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Motor Theory and Practice

Medical Robotics: Hardware Development

3th & 5th of February 2020

Miguel Xochicale, PhD

School of Biomedical Engineering & Imaging Sciences
King's College London

Outline

1. Introduction
2. Background
3. Motors Types
4. Motor Modelling
5. Motor Control
6. Summary

INTRODUCTION

Introduction to Motors

Motors of all types serve to convert electrical energy into mechanical energy.

Introduction to Motors



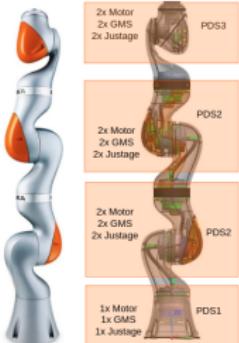
Some Motors in the School of BMEIS



KUKA



LBR iiwa - Mechatronics



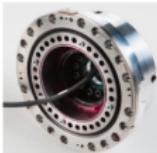
High level of integration of electronics and mechanics



Joint Torque Sensor



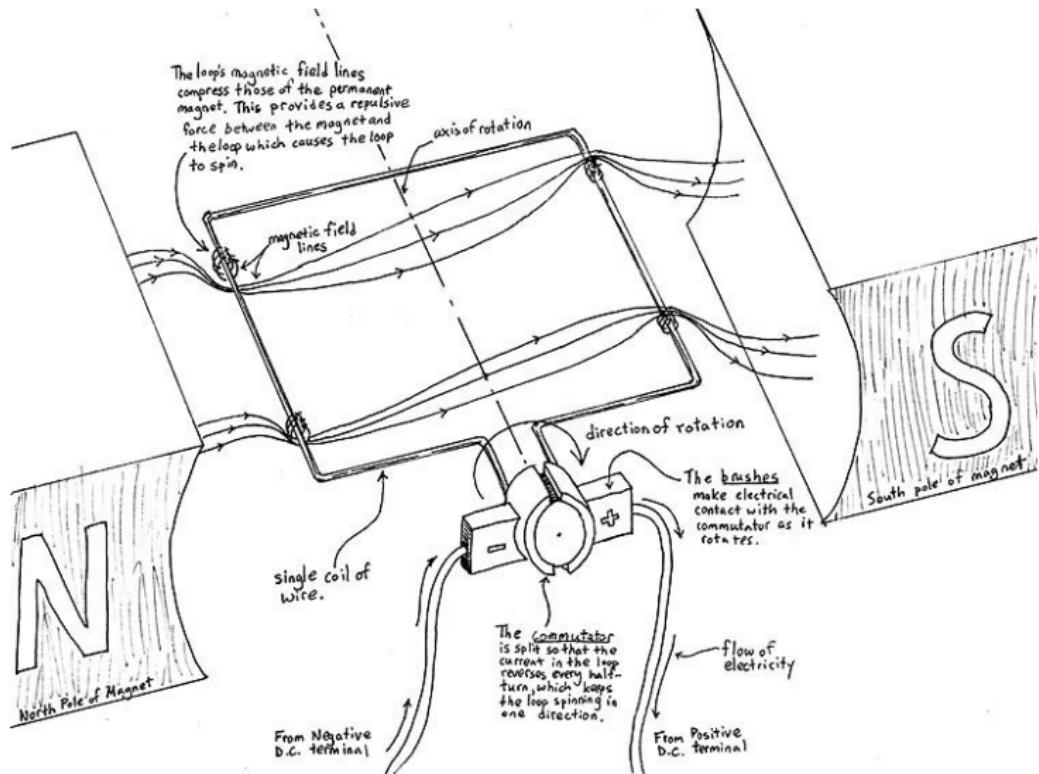
Motors



Gears

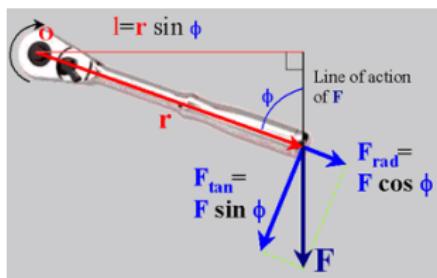
BACKGROUND

How do a motor work

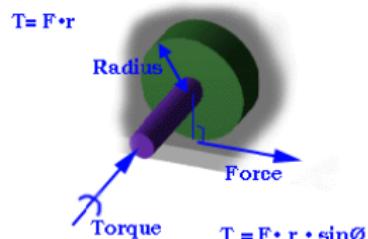


Torque

Torque, also known as momentum, is the term used to talk about forces that cause or change rotational motion.



