MASTER OF FINANCIAL ENGINEERING
UCLA Anderson School
Credit Risk
Prof. Holger Kraft
Problem Set 3
Due: Oct 20, 8am (before TA session)

Problem 7 (Default and Duration)

We consider a corporate zero-coupon bond that matures in one year. Assume for simplicity that the risk-free interest rate r, the default intensity and the recovery rate of the company are constant. Besides, suppose that we can write the default intensity as

$$\lambda = a + br,$$

where a and b are constants.

(a) Which signs of a and b do you expect? Explain your answer.

(b) Write down the corporate bond price assuming

  (i) recovery of market value and

  (ii) recovery of Treasury (aka recovery of par at maturity).

The sensitivity of a Treasury bond w.r.t. interest rate changes is defined as

$$D = -\frac{\frac{\partial p(0,1)}{\partial r_0}}{p(0,1)}$$

where r_0 is the initial value of the short rate.

(c) Calculate the corresponding sensitivity of the corporate bond for the models in (b).

(d) Is the sensitivity of a corporate bond bigger or smaller than the sensitivity of a Treasury bond with the same maturity? Why? Which parameter drives the results?

(e) Relate your findings to the interpretation of the classical duration (Macauley duration).

Problem 8 (CDS)

On October 01 the 5y CDS spread of the XYZ company was 100bp. We make the following simplifying assumptions:

- The recovery rate is 40%.
- Interest rates are zero.
- The default intensity of XYZ is constant.

(a) Suppose that the fee leg is paid continuously. What is the implied risk-neutral default intensity of XYZ?

(b) Calculate the corresponding one-year survival and default probabilities.

(c) Suppose that the 2y CDS spread is also 100bp and that you enter such a contract as the protection buyer. Assume further that in one year the 1y CDS spread will be 110bp if fees were paid continuously. What will be the value of your contract in one year if you pay your fees semi-annually?

(d) Now, we consider a 1y CDS contract where fees are paid annually and the protection payment (if any) is made *at the maturity of the CDS contract*. What is the default intensity of the underlying if you observe a CDS spread of 110 bp?

Problem 9 (Bond Pricing)

Assume that the risk-free rate is $r = 0.01$. Unless otherwise stated, consider a CIR model

$$d\lambda_t = \kappa(\theta - \lambda_t)\, dt + \sigma\sqrt{\lambda_t}\, dW_t.$$

where the parameters are given by $\kappa = 1$, $\theta = 0.02$, $\sigma = 0.15$, and $\lambda_0 = 0.01$.

(a) For affine models, it is known that

$$\mathrm{E}[e^{-\int_0^t \lambda_u\, du}] = e^{A(t) - B(t)\lambda_0}.$$

Implement the functions $A(t)$ and $B(t)$.

(b) Calculate the spreads of zero-coupon bonds with maturities $T = 1, ..., 10$. Assume zero recovery, $R = 0$.

(c) What do you observe when $T \to 0$? Explain. What is different compared to the firm value model by Merton (1974)?

(d) Compute the fair prices of defaultable coupon bonds with recovery of par and $R = 0.5$ for maturities $T = 1, ..., 10$. Assume that coupons of $c = 0.02$ are payed semi-annually. Please disregard accrued payments.