A New Method for Design Narrow Band Lowpass FIR Filters Using a Scale Function

M. A. Pérez-Xochicale and G. Jovanovic-Dolecek
Department of Electronics, Institute INAOE, P.O. Box 51 & 216, Puebla, Mexico
E-mail: xoch@inaoep.mx, gordana@inaoep.mx

Abstract

A simple method for design of computationally efficient finite impulse (FIR) lowpass filter is presented. The design method is based on the sharpening and rounding techniques. At first, the filter which satisfies the specification is designed by Remez algoritm. In the next step the filter coefficients are rounded to the nearest integers. The sharpening technique is applied to the rounded filter to improve the magnitude characteristic and to satisfy the specification. In order to get less complex filter the proposed scale function is used.

1. Introduction

FIR filters are often preferred over IIR filters because they have several very desirable properties such as linear phase, stability, and absence of limit cycle. The main disadvantage of FIR filters is that they involve a higher degree of computational complexity compared to IIR filters with equivalent magnitude response [1] .

FIR filters of length N require $(N+1)/2$ multipliers if N is odd, $N/2$ multipliers if N is even, and N-1 adders and N-1 delays. The complexity of the implementation increases with the increase in the number of multipliers.

Different methods have been proposed to reduce the complexity of the FIR filters like [2-10].

In this paper we propose to reduce the complexity of the FIR filters using the rounding and sharpening techniques. In that way the filter coefficients are presented as integers. Considering that the integer coefficients can be realized with only shift-and-add operations, the sharpened rounded impulse response filters are multiplier-free. Additionaly, unlike to the method [4] we propose here to include the scale function in order to further decrease the complexity of the design.

The paper is organized as follows: In next section we briefly review the rounding and the sharpening techniques. The proposed method is described in Section 3 and illustrated with two examples.

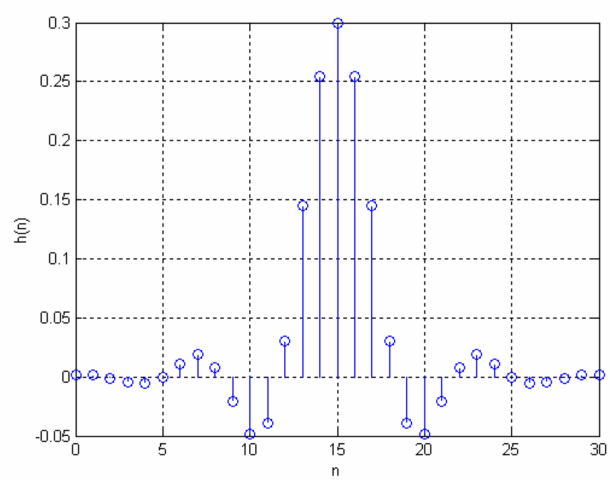
2. Rounding-Sharpening Technique

2.1. Rounding technique

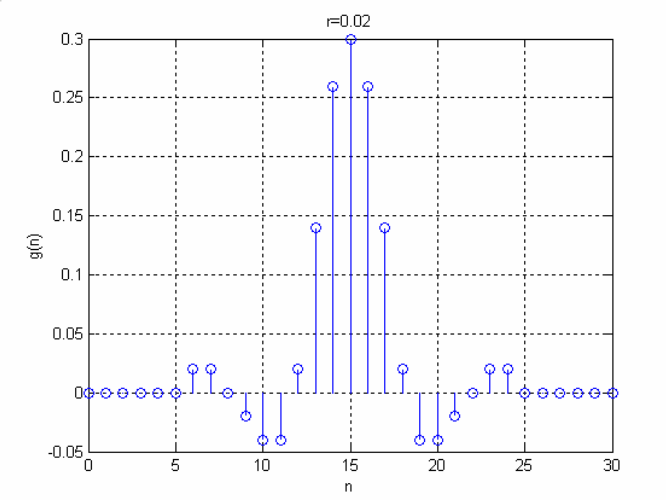
The coefficients of the given impulse response *h(n)* are rounded to the nearest integer values using rounding,

$$h_r(n) = r \cdot h_I(n) = r \cdot round(h(n)/r) \qquad (1)$$

where $h(n)$ is an equiripple type FIR filter which satisfies given specification, $h_I(n)$ is the new impulse response derived by rounding all coefficients of $h(n)$ to the nearest integer, and *round*(.) means the rounding operation. The rounded impulse response $h_I(n)$ is scaled by r in order that gain in dB of the rounded filter $h_r(n)$ has the value $(0 \pm R_p)$ dB in the passband, where R_p is the specified passband ripple. The rounding constant r determines the precision of the approximation of $h_r(n)$ to $h(n)$. Fig. 1 illustrates an original impulse response $h(n)$ and the new impulse response $h_r(n)$ obtained using the rounding constant r=0.02.



a. The impulse response $h(n)$.



b. Rounded impulse response, r=0.02.
Fig.1. Rounding.