For the case of a wheel or winch the force is always tangent.



$$T = r \cdot F \cdot \sin(\phi)$$

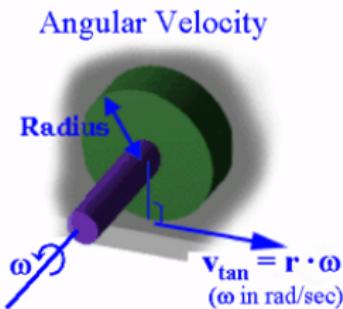
$$T = r \cdot F_{tan}$$

Units of torque

- * SI: newton-meters (N·m)
- * English: inch-pounds (in-lb), foot-pounds (ft-lb), inch-ounces (in-oz)

Speed

The rate of rotation around an axis usually expressed in radians or revolutions per second or per minute.



$$1\text{ revolution} = 360\text{deg}$$

$$1\text{ revolution} = 2\pi\text{radians}$$

$$1\text{radian} = (180/\pi)\text{deg}$$

$$1\text{deg} = (\pi/180)\text{radians}$$

Units of speed:

* radians/second (rad/s)

* revolutions/second (rps)

* revolutions/minute (rpm)

From the angular velocity ω , we can find the tangential velocity with $v_{tan} = r \cdot \omega$ anywhere in the rotating body.

Power

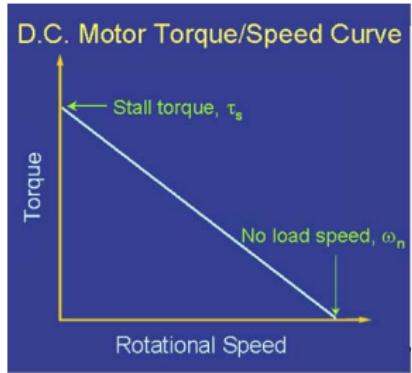
When a torque T (with respect to the axis of rotation) acts on a body that rotates with angular velocity ω , its power (rate of doing work) is the product of the torque and angular velocity.

$$P_{rot} = T \cdot \omega$$

Units of power:

- * SI: Watts (W), newton-meters per second (N-m/s)
- * English: foot-pound per second (ft-lb/s), horsepower (hp)

Torque/Speed Curves

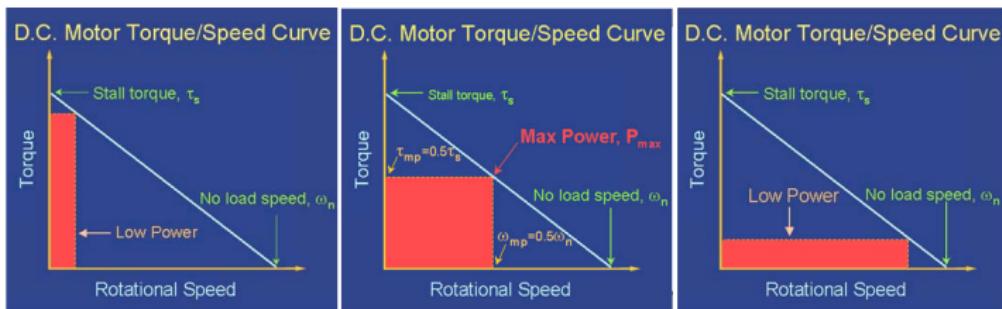


There is a tradeoff between how much torque a motor delivers, and how fast the output shaft spins.

