Why this structure?
Probably too broad - should look at focus on a couple of specifics.
What is the conclusions that could be taken by practitioner or analyst from this?

The Application of ~~Strengths and Weaknesses of~~ Recurrent Quantification Analysis ~~in the context of~~ to Human-Humanoid Interaction

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ABSTRACT

Human movement variability occurs in motor performance across multiple repetitions of a task and such behaviour is an inherent feature within and between each persons' movement. Quantifying movement variability is still an open problem, particularly when traditional methods in time domain and frequency domain fail to detect tiny modulations in frequency or phase for time series. For this work, we hence investigate methodologies from nonlinear dynamics such as reconstructed state space (RSS), uniform time-delay embedding (UTDE), recurrence plots (RPs) and recurrence quantification analysis (RQA) metrics. Particularly, we are interested in the weaknesses and robustness of nonlinear dynamics tools from raw and post-processed data of wearable inertial sensors (IMUs). In the reported experiment, twenty right-handed healthy participants imitated simple vertical and horizontal arm movements in normal and faster speed from an humanoid robot. With four window lengths and three levels of smoothed time series, we found visual differences in the patterns of RSSs and RPs. Then using metrics of RQA, we find out that the type of movements and the level of smoothness affects those metrics. Our contribution for this work is that entropy values from RQA were well distributed and presenting variation in all the conditions for time series. This work have then the potential to enhance the development of better diagnostic tools for various applications rehabilitation, sport science or for new forms of human-robot interaction.

Not a conclusion just an observation

Introduction

Challenge of managing variability of movement (to maintain consistency) within a variable changing environment (to give adaptability).

Human movement ~~variability is~~ requires a complex system where ~~not only~~ multiple joints and limbs are involved for a specific task in a determined environment ~~but also how we process the external information with all of our available senses and use our experiences~~[1]. ~~Recent~~ Studies ~~in~~ of human motion ~~recognition have~~ revealed the possibility to estimate features from lower dimension signals to distinguish differences between styles of pedalling motion[2,3] and gait identification[4,5] ~~or pattern recognition from biological signals~~ systems[6]. The lower dimension signals from biological ~~signals~~ systems are time series of 1−dimension in $\mathbb{R}$ which commonly are noisy, nonlinear and non-stationary[6]. Hence, ~~we are hypothesising that~~ nonlinear dynamics can be used to objectively quantify ~~such human movement~~ variability of lower dimension signals[2–8]. For instance, Bradley et al. 2015[9] reviewed methods for nonlinear time series analysis based on the appropriate estimation of the embedding parameters (m embedding dimension and τ embedding delay) to reconstruct the state space, where an n-dimensional reconstructed state space using 1−dimensional time series, can preserve the topological properties of an unknown M-dimensional state space[10]. ~~Also,~~ Bradley et al. 2015[9] also reviewed the use of Recurrence Plots (RPs), a graphical representation of a two-dimensional map which show black and white dots as recurrences in a given n-dimensional system, and Recurrence Quantification Analysis (RQA) metrics ~~can pick out important directions and~~ statistics in RPs. In general, RPs and RQA provide ~~a more~~ intuitive meaning of the time series, for instance, RQA is quantitatively and qualitatively independent of embedding dimension (also verified experimentally[11]). However, the estimation of embedding parameters and the selection of the right parameters to perform RQA is still an open problem. Bradley et al. 2015[9] pointed out that there is no general technique that can be used to compute the embedding parameters since ~~the~~ time series are system-dependent which means that ~~once~~ computing the values for embedding parameters~~, these~~ may only work for one purpose (e.g., prediction) and may not work well for another purpose (e.g., computing dynamical invariants). Additionally, the methodologies of nonlinear dynamics for computing the embedding parameters e.g., autocorrelation, mutual information, and nearest neighbour require data which is well sampled and with little noise[12] or require purely deterministic signals[13]. ~~Similarly, these~~ Thus nonlinear analysis methods for computing the embedding parameters can break down with real-world datasets which have generally different length, different values of accuracy and precision (rounding errors due to finite precision of the measurement apparatus which include frequency acquisition[5]), and data may be contaminated with different sources of noise[12]. It is surprising that even with the previous constraints with regard to the quality of data, and the problem with the estimation of embedding parameters, the use of nonlinear dynamics have proven to be helpful to understand

- rewrite

The same approaches can apply to

discern key

In contrast, variability in humanoid ~~activity~~ movement is usually very small, as a result of the control systems used. This means that, while humanoids solve the degrees of freedom problem through joint design and algorithms, humans tend to have a more fluid and flexible approach. Consequently, one can see much variability in the performance of even the simplest of tasks.

and characterise dynamics of time series[2–6,6–9].

For this work, we are interested in exploring and investigating the effects of different features of time series (e.g. levels of smoothness, window data length, structures of time series based on types of movements, types of sensors, participants and velocities) to estimate embedded parameters for uniform time-delay embedding, recurrence plots and recurrence quantification analysis. Hence, we conducted an experiment with twenty right-handed healthy participants in the context of human-humanoid imitation activities where participants were asked to imitate simple arm movements performed by a humanoid robot. The primary aim of this work is to explore the following questions:

- What are the effects on three nonlinear methods (reconstructed state space with uniform time-delay embedding, Recurrence Plots, and Recurrence Quantification Analysis), for different embedding parameters, different recurrence thresholds and different characteristics of time series (structure of the signal, window length and smoothness of the signal)?, and
- What are the strengths and weaknesses of Recurrence Quantification Analysis for the previous conditions of nonlinear analysis methods?

Results

Methods

To give insight to the research questions, we considered various conditions of time series collected for this work (see Experiment section for more details) which are described as follows

- Three levels of smoothness for the normalised data (sg0zmuv, sg1zmuv and sg2zmuv), computed from two different filter lengths (29 and 159) with the same polynomial degree of 5 using the function `sgolay(p,n,m)`[14],
- four velocities arm movement activities: horizontal normal (HN), horizontal faster (HF), vertical normal (VN), and vertical faster (VF), and
- four window lengths: 2-sec (100 samples), 5-sec (250 samples), 10-sec (500 samples) and 15-sec (750 samples).