We can observe the following:

- The process of rounding introduces some zero-valued coefficients in the rounded impulse response. The number of nonzero integer coefficients corresponds to the number of the sums and decreases with the increase of the constant r .
- Some nonzero coefficients of the rounded filter have the same values. Therefore the number of integer multiplications corresponds to the number of a different positive integer coefficients values. (The values 1 and the corresponding negative values are not counted). This number also decreases with the increase of the constant r.

The rounding introduces the distortion in the magnitude response. We propose to use the sharpening technique to improve the magnitude response of the rounded filter.

2.2. Sharpening technique

The filter sharpening technique introduced by Kaiser and Hamming [11], can be used for simultaneous improvement of both pass band and stop band characteristics of a linear-phase FIR digital filter. The technique uses amplitude change function (ACF). An ACF is a polynomial relationship of the form $H_0=f(H)$ between the amplitudes of the overall and the prototype filters, H_0 and H, respectively. The improvement in the pass band, near H=1, or in the stop band, near H=0, depends on the order of tangencies m and n of the ACF at H=1 or at H=0.

The expressions for ACF, proposed by Kaiser and Hamming for the m^{th} and n^{th} order tangencies of the ACF at H = 1 and H = 0, respectively, are given as,

$$Sh\{H\} = H^{n+1}\sum_{k=0}^{m}\frac{(n+k)!}{n!k!}(1-H)^k$$
$$= H^{n+1}\sum_{k=0}^{m}C(n+k,k)(1-H)^k \quad ,(2)$$

where $C(n+k, k)$ is the binomial coefficient.

The values of the ACF for some typical values of m and n are given in Table I.

TABLE I: ACF polynomials for m = 1, 2, 3 and n=1, 2, 3.

m	***n***	***ACFpolynomials***
1	0	$2H-H^2$
1	1	$3H^2-2H^3$
1	2	$4H^3-3H^4$
1	3	$5H^4-4H^5$
1	4	$6H^5-5H^6$
2	0	H^3-3H^2+3H
2	1	$3H^4-8H^3+6H^2$
2	2	$6H^5-15H^4+10H^3$
2	3	$10H^6-24H^5+15H^4$
2	4	$15H^7-35H^6+21H^5$
3	0	$-H^4+4H^3-6H^2+4H$
3	1	$-4H^5+15H^4-20H^3+10H^2$
3	2	$-10H^6+36H^5-45H^4+20H^3$
3	3	$-20H^7+70H^6-84H^5+35H^4$

3. Description of the method

In order to decrease the complexity of the designed filters we propose to change the stopband frequency of the sharpened filter using the following scaling function

$$\omega_{Ps} = \omega_s + \gamma\left(\Delta\omega\right), \quad (3)$$

where the scale function is given by $\gamma\left(\Delta\omega\right)$ and is determined by

$$\gamma\left(\Delta\omega\right) \approx \frac{\Delta\omega^{(\Delta\omega+1)}}{3}. \quad (4)$$

This scaling function is obtained empirically, and some typical values are given in Table II.

TABLE II: The typical values of the scale function.

$\Delta\omega$	$\gamma(\Delta\omega)$
0.1	$\Delta\omega^{(\Delta\omega+1)}/3 \approx 0.027$
0.09	$\Delta\omega^{(\Delta\omega+1)}/3 \approx 0.023$
0.08	$\Delta\omega^{(\Delta\omega+1)}/3 \approx 0.02$
0.07	$\Delta\omega^{(\Delta\omega+1)}/3 \approx 0.019$
0.06	$\Delta\omega^{(\Delta\omega+1)}/3 \approx 0.016$
0.05	$\Delta\omega^{(\Delta\omega+1)}/3 \approx 0.014$
0.04	$\Delta\omega^{(\Delta\omega+1)}/3 \approx 0.011$
0.03	$\Delta\omega^{(\Delta\omega+1)}/4 \approx 0.0067$
0.02	$\Delta\omega^{(\Delta\omega+1)}/4 \approx 0.0046$
0.01	$\Delta\omega^{(\Delta\omega+1)}/4 \approx 0.0023$

In the following we describe the steps of the proposed design.

1) Design the prototype equiripple filter $H(z)$ using Remez algorithm.
2) Choose the value of the rounding constant r. As a good starting point choose $r = 0.01$. Round the coefficients of the filter $H(z)$ using Eq. (1) to obtain the filter $H_r(z)$.
3) Apply the sharpening technique to the rounded filter $H_r(z)$, to obtain the filter $Sh\{H_r(z)\}$.
4) Use the scale function to define new transition band of the sharpened filter $Sh\{H_r(z)\}$.
5) Check if the specification is satisfied. If not, go to the Step 2, and decrease the rounding constant.

