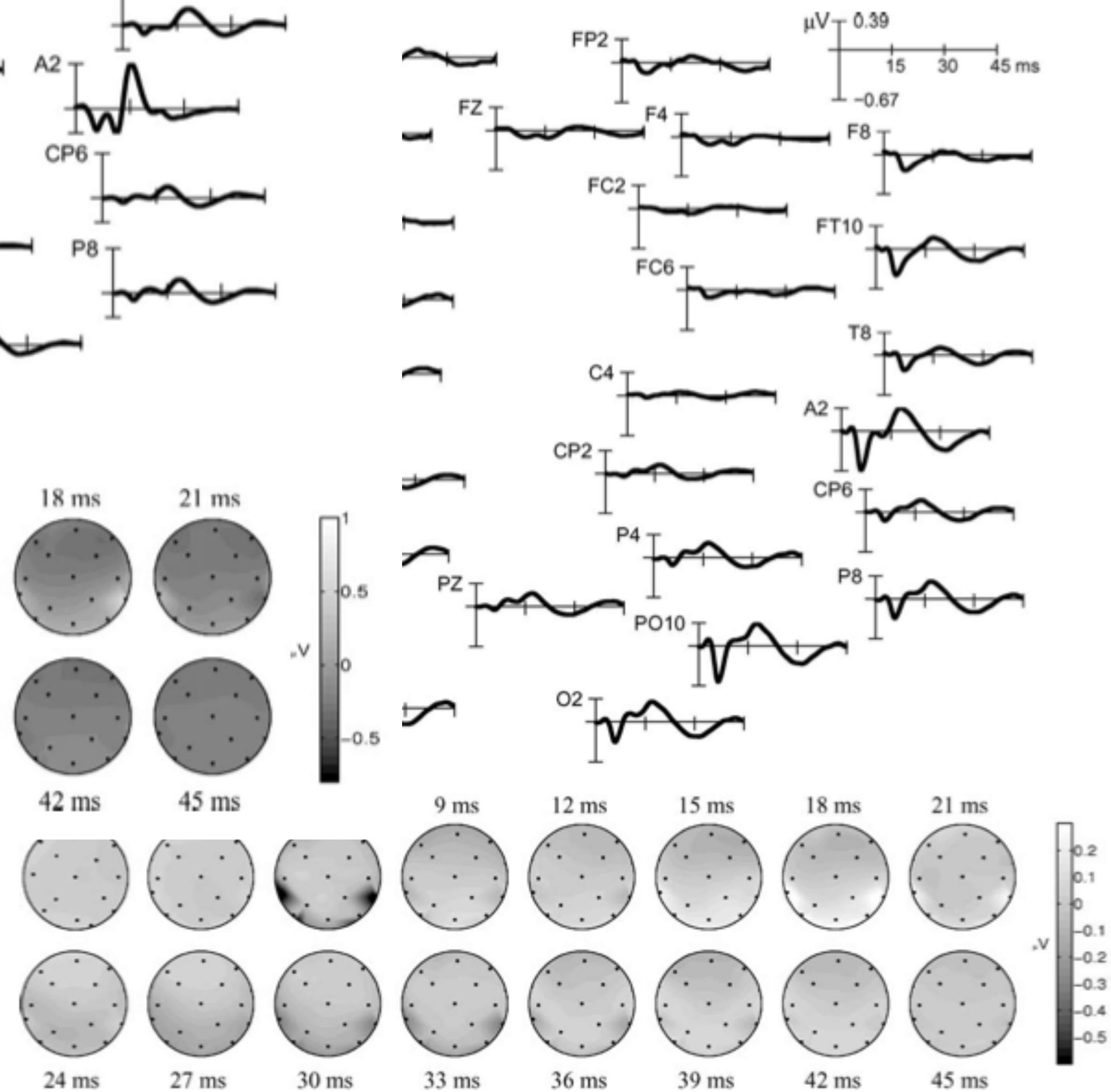
Event-related components in EEG

(Hutt and Riedel, Physica D 177:203-232 (2003))



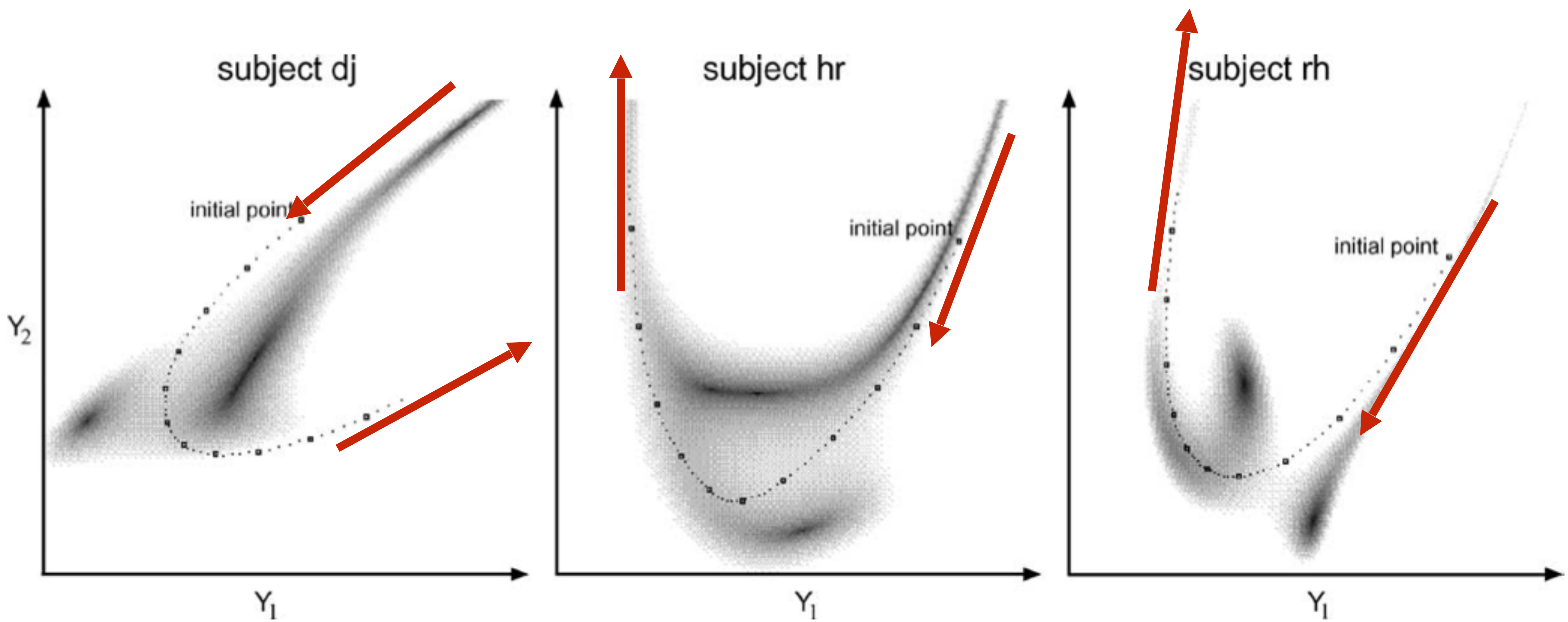
different subjects





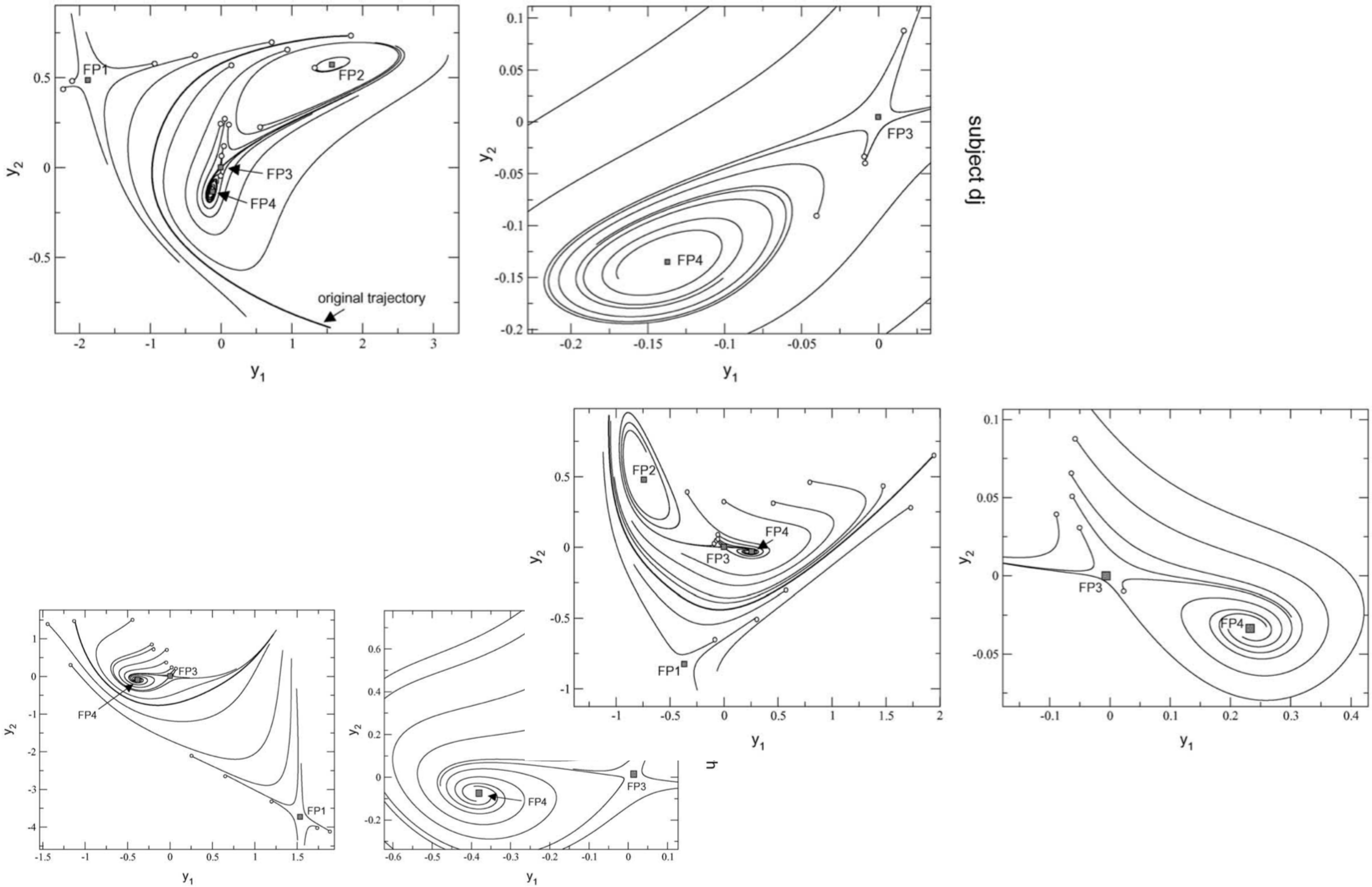
- Non-orthogonal PCA on two modes of ERP-component P30 at [27ms;32ms]

- optimal fit of ordinary differential equation system

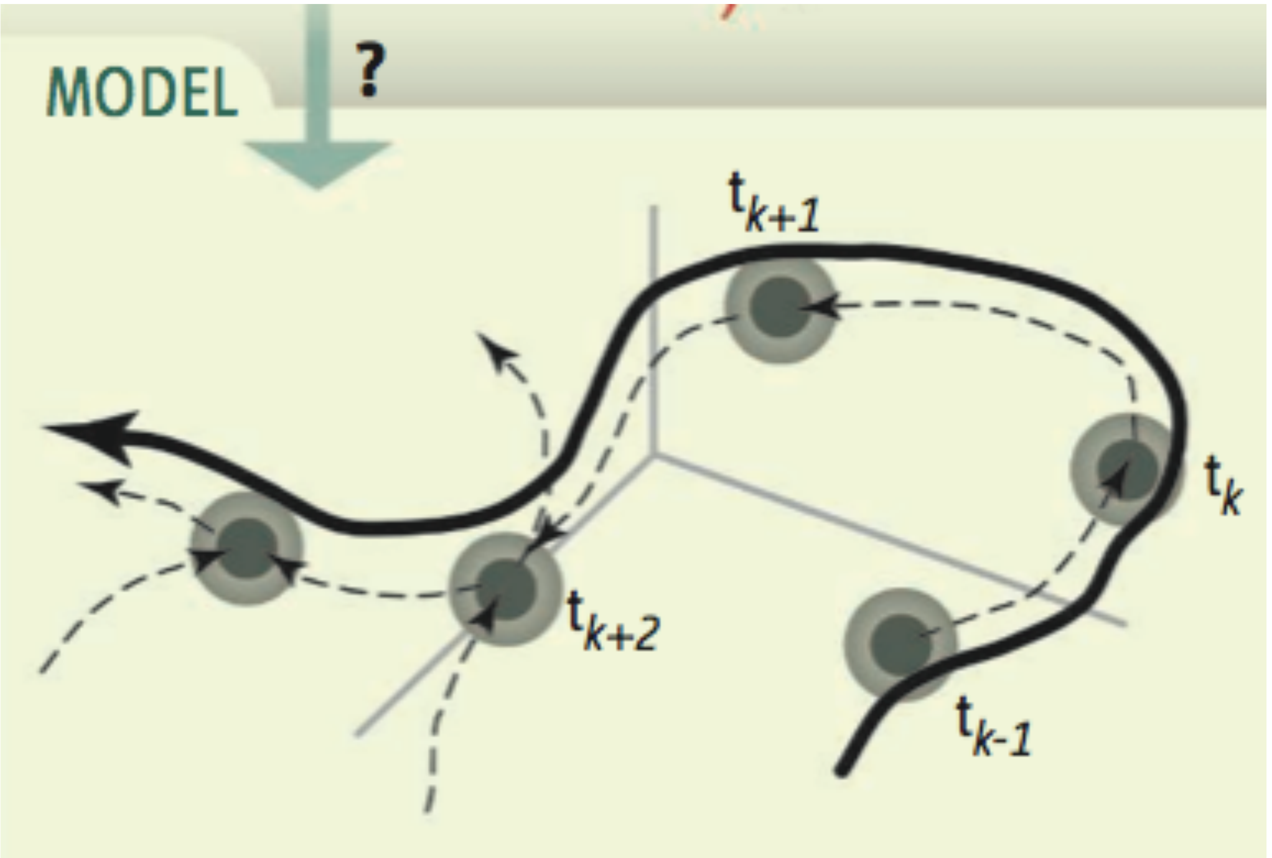


extraction of saddles comprising three fixed points

identical dynamical topology of ERP-components in different subjects



heteroclinic sequence of attractors



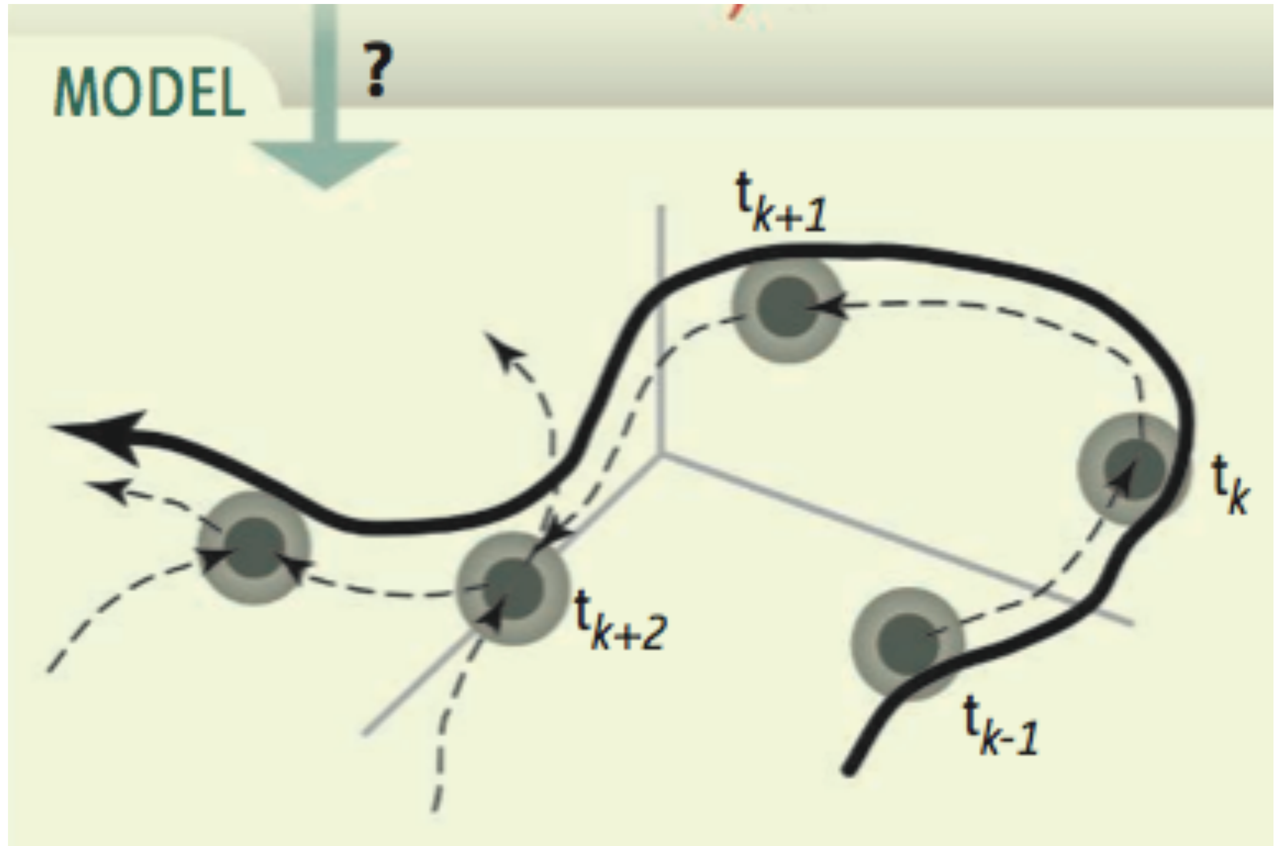
(Rabinovich et al., Science 321:48 (2008))