- * The stall torque (T_s) point in the curve where torque is at its maximum but the shaft is not rotating.
- * The no load speed (ω_n) maximum output speed of the motor (where no torque is applied to the shaft)

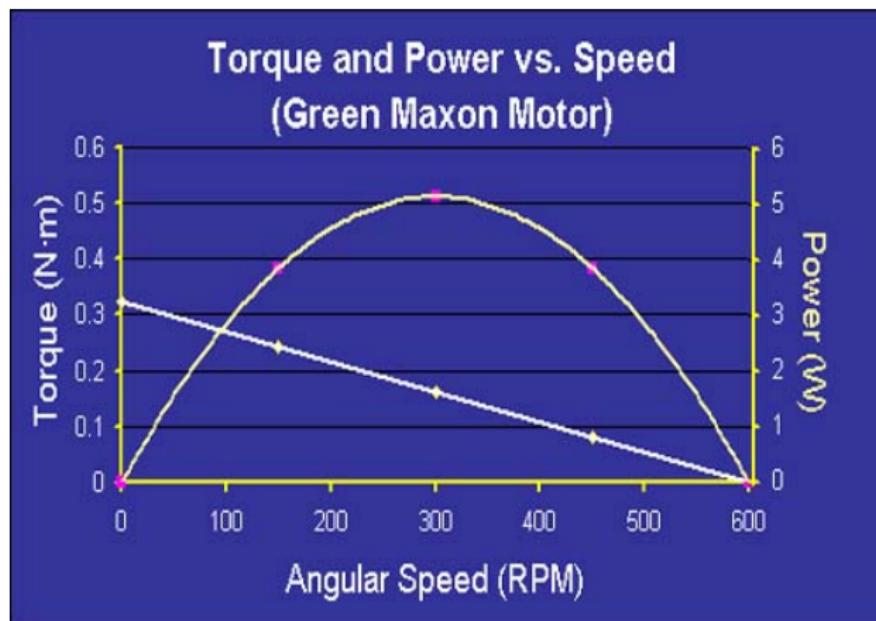
Torque/Speed Curves

The area under the curve is the power given by the product of torque and angular velocity.



- * Due to the linear relationship of torque and angular velocity, the maximum power occurs at the point where $\omega = 1/2\omega_n$, $T = 1/2\tau_s$.

Power/Torque and Power/Speed Curves



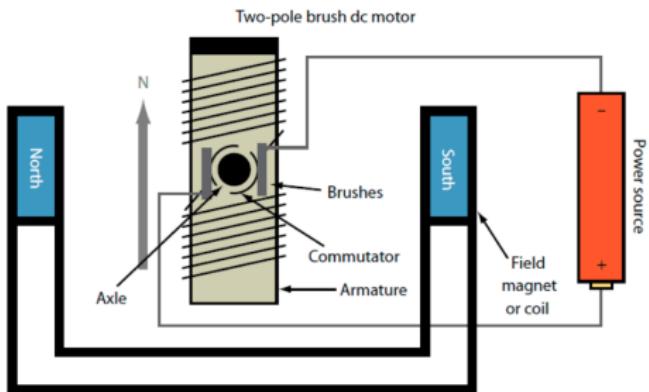
MOTOR TYPES

Choosing an electric motor

- Torque
- Speed
- Precision and Accuracy
- Voltage
- Cost
- Form factor

1. Brushed DC Motor
2. DC gear motors
3. Brushless motors
4. Servo motors
5. Stepper motors

Brushed DC Motor



Advantages:

- * Inexpensive
- * Lightweight
- * Reasonable Efficient
- * Good low-speed torque

Limitations:

audible whine from the commutator brushes,
create electrical noise

Applications:

- * Toys
- * RC Servos
- * Gear Motors

DC gear motors



Advantages:

- * Speed reduction
- * Increase torque

Limitations:

The extra resistance can make these gear-trains balky at low speeds.

