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<https://github.com/mxochicale/nts2020>

# Nonlinear Analysis of Time-series for Inertial Sensor-based Data

Big Data Institute  
University of Oxford  
20th January 2020

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School of Biomedical Engineering Imaging Sciences

Movement Variability  
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Nonlinear Analysis  
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Experiment  
ooo

Results  
oooooooooooo

Inertial Sensors  
oooo

Conclusions  
oooooo

## Outline

1. Movement Variability
2. Nonlinear Analysis
3. Experiment
4. Results
5. Inertial Sensors
6. Conclusions

## EDUCATION

- Ph.D. in Computer Engineering  
University of Birmingham (2014-2018)
- M.Sc. in Digital Signal Processing  
INAOE, México (2004 - 2006)
- B.Eng. in Electronics  
Puebla Institute of Technology, México (1999 - 2004)

## EXPERIENCE

- 1 year as a Research Associate in KCL
  - \* Ultrasound Guidance Interventions, GE US, verification of needle tip tracking, linear stages, etc.
- 5 years in Human-Robot Interaction (2013-2018)
  - +1 Research Assistant in Robotics and +4 PhD in HRI
- 17 years in Computer Engineering (1999-2018)
  - +4 Electronics, +2 Digital Filters, +6 Mechatronics and +5 Computer Engineering

## Teaching Associate (2014-2018, UK)

- Engineering Maths 2  
Lecturers: Professor Martin Russell, Dr Carl Anthony
- Small Embedded Systems  
Lecturer: Professor Chris Baber
- Computing for Engineering  
Lecturer: Dr Sridhar Pammu
- Matlab Laboratories  
Lecturer: Dr Edward Tarte

## Junior Lecturer in Mechatronic and Electronics (2006-2012, México)

- Computer Engineering, Linear Algebra and Research Projects  
Madero University, México (2012)
- Stochastic Processing and Digital Filters  
Iberoamerica University, México (2006-2011)
- Electricity and Magnetism, Electronics and Numerical Methods  
Atlixco Institute of Technology (2006-2007)

## GNU/Linux

- \* Ubuntu 12.04/14.04/16.04/18.04 (32/64 bit)

- \* Raspbian (Wheezy/Jessie/Streech)

## Single-board computers

- \* Arduino (miniarduino, uno)

- \* Beaglebone (720MHz ARM/2x46 headers)

- \* RaspberryPi (2/3B+[1.2GHz/64bit])



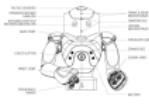
## Robots

- \* OWI 535



- \* PatrolBot

- \* NAO V4 T2



## SLAM

- \* PatrolBot

(800 wheel encoders/  
Laser sick lms200 [~4Hz] )

- \* Navigation (Aria/MRPT)

- \* Montecarlo Localization

## ROS

- \* fuerte/groovy/kinetic/melodic  
on Ubuntu 12.04/14.04/16.04

- \* Packages:

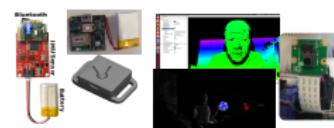
razor\_imu\_9dof/openface\_ros

## Sensors

- \* IMUs (razor9dof, MUSE)

- \* Cameras

(Kinect 1 [color/depth],  
pi-camera 2.1 [8Mp],  
Logitech c930e [1080p])



## Deep Learning

- \* GANs, CNNs

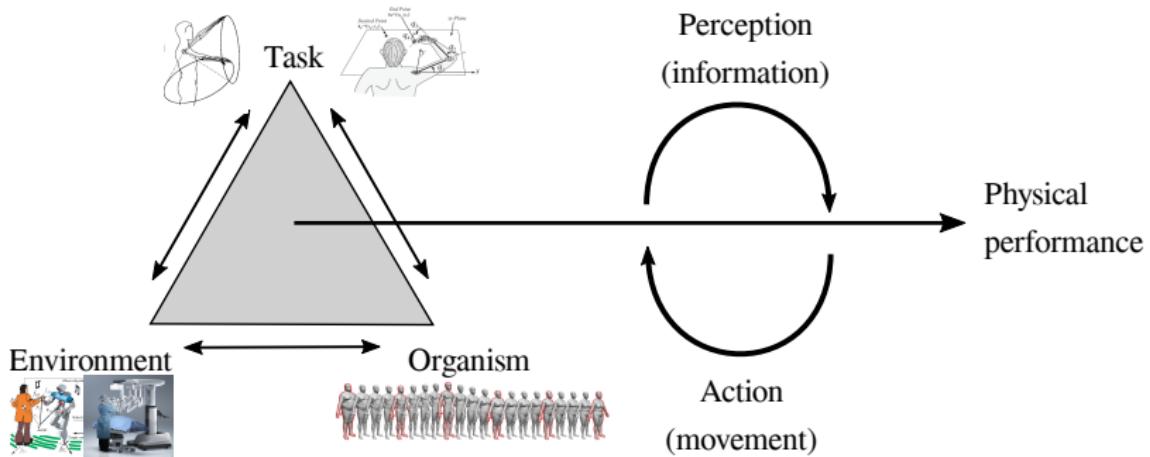
- \* python 3.6, pytorch, tensorflow

- \* GPU [GeForce GTX960]

- \* CUDA 9.0, cuDNN

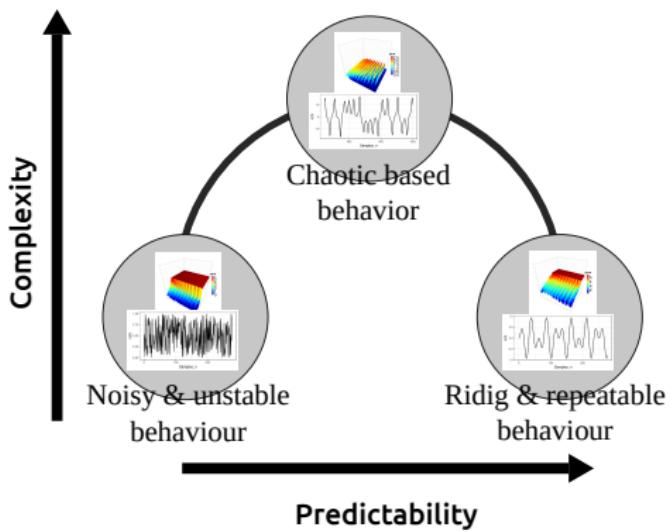
# MOVEMENT VARIABILITY

## Modelling Movement Variability



**Figure 1:** Newell's model of movement constrains

## Modelling Movement Variability



**Figure 2:** Theoretical Model of Optimal Movement Variability

## Nonlinear Analysis

There is no best tool to quantify MV and unification of tools is still an open question (Caballero et al. 2014; Wijnants et al. 2009) which led me (i) to explore different nonlinear analysis to measure MV and (ii) to understand their strengths and weaknesses.

- Reconstructed State Space (Takens 1981)
  - Recurrence Plots (Eckmann et al. 1987, Marwan et al. 2007)
  - Approximate Entropy (Pincus 1991, 1995)
  - Sample Entropy (Richman and Moorman, 2000)
  - Largest Lyapunov exponent (Stergiou, 2016)
  - Recurrence Quantification Analysis (Zbilut and Webber et al., 1992)

## Research Questions

- What are the effects on RSSs, RPs, and RQA metrics of different embedding parameters, different recurrence thresholds and different characteristics of time series (structure, smoothness and window length size)?
- What are the weaknesses and strengths of RQA metrics when quantifying movement variability?
- How does the smoothing of raw time series affect methods of nonlinear analysis when quantifying movement variability?