idea: attractors …

… are low-dimensional

… reflect
information processing

heteroclinic sequence of attractors



(Rabinovich et al., Science 321:48 (2008))

idea: attractors …

… are low-dimensional

… reflect
information processing

idea: transients …

… are high-dimensional

… do not reflect
information processing

Bird songs

tone

=

oscillatory metastable attractor

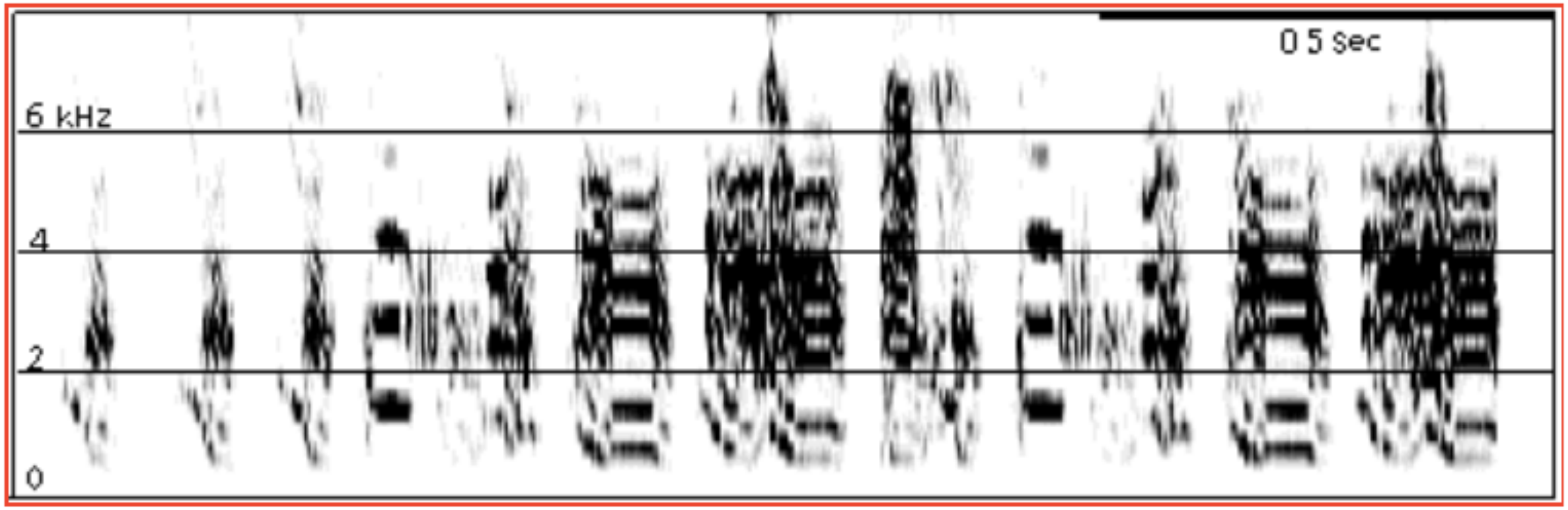
(Yildiz et al. (2013),PLoS Computational Biology 9(9) ;
Yildiz and Kiebel (2011), PLoS Computational Biology 7(12))

Bird songs

tone
=
oscillatory metastable attractor

(Yildiz et al. (2013),PLoS Computational Biology 9(9) ;
Yildiz and Kiebel (2011), PLoS Computational Biology 7(12))

sonogram of zebra fink



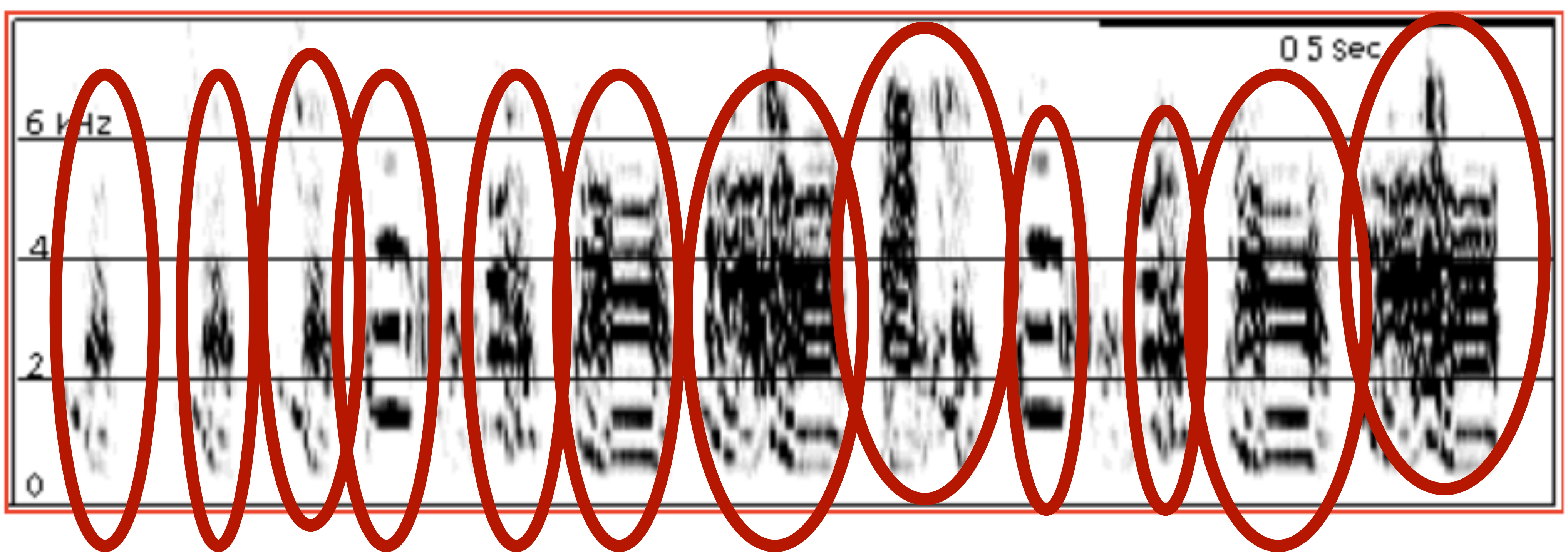
(taken from http://www.cs.cmu.edu/afs/cs/academic/class/15883-f13/lectures/birdsong/zfsong.html)

Bird songs

tone
=
oscillatory metastable attractor

(Yildiz et al. (2013),PLoS Computational Biology 9(9) ;
Yildiz and Kiebel (2011), PLoS Computational Biology 7(12))

sonogram of zebra fink



(taken from http://www.cs.cmu.edu/afs/cs/academic/class/15883-f13/lectures/birdsong/zfsong.html)

in general, sequences of metastable attractors are:

- high dimensional
- nonlinear
- non-stationary in duration and properties

in general, sequences of metastable attractors are:

- high dimensional
- nonlinear
- non-stationary in duration and properties

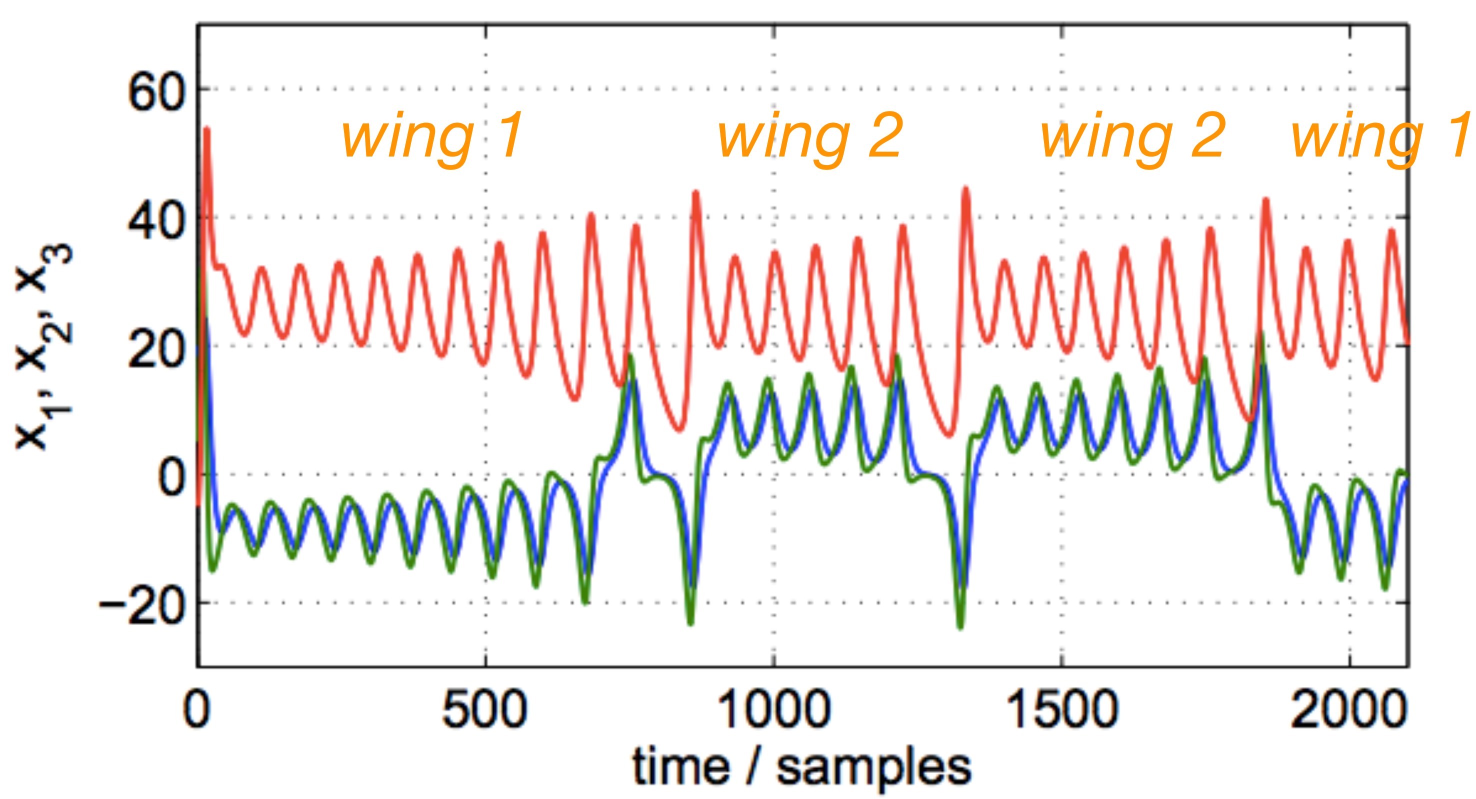
→ attractors are difficult to detect

in general, sequences of metastable attractors are:

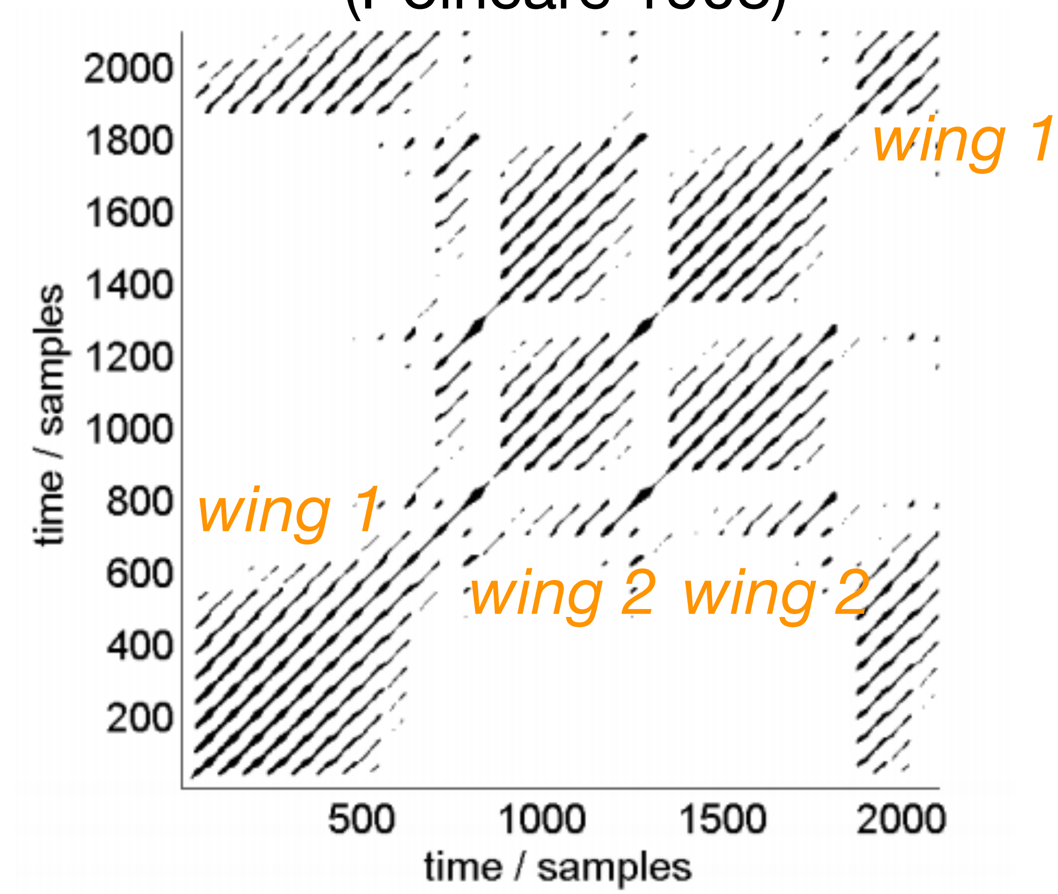
- high dimensional
- nonlinear
- non-stationary in duration and properties

<u>idea:</u>

detection of metastable attractors by recurrence plots computed from time series



(Poincare 1908)