Applications:

- * Robot Drive Trains
- * Radio Control Vehicles
- * Cordless Tools

Brushless motors



Advantages:

- * Quiet
- * Efficient

Limitations:

Some types of brushless motors require a separate controller

Applications:

Multicopters, Drones,
Radio Control Vehicles,
Disk Drives, Fans,
Industrial Servos, Hybrid
Vehicles, High-End
Gearmotors

Servo motors



Advantages:

- * Low cost - (RC Servos)
- * Variety - There is a wide range of sizes and torque ratings
- * Simple to control - using logic level pulses from a microcontroller or a dedicated servo controller

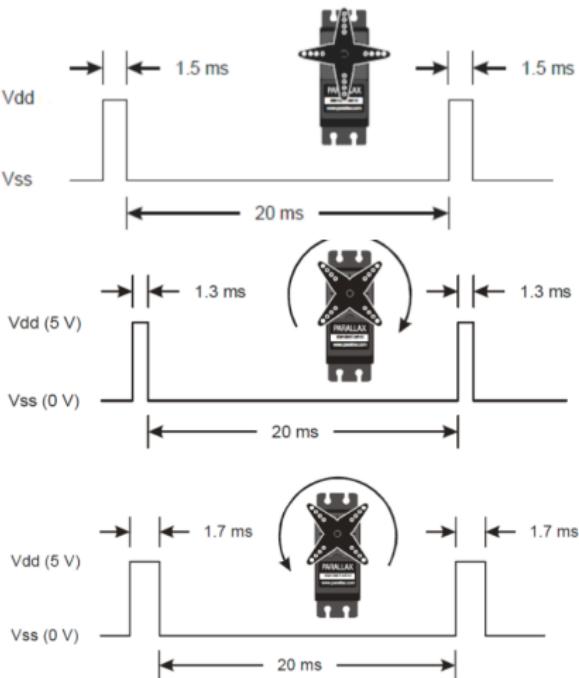
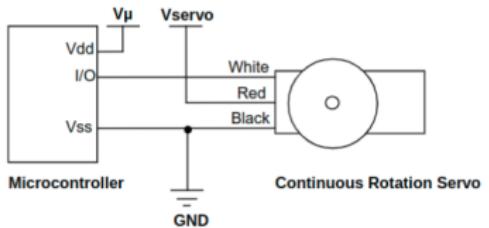
Limitations:

Most RC servos are limited to 180 degrees of motion and positioning accuracy and repeatability of +/- 1 degree is typical.

Applications:

Robotics, Animatronics, Radio Control Cars/Boats/Planes

Parallax Servo motors



Stepper motors



Advantages:

- * Precise repeatable positioning
- * Precise speed control
- * Excellent low-speed torque
- * Excellent 'holding torque' to maintain position

Limitations:

- * Low efficiency
- * May need encoder or limit switch to establish a reference position
- * Subject to missed steps if overloaded

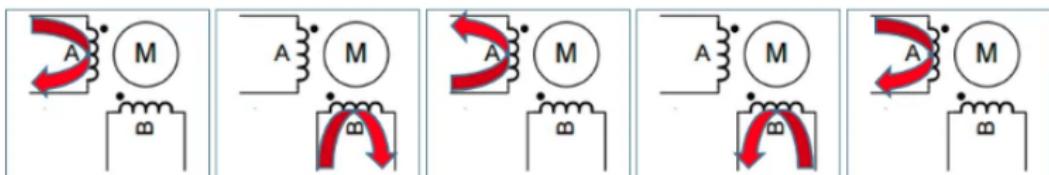
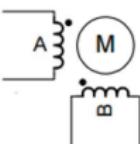
Applications:

3D Printers, CNC Machines, Camera rigs Robotics, Printers, Precision Gearmotors

Stepper motors

4 LEAD WIRES	1	2	3	4
Color Code 1	Red	Blue	Green	Black
Color Code 2	Brown	Orange	Red	Yellow
Color Code 3	Red	Red White Stripe	Green	Green White Stripe
Bipolar Driver	A	—A	B	—B

Bipolar Motors:



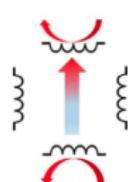
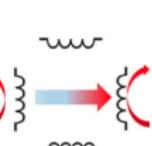
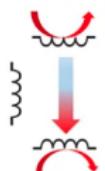
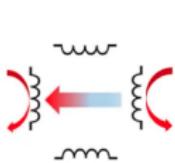
Step 1