# NONLINEAR ANALYSIS

## State Space Reconstruction Theorem

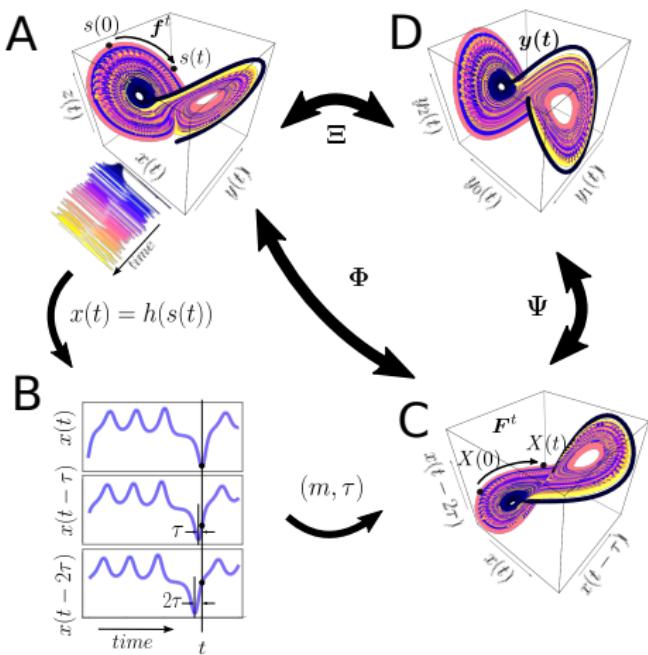


Figure is adapted from (Casdagli et al. 1991; Quintana-Duque (2012); Uzal et al. 2011)

## Takens's Theorem

$$s(t) = f^t[s(0)]$$

- $s$  represents a trajectory which evolves in an unknown  $d$ -dimensional manifold  $M$
  - $f^t$  is a evolution function with time evolution  $t$

Then

$$x(t) = h[s(t)]$$

- $x(t)$  scalar time series in  $\mathbb{R}$
  - $h$  is a function defined on the trajectory  $s(t)$

# State Space Reconstruction Theorem

## Uniform time-delay embedding matrix

$X(t) = \{x(t), x(t - \tau), \dots, x(t - (m - 1)\tau)\}$  defines a map  $\Phi : M \rightarrow \mathbb{R}^m$  such that

$$X(t) = \Phi(s(t))$$

where  $\Phi$  is a diffeomorphic map whenever  $\tau > 0$  and  $m > 2d_{box}$  and  $d_{box}$  is the box-counting dimension of  $M$ .

## Uniform Time-Delay Embedding (UTDE)

For a given discrete time series  $\{x_n\}_{n=1}^N = [x_1, x_2, \dots, x_N]$  of sample length  $N$ , a uniform time-delay embedding matrix is defined as

$$\mathbf{X}_\tau^m = \begin{pmatrix} \tilde{x}_n \\ \tilde{x}_{n-\tau} \\ \vdots \\ \tilde{x}_{n-(m-1)\tau} \end{pmatrix}^T$$

where  $m$  is the **embedding dimension** and  $\tau$  is the **embedding delay**.

The sample length for  $\tilde{x}(n - i\tau)$ , where  $0 \leq i \leq (m - 1)$ , is  $N - (m - 1)\tau$ , and the dimensions of  $\mathbf{X}_\tau^m$  are  $(m, (N - (m - 1)\tau))$ .

## Estimation of Embedding Parameters

## False Nearest Neighbours (FNN) for $m$

Unfold the attractor (i.e. evolving trajectories in a state space).

## Average Mutual Information (AMI) for $\tau$

Maximize the information in the RSSs.

# False Nearest Neighbours (FNN) for embedding dimension

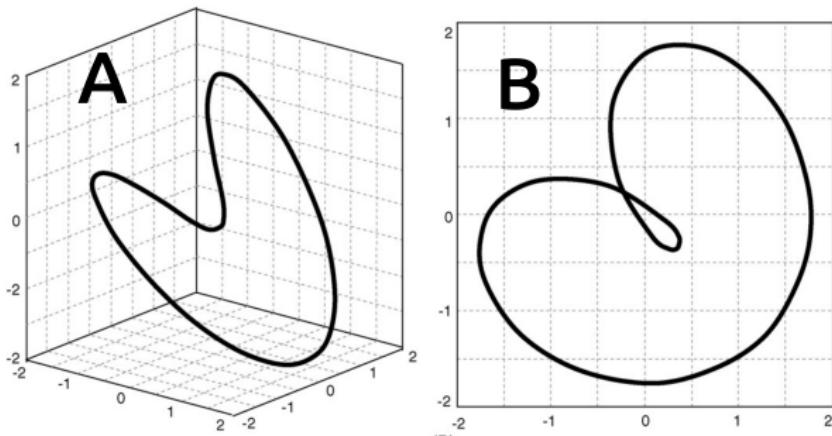


Figure 2 England 2007.

**Figure 3:** (A) Plot of  $x(t) = \sin(2\pi t) + \cos(2\pi t)$  with embedding dimension  $n = 3$  and (B)  $n = 2$ . Note that the false-nearest-neighbor illustrated by the intersection (-0.25, 0.25) when  $n = 2$ .

## False Nearest Neighbours (FNN) for embedding dimension

## False Nearest Neighbours (FNN)

$$E(m) = \frac{1}{N - m\tau} \sum_{i=1}^{N-m\tau} \frac{\|X_i(m+1) - X_{n(i,m)}(m+1)\|}{\|X_i(m) - X_{n(i,m)}(m)\|}$$

$E_1(m)$  and  $E_2(m)$

$$E_1(m) = \frac{E(m+1)}{E(m)} \quad E_2(m) = \frac{E^*(m+1)}{E^*(m)}$$

## False Nearest Neighbours (FNN) for embedding dimension

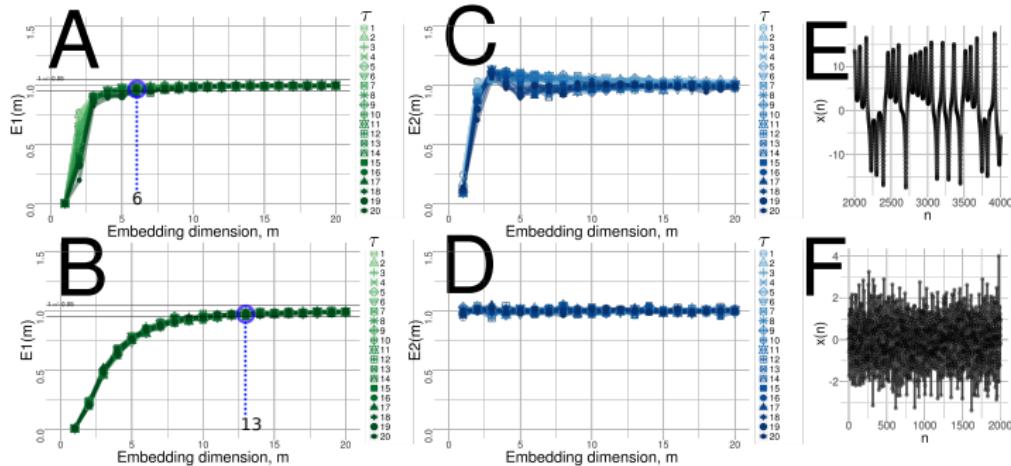


Figure is adapted from Cao L 1997 in *Physica D*

**Figure 4:** (A,B)  $E_1(m)$  and (C, D)  $E_2(m)$  values for (E) chaotic and (F) random time series

### Average Mutual Information (AMI) for embedding delay

In order to obtain  $\tau_0$ , "it has to be found in the first minimum of  $I(\tau)$  where  $x(n + \tau)$  adds maximal information to the knowledge from  $x(n)$ " meaning that the redundancy between  $x(n + \tau)$  and  $x(n)$  is the least .

