Math 578 Assignment 1

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Question 2

Signum function

For all polynomial degrees the order of error was constant with respect to h. This is reasonable; the discontinuity at 0 affects the *sup* norm strongly.

Sine

The error for sine showed good agreement with the theory, until it reached our effective machine epsilon around 10^{-16}. The order of the error was about h^3, h^9, and h^17 for the degree 2, 7, and 16 polynomial interpolations. Note that the degree 16 converged rapidly the machine epsilon, so we needed to toss out many small h-values to obtain the 'true' order of error.

Absolute value

The absolute value had order h convergence for all degrees. This agrees with the theory because abs is in C^0 but not C^1.

Quintic

At first glance, this is garbage. The error should obviously plummet to 0 for the degree 7 and 16 polynomials. However, there is some itsy-bitsy instability in the calculation of the coefficients for P^n, hence we pick up some rounding error.

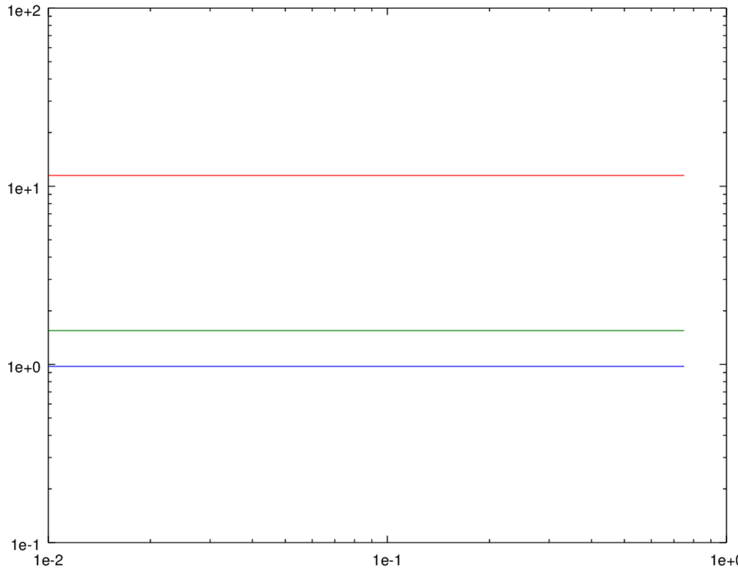


Figure 1:

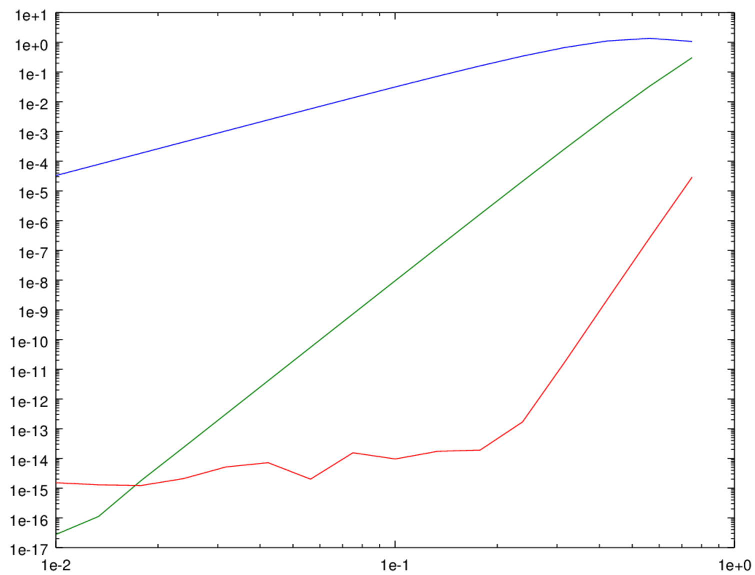

1e+1
1e+0
1e-1
1e-2
1e-3
1e-4
1e-5
1e-6
1e-7
1e-8
1e-9
1e-10
1e-11
1e-12
1e-13
1e-14
1e-15
1e-16
1e-17
1e-2
1e-1
1e+0