(I) metastable dynamics

(II) feature detection and model

(III) applications

(IV) extensions

(II) feature detection and model

(1) detection of attractors

(2) model of attractor sequence

Recurrence plot

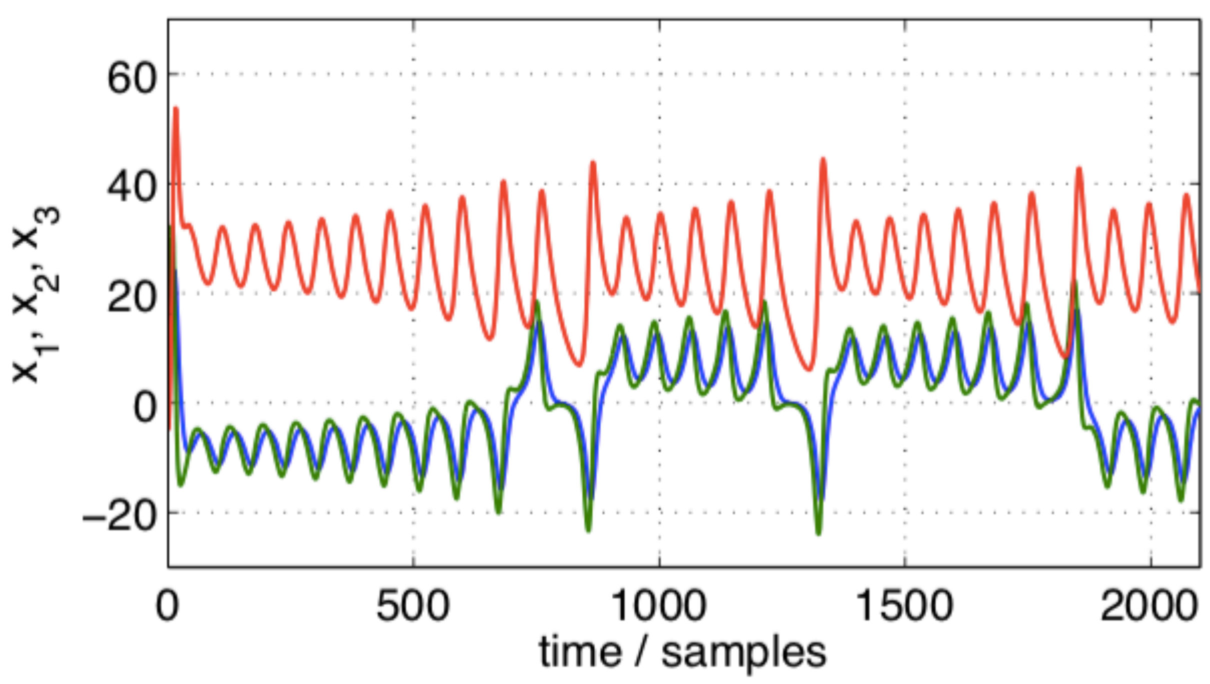
state space location is recurrent, if trajectory returns to phase space location

$$R(i,j) = \begin{cases} 1 & \text{if} \quad \|\vec{x}(i) - \vec{x}(j)\| \leq \varepsilon \\ 0 & \text{otherwise,} \end{cases}$$

Recurrence plot

state space location is recurrent, if trajectory returns to phase space location

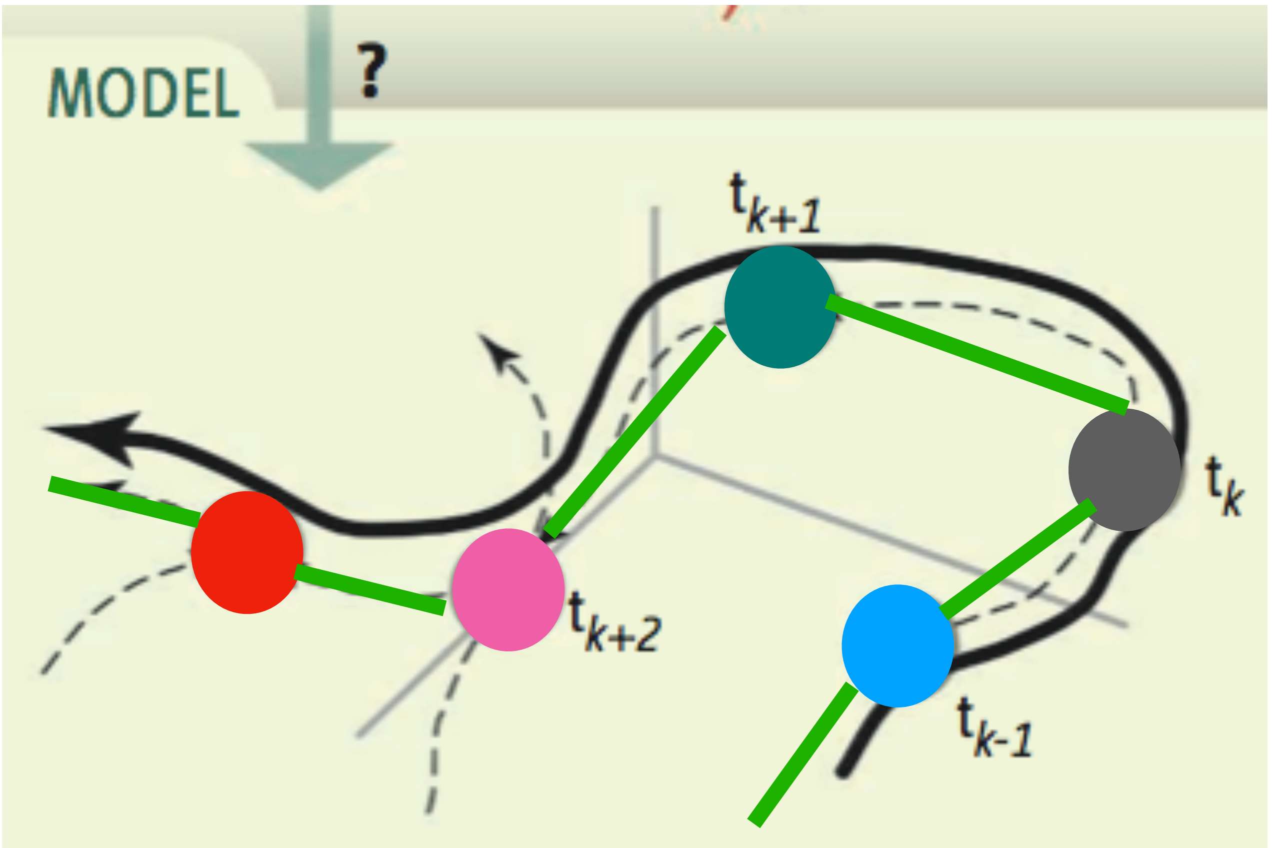
$$R(i,j) = \begin{cases} 1 & \text{if} \quad \|\vec{x}(i) - \vec{x}(j)\| \leq \varepsilon \\ 0 & \text{otherwise,} \end{cases}$$



symbolic dynamics
identifies metastable attractors

(Beim Graben and Hutt (2013), Physical Review Letters 110, 154101)

result:



identification of attractors

maps dynamics to

chain of attractors

long-time open question : how to choose ε ?

(1) equipartition model

$$H(\varepsilon) = -\sum_{k}^{M(\varepsilon)} p_k \log p_k$$

p_k: relative frequency of symbol k

M: number of symbols

$$h(\varepsilon) = \frac{H(\varepsilon)}{M(\varepsilon)} \qquad \varepsilon^* = \arg\max_{\varepsilon} h(\varepsilon)$$

long-time open question : how to choose ε ?

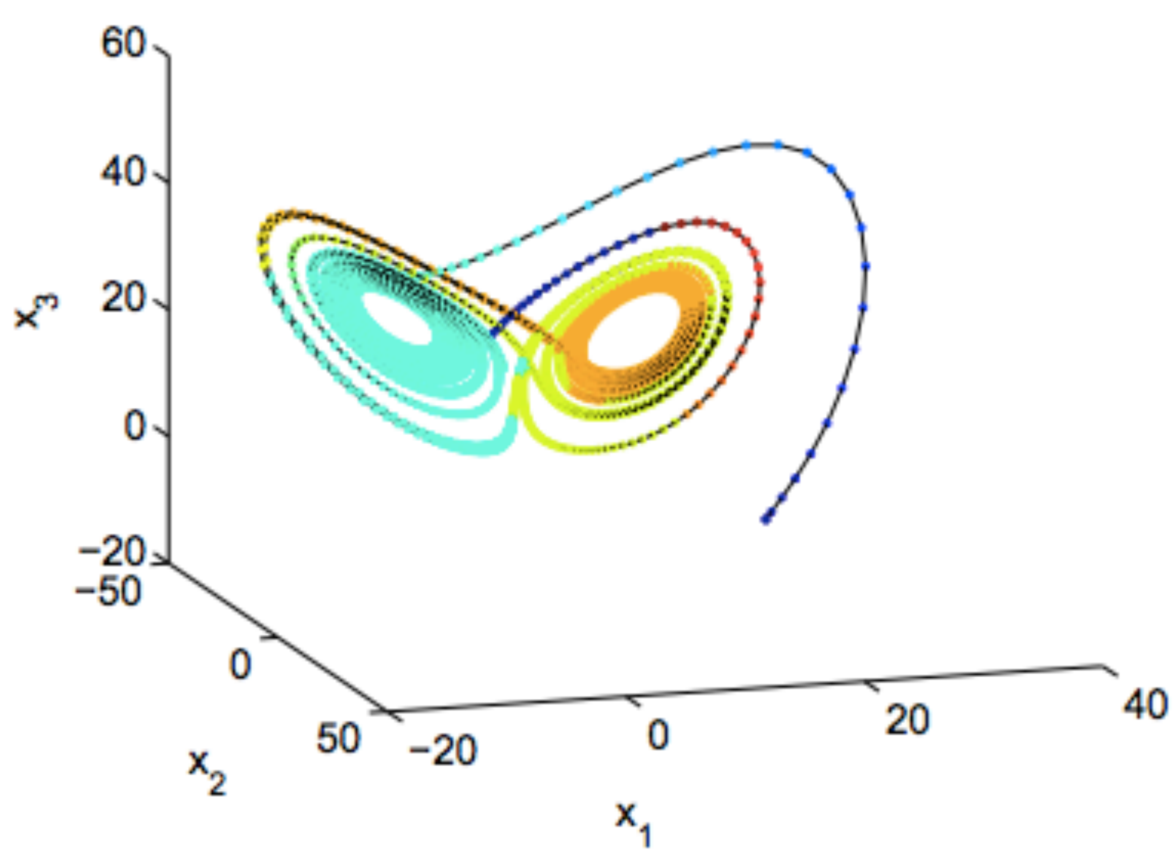
(1) equipartition model

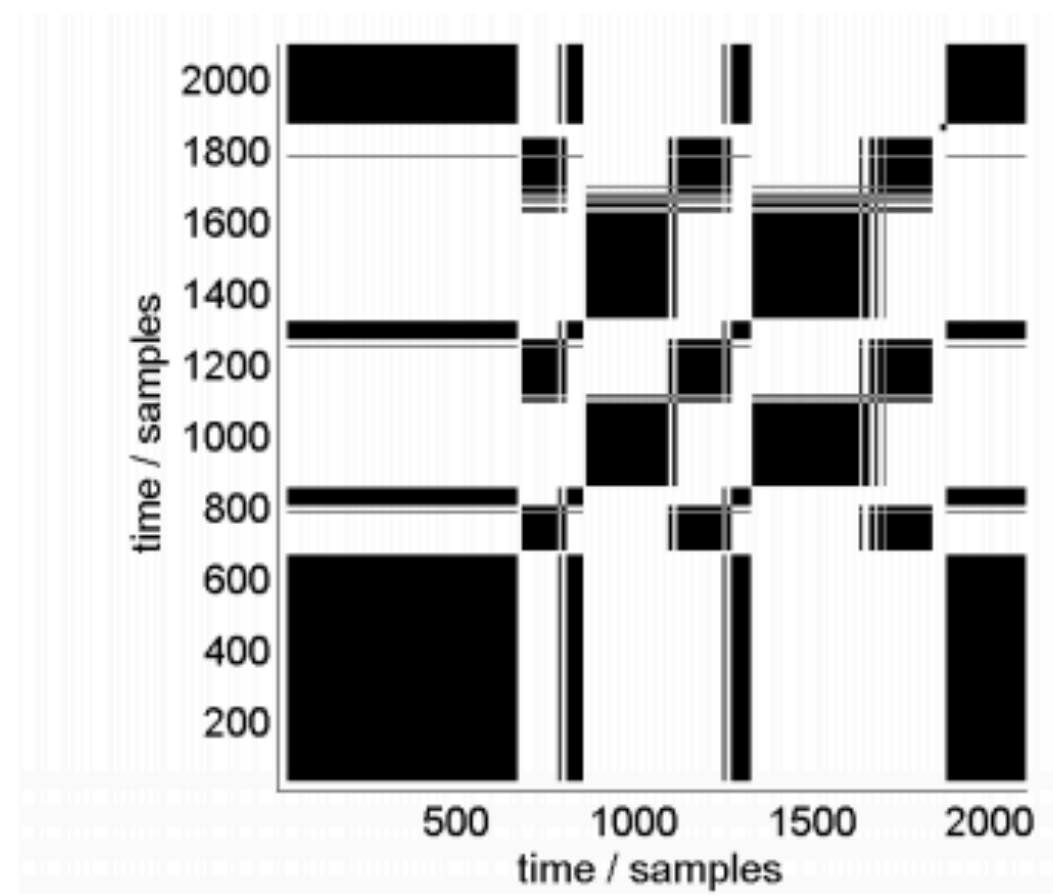
$$H(\varepsilon) = -\sum_{k}^{M(\varepsilon)} p_k \log p_k$$

p_k: relative frequency of symbol k

M: number of symbols

$$h(\varepsilon) = \frac{H(\varepsilon)}{M(\varepsilon)} \qquad \varepsilon^* = \arg\max_{\varepsilon} h(\varepsilon)$$

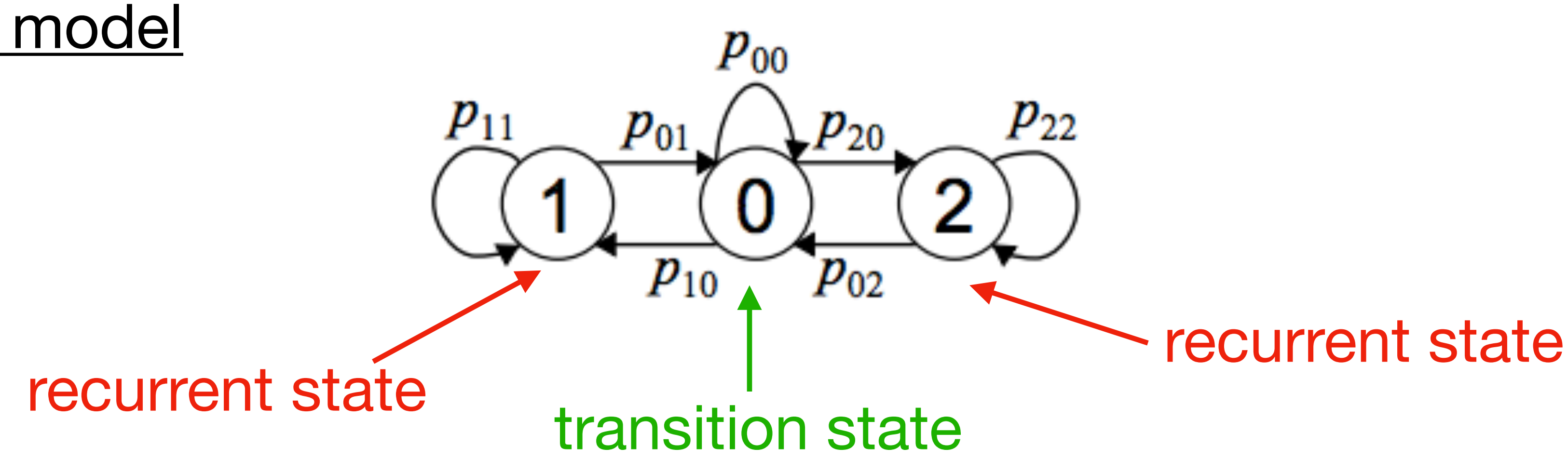




(Beim Graben and Hutt (2013), Physical Review Letters 110, 154101)

long-time open question : how to choose ε ?