# Average Mutual Information (AMI) for embedding delay

## Average Mutual Information (AMI)

$$I(\tau) = \sum_{i,j}^N p_{ij} \log_2 \frac{p_{ij}}{p_i p_j}.$$

where:  $p_i$  is the probability that  $x(n)$  has a value inside the  $i$ -th bin of the histogram,  $p_j$  is the probability that  $x(n + \tau)$  has a value inside the  $j$ -th bin of the histogram and  $p_{ij}(\tau)$  is the probability that  $x(n)$  is in bin  $i$  and  $x(n + \tau)$  is in bin  $j$ .

## Average Mutual Information (AMI) for embedding delay

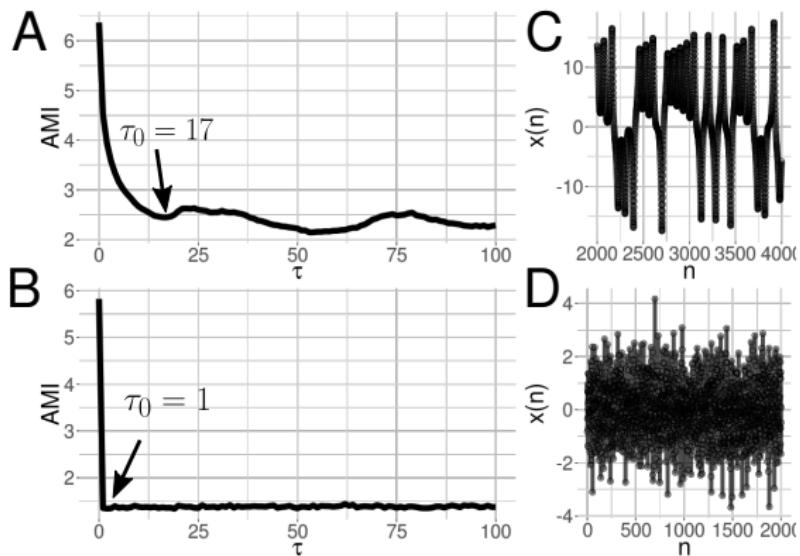


Figure is adapted from Kabiraj et al. 2012 in Chaos

**Figure 5:** (A, B) AMI values for (C) chaotic and (D) noise time series.

# Recurrence Plots (RP)

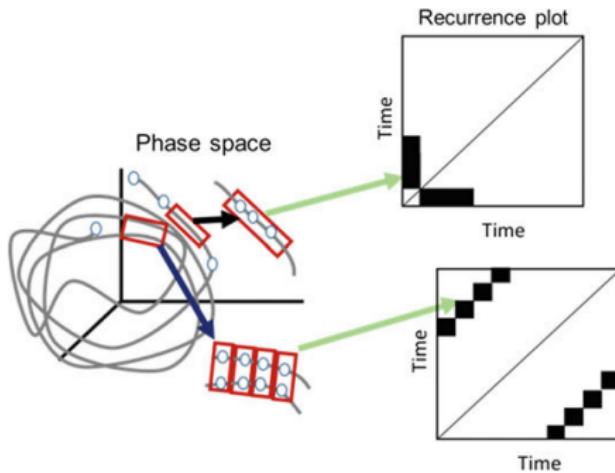


Figure is from Pawar 2018.

**Figure 6:** The vertical lines in the RP show that more than one point of the same trajectory are recurring at the same time. The diagonal lines in the RP depict that two trajectories are running in parallel to each other.

# Recurrence Plots

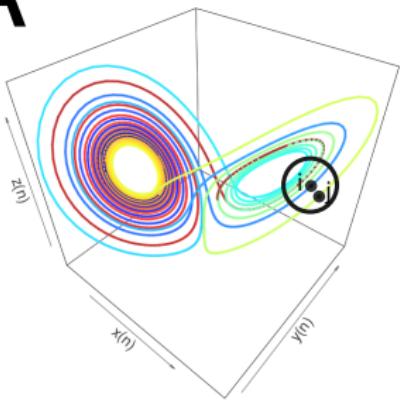
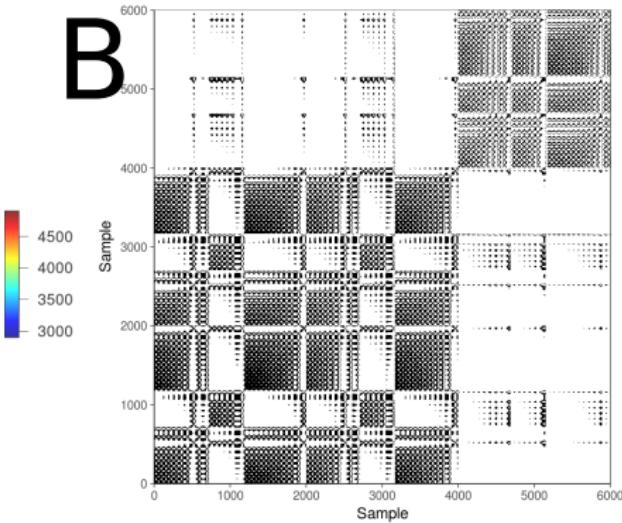
**A****B**

Figure is adapted from (Marwan et al. 2007)

**Figure 7:** (A) State space for Lorenz systems, and (B) Recurrence plot with embeddings ( $m = 1$ ,  $\tau = 1$ ) and  $\epsilon = 5$

# Recurrence Plots

$\mathbf{R}_{i,j}^m(\epsilon)$  is two dimensional plot of  $N \times N$  square matrix defined by

$$\mathbf{R}_{i,j}^m(\epsilon) = \Theta(\epsilon_i - \|X(i) - X(j)\|), \quad i, j = 1, \dots, N$$

where  $N$  is the number of considered reconstructed states of  $X(i)$  ( $X(i) \in \mathbb{R}^m$ ),  $\epsilon$  is a threshold distance,  $\|\cdot\|$  a norm, and  $\Theta(\cdot)$  is the Heaviside function.

# Recurrence Plot Patterns

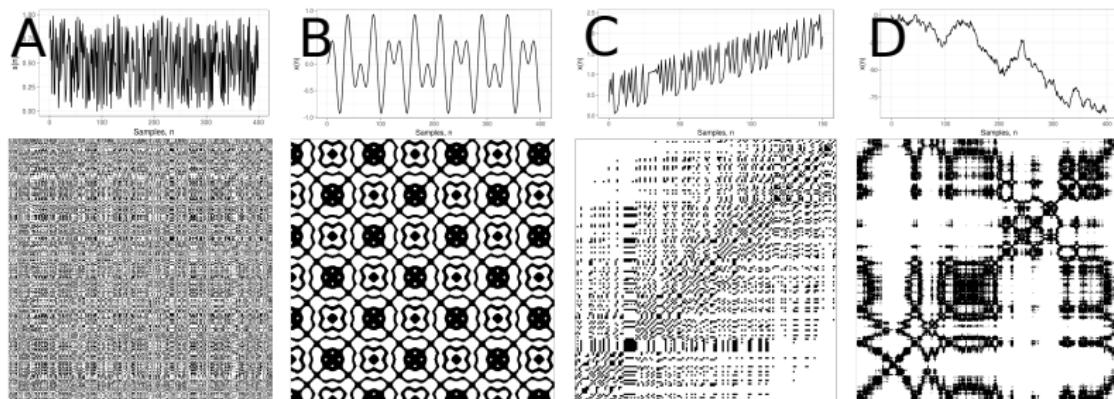


Figure is adapted from (Marwan et al. 2007)

**Figure 8:** Recurrence plots for (A) uniformly distributed noise, (B) super-positioned harmonic oscillation, (C) drift logistic map with a linear increase term, and (D) disrupted brownian motion.

