# Homework 4

The questions can be done informally or in Agda.

## Question 1

Consider the following language of arithmetic expressions with a predecessor function

$$e \ ::= \ \mathsf{zero} \mid \mathsf{pred}\ e \mid \mathsf{suc}\ e$$

The values are defined by

$$v \ ::= \ \mathsf{zero} \mid \mathsf{suc}\ v$$

Define recursively the value $[\![e]\!]$ of an expression $e$ as a natural number.

If $n$ is a natural number, we can define $\mathsf{num}\ n = \mathsf{suc}^n\ \mathsf{zero}$.

Give a big step presentation of the evaluation $e \Downarrow v$.

Show then that we have $\forall e\ \ e \Downarrow \mathsf{num}\ [\![e]\!]$.

(Hint: you can have a Lemma stating that if $e \Downarrow \mathsf{num}\ n$ then $\mathsf{pred}\ e \Downarrow \mathsf{num}\ (\mathsf{pred}\ n)$ where $\mathsf{pred}$ is the predecessor function on natural numbers.)

## Question 2

Show that if we have (universal quantifications are implicit)

$$(a \to b \wedge a \to^* c) \Rightarrow \exists d\ (b \to^* d \wedge c \to d)$$

then $\to$ is confluent i.e.

$$(a \to^* b \wedge a \to^* c) \Rightarrow \exists d\ (b \to^* d \wedge c \to^* d)$$

As usual $\to^*$ is the reflexive transitive closure of $\to$.

Deduce from this that if we have

$$(a \to b \wedge a \to c) \Rightarrow \exists d\ (b \to d \wedge c \to d)$$

then $\to$ is confluent.