Step 2

Step 3

Step 4

Repeat



Step 1

Step 2

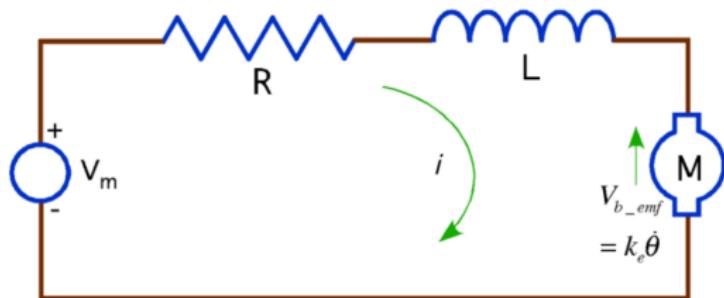
Step 3

Step 4

MOTOR MODELLING

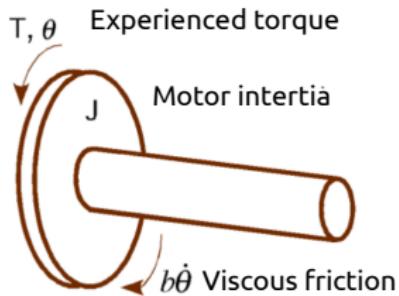
Motor Modelling

Electrical part



Motor stator

Mechanical part



Motor rotor

DC motor modelling: electrical part

By Kirchhoff's Voltage Law, we have

$$\begin{aligned} V_m - V_R - V_L - V_{emf} &= 0 \\ V_m - Ri - L \frac{di}{dt} - k_e \dot{\theta} &= 0 \end{aligned} \tag{1}$$

Assuming the circuit has a very fast response, Eq (1) with $L \approx 0$ is presented as

$$\begin{aligned} V_m - Ri - k_e \dot{\theta} &= 0 \\ \frac{V_m}{R} - \frac{k_e}{R} \dot{\theta} &= i \end{aligned} \tag{2}$$

DC motor modelling: mechanical part

By Newton's Law (torque aka energy balance), we have

$$T_e - T_{\dot{\theta}} - T_b - T_L = 0 \quad (3)$$

Where T_e is the electromagnetic torque. $T_{\dot{\theta}}$, is torque generated from the rotational acceleration of the rotor. T_b , is the torque due to the friction and angular velocity in the motor. T_L is the torque of the mechanical load (external load).

$$K_t i - J\ddot{\theta} - b\dot{\theta} - T_L = 0 \quad (4)$$

DC motor modelling: complete

Assume the motor is not connected to the load (i.e. $T_L = 0$) in Eq 4, we have

$$\begin{aligned} J\ddot{\theta} + b\dot{\theta} &= K_t i \\ \frac{J}{K_t}\ddot{\theta} + \frac{b}{K_t}\dot{\theta} &= i \end{aligned} \tag{5}$$

DC motor modelling: complete

Substituting i from (2) in (5), we then have

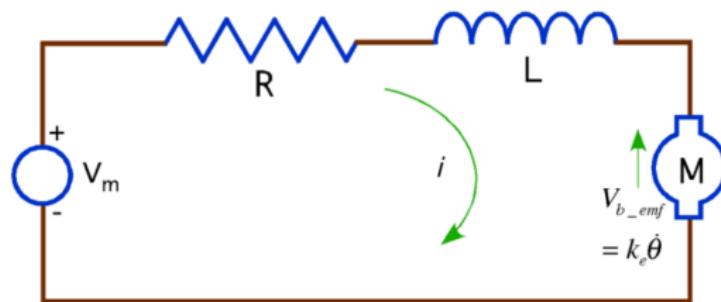
$$\begin{aligned}\frac{J}{K_t} \ddot{\theta} + \frac{b}{K_t} \dot{\theta} &= \frac{V_m}{R} - \frac{k_e}{R} \dot{\theta} \\ \frac{J}{K_t} \ddot{\theta} + \left(\frac{b}{K_t} + \frac{k_e}{R} \right) \dot{\theta} &= \frac{V_m}{R} \\ J \ddot{\theta} + \left(b + \frac{k_t k_e}{R} \right) \dot{\theta} &= V_m \frac{k_t}{R}\end{aligned}\tag{6}$$