(2) Markov model



$$u(\varepsilon) = \frac{1}{n+2}\left[\operatorname{tr} \boldsymbol{P}(\varepsilon) + h_r(\varepsilon) + h_c(\varepsilon)\right]$$

$$\varepsilon^* = \arg\max_{\varepsilon} u(\varepsilon)$$

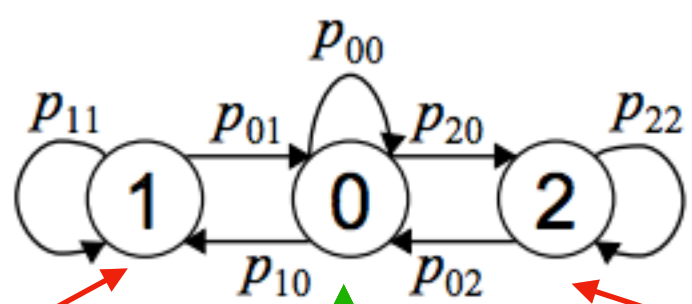
$$h_r = -\frac{1}{\log(n-1)} \sum_{j=1}^{n-1} p'_{0j} \log p'_{0j}$$

$$h_c = -\frac{1}{\log(n-1)} \sum_{i=1}^{n-1} p'_{i0} \log p'_{i0}$$

P: transition probability matrix

long-time open question : how to choose ε ?

(2) Markov model



recurrent state

transition state

recurrent state

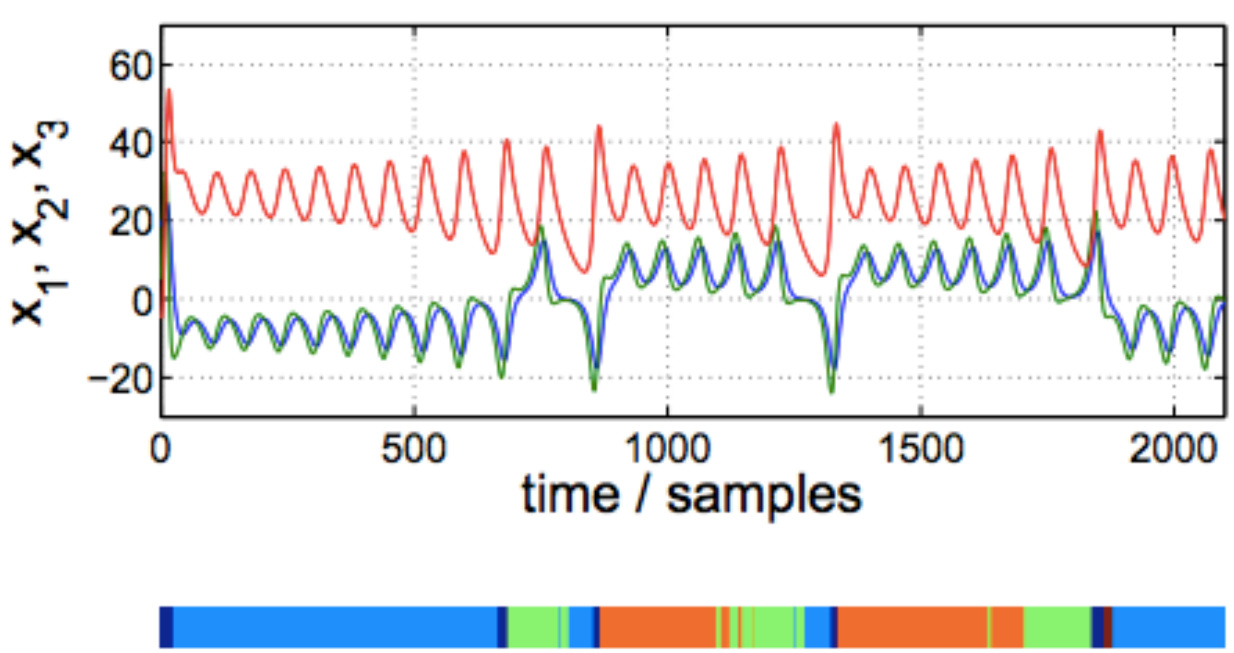
$$u(\varepsilon) = \frac{1}{n+2}\left[\operatorname{tr} \boldsymbol{P}(\varepsilon) + h_r(\varepsilon) + h_c(\varepsilon)\right]$$

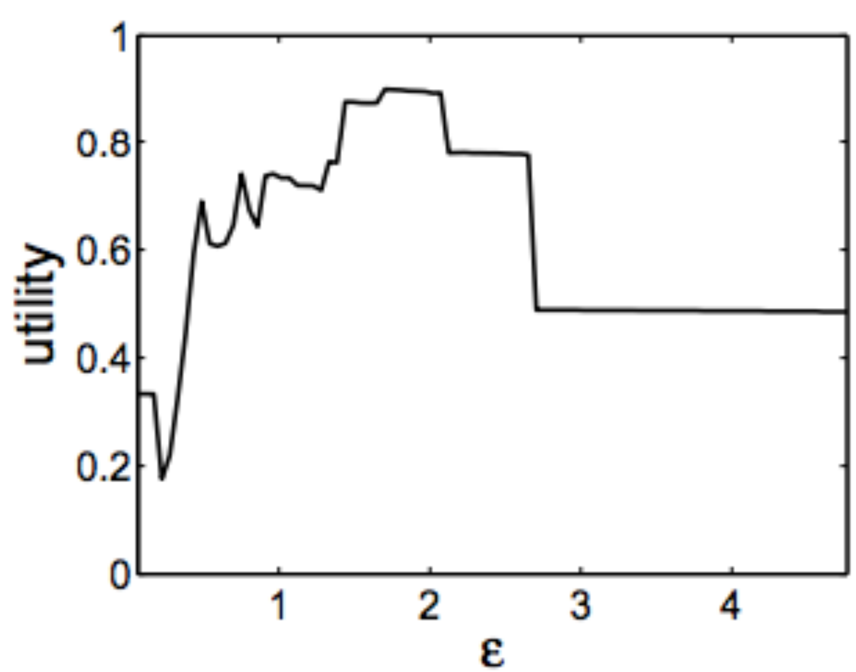
$$\varepsilon^* = \arg\max_{\varepsilon} u(\varepsilon)$$

$$h_r = -\frac{1}{\log(n-1)} \sum_{j=1}^{n-1} p'_{0j} \log p'_{0j}$$

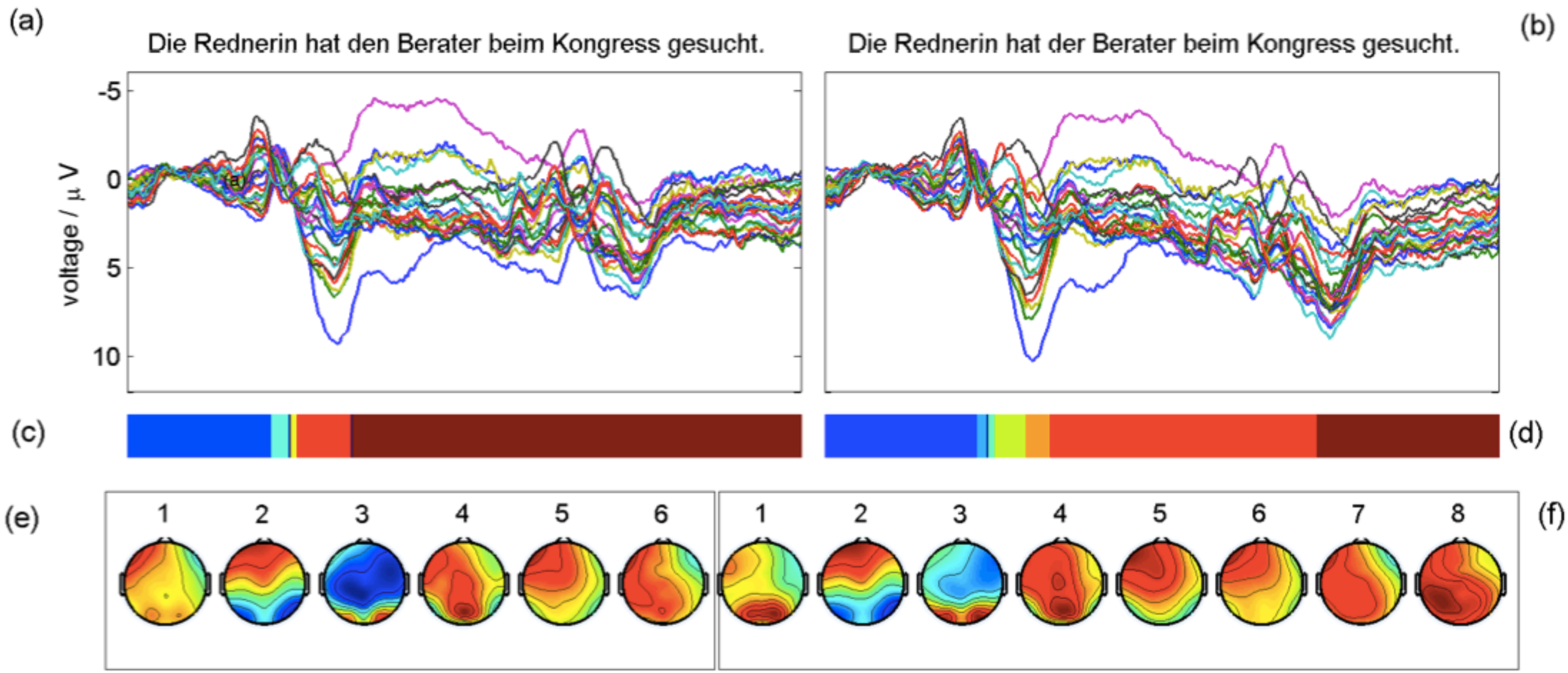
$$h_c = -\frac{1}{\log(n-1)} \sum_{i=1}^{n-1} p'_{i0} \log p'_{i0}$$

P: transition probability matrix





(Beim Graben et al. (2016), Europhys. Lett. 114(3): 38003) example: multi-channel event-related potentials in linguistic EEG experiments



sequence of states / meta-stable attractors

(ERP data from P.beimGraben et al.,Cogn. Neurodyn.(2008))

(II) feature detection and model

(1) detection of attractors

(2) model of attractor sequence

model template for heteroclinic sequence

$$\frac{\mathrm{d}\xi_k}{\mathrm{d}t} = \xi_k \left(\sigma_k - \sum_{j=1}^{n} \rho_{kj} \xi_j \right)$$

generalized Lotka-Volterra system in n dimensions

$$u(x,t) = \sum_{k=1}^{n} \alpha_k(t) v_k(x)$$

$$\alpha_k(t) = \frac{\xi_k}{\sigma_k}$$

amplitudes of attractors $v_k(x)$

model template for heteroclinic sequence

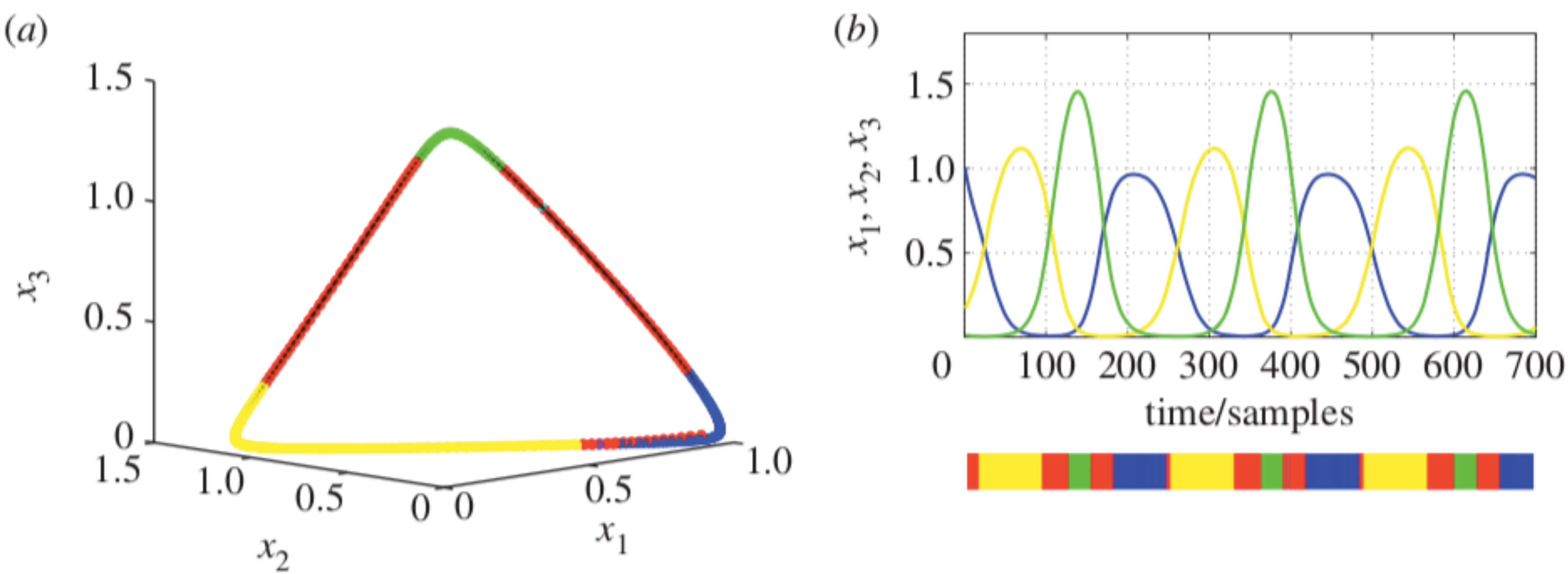
$$\frac{d\xi_k}{dt} = \xi_k \left(\sigma_k - \sum_{j=1}^{n} \rho_{kj}\xi_j \right)$$

generalized Lotka-Volterra system in n dimensions

$$u(x,t) = \sum_{k=1}^{n} \alpha_k(t) v_k(x) \qquad \alpha_k(t) = \frac{\xi_k}{\sigma_k}$$

amplitudes of attractors $v_k(x)$

n=3:



construction of neural field model from attractor model

$$u(x,t) = \sum_{k=1}^{n} \alpha_k(t) v_k(x)$$

u: mean potential of neural population

v_k: attractor state, e.g. spatial pattern

α_k: amplitude of attractor state

construction of neural field model from attractor model

$$u(x,t) = \sum_{k=1}^{n} \alpha_k(t) v_k(x)$$

u: mean potential of neural population

v_k: attractor state, e.g. spatial pattern

α_k: amplitude of attractor state

$$\boxed{\begin{aligned}\frac{\partial u(x,t)}{\partial t} &= -u(x,t) + \int_\Omega w_1(x,y) u(y,t)\,\mathrm{d}y \\ &\quad + \int_\Omega \int_\Omega w_2(x,y,z) u(y,t) u(z,t)\,\mathrm{d}y\,\mathrm{d}z\end{aligned}}$$

$$w_1(x,y) = \sum_k (\sigma_k + 1) v_k(x) v_k^+(y)$$

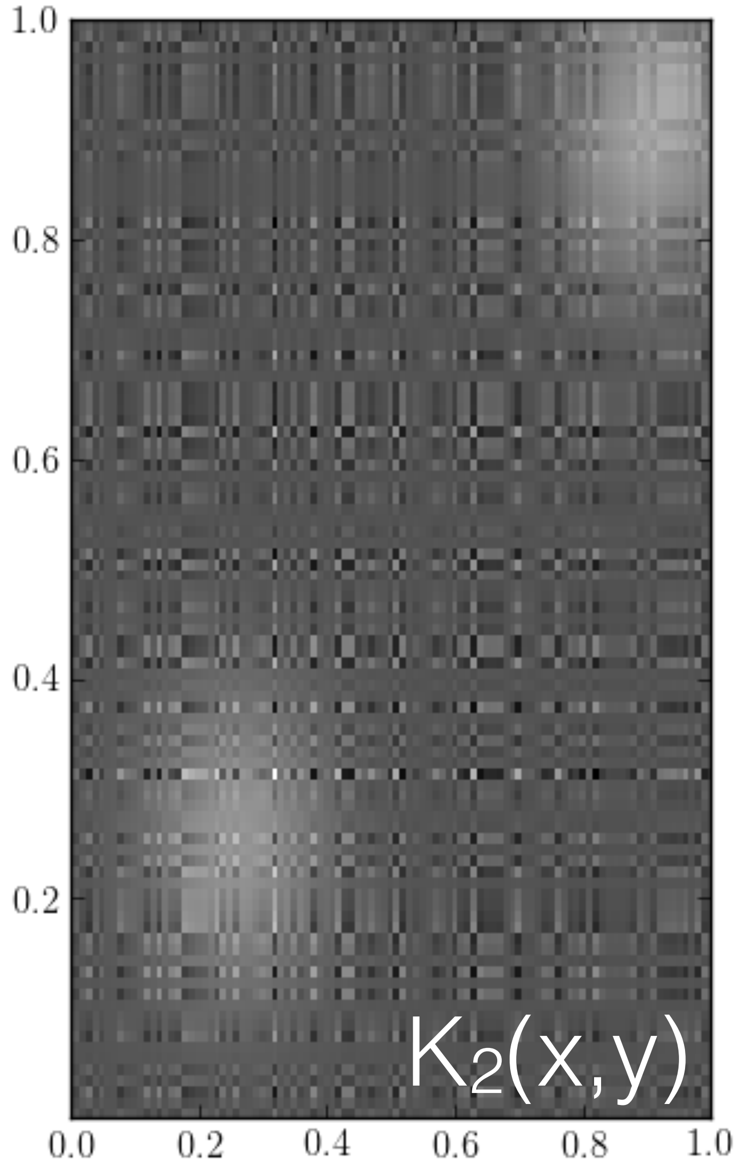
$$w_2(x,y,z) = -\sum_{kj} \sigma_j \rho_{kj} v_k(x) v_k^+(y) v_j^+(z)$$