The procedure is illustrated in the following examples.

Example 1.

We design the filter with the following specification: the normalized passband and stopband edges are 0.7 and 0.79, respectively. The passband ripple is 0.1 dB, while the minimum stopband attenuation is 90 dB.

1) We design the corresponding filter using Remez algorithm. The length of the equiripple filter is N=120. The corresponding magnitude response is given in Fig.2.
2) The value of the rounding constant $r = 0.003$ is chosen to obtain filter $H_r(z)$. The magnitude response of the rounded filter is also shown in Figure 2.
3) The sharpening technique is applied to the filter $H_r(z)$, using the polynomial m=1, n=2, from Table I. The sharpened filter $H_{rs}(z) = Sh\{H_r(z)\}$ is also given in Fig. 2. Figure 2 indicates that we need to change the stopband frequency.
4) Using the Table II the new stopband cutoff frequency of the filter is 0.813.
5) Figure 3 shows the resulting magnitude response along with the responses of the prototype, rounded and rounded-sharpened filters. Note that the specification is satisfied.

Table III compares the number of necessary multipliers using Remez algorithm, the method [4], and the proposed method .

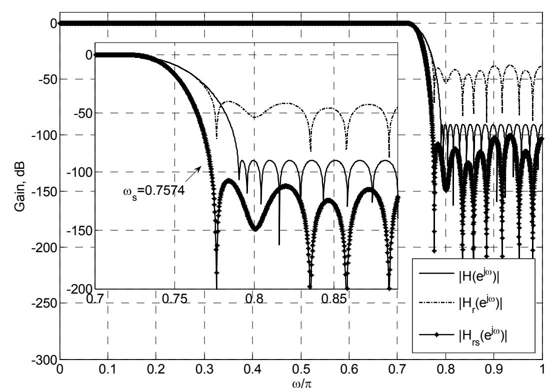


Fig. 2. Gain responses of the original, rounded and rounded-sharpening filters.

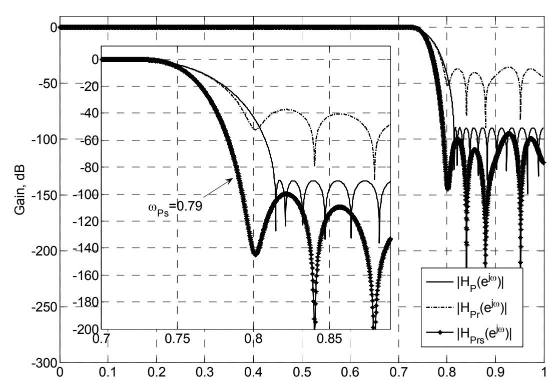


Fig. 3. Gain responses of the original, rounded, and proposed filters.

Table III. Number of multipliers in example 1.

Method	Number of Multiplications
Equiripple filter (Remez)	61 (floatings)
Rounded-Sharpened [4]	56 (Integers)
Proposed	48 (Integers)

Example 2.

In this example we design a more complex filter having the normalized passband and the stopband frequencies 0.7 and 0.72, respectively. The minimum required stopband attenuation is 90 dB, and the maximum passband ripple is 0.1 dB.
The proposed design is given in the following steps.

1) Using the Remez algorithm we get the equiripple filter of the order *N*=532.
2) The value of the rounding constant $r = 0.0015$ is chosen, and the rounded filter is obtained. Its magnitude response is given in Figure 4.
3) The sharpening polynomial *m*=1, *n*=2 (Table I) is applied to the rounded filter $G_r(z)$, to obtain the sharpened rounded filter, also shown in Figure 4. Note that we need to change the stopband frequency.
4) Using the Table II, the new stopband cutoff frequency of the filter is 0.7246.
5) Figure 5 shows the magnitude response of the resulting filter along with ones of the prototype, rounded and rounded-sharpened filters.

Table IV provides the comparison of the number of multipliers using the proposed, the Remez algoritm, and the method [3].

Table IV. The number of multipliers in example 2.