After the data collection, raw time series were windowed, normalised and smoothed. Then, due to space limitations and to have simple visualisation, we only present 10-sec (500 samples) window length time series for three participants (p01, p01 and p03) performing horizontal arm movements (axis GyroZ) and vertical arm movements (axis GyroY) (Figs 1).

~~Reconstructed State Spaces~~

One of the ~~main~~ challenges of the implementation of Uniform Time-Delay Embedding to Reconstructed State Spaces is the selection of embedding parameters because each time series is unique in terms of its structure (modulation of amplitude, frequency and phase)[4,5,9]. With that in mind, the problem for this work is not to compute individual embedding parameters for each of the time series but to deal with a selecting of two parameters that can represent all the time series. Our solution for that problem was to compute a sample mean over all values in each of the conditions of the time series for minimum dimension values and for minimum delay values as follows.

Minimum embedding parameters

Minimum embedding parameters were firstly computed with False Nearest Neighbour and Average Mutual Information. Figures 2(A) show that minimum embedding values for sensor HS01 appear to show more variations as ~~their~~ interquartile range ~~of the box plots are~~ is near to 1 with three exceptions (HF sg0, VN sg0, and VF sg1). Minimum embedding values appear to be constant for sensor RS01 as ~~their~~ interquartile range ~~in the box plots are~~ is near to 0.1 with the exception of two axis (HF sg0 and VF sg1). Additionally, it can be seen in Figs 2(A) a decrease of mean values (rhombus) in the box plots as smoothness of time series increase. Similarly, the first minimum values of AMI values are shown in the box plots of Figs 2(B). It can be seen that values for HS01 tend to be more spread as the smoothness of the time series is increasing (see the increase of both mean (rhombus) and interquartile range). However, AMI values for RS01 do not show such increase in relation with the increase of smoothness except~~ing~~ for HF and VF (Figs 2(B)).

We also computed an overall minimum embedding parameters that represent participants, activities, sensors and levels of smoothness, using a sample mean of all values in Figs 2. The sample mean for the minimum values of $E_1(m)$ from Figs 2(A) is $\overline{m_0} = 6$ and the sample mean for minimum values of AMIs from Figs 2(B) is $\overline{\tau_0} = 8$.

Uniform Time-Delay Embedding

Uniform time-delay embedding were computed with embedding parameters ($\overline{m_0} = 6$, $\overline{\tau_0} = 8$) and the first three axis of the rotated data of the PCA are shown for the reconstructed state spaces in Figs 3 for horizontal arm movements and Figs 4 for vertical arm movements.

~~Evidently,~~ It is easy to observe by eye the differences in each of the trajectories in the reconstructed state spaces (Figs 3, 4), however one might be not objective when quantifying those differences since those observation might vary from person to person. With that in mind, we tried to objectively quantify those differences using euclidean distances between the origin to each of the points in the trajectories, however these results created a suspicious metric, specially for trajectories which looked very messy. With that in mind, we computed Recurrence Plots and Recurrence Quantification Analysis to objectively quantify the differences in each of the cases of the time series.

Recurrences Plots

Recurrence Plots (RP) were computed for horizontal arm movements (Fig 5) and vertical arm movements (Fig 6) using the average embedding parameters ($m = 6$, $\tau = 8$) and an recurrence threshold of $\varepsilon = 1$.

Recurrence Quantification Analysis

Similarly to RP, we computed four RQA metrics (REC, DET, RATIO and ENTR) with embedding parameters $m = 6$, $\tau = 8$ and recurrence threshold $\varepsilon = 1$.

REC values

It can be seen in the box plots of Figs 7(A) that REC values, representing the % of black dots in the RPs, are more spread for HN and VN movements (higher interquartile range) than HF and VF movements (lower interquartile range) for HS01 sensor. In contrast, REC values for RS01 sensor present little variation (interquartile range of 0.01). With regard to the increase of smoothness of time series (sg0, sg1 and sg2), REC values present little variation as the smoothness is increasing for time series from HS01 (changes of mean values (rhombus)) while REC values are more affected with the smoothness for data from RS01 (see the incremental changes of mean values (rhombus)).

DET values

Figs 7(B) illustrate DET values, representing predictability and organisation of the RPs, which change very little (interquartile range is around 0.1) for type of movement, level of smoothness or type of sensor. There is also increase of DET values as the smoothness of the signal increase (see the incremental changes of mean values (rhombus)). It can be noted that the interquartile range for faster movements (HF and VF) with no smoothing (sg0) is lower than the other levels of smoothness (sg1 and sg2).

RATIO values

Figs 7(C) present RATIO values, representing dynamic transitions, for horizontal and vertical movements. It can be seen that RATIO values for HS01 sensor vary less for HN movements (interquartile range around 2) than HF movements (interquartile range around 5). For faster movements (HF and VF), it can be noted a decrease of RATIO values as the smoothness of the time series is increasing (rhombus). For normal movements (HN and VN), the decrease of RATIO values is less evident than faster movements (see rhombus).

ENTR values

Figs 7(D) show ENTR values, Shannon entropy values which represent the complexity of the structure the time series, for type of movement, level of smoothness or type of sensor. ENTR values for HS01 sensor show a higher variation (interquartile range around 0.5) than ENTR values for RS01 sensor which appear to be more constant (interquartile range 0.1). It can also be noted the increase of smoothness of time series also is presented with an increase of ENTR values (see gray rhombus).