First-order differential equation in angular velocity

$$J \dot{\omega} + \left(b + \frac{k_t k_e}{R} \right) \omega = V_m \frac{k_t}{R}$$

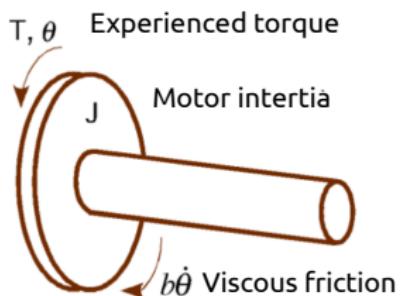
Motor Modelling

Electrical part



Motor stator

Mechanical part



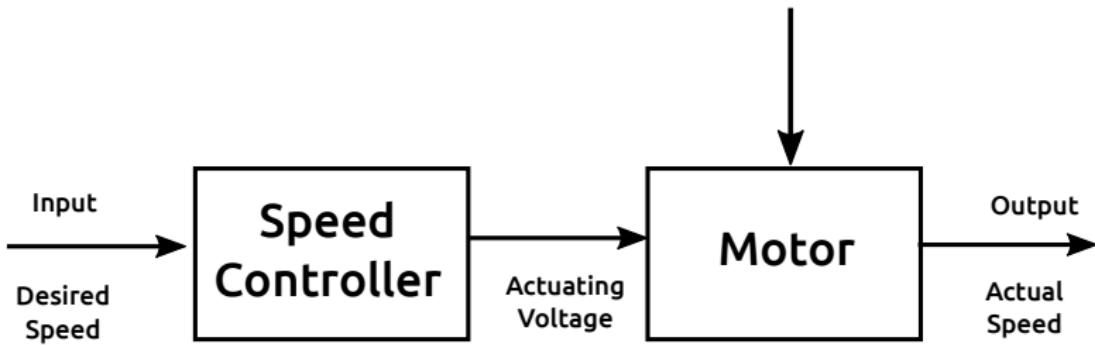
Motor rotor

$$J\ddot{\theta} + (b + \frac{k_t k_e}{R})\dot{\theta} = V_m \frac{k_t}{R}$$

MOTOR CONTROL

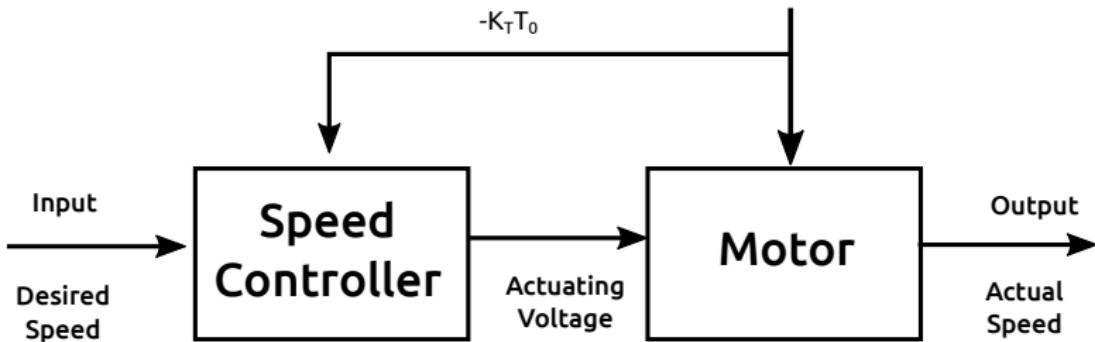
Open loop

Disturbances



Closed loop

Disturbances



REFERENCES

References

-  Andrew D. Lewis
» A Mathematical Approach to Classical Control «
Open Book (2003)
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