You can download the sources of this presentation here:
https://github.com/mxochicale/mrhd

Motor Theory and Practice

Medical Robotics: Hardware Development

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Miguel Xochicale, PhD

School of Biomedical Engineering & Imaging Sciences
King's College London

Outline

INTRODUCTION

Introduction to Motors

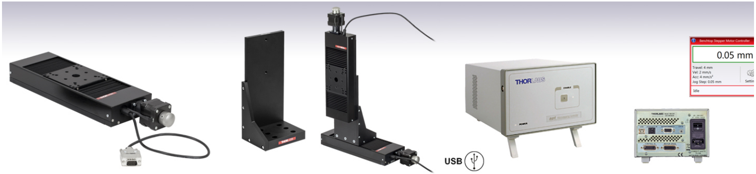
Motors of all types serve to convert electrical energy into mechanical energy.

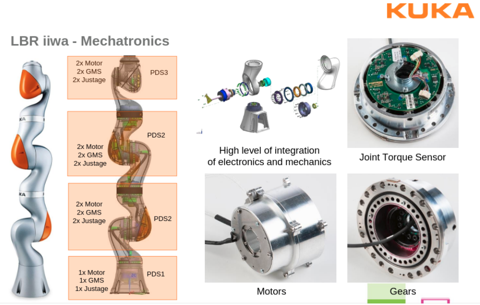
(Page M. 1999 in **http://lancet.mit.edu/motors/motors1.html**)

Introduction to Motors

(taifur 2016 in **instructables/Complete-Motor-Guide-for-Robotics**)

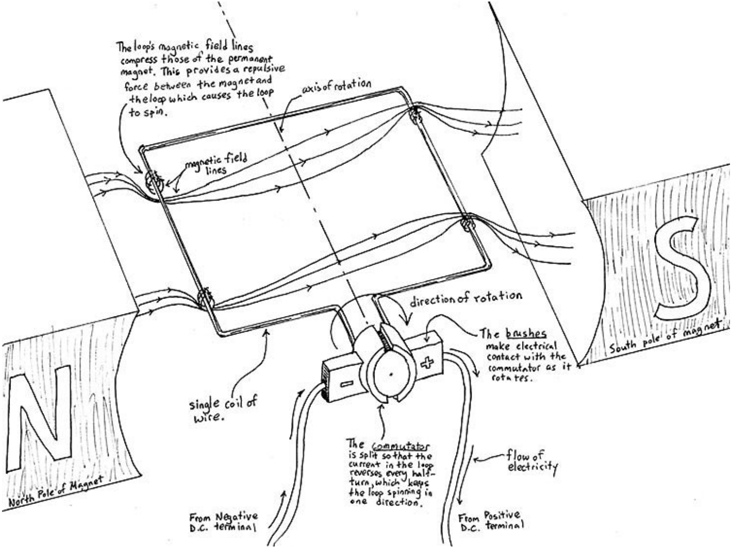
Some Motors in the School of BMEIS





BACKGROUND

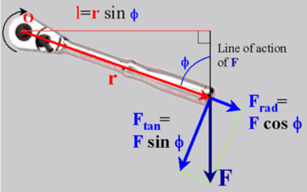
How do a motor work

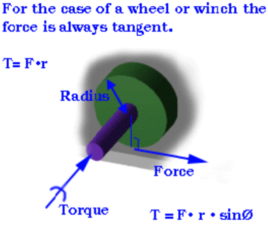


(Page M. 1999 in **http://lancet.mit.edu/motors/motors2.html**)

Torque

Torque, also known as momentum, is the term used to talk about forces that cause or change rotational motion.





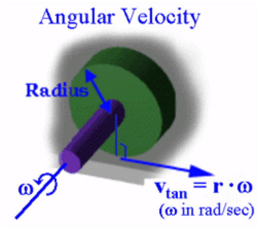
$$T = r \cdot \mathbf{F} \cdot \sin(\phi)$$
$$T = r \cdot \mathbf{F}_{tan}$$

Units of torque
* SI: newton-meters (N-m)
* English: inch-pounds (in-lb), foot-pounds (ft-lb), inch-ounces (in-oz)

(Page M. 1999 in **http://lancet.mit.edu/motors/motors3.html**)

Speed

The rate of rotation around an axis usually expressed in radians or revolutions per second or per minute.



$$1 revolution = 360 deg$$
$$1 revolution = 2\pi radians$$
$$1 radian = (180/\pi) deg$$
$$1 deg = (\pi/180) radians$$

Units of speed:
* radians/second (rad/s)
* revolutions/second (rps)
* revolutions/minute (rpm)

From the angular velocity ω, we can find the tangential velocity with $v_{tan} = r \cdot \omega$ anywhere in the rotating body.