## Recurrence Quantification Analysis (RQA)

**REC** enumerates the black dots in the RP.

$$REC(\epsilon, N) = \frac{1}{N^2 - N} \sum_{i \neq j=1}^N \mathbf{R}_{i,j}^m(\epsilon)$$

**DET** fraction of recurrence points that form diagonal lines.

(interpreted as the predictability where, for example, periodic signals show longer diagonal lines than chaotic ones. )

$$DET = \frac{\sum_{l=d_{min}}^N lH_D l}{\sum_{i,j=1}^N \mathbf{R}_{i,j}^m(\epsilon)}$$

## Recurrence Quantification Analysis (RQA)

**RATIO** is the ratio of DET to REC.

(useful to discover dynamic transitions).

**ENTR** Shannon entropy of the frequency distribution of the diagonal line lengths. (*useful to represent the complexity of the structure of the time series*)

$$ENT = - \sum_{l=d_{min}}^N p(l) \ln p(l),$$

where

$$p(l) = \frac{H_D(l)}{\sum_{l=d_{min}}^N H_D(l)}$$

## RQA

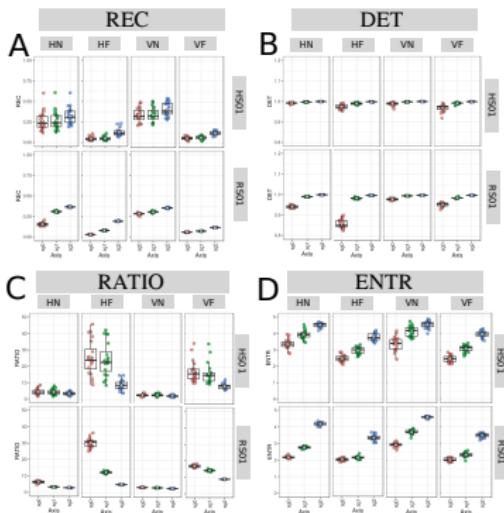


Figure is adapted from Xochicale 2019

**Figure 9:** Recurrence Quantification Analysis with  $m_0 = 6$ ,  $\tau_0 = 8$  and  $\epsilon = 1$ .

# 3D surface plots of RQA (preliminary approach)

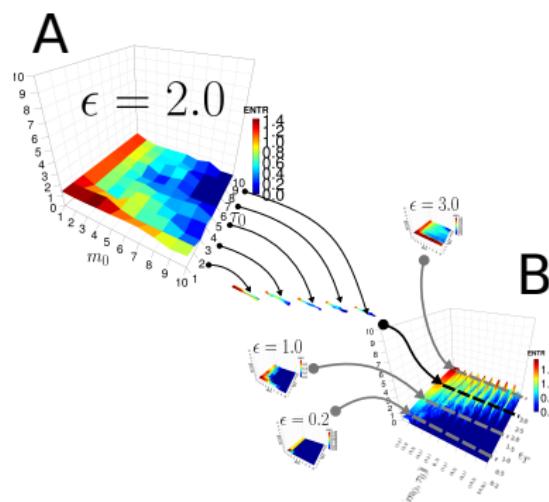


Figure is adapted from Xochicale 2019

**Figure 10:** (A) 3D surface plots for with increasing pair of embedding parameters ( $0 \leq m \leq 10$ ,  $0 \leq \tau \leq 10$ ) and  $\epsilon = 3.0$ . (B) Surface plot A with decimal increase of 0.1 for recurrence thresholds ( $0.2 \geq \epsilon \leq 3$ ).

## 3D surface plots of RQA (preliminary results)

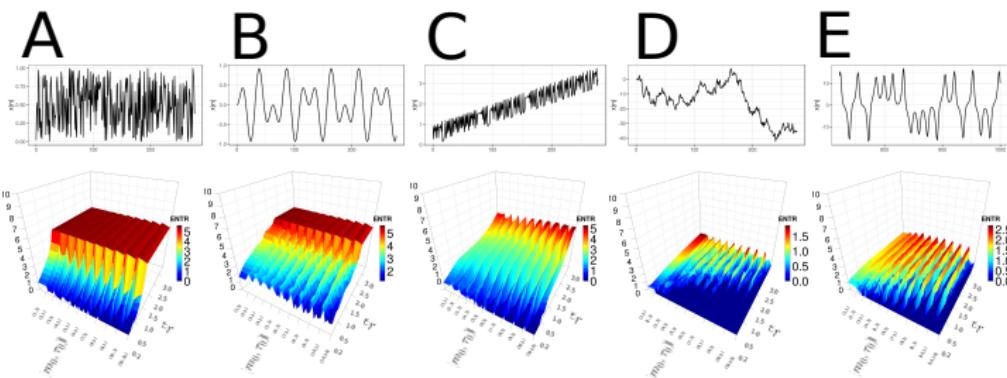


Figure is adapted from Xochicale 2019

**Figure 11:** 3D surface plots for (A) uniformly distributed noise, (B) super-positionet harmonic oscillation, (C) drift logistic map with a linear increase term, (D) disrupted brownian motion, and (E) Lorenz system.

# EXPERIMENT

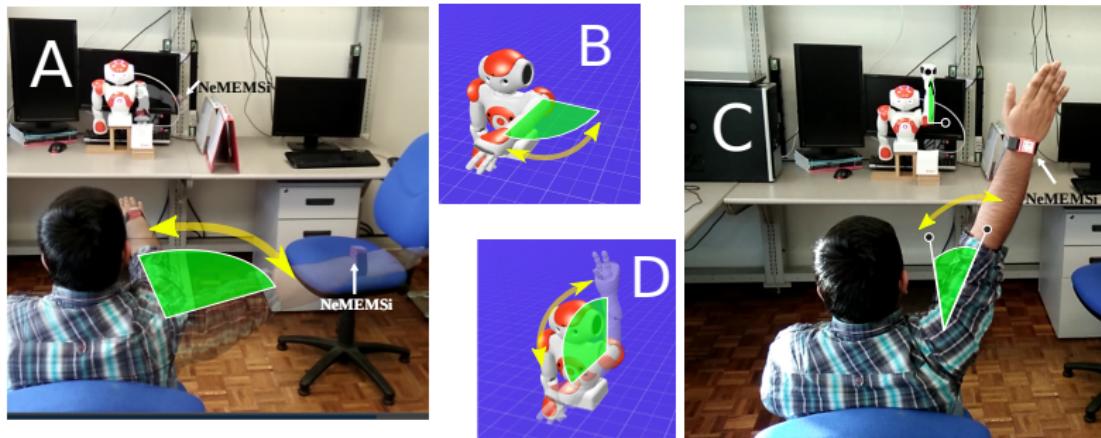
## Participants

### Human-Humanoid Imitation Activity\*

20 participants with mean and standard deviation (SD) age of mean=19.8 (SD=1.39) years, being four females and sixteen males.

\*Originally, 23 right-handed healthy participants were invited for two experiments, however some of these were not considered in the analysis due to technical problems with IMU's.

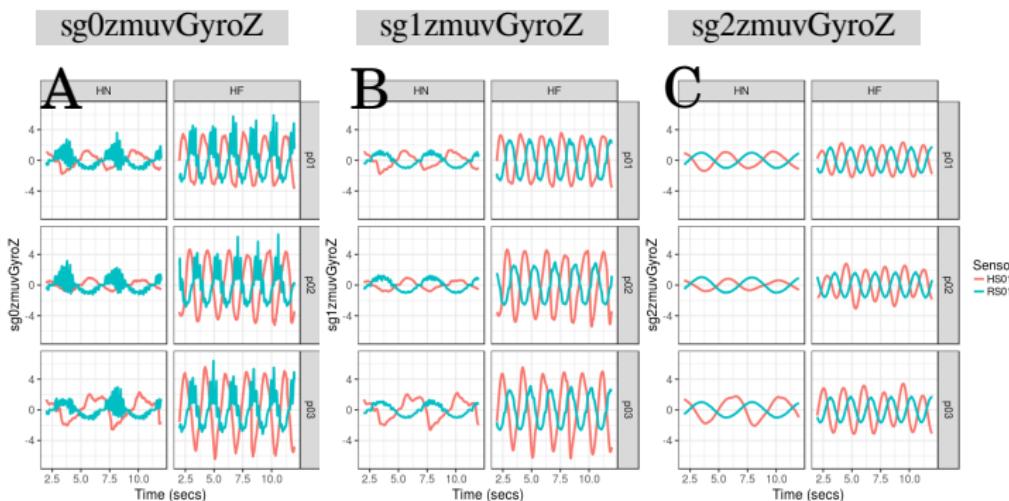
# Human-Humanoid Imitation Activities



**Figure 12:** (A/C) Front-to-Front Human-Humanoid Imitation Activities of Horizontal/Vertical Movements, (B/D) NAO, humanoid robot, performing Horizontal/Vertical arm movements.