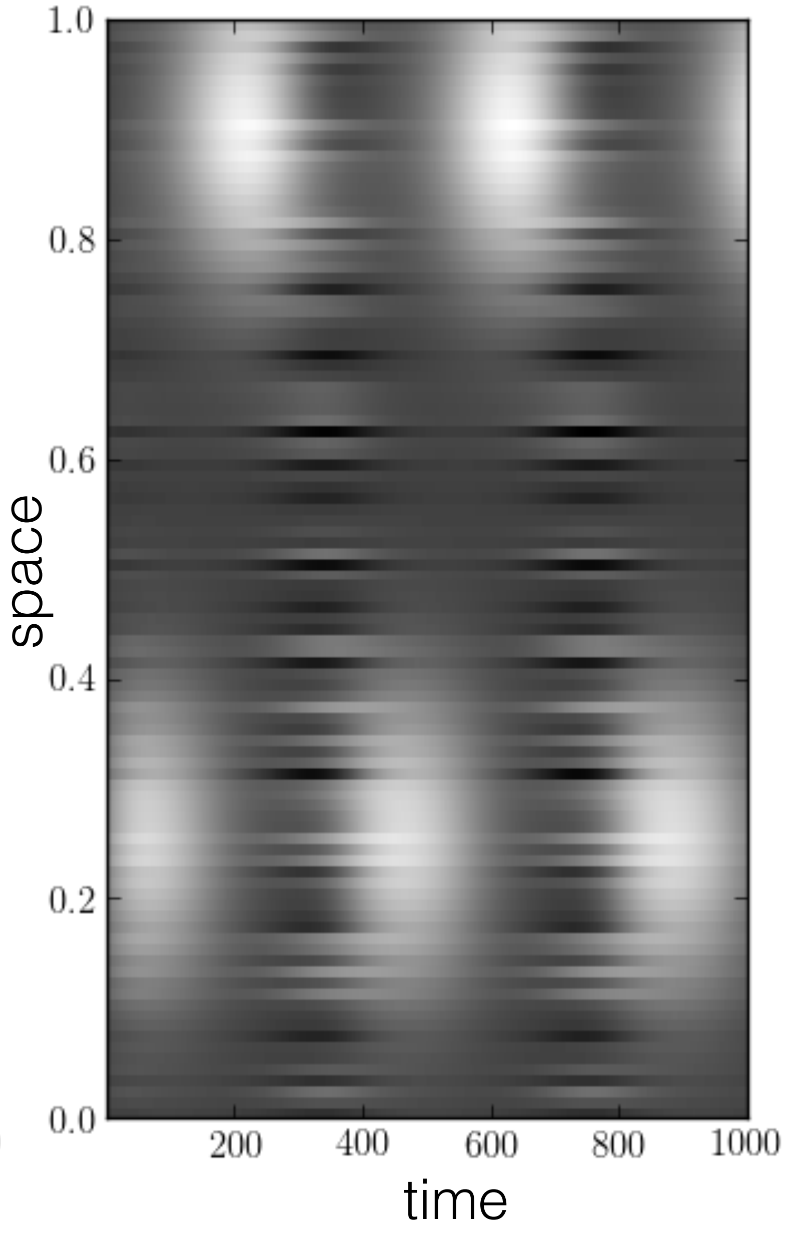
Hebbian kernels

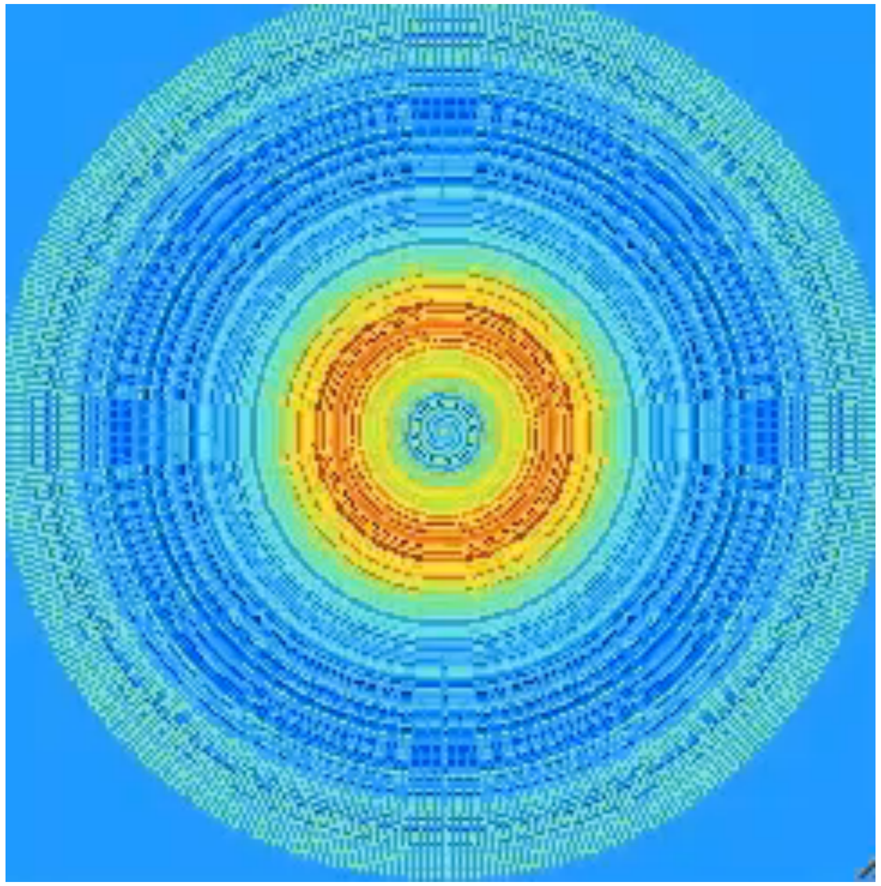
(Hutt and beim Graben (2017), Front. Appl. Math. Stat. 3:11 ;
Schwappach et al. (2015), Front. Syst. Neurosci. 9, 97)

Example: simulation results with 3 saddle attractors

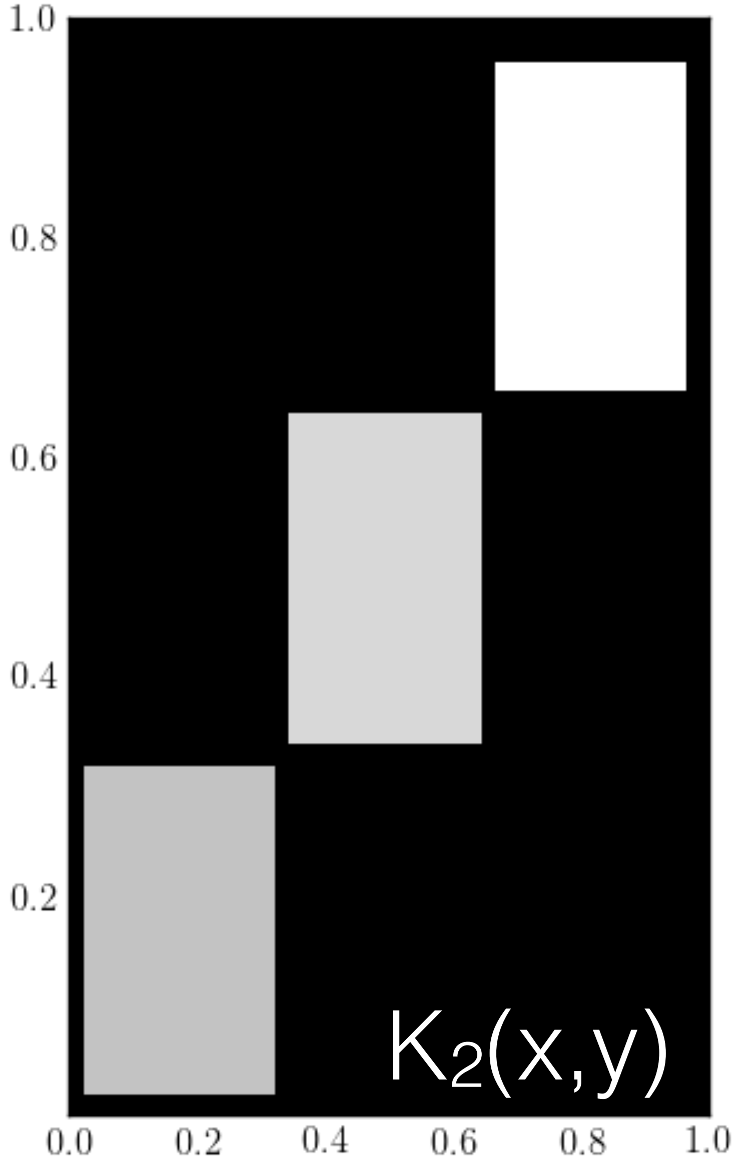
1. stored patterns:
   two spatially separated bell functions + global noise

