# One Method for Design of Wide-band FIR Filters Without Multipliers

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## **Abstract**

This paper presents the method for the wide-band FIR filters design without multipliers. The proposed method is based on the masking method and the techniques for rounding and sharpening. The coefficient values of the model and masking filters are represented as integers using the rounding technique. The sharpening technique is applied to the masking filters to improve the overall magnitude characteristic and to satisfy the given specification.

## 1. Introduction

Linear-phase finite impulse response (FIR) filters of length N require (N+1)/2 multipliers, N-1 adders and N-1 delays. The complexity of the implementation increases with the increase in the number of multipliers. Over the past years there have been a lot of attempts to reduce the number of multipliers [1-4].

Another approach is a true multiplier-free design where the coefficients are reduced to simple integers or to simple combinations of powers of two, for example, [5].

The application of frequency-response masking technique for design a sharp FIR filter with a wide-bandwidth was introduced in [4]. The purpose of the method is the reduction of the overall FIR filter. The less order model and masking filters are designed instead of a high order FIR filter. In this paper we propose to further reduce the complexity of the masking method by presenting the filter coefficients as integers using the rounding technique. As a consequence the magnitude characteristic of the overall filter is deteriorated. To resolve this problem we propose to apply the sharpening technique.

Considering that the integer coefficient multiplications can be accomplished with only shift-and-add operations, the sharpened rounded impulse response filters are multiplier-free [5].

The rest of the paper is organized in the following form. Next Section briefly describes the masking method. Second Section presents the rounding and sharpening techniques. The proposed method is described in Section 4 and illustrated with one example.

# 2. Frequency masking method

The structure of the FIR filter synthesized using frequency-response masking approach is shown in Fig.1.

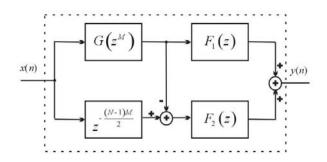


Figure1: Block diagram of frequency masking filter.

The transfer function of the overall filter is given as [4], [6]

$$H(z) = G(z^{M})F_{1}(z) + G_{c}(z^{M})F_{2}(z),$$
 (1)

where  $G(z^M)$  and  $G_c(z^M)$  are referred to as the periodic model filter and periodic complementary model filter, respectively, and  $F_1(z)$  and  $F_2(z)$  as masking filters.

The periodic model filter  $G(z^M)$  is obtained from the model filter G(z) by changing each delay with M



delays. The periodic complementary model filter  $G_c(z^M)$  is given as [4], [6]

$$G_c(z) = z^{-M(N-1)/2} - G(z^M)$$
. (2)

This filter can be implemented by subtracting the output of  $G(z^M)$  from the delayed version of the input as shown in Figure 1.

Usually M is chosen first to minimize the overall filter complexity [6]. The passband and the stopband frequencies  $\alpha$  and  $\beta$  of the model filter are defined using the given passband and stopband frequencies  $\omega_p$  and  $\omega_s$ , [4], [6]

$$\alpha = \omega_D M - 2k\pi$$
,  $\beta = \omega_S M - 2k\pi$ , (3)

where

$$k = \left| \frac{\omega_p M}{2\pi} \right| \tag{4}$$

and [•] denotes the nearest integer from the lower side.

The passband and the stopband frequencies for the masking filter  $F_1(z)$  and  $F_2(z)$  are, respectively, [4], [6]

$$\alpha_1 = \frac{2k\pi + \alpha}{M}, \quad \beta_1 = \frac{2(k+1)\pi - \beta}{M}$$
 (5)

$$\alpha_2 = \frac{2k\pi - \alpha}{M}, \quad \beta_2 = \frac{2k\pi + \beta}{M} \quad . \quad (6)$$

The lengths of masking filters must either be both even or both odd [4], [6].

# 3. Rounding and sharpening techniques

We next use the approach proposed in [1-3] for the impulse response rounding given by

$$h_r(n) = r \cdot h_I(n) = r \cdot round(h(n)/r), \tag{7}$$

where h(n) is an equiripple type FIR filter which satisfies given specification,  $h_I(n)$  is the new impulse response derived by rounding all coefficients of h(n) to the nearest integer, and round(.) means the rounding operation. The rounded impulse response  $h_I(n)$  is scaled by r in order that gain in dB of the rounded filter  $h_r(n)$  has the value  $(0 \pm R_D)$  dB in the

passband, where  $R_p$  is the specified passband ripple. The rounding constant r determines the precision of the approximation of  $h_r(n)$  to h(n). Considering that the integer coefficient multiplications can be implemented with only shift-and-add operations, the rounded impulse response filter is multiplier-free.

Using the results from [2-3] we notice that:

- The process of rounding introduces some zerovalued coefficients in the rounded impulse response. The number of nonzero integer coefficients corresponds to the number of the sums and decreases with the increase of the constant r.
- Some of the nonzero coefficients have the value 1 or have the same absolute values. As a result the number of integer multiplications is further decreased. This number also decreases with the increase of the constant *r*.

To improve the gain response characteristics, we propose to use the sharpening technique for simultaneous improvements of both the passband and stopband characteristics [7]. The technique uses the amplitude change function (ACF) which is a polynomial relationship of the form  $H_{sh} = f(H)$  between the amplitudes of the overall and the prototype filters,  $H_{sh}$  and H, respectively.