(Page M. 1999 in **http://lancet.mit.edu/motors/motors3.html**)

Power

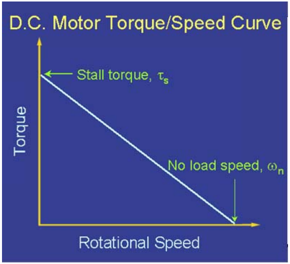
When a torque T (with respect to the axis of rotation) acts on a body that rotates with angular velocity ω, its power (rate of doing work) is the product of the torque and angular velocity.

$$P_{rot} = T \cdot \omega$$

Units of power:
* SI: Watts (W), newton-meters per second (N-m/s)
* English: foot-pound per second (ft-lb/s), horsepower (hp)

(Page M. 1999 in **http://lancet.mit.edu/motors/motors3.html**)

Torque/Speed Curves

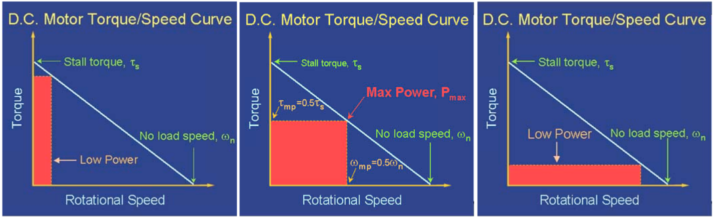


There is a tradeoff between how much torque a motor delivers, and how fast the output shaft spins.
* The stall torque (T_s) point in the curve where torque is at its maximum but the shaft is not rotating.
* The no load speed (ω_n) maximum output speed of the motor (where no torque is applied to the shaft)

(Page M. 1999 in **http://lancet.mit.edu/motors/motors3.html**)

Torque/Speed Curves

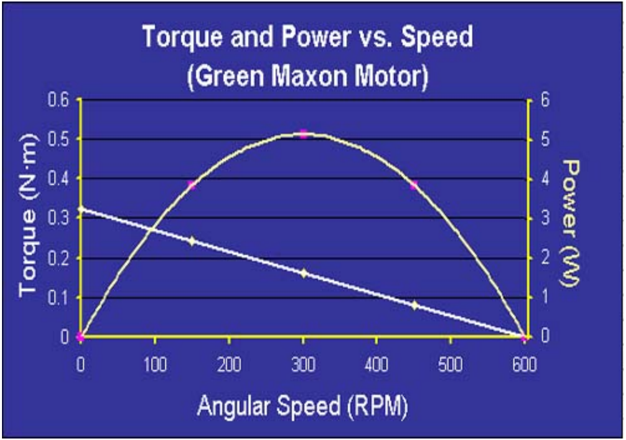
The area under the curve is the power given by the product of torque and angular velocity.



* Due to the linear relationship of torque and angular velocity, the maximum power occurs at the point where $\omega = 1/2_n$, $T = 1/2T_s$.

(Page M. 1999 in **http://lancet.mit.edu/motors/motors3.html**)

Power/Torque and Power/Speed Curves



(Page M. 1999 in **http://lancet.mit.edu/motors/motors3.html**)

MOTOR TYPES

Choosing an electric motor

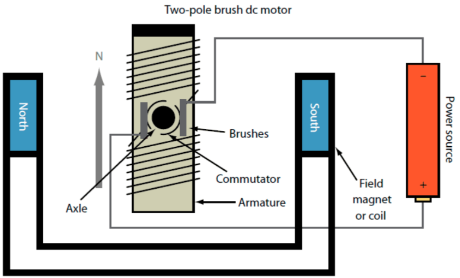
- Torque
- Speed
- Precision and Accuracy
- Voltage
- Cost
- Form factor

(taifur 2016 in **instructables/Complete-Motor-Guide-for-Robotics**)

1. Brushed DC Motor
2. DC gear motors
3. Brushless motors
4. Servo motors
5. Stepper motors

(taifur 2016 in **instructables/Complete-Motor-Guide-for-Robotics**)

Brushed DC Motor



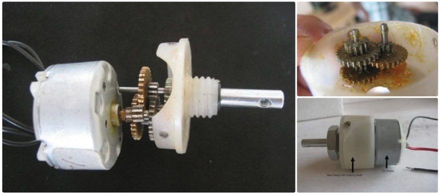
Advantages:
* Inexpensive
* Lightweight
* Reasonable Efficient
* Good low-speed torque