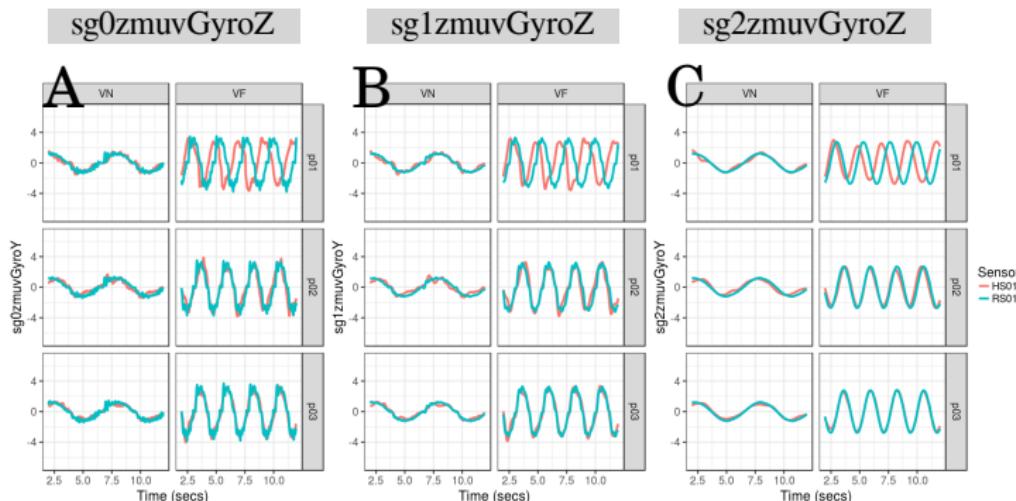
# RESULTS

# From Raw to Smoothed Time Series



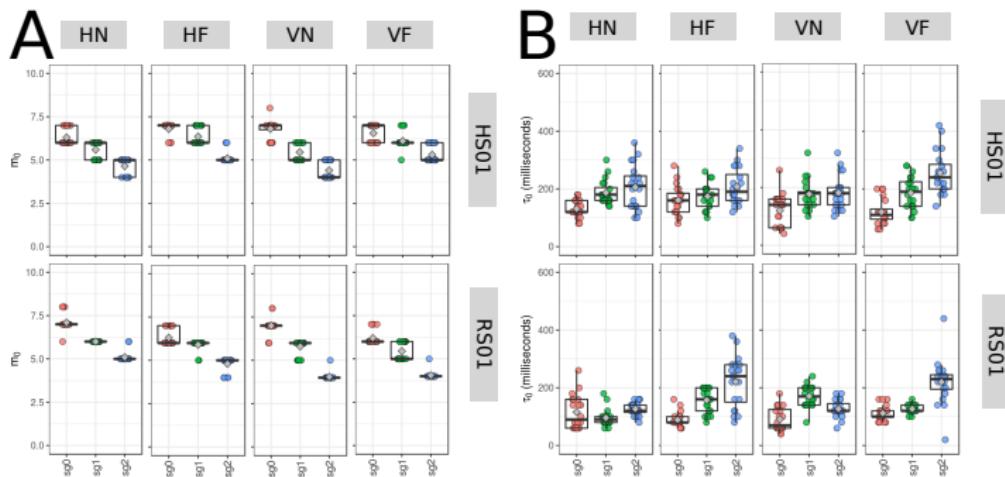
**Figure 13:** Time-series of horizontal movements for (A) normalised, (B) sgolay( $p=5, n=25$ ), and (C) sgolay( $p=5, n=159$ ).

# From Raw to Smoothed Time Series



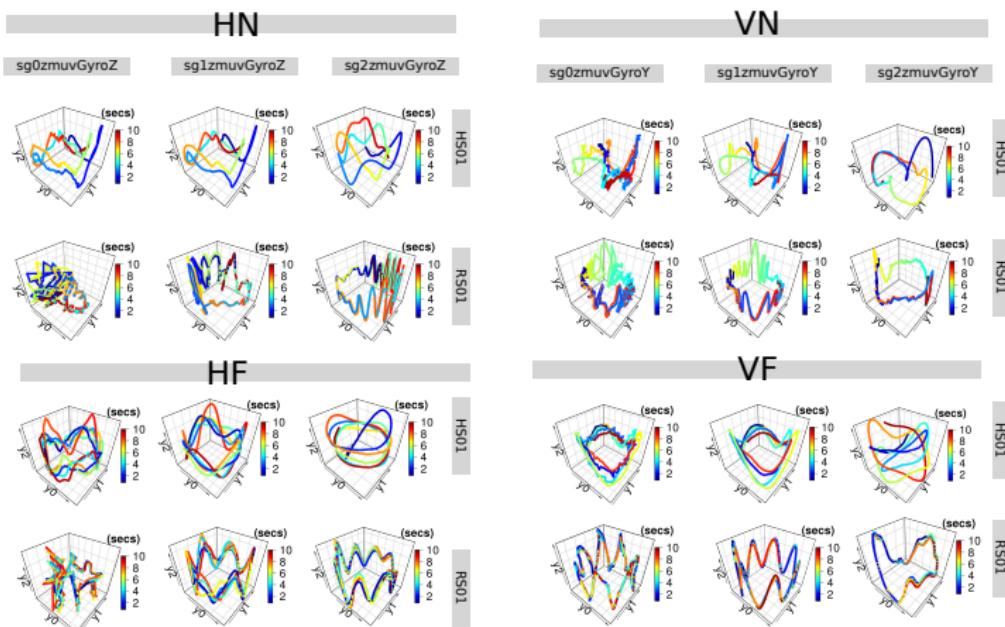
**Figure 14:** Time-series of vertical movements for (A) normalised, (B) sgolay( $p=5, n=25$ ), and (C) sgolay( $p=5, n=159$ ).