Figure 2:

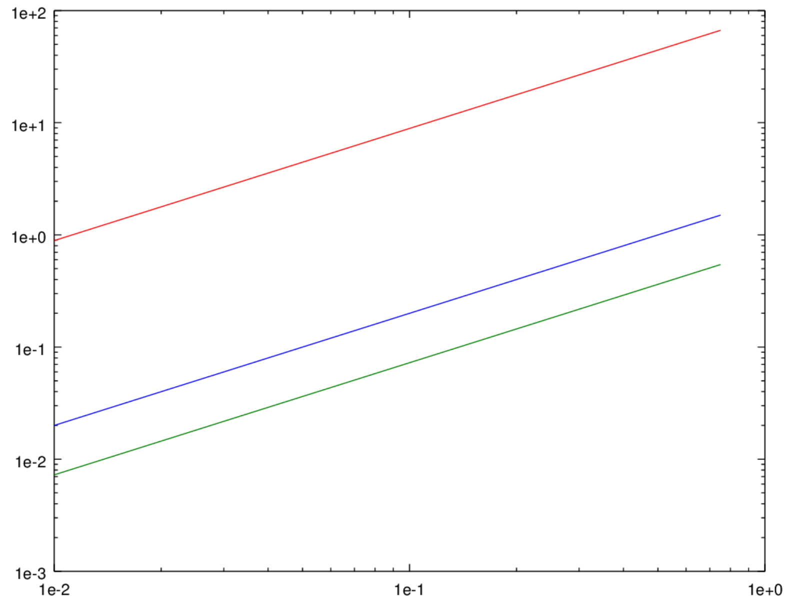


Figure 3:

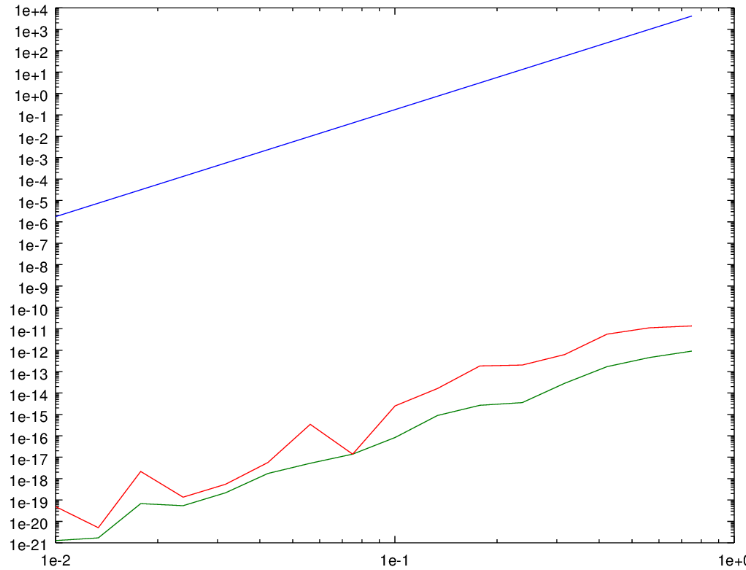

1e+4
1e+3
1e+2
1e+1
1e+0
1e-1
1e-2
1e-3
1e-4
1e-5
1e-6
1e-7
1e-8
1e-9
1e-10
1e-11
1e-12
1e-13
1e-14
1e-15
1e-16
1e-17
1e-18
1e-19
1e-20
1e-21
1e-2
1e-1
1e+0

Figure 4:

Question 3

Equidistant

Behold! We observe the Runge phenomenon for large n: more equidistant points fail to guarantee convergence if the derivatives of a function are poorly behaved. Note, however, that error still decreases with respect to h.