Limitations:
audible whine from the commutator brushes, create electrical noise

Applications:
* Toys
* RC Servos
* Gear Motors

(taifur 2016 in **instructables/Complete-Motor-Guide-for-Robotics**)

DC geat motors



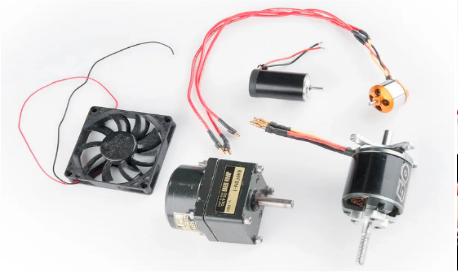
Advantages:
* Speed reduction
* Increase torque

Limitations:
The extra resistance can make these gear-trains balky at low speeds.

Applications:
* Robot Drive Trains
* Radio Control Vehicles
* Cordless Tools

(taifur 2016 in **instructables/Complete-Motor-Guide-for-Robotics**)

Brushless motors



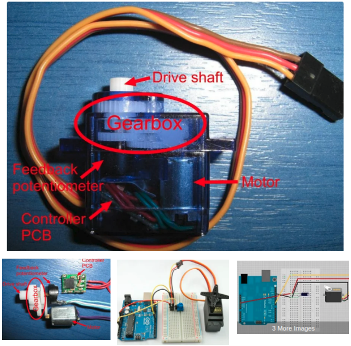
Advantages:
* Quiet
* Efficient

Limitations:
Some types of brushless motors require a separate controller

Applications:
Multicopters, Drones, Radio Control Vehicles, Disk Drives, Fans, Industrial Servos, Hybrid Vehicles, High-End Gearmotors

(taifur 2016 in **instructables/Complete-Motor-Guide-for-Robotics**)

Servo motors



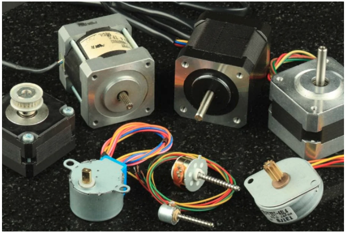
Advantages:
* Low cost - (RC Servos)
* Variety - There is a wide range of sizes and torque ratings
* Simple to control - using logic level pulses from a microcontroller or a dedicated servo controller

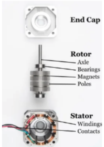
Limitations:
Most RC servos are limited to 180 degrees of motion and positioning accuracy and repeatability of +/- 1 degree is typical.

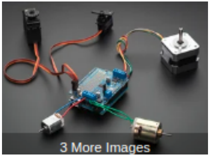
Applications:
Robotics, Animatronics, Radio Control Cars/Boats/Planes

(taifur 2016 in **instructables/Complete-Motor-Guide-for-Robotics**)

Stepper motors







Advantages:
* Precise repeatable positioning
* Precise speed control
* Excellent low-speed torque
* Excellent 'holding torque' to maintain position

Limitations:
* Low efficiency
* May need encoder or limit switch to establish a reference position
* Subject to missed steps if overloaded

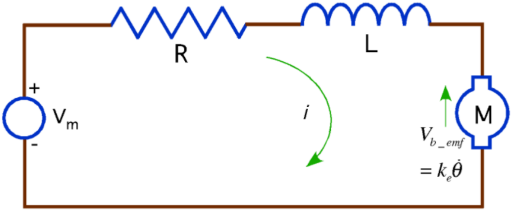
Applications:
3D Printers, CNC Machines, Camera rigs Robotics, Printers, Precision Gearmotors

(taifur 2016 in **instructables/Complete-Motor-Guide-for-Robotics**)

MOTOR MODELLING

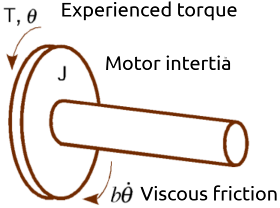
Motor Modelling

Electrical part



Motor stator

Mechanical part



Motor rotor

DC motor modelling: electrical part

By Kirchhoff's Voltage Law, we have

$$
\begin{aligned}
V_m - V_R - V_L - V_{emf} &= 0 \\
V_m - Ri - L\frac{di}{dt} - k_e\dot{\theta} &= 0
\end{aligned}
\tag{1}
$$

Asumming the circuit has a very fast response, Eq (1) with $L \approx 0$ is presented as

$$
\begin{aligned}
V_m - Ri - k_e\dot{\theta} &= 0 \\
\frac{V_m}{R} - \frac{k_e}{R}\dot{\theta} &= i
\end{aligned}
\tag{2}
$$