## Minimum Embedding Parameters



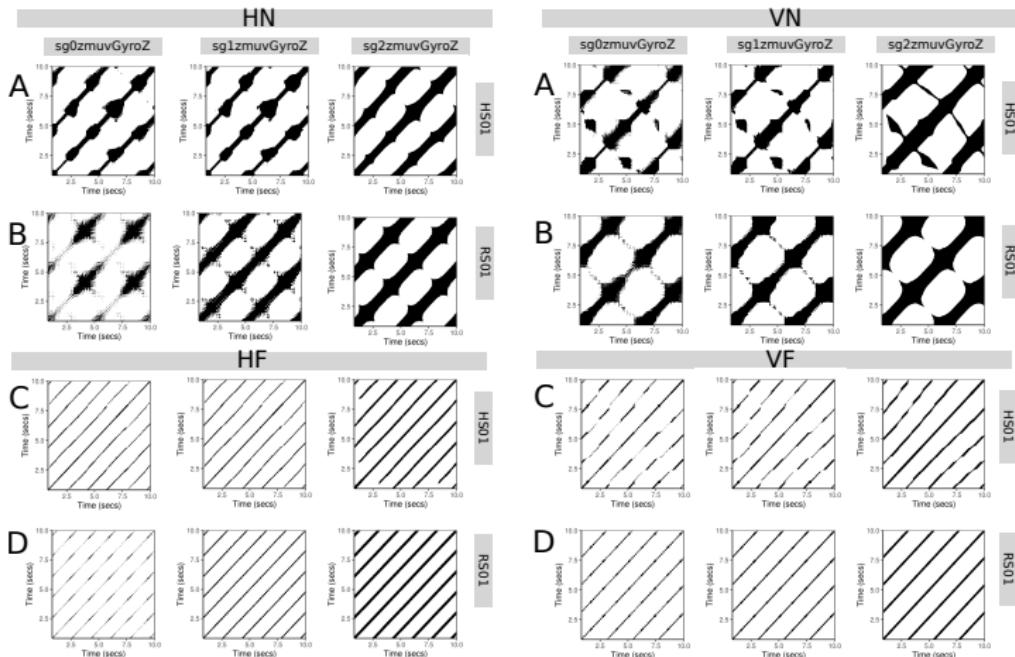
**Figure 15:** (A) Minimum Embedding Dimension (B) First Minimum AMI

# Reconstructed State Spaces



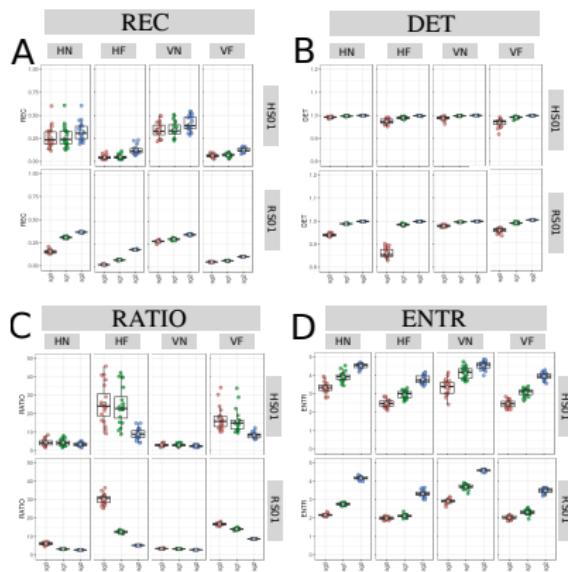
**Figure 16:** RSS for participant 01 computed with ( $m = 6$ ,  $\tau = 8$ ) for different activities, signals and source of time-series data.

## Recurrence Plots



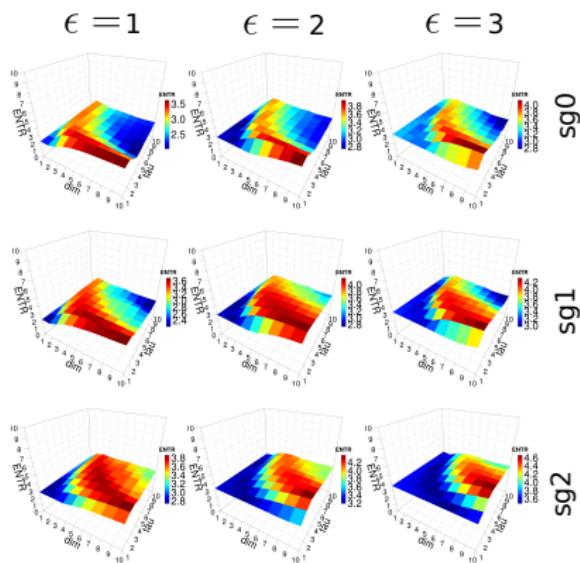
**Figure 17:** RP for participant 01 computed with  $(m = 6, \tau = 8, \epsilon = 1)$  for different activities, signals and source of time-series data.

# Recurrence Quantification Analysis



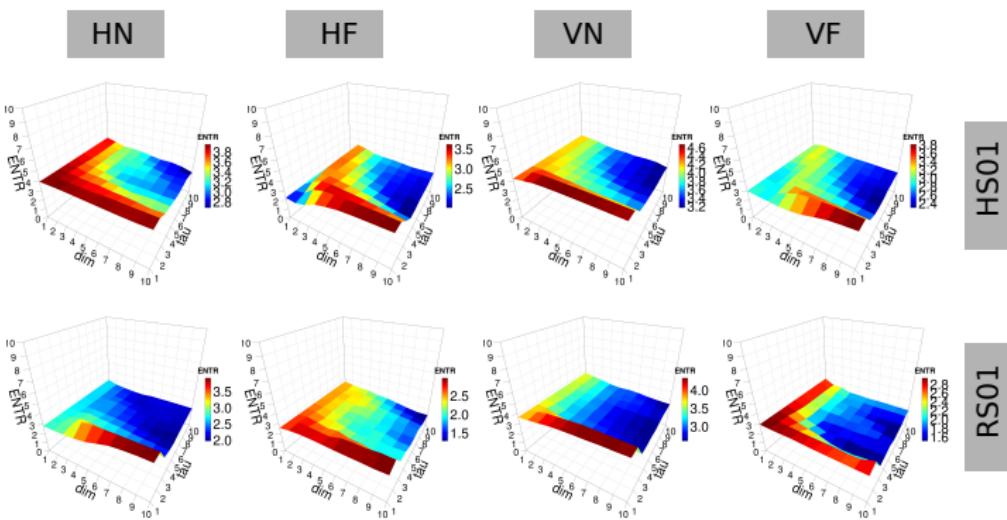
**Figure 18:** Box values of RQA computed with ( $m = 7$ ,  $\tau = 5$ ,  $\epsilon = 1$ ). These values are for 20 participants.

RQA ENTR for  $\epsilon$  thresholds & smoothness (in preparation)



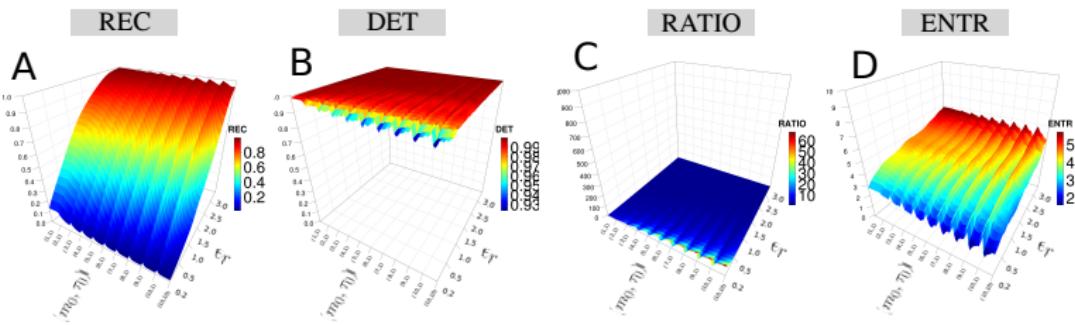
**Figure 19:** RQA ENTR values are for  $p03$ , sensor  $HS01$ , of a window size of 10-secs (500 samples).

