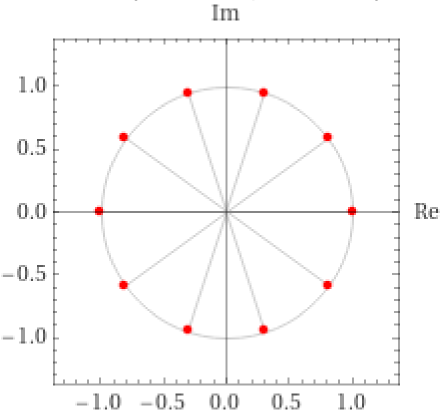
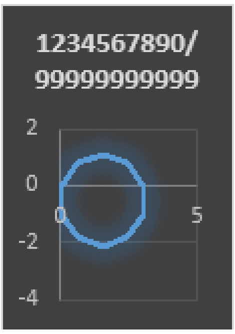
In your chalkdust article you mention that the fraction 1/61 makes a perfect loop and ends up back at its starting point. I've realised why and it's quite a lucky coincidence that it occurs at such a low number.




Each bearing can be considered as a complex number, specifically one of the 10th roots of unity i.e solutions to $z^{10} = 1$ (shown above).
When we take a step from one digit to the next, it is simply the same as the addition of these two complex numbers. The resulting complex number gives the point obtained after these two steps.
If we allow the first complex root to be *w* (= $e^{2i\pi/10}$), then each digit *k* of the number gives the bearing as a complex number in the form $w^k$.

If we consider the sum of all of the digits $w^0 + w^1 + w^2 + \ldots + w^9$, this forms a finite geometric series. Note that $w^{10} = w^0 = 1$.
The sum, using the formula for the sum of a finite geometric sequence, is thus equal to $(w^{10} - 1)/(w - 1) = 0$.
Remember the decagon formed by 1234567890/99999999999? This is essentially the same thing as what I have just described: the sum of $w^0 + w^1 + w^2 + \ldots + w^9 +$ = 0, so we end up back at the origin.




It just so happens that in the expansion of 1/61, which is 60 digits long, each digit 0 to 9 occurs *exactly* 6 times.
Thinking in terms of complex numbers, this means that the frawk of 1/61 is equivalent to $6 \times (w^0 + w^1 + w^2 + \ldots + w^9)$, which as shown, is equal to $6 \times 0$.
Hence 1/61 ends up back at the origin after its entire 60 fraction cycle, and so it restarts from the beginning.

So, to make any fraction you'd like that ends up back at the origin perfectly, it has to:

- Have a period divisible by $10*n$, where *n* is a positive integer
- Use each digit 0...9 exactly *n* times.

I wonder what the next number satisfying these in the form 1/k would be? It is probably quite rare that those two requirements are satisfied AND the number is a factor of $(10^n - 1)$.