Table 1: Errors for n-degree interpolants of Runge function, varying with h.

h	n=2	n=7	n=16	n=100
1.0000e+00	8.0957e-01	6.1306e-01	7.3637e+01	1.9667e+06
5.0000e-01	6.4491e-01	2.4631e-01	1.4373e+01	6.2507e+03
2.5000e-01	3.9445e-01	9.6406e-02	4.0398e-01	1.4555e+00
1.2500e-01	1.4045e-01	1.2113e-02	6.3266e-04	1.0609e-06
6.2500e-02	2.3066e-02	2.7710e-04	4.6669e-08	5.8000e-08
3.1250e-02	2.0714e-03	2.0345e-06	4.9116e-13	1.4652e-08
1.5625e-02	1.4353e-04	9.4982e-09	6.2172e-15	5.5565e-09
7.8125e-03	9.2139e-06	3.8854e-11	1.2212e-14	2.8005e-08
3.9062e-03	5.7977e-07	1.5343e-13	9.3259e-15	4.0407e-08
1.9531e-03	3.6297e-08	7.7716e-16	2.3315e-15	3.5323e-08

Chebyshev

Sampling the Chebyshev nodes should guarantee convergence of the *sup* norm with respect to n; below, we largely see that trend hold, except in the upper right hand corner. This is attributable to roundoff error from a poorly conditioned for large n

Table 2: Errors for various n-degree Chebyshev interpolants of Runge function, varying with h.

h	n=2	n=7	n=16	n=100
1.0000e+00	7.8366e-01	7.4327e-01	1.8882e-01	*1.6040e-01*
5.0000e-01	5.9990e-01	3.9059e-01	3.2580e-02	2.2293e-03
2.5000e-01	3.3651e-01	8.7943e-02	1.3095e-03	2.4638e-07
1.2500e-01	1.0967e-01	5.6849e-03	3.8888e-06	4.7828e-13
6.2500e-02	2.1216e-02	9.1388e-05	5.4007e-10	1.1102e-15
3.1250e-02	2.0238e-03	5.8748e-07	7.6605e-15	4.6629e-15
1.5625e-02	1.4284e-04	2.6401e-09	4.4409e-16	3.4417e-15
7.8125e-03	9.2145e-06	1.0700e-11	3.3307e-16	1.9984e-15
3.9062e-03	5.8053e-07	4.2299e-14	4.4409e-16	3.9968e-15