RQA ENTR for sensors and activities. (in preparation)



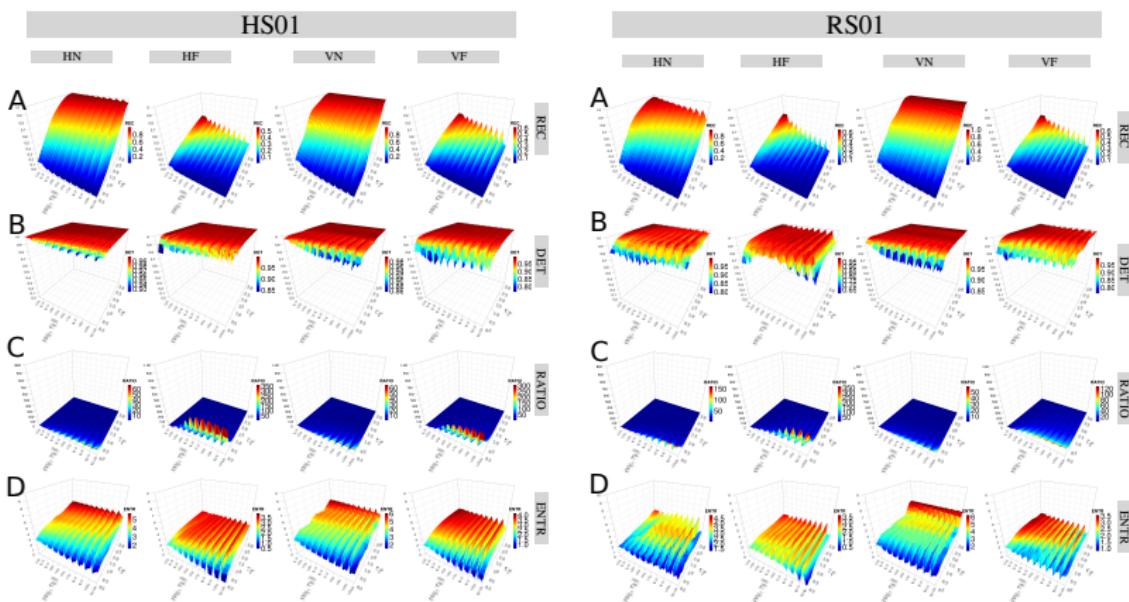
**Figure 20:** RQA ENTR values are for  $p03$ ,  $sg0$  and window size of 10-secs (500 samples).

## 3D surfaces plots of RQA (in preparation)



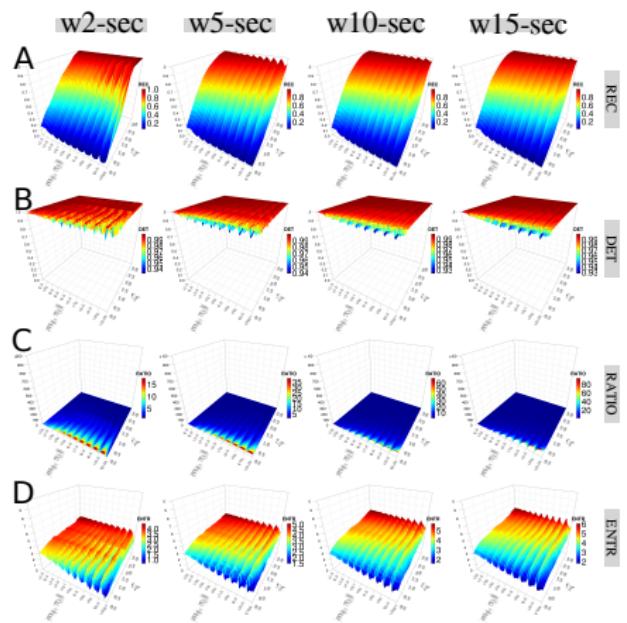
**Figure 21:** 3D RQA surfaces with increasing pair of embedding parameters ( $0 \leq m \leq 10$ ,  $0 \leq \tau \leq 10$ ) and recurrence thresholds ( $0.2 \leq \epsilon \leq 3$ ).

# Sensors and activities (in preparation)



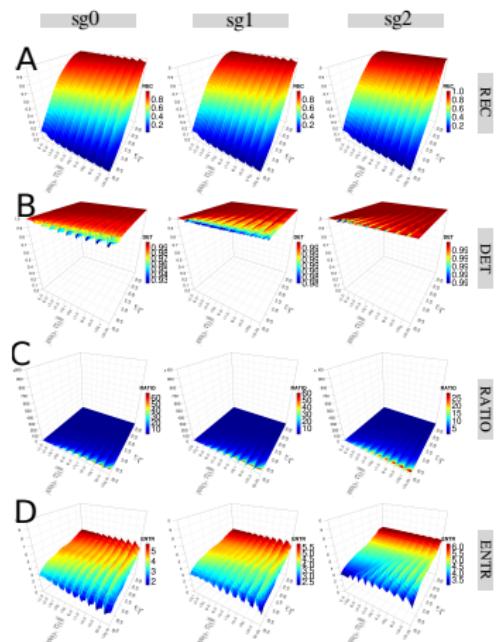
**Figure 22:** 3D surface plots of RQA for different sensors and activities.

# Window size lengths (in preparation)



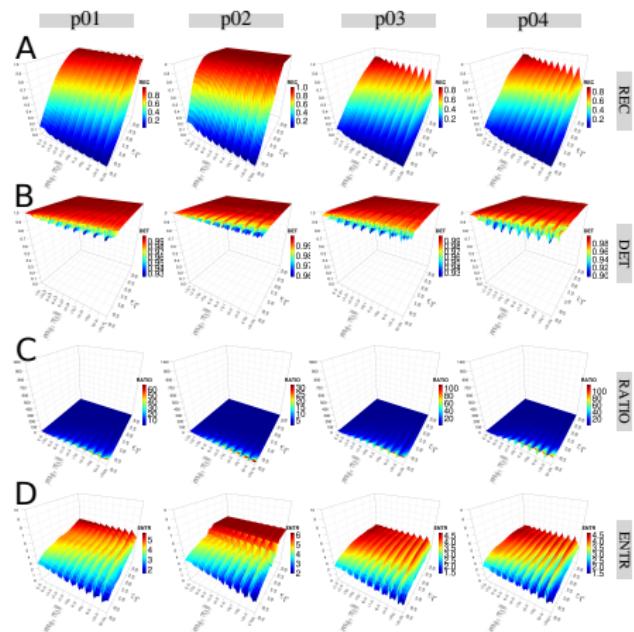
**Figure 23:** Window length size effect on 3D surface plots of RQA.

# Smoothness (in preparation)



**Figure 24:** Smoothness effect on 3D surface plots of RQA.

# Participants (in preparation)



**Figure 25:** Participants differences of 3D surface plots of RQA.

# INERTIAL SENSORS

# NeMEMSi IMU sensor



Figure 4. Test setup used for the comparison of neMEMSi with respect to the MTI-30 module.

TABLE II. MAIN FEATURES OF NEMEMSI WITH RESPECT TO STATE-OF-THE-ART DEVICES

Device	Power Cons.	Max ODR [Hz]	Size [mm3]	Wireless
MTI 30 [2]	160 mA @ 3.3V	100	57 × 42 × 23	no
MTI 300 [5]	245 mA @ 3.3 V	100	57 × 42 × 23	no
Mtw [6]	n.a.	120	35 × 58 × 15	802.15.4
X-IMU [7]	100 mA @ 3.6 V	512	57 × 38 × 21	BT Class I
VN100 [8]	70 mA @ 5.0 V	300	36 × 33 × 9	no
<b>neMEMSi</b>	<b>38.5 mA @ 3.8 V</b>	<b>150</b>	<b>30 × 30 × 8</b>	<b>BT Class I</b>

TABLE I. OVERALL POWER CONSUMPTION OF NEMEMSI

ODR <sup>a</sup> [Hz]	Radio Mode	Current Drawn [mA]	Power Cons. @ 3.8V <sup>b</sup> [mW]
0	WAIT	14.8	56.2
25	ON	26.5	100.7
50	ON	29.0	110.2
100	ON	34.0	129.0
150	ON	38.5	146.3

a. Output Data Rate of the sensor fusion algorithm.