Method	Multiplications
Remez	267 (floatings)
Rounded-Sharpened [4]	72 (Integers)
Proposed	48 (Integers)

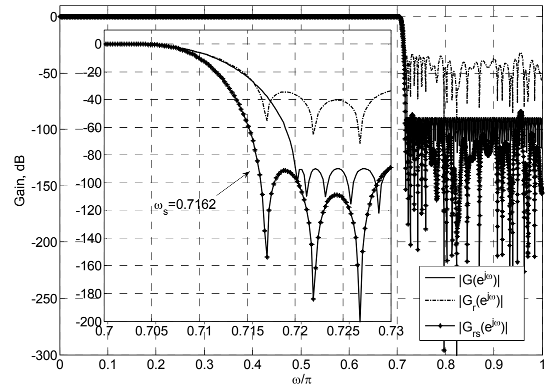


Fig. 4. Gain responses of the original, rounded and rounded-sharpening filters.

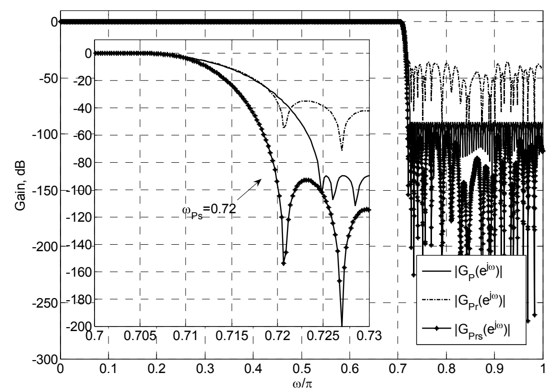


Fig. 5. Gain responses of the original, rounded, and proposed filters.

4. Conclusions

A simple multiplerless FIR design method based on rounded-sharpened technique and the proposed scale function is presented. The method uses the rounding of the coefficients of the equiripple filter which satisfies the desired specification. In the next step the sharpening technique is used to improve the magnitude characteristic and to satisfy the magnitude specification. The complexity of the rounded filters (number of the sums and the integer multiplications) depends on the choice of the rounding constant. Less complexity (higher values of *r*) results in more distortion in the corresponding magnitude response and consequently needs more complex sharpening polynomials.

Unlike to the method proposed in [4] we propose here to use the scale factor to increase the corresponding transition band of the sharpened rounded filter and consequently to decrease the complexity of the design. As the provided examples illustrate the proposed designs result in less number of integer multipliers for the same specifications.

Acknowledgements

This work is supported by CONACYT grant No. 49640.

References

[1] S. K. Mitra, *Digital Signal Processing – A Computer-Based Approach*, 3nd ed. New York: McGraw-Hill Book Company, 2005.

[2] A. Bartolo, B. D. Clymer, R. C. Burgess, and J. P. Turnbull, "An efficient method of FIR filter based on impulse response rounding", *IEEE Transactions on Signal Processing*, vol. 46, No. 8, pp. 2243-2248, August 1998.

[3] M. Bhattacharya and T. Saramaki, "Some observation leading to multiplierless implementation of linear phase FIR filters", *Proc. ICASSP,* pp. II-517-II-520, 2003.

[4] G. J. Dolecek and S. K. Mitra, "Computationally efficient FIR filter design based on impulse response rounding and sharpening", *Proccedings of the Fifth IEEE International Caracas Conference on Devices, Circuits and Systems, Dominican Republic*, pp. 249-253, Nov 2004.

[5] Adams, J.,W., and Wilson, A. N., A new approach to FIR digital filter design with fewer multipliers and reduced sensitivity, *IEEE Trans. Circuits and Systems*, 1983, vol. 30, pp.277-283.

[6] Adams, J.,W., and Wilson, A. N., Some efficient digital prefilter structure, *IEEE Trans. Circuits and Systems*, 1984, vol. 31, pp.260-265.

[7] Bartolo, A, Clymer, B. D., Burgess, R. C., and Turnbull, J. P., An efficient method of FIR filtering based on impulse response rounding, *IEEE Trans. on Signal Processing*, 1988, vol. 46, pp. 2243-2248.

[8] Coleman, J. O., Factored computing structures for multiplier less FIR filters using symbol-sequence number systems in linear spaces, *2002 IEEE International Conference on Acoustics, Speech, and Signal Processing,* vol. 3, pp. 3132-3135.

[9] Liu M. C., Chen, C. L., Shin, D. Y., Lin, C. H., and Jou S. J., Low-power multiplier less FIR filter synthesizer based on CSD code, *The 2001 International Symposium on Circuits and Systems, ISCAS 2001*, 4, 666-669.

[10] Yli-Kaakinen J, and Saramaki T., A systematic algorithm for the design of multiplier less FIR filters, *The 2001 International Symposium on Circuits and Systems, ISCAS 2001*, vol. 2, pp. 185-188.

[11] J. Kaiser and R. Hamming, "Sharpening the response of a symmetric nonrecursive filter by multiple use of the same filter", *IEEE Transactions on Acoustic, Speech, Signal, Processing*, vol. 5, no 25, pp. 457-467, May 1977.