h	n=2	n=7	n=16	n=100
1.9531e-03	3.6356e-08	3.3307e-16	2.2204e-16	8.5487e-15

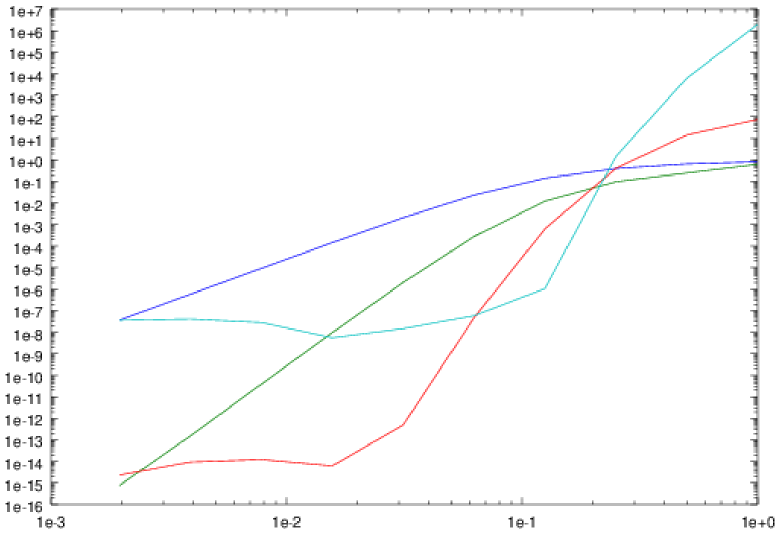


Figure 5: loglog plot of error vs. h, for equidistant points

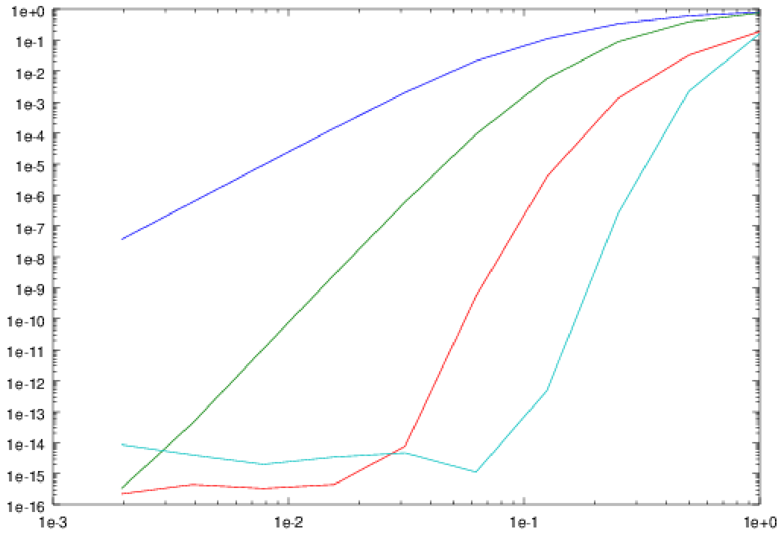
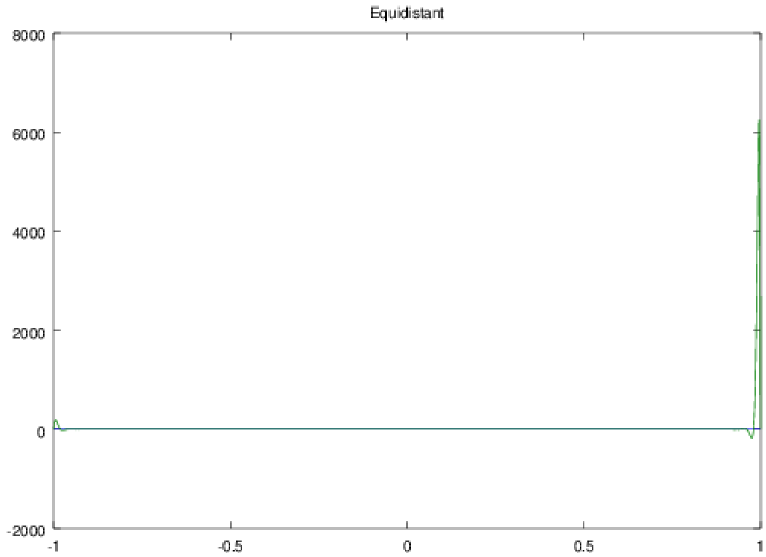
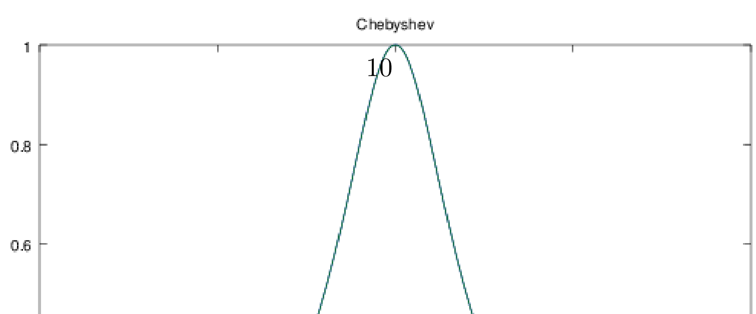


Figure 6: loglog plot of error vs. h, for Chebyshev points


Equidistant
8000
6000
4000
2000
0
-2000
-1
-0.5
0
0.5
1


Chebyshev
1
0.8
0.6

Question 4

Briefly put, you can't make chicken pie out of chicken feed. Regardless of the method chosen, irregular functions like signum and abs will not be well-approximated by a polynomial scheme, and their order of convergence is independent of the polynomial degree past their level of continuity. Smooth functions with bounded derivatives like sin and x^5 behave nicely, and converge more quickly with higher-degree schemes (nearest < linear < cubic < spline).