TABLE III. AVERAGES AND STANDARD DEVIATIONS OF THE ABSOLUTE VALUE OF THE DIFFERENCE ON THE ROLL, PITCH AND YAW ANGLES

$ \epsilon $	Rotation		
	Roll	Pitch	Yaw
Avg [deg]	0.37	0.54	0.75
$\sigma$ [deg]	0.29	0.30	0.52

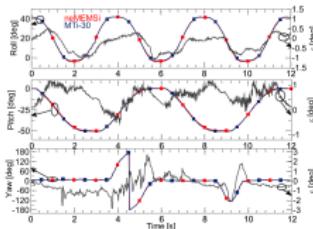
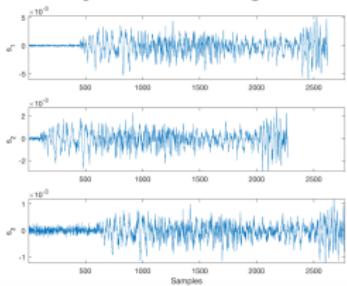


Figure 5. Sample results of single rotations around the x (roll), y (pitch) and z (yaw) axis, together with the relative difference.

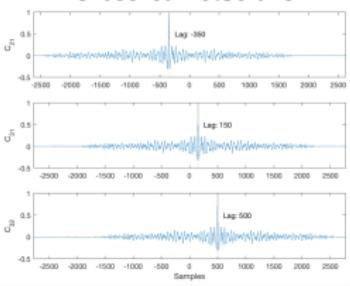
For 2 minutes of data collection there were issues of  
 \* drift in time when using 4 inertial sensors, and  
 \* disconnections to bluetooth module

# Data Synchronisation of Multidimensional Time-Series

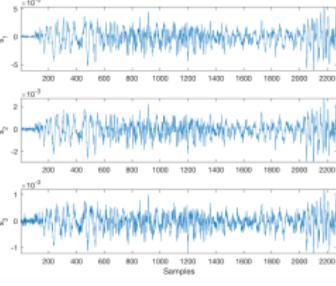
## Asynchronous signals



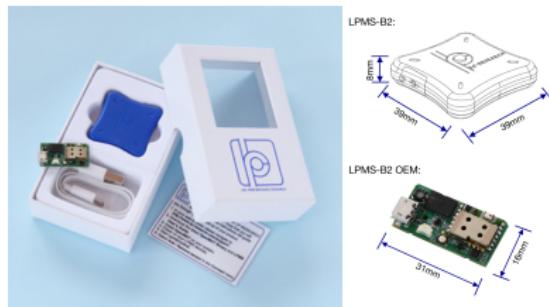
## Cross-correlations



## Line up signals



## LPMS-B2 inertial sensor



- Customers: 42 customers (e.g. HONDA, Canon, ETH Zürich, Newcastle University, etc)
- Citations for lpms imu: About 127 results (20-01-2020)
- Programming Libraries: C, C++, C, Java, MATLAB, ROS (Robot Operating System)
- SDK: Windows, Linux, Android
- Price (UK): £264.78

# CONCLUSIONS

## Research Questions

- **Q1: What are the effects on RSSs, RPs, and RQA metrics of different embedding parameters, different recurrence thresholds and different characteristics of time series (structure, smoothness and window length size)?**  
*Nonlinear analysis tools can quantify different data time-series. That said, the main contribution of this work is to find that measurements of entropy with 3D plot surfaces of RQA appear to be robust for real-word data (i.e. different time series structures, window length size and levels of smoothness).*

## Research Questions

- **Q2: What are the weaknesses and strengths of RQA metrics when quantifying movement variability?**

*WEAKNESSES: (i) requirement of an expert for interpretation and computation of nonlinear analysis results, (ii) laborious implementation, and (iii) nonlinear analysis does not give the best representation of the dynamics of time-series data.*

*STRENGTHS: (i) little setup of parameters for 3D plot surfaces, and (ii) 3D plot surfaces can provide insight into the understanding of the dynamics of time-series data.*

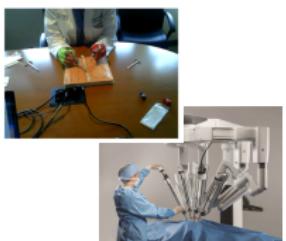
## Research Questions

- **Q3: How does the smoothing of raw time series affect methods of nonlinear analysis when quantifying movement variability?**

*Smoothing raw time series can create well defined trajectories or patterns in RSS or RP, however such increase of smoothness can also create more complex (i.e. not well defined) trajectories or patterns in nonlinear analysis.*

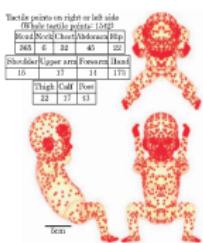
# Applications of Nonlinear Dynamics

## Quantification of skill learning

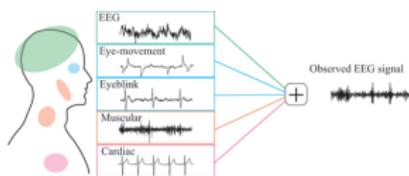


- \* Surgical Skills Assessment
- \* Robot-Assisted Surgery

## Fetal behavioral development



## Nonlinear Biomedical Signal Processing



- \* General movements
- \* Arm/Legs Movs
- \* Hand/Face Contacts
- \* EEG time series
- \* Heart rate variability
- \* Eye Movements

# Future Work

## Investigate

- other derivatives of acceleration data to have better understanding of the nature of human movement,
- other methodologies for state space reconstruction,
- the robustness of Entropy measurements with RQA, and
- variability in perception of velocity.

**In the context of human-humanoid interaction, the proposed method can be applied to**

- evaluate improvement of movement performance,
- provide feedback of level of skillfulness, and
- quantify motor control problems and pathologies.

# OA Publications

## PEER-REVIEW CONFERENCE PAPERS

- *Towards the Analysis of Movement Variability in Human-Humanoid Imitation Activities* (HAI2017)
- *Towards the Quantification of Human-Robot Imitation Using Wearable Inertial Sensors* (HRI2017)
- *Analysis of the Movement Variability in Dance Activities using Wearable Sensors* (WeRob2016)
- *Understanding Movement Variability of Simplistic Gestures Using an Inertial Sensor* (PerDis2016)

## PREPRINTS & in preparation

- *Strengths and weaknesses of Recurrence Quantification Analysis in the context of human-humanoid interaction* (ArXiv, October 2018) for Scientific Reports.
- *3D surface plots of RQA Shannon Entropy*  
for Frontiers in Applied Mathematics and Statistics.

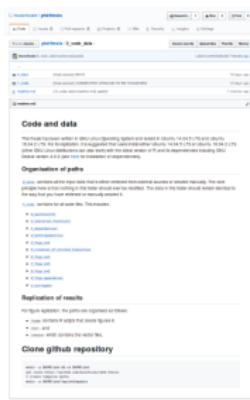
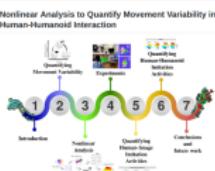
## TALKS

- *Quantifying the Inherent Chaos of Human Movement Variability*  
15th Experimental Chaos and Complexity Conference
- *Towards the Analysis of Movement Variability for Facial Expressions with Nonlinear Dynamics*  
The 7th Consortium of European Research on Emotion Conference

FIRST Open Access PhD Thesis at UoB (since 1900)



<https://github.com/mxochicale-phd/thesis>



QA PhD Thesis

- \* LaTeX project  
\* Vector files

OA DATA

- \* Multidimensional Times-series  
22 participants,  
4 IMUs (6 axis), and  
4 Activities.

OA SOFTWARE

- ```
* R version 3.4.4 (2018-03-15)
* R packages:
  data.table
  ggplot2
  tseriesChaos
  nonlinearTseries
  RccArmadillo
* GNU Octave 4.0.2
```

## References



Xochicale Miguel

» Nonlinear Analysis to Quantify Movement Variability in  
Human-Humanoid Interaction «

Open Access Ph.D. Thesis (2019)

<https://github.com/mxochicale-phd/thesis>



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