DC motor modelling: mechanical part

By Newton's Law (torque aka energy balance), we have

$$T_e - T_{\dot{\theta}} - T_b - T_L = 0 \tag{3}$$

Where T_e is the electromagnetic torque. $T_{\dot{\theta}}$, is torque generated from the rotational acceleration of the rotor. T_b, is the torque due to the friction and angular velocity in the motor. T_L is the torque of the mechanical load (external load).

$$K_t i - J\ddot{\theta} - b\dot{\theta} - T_L = 0 \tag{4}$$

DC motor modelling: complete

Assume the motor is not connected to the load (i.e. $T_L = 0$) in Eq 4, we have

$$
\begin{aligned}
J\ddot{\theta} + b\dot{\theta} &= K_t i \\
\frac{J}{K_t}\ddot{\theta} + \frac{b}{K_t}\dot{\theta} &= i
\end{aligned}
\tag{5}
$$

DC motor modelling: complete

Subtituting i from (2) in (5), we then have

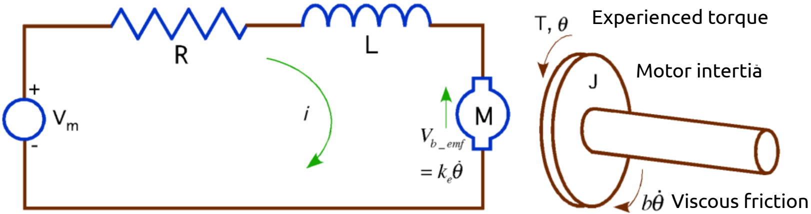
$$\begin{aligned}\frac{J}{K_t}\ddot{\theta} + \frac{b}{K_t}\dot{\theta} &= \frac{V_m}{R} - \frac{k_e}{R}\dot{\theta} \\ \frac{J}{K_t}\ddot{\theta} + (\frac{b}{K_t} + \frac{k_e}{R})\dot{\theta} &= \frac{V_m}{R} \\ J\ddot{\theta} + (b + \frac{k_t k_e}{R})\dot{\theta} &= V_m\frac{k_t}{R}\end{aligned} \tag{6}$$

First-order differential equation in angular velocity

$$J\dot{\omega} + (b + \frac{k_t k_e}{R})\omega = V_m\frac{k_t}{R}$$

Motor Modelling

Electrical part **Mechanical part**



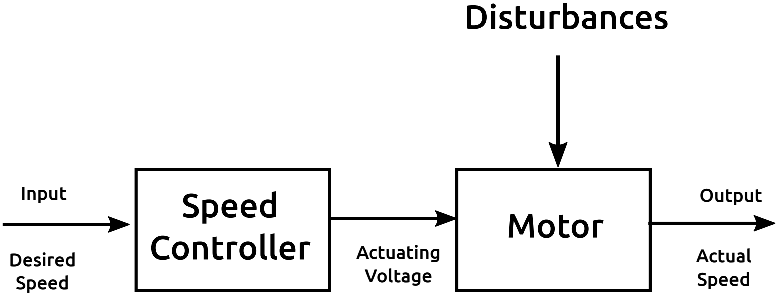
Motor stator Motor rotor

First-order differential equation in angular velocity

$$J\dot{\omega} + (b + \frac{k_t k_e}{R})\omega = V_m \frac{k_t}{R}$$

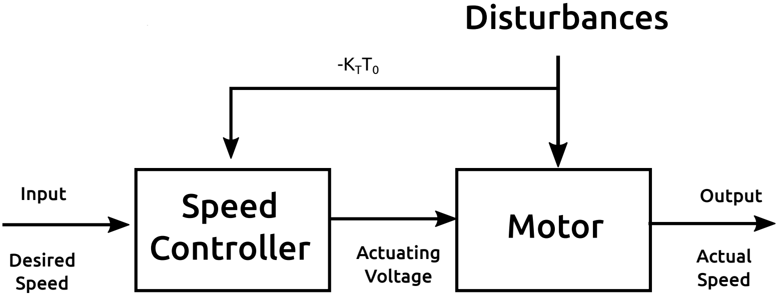
MOTOR CONTROL

Open loop



(Lewis A. D. 2003)

Closed loop



(Lewis A. D. 2003)

REFERENCES

References

Andrew D. Lewis
» A Mathematical Approachto Classical Control «
Open Book (2003)
freescience.info/go.php?pagename=books&id=1740