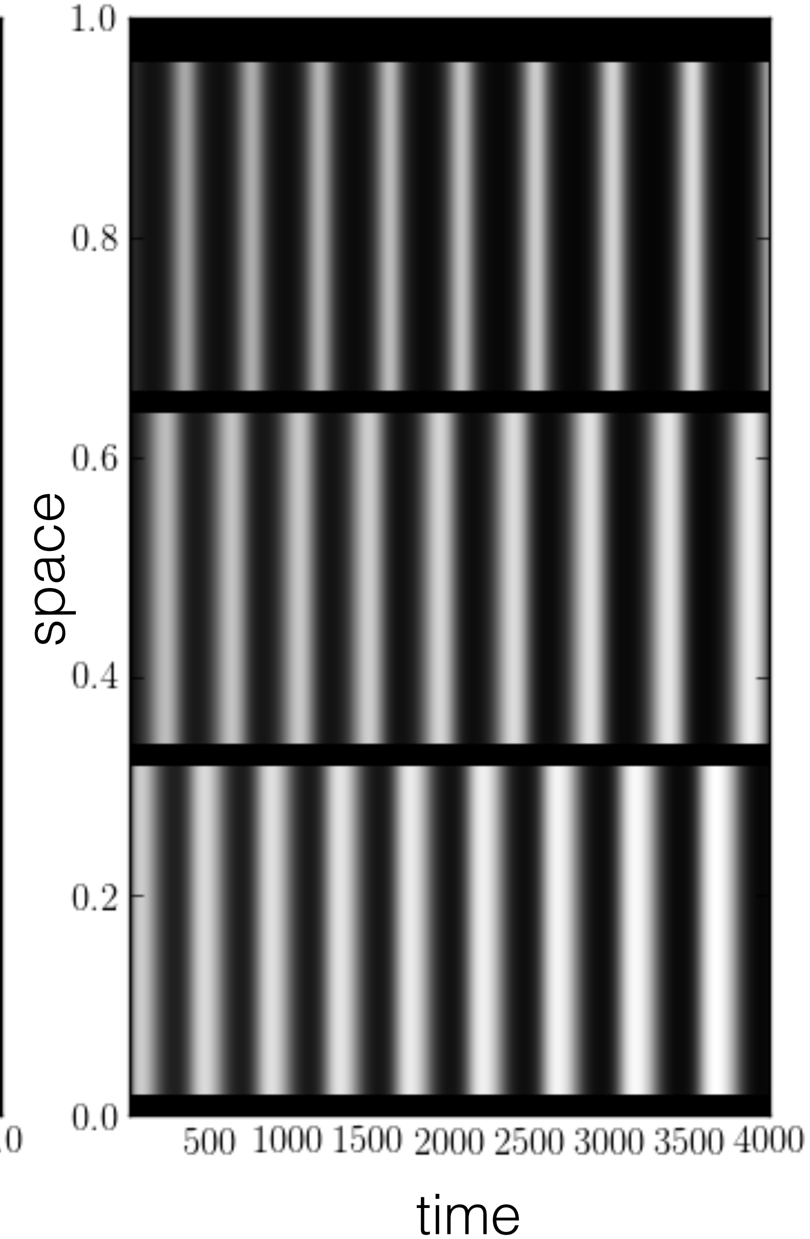


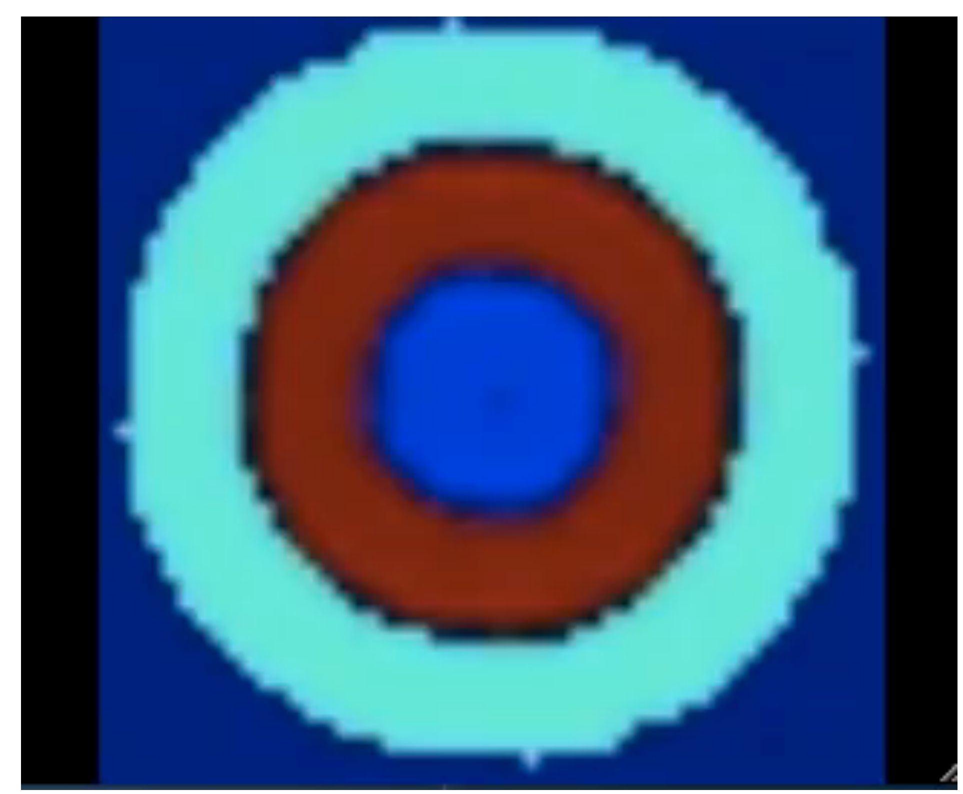




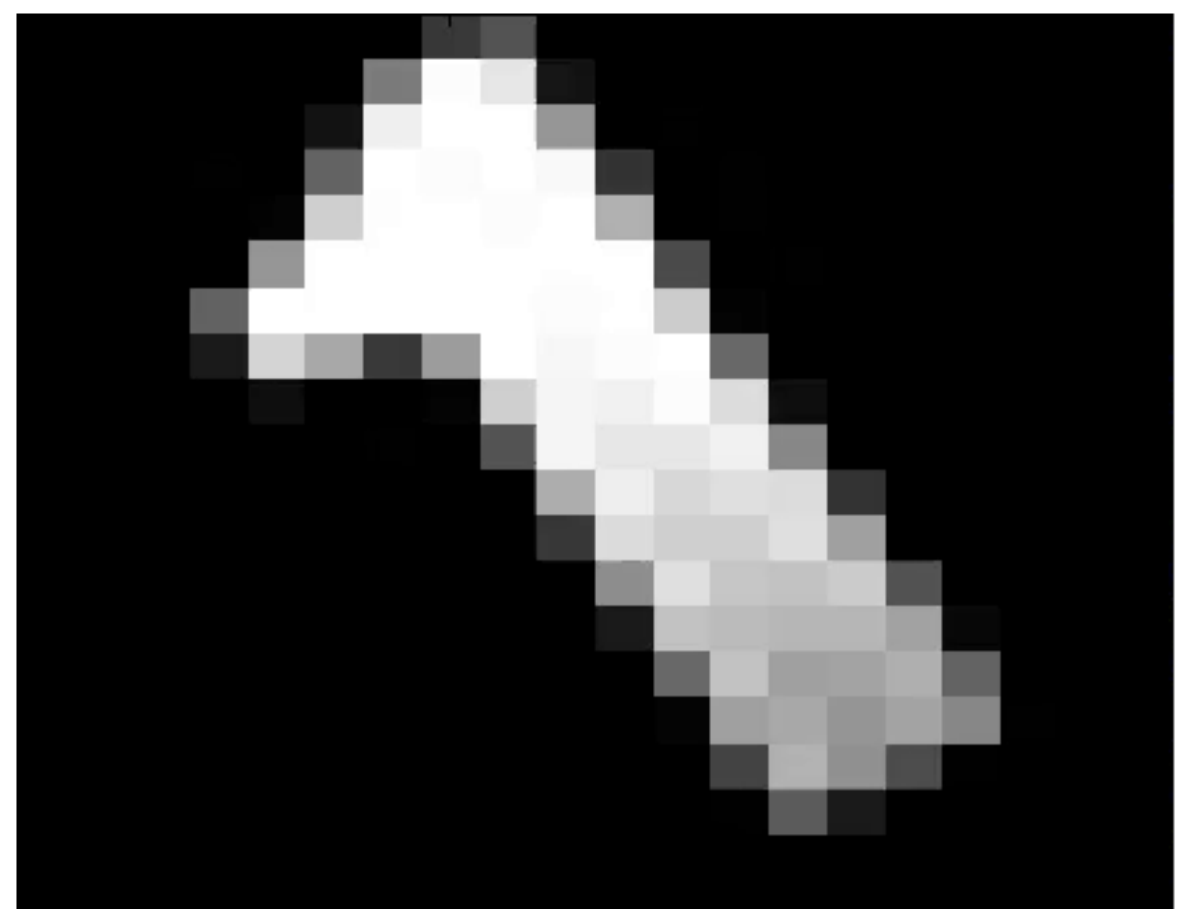
2. stored patterns:
three displaced blocks







3. 2-dimensional patterns: artificial data



(Schwappach et al. (2015), Front. Syst. Neurosci. 9, 97)