I suppose the only two points of interest are that `pchip` is the same as `cubic`[1] and `spline` yielded quicker convergence than the `cubic`. Consulting the documentation, `spline` to MATLAB means matching 2nd derivatives as well as the first; which means we should expect order 7 convergence from `spline`. Digging through the raw numbers doesn't clarify things much, as in no case do we run up against the machine epsilon in such a way as to throw off the line of best fit.

Truly, mysterious.

Table 3: Slope of best-fit line for various functions and interpolation methods.

function	nearest	linear	spline	pchip	cubic
signum	NaN	0.012206	0.014586	0.018865	0.018865
sine	1.036637	1.993232	3.907034	3.019374	3.019374
abs	1.044457	1.056662	1.081153	1.056662	1.056662
quintic	0.944555	1.912727	3.914612	2.933812	2.933812
runge	0.970954	1.893706	3.857821	1.895098	1.895098

[1] indeed, consult http://blogs.mathworks.com/cleve/2012/07/16/splines-and-pchips/.

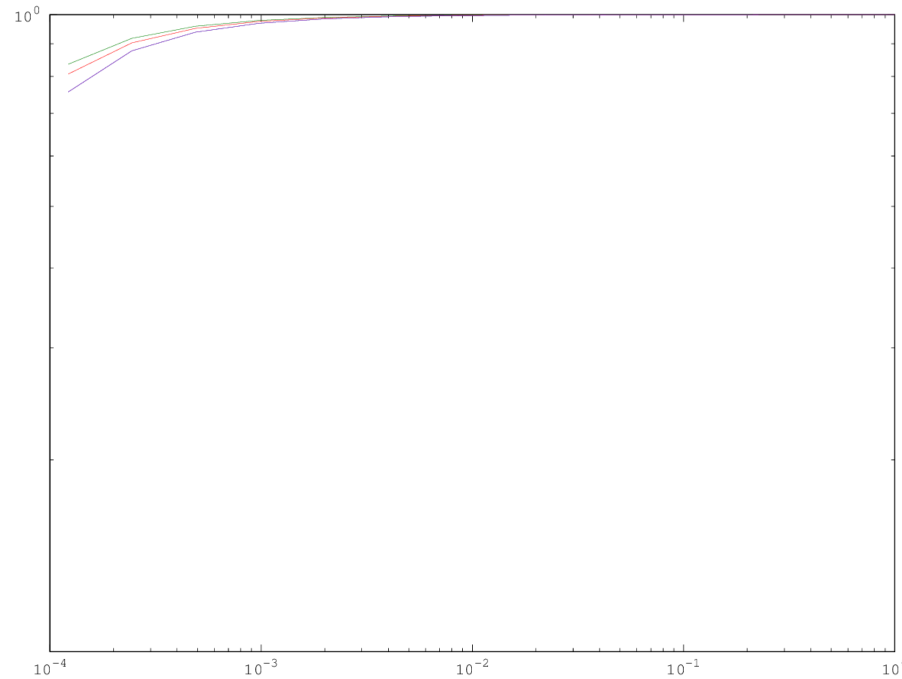


Figure 7:

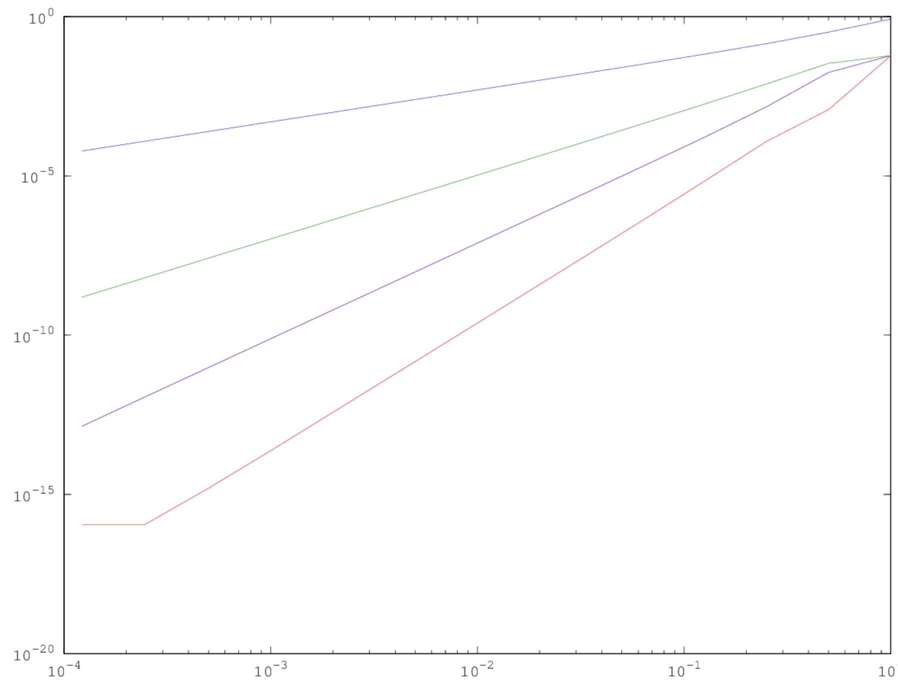


Figure 8:

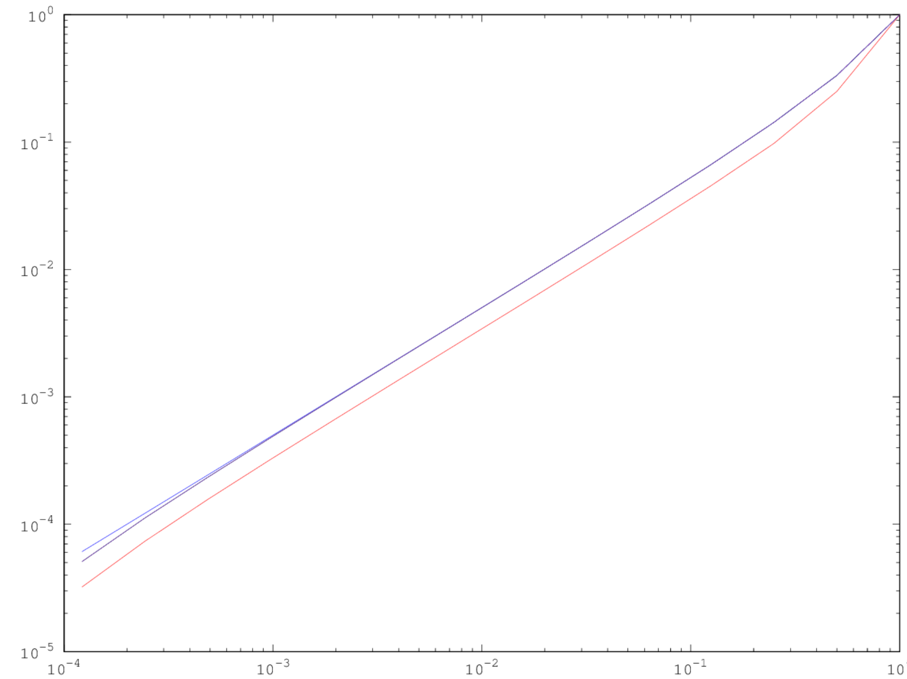


Figure 9:

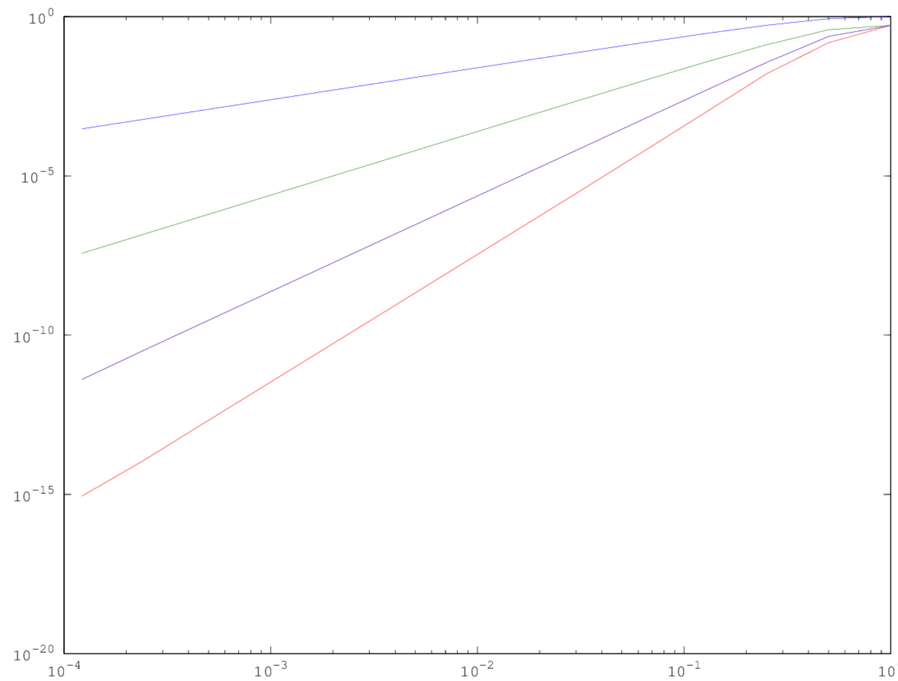


Figure 10:

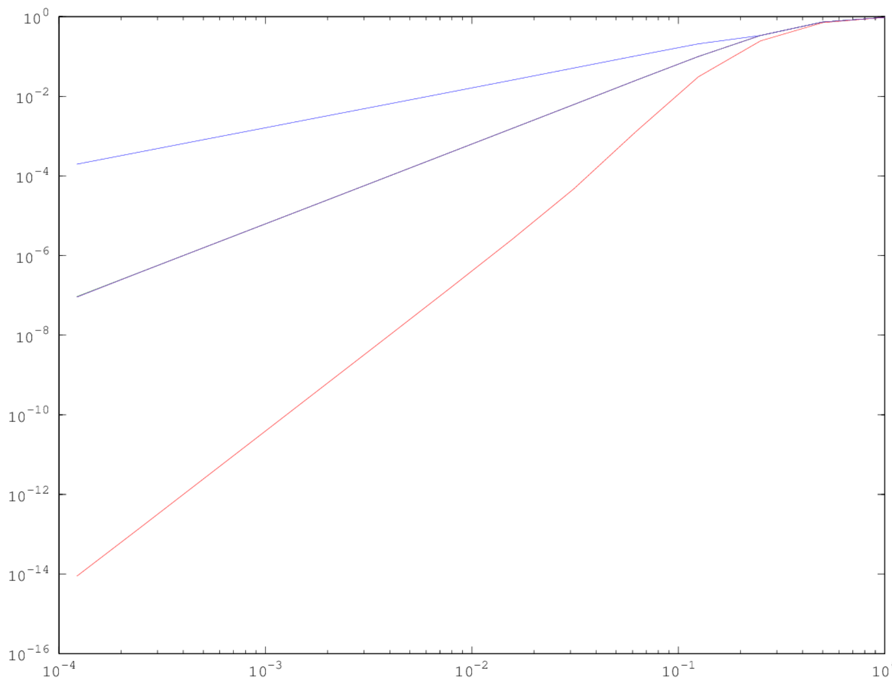


Figure 11:

Question 5

Part 1

$$A = \begin{bmatrix} A1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \\ -1 & 1 & -1 & 1 \\ 3 & -2 & 1 & 0 \end{bmatrix}$$

$$A^1 = \frac{1}{4} \begin{bmatrix} A1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \\ -1 & 1 & -1 & 1 \\ 3 & -2 & 1 & 0 \end{bmatrix}$$

Part 2

Part 3