3D RQA ENTR

Choosing an appropriate recurrence threshold is crucial to get meaningful representations in RP and RQA, however, as previously shown, one can selected the default recurrence threshold of $\varepsilon = 1$, as long as it is able to represent the dynamical transitions in each of the time series[7]. However, to show how that the selection of recurrence threshold affects RQA values, we computed ENTR values of RQA metrics for different embedding parameters $[1,10] = \{m \in \mathbb{R} | 1 \leq m \leq 10\}$, $[1,10] = \{\tau \in \mathbb{R} | 1 \leq \tau \leq 10\}$ with an increase of 1 and different recurrence thresholds $[0.2,3] = \{\varepsilon \in \mathbb{R} | 0.2 \leq \varepsilon \leq 3.0\}$ with increments of 0.1. For easily visualisation, Figs 8 only show 3D RQA for three recurrence thresholds $\varepsilon = 1,2,3$ with three levels of smoothness (sg0, sg1, sg2). It can be noted in Figs 8 that the increase of recurrence threshold is associated to the increase of ENTR values in any of the levels of smoothness (see values of the ENTR bar). One can also see that the maximum values of ENTR for $\varepsilon = 1$ and sg0 are for embedding dimensions of 2 and then as ε increases the maximum values of ENTR are for an associated increase of in the embedding dimension. It can also be noted that the increase of level of smoothness (from sg0 to sg2) is associated with both the increase of ENTR values and its smoothed 3D shape.

Other effects on 3D RQA ENTR values

We also computed 3D surfaces of RQA metrics for different sensors and different activities (Fig 9). From the maximum values of ENTR (lateral bars), one can see a decrease of ENTR values for activities going from normal to faster velocity and from human sensor (HS01) to robot sensor (RS01) (See Fig 9).

Discussion

It is evident~~ly~~ that time series from different sources (participants, movements, axis type, window length or levels of smoothness) presents visual differences for embedding parameters and therefore for RRSs. For which, the selection of embedding parameters was our first challenge where we computed embedding parameters for each time series (Fig) and then computed a sample mean over all time series in order to get two embedding parameters to compute all RRSs (Figs and). Then we found that the quantification of variability with regard to the shape of the trajectories in RSSs requires more investigation since our original proposed method base on euclidean metric failed to quantify those trajectories. Specially, for trajectories which were not well unfolded. With that in mind, we ~~proceed to~~ apply Recurrence Quantification Analysis (RQA) metrics (REC, DET, RATIO and ENTR) in order to avoid any subjective interpretations or personal bias with regard to the representation of the trajectories in RSSs.

RQA metrics with fixed parameters

Considering that RQA metrics were computed with fixed embedding parameters ($m = 6$ and $\tau = 8$) and recurrence thresholds ($\varepsilon = 1$), we found the following. REC values, which represents the % of black points in the RPs, were more affected with and increase in normal speed movements (HN and VN) than faster movements (HF and VF) for the sensor attached to the participants (HS01). Such decrease of REC values from normal speed to faster speed movements is also presented in data from sensor attached to the robot (RS01), and little can be said with regard to the dynamics of the time series coming from RS01 (Fig 7A). Similarly, DET values, representing predictability and organisation in the RPs, present little variation in the any of the time series where little can be said (Fig 7B). In contrast, RATIO values, which represent dynamic transitions, were more variable for faster movements (HF and VF) than normal speed movements (HN and VN) with sensors attached to the participants (HS01). For data coming from sensors attached to the robot (RS01), RATIO values from horizontal movements (HN, HF) appear to vary more than values coming from vertical movmentes (VN, VF) (Fig 7C). With that, it can be said that RATIO values can represent a bit better than REC or DET metrics for the variability of imitation activities in each of the conditions for time series. Similarly, ENTR values for HN were higher than values for HF and ENTR values varied more for sensor attached to participants than ENTR values for sensors of the robot. It is evidently that the higher the entropy the more complex the dynamics are, however, ENTR values for HN appear a bit higher than HF values, for which we believe this happens because of the structure the time series which appear more complex for HN than HF movements which presented a more consistence repetition (Fig 7D).

Additionally, we observed that some RQA metrics are affected by the smoothness of data. For instance, REC and DET values were not completely affected by the smoothness of time-series since these RQA values seemed to be constants. However, for RATIO values, the effect of smoothness can be noticed with a slight decrease of amplitude in any of the time series conditions which is also presented with ENTR values.

RQA metrics with different parameters

Iwansky et al.[11] stated that patterns in RPs and metrics for RQA are independent of embedding dimension parameters, however, that is not the case when using different recurrence thresholds. Such changes of recurrence threshold values can modify the patterns in RPs and therefore the values of RQA metrics. We therefore computed 3D surfaces to explore the sensibility and robustness of embedding parameters and recurrence threshold in RQA metrics. Following the same methodology of computing 3D surfaces, we also considered variation of window length size to present RQA metrics dependencies with embedding parameters, recurrence thresholds and window length size.

Conclusions

Generally, we can conclude that using a different level of smoothness for time series help us to visualise and to quantify the variation of movements between participants using RSSs, RPs and RQA. It is important to mention that some RQA's metrics (e.g. DET and ENTR) are more robust to the effect of smoothness of time series. ~~However, using determined~~ RQA metrics will be depend on what ~~one want~~ to quantify, for instance, one can compute predicability, organisation, dynamics transitions, or complexity and determinism of the input time series. However, RQA metrics show certain constrains with regard to activity type, window length and structure of the time series. For instance, RATIO and ENTR are helpful to distinguish differences in any of the categories of the time series (sensor, activity, level of smoothness and number of participant), however ~~for certain~~ time series such as ~~time series~~ data from the sensor attached to the robot seemed to have little variation~~, between each of the~~

Choice of

humanoid

~~participants. The latter phenomena was in a way evidently~~ as robot degrees of freedom did not allow it to move with a wide range of variability.

Additionally, we can point out that even though our experiment is limited to twenty healthy right-handed participants of a range age of mean 19.8 SD=1.39, RQA metrics show the potential to quantify human movement variability in the context of human-humanoid imitation activities. → You need a sentence or two here to explain why this is claimed.