(I) metastable dynamics

(II) feature detection and model

(III) applications

(IV) extensions

(III) applications

(1) univariate data

(2) multivariate data

if univariate time series are given,

recurrence plot is not meaningful

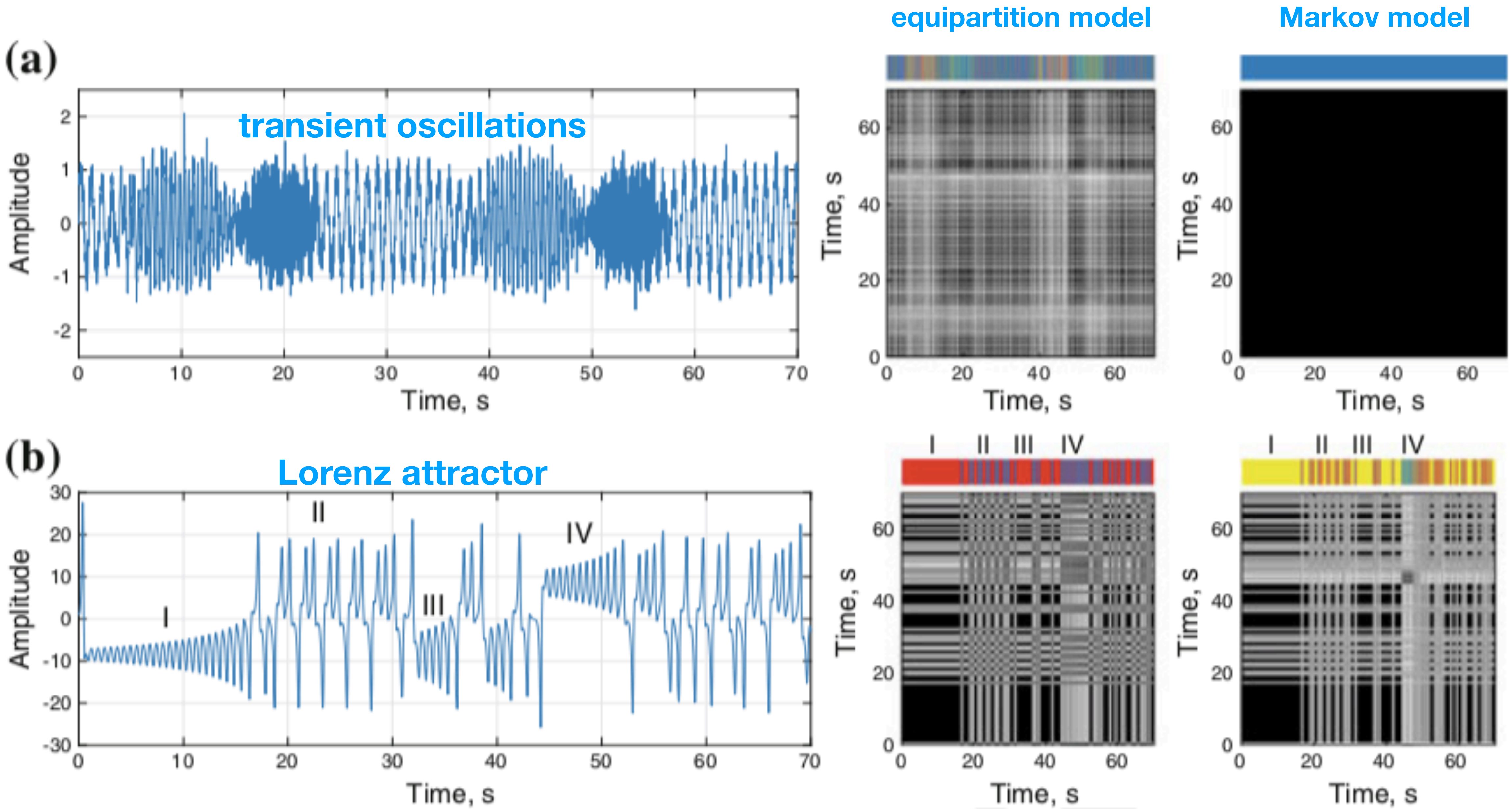
→ expansion of dimension is mandatory

if univariate time series are given,

recurrence plot is not meaningful

→ expansion of dimension is mandatory

first approach: delayed embedding

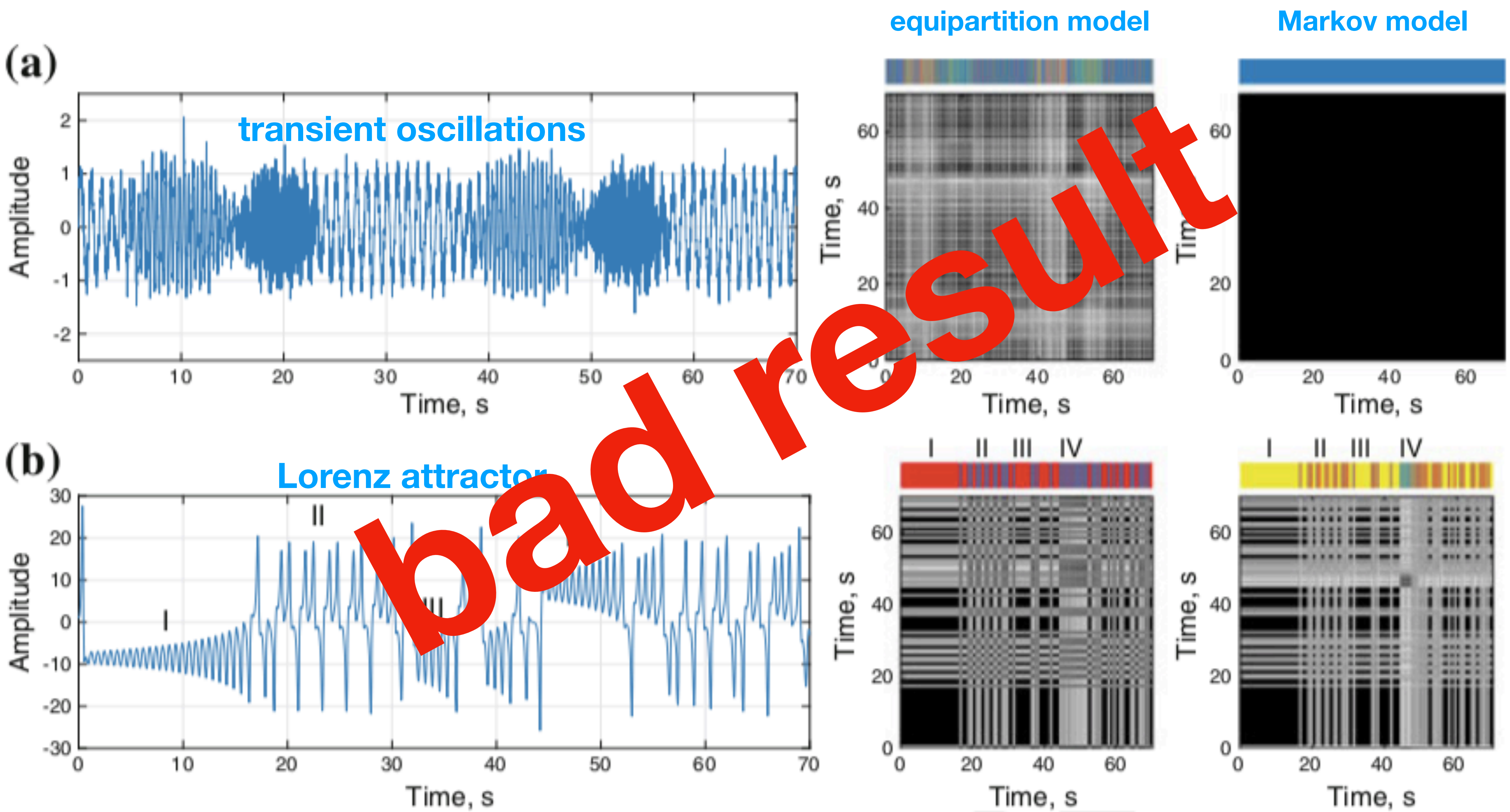


if univariate time series are given,

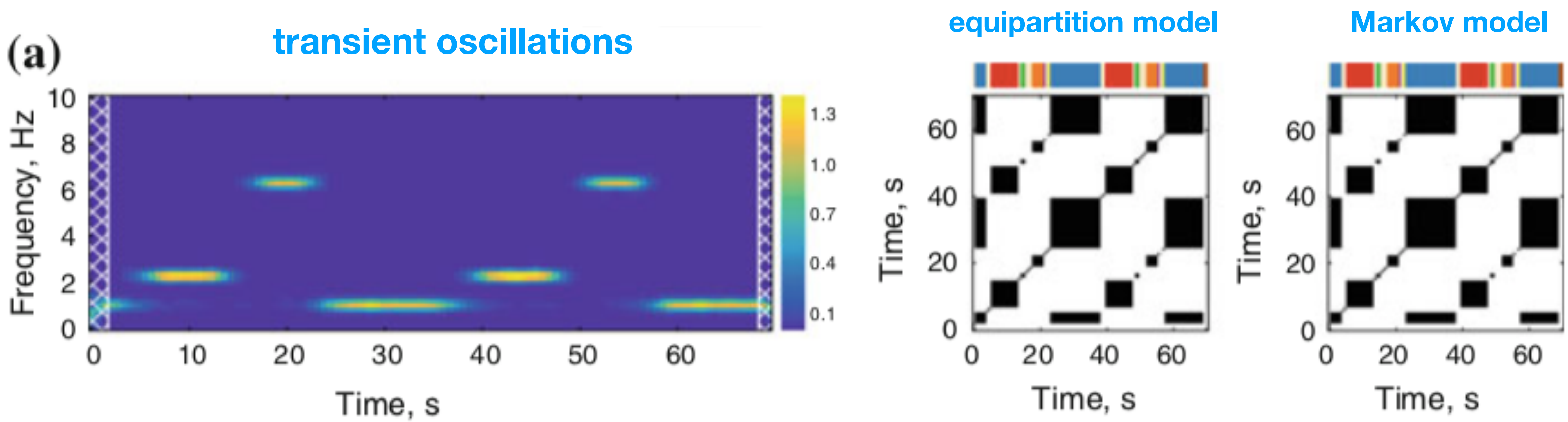
recurrence plot is not meaningful

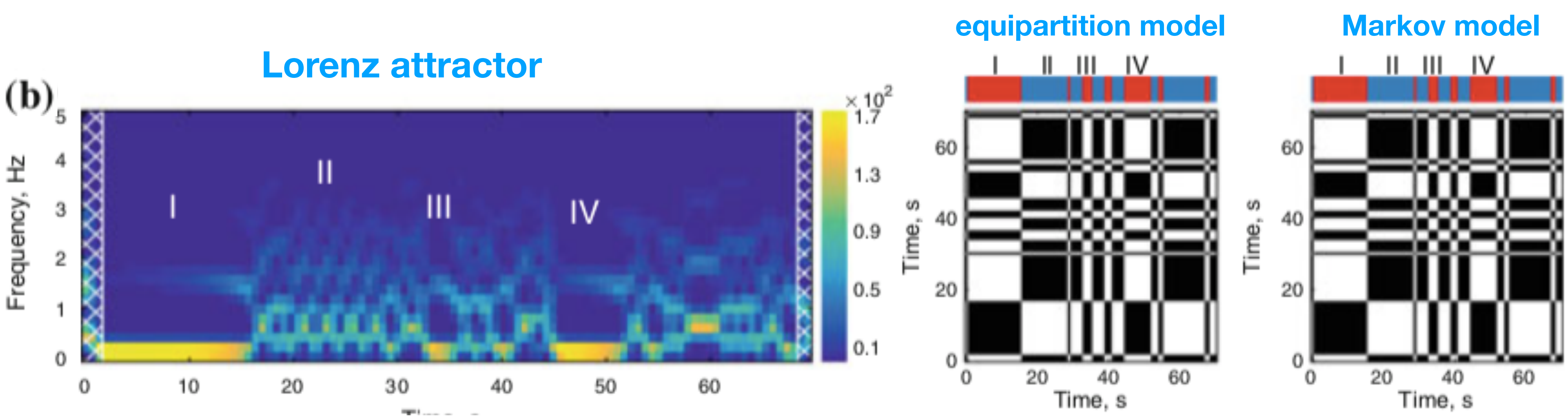
→ expansion of dimension is mandatory

first approach: delayed embedding



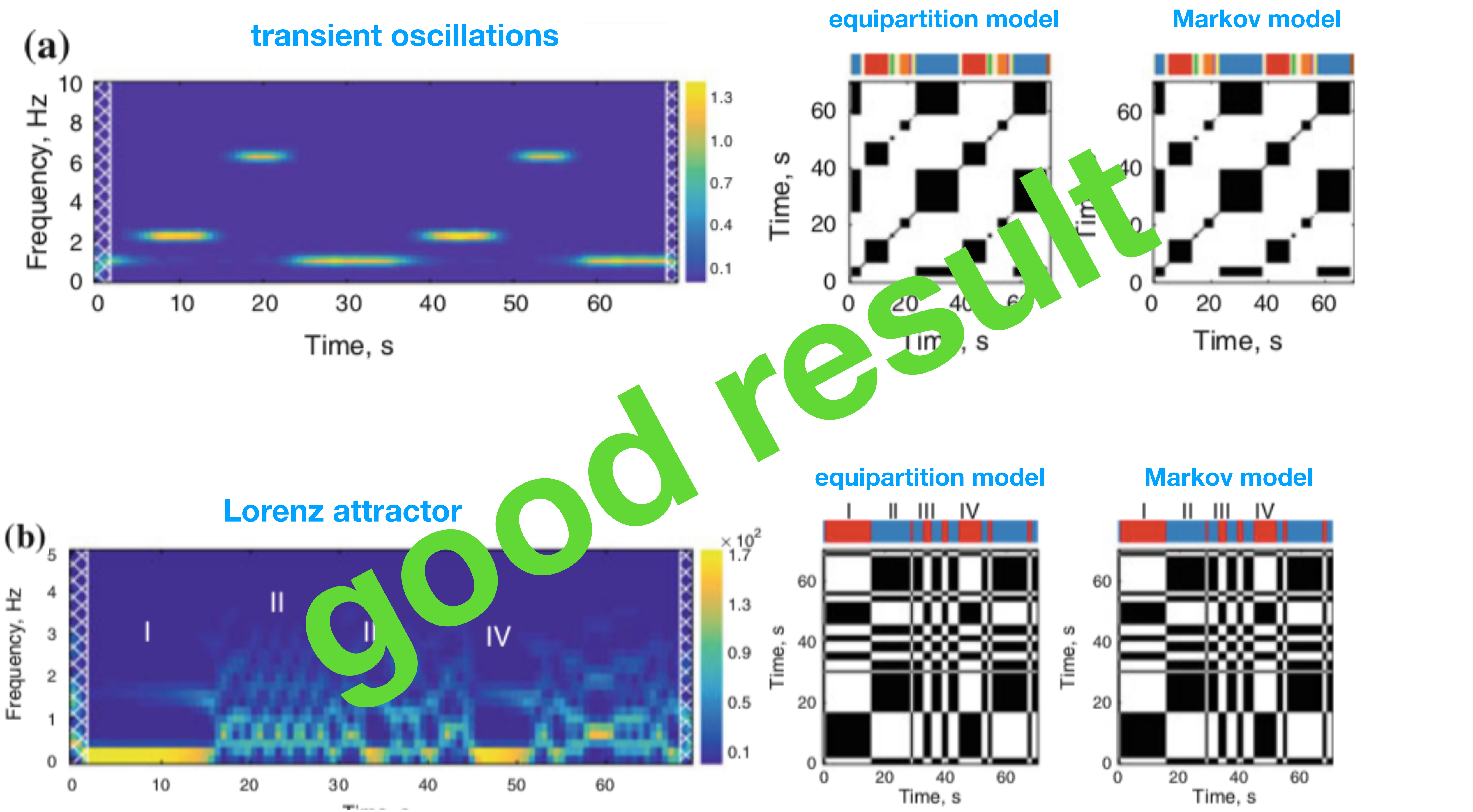
improved approach:
time-frequency spectral power distribution





(Fedotenkova et al. (2017), In: International Work-Conference on Time Series Analysis ITISE 2016: 89-102)

improved approach:
time-frequency spectral power distribution



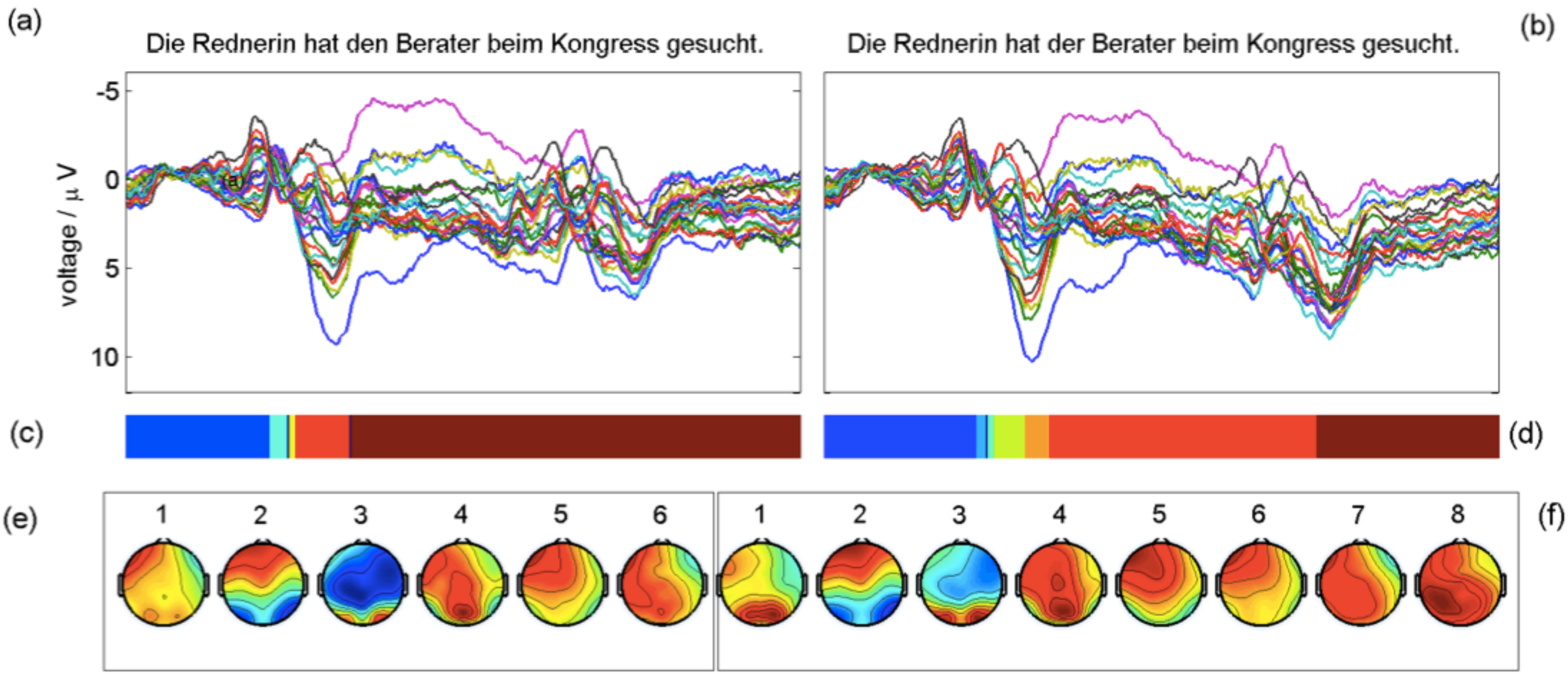
(Fedotenkova et al. (2017), In: International Work-Conference on Time Series Analysis ITISE 2016: 89-102)

(III) applications

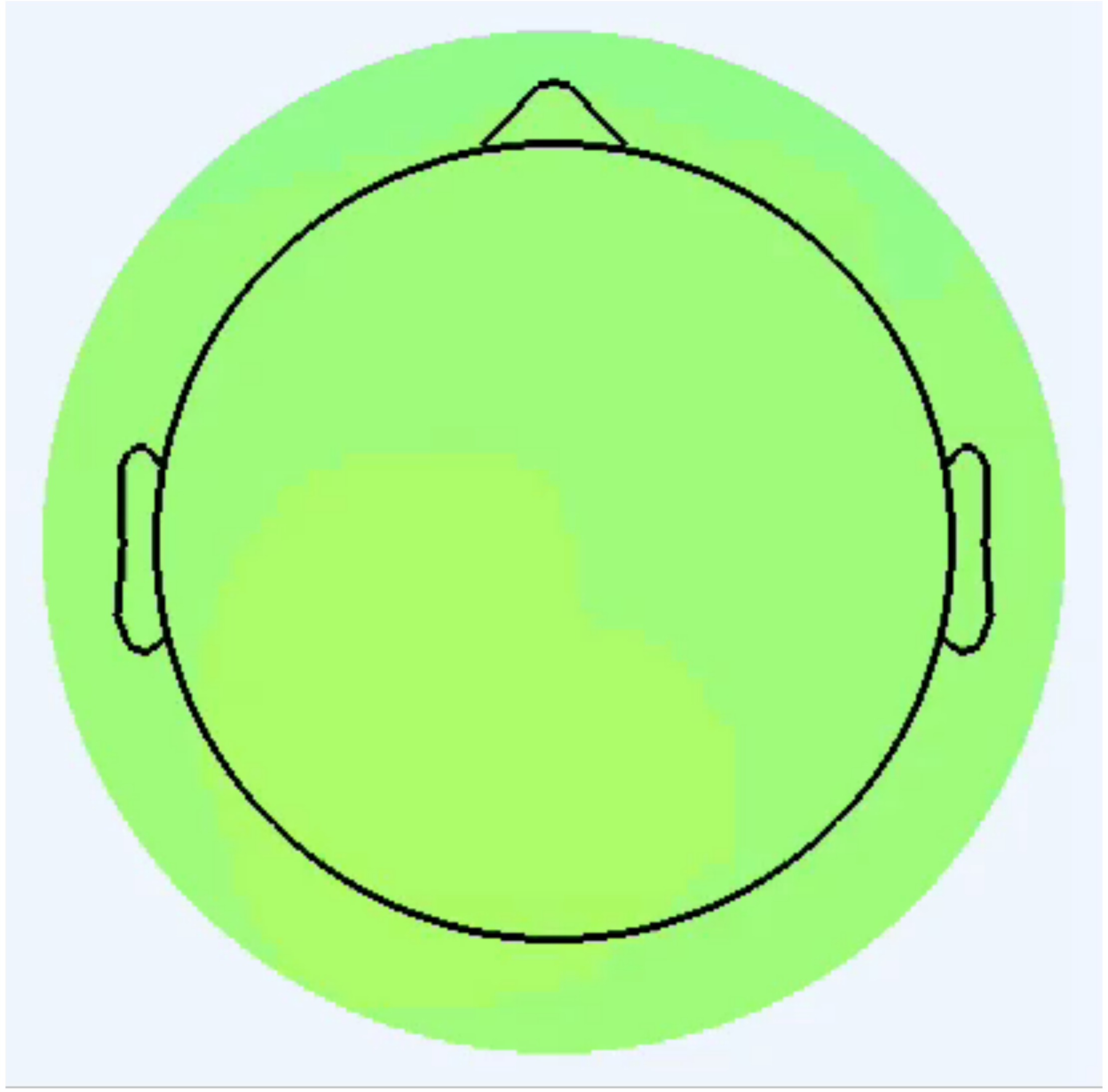
(1) univariate data

(2) multivariate data

Example for model of event-related potentials



(Beim Graben and Hutt (2015), Phil. Trans. Royal Soc. A 373:20140089)