In order to not increase the overall complexity in this paper we propose to use the simple polynomial as

$$H_{sh} = 3H^{2l} - 2H^{3l} \tag{8}$$

where l is the number of the cascaded filters H.

# 4. Description of the design procedure

In this Section we describe the procedure of the design.

1. Step:

We choose the interpolation factor M and design the model and masking filters using (3)-(6). The model filter is rounded and expanded by M, resulting in  $G_r(z^M)$ . The starting rounded constant is chosen as

 $G_r(z^M)$ . The starting rounded constant is chosen as r=0.001.

2. Step:

Using (2) we find the rounded complement of the filter  $G_r(z^M)$ , denoted as  $G_{rc}(z^M)$ .

3. Step

We apply the rounding technique in the masking filters resulting in rounded masking filters  $F_{1r}(z)$  and  $F_{2r}(z)$ . Higher rounding constant is used for less order filter. A good starting point is r=0.01.

4. Step:

The sharpening technique is applied to the filters  $F_{1r}(z)$  and  $F_{2r}(z)$ .

5. Step

We find the resulting filter using



$$H_m(z) = G_r(z^M)Sh\{F_{1r}(z)\}$$
  
  $+G_{rc}(z^M)Sh\{F_{2r}(z)\}$  , (9)

as illustrated in Figure 2, where Sh{.} denotes sharpening.

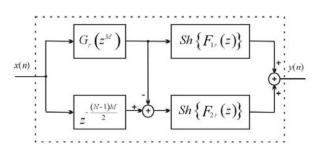


Figure 2: Modified masking filter.

The method is illustrated in the following example.

## Example 1:

We design the FIR filter with the normalized passband and the stop band frequencies  $\omega_p=0.7, \omega_s=0.73$ , and the passband ripple  $\mathit{Rp}=0.2\,$  dB and the minimum stopband attenuation  $\mathit{As}=70\mathrm{dB}.$ 

The prototype filter designed by Remez algorithm has an order of 190.

#### 1 step:

We choose the interpolation factor *M*=4 and design the model and masking filters.

We use the rounding constant r=0.001 and find the rounded expanded model filter.

# Step 2:

We find the complement filter  $G_{rc}(z^M)$ .

The magnitude responses of the rounded expanded model filter and its complement are shown in Fig.3. Step 3.

We apply the rounding of the masking filters using r=0.01.

## Step 4:

We apply the sharpening technique for masking filters using l=1 for the filter  $F_1(z)$  and l=2 for the filter  $F_2(z)$ . The magnitude responses of the sharpened rounded masking filters are shown in Fig. 4. Step 5:

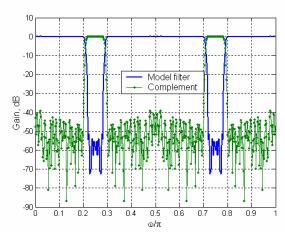


Figure 3: Expanded model filter and its complement.

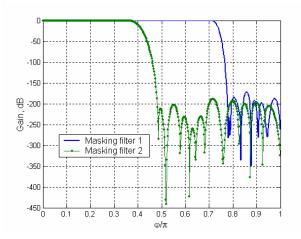


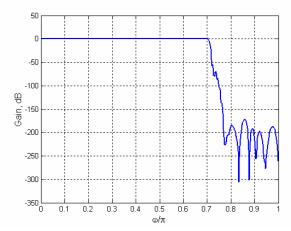
Figure 4: Sharpened rounded masking filters.

We find the resulting filter and check if the specification is satisfied. The resulting magnitude response is given in Fig.5.a. The passband and the stopband details demonstrate that the specification is satisfied. The number of adders and the integer multiplications are given in Table 1.

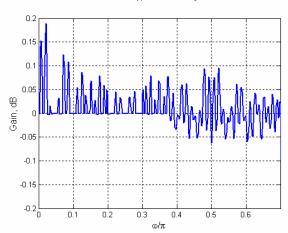
Table 2 presents the numbers of additions and the integer multiplications for the sharpened rounded masking filters.

Therefore the total number of the additions is 304 and the total number of the integer multiplications is 110 in the designed filter. The corresponding overall filter designed using the Remez algorithm has 189 adders and 95 multipliers, while the masking filter has 166 adders and 85 multipliers.





#### a. Overall magnitude response.



b. Passband.

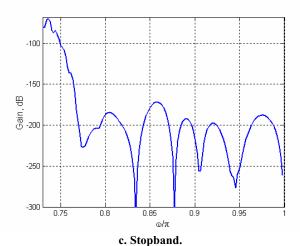


Figure 5: Example 1.

	Sharp. Masking 1	Sharp. Masking 2
Number of adders	141	91
Number of int. multiplications	42	50

Table 1: Number of adders and multiplications.

	Model	Mask. 1	Mask. 2
Number of adders	72	70	15
Number of int.	18	20	8
multiplications			

Table 2: Number of adders and integer multiplications.

## 5. Conclusion

This paper presents method for a wideband FIR filter design without multipliers. The proposed design is based on the masking—frequency technique. The rounding technique approximates the corresponding impulse response with the scaled rounded impulse response with the integer coefficients. The sharpening technique is applied to rounded masking filters to improve the overall magnitude characteristic. The price for introducing the integer multiplications is paid by increasing the number of adders.

## 6. References

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