Future Work

Inertial Sensors

better understand the

To ~~have more fundamental understating of nature of~~ signals collected through inertial sensors in the context of human-robot interaction, we ~~are considering to~~ apply derivates to the acceleration data. We can then explore the jerkiness of movements and therefore the nature of arm movements which typically have minimum jerk[16], its relationship with different body parts[17,18] or the application of higher derivatives of displacement with respect time such as snap, crackle and pop[19]. humanoid could

RQA ENTR

Having shown that RQA ENTR metrics are robust against different data time series, we believe that further investigation is required ~~to be done~~. For example, Marwan et al.[20,21] reviewed different aspects to compute RPs using different criteria for neighbours, different norms (L_{1-norm}, L_{2-norm}, or $L_{\infty-norm}$) or different methods to select the recurrence threshold ε, such as using certain percentage of the signal[22], the amount of noise or using a factor based on the standard deviation of the observational noise among many others[20].

Methods

State Space Reconstruction

has been applied in

The method of state space reconstruction ~~was originally proposed by~~[23] ~~and formalised by~~[10] ~~. Since then, different investigations and~~ disciplines ~~have benefited from the use of the method of state space reconstruction~~[3–5,8,24]. The method of state space reconstruction is based on uniform time-delay embedding methodology which is a simple matrix implementation that can reconstruct an unknown $d-$dimensional manifold M from a scalar time series (e.g. one-dimensional time series in $\mathbb{R}$). A manifold, in this context, is a multidimensional curved surface within a space (e.g. a saddle)[25]. many

The use of a scalar time series is the main advantage of the method of state space reconstruction which in essence preserve dynamic invariants such as correlation dimension, fractal dimension, Lyapunov exponents, Kolmogorov-Sinai entropy and detrended fluctuation analysis[2,3,9,26,27]. However, selecting appropriate embedding parameters which are sued to apply the state space reconstruction is still an open challenge for which we present introductions for the methodologies to compute such embedding parameters With that in mind, in the following subsections, we describe in more detail the state space reconstruction theorem (RSSs), uniform time-delay embedding theorem (UTDE), false nearest neighbours (FNN) and average mutual information (AMI).

State Space Reconstruction Theorem

Following the notation employed in[10,12,28–31], state space reconstruction is defined by:

$$s(t) = f^t[s(0)], \tag{1}$$

where s, $s : A \rightarrow M$ given that $A \subseteq \mathbb{R}$ and $M \subseteq \mathbb{R}^d$, represents a trajectory which evolves in an unknown $d-$dimensional manifold M, $f : M \rightarrow M$ is an evolution function and f^t, with time evolution $t \in \mathbb{N}$, is the t-th iteration of f that corresponds to an initial position $s(0) \in M$[10]. Then, a point of a one-dimensional time series $x(t)$ in $\mathbb{R}$, can be obtained with

$$x(t) = h[s(t)], \tag{2}$$

where h is a function, $h : M \rightarrow \mathbb{R}$, defined on the trajectory $s(t)$. Reconstructed state space can then be described as an $n-$dimensional state space defined by $y(t) = \Psi[X(t)]$ where $X(t) = \{x(t), x(t-\tau), ..., x(t-(m-1)\tau)\}$ is the uniform time-delay embedding with a dimension embedding m and delay embedding τ and $\Psi : \mathbb{R}^m \rightarrow \mathbb{R}^n$ is a further transformation of dimensionality (e.g. Principal Component Analysis, Singular Value Decomposition, etc) being $n \leq m$. With that in mind, uniform time-delay embedding, $X(t)$, defines a map $\Phi : M \rightarrow \mathbb{R}^m$ such that $X(t) = \Phi(s(t))$, where Φ is a diffeomorphic map[10] whenever $\tau > 0$ and $m > 2d_{box}$ and d_{box} is the box-counting dimension of M[12]. Then, if Φ is an embedding of evolving trajectories in the reconstructed space then a composition of functions represented with F^t is induced on the reconstructed state space determined:

$$X(t) = F^t[X(0)] = \Phi \circ f^t \circ \Phi^{-1}[X(0)]. \tag{3}$$

With this in mind, an embedding is defined as "a smooth one-to-one coordinate transformation with a smooth inverse" and the total reconstruction map is defined as $\Xi = \Psi \circ \Phi$[28]. Fig 10 illustrates the state space reconstruction.

Uniform Time-Delay Embedding (UTDE)

Frank et al. and Sama et al. refer to the state space reconstruction as "time-delay embeddings" or "delay coordinates"[4,5]. However, we consider the term "uniform time-delay embedding" as more descriptive and appropriate terminology for our work. Hence, the uniform time-delay embedding is represented as a matrix of uniform delayed copies of the time series $\{x_n\}_{n=1}^{N}$ where N is the sample length of $\{x_n\}$ and n is index for the samples of $\{x_n\}$. $\{x_n\}_{n=1}^{N}$ has a sample rate of T. The delayed copies of $\{x_n\}$ are uniformly separated by τ and represented as $\{\tilde{x}_{n-i\tau}\}$ where i goes from $0, 1, \ldots, (m-1)$. Generally speaking, $\{\tilde{x}_{n-i\tau}\}$ contains information of unobserved state variables and encapsulates the information of the delayed copies of the available time series in the uniform time-delay embedding matrix X_{τ}^{m}, $X_{\tau}^{m} \in \mathbb{R}^{m}$, defined as

$$X_{\tau}^{m} = \begin{pmatrix} \tilde{x}_n \\ \tilde{x}_{n-\tau} \\ \tilde{x}_{n-2\tau} \\ \vdots \\ \tilde{x}_{n-(m-1)\tau} \end{pmatrix}^{\intercal}, \tag{4}$$

where m is the embedding dimension, τ is the embedding delay and $^{\intercal}$ denotes the transpose. m and τ are known as embedding parameters. The matrix dimension of X_{τ}^{m} is defined by $N-(m-1)\tau$ rows and m columns and $N-(m-1)\tau$ defines the length of each delayed copy of $\{\tilde{x}_n\}$ in X_{τ}^{m}.