(I) metastable dynamics

(II) feature detection and model

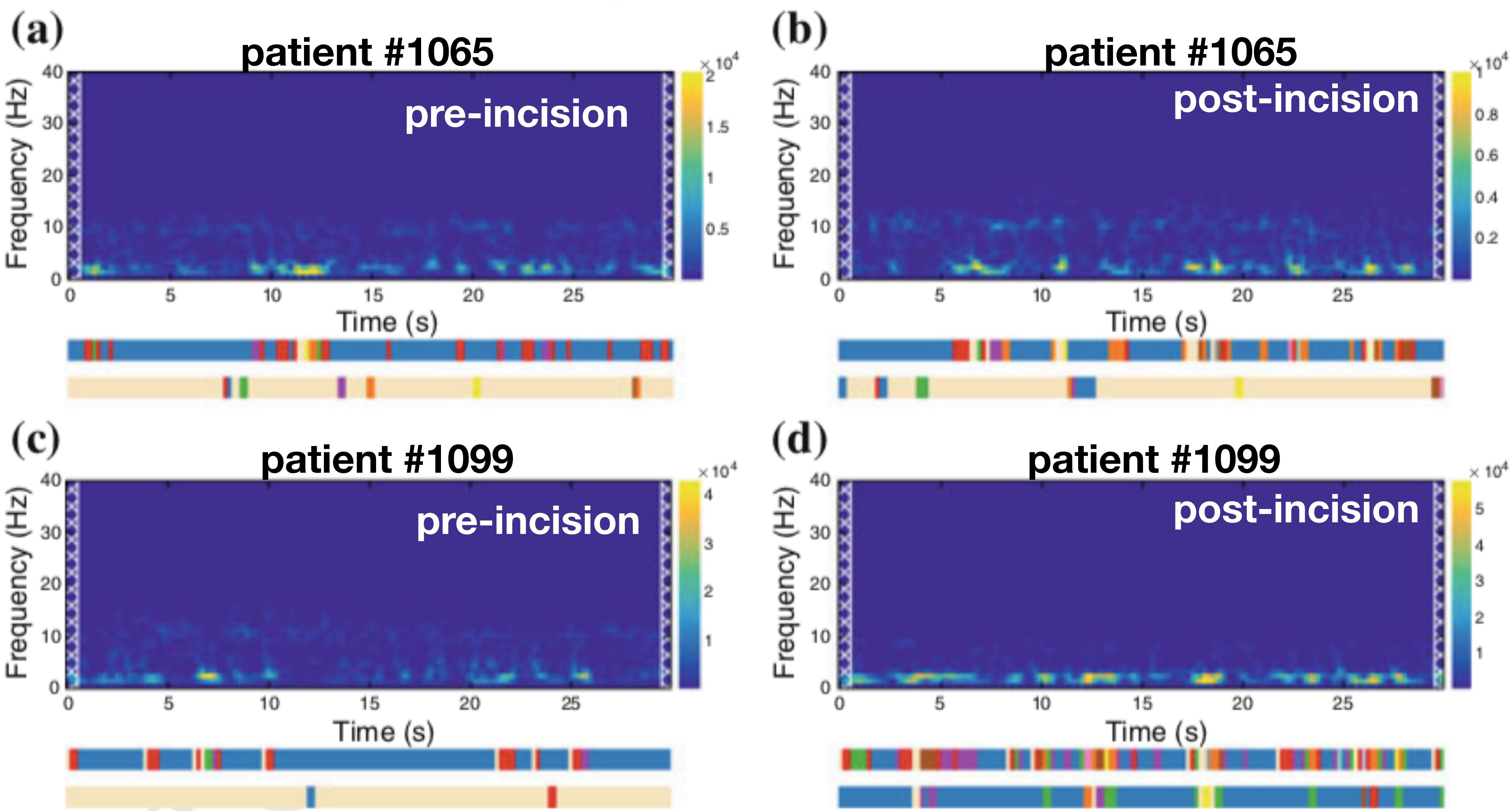
(III) applications

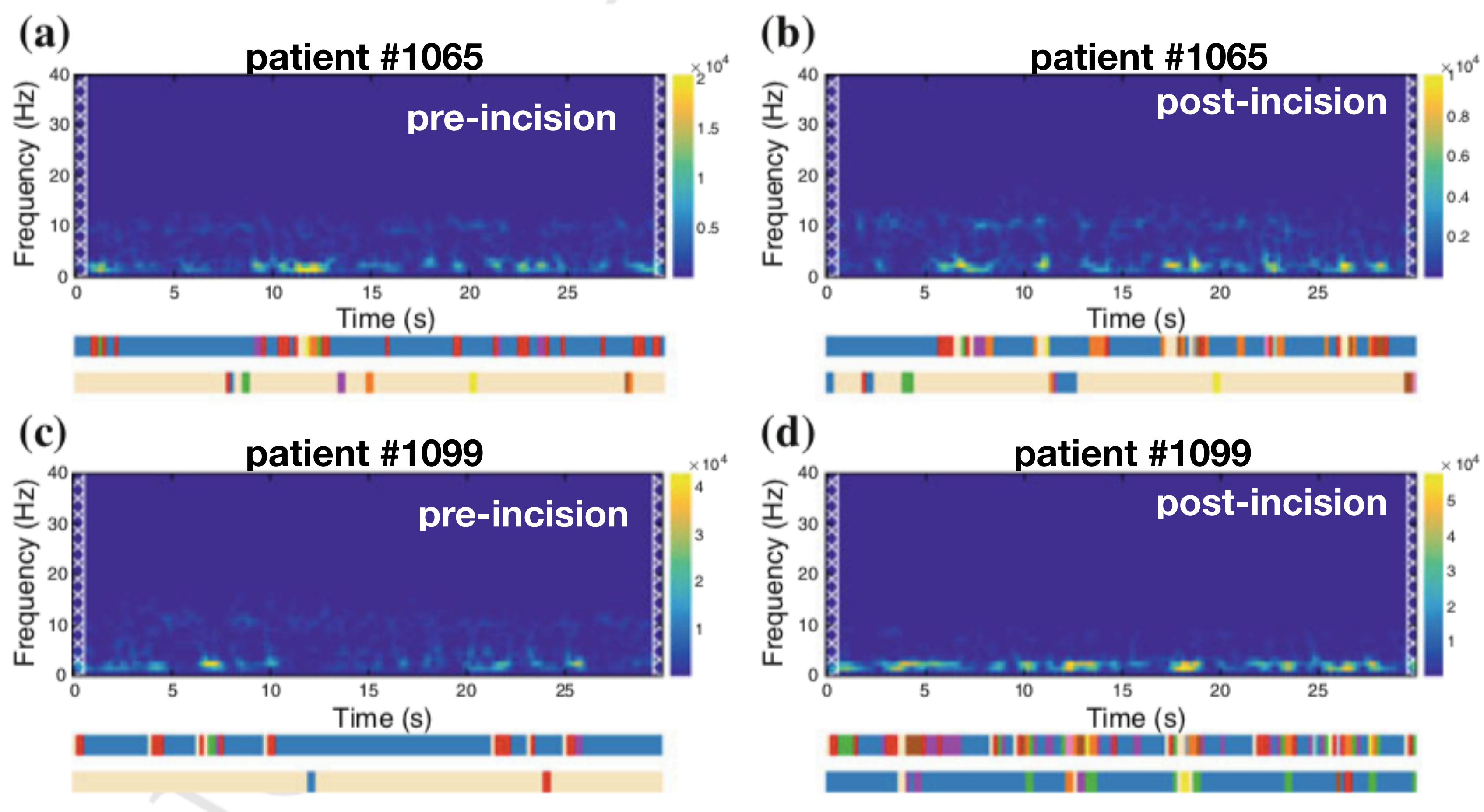
(IV) extensions

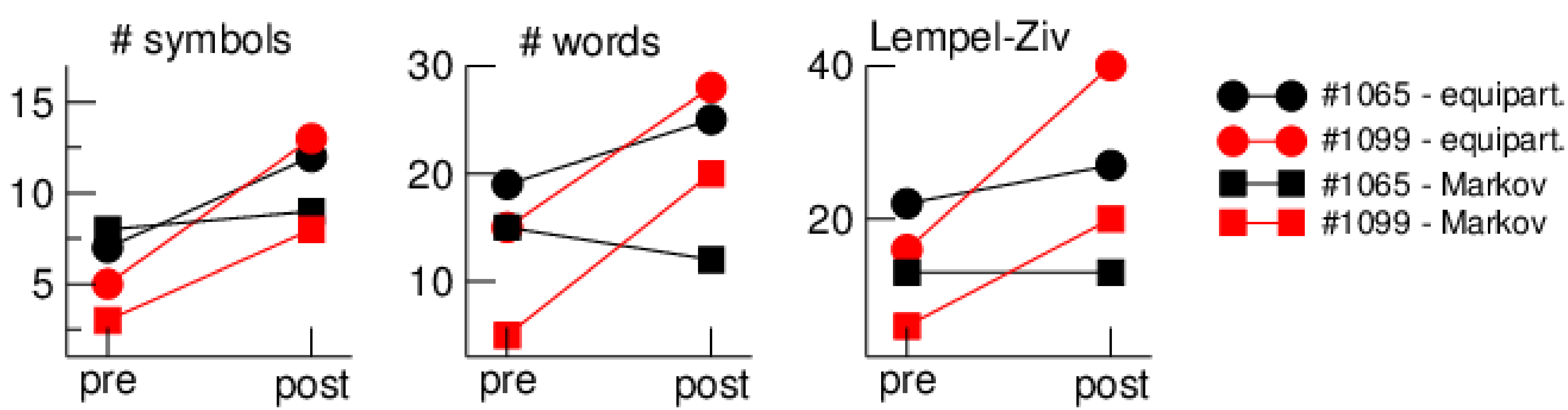
Symbolic dynamics allows to compute recurrence complexity

Example 1:

- EEG (electrode Fz) from patients during surgery
- before (*pre*) and after (*post*) incision
- Fedotenkova et al. (2017), In: ITISE 2016: 89-102







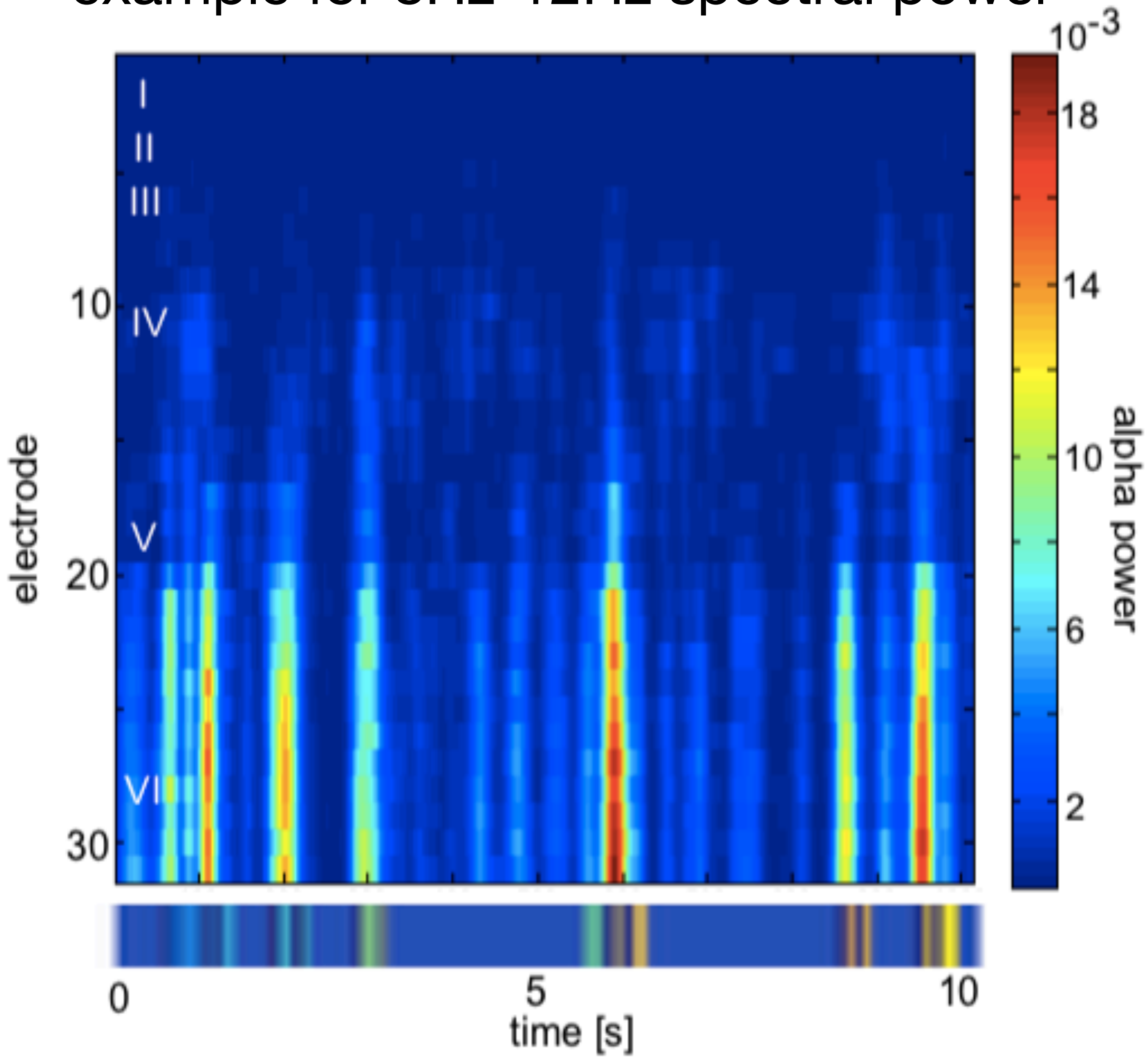
recurrence complexity increases with incision

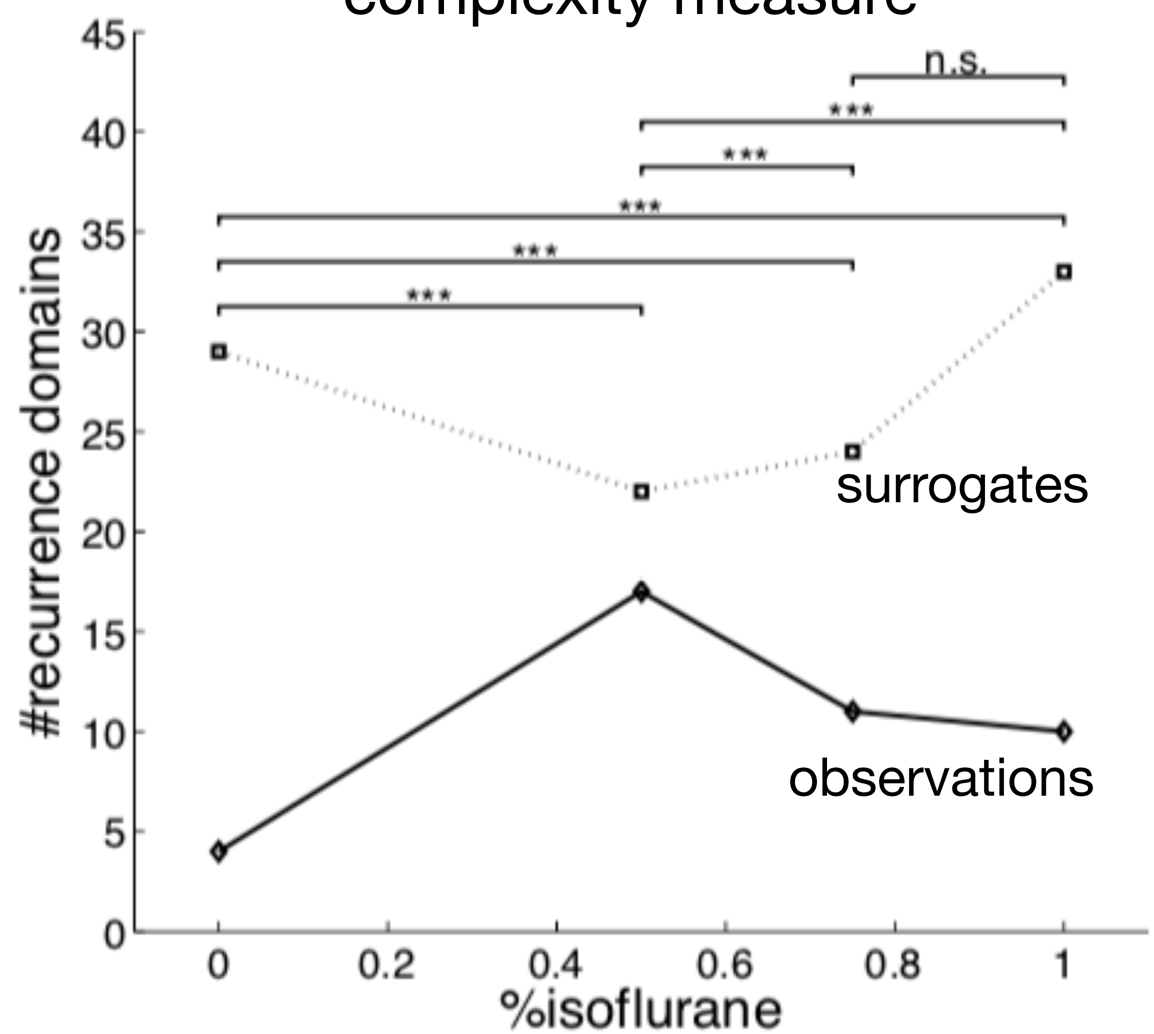
Symbolic dynamics allows to compute recurrence complexity

Example 2:

- intracranial Local Field Potentials in ferrets during anaesthesia
- increasing concentration of anaesthetic *isoflurane*
- Beim Graben et al. (2016), Europhys. Lett. 114(3): 38003

example for 8Hz-12Hz spectral power





characteristic change of recurrence complexity

what does changing recurrence complexity tell us ?

what does changing recurrence complexity tell us ?

- **degree of irregularity**

what does changing recurrence complexity tell us ?

- **degree of irregularity**

what is it good for ?

what does changing recurrence complexity tell us ?

- degree of irregularity

what is it good for ?

- feature measure to distinguish different dynamical states

what does changing recurrence complexity tell us ?

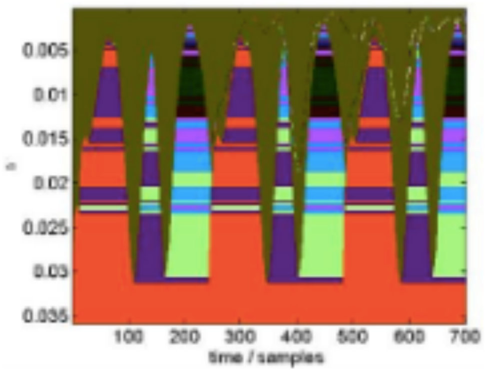
- degree of irregularity

what is it good for ?

- feature measure to distinguish different dynamical states

- future studies on patient data will tell …..

Announcement of Frontiers Research Topic



Research Topic

Recurrence Analysis of Complex Systems Dynamics

Guest Editors: P. beim Graben, N. Marwan, C. Uhl, CL Webber Jr. and A. Hutt

Participating Journals

Manuscripts can be submitted to this Research Topic via the following journals:

Frontiers in
Applied Mathematics and Statistics
Dynamical Systems

Frontiers in
Physics
Interdisciplinary Physics
Biophysics

Frontiers in
Physiology
Biophysics

Frontiers in
Environmental Science
Interdisciplinary Climate Studies

Frontiers in
Earth Science
Interdisciplinary Climate Studies

Frontiers in
Ecology and Evolution
Interdisciplinary Climate Studies

Frontiers in
Computational Neuroscience