Estimation of Embedding Parameters

The estimation of the embedding parameters (m and τ) is a fundamental step for the state space reconstruction with the use of uniform time-delay embedding method. With this in mind, we review two of the most common algorithms, which will be used in our work, to compute the embedding parameters: the false nearest neighbour (FNN) and the average mutual information (AMI).

False Nearest Neighbours

To select the minimum embedding dimension m_0, Kennel et al.[32] used the method of false neighbours which can be understood as follows: on the one hand, when the embedding dimension is too small to unfold the attractor "not all points that lie close each other will be neighbours and some points appear as neighbours because of the attractor has been projected down into an smaller space", on the other hand, when increasing the embedding dimension "points that are near to each other in the sufficient embedding dimension should remain close as the dimension increase from m to $m+1$[27]". From a mathematical point of view, the state space reconstruction theorem is done when the attractor is unfolded with either the minimum embedding dimension, m_0, or any other embedding dimension value where $m \geq m_0$[32]. On the contrary, any large value of m_0 leads to excessive computations[9]. With this in mind, Cao[33] proposed an algorithm based on the false neighbour method where only the time-series and one delay embedding value are necessary to select the minimum embedding dimension. Cao's algorithm is based on $E(m)$ which is the mean value of all $a(i,m)$, both defined as follows

$$\begin{aligned} E(m) &= \frac{1}{N-m\tau} \sum_{i=1}^{N-m\tau} a(i,m) \\ &= \frac{1}{N-m\tau} \sum_{i=1}^{N-m\tau} \frac{||X_i(m+1) - X_{n(i,m)}(m+1)||}{||X_i(m) - X_{n(i,m)}(m)||} \end{aligned} \tag{5}$$

where $X_i(m)$ and $X_{n(i,m)}(m)$ are the time-delay embeddings with $i = 1, 2, \ldots, N-(m-1)\tau$ and $n(i,m) = 1 \leq n(i,m) \leq N - m\tau$. From Eq. 5, it can be seen that $E(m)$ is only dependent on m and τ for which $E_1(m)$ is defined as

$$E_1(m) = \frac{E(m+1)}{E(m)}. \tag{6}$$

$E_1(m)$ is therefore considered to investigate the variation from m to $m+1$ in order to find the minimum embedding dimension m_0 (Eq. 6). As described in[33]: "$E_1(m)$ stops changing when m is greater than some m_0, if the time series comes from a multidimensional state space then $m_0 + 1$ is the minimum dimension". Additionally, Cao proposed $E_2(m)$ to distinguish deterministic signals from stochastic signals. $E_2(m)$ is defined as

$$E_2(m) = \frac{E^*(m+1)}{E^*(m)}, \tag{7}$$

where

$$E^*(m) = \frac{1}{N-m\tau} \sum_{i=1}^{N-m\tau} |X_i(m+1) - X_{n(i,m)}(m+1)|. \tag{8}$$

For instance, when the signal comes from random noise (values that are independent from each other), all $E_2(m)$ values are approximately equal to 1 (e.g. $E_2(m) \approx 1$). However, for deterministic data $E_2(m)$ is not constant for all m (e.g. $E_2(m) \neq 1$).

As an example of the use of $E_1(m)$ and $E_2(m)$ values, we consider two time series: the solution for the x variable of the Lorenz system (Fig 11E), and a Gaussian noise time series with zero mean and a variance of one (Fig 11F). We then compute $E_1(m)$ and $E_2(m)$ values for each time series. The $E_1(m)$ values for the chaotic time series appear to be constant after the dimension is equal to six. The determination of six is given that any value of m can be used as they are within the threshold of 1 ± 0.05 (Fig 11A). $E_2(m)$ values, for chaotic time series, are different to one (Fig 11C), for which, it can be concluded that for the chaotic time series the minimum embedding dimension the time series comes from a deterministic signal. With regard to the noise time series, $E_1(m)$ values appeared to be constant when m is close to thirteen, which is defined by the threshold of 1 ± 0.05 (Fig 11B). $E_1(m)$ values then indicate the minimum embedding dimension of the noisy time series is thirteen, however all of the $E_2(m)$ values are approximately equal to one (Figure 11D) for which it can be concluded that noise time series is a stochastic signal.

It is important to note that for this work not only $E_1(m)$ and $E_2(m)$ are computed but also a variation of τ from 1 to 20 is presented. The purpose of such variation for τ is to show its independence with regard to $E_1(m)$ and $E_2(m)$ values as τ is increasing (Fig 11A,B,C, and D). However, one negative of the Cao's algorithm[33] is the definition of a new threshold where m values appear to be constant in $E_1(m)$. In the case of the given examples and reported results, we defined such threshold as 0.05. Further investigation is required for the selection of the threshold in the $E_1(m)$, as the selection of the threshold in this work is base on no particular method but visual inspection.

Average Mutual Information

When selecting the delay dimension parameter, τ, one can consider the following two cases: (i) when τ is too small, the elements of time-delay embedding will be along the bisectrix of the phase space and the reconstruction is generally not satisfactory, (ii) on the contraty, when τ is too large the elements of the uniform time-delay embedding will become spread and uncorrelated which makes recovering the underlying attractor more difficult if not impossible[28,34,35]. With regard to the algorithms to compute τ, Emrani et al.[34], for instance, used the autocorrelation function in which the first zero crossing is considered as the minimum delay embedding parameter. However, the autocorrelation function is a linear statistic over which the Average Mutual Information (AMI) algorithm is preferred because the AMI takes into account the nonlinear dynamical correlations[27,36]. With this in mind, the AMI algorithm is described below to estimate the minimum delay embedding parameter, τ_0.

To compute the AMI, an histogram of $x(n)$ using n bins is calculated and then a probability distribution of data is computed[13]. AMI is therefore denoted by $I(\tau)$ which is the average mutual information between the original time series, $x(n)$, and the delayed time series, $x(n-\tau)$, delayed by τ[37]. AMI is defined by

$$I(\tau) = \sum_{i,j}^{N} p_{ij} log_2 \frac{p_{ij}}{p_i p_j}. \tag{9}$$

Probabilities are defined as follows: p_i is the probability that $x(n)$ has a value inside the i-th bin of the histogram, p_j is the probability that $x(n+\tau)$ has a value inside the j-th bin of the histogram and $p_{ij}(\tau)$ the probability that $x(n)$ is in bin i and $x(n+\tau)$ is in bin j. The AMI is measured in bits (base 2, also called shannons)[13,38]. For small τ, AMI will be large and it will then decrease more or less rapidly. As τ increase and goes to a large limit, $x(n)$ and $x(n+\tau)$ have nothing to do with each other and $p_(ij)$ is factorised as $p_i p_j$ for which AMI is close to zero. Then, in order to obtain τ_0, "it has to be found the first minimum of $I(\tau)$ where $x(n+\tau)$ adds maximal information to the knowledge from $x(n)$, or, where the redundancy is the least"[13].

For example, we compute the AMI for two time series: A) the x solution of the Lorenz system, and B) a noise time series using a normal distribution with mean zero and standard deviation equal to one. From Fig 12, it can then be concluded that the amount of knowledge for any noise time series is zero for which the first minimum embedding parameter is $\tau_0 = 1$. On the contrary, the first minimum of the AMI for the chaotic time series is $\tau_0 = 17$ which is the value that maximize the independence between $x(n)$ and $x(n+\tau)$ in the reconstructed state space[9]. Similarly as Cao's algorithm negatives, AMI's algorithm is not an exception for negatives, which are worthwhile to mention for further investigations. For instance, (i) is not clear why the choose of the first minimum of the AMI is the minimum delay embedding parameter[13] and (ii) the probability distribution of the AMI function is computed with the use of histograms which depends on a heuristic choice of number of bins for which AMI depends on partitioning[35].

Recurrence Quantification

Recurrence Plots

Originally, Henri Poincaré in 1890 introduced the concept of recurrences in conservative systems, however such discovery was not put into practice until the development of faster computers[20], for which Eckmann et al.[39] in 1987 introduced a method where recurrences in the dynamics of a system can be visualised using Recurrence Plots. The intention of Eckmann et al.[39] was to propose a tool, called Recurrence Plot (RP), that provides insights into high-dimensional dynamical systems where trajectories are very difficult to visualise. Therefore, "RP helps us to investigate the $m-$dimensional phase space trajectories through a two-dimensional representation of its recurrences"[21]. Similarly, Marwan et al.[21] pointed out that additionally to the methodologies of the state space reconstruction and other dynamic invariants such as Lyapunov exponent, Kolmogorov-Sinai entropy, the recurrences of the trajectories in the phase space can provide important clues to characterise natural process that present, for instance, periodicities (as Milankovitch cycles) or irregular cycles (as El Niño Southern Oscillation). Such recurrences can not only be presented visually using Recurrence Plots (RP) but also be quantified with Recurrence Quantification metrics, which leads to applications of these in various areas such as economy, physiology, neuroscience, earth science, astrophysics and engineering[20].

For the creation of a recurrence plot based on time series $\{x_n\}$, it is first computed the state space reconstruction with uniform time-delay embedding $X(i) = \{\tilde{x}_n, \ldots, \tilde{x}_{n-(m-1)\tau}\}$ where $i = 1, \ldots, N$, N is the number of considered states of $X(i)$ and $X(i) \in \mathbb{R}^{m}$[39]. The recurrence plot is therefore a two-dimensional $N \times N$ square matrix, $\mathbf{R}$, where a black dot is placed at (i, j) whenever $X(i)$ is sufficiently close to $X(j)$:

$$\mathbf{R}^{m}_{i,j}(\varepsilon) = \Theta(\varepsilon_i - ||X(i) - X(j)|| \tag{10}$$

where $i, j = 1, \ldots, N$, ε is a threshold distance, $||\cdot||$ a norm, and $\Theta(\cdot)$ is the Heaviside function (i.e. $\Theta(x) = 0$, if $x < 0$, and $\Theta(x) = 1$ otherwise) (Fig 13)[20,21,39]. RP is also characterised with a line of identity (LOI) which is a diagonal line due to $R_{i,j} = 1 (i, j = 1, \ldots, N)$.

Structures of Recurrence Plots

Pattern formations in the RPs can be designated either as topology for large-scale patterns or texture for small-scale patterns. In the case of topology, the following pattern formations are commonly presented: (i) homogeneous where uniform recurrence points are spread in the RP e.g., uniformly distributed noise (Fig 14A), (ii) periodic and quasi-periodic systems where diagonal lines and checkerboard structures represent oscillating systems, e.g., sinusoidal signals (Fig 14B), (iii) drift where paling or darkening recurrence points away from the LOI is caused by drifting systems, e.g., logistic map (Fig 14C), and (iv) disrupted where recurrence points are presented white areas or bands that indicate abrupt changes in the dynamics, e.g. Brownian motion (Fig 14D)[21,39]. Texture patterns in RPs can be categorised as: (i) single or isolated recurrence points that represent rare occurring states, and do not persist for any time or fluctuate heavily, (ii) dots forming diagonal lines where the length of the small-scale parallel lines in the diagonal are related to the ratio of determinism or predictability in the dynamics of the system, and (iii) dots forming vertical and horizontal lines where the length of the lines represent a time length where a state does not change or change very slowly and these patterns formation represent discontinuities in the signal, and (iv) dots clustering to inscribe rectangular regions which are related to laminar states or singularities[21].

Although, each of the previous pattern descriptions of the structures in the RP offer an idea of the characteristics of dynamical systems, these might be misinterpreted and conclusions might tend to be subjective as these require the interpretation of a particular researcher(s). Because of that, recurrence quantification analyis (RQA) offer objective methodologies to quantify such visual characteristics of previous recurrent pattern structures in the RP[40].

Recurrence Quantifications Analysis (RQA)

Originally, Zbilut et al.[40] proposed metrics to investigate the density of recurrence points in RPs, then histograms of lengths for diagonal lines in RPs were studied by[41] which were the introduction to the term recurrence quantification analysis (RQA)[42]. RQA has been applied in many fields such as life science, engineering, physics, and others[42]. Particularly in human movement to investigate noise and complexity of postural control[43], postural control[44] or interpersonal coordination[45]. The success of RQA is not only due to its simple algorithmic implementation but also to its capacity to detect tiny modulations in frequency or phase which are not detectable using standard methods e.g. spectral or wavelet analysis[7], and that RQA's metrics are quantitatively and qualitatively independe]nt of embedding dimension which is verified experimentally by[11]. RQA metrics comprehend percentage of recurrence, percentage of determinism, ratio, Shannon entropy of the frequency distributions of the line lengths, maximal line length and divergence, trend and laminariy[20,21]. For this work, we considered only four RQA metrics, due to its consistency with our preliminary experiments, which are described below. Such metrics are computed the nonlinearTseries R package[38].

REC values

The percentage of recurrence (REC) is defined as

$$REC(\varepsilon,N) = \frac{1}{N^2-N}\sum_{i\neq j=1}^{N} \mathbf{R}^m_{i,j}(\varepsilon), \tag{11}$$

which enumerates the black dots in the RP excluding the line of identity. REC is a measure of the relative density of recurrence points in the sparse matrix[21].

DET values

The percent determinism (DET) is defined as the fraction of recurrence points that form diagonal lines and it is determined by

$$DET = \frac{\sum_{l=d_{min}}^{N} lH_D l}{\sum_{i,j=1}^{N} \mathbf{R}_{i,j}(\varepsilon)}, \tag{12}$$

where

$$H_D(l) = \sum_{i,j=1}^{N} (1-\mathbf{R}_{i-1,j-1}(\varepsilon))(1-\mathbf{R}_{i+l,j+l}(\varepsilon))\prod_{k=0}^{l-1}\mathbf{R}_{i+k,j+k}(\varepsilon) \tag{13}$$

is the histogram of the lengths of the diagonal structures in the RP. DET can be interpreted as the predictability of the system for periodic signals which, in essence, have longer diagonal lines than the short diagonals lines for chaotic signals or absent diagonal lines for stochastic signals[20,21]. Similarly, DET is considered as a measurement for the organisation of points in RPs[11].

RATIO values

RATIO is defined as the ratio between DET and REC and it is calculated from the frequency distributions of the lengths of the diagonal lines. RATIO is useful to discover dynamic transitions[21].

ENT values

ENT is the Shannon entropy of the frequency distribution of the diagonal line lengths and it is defined as

$$ENT = -\sum_{l=d_{min}}^{N} p(l)lnp(l) \quad with \quad p(l) = \frac{H_D(l)}{\sum_{l=d_{min}}^{N} H_D(l)}. \tag{14}$$

ENT reflects the complexity of the deterministic structure in the system. For instance, for uncorrelated noise or oscillations, the value of ENT is rather small and indicates low complexity of the system, therefore "the higher the ENT is the more complex the dynamics are"[20,21].

Sensitivity and robustness of RPs and RQA.

RP and RQA are a very young field in nonlinear dynamics and many questions are still open, for instance, different parameters for window length size of the time series, embedding parameters or recurrence threshold can generate different results in RQA's metrics[7,39].

The selection of recurrence threshold, ε, can depend on the system that is analysed. For instance, when studying dynamical invariants ε require to be very small, for trajectory reconstruction ε requires to have a large thresholds or when studying dynamical transition there is little importance about the selection of the threshold[7]. Other criteria for the selection of ε is that the recurrence threshold should be five times larger than the standard deviation of the observational noise or the use of diagonal structures within the RP is suggested in order to find the optimal recurrence threshold for (quasi-)periodic process[7].

Similarly, Iwanski et al.[11] highlighted the importance of choosing the right embedding parameters to perform RQA for which many experiments have to be performed using different parameters in order to have a better intuition of the nature of the time series and how this is represented by using RQA.

With that in mind, this work explores the sensitivity and robustness of the window size of time series, embedding parameters for RSS with UTDE and recurrence threshold for RP and RQA in order to gain a better insight into the underlying time series collected from inertial sensors in the context of human-humanoid imitation activities.

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Experiment

We conducted an experiment in the context of human-humanoid imitation (HHI) activities where participants were asked to imitate simple horizontal and vertical arm movements performed by NAO, a humanoid robot[46]. Such simple movements were repeated ten times for the participant who copied NAO's arm movements in a face-to-face imitation activity. Also, wearable inertial measurement unit (IMU) sensors were attached to the right hand of the participant and to the left hand of the robot (Figure 15 A,C). Data were then collected with four NeMEMSi IMU sensors with sampling rate of 50Hz provinding tri-axial data of the accelerometer, gyroscope and magnetometer sensors and quaternions[47].

Participants

Twenty-three participants, from now on defined as *pN* where *N* is the number of participant, were invited to do the experiment. However, data for three participants were not used because the instructions for $p01$, who was the only left-handed, were mistakenly given in a way that movements were performed different from what had been planned, and for participants $p13$ and $p16$ data were corrupted because bluetooth communications problems with the sensors. With that in mind, data for twenty participants were analysed in this work.

Of the twenty participants, all of them are right-handed healthy participants of whom four are females and sixteen are males with a mean and standard deviation (SD) age of mean=19.8 (SD=1.39). All participants provided informed consent forms prior to participation in the experiment.

Human-humanoid imitation activities

For human-humanoid imitation (HHI) activities four neMEMSi sensors were used, two of which were attached to the right hand of the participant and the other two to the left hand of the humanoid robot. Then, each participant was asked to imitate repetitions of simple horizontal and vertical arm movements performed by the humanoid robot in the following conditions: (i) ten repetitions of horizontal arm movement at normal (HN) and faster (HF) speed (Figure 15 A), and (ii) ten repetitions of vertical arm movement at normal (VN) and faster (VF) speed (Figure 15 C). The normal and faster speed of arm movements is defined by the duration in number of samples of one repetition of NAO's arm movements. We select NAO's arm movements duration to distinguish between normal and faster arm movements as NAO's movements have less variation between repetition to repetition. The duration for one repetition of the horizontal arm movement at normal speed, HN, is about 5 seconds considering that each repetition last around 250 samples. For horizontal arm movement at faster speed, HF, each repetition were performed in around 2 seconds which correspond to 90 samples of data. The vertical arm movement at normal speed, VN, were performed in 6 seconds which is around 300 samples of data. For vertical arm movement at faster speed, VF, each repetition lasts about 2.4 seconds which correspond to 120 samples of data. To visualise the distinction between normal and faster speed for horizontal and vertical arm movements, Fig 16 shows smoothed time series for axes Z and Y of the gyroscope sensors with four window lengths: 2-sec (100-samples), 5-sec (250-samples), 10-sec (500-samples) and 15-sec (750-samples).

Data from Inertial Measurement Units

Raw data

Considering the work of[48] which provided evidence of an improvement in recognition activities when combining data from accelerometer and gyroscope. We focus our analysis from data of the accelerometer and gyroscope of the NeMEMsi sensors[47] and leave the data of the magnetometer and quaternions for further investigation because of their possible variations with regard to magnetic disturbances.

Data from the accelerometer is defined by triaxial time series $A_x(n)$, $A_y(n)$, $A_z(n)$ which forms the matrix A (Eq. 15), and the same for data from the gyroscope which is defined by triaxial time-series of $G_x(n)$, $G_y(n)$, $G_z(n)$ representing the matrix G (Eq. 16). Both triaxial time series of each sensor, a and g, are denoted with its respective axes subscripts x, y, z, where n is the sample index and N is the same maximum length of all axes for the time series. Matrices A and G are represented as follows

$$A = \begin{pmatrix} A_x(n) \\ A_y(n) \\ A_z(n) \end{pmatrix} = \begin{pmatrix} a_x(1), a_x(2), \ldots, a_x(N) \\ a_y(1), a_y(2), \ldots, a_y(N) \\ a_z(1), a_z(2), \ldots, a_z(N) \end{pmatrix}, \tag{15}$$

and

$$G = \begin{pmatrix} G_x(n) \\ G_y(n) \\ G_z(n) \end{pmatrix} = \begin{pmatrix} g_x(1), g_x(2), \ldots, g_x(N) \\ g_y(1), g_y(2), \ldots, g_y(N) \\ g_z(1), g_z(2), \ldots, g_z(N) \end{pmatrix}. \tag{16}$$

Postprocessing data

After the collection of raw data from four NeMEMsi sensors, time synchronisation alignment and interpolation were performed in order to create time series with the same length and synchronised time. We refer the reader to[47] for further details about the time synchronisation process.

Data normalization

Data is normalised to have zero mean and unit variance using sample mean and sample standard deviation. The sample mean and sample standard deviation using $x(n)$ is given by

$$\mu_{x(n)} = \frac{1}{N}(\sum_{i=1}^{N} x(i)), \quad \sigma_{x(n)} = \sqrt{\frac{\sum_{1=1}^{N}(x(i)-\mu_{x(n)})^2}{N-1}}, \tag{17}$$

and the normalised data, $\hat{x}(n)$, is computed as follows

$$\hat{x}(n) = \frac{x(n)-\mu_{x(n)}}{\sigma_{x(n)}}. \tag{18}$$

Smoothing data

Commonly, a low-pass filter is the method either to capture the low frequencies that represent %99 of the human body energy or to get the gravitational and body motion components of accelerations[49]. However, for this work the elimination of certain range of frequencies is not the main focus but the conservation of the structure in the time series in terms of the width and heights where, for instance, Savitzky-Golay filter can help to accomplish such task[50]. Savitzky-Golay filter is based on the principle of moving window averaging which preserves the area under the curve (the zeroth moment) and its mean position in time (the first moment) but the line width (the second moment) is violated and that results, for example, in the case of spectrometric data where a narrow spectral line is presented with reduced height and width. With that in mind, the aim of Savitzky-Golay filtering is to find filter coefficients c_n that preserve higher momentums which are based on local least-square polynomial approximations[50–52]. Therefore, Savitzky-Golay coefficients are therefore computed using an R function `sgolay(p,n,m)` where `p` is the filter order, `n` is the filter length (must be odd) and `m` is the m-th derivative of the filter coefficients[14]. Smoothed signal is represented with a tilde over the original signal: $\tilde{x}(n)$.

Window size data

With regard to the window size,[48] investigated its effects using seven window lengths (2, 5, 10, 15, 20, 25, 30 seconds) and combination of inertial sensors (accelerometer, gyroscope and linear acceleration sensor) to improve the activity recognition performance for repetitive activities (walking, jogging and biking) and less repetitive activities (smoking, eating, giving a talk or drinking a coffee). With that in mind, Shoaib et al.[48] concluded that the increase of window length improve the recognition of complex activities because these requires a large window length to learn the repetitive motion patterns. Particularly, one of the recommendations is to use large window size to recognise less repetitive activities which mainly involve random hand gestures. Therefore, for the four activities (HN, HF, VN, and VF) in this work, which are mainly repetitive, we select only four window sizes for analysis: 2-s window (100 samples), 5-s window (250 samples), 10-s (500 samples) and 15-s window (750 samples) (